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Examining the Validity of the Spring 2007 Math 1050 Common Final

Angela K. Brock

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April 24, 2008
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I began this study wanting to know more about the Math 1050 Common Final. I've heard so many negative things about the class from students, I wondered if the common final, which has so much influence on students' grades, is actually valid. A measurement is valid to the degree that it is both reliable and relevant, so I needed to address both relevance and reliability. To do this, I began by finding a reliability coefficient for the multiple choice section. I then analyzed each item with respect to difficulty, discrimination and efficiency. To determine content and learning level relevance, I used the Measurement Analysis Coding Form.

To begin, I collected the final exams from one Spring 2007 Math 1050 section. The final consists of 20 Multiple Choice items worth 7 pts each and 5 free response items worth 12 pts each. I then recorded the scores of each multiple choice question and, using these scores, I calculated a reliability coefficient. When doing this, I calculated mc as 1 pt each so as not to artificially inflate the variance.

Using coefficient KR20 to compute a reliability coefficient:

\[ KR20 = \frac{k}{k-1} \left(1 - \frac{\sum_{j=1}^{k} p_j(1-p_j)}{\sigma^2}\right) = 0.725 \]

where \( KR20 \) is the reliability coefficient, \( k \) is the number of test items, \( \sigma^2 \) is the variance of the test scores, and \( p_j \) is the proportion of students that scored 1 point for Item \( \#j \). KR20 is restricted for use with tests that only include items that are dichotomously scored. To interpret this value, we can use a definition of reliability: A measurement is reliable to the degree to which it can be depended upon to provide non-contradictory information (Cangelosi, 184).

Assessment Strategies for Monitoring Student Learning (p620) indicates that for reliability coefficient values of greater than 0.75 we usually can agree that the measurement has a satisfactory degree of internal consistency. Values of \( r \) less than 0.65 indicate a level of unsatisfactory internal consistency, while values between 0.65 and 0.75 require more inference.

To determine what an alpha value of 0.725 means with respect to this measurement I considered the types of prompts and the learning level of the items. Since all the items are multiple choice, there should not be any additional inconsistencies introduced by the scorer. This should also improve internal consistency because students do not have to change thinking to answer different types of items.
Using the Measurement Analysis Coding Form, I found that most of these items are also at the same learning level: 60% (12 items) Algorithmic skill, 30% (6 items) Comprehension and Communication, 5% (1 item) Simple knowledge, 5% (1 item) Application. Because the prompts are not stimulating thinking at a wide variety of learning levels, this eliminates one factor that reduces internal consistency. For these reasons, I conclude that 0.725 is not a satisfactory level of reliability.

In theory, the Standard Error of Measurement (SEM) is due to test-retest reliability. If we could give the test to the same students repeatedly (without the student remembering) then the differences in scores would be due to the test having imperfect reliability. The SEM is the standard deviation of the distribution of test-retest reliability. For this test, I found the SEM = 0.524.

To understand the test better, I wanted to know more about each multiple choice items’ effectiveness. First I had to select a high group \( PH \) and a low group \( PL \). 49 students took the test so I used the top 15 (with respect to mc score) as the high group and the bottom 15 as the low group. “The trick is to make both \( N_h \) and \( N_l \) large enough to mitigate the influence of change factors, but small enough to have reasonably large numbers of students with in-between scores excluded from both Groups H and L.” (p 649) Although I am not confident the test is valid, as a group, I can assume those with higher scores have higher levels of achievement. As I continue the item analysis, keep in mind that the strength of the interpretation of the data as a measure of individual item effectiveness depends on the overall string of scores being valid.

Then I calculated the proportion of students in each group that got each MC item correct. Comparing the \( PH_j \) and \( PL_j \) of each item \( j \), I found that for all 20 items, \( PH_j > PL_j \), which is a good indicator that the items are effective to some degree.

Since tests are designed to discriminate between achievement levels of students, there needs to be a range of easy and hard items. We can design items to have a certain level of difficulty, but then to see which problems students actually found easier than others we simply look at the proportion of students who got each item right. The items with the highest proportion of correct answers are the easiest, while the items with a low proportion are the more difficult items.

On this test, there is a range of \( P_j \), index of difficulty, of \( 0.4 \leq P_j \leq 0.83 \). The closer \( P_j \) is to 0, the harder the item; the closer \( P_j \) is to 1, the easier the item. This test has one item with \( P_j = 0.4 \), item 20 (partial fraction decomposition). Because this item has the lowest \( P_j \) value, it is the hardest item on the test. Items 12, 13, and 19 have \( P_j \) values of 0.53 and are the next hardest items on down to items 3 and 11 which have \( P_j \) values of 0.83 and are the easiest items. Although most standardized tests have most items
with $P_i = .5$, the literature recommends that on a test like this there be 25% hard items, 50% moderate and 25% easy items. This helps to discriminate the differences in the high scores and the differences in the low scores, instead of just the differences between the high and low scores. Because the lowest $P_i$ on this test is 0.4, psychometrically, there is no hard item.

This becomes interesting when you consider the overall grading scheme for math 1050. The pre-set grading scale influences the writer to make the test easier than it might be otherwise. But then these tests and items don’t discriminate between high levels of achievement because there are not hard enough items.

If a test is very hard, students usually receive lower overall grades. Then these tests scores influence the final grades to be lower. This is why the literature recommends no pre-set grading scheme. Instead, determine grades after all measurements have been scored. When computing grades, teachers should not average numbers, but instead average the actual A, B, C, grade on the assignment or assessment.

To find out more about each items’ effectiveness I calculated $D_i = PH_i - PL_i$, the index of discrimination, for each item. The closer $D_i$ is to 1, the more effective an item. Depending on the difficulty level of items, $D_i$ can have a range up to $-1 \leq D_i \leq 0.1$. This happens when $P_i = .5$, which is on most standardized tests. So, for a very easy or very hard item, $D_i \approx 0$.

I found the range of $D_i$ for this test to be $0.2 \leq D_i \leq 0.8$. Some rough guidelines for interpreting this statistic (1) $0.4 \leq D_i \leq 1.00$ suggests an item is very effective, (2) $0.3 \leq D_i \leq 0.4$ suggests an item is reasonably effective, but could be improved (3) $0.2 \leq D_i \leq 0.3$ suggests that the item is only marginally effective and should probably be refined and (4) $-1.00 \leq D_i \leq 0.2$ suggests that the item is inadequately effective and should be modified or rejected. But there is one large disclaimer: these guidelines are only good for moderately hard items $0.25 \leq P_i \leq 0.75$.

According to $D_i$ values, this test has no items that should be rejected, but there are some for which revision should be considered. Items 2, 6, and 11 have $D_i$ values of 0.27, 0.27, and 0.2 respectively. These should be the first items considered for revision. Items 1, 3 and 7 all have $D_i$ values of 0.33 and could also be subject to improvement. All other items have $D_i$ values of at least 0.4 and can be considered effective. $D_i$ is highly biased because of the difficulty of items; this statistic depends on the proportion of students who got the item right.
$E_i$ reduces that bias by compensating for the difficulty of an item. $E_i = \frac{D_i}{\text{Max } D_i}$. 

Calculating $E_i$, the index of item efficiency, for each item gives me a better indication of which items are more effective. The higher the $E_i$, the more effective an item. An effective item discriminates between responses of students with high scores and responses from those with low scores. Ideally, students with higher scores would have better mastery of the material than students with low scores. But this assumes the test is perfectly valid. Since test will never be perfect, students score is the best indicator of what a student has learned. So we have to work under the assumption that higher scores indicate a better understanding of the material. The $E_i$'s are a form of item correlation with overall scores. This number indicates which items contribute or detract from internal consistency.

Students who have a better mastery of the material and therefore receive a higher score on the measurement should get effective items correct while those students who have lower levels of mastery and lower scores, should get these questions wrong.

Using $E_i$ to rank the MC questions from most effective to least:

3 10 14-18 with $E_i=1$,

5 9 20 with $E_i=0.83$

19,7,11,13,8,2,12,1,4,6 with $E_i=0.25$

These numbers result from looking at the proportion of students in the high group that got the question correct and comparing this with the number of people in the low group that correctly answered the question. Having all values for $E_i$ being positive is a good thing because it says that the problem is effective in discriminating higher and lower scores. In other words, students who scored higher on the MC portion correctly answered the problems while students who scored lower did not. If $E_i$ was negative, it would mean that for that particular question, more students with low scores correctly answered it than did students with high scores. A negative $E_i$ calls attention to items that should be considered for refinement.

Now considering both $D_i$ and $E_i$ values, I can better decide which items should be considered for revision. Because items 2, 6 & 11 have low values for both $D_i$ and $E_i$, I believe they should be considered for revision. I no longer think item 3 needs to be considered because of its high $E_i$ value, but I recommend possible revision for items 1 & 7.
These are the three items that should first be considered for revision.

2. Which interval represents the solution to the inequality: \( x(x - 4) \geq 12 \)

   (a) \([6, \infty)\)  \hspace{1cm}  (b) \((-\infty, 0] \cup [4, \infty)\)  \hspace{1cm}  (c) \((-\infty, -6]\)  \hspace{1cm}  (d) \((-\infty, -2] \cup [6, \infty)\)  \hspace{1cm}  (e) \([4, \infty)\)

6. How many real solutions does the equation have: \( 2x - 2\sqrt{x} - 4 = 0 \) (note that \( x = \left(\sqrt{x}\right)^2 \))

   (a) 0  \hspace{1cm}  (b) 1  \hspace{1cm}  (c) 2  \hspace{1cm}  (d) 3  \hspace{1cm}  (e) 4

11. Determine the factored form of the polynomial function \( p(x) = x^4 - 2x^3 + 4x^2 - 8x \), given that \( x = 2i \) is a zero of the function.

   (a) \( p(x) = x(x - 2)(x + 2)^2 \)  \hspace{1cm}  (b) \( p(x) = x(x^2 + 4)(x - 2) \)  \hspace{1cm}  (c) \( p(x) = x(x^2 + 4)(x - 2)^2 \)

   (d) \( p(x) = x(x + 2)(x - 2)^2 \)  \hspace{1cm}  (e) \( p(x) = (x^3 - 4)(x - 2) \)

Also to be considered:

1. Simplify the expression, and write the result in the form \( a + bi \): \(-2\sqrt{-4} + i(5 - i)\)

   (a) \( 5 + 5i \)  \hspace{1cm}  (b) \( 1 + i \)  \hspace{1cm}  (c) \( -1 + i \)  \hspace{1cm}  (d) \( 3 + 5i \)  \hspace{1cm}  (e) \( 1 + 9i \)

7. The graph of the function \( f(x) = x^2 \) is to be transformed as follows: First, the graph is to be reflected about the x-axis. Then the graph is to be shifted vertically 2 units upward. Which of the functions below would be appropriate for the graph that results from these transformations?

   (a) \( g(x) = x^2 - 2 \)  \hspace{1cm}  (b) \( g(x) = -x^2 + 2 \)  \hspace{1cm}  (c) \( g(x) = -(x^2 + 2) \)  \hspace{1cm}  (d) \( g(x) = -(x + 2)^2 \)  \hspace{1cm}  (e) \( g(x) = -(x - 2)^2 \)

One distracter deserves some attention. On exercise 9 alternative (e) states "Both (b) and (d) are correct responses. This introduces an entirely different way of finding an answer. For this distracter, test wise-ness of the student is influencing the scores, not just their knowledge of the material. Instead of just testing if a student can determine functions, we are asking if a student is a good test taker. The literature clearly outlines that this is not good test writing practice. "Multiple-choice prompts should be designed so that the alternatives are all parallel. Two alternatives are parallel if they pose the same question to students-a question about a specific subject-content." (Cangelosi, 339).
On most multiple choice items, students are asked to compare the alternatives and decide which is correct. But on items such as exercise 9, students are really being asked 4 or 5 different True/False questions. Distracters such as (e) ask students a new question because students are no longer comparing a list of answers.

For timed sections of tests, Assessment Strategies also recommends formatting a test such that the problems are sequenced from easier items to the more difficult items. This will help manage students time so that the time spend on harder problems does not prevent students from answering easier problems.

I used the Measurement Analysis Coding Form to determine content and learning level relevance. Before doing this, I spoke with the author of the test. He told me that the test is written in such a way that if a student doesn’t know the fundamentals that will be required in later classes, the student should not be able to pass the final. To me, this seemed very vague.

As described in Assessment Strategies, tests should be constructed with objectives in mind. To be relevant, test items should be constructed before the test is assembled. The items should be designed to assess the specific objectives that the lessons will attempt to teach. Since each objective will be categorized as a specific learning level, test items constructed with the objective in mind will assess the student at the level of the objective. When designing a test, it is best to begin with weighted objectives and use a test blueprint to determine the items that should make up the test. This test blueprint does not begin with a predetermined number of points. Instead the weighted objectives dictate where points will be assigned and eventually will determine the total number of points for the test.

After conducting this analysis, I have a few recommendations. I first recommend that those supervising the course and writing the final come to a conclusion about the real purpose of this final. Is the test meant to differentiate between individual student levels of achievement or to simply discriminate between students who have performed ‘well enough’ to pass the course from those who have not? For the test to differentiate between individual achievement levels, harder items \((P, \leq 0.25)\) must appear on the final. In general, specific content and learning level objectives for the final need to be defined before it is constructed. This would help both instructors of the course as well as the author(s) of the common final. The test writing strategies mentioned here and suggested by the literature should be used when designing this common final in the future.
References:

### Measurement Analysis Coding Form

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1. Simplify the expression, and write the result in the form $a+bi$: $-2\sqrt{-4} + i(5-i)$
   (a) $5+5i$  (b) $1+i$  (c) $-1+i$  (d) $3+5i$  (e) $1+9i$

2. Which interval represents the solution to the inequality: $x(x-4) \geq 12$
   (a) $[6, \infty)$  (b) $(-\infty, 0] \cup [4, \infty)$  (c) $(-\infty, -6]$  (d) $(-\infty, -2] \cup [6, \infty)$  (e) $[4, \infty)$

3. How many real solutions does the equation have: $x^5 - 4x^3 - 5x = 0$
   (a) 0  (b) 1  (c) 2  (d) 3  (e) 5

4. Which interval represents the solution to the inequality: $\frac{x}{x+20} \geq 0$
   (a) $[0, \infty)$  (b) $(0, \infty)$  (c) $(-\infty, -20) \cup [0, \infty)$  (d) $(-\infty, 0] \cup [20, \infty)$  (e) $(-\infty, -20)$

5. Determine the domain of the function: $f(x) = \frac{x}{x^2 + 400}$
   (a) The set of real numbers $x \neq 0$  (b) The set of real numbers $x \neq -20$  (c) The set of real numbers $x \neq -20, 20$  (d) The set of real numbers $x \neq 0, -20, 20$  (e) The set of real numbers

6. How many real solutions does the equation have: $2x - 2\sqrt{x} - 4 = 0$ (note that $x = (\sqrt{x})^2$)
   (a) 0  (b) 1  (c) 2  (d) 3  (e) 4

7. The graph of the function $f(x) = x^2$ is to be transformed as follows: First, the graph is to be reflected about the x-axis. Then the graph is to be shifted vertically 2 units upward. Which of the functions below would be appropriate for the graph that results from these transformations?
   (a) $g(x) = x^2 - 2$  (b) $g(x) = -x^2 + 2$  (c) $g(x) = -(x^2 + 2)$  (d) $g(x) = -(x+2)^2$  (e) $g(x) = -(x-2)^2$
8. Determine the inverse of the function, if it exists: \( f(x) = \sqrt[3]{x} - 2 \)

(a) \( f^{-1}(x) = x^3 + 2 \)  
(b) \( f^{-1}(x) = x^3 - 8 \)  
(c) \( f^{-1}(x) = (x + 2)^3 \)  
(d) \( f^{-1}(x) = x^3 + 8 \)  
(e) The function has no inverse

9. Given the function \( h(x) = \frac{2}{x^2 + 4} \), determine the two function \( f(x) \) and \( g(x) \) such that 
\[
(f \circ g)(x) = h(x)
\]

(a) \( f(x) = \frac{2}{x^2}; g(x) = x + 2 \)  
(b) \( f(x) = x^2 + 4; g(x) = \frac{2}{x} \)  
(c) \( f(x) = 2; g(x) = \frac{1}{x^2 + 4} \)

(d) \( f(x) = \frac{2}{x}; g(x) = x^2 + 4 \)  
(e) Both (b) and (d) are correct responses

10. The height above the ground, \( h \), of a bullet \( t \) seconds after it is fired vertically into the air from the top of an 800-foot tall building is given by the function 
\( h(t) = -16t^2 + 384t + 800 \) feet, \( t \geq 0 \). According to this model, what is the maximum height above ground that the bullet will reach?

(a) 3040 feet  
(b) 1184 feet  
(c) 3104 feet  
(d) 1168 feet  
(e) 2512 feet

11. Determine the factored form of the polynomial function \( p(x) = x^4 - 2x^3 + 4x^2 - 8x \), given that \( x = 2i \) is a zero of the function.

(a) \( p(x) = (x - 2)(x + 2)^2 \)  
(b) \( p(x) = x(x^2 + 4)(x - 2) \)  
(c) \( p(x) = x(x^2 + 4)(x - 2)^2 \)

(d) \( p(x) = x(x + 2)(x - 2)^2 \)  
(e) \( p(x) = (x^3 - 4)(x - 2) \)

12. Which of the following is a factor of the polynomial function 
\( p(x) = x^4 - x^3 - 10x^2 - 2x - 24 \)

(a) \( x - \sqrt{2} \)  
(b) \( x - 3 \)  
(c) \( x + 4 \)  
(d) \( x - 4 \)  
(e) \( x + \sqrt{2} \)
13. Which of the following rational functions would be appropriate for a rational function that has the following characteristics:

(1) A horizontal asymptote given by the line \( y = 0 \) (the x-axis); and

(2) Vertical asymptotes given by the lines \( x = 0 \) and \( x = -20 \)

(a) \( f(x) = \frac{x^2}{x^2 + 20x} \)  
(b) \( f(x) = \frac{2}{x + 20} \)  
(c) \( f(x) = \frac{x}{x^2 + 20x} \)

(d) \( f(x) = \frac{2}{x^2 - 20x} \)  
(e) \( f(x) = \frac{x + 2}{x^2 + 20x} \)

14. Determine the equation of the slant asymptote for the graph of the function

\( f(x) = \frac{2x^2 + 2}{x} \)

(a) \( y = 2 \)  
(b) \( y = 2x \)  
(c) \( y = \frac{2}{x} \)  
(d) \( y = 2x + 2 \)  
(e) The graph of the function has no slant asymptote

15. Which of the following expression is equivalent to: \( \ln\left(\frac{1}{e^2}\right) + \ln(x^3) \)

(a) \(-2 + 3\ln(x)\)  
(b) \(\frac{1}{2} + 3\ln(x)\)  
(c) \(-1 + \ln(x)\)  
(d) \(2 + 3\ln(x)\)  
(e) \(-2 + 3x\)

16. Solve the equation for \( x \):

\( \frac{600}{e^{2x}} = 100 \)

(a) \( x = \ln 6 \)  
(b) \( x = \ln 3 \)  
(c) \( x = \frac{\ln 500}{2} \)  
(d) \( x = \frac{300}{\ln 100} \)  
(e) \( x = 3 \)

17. Solve the equation for \( x \):

\( \ln(x) + \ln 5 = \ln(e^3) \)

(a) \( x = \frac{3e}{5} \)  
(b) \( x = \frac{e^3}{5} \)  
(c) \( x = e^3 - 5 \)  
(d) \( x = 5e^4 \)  
(e) The equation has no solution

18. Simplify the expression and write the result as a single logarithm:

\( \log(8x^2) - \log(2x) + \log(1) = \)

(a) \( \log(8x^2 - 2x) \)  
(b) \( \log(16x^3) \)  
(c) \( \log(4x) \)  
(d) \( 2\log(6x) \)  
(e) \( \log(4x + 1) \)
19. A company that produces digital cameras wishes to use a **quadratic function** to represent the monthly profits, $P$, in terms of $x$, the quantity of cameras produced and sold each month. They have determined that the **maximum profit** is $9500 when $x = 100$ cameras produced. Also, if $x = 0$, the profit is $P = -500$ (a loss). Determine a function, $P(x)$, in **standard form**, that is appropriate for these conditions. Note: The standard form of a quadratic function is given by: $y = a(x-h)^2 + k$.

(a) $P(x) = -(x+100)^2 - 9500$  
(b) $P(x) = (x-100)^2 + 9500$  
(c) $P(x) = -(x-100)^2 + 9500$  
(d) $P(x) = -(x+100)^2 + 9500$  
(e) $P(x) = -500(x-100)^2 + 9500$

20. In the **partial fraction decomposition** of $\frac{4}{x^2 - 4}$, which of the following is the **numerator** of the term that has a denominator of $x + 2$?

(a) 2  
(b) -2  
(c) 1  
(d) -1  
(e) 4