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Uncertainty and the Voluntary Provision of a Pure Public Good in a Two-Moment Decision Model

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Abstract

In this paper, we explore the potential benefits of uncertainty that may arise in a two-moment model of the voluntary provision of a pure public good. We find that an increase in a given contributor i's risk associated with the aggregate contribution level of the other contributors (i.e., an increase in social uncertainty) induces that contributor to increase his own contribution level if and only if the uncertainty's incremental effect on the expected value of his net marginal utility is negative. Contributor i's welfare likewise increases when a closely related condition is met, namely that the uncertainty's marginal effect on his expected marginal utility value of the public good exceeds its countervailing effect on the numeraire. Further, the corresponding aggregate contribution to the public good increases in the presence of free-riding if and only if the incremental effect of contributor i's contribution on the aggregate expected value of all other contributors' net marginal utilities is small-enough positive. We derive similar conditions for the case of private uncertainty, where the increase in contributor i's risk is associated with his own marginal valuation of the public good. A simple example illustrates these conceptual results. Numerical analysis demonstrates that an increase in private uncertainty can have a nonmonotonic impact on contributor i's welfare.

1. Introduction

In this paper, we investigate the potential benefits of uncertainty that may arise in the context of the voluntary provision of a pure public good; a context that to date has received little, albeit increasing attention. Early studies considered uncertainty associated with a discrete public good's provision point for both contribution and subscription games.[1] For example, McBride(2006) characterizes conditions for a contribution

[1] Contributions are not refunded in a contribution game if the project is ultimately not funded, while in a subscription game they are.

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game where a contributor's contribution level and attendant welfare are each positively related to an uncertain provision threshold, as determined by the second-order stochastic dominance (SOSD) criterion. Positive relationships exist for a large class of threshold probability distributions when the public good's value is sufficiently high and/or the corresponding probability density functions satisfy a single-crossing property.[2] Menezes, Monteiro, and Temimi (2001) adopt a continuous contribution framework and find that public good provision levels in both contribution- and subscription-game equilibria are inefficiently low for a wide range of provision costs in the presence of "social uncertainty," where contributors are uncertain about the aggregate contribution levels of all other contributors.[3]

Similar to Menezes *et al.* (2001), Barbieri and Malueg (2010) consider discrete public good provision in a continuous-contribution, uncertain-threshold, subscription-game framework. The authors show that in the presence of uniformly distributed threshold uncertainty, an increase in the dispersion of a given contributor's valuation of the public good induces an expected increase in that contributor's own contribution level, which in turn outweighs the expected aggregate decrease in the contribution levels of all other contributors. The expected (net) utility of the more-uncertain contributor declines, while the expected utilities of all other contributors increase.[4] We obtain similar results for our model of a continuous public good sans threshold (*à la* Bergstrom, Blume, and Varian 1986) under certain circumstances.[5]

We derive a necessary and sufficient condition for the class of mean-variance (or two-moment) preferences defined over a pure public good that determines when a given contributor (henceforth contributor i) chooses to monotonically increase his own contribution level in the presence of increased risk associated with Menezes *et al.*'s (2001) notion of social uncertainty.[6] The condition is that the incremental

[2] As McBride (2006) shows, a sufficiently highly valued public good increases the probability of a given contributor being pivotal in the face of threshold uncertainty, which in turn increases the probability that s/he will contribute.

[3] Specifically, Menezes *et al.* (2001) show that the probability of public good provision in either symmetric or asymmetric equilibria is less than one in both contribution and subscription games. However, since contributing zero is the unique equilibrium of a contribution game for a wide range of provision costs, the subscription game results in a superior equilibrium outcome. As Wit and Wilke (1998) show, social uncertainty in an experimental public goods game decreases cooperation solely in cases of relatively high uncertainty. Palfrey and Rosenthal (1988, 1991) examine the related issues of uncertainty about other contributors' altruism levels and other's contribution costs, respectively, in an experimental framework.

[4] Barbieri and Malueg (2010) show the more general result that convex strategies, rather than uniform threshold uncertainty *per se*, is necessary for these results. However, they do not provide any indication of what other threshold distributions are consistent with convex strategies. Curvature conditions on contributor strategies play a similarly important role in Barbieri and Malueg (2014) regarding the comparative static effects associated with an increase in the riskiness of a contributor's *ex ante* income distribution as well as his cost of contributing. Here, the authors provide examples of utility functions that generate linear strategies.

[5] As will be shown in Section 3, there are other notable differences between our model and that of Barbieri and Malueg (2010). Foremost among these differences are (1) we do not assume common knowledge among contributors regarding the key probability distributions characterizing risk in the model; in Barbieri and Malueg (2010) the distributions are defined independently over the provision threshold and the value obtained from consuming the public good, while in this paper the probability distribution is defined subjectively by a given contributor solely over the other agent's contribution levels (in Section 3.1.) and over her own valuation of the public good (in Section 3.2), respectively, and (2) we restrict our attention to two-moment decision models.

[6] In other words, we derive the necessary and sufficient condition for contributor i's "contribution reaction function" to be upward sloping with respect to social uncertainty.

effect of social uncertainty on the expected value of contributor i's net marginal utility must be negative.[7] If the respective risks incurred by all other contributors are in turn independent of contributor i's contribution level, then their contribution levels remain constant in the face of increased social uncertainty experienced by contributor i, and the aggregate contribution level thus rises one-for-one with the increase in contributor i's contribution. Otherwise, a necessary and sufficient condition for the aggregate contribution level to increase in the face of rising social uncertainty (as experienced by contributor i) and free-riding (as practiced by all other contributors) is that the incremental effect of contributor i's contribution on the aggregate expected value of all other contributors' net marginal utilities be small-enough positive. We derive similar necessary and sufficient conditions for both contributor i's expected utility and aggregate welfare to increase with increased social uncertainty.

Further, we find that an increase in an individual's risk associated with his own marginal valuation of the public good (i.e., an increase in "private uncertainty" similar to Barbieri and Malueg 2010) induces the individual to increase his contribution level only for a large-enough mean marginal valuation of the public good and/or small-enough associated variance. Numerical analysis demonstrates that increases in social uncertainty can induce a monotonic reduction in aggregate welfare as well. Whereas increases in private uncertainty can similarly result in monotonic reductions in aggregate welfare, the effect on the individual's welfare level can be shown to be nonmonotonic.

To set the stage for our main analysis, we begin in Section 2 with a presentation of our basic model in a deterministic setting. Section 3 then presents our main results in the context of a generalized two-moment model of uncertainty. Section 4 provides an example of uncertain public good provision in the context of a standard mean-variance specification of preferences. In this section, the effects of social and private uncertainty on individual and aggregate contribution and expected-welfare levels are examined both analytically and numerically. Section 5 addresses two issues pertaining to the generality of the two-moment model. Section 6 concludes.

2. Deterministic Public Good Provision

Consider contributor i's preferences for numeraire, x_i, and pure public good, G, represented by strictly quasi-concave utility function $u^i(x_i, G)$, $i = 1, \ldots, N$. Contributor i faces budget constraint $w_i = x_i + pg_i$, where w_i represents his wealth level, p the constant per-unit cost of providing G (in terms of x_i foregone), and g_i his (continuous) contribution level, $G = \sum_i g_i$. At an interior solution to this problem $x_i^* < x_i^N$, $g_i^* > g_i^N$, and $G^* > G^N$, where superscripts * and N, refer to Pareto-efficient and Nash equilibria, respectively.[8] Necessary and sufficient conditions for $\partial g_i/\partial g_j < 0$ (i.e., classic free-riding) and $\partial G^N/\partial g_j > 0$ (free-riding is nevertheless too weak to reduce the

[7] The notion of net marginal utility is first discussed in Section 2 for the case of deterministic public good provision. In Section 3, we provide a definition of net marginal utility for the case of uncertain public good provision.

[8] Bergstrom *et al.* (1986) provide existence and uniqueness conditions for the continuous, voluntary provision problem; conditions that the reaction functions in our model(s), for example, as represented by Equations (1) and (2) in this section and (5) and (8) in Section 3, are assumed to satisfy. The authors show that single-valued, continuously differentiable demand for the public good guarantees existence. Uniqueness requires that both the public and private good be normal.

aggregate level of the public good in a Nash equilibrium), respectively, for any $i \neq j$, are $p\,u^i_{xG} - u^i_{GG} > 0$ and

$$0 < \sum_{i \neq j} \left(\frac{p\,u^i_{xG} - u^i_{GG}}{H_i} \right) < 1, \tag{1}$$

where H_i is the determinant of contributor i's bordered Hessian matrix ($H_i > 0$ by the sufficiency condition for utility maximization) and all functions are evaluated at the utility-maximizing solution.[9] Here, $pu^i_{xG} - u^i_{GG}$ represents what was referred to in Section 1 as contributor i's "value of net marginal utility," in this case as determined by the incremental effect of G rather than by social uncertainty.

By way of a simple example, consider the commonly used exponential specification for the utility function, $u^i(x_i, G) = x_i^{\delta_i} - e^{-\theta_i G}$, where $0 < \delta_i < 1$ and $\theta_i > 0$ are both constants and e is Euler's number. In the case of certain G_{-i}, where the $-i$ subscript denotes the aggregate public good contribution across all $j \neq i$, we find

$$\frac{\partial g_i}{\partial g_j} = \frac{-\theta_i^2 e^{-\theta_i G}}{H_i} < 0, \quad i \neq j, \quad i, j = 1, \ldots, N, \tag{2}$$

where $H_i = \theta_i^2 e^{-\theta_i G} - p^2 \delta_i (\delta_i - 1) x_i^{\delta_i - 2} > 0$.[10] Although $\frac{\partial g_i}{\partial g_j} > -1$ for all $i \neq j$ and any j, it is not necessarily the case that $\sum_{i \neq j} \frac{\theta_i^2 e^{-\theta_i G}}{H_i} < 1$. Thus, we cannot unambiguously determine whether $dG/dg_j > 0$ in a Nash equilibrium for any $j \neq i$.

3. Uncertain Public Good Provision in a Generalized Two-Moment Model

3.1. Social Uncertainty

In its most general form, a two-moment model of contributor i's preferences under social uncertainty extends naturally from the utility function presented in Section 2. Contributor i's expected utility is represented as $E(u^i) = \tilde{u}^i(x_i, G; \theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}})$, $i = 1, \ldots, N$, where parameter $\mu_{G_{-i}}$ is the mean of i's probability distribution over G_{-i} and $\sigma^2_{G_{-i}}$ is its variance.[11] Function $\tilde{u}^i$ is assumed strictly concave in variables x_i and G (implying risk aversion), positive in parameters θ_i and $\mu_{G_{-i}}$, and negative in $\sigma^2_{G_{-i}}$ (Bigelow 1993). An increase in social uncertainty experienced by contributor i is therefore conveniently represented as an increase in $\sigma^2_{G_{-i}}$.

Using the same basic approach as in Section 2 to determine relevant comparative statics, we state our first proposition regarding voluntary provision of the public good in a two-moment model with social uncertainty.

PROPOSITION 1: *For two-moment preferences, (i) contributor i's private contribution to the public good increases with social uncertainty if and only if the incremental effect of social uncertainty*

[9] Subscripts on function u^i represent partial derivatives.

[10] In this case, contributor i's net marginal utility, which is the numerator of Equation (2), reduces to $-u^i_{GG} > 0$.

[11] We could just as easily denote the second moment as standard deviation $\sigma_{G_{-i}}$. Including constant $\theta_i > 0$ in this specification for social uncertainty is merely for comparison purposes with the case of private uncertainty presented in Section 3.2. Recall from Section 2 that θ_i is a factor determining the marginal utility (in this case the expected marginal utility) of the public good.

on the expected value of his net marginal utility is negative at his utility-maximizing solution, and (ii) the aggregate contribution to the public good simultaneously increases in the presence of free-riding if and only if the incremental effect of contributor i's contribution on the aggregate expected value of all other contributors' net marginal utilities is small-enough positive at the Bayes–Nash Equilibrium.

The proof of Proposition 1 is contained in the online Supporting Information. For Part (i) of the proposition, we note from the numerator of Equation (S2) in Appendix 1 that a negative incremental effect of social uncertainty on the expected value of contributor i's net marginal utility can be rewritten as $\tilde{u}^i_{G\sigma^2_{G_{-i}}} > p\,\tilde{u}^i_{x\sigma^2_{G_{-i}}}$, which conveys a simple intuition. Contributor i's contribution to the public good increases with social uncertainty when the uncertainty's effect on the expected marginal value of the public good exceeds its effect on the expected marginal value of the numeraire good—in other words, when the expected marginal value of a private investment in the public good exceeds that of the numeraire good.

Two observations about this condition merit further mention. First, because it merely requires the marginal value of one investment to exceed that of another, nothing precludes $\tilde{u}^i_{G\sigma^2_{G_{-i}}}$ or $\tilde{u}^i_{x\sigma^2_{G_{-i}}}$ from being nonpositive at contributor i's utility-maximizing solution.[12] Second, it is interesting to note the parallel between our result and that of McBride (2006). As mentioned in Section 1, McBride (2006) finds that increased threshold uncertainty increases the probability of a given contributor being pivotal in the case of a discrete public good. This in turn increases the probability that s/he will choose to contribute when the public good's value is sufficiently high relative to the cost of contributing. This is a close analogy to what we find in our two-moment model. Thus, in a loose way our result can be thought of as a continuous analogue of McBride's (2006).[13]

Part (ii) of the proposition conveys a similarly simple intuition concerning the potential effect of free-riding motivated by contributor i's increased contribution level. From Equation (S3) in Appendix 1, we note that the Bayes–Nash equilibrium aggregate contribution to the public good simultaneously increases in the presence of free-riding for a small-enough decline in the magnitude of the aggregate expected marginal value of the public good across all other contributors (i.e., $\sum_{j\neq i}\tilde{u}^j_{GG}$), where "small-enough" in this instance is in relation to the rate at which the aggregate expected marginal value of the numeraire good across all other contributors (i.e., $\sum_{j\neq i}\tilde{u}^j_{xG}$) changes. In other words, free-riding is outweighed by the increased contribution level of contributor i for a small-enough diminishment in the expected marginal value of that increased contribution level across all other contributors.

The condition governing the direction and magnitude of change in contributor i's expected utility in the presence of social uncertainty is tied directly to the comparative static effects of this uncertainty on both his consumption of the numeraire good and his contribution to the public good. To see this, we first define contributor i's optimal-value function, or indirect expected utility, as

[12] However, if $\tilde{u}^i_{G\sigma^2_{G_{-i}}} < 0$, then it must also be the case that $\tilde{u}^i_{x\sigma^2_{G_{-i}}} \leq 0$ for the condition to hold.

[13] How loose of an analogy is of course an open question. Is a contributor who questions the degree to which s/he is pivotal in the face of increasing threshold uncertainty really that different than a contributor who questions the extent of others' aggregate contribution in the face of increasing social uncertainty?

$\tilde{u}^{*i}(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}) = \tilde{u}^i(x_i(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}), (g_i(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}) + G_{-i}))$, where functions $x_i(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}})$ and $g_i(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}})$ represent contributor i's respective ordinary demands for the numeraire and public good. Clearly, contributor i's welfare increases with social uncertainty if and only if

$$\frac{\partial \tilde{u}^{*i}}{\partial \sigma^2_{G_{-i}}} = \frac{\partial \tilde{u}^i}{\partial x_i}\frac{\partial x_i}{\partial \sigma^2_{G_{-i}}} + \frac{\partial \tilde{u}^i}{\partial g_i}\frac{\partial g_i}{\partial \sigma^2_{G_{-i}}} > 0, \tag{3}$$

i.e., when the total marginal effect on indirect expected utility of a change in $\sigma^2_{G_{-i}}$ is positive. Note that Part (i) of Proposition 1 is met for an increase in g_i—leading to a positive second term on the right-hand side of Equation (3)—and contributor i's budget constraint ensures that the first term on the right-hand side is negative. Thus, contributor i's welfare increases with social uncertainty if and only if the uncertainty's marginal effect on the expected marginal utility value of g_i exceeds its countervailing effect on x_i.

3.2. Private Uncertainty

The two-moment model of contributor i's preferences under private uncertainty extends naturally from the utility function presented in Section 3.1 for social uncertainty. Contributor i's expected utility is now represented as $E(u^i) = \tilde{u}^i(x_i, G; \theta_i, \mu_{\theta_i}, \sigma^2_{\theta_i})$, $i = 1, \ldots, N$, where μ_{θ_i} is the mean of i's probability distribution over θ_i, and $\sigma^2_{\theta_i}$ is its variance. Function $\tilde{u}^i$ is again assumed strictly concave in variables x_i and G and positive in parameter θ_i, and now positive in μ_{θ_i} and negative in $\sigma^2_{\theta_i}$. An increase in private uncertainty is therefore represented as an increase in $\sigma^2_{\theta_i}$; that is, contributor i is uncertain about his own marginal valuation of the public good rather than the aggregate contribution level of the other contributors.

The formal proposition regarding voluntary provision of the public good in a two-moment model with private uncertainty closely mimics Proposition 1.

PROPOSITION 2: *For two-moment preferences, (i) contributor i's private contribution to the public good increases with private uncertainty if and only if the incremental effect of private uncertainty on the expected value of his net marginal utility is negative at his utility-maximizing solution, and (ii) the aggregate contribution to the public good simultaneously increases in the presence of free-riding if and only if the incremental effect of contributor i's contribution on the aggregate expected value of all other contributors' net marginal utilities is small-enough positive at the Bayes–Nash equilibrium.*

The proof for Proposition 2 similarly mimics that for Proposition 1. As a result, the intuition for Proposition 2 is identical to that for Proposition 1 regarding social uncertainty, as is the main result regarding the associated change in contributor i's expected utility.[14] Therefore, of interest here is not a comparison of the findings for private and social uncertainty in the context of our two-moment model, but rather a comparison of our findings for private uncertainty with those of Barbieri and Malueg (2010).

[14] Contributor i's welfare increases with private uncertainty if and only if $\frac{\partial \tilde{u}^{*i}}{\partial \sigma^2_{\theta_i}} = \frac{\partial \tilde{u}^i}{\partial x_i}\frac{\partial x_i}{\partial \sigma^2_{\theta_i}} + \frac{\partial \tilde{u}^i}{\partial g_i}\frac{\partial g_i}{\partial \sigma^2_{\theta_i}} > 0$. The proof for Proposition 2 is archived as a separate appendix with the journal.

As mentioned in Section 1, Barbieri and Malueg (2010) find that an increase in contributor i's private uncertainty induces an expected increase in his private contribution level, as well as the aggregate contribution level, in the presence of uniformly distributed threshold uncertainty. Further, contributor i's expected utility declines while the expected utilities of all other contributors increase. Ultimately what drives Barbieri and Malueg's (2010) results are conditions placed on the relative slopes of the respective contributors' "expected-contribution reaction functions," which are met by the assumption of uniform threshold uncertainty. Uniform threshold uncertainty plays a key role in determining the slope of the reaction function represented by Barbieri and Malueg's (2010) Equation (12), which in turn is crucial in proving their Lemma 2.[15] Their Lemma 2 is then used to prove their proposition 5, which compiles their findings reported on here. As our Proposition 2 and the ensuing discussion regarding contributor i's expected utility demonstrate, similar results occur in the context of our general two-moment model only under well-defined circumstances.

In an attempt to reach more definitive conclusions about outcome(s) associated with increased uncertainty in our model, we turn to an example with more restrictive conditions placed on contributor i's utility function, as well as the functional forms of the probability distributions representing both his social and private uncertainty.

4. An Example

4.1. Social Uncertainty

Let contributor i's von-Neumann–Morgenstern preferences be specified as $u^i(x_i, G) = v_i(x_i) - e^{-\theta_i G}$, where $v_i(x_i)$ is increasing and concave in x_i (e.g., Varian 1992; McLaren 2009).[16] If we further assume contributor i's probability distribution over G_{-i} is truncated normal (from the left), that is, $G_{-i} \sim N_T(\mu_{G_{-i}}, \sigma^2_{G_{-i}}, a, b)$, with $a = 0$ and $b = +\infty$, then G_{-i}'s associated density function is defined as

$$f_i(G_{-i}) = \frac{e^{\alpha}}{\sigma_{G_{-i}}\sqrt{2\pi}\,(1 - \Phi(\beta))}, \tag{4}$$

where $\alpha = (\frac{-(G_{-i}-\mu_{G_{-i}})^2}{2\sigma^2_{G_{-i}}})$, $\beta = (\frac{a-\mu_{G_{-i}}}{\sigma_{G_{-i}}}) = -\frac{\mu_{G_{-i}}}{\sigma_{G_{-i}}}$, and $\Phi(\beta) = \frac{1}{2\pi}\int_{-\infty}^{\beta} e^{-t^2/2}dt$ is the corresponding cumulative normal distribution function. Following Varian (1992) and McLaren (2009), it can be shown that contributor i's expected utility results in the closed-form expression,

$$E(u^i) = \tilde{u}^i\left(x_i, G; \theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}\right) = v_i(x_i) - e^{-\theta_i(g_i+\Omega_i)}, \tag{5}$$

[15] This "key role" has to do with a degree of constancy that the uniform threshold assumption lends to Barbieri and Malueg's (2010) Equation (12).

[16] The negative exponential form in our particular context requires $u^i(x_i, G)$ to be separable in x_i and G.

where $\Omega_i = \mu_{G_{-i}} - \frac{\theta_i \sigma^2_{G_{-i}}}{2}$, implying that contributor i's expected utility is of mean-variance form and independent of G_{-i}.[17] Given (5), it follows that

$$\frac{\partial g_i}{\partial \sigma^2_{G_{-i}}} = \frac{\theta_i^2 e^{-\theta_i (g_i + \Omega_i)}}{2\tilde{H}_i} > 0 \tag{6a}$$

and

$$\frac{\partial x_i}{\partial \sigma^2_{G_{-i}}} = 0\,, \tag{6b}$$

where again $\tilde{H}_i > 0$ and all functions are evaluated at their respective utility-maximizing values.[18] Further, $\partial g_i / \partial \sigma^2_{G_{-i}} < 1$. Because for this problem, $\partial g_i / \partial g_j = 0$ for all i and any $j,\ i \neq j$ (which comes about via contributor i's expected utility in (4) being independent of G_{-i}), it is also the case that

$$\frac{dG}{d\sigma^2_{G_{-i}}} = \frac{\partial g_i}{\partial \sigma^2_{G_{-i}}} > 0. \tag{7}$$

In other words, with negative-exponential utility defined over a normally distributed random variable, an increase in social uncertainty induces contributor i to voluntarily increase his contribution, thereby increasing the aggregate amount of the public good one-for-one in a Bayes–Nash equilibrium.

To the contrary, social uncertainty is potentially disadvantageous for both individual and aggregate expected welfare levels. To see this, we first delineate contributor i's indirect expected-utility function, as defined in Section 3.1, as

$$\tilde{u}^{*i}\left(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}\right) = v_i\left(x_i\left(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}\right)\right) - e^{-\theta_i\left(g_i\left(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}\right) + \theta_i\left(\mu_{G_{-i}} - \frac{\theta_i \sigma^2_{G_{-i}}}{2}\right)\right)}. \tag{8}$$

Differentiating $\tilde{u}^{*i}(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}})$ in (8) with respect to $\sigma^2_{G_{-i}}$ and noting (5b) results in

$$\frac{\partial \tilde{u}^{*i}\left(\theta_i, \mu_{G_{-i}}, \sigma^2_{G_{-i}}\right)}{\partial \sigma^2_{G_{-i}}} \gtreqless 0 \Leftrightarrow \frac{\partial g_i}{\partial \sigma^2_{G_{-i}}} \gtreqless \frac{\theta_i}{2}. \tag{9}$$

Thus, social uncertainty is welfare-improving for contributor i only for large-enough $\partial g_i / \partial \sigma^2_{G_{-i}}$, which from Equation (6a) we know cannot be greater than one. Again, because all other contributors' expected utilities represented by Equation (5) (and their indirect expected utilities represented by (8)) are independent of G_{-i}, the expected welfare levels of all other contributors are ultimately unaffected by the increase in contributor i's contribution level. Thus, aggregate expected welfare changes one-for-one with the change in contributor i's welfare when he experiences an increase in social uncertainty.

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[17] As McLaren (2009) points out, negative-exponential preferences exhibit constant absolute risk aversion (ARA). Further, the constant relative risk aversion (CRRA) utility function, which in the context of our model would be written as $u^i(x_i, G) = v_i(x_i) + \frac{G^{1-\gamma}}{1-\gamma}$, $\gamma > 0$, exhibits decreasing ARA. While it has a negative effect on negative-exponential (expected) utility in the two-moment model, social uncertainty has a positive effect on CRRA utility.

[18] Note that the numerator in (6a), $\theta_i^2 e^{-\theta_i(g_i+\Omega_i)}$, represents this example's specific functional form of $-\tilde{u}^i_{G\sigma^2_{G_{-i}}} > 0$ from Section 3.1. Also, for this example $p\,\tilde{u}^i_{x\sigma^2_{G_{-i}}} = 0$.

4.2. Private Uncertainty

Next, we assume that contributor i's (truncated normal) probability distribution is defined over parameter θ_i rather than G_{-i}, that is, $\theta_i \sim N_T(\mu_{\theta_i}, \sigma^2_{\theta_i}, 0, +\infty)$. Rewriting (5) accordingly, contributor i' s expected utility is

$$E(u^i) = v_i(x_i) - e^{-G\left(\mu_{\theta_i} - \frac{G\sigma^2_{\theta_i}}{2}\right)}. \quad (10)$$

After a bit of algebra we find that

$$\frac{\partial g_i}{\partial \sigma^2_{\theta_i}} = \frac{\left(\left(\mu_{\theta_i} - G\sigma^2_{\theta_i}\right)\frac{G^2}{2} - G\right) e^{-G\left(\mu_{\theta_i} - \frac{G\sigma^2_{\theta_i}}{2}\right)}}{H_i} > 0\,, \quad (11)$$

if and only if

$$\left(\mu_{\theta_i} - G\sigma^2_{\theta_i}\right)\frac{G}{2} > 1.^{19} \quad (12)$$

Unlike in Section 4.1, $\partial g_i/\partial g_j$ does not necessarily equal zero for all i and any j, $i \neq j$. In particular,

$$\frac{\partial g_i}{\partial g_j} = -\frac{\left(\left(\mu_{\theta_i} - G\sigma^2_{\theta_i}\right)\left(\frac{G\sigma^2_{\theta_i}}{2} - \left(\mu_{\theta_i} - \frac{G\sigma^2_{\theta_i}}{2}\right)\right) - \sigma^2_{\theta_i}\right) e^{-G\left(\mu_{\theta_i} - \frac{G\sigma^2_{\theta_i}}{2}\right)}}{H_i} \gtreqless 0$$

$$\text{as } \sigma^2_{\theta_i} + \mu_{\theta_i}\left(\mu_{\theta_i} - \frac{G\sigma^2_{\theta_i}}{2}\right) \gtreqless \frac{3G\sigma^2_{\theta_i}}{2}. \quad (13)$$

As Equations (11)–(13) reveal, the probability that contributor i's contribution increases in private uncertainty is, all else equal, increasing in the mean of contributor i's (truncated normal) probability distribution over his own marginal valuation of the public good and decreasing with respect to the distribution's variance.[20] The effect on the aggregate level of the public good is indeterminate.[21]

Thus, even in our simple example (and unlike in Barbieri and Malueg 2010) an increase in contributor i's level of private uncertainty induces him to voluntarily increase his public good contribution only under certain conditions; conditions that ultimately relate to the mean and variance of the underlying probability distribution defined over the individual's own marginal valuation of the public good.[22] Further, contrary to the case of social uncertainty, we are unable to say for certain whether private uncertainty is beneficial with respect to individual contribution levels and the aggregate amount of the public good.

[19] Using the problem's first-order conditions, it can be shown that $\mu_{\theta_i} - \frac{G\sigma^2_{\theta_i}}{2} > \mu_{\theta_i} - G\sigma^2_{\theta_i} > 0$.

[20] To see this, note that since the left-hand side of (12) is increasing (decreasing) in μ_{θ_i} $(\sigma^2_{\theta_i})$, the probability that the inequality in (11) will be satisfied is likewise increasing (decreasing).

[21] This follows directly from (13).

[22] Note that, all else equal, the probability of $\partial g_i/\partial \sigma^2_{\theta_i} > 0$ holding is also increasing in G if and only if $G < \frac{\mu_{\theta_i}}{2\sigma^2_{\theta_i}}$.

Table 1: Parameters for numerical analysis of Section 4.2's results

Parameter	Description	Value
δ_i	Substitutability coefficient for $x_i, i = 1, 2$	0.15
p	Per-unit price of $g_i, i = 1, 2$	1.5
w_i	Individual i's level of wealth, $i = 1, 2$	10
$\mu^i_{\theta_i}$	Mean of individual i's subjective guess of θ_i	5 and 4.75
$\sigma^2_{\theta_i}$	Variance of individual i's subjective guess of θ_i	2.5

4.3. Numerical Analysis

This section summarizes a numerical analysis of results derived in Section 4.2, where we assume that $v_i(x_i) = x_i^{\delta_i}$, $0 < \delta_i < 0$, $i = 1, 2$.[23] Since our previous results do not depend on the total number of contributors to the public good, we restrict the ensuing analysis to the case of $N = 2$. Our focal individual is contributor 1. Table 1 lists the parameter values used in the analysis of Section 4.2's results.[24]

Note the alternate values of 4.75 and 5 for $\mu^i_{\theta_i}$, which enable a simple numerical test of Section 4.2's claim that increases in contributor i's private uncertainty result in an increase in his private contribution when Equation (12) holds, that is, when $(\mu_{\theta_i} - G\sigma^2_{\theta_i})\frac{G}{2} > 1$. Table 2 presents the results for this analysis.

We note from Table 2 that except for the first two variance levels—$\sigma^2_{\theta_1}$ equals 2.5 and 2.55—a larger g_1 is associated with a larger $\mu^i_{\theta_i}$. The value of g_1 increases with increases in $\sigma^2_{\theta_1}$, but only up to $\sigma^2_{\theta_1} = 2.55$ for $\mu^i_{\theta_i} = 4.75$ and up to $\sigma^2_{\theta_1} = 2.8$ for $\mu^i_{\theta_i} = 5$, respectively, beyond which g_1 decreases. The same nonmonotonic patterns vis-à-vis $\sigma^2_{\theta_1}$ are displayed by G. Interestingly, individual 1's expected utility, $\tilde{u}^1$, first decreases and then increases with successive increases in $\sigma^2_{\theta_1}$. The turning points for this inverted-U relationship occur at $\sigma^2_{\theta_1} = 2.8$ for $\mu^i_{\theta_i} = 4.75$ and $\sigma^2_{\theta_1} = 3.1$ for $\mu^i_{\theta_i} = 5$. Aggregate welfare decreases monotonically with increases in $\sigma^2_{\theta_1}$, irrespective of the value of $\mu^i_{\theta_i}$.

5. Generality of the Two-Moment Model

Two issues pertaining to the generality of the two-moment decision model beg further mention. The first concerns the generality of the two-moment modeling framework. The second relates to the relationship between variance or standard deviation as a risk criterion and that of SOSD, which, as mentioned in Section 1, has been the criterion most commonly adopted in the literature.

Meyer (1987) was one of the first to examine the conceptual relationship between two-moment decision models and expected utility maximization, identifying what he called the location and scale (LS) condition for determining when an expected-utility ranking of a choice set can instead be represented by a two-moment ordering. As Meyer (1987) shows, the LS condition—which identifies random variables whose cumulative distribution functions differ from one another only by LS parameters—puts

[23] Numerical analysis of the conceptual model presented Section 4.1 provides no additional insights into the results obtained therein. Therefore, to save space, this analysis is archived with the journal as a separate appendix.

[24] Several parameter specifications were tested other than those presented in Table 1. In each respective case, the results are qualitatively similar to those presented later.

Table 2: Numerical results for Section 4.2

	g_1		G		$\tilde{u}^1$		U	
$\sigma^2_{\theta_1}$	$\mu^i_{\theta_i} = 5$	$\mu^i_{\theta_i} = 4.75$	$\mu^i_{\theta_i} = 5$	$\mu^i_{\theta_i} = 4.75$	$\mu^i_{\theta_i} = 5$	$\mu^i_{\theta_i} = 4.75$	$\mu^i_{\theta_i} = 5$	$\mu^i_{\theta_i} = 4.75$
2.5	0.5793	0.6043	1.1586	1.2087	1.3771	1.3726	2.7542	2.7452
2.55	0.5999	0.6077	1.1595	1.2089	1.3758	1.3717	2.7536	2.7445
2.6	0.6175	0.6063	1.1602	1.2088	1.3747	1.3710	2.7531	2.7437
2.65	0.6320	0.5998	1.1608	1.2085	1.3736	1.3704	2.7524	2.7429
2.7	0.6431	0.5880	1.1612	1.2080	1.3726	1.3700	2.7519	2.7421
2.75	0.6507	0.5704	1.1615	1.2073	1.3717	1.3697	2.7513	2.7412
2.8	0.6544	0.5469	1.1617	1.2063	1.3710	1.3696	2.7506	2.7402
2.85	0.6542	0.5171	1.1617	1.2050	1.3703	1.3697	2.7499	2.7392
2.9	0.6497	0.4808	1.1615	1.2034	1.3697	1.3699	2.7492	2.7382
2.95	0.6407	0.4376	1.1611	1.2016	1.3692	1.3703	2.7485	2.7371
3.0	0.6270	0.3874	1.1606	1.1994	1.3690	1.3709	2.7477	2.7359
3.05	0.6084	0.3300	1.1598	1.1969	1.3688	1.3716	2.7469	2.7346
3.1	0.5846	0.2653	1.1589	1.1940	1.3688	1.3726	2.7461	2.7332
3.15	0.5553	0.1931	1.1577	1.1908	1.3689	1.3737	2.7452	2.7317
3.2	0.5205	0.1134	1.1562	1.1872	1.3692	1.3750	2.7443	2.7301

"useful rather than overly restrictive structure on preferences" in two-moment space. This structure is substantially more flexible than that imposed by the quadratic and exponential utility functions, or normally and uniformly distributed random variables, and is consistent with many uncertain economic decision problems. Bigelow's (1993) finding that preferences admitting normalized risk comparability (NRC) across lotteries also result in the expected-utility ranking of the choice set being "mean-variance consistent" echoes Meyer's (1987) claim that two-moment models do not overly restrict preferences.[25] Therefore, the general consensus seems to be that economic implications stemming from two-moment and expected-utility models are consistent with one another.[26]

Lastly, we note that relying on a distribution's second moment to represent risk rather on than the SOSD criterion can also be considered a venial simplification. It is well known that in order for one distribution to SOSD another it must be the case that that distribution's variance is also less than the other's (cf., Eeckhoudt, Gollier, and Schlesinger 2005). Thus, there is a tight relationship between the SOSD and second-moment criteria. In turn, this effectively nests two-moment models in the broader context of expected utility maximization when it comes to assessing the impacts of risk on individual decision making.

6. Conclusions

This paper answers the question of when uncertainty is beneficial in a two-moment model of voluntary public good provision. We find that an increase in contributor i's risk associated with the aggregate contribution level of the other contributors (i.e., an increase in social uncertainty) induces an increase in his own contribution level if and only if the incremental effect of social uncertainty on the expected value of his net marginal utility is negative. Contributor i's welfare likewise increases when a closely related condition is met, namely that the uncertainty's marginal effect on his expected marginal utility value of the public good exceeds its countervailing effect on the marginal-utility value of the numeraire. Society's corresponding aggregate contribution to the public good increases in the presence of free-riding if and only if the incremental effect of contributor i's contribution on the expected value of all other contributors' net marginal utilities is small-enough positive.

We derive similar conditions for the case of private uncertainty, where the increase in contributor i's risk is associated with his own marginal valuation of the public good. A simple example illustrates our conceptual results. Numerical analysis further demonstrates that an increase in private uncertainty can have a nonmonotonic impact on contributor i's welfare.

As McBride (2006) points out, given its potential effect on public-good contribution levels and the attendant welfare of contributors, there may be a strategic role to be played by uncertainty in the mechanism-designer's decision framework. For example,

[25] NRC means that an individual's preference over any set of lotteries is the same before and after the lotteries' respective mean values are normalized to zero. Konrad (1993) links Bigelow's (1993) NRC condition with Meyer's (1987) LS condition, the latter of which he shows is consistent with the class of linear probability distributions. Bar-Shira and Finkelshtain (1999) point out that Meyer's LS condition does not place restrictions on either class of utility functions or probability distribution function. Rather, the condition depends solely upon the decision problem's structure.

[26] Bar-Shira and Finkelshtain (1999) argue that the two-moment model may actually be considered more general than the expected-utility model because the former accommodates nonlinearities in the probabilities.

organizers of pledge drives (e.g., for public radio and television) and capital campaigns (e.g., by alumni associations) may benefit from providing potential contributors not only with information about the average pledge or contribution level, but also about the corresponding probability distributions associated with the contributions. This added information about the risk (in this case, social uncertainty) faced not only by the organizers, but also by the contributors themselves, may lend an added sense of efficacy (or perhaps urgency) to the average contributor's decision-making process. In the end, helping potential contributors better understand this risk could also help enhance the welfare they derive from the public good itself. As such, it is clear that the question of how social uncertainty might best become a tool in the proverbial toolbox of planners is ripe for both laboratory and field experimentation.

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Supporting Information

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Online Appendix