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Hands-on Activities and Activities Involving Technology to Help Students Construct Concepts and Gain a Deeper Understanding and Appreciation for Mathematical Concepts Based on the Utah Core Curriculum for Algebra II

BrookeAnn Watterson
Utah State University
Abstract

Research shows there are several methods that expand students’ understanding, appreciation for, and interest in mathematics by following teaching strategies. These strategies include incorporating hands-on activities, technology, discovery learning, cooperative learning, and having activities be applicable to real world contexts. This project focuses specifically on activities based on objectives from the Utah State Core for Algebra II that incorporate such strategies in five units: (1) absolute value, (2) exponential growth/decay and logarithms, (3) trigonometric functions, (4) probability, permutations and combinations, and (5) statistics.
Activities and Lessons Based on the Utah Core Curriculum Objectives for Algebra II

The majority of mathematics teachers at the secondary and post-secondary level teach mathematics strictly through lecture driven lessons, teaching the students algorithms to perform then the students repeat those algorithms using thirty or more practice problems to memorize and understand the process. Cangelosi states, “Unfortunately, most mathematics teachers tend to view mathematics narrowly as a school-bound sequence of vocabulary and symbol meanings, rules, algorithms, and theorems that are not applicable to the outside-of-school interests of adolescents” (1992, p. 6). Students in such an environment generally never gain an understanding of how to apply mathematics in their own lives. According to learning expert, Eric Jensen, “Given what the research shows, it should be apparent that presenting more content per minute or moving from one piece of learning to the next too rapidly, virtually guarantees that little will be learned or retained” (2005, p. 43). It is to the advantage of the students to present the information in a manner that will be motivating and help them gain problem solving skills that they can apply in their own lives. When students understand a concept initially then less time needs to be spent on reteaching or reviewing concepts they should have learned previously but did not. By including more engaging activities to keep students actively involved they will retain and understand concepts more effectively.

Research has shown that applying constructivist methods in the mathematics classroom leads students to have an increased value of mathematics and a greater understanding of mathematical concepts. Constructivism is the philosophy or belief that learners create their own knowledge based on interactions with their environment, including their interactions with other people. The goals of a constructivist pedagogy focus on individual students developing deep understandings within the subject matter and fostering habits that will aid in future learning.
While employing constructivism, the focus is on the process of gaining knowledge and not the end product.

Children learn effectively through interactions with experiences in their natural environment. Mathematics teaching consists primarily of the mathematical interactions between a teacher and children. This indirect approach to instruction effectively allows the student to learn in the context of meaningful activities. Learning is an endless, lifelong process that results from interactions with a multitude of situations. The constructivist approach does not solely focus on the action of the teacher or the learner, but on the interactions between the two. The constructivist classroom creates an environment that encourages learning. The teacher should create surroundings where students can make sense of mathematics as it relates to the world. Constructivism focuses on student-centered instruction (Brumbaugh, 2001, pp. 25-26).

By providing meaningful activities for students to engage in, students not only learn the mathematical concepts but gain invaluable problem solving skills that they can apply throughout their lives. "It is important to recognize that students' conceptual construction comes about from engagement in the task environment created by the teacher" (Kirshner, 2002, p. 52). Therefore in a constructive environment students are actively engaged in meaningful tasks and not merely empty vessels waiting to be filled with knowledge provided by their teacher. Students are active learners that can come up with questions, solve problems, and construct theories and knowledge. Creating activities that keep students actively engaged in meaningful tasks while meeting state core objectives give a specific purpose to the activities and help students learn more effectively than teaching by a lecture driven approach.

There are many methods that fall under the constructivism theory. The activities included
in my project contain constructivist strategies including: hands-on activities, incorporating technology in the classroom, guided discovery learning, cooperative learning, and contextual real world application. These strategies help students experience success and gain a greater appreciation for the application of mathematics.

A major component in making math more meaningful and engaging is the hands-on aspect that has been incorporated within these activities. Hands-on activities help students be more involved, make connections with concepts being taught and construct personal, relevant meanings. According to Cangelosi,

Older students need experiences working with manipulatives and concrete models to gain conceptual-level understanding of mathematical concepts and relationships. Once secondary teachers begin involving their students in hands-on activities with concrete objects, they tend to continue doing so because students prefer them to more passive paper-and-pencil exercises and, consequently, are more cooperative, appreciative, and easier to manage (1992, p. 43).

This does not imply that a teacher simply has their students involved in hands-on activities every day. The teacher needs to make sure that any activity the students participate in is directly connected to the state core objectives, ensuring that there is a specific purpose to the activity.

As I have taught I have seen how the same lesson taught with hands-on activities compared to teaching strictly through lecture has greatly increased students’ motivation, engagement, and understanding.

English learners make more rapid progress in mastering content objectives when they are provided with multiple opportunities to practice with hands-on materials and/or manipulatives. These may be organized, created, counted, classified, stacked,
experimented with, observed, rearranged, dismantled, and so forth. Manipulating learning materials is important for ELs because it helps them connect abstract concepts with concrete experiences (Echevarria, Vogt, & Short, 2008, pp. 139-140).

Hands-on activities increase all students learning, but are especially beneficial to English learners (ELs) or students with disabilities. Through hands-on activities all students have something concrete to work with that helps minimize any language barrier that may exist.

Hands-on activities help students connect to the math they are learning and provide greater opportunity for modeling real-world application of mathematical concepts. Through technology, many mathematical concepts become hands-on as technology provides a means to manipulate, experiment, and observe results that previously have been difficult to illustrate. With the passage of time there have been many great technological advances that can be employed in the classroom to help students be more engaged in mathematics, visualize and gain a better understanding of concepts being taught, and provide opportunities for interaction with mathematical concepts. Technology is used “in the broadest sense to include computers, probeware, graphing calculators, scientific calculators, four-function calculators, and even manipulatives” (Huetinck & Munshin, 2000). There are differing views on the effect of technology in the classroom.

The use of calculators has long been debated. Advocates say that calculator use will pare the time spent on routine computations and help students concentrate on mathematical concepts and applications. Critics contend that calculators will allow students to avoid learning the basics of mathematics they will need beyond their school years. . . Contrary to the fears of many, there is no evidence that the availability of calculators and computers makes students dependent on them for simple calculations. “As long as the
primary focus is their use as a tool, they should pose no threat to mathematics education,” says Chancey Jones, principal measurement specialist at ETS. In fact, he notes, today’s calculators have already progressed far beyond simple arithmetic functions. “They can be a teaching instrument because the students can see on the screen what it means to approach a limit or draw the curve of an equation. It’s right in front of them; they can interact with the material. For all practical purposes, a lot of the graphing calculators today are really minicomputers.” The public’s fears are based on a general misunderstanding of mathematics itself. They think of mathematics as learning to add, but that’s not mathematics – that kind of operation is a very small piece of mathematics . . . Graphics have vastly improved the way people can communicate mathematical ideas and they therefore need to be included in the mathematics classrooms (Carlson, 1992, p.14).

Technology is a great way to be able to have students visualize concepts, manipulate equations to see how various parameters effect graphs, gain a greater understanding of mathematics and be able to see how mathematics applies to the real world. “Employing technology’s ability to rapidly produce alternate representations, specifically graphs and tables can strengthen students’ awareness of symbolic expressions and equations as well” (Rider, 2007 p. 494). Technology makes teaching certain concepts quicker and more precise. Technology also helps bridge what students are familiar with to what they are not familiar with. Most of today’s students are very comfortable using a computer or other technology. “Research shows the use of technology in the mathematics environment causes students to: become better problem solvers, better understand the concepts that are taught, demonstrate increased performance levels on standardized tests, and achieve higher ACT and SAT scores” (Brumbaugh
As a result of incorporating technology within the classroom students are not only more motivated to engage in the learning tasks but increase the level of understanding concepts taught.

I have created and compiled activities that fit into five units for an Algebra II class. These five units are (1) absolute value, (2) exponential growth/decay and logarithms, (3) trigonometric functions, (4) probability, permutations and combinations, and (5) statistics. Each of these units includes hands-on activities, many of which use technology. In the school I have been teaching at, all of the computers in the computer lab have Geometer’s Sketchpad installed on them. “The Geometer's Sketchpad is a dynamic construction, demonstration, and exploration tool that adds a powerful dimension to the study of mathematics. You . . . can use this software program to build and investigate mathematical models, objects, figures, diagrams, and graphs” (from http://www.keypress.com/x5521.xml). This is a great tool to enable students to manipulate graphs and equations and to observe the results.

Another teaching strategy included throughout these units is guided discovery learning. Discovery learning includes activities through which students observe, formulate questions, devise ways to answer those questions, generate solutions and new knowledge, and test the reliability of their solutions.

The open-endedness that characterizes inquiry requires extreme flexibility in terms of curriculum content and choices. A teacher will often need to deviate from the original lesson plan in order to follow a new lead, pursue valuable questions raised by the students, or let the class fully engage in a debate stimulated by a difference in opinion or different solutions (Borasi, 1991, p. 202).

Due to the complexity of the classroom and state core objectives that are required to be covered
during the school year, I will employ a guided discovery approach to guide students to make inquiries that will lead them to discover specific mathematical concepts. This approach allows the use of the inquiry and discovery method while staying focused on the state core objectives. As students are actively involved in discovering solutions they have more of an “ownership” with the concepts being taught which results in greater understanding and ability to recall concepts learned. Research has shown that when teachers use inquiry based approaches students’ achievement is significantly higher compared to those taught using a lecture format (Borasi, 1991).

Employing discovery approaches within a class “is not always easy to accomplish because for years the students has been told when and how to do things. That student has become dependent on being told. It is a lot easier to be told what to do than it is to figure out what to do. Therefore, students resist discovery – it is too much work” (Brumbaugh & Rock, 2001, p. 196). While incorporating discovery learning I will allow students to struggle in order to gain benefits from inquiry and discovery lessons and be able to gain skills that will help them apply math in a real world setting. Discovery learning needs to be reinforced by application. Huetinck and Munshin say that by requesting “students to further exercise their generalization, it is more likely that they will transfer knowledge and retain concepts in addition to providing you with feedback as to the level of student understanding” (2000, p. 236). Throughout the units I have planned, students have many activities that incorporate guided discovery learning. These activities are followed with reinforcement of generalizations the class reached during an activity. This provides greater opportunity for students to transfer and retain skills gained during the activity. The most open unit to discovery is the statistics unit where the students will decide what data to collect and how to collect it, followed by instruction of various statistics that can be
During the units, students will have an opportunity to complete assignments individually or through cooperative learning. According to Cangelosi, cooperative learning occurs in mathematics when groups of students explore mathematical ideas, form conjectures, discuss results, and compare mathematical strategies together. For cooperative learning activities to be productive and effective, they should include: clearly defined tasks with a specific objective in mind, an advanced organizer or task sheet, monitoring groups’ activities, providing guidance as needed without usurping the groups’ responsibilities for the designated task, modeling active listening techniques, providing formative feedback to students, a plan for closure points, and follow up the concepts learned in subsequent learning activities.

Huetinck and Munshin mention several benefits of cooperative learning:

- Concepts and problem-solving approaches are clarified and extended by students needing to communicate them to one another;
- Confidence is built through oral presentations and written discussions of procedures and processes involved in doing the mathematics;
- Critical thinking is developed by seeing and analyzing a variety of approaches to a given problem situation;
- Appreciation of and respect for other ways of thinking and learning is enhanced;
- Positive attitudes about mathematics and the mathematics classroom are fostered.

(2000, p. 15)

When having students work cooperatively, it is important to take into consideration how the students will be grouped, both the size of the group and whether students will be grouped with similar or differing levels of ability.
Varying grouping configurations – by moving from whole to small group, whole group to partners, small group to individual assignments – provides students with opportunities to learn new information, discuss it, and process it. Organizing students into smaller groups for instructional purposes provides a context that whole-group, teacher-dominated instruction doesn’t offer. Students can use groups to critique or analyze material, create graphic representations of vocabulary terms or concepts, or summarize material. All these activities are more meaningful and increase learning (Echevarria, Vogt, & Short, 2008, pp. 122-124).

When using cooperative learning it is also important to consider what students to put together. There are times that a heterogeneous grouping will be more beneficial (students with various ability levels) and times when a homogeneous grouping is preferable. There will be times when I want students with similar ability grouped together in order to provide more or less support to the whole group, and other times when I will have students grouped together with various abilities to provide opportunity for them to learn from and instruct each other. A variety of grouping helps to maintain students’ interest and helps meet the various learning preferences that students have. Some learn best working with a partner, others with interacting in a larger group setting, and still others working individually. By varying how students interact there is greater opportunity for all students to be in an environment that fits their learning preferences. When students are involved in cooperative learning activities, I will monitor the class to ensure each student is participating and that the work is not just being done by one or two students within the group. Initially I will need to monitor cooperative learning activities more as students learn how to effectively interact with each other and contribute to the group.

All students benefit from instruction that frequently includes a variety of grouping
configurations. Whole-class groups are beneficial for introducing new information and concepts, modeling processes, and review. Flexible small groups promote the development of multiple perspectives and encourage collaboration. Partnering encourages success because it provides practice opportunities, scaffolding, and assistance for classmates (Echevarria, Vogt, & Short, 2008, p. 126).

Through cooperative learning activities, students of different academic levels interact, support, and learn from each other, increasing everyone’s understanding. According to Artz & Newman, research has found that cooperative learning can facilitate greater student participation, promote positive attitudes about learning mathematics, and thereby increase student achievement in mathematics.

With all of the above strategies, it is also important to have mathematical concepts be meaningful to the students.

Mathematics is learned most effectively when it is placed in a personal context . . . When students are able to link theory with reality, Lesh explains, they are better able to understand that math is not just “a bunch of rules that people use for doing calculations with numbers. Mathematics is really about models for thinking about the world – models that deal with quantities, shapes, relationships, and other mathematical things.” At its best, mathematics education helps people understand these models so they can apply them in different situations (Carlson, 1992, p. 6).

Several of the activities within the five units have students gather data through hands-on activities and then apply mathematics to be able to model that data graphically and with a mathematical function. This not only allows students to learn the mathematical objectives but to be able to see mathematics in a meaningful, applicable way.
I will now go through each of the five units created for this project, giving the Utah state core objectives for Algebra II that each unit covers, a brief daily outline, worksheets and activities that students will complete along with an explanation of each. The Utah state core objectives for Algebra II can be found online at http://www.schoo.ls.utah.gov/curr/math/sec/PDF/Core/Secondary_Mathematics_Core_2007_Alg2.pdf. Many of the lessons include textbook problems to help reinforce concepts students have constructed through hands-on activities and to provide various ways problems can be presented. These assignments come from Prentice Hall Mathematics: Algebra 2 (2007). When assignments come from the textbook or practice worksheets, they will be indicated in the form 2-7 meaning the assignment comes from chapter 2 section 7. Also during the teaching sequence students will have opportunities to practice new skills through the use of individual whiteboards and dry erase markers. As I have taught I have found this to be an effective way of evaluating how well students understand concepts. This provides instant feedback to help me know if I need to modify lessons or if the students appear to understand concepts well enough to move on. I have also found that when students work problems on individual whiteboards they are more engaged than when assigned traditional pencil-and-paper work and learn more effectively as they are able to obtain instant feedback.

**ABSOLUTE VALUE UNIT:** The state standards and objectives that the activities for this unit cover:

**Standard I:** Students will use the language and operations of algebra to evaluate, analyze and solve problems.

**Objective 1:** Evaluate, analyze and solve mathematical situations using algebraic properties and symbols.

a. Solve and graph first-degree absolute value equations of a single variable.
c. Solve absolute value and compound inequalities of a single variable.

**Objective 2:** Solve systems of equations and inequalities.

a. Solve systems of... absolute value... equations algebraically and graphically.

b. Graph the solutions of... absolute value... inequalities on the coordinate plane.

**Standard III:** Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

**Objective 1:** Examine the behavior of functions using coordinate geometry.

a. Identify the domain and range of the absolute value... function

b. Graph the absolute value... functions.

c. Graph functions using transformations of parent functions.

**Scope and Sequence of Unit:**

Day 1: Define absolute value, introduce students to the concept of absolute value, have students write absolute value equations, and begin solving basic absolute value equations in one variable. Connect to students and build background knowledge through the use of Google Earth and finding the distance from the high school to other places nearby familiar to the students.

Day 2: Students solve multi-step absolute value equations (work on whiteboards, 1-5).

Day 3: Students are introduced to absolute value inequalities. Students complete a hands-on activity to learn about tolerance involving measuring various items’ weight, circumference, quantity or volume. From this data students write compound inequalities, graph these inequalities, and rewrite the compound inequality as an absolute value inequality.
Day 4: Students solve and graph multi-step absolute value equations and inequalities and determine when an absolute value equation has extraneous solutions (Practice 1-5).

Day 5: Students are introduced to graphing absolute value functions (in two variables) on the coordinate plane using Geometer’s Sketchpad to manipulate the standard form of an absolute value function to discover what effect each value has on the graph.

Day 6: Class discussion of results from previous day. Students summarize how to graph and absolute value equation and how to translate that function. Students graph absolute value function in two variables (pencil-and-paper, reinforcing the concepts learned during day 5’s activity).

Day 7: Students write their own problem involving absolute values (1 and 2 variable), solve equations and graph functions (2-5).

Day 8: Students graph absolute value inequalities in two variables on the coordinate plane (2-7).

Day 9: Students review with an activity to prepare for test.

Day 10: Test given with a variety of questions: Some multiple choice (help prepare for state tests generally given in this format), free response questions, essays (explain how to ... ) and problems using Geometer’s Sketchpad.

The “Introduction to Solving Absolute Value Equations” is set up to provide scaffolding to students as it begins very simple and gradually gets more complex. When I have taught absolute value before, and even when tutoring college students on absolute value, I have found that a common misconception is if one answer to an absolute value equation is 3, the other answer is automatically -3. This worksheet really focuses on the definition of absolute value as distance and a graphical representation to help students better visualize what absolute value is. Students begin by finding distances between values and finding what value is a specific distance
from a given value before being introduced to the term “absolute value”. The lesson will be introduced by showing an image from Google Earth with the school located at the center. The class will discuss and find distances to various known locations. I will then propose the question, “If I know Mark lives 3 miles away from the school, can I find where he lives?” The class will discuss this and then I will tie in a number line, overlapping the image and change the question to be, “If I know Mark lives directly east or west of the school, where does he live?” There will be more discussion, followed by modeling the first problem from the worksheet.

Students will be assigned to work individually, then discuss and compare answers with a partner, and finally discuss their results with the class (focusing on problem 9 where students described how they found specific values). After the class discussion on specific answers, students will then be introduced to the term absolute value (In the classroom there will be a word wall. Whenever a new word is introduced a student will write the word to be displayed on the word wall and all students will write the definition in a personal dictionary). Students will be instructed to write each of the 15 problems as an absolute value problem (several examples will be given and modeled for students before they are assigned this).
Introduction to Solving Absolute Value Equations

On the number line provided in each problem, graph the numbers given and find the distance between the two numbers.

1. 5 and 2: Distance: _____ units

2. -1 and 1: Distance: _____ units

3. -2 and -2: Distance: _____ units

4. 4 and 0: Distance: _____ units

5. -4 and 0: Distance: _____ units

6. 7 and 24: Distance: _____ units

7. 8 and -3: Distance: _____ units

8. -6 and -9: Distance: _____ units

9. Describe how you found the distance between the two values given in the previous 8 problems.
Find the value(s) for $x$ that make the following true. Graph the given value and the distance indicated.

10. a) $x$ is 2 units away from 7

Value(s) for $x$: ____________

b) How many values make this true? Explain.

11. The distance between $x$ and 4 is 3 units.

Value(s) for $x$: ____________

12. The distance between 56 and $x$ is 11 units.

Value(s) for $x$: ____________

13. The distance between $x$ and 8 is 9 units.

Value(s) for $x$: ____________

14. The distance between $x$ and -4 is 0 units.

Value(s) for $x$: ____________

15. The distance between $2x$ and 5 is 7 units.

Value(s) for $x$: ____________ (hint: start by finding $2x$ then solve for $x$ from there)
On the third day of the unit, there will be a hands-on activity. Students will be assigned to groups (3-4 students per group). Each group will have different items that they will be finding measurements for. The size and number of groups will be used based on the size of the class. The items will depend on what I can get donated or what students can bring in:

- Weigh several cans of soup (also bags of potato chips, cereal, cotton, etc).
- Count the number of items in a bag (skittles, cotton, q-tips, popcorn etc.).
- Measure the volume of water in each water bottle.
- Measure the circumference of each basketball.

Each group will have at least 10 similar items to measure. They will record the results for each item’s measurement, create a compound inequality to represent the range of values they recorded, and compare this to the manufacturer’s given value measuring the same characteristic. Each groups’ worksheet will have slightly different instructions/problems based on what they are measuring. This can easily be changed based on items being used for this activity.

Each group will share their results with the class and we will discuss similarities and differences and talk about manufacturing practices for the various characteristics measured (the definition of tolerance is introduced). Each group writes down the inequalities from all the groups with a brief explanation of what it represents, then graph each inequality and write the inequality as an absolute value inequality. Results are shared and discussed with the class and compared to the manufacturer’s given data. The class will have a discussion on what the absolute value equation would look like if it were based on the students tolerance found and the given information from the manufacturer. The next two pages are the worksheets to be given to the students, though worksheets will vary based on the items available.
Tolerance and Absolute Value Activity

With your group, look at each can of soup. How are they similar?

How are they different?

How much does each can of soup weigh according to the manufacturer (in ounces and grams)?

Record the weight each can of soup in grams (to the nearest hundredth) in the table below.

<table>
<thead>
<tr>
<th>ITEM</th>
<th>WEIGHT</th>
<th>ITEM</th>
<th>WEIGHT</th>
<th>ITEM</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>11</td>
<td></td>
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<tr>
<td>2</td>
<td>7</td>
<td>7</td>
<td>12</td>
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<td>3</td>
<td>8</td>
<td>8</td>
<td>13</td>
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<td>5</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Why do you think these values vary from what the manufacturer states each can weighs?

Write a compound inequality to express the range of values for the weight of the cans of soup.
For each of the equations given by the other groups, write the compound inequality below, graph the inequality on the provided number lines and write an absolute value equation to represent each one. Label each graph according to the specific item and units of measure. The tick marks on each number line should be based on what the degree of accuracy is (example, the cans of soup were measured in grams to the nearest hundredth, so each tick mark should represent a change of .01).

<table>
<thead>
<tr>
<th>Compound Inequality</th>
<th>Graph</th>
<th>Absolute Value Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

Explain how you found the absolute value inequality for each of the graphs/equations above.

Based on the class discussion in your own words write a definition of tolerance.
On day four of the unit, the class will discuss the previous day’s activities and review concepts learned. Students will practice problems on their whiteboards so I can check for understanding, beginning with simple problems and then getting more complex. Students are also taught about extraneous solutions. Students will then be assigned to complete the Practice 1-5 worksheet.

Students learn about graphing absolute value equations with two variables on day five of the unit. The worksheet is set up to be a guided discovery activity. I will begin by reviewing how to graph linear equations and have students practice examples to help them recall/review this. Then students will complete the “Introduction to Graphing Absolute Value Equations” worksheet. This worksheet begins by having students graph a linear equation then graph what they think the graph of the absolute value of that same equation would look like. They then graph the absolute value equation on Geometer’s Sketchpad and compare with their graph. The rest of the worksheet leads students through an activity that allows them to discover how each term in the equation \( y = m|x - h| + k \) affects the graph of the function and how to graph and absolute value function or how to write and equation based on the graph. The activity sketch used in Geometer’s Sketchpad for this activity, VGRAPH.gsp, comes from “Exploring Algebra 2 with The Geometer’s Sketchpad”. The questions are based on an activity found on pages 153 and 154 of this book.

On the sixth day students will review what they learned from this activity and discuss how each value in the equation \( y = m|x - h| + k \) affects the graph. Students will complete problems on their whiteboards so I can check for understanding and then students will complete the worksheet “Graphing Absolute Value Functions” (without use of technology) to practice and reinforce the concepts learned during the fifth day’s activity.
Introduction to Graphing Absolute Value Equations

1. Sketch the graph of \( y = x \)

2. Think about what \( y = |x| \) means, then sketch what you think the graph might look like.

3. Now use your calculator or geometer’s sketchpad to graph \( y = |x| \) and \( y = |2x - 4| \)
4. Describe how the graphs of \( y = 2x - 4 \) and \( y = |2x - 4| \) compare. Discuss their shapes, ranges, and other features you notice.

5. In Geometer’s Sketchpad, open VGRAPH.gsp. You should see the graph of an equation in the form 
\[ y = m|x - h| + k \] 
and sliders \( m, h, \) and \( k. \)

\[ y = 3.00|x - (2.00)| + (-1.60) \]

\( m = 3.00 \)
\( h = 2.00 \)
\( k = -1.60 \)

a) Adjust the slider \( m. \) Describe the effect \( m \) has on the graph when \( m \) is a large positive number? a small positive number? zero? a negative value?

6. Changing \( m \) moves the entire \( V \)-graph except for one point. This point is called the **vertex**. Adjust the sliders for \( h \) and \( k. \) How does the location of the vertex relate to the values \( h \) and \( k? \)
7. Write an equation in the form $y = m|x-h| + k$ for each of the following. Make sure your answer is correct by adjusting the sliders.
   a. Vertex at (-1, 2) and contains the point (0, 4)
   b. Vertex at (2, 3) and contains the point (4, 0)
   c. Vertex at (-3, -1) and has the same shape as $y = 3|x-2| + 5$
   d. X-intercepts at (6, 0) and (-6, 0) and contains the point (1, -2)
   e. X-intercepts at (3, 0) and (-5, 0) and contains the point (2, 1)
   f. Having the same vertex as $y = 3|x-2| + 5$ and containing the point (0, 0)

8. Do all graphs of the form $y = m|x-h| + k$ have x-intercepts? Explain.

9. Write a paragraph summarizing how each value m, h, and k effect the graph of $y = m|x-h| + k$. 
Graphing Absolute Value Functions

Utilize the findings from the previous exploration to complete the following exercises regarding the general form of the absolute value function. $f(x) = m\,|x - h| + k$.

1. $f(x) = 2|x + 3| - 2$
   
   $m =$
   
   $h =$
   
   $k =$

   Direction of Opening
   
   Coordinates of the Vertex
   
   Slope of Graph on Left Side
   
   Slope of Graph on the Right Side

2. $f(x) = \frac{-2}{3}|x - 1| + 2$
   
   $m =$
   
   $h =$
   
   $k =$

   Direction of Opening
   
   Coordinates of the Vertex
   
   Slope of Graph on Left Side
   
   Slope of Graph on the Right Side
3. \[ f(x) = -|x - 3| \]

\[ m = \]
\[ h = \]
\[ k = \]
Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side

4. \[ f(x) = \frac{1}{2}|x| - 2 \]

\[ m = \]
\[ h = \]
\[ k = \]
Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side
5. \[ f(x) = \frac{1}{4} |x - 1| - 3 \]

\( m = \)

\( h = \)

\( k = \)

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side

6. \[ f(x) = 3|x + 2| - 5 \]

\( m = \)

\( h = \)

\( k = \)

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side
7. \( f(x) = |x + 3| - 2 \)

\[ m = \]

\[ h = \]

\[ k = \]

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side

8. \( f(x) = \frac{4}{3} |x + 2| - 4 \)

\[ m = \]

\[ h = \]

\[ k = \]

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side
9. \( f(x) = -\frac{1}{2}|x-1| + 5 \)

\( m = \)
\( h = \)
\( k = \)

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side

10. \( f(x) = \frac{1}{3}|x-3| - 6 \)

\( m = \)
\( h = \)
\( k = \)

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side
11. \( f(x) = -2|x-3|-4 \)

\[ m = \]
\[ h = \]
\[ k = \]

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side

12. \( f(x) = -4|x-2|+8 \)

\[ m = \]
\[ h = \]
\[ k = \]

Direction of Opening
Coordinates of the Vertex
Slope of Graph on Left Side
Slope of Graph on the Right Side
EXPONENTIAL GROWTH/DECAY AND LOGARITHMS UNIT: The state standards and objectives that the activities for this unit cover:

**Standard II:** Students will understand and represent functions and analyze function behavior.

**Objective 1:** Represent mathematical situations using relations using algebraic properties and symbols

a. Model real-world relationships with functions.

b. Describe a pattern using function notation.

**Objective 3:** Define and graph exponential functions and use them to model problems in mathematical and real-world contexts.

a. Define exponential functions as functions of the form \( y = ab^x, \ b > 0, \ b \neq 1 \).

b. Model problems of growth and decay using exponential functions.

c. Graph exponential functions.

**Objective 4:** Define and graph logarithmic functions and use them to solve problems in mathematics and real-world contexts.

a. Relate logarithmic and exponential functions.

b. Simplify logarithmic expressions.

c. Convert logarithms between bases.

d. Solve exponential and logarithmic equations.

e. Graph logarithmic functions.

f. Solve problems involving growth and decay.

**Scope and Sequence of Unit:**

**Day 1:** Hands-on activity to explore and model the concept and graph of exponential decay.

The students are divided into 8 groups (3-4 people per group depending on class size), 4 groups will begin with 500 pennies (or bingo chips marked on one side) and the other 4
groups will start the activity with 50 dice. See the worksheets for the specific instructions on the following pages. Once one group has finished with pennies they will switch with a group that has finished with the dice and vice versa. Students fill in charts with data from their experiment, graph the data, and compare different rates of decay based on their experiment. The class will have a discussion having students comment on what they learned and what real-world phenomena they think follows a similar model.

**Day 2:** Students learn how to find an exponential regression equation to model data from day 1 activity in the form \( y = ab^x \) using graphing calculators. As a class we will discuss how \( a \) and \( b \) relate to the experiment parameters from the previous.

**Day 3:** Hands-on activity to explore and model the concept and graph of exponential growth (set up similar to day 1).

**Day 4:** Students solve and graph exponential equations including growth and decay problems (Prentice Hall 8-1).

**Day 5:** Students introduced to \( e \) and solve exponential problems (8-2). Quiz.

**Day 6:** Students are introduced to logarithms by comparisons to exponents of base 10 (also review of properties of exponents). Students solve for unknown exponents (base 10). The initial problems are simple problems that students can solve based on their understanding of exponents learned previously (to be reviewed at the beginning of class). As the problems get more complex students solve through trial and error for \( x \) in the equations \( 10^x = 50 \) and \( 10^x = 150 \). After the worksheet is completed the class will discuss difficulties in solving for \( x \) and logarithms will be introduced as a way to solve for unknown exponents (emphasizing that logarithmic functions are the inverse of exponential functions).
Day 7: Students begin project to find real world contexts that involve exponential growth/decay or logarithms (in the library students look up real world contexts involving applications of exponents or logarithms).

Day 8: Students convert equations between exponential form and logarithmic form and evaluate basic logarithms (8-3).

Day 9: Students complete an activity on their calculator to learn the properties of logarithms. This activity is found online at Tlalgebra.com and is specifically for use with TI-83 or TI-84 calculators. This activity will be done as a class. Students will be given time between problems to figure them out on their own or with a partner before discussing with the whole class.

Day 10: Students complete problems to reinforce the properties of logarithms they learned during the activity on day 9 (8-4).

Day 11: Students complete an activity on their calculator to learn the change of base equation for logarithms. This activity is found online at Tlalgebra.com and is specifically for use with TI-83 or TI-84 calculators. This activity will be done as a class. Students will be given time between problems to figure them out on their own or with a partner before discussing with the whole class.

Day 12: Students solve logarithmic equations (8-5).

Day 13: Students solve and graph logarithmic and exponential equations/functions (8-5).

Day 14: Students solve and graph natural logarithmic equations (8-6).

Day 15: Review worksheet, time to finish project.

Day 16: Review/finish project.

Day 17: Assessment.
Exponential Decay Activity

Materials needed: 500 pennies or bingo chips marked on one side.

1. Before beginning, read through the instructions for problem 2 to estimate how many turns you think it will take to have the following number of pennies remaining (explain your reasoning for each):
   a. 100
   b. 10
   c. None

2. Place the 500 pennies in the pizza box, close the lid then shake the box. After each shake remove the pennies with “heads” showing. Record the number of pennies remaining and the number of pennies with a heads showing in the table below. Repeat until all pennies have been removed. If more rows are needed than 15, write them to the side.

3. Graph the results of your experiment on the graph below to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># of heads</th>
<th>Total coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>500</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td></td>
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<tr>
<td>15</td>
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</tbody>
</table>

4. Is this graph linear? Explain your reasoning on why it is linear or why it is not.
5. Find a group who used dice instead of pennies that is finished with their activity and exchange your pennies for their dice to complete the rest of the worksheet.

6. Read through the instructions in problem 7 to estimate how many turns you think it will take to have the following number of dice remaining (explain your reasoning):
   a. 20
   b. 3
   c. None

7. Place all the dice in the plastic box, put on the lid. Shake the box and remove the lid. Any dice landing with a 1 face up, remove. Record the results in the table below of how many dice landed with a 1 face up and how many dice were left after each turn. Repeat until all dice are removed.

8. Graph the results of your experiment below on the graph to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># 1s</th>
<th>Total dice</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>50</td>
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<tr>
<td>1</td>
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<td>15</td>
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</tbody>
</table>

9. Is this graph linear? Explain your reasoning on why it is linear or why it is not.

10. Compare the two graphs created in problems 3 and 8.
   a. Explain how the two graphs are similar?

   b. Explain how the two graphs are different?
Exponential Decay Activity

Materials needed: 50 dice.

1. Read through the instructions for problem 2 to estimate how many turns you think it will take to have the following number of dice remaining (explain your reasoning for each):
   a. 20
   b. 3
   c. None

2. Place all the dice in the plastic box, put on the lid. Shake the box then remove the lid. Any dice landing with a 1 face up, remove. Record the results in the table below of how many dice landed with a 1 face up and how many dice were left after each turn. Repeat until all dice are removed.

3. Graph the results of your experiment below on the graph to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># 1s</th>
<th>Total dice</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>50</td>
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<tr>
<td>1</td>
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<td>?</td>
</tr>
</tbody>
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4. Is this graph linear? Explain your reasoning on why it is linear or why it is not.

5. Find a group who used pennies instead of dice that is finished with their activity and exchange your dice for their pennies to complete the rest of the worksheet.
6. Read through the instructions for problem 7 to estimate how many turns you think it will take to have the following number of pennies remaining (explain your reasoning for each):
   a. 100
   b. 10
   c. None

7. Place the 500 pennies in the pizza box, close the lid then shake the box. After each shake remove the pennies with “heads” showing. Record the number of pennies remaining and the number of pennies with a heads showing in the table below. Repeat until all pennies have been removed. If more rows are needed than 15, write them to the side.

8. Graph the results of your experiment on the graph below to the right.

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<tr>
<th>Turn #</th>
<th># of heads</th>
<th>Total coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>500</td>
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<td>1</td>
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<td>15</td>
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</tbody>
</table>

9. Is this graph linear? Explain your reasoning on why it is linear or why it is not.

10. Compare the two graphs created in problems 3 and 8.
   a. Explain how the two graphs are similar?
   b. Explain how the two graphs are different?
Finding Exponential Equations for Data

Follow along with Ms. Watterson to complete the following:

Before beginning, clear any functions from the [GRAPH] menu.

To complete this, you need your worksheets with the information from
the exponential growth and decay activity.

Open the List Editor by pressing [STAT], Edit. Enter the number of
trial, 0, in L1 and enter the number of pennies you started with (500) in
L2. Fill in the rest of L1 (going until the second to last trial), fill in L2
corresponding to how many pennies you had after each turn.

* Do not record the last trial with no pennies remaining.

Make a scatter plot of the data. Open Plot1 by pressing 2ndY=. Turn
the plot on, choose scatter as the type, L1 for the XList and L2 for the YList.
Adjust the window settings by pressing WINDOW. The x-axis should run
from 0 to an even number greater than the number of trials. The y-axis
should run from 0 to an even number greater than the number of pennies
that you started with. Then press [GRAPH] to view the scatter plot.

Now find an exponential model for the data. Press [STAT] and move
the cursor to the CALC menu. Press the up or down arrow to
select ExpReg and press [ENTER].

Press 2nd[L1] □ 2nd[L2] □. Then press [VARS] and move the
cursor to the Y-VARS menu. Select 1:Function by pressing
[ENTER].

Select 1:Y1 by pressing [ENTER].

Press [ENTER] to calculate the exponential regression. You should
get an equation in the form \( y = ab^x \).

Record this equation (substituting the values for \( a \) and \( b \)) on your paper
where indicated. Repeat the process for the remaining experiment done
in class.
Finding Exponential Equations for Data

Find the equation that best fits your data from each of the experiments done in class (you should have all the data entered in on your worksheets we did in class yesterday). Follow the directions on the first page (Ms. Watterson will walk you through the first example).

EQUATIONS: Write equations in the form $y = ab^x$ (round $a$ and $b$ to the nearest hundredth).

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pennies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dice</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) As a class, combine the results for the penny decay experiment.

<table>
<thead>
<tr>
<th>Group</th>
<th>$a$ value (rounded to nearest hundredth)</th>
<th>$b$ value (rounded to nearest hundredth)</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>8</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CLASS AVERAGE:</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

2) These results are based on the class’ experiment. How do you think the values of $a$ and $b$ relate to the original problem?

3) Explain what the theoretical values of $a$ and $b$ should be based upon the problem given.

4) What do the values $x$ and $y$ represent in the function $y = ab^x$?
5) Repeat the process for the dice problem: As a class, combine the results for the dice decay experiment.

<table>
<thead>
<tr>
<th>Group</th>
<th>$a$ value (rounded to nearest hundredth)</th>
<th>$b$ value (rounded to nearest hundredth)</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>CLASS AVERAGE:</td>
<td></td>
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</tbody>
</table>

6) These results are based on the class’ experiment. How do you think the values of $a$ and $b$ relate to the original problem?

7) Explain what the theoretical values of $a$ and $b$ should be based upon the problem given.

8) What do the values $x$ and $y$ represent in the function $y = ab^x$?

9) Compare the two functions for the penny decay and the dice decay. Explain how they are similar and how they are different.

10) What do you think the value of $b$ would be for an exponential growth function? Explain your response.
Exponential Growth Activity

Materials needed: 50 dice.

1. Read through the instructions for problem 2 to estimate how many dice you think there will be in the box after: (explain your reasoning for each):
   a. 3
   b. 10

2. Place 10 of the dice in the plastic box then put on the lid. Shake the box then remove the lid. Count the number of dice landing with a 6 face up. For each dice landing with a 6 face up, add another die (example, if I shake the 10 dice and 2 of them land with a 6 face up I add 2 dice to the others and now have a total of 12 dice). Record the results in the table below of how many dice landed with a 1 face up and how many dice there are after each turn. Repeat for a total of 10 times.

3. Graph the results of your experiment below on the graph to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># of 6's showing</th>
<th>Total dice</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td></td>
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<td>2</td>
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<td></td>
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<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. How does this graph compare to that when dice were removed?

5. Find a group who used pennies instead of dice that is finished with their activity and exchange your dice for their pennies to complete the rest of the worksheet.
6. Read through the instructions for problem 7 to estimate how many pennies you will have after the given number of turns (explain your reasoning for each):
   a. 3
   b. 10

7. Place 10 pennies in the pizza box. Close the lid then shake the box. After each shake add a penny for each penny with a "tails" showing. Record the number of pennies at the end of each turn along with the number of tails. Repeat until 10 turns are completed.

8. Graph the results of your experiment on the graph below to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># of tails showing</th>
<th>Total coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
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<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. Compare this graph to the one in problem 3. How are they similar? How are they different?

10. Using a calculator, come up with an equation that best fits the data from your graph above (refer to handout from yesterday if needed).

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pennies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dice</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. Theoretically, what should the value of a and b be for each of the equations? Explain.
Exponential Growth Activity

Materials needed: 500 pennies.

1. Read through the instructions for problem 2 to estimate how many pennies you will have after the given number of turns (explain your reasoning for each):
   a. 3
   b. 10

2. Place 10 pennies in the pizza box. Close the lid then shake the box. After each shake add a penny for each penny with a "tails" showing. Record the number of pennies at the end of each turn along with the number of tails. Repeat until 10 turns are completed.

3. Graph the results of your experiment on the graph below to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># of tails showing</th>
<th>Total pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
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<tr>
<td>7</td>
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<td>8</td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. How does this graph compare to that when pennies were removed?

5. Find a group who used dice instead of pennies that is finished with their activity and exchange your pennies for their dice to complete the rest of the worksheet.
6. Read through the instructions for problem 7 to estimate how many dice you think there will be in the box after the following number of turns: (explain your reasoning for each):
   a. 3
   b. 10

7. Place 10 of the dice in the plastic box then put on the lid. Shake the box then remove the lid. Count the number of dice landing with a 6 face up. For each dice landing with a 6 face up, add another die (example, if I shake the 10 dice and 2 of them land with a 6 face up I add 2 dice to the others and now have a total of 12 dice). Record the results in the table below of how many dice landed with a 6 face up and how many dice there are after each turn. Repeat for a total of 10 times.

8. Graph the results of your experiment below on the graph to the right.

<table>
<thead>
<tr>
<th>Turn #</th>
<th># of 6's showing</th>
<th>Total dice</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>----</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
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<td></td>
</tr>
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<td></td>
</tr>
</tbody>
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9. Compare this graph to the one in problem 3. How are they similar? How are they different?

10. Using a calculator, come up with an equation that best fits the data from your graph above (refer to handout from yesterday if needed).

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
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<td></td>
</tr>
<tr>
<td>Dice</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. Theoretically, what should the value of $a$ and $b$ be for each of the equations? Explain.
SOLVING FOR AN EXPONENT

**Directions:** Solve for $x$ in each of the problems. If the answer is not a whole number, round the answer to 6 decimal places. Check your answer (show this step).

<table>
<thead>
<tr>
<th>EQUATION (solve for $x$)</th>
<th>$x$</th>
<th>Check your work</th>
<th>Explain how you got this answer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $10^x = 100$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) $10^x = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) $10^x = \frac{1}{10}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) $10^x = 10,000$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) $10^x = \frac{1}{100}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) $10^x = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) $10^x = 100$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8) $10^x = 50$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9) $10^x = 150$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10) Which of the above problems were the easiest? Explain.

11) Which were the most difficult? Explain.

12) For numbers 8 and 9, you may need to find the value by trial and error, for example if I were doing problem number 8, I don’t know a value for $x$ that will solve for $10^x = 50$, but I know that 50 is between 10 ($10^1$) and 100 ($10^2$). From this I know that the value of $x$ must be between 1 and 2. Find a value for $x$ accurate to 6 decimal places (one option is by trial and error) Show your work for problems 8 and 9 on the following pages.
<table>
<thead>
<tr>
<th>$x$-value</th>
<th>Value of $10^x$ (rounded 6 decimal places)</th>
<th>Reason for trying this value for $x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>$10^{1.5} = 31.622777$</td>
<td>I knew $x$ had to be between 1 and 2 since 50 is between 10 and 100, so I chose 1.5 since it is in the middle of 1 and 2.</td>
</tr>
<tr>
<td>1.6</td>
<td>$10^{1.6} = 39.810717$</td>
<td>In the last trial, 1.5 was not large enough to give 50 so I tried 1.6 since it is a larger value.</td>
</tr>
</tbody>
</table>
### Trial and Error to solve for $x$ in $10^x = 150$

<table>
<thead>
<tr>
<th>$x$-value</th>
<th>Value of $10^x$ (rounded 6 decimal places)</th>
<th>Reason for trying this value for $x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Recall some of the laws of exponents:

\[ a^m \cdot a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \]

\[ a^0 = 1 \quad (a^m)^n = a^{mn} \]

Use these laws to solve for \( x \) in the following problems:

13) \( 10^x = \frac{50}{150} \)

14) \( 10^x = \frac{150}{150} \)

15) \( 10^x = \frac{150}{50} \)

16) \( 10^x = (50)(150) \)

17) \( 10^x = (50)^2 \)
Properties of Logarithms

ID: 9606

Time required
45 minutes

Topic: Exponential & Logarithmic Functions & Equations

• Derive algebraically the rules for products, quotients, and powers of logarithmic expressions.
• Verify graphically the rules for products, quotients, and powers of logarithmic expressions.

Activity Overview

In this activity, students use a combination of algebra and graphing to discover the properties of logarithms. Given an equation in a and b such as $b = a^2$ and a table of values for a, students define $x = \log a$ and $y = \log b$ and graph the resulting sets on the coordinate plane, graphically "taking the log of both sides." Finding the equation of this graph and working backwards reveals an equation that expresses one of the properties of logarithms. Seeing a power function transformed into a linear function visually reinforces the effect of the log. Problems 1 and 2 cover the Power and Product Properties, respectively; Problem 3 covers the Quotient Property in addition to introducing log-log "paper", a grid with a logarithmic scale on each axis.

Teacher Preparation

• This activity is appropriate for students in Algebra 2 or as a review for Pre-calculus. Prior to beginning this activity, students should be familiar with the definition of a logarithm and have experience with simple exponential equations of the form $a^x = b$, as well as the exponent rules.
• Before beginning the activity, students should download the Cabri Jr file LOGLOG to their graphing calculators.
• To download the Cabri Jr. file LOGLOG, go to eduction.ti.com/exchange and enter “9606” in the quick search box.

Classroom Management

• This activity is designed to be teacher-led with periods of independent work. The worksheet helps guide students through the activity.

TI-84 Plus Applications

Cabri Jr
Properties of Logarithms  ID: 9606

In this activity, you will explore:

- Creating a table of values and graphing log functions
- The properties of logarithms
- Transforming equations via substitution

Logarithms are just another way of writing exponents. Just like exponents, logarithms have properties that allow you to simplify expressions and solve equations. In this activity, you will discover some of these properties with graphing and with algebra.

Problem 1 – The Power Property of Logarithms

Suppose you wanted to simplify the logarithm of a power, like \( \log a^2 \). How might you do it?

Let’s start by defining a new variable \( b = a^2 \).

We can make a table of values for \( a \) and \( b \) using the calculator’s List feature. Open the lists by pressing STAT then ENTER.

If necessary, clear any data from the first list by using the arrow keys to move to the top of L1 and pressing CLEAR – ENTER.

Enter at least 10 values for \( a \) in L1. We will be taking the logarithm of these values later, so choose only values that are in the domain of the logarithm function.

Move to the top of L2. Enter a formula that will calculate \( b = a^2 \) from the values of \( a \) you entered in L1. To square the values in L1, press 2nd + 1 + x^2.

Make a scatter plot of these values. Press 2nd + Y= to open the StatPlot screen and adjust the settings to display the \( a \) values in L1 along the x-axis and the \( b \) values in L2 along the y-axis.
Go to **Zoom > ZoomStat** to view the plot in an appropriate window.

- What is the shape of the graph? Is this what you expected?

Now we will define two new variables, $x$ and $y$. Let

$x = \log a$ and $y = \log b$. Enter a formula at the top of $L3$ that calculates $x$ from the values of $a$ in $L1$. Then enter a formula at the top of $L4$ that calculates $y$ from the values of $b$ in $L2$.

Make a scatter plot of $x$ vs. $y$. Go to **Zoom > ZoomStat** to view the plot in an appropriate window.

- What is the shape of the graph? Is this what you expected?

The graph appears to be linear. Find the equation of a line through these points with the **LinReg** command, found in the **Stat > Calc** menu. After the command, type the $x$ list, then a comma, then the $y$ list.

- What is the equation of the line through these points?

You should find that $y = 2x$. What does this mean? We can substitute to rewrite this as an equation in terms of $a$. The explanation for each step is given to the right.

$$y = 2x$$

$$(equation \ of \ the \ line)$$

$$\log b = 2 \log a$$

$$(x = \log a \ and \ y = \log b)$$

$$\log a^2 = 2 \log a$$

$$(b = a^2)$$

This last line is an example of the **Power Property of Logarithms**. In general, for $a > 0$,

$$\log a^b = b \log a.$$ 

The power property of logarithms can be used to rewrite logarithmic expressions.

$log x^3$ can be written as $3 \log x$

$8 \log x$ can be written as $\log x^8$
Problem 2 – The Product Property of Logarithms

Now suppose you wanted to simplify the logarithm of a product, like log 6a. How might you do it?

We can use the same steps you used in Problem one. Start by defining a new variable \( b = 6a \).

Clear all the data from \( L2-L4 \).

Now you have at least 10 values for \( a \) in \( L1 \). These values were chosen from the domain of the logarithm function.

Move to the top of \( L2 \). Enter a formula that will calculate \( b = 6a \) from the values of \( a \) you entered in \( L1 \).

Make a scatter plot of \( a \) vs. \( b \). Go to Zoom > ZoomStat to view the plot in an appropriate window.

- What is the shape of the graph? Is this what you expected?
- Imagine a line through this data. What would be the slope and \( y \)-intercept? (If you have trouble imagining the line, use the LinReg command to find the equation of a line through this data.)
- Are the slope and \( y \)-intercept what you expected?

Now define \( x = \log a \) and \( y = \log b \). Enter a formula at the top of \( L3 \) that calculates \( x \) from the values of \( a \) in \( L1 \). Then enter a formula at the top of \( L4 \) that calculates \( y \) from the values of \( b \) in \( L2 \).

Make a scatter plot of \( x \) vs. \( y \). Go to Zoom > ZoomStat to view the plot in an appropriate window.

The data appear linear. Find the equation of a line through these points with the LinReg command, found in the Stat > Calc menu. After the command, type the \( x \) list, then a comma, then the \( y \) list.

- What is the equation of the line through these points?
- What is the \( y \)-intercept of the line?
You should have found that the equation of the line was $y = x + 0.778151$. Where did this $y$-intercept come from? Here’s a hint. Try raising 10 to the 0.778151 power.

- What is 0.778151?

- Since $10^{0.778151} \approx \ldots$, $\log(6) \approx \ldots$.

You have found that $y = \log 6 + x$. What does this mean? Substitute to rewrite this as an equation in terms of $a$. The explanation for each step is given to the right.

\[
y = \log 6 + x
\]

(equation of the line)

\[
(x = \log a \text{ and } y = \log b)
\]

\[
(b = 6a)
\]

This last line is an example of the **Product Property of Logarithms**. In general, for $a > 0$, and $b > 0$

\[
\log ab = \log a + \log b.
\]

The Product Property of Logarithms can be used to rewrite logarithmic expressions.

$log xy$ is written in **expanded form** as $\log x + \log y$.

$log 7 + \log z$ is written as a single logarithm as $\log 7z$.

**Problem 3 – The Quotient Property of Logarithms**

In problems 1 and 2, you used substitution to identify two properties of logarithms. There is a special kind of graph paper called log-log paper that lets you make these substitutions graphically, by hand. In this problem, you will use log-log “paper” to simplify the expression $\frac{8}{a}$.

To see an example of log-log paper, launch the CabriJr app and open the file LOGLOG.

Examine the grid. Log-log paper is made by taking the log of each value of $x$ and $y$ and then renumbering the axes.

- Why does the log scale start at 1 instead of 0?

Note that the spaces between the lines are not equal. That is because the distance from 1 to 2 on a log scale is equal to $\log 2$ units, and the distance from 1 to 3 is equal to $\log 3$ units.

To get a better idea of how this works, find the values to:

\[
\begin{align*}
\log 1 &= \\
\log 2 &= \\
\log 3 &= \\
\log 4 &= \\
\log 5 &= \\
\log 6 &= \\
\log 7 &= \\
\log 8 &= \\
\end{align*}
\]
Define \( b = \frac{8}{a} \). Use the Point tool (found in the F2: Creation menu) to plot points from the function \( y = \frac{8}{x} \) on the log-log paper.

Graph only points with whole number coordinates.

- What is the shape of the graph?

Draw a line (F2: Creation > Line) through the points. The equation of this line with respect to the linear scale (not the log scale) is \( y = 0.90309 - x \).

Where did this \( y \)-intercept come from? Can you guess? Press \( 2^{nd} + \) MODE to exit CabriJr and make a calculation to confirm your guess.

- What is 0.90309?

- Since \( 10^{0.90309} \approx \), \( \log (8) \approx \).

You have found that \( y = \log 8 - x \). What does this mean? Substitute to rewrite this as an equation in terms of \( a \). The explanation for each step is given to the right.

\[
y = \log 8 - x.
\]

(equation of the line)

\[
(x = \log a \text{ and } y = \log b)
\]

\[
(b = 8/a)
\]

This last line is an example of the **Quotient Property of Logarithms**. In general, for \( a > 0 \), and \( b > 0 \)

\[
\log \frac{b}{a} = \log b - \log a.
\]

The Quotient Property of Logarithms can be used to rewrite logarithmic expressions.

\[
\log \frac{x}{y} \text{ is written in expanded form as } \log x - \log y.
\]

\[
\log 7 - \log z \text{ is written as a single logarithm as } \log \frac{7}{z}.
\]

The base 10 logarithms were used throughout this activity for convenience only.

These properties apply to logarithms with any base.
Change of Base
ID: 9468

Time required
40 minutes

Topic: Logarithmic Functions

- Derive and use the equation \( \log_{a}x = \log_{b}b \cdot \log_{a}x \) to convert a logarithm to another base.

Activity Overview

In this activity, students discover the change of base rule for logarithms by examining the ratio of two logarithmic functions with different bases. It begins with a review of the definition of a logarithmic function, as students are challenged to guess the base of two basic logarithmic functions from their graphs. The goal of applying the properties of logarithms to add these two functions is introduced as a motivator for writing them in the same base. Students explore the hypothesis that the two functions are related by a constant first by viewing a table of values, then by exploring different values for the two bases. Finally, they prove the change of base rule algebraically and apply it to find the sum of the two original functions.

Teacher Preparation

- This activity is designed to be used in an Algebra 2 or Precalculus classroom. This investigation could be used to introduce the change of base rule for logarithms, or to offer a deeper understanding to students who have encountered it already.
- Prior to beginning this activity, students should have experience applying the other properties of logarithms and solving simple logarithmic equations.
- This activity also offers deeper insight into the definition of a function through a discussion of what it means for two functions to be equal.
- To download the DIFFBASE program, go to education.ti.com/exchange and enter “9468” in the quick search box.

Classroom Management

- This activity is intended to be mainly teacher-led with brief periods of independent student work. This worksheet helps guide students through the activity and poses discussion questions.

TI-84 Plus Applications

none
Change of Base
ID: 9468

In this activity, you will explore:
• finding the base of a log function from a graph
• calculating the ratio of one log function to another
• deriving a formula for changing the base of a log

Before beginning the activity, clear all Y= entries and all lists.

Problem 1- Relating log functions with different bases

Execute the DIFFBASE program. Press PRGM and choose it from the list. Press ENTER

Choose SeeGraphs from the menu. This program displays the graphs of two logarithmic functions with different bases.

\[ f_1(x) = \log_a x \quad \text{and} \quad f_2(x) = \log_b x \]

You can determine \( a \) and \( b \) by examining points on each curve. This is a picture file, not an actual graph, so you cannot use the Trace feature, but you can move the cursor along the curve yourself to approximate the coordinates of points on each curve.

• What are \( a \) and \( b \)?
• What points on the graph are the best clues to the base of the logarithmic functions?

Once you think you know you \( a \) and \( b \), run the DIFFBASE program again and choose GuessBases from the menu. Enter the values of \( a \) and \( b \) that you found.

The program graphs two logarithmic functions with bases you entered as thick lines on top of the original graph. If you choose \( a \) and \( b \) correctly, your graph will look like the one shown.

If you see more than 2 curves on your graph, try different values for \( a \) and \( b \).
Suppose we were interested in the sum of these functions,
\[(f_1 + f_2)(x) = \log_a x + \log_b x.\]

How can we write this as a single logarithmic expression?

We need to rewrite the functions with the same base.

We want to find a function that is equal to \(f_1\), but involves logarithms base \(b\) instead logarithms base \(a\).

We could hope that there is a constant \(c\) that relates the two functions, like:
\[c \cdot f_1(x) = f_2(x).\]

Then we would have
\[f_1(x) = \frac{f_2(x)}{c} = \frac{1}{c} \cdot \log_b(x)\]
which is a logarithmic function with base \(b\).

We cannot be sure there is such a constant, but that doesn’t have to stop us from looking for one.

The DIFFBASE program will calculate \(c = \frac{f_2(x)}{f_1(x)} = \frac{\log_b x}{\log_a x}\) for you. Run the DIFFBASE program again and choose GraphC from the menu. The program calculates \(c = \frac{f_2(x)}{f_1(x)} = \frac{\log_b x}{\log_a x}\) and stores the result in \(Y3\).

Examine the graph of \(Y3\) and then view the \(Y3\) function table. What is \(c\)?

**Problem 2 – A closer look at \(c\)**

Is \(c\) always the same? Run the DIFFBASE program and choose CalculateC from the menu. Given \(a\) and \(b\), the program calculates
\[c = \frac{f_2(x)}{f_1(x)} = \frac{\log_b x}{\log_a x}\]
and displays the value. Try two different values of \(a\) and \(b\). What is \(c\) now?
Continue to choose CalculateC from the menu to experiment with different values of \(a\) and \(b\). As you try different values, the program records the results in the Lists. (Values of \(a\) are stored in \(L1\), \(b\) values in \(L2\), and \(c\) values in \(L3\).)

Be sure to try some powers of \(a\) and \(b\) such that one is a power of the other, like 2 and 8 or 9 and 3.

After you have tried at least 10 different values for \(a\) and \(b\), exit the program and view the data in the lists. (Press STAT then ENTER.) Can you guess a formula for \(c\)?

Hint: Try calculating \(1/c\) in \(L4\).

We are convinced now that there is a constant that relates \(\log_a x\) to \(\log_b x\) and that the constant depends on the values of \(a\) and \(b\). We may even have an idea how to calculate the constant \(c\) given \(a\) and \(b\).

Time to use some algebra to find out for sure.

Two functions are equal if and only if their values are equal for every \(x\) value in their domain. Let’s pick a generic point \((x, y)\) on the graph of \(f1\). For this \((x, y)\),

\[\log_a x = y.\]

If we can write \(y\) in terms of logarithms base \(b\), we will have our function.

- Rewrite \(\log_a x = y\) as an exponential expression.
- We want an expression with base \(b\) logarithms, so take \(\log_b\) of both sides.
- Simplify using the properties of logarithms.
- Solve for \(y\).

This is the relationship! You should have found that

\[\log_a(x) = \frac{\log_b(x)}{\log_b(a)}.\]

You have found a formula for changing the base of a logarithm. To change a logarithm base \(a\) to a logarithm base \(b\), simply divide the expression by \(\log_b a\).

- Use this formula to find \((f1 + f2)(x)\) if \(f1(x) = \log_3(x)\) and \(f2(x) = \log_5(x)\).
Solutions

- \(a = 3\) and \(b = 10\)

- The most informative points on the graphs will be those where \(y = 1\) or some other whole number. (When \(y = 1\), \(x\) is equal to the base of the logarithm.)

- Rewrite \(\log_{a} x = y\) as an exponential expression.
  \[a^y = x\]

- We want an expression with base \(b\) logarithms, so take \(\log_{b}\) of both sides.
  \[\log_{b}(a^y) = \log_{b} x\]

- Simplify using the properties of logarithms.
  \[y \log_{b} a = \log_{b} x\]

- Solve for \(y\).
  \[y = \frac{\log_{b} x}{\log_{b} a}\]

- Use this formula to find \((f1 + f2)(x)\) if \(f1(x) = \log_{3}(x)\) and \(f2(x) = \log_{10}(x)\).
  \[
  \log_{3} x + \log_{10} x \\
  = \frac{\log_{10} x}{\log_{10} 3} + \log_{10} x \\
  = \frac{1}{\log_{10} 3} \left(\log_{10} x + \log_{10} 3 \cdot \log_{10} x\right) \\
  = \frac{1}{\log_{10} 3} \left(\log_{10} x + \log_{10} x^{\log_{10} 3}\right) \\
  = \frac{\log_{10} x^{1+\log_{10} 3}}{\log_{10} 3}
  \]
TRIGONOMETRIC UNIT: The state standards and objectives that the activities for this unit cover:

**Standard III:** Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

**Objective 1:** Examine the behavior of functions using coordinate geometry.
   a. Identify the domain and range of the \( \ldots \) sine and cosine functions.
   b. Graph the \( \ldots \) sine and cosine functions.
   c. Graph functions using transformations of parent functions.

**Objective 2:** Determine radian and degree measures for angles.
   a. Convert angle measurements between radians and degrees.
   b. Find angle measures in degrees and radians using inverse trigonometric functions, including exact values for special triangles.

**Objective 3:** Determine trigonometric measurements using appropriate techniques, tools, and formulas.
   a. Define the sine, cosine, and tangent functions using the unit circle.
   b. Determine the exact values of the sine, cosine, and tangent functions for the special angles of the unit circle using reference angles.
   c. Find the length of an arc using radian measure.
   d. Find the area of a sector in a circle using radian measure.

**Scope and Sequence of Unit:**

Day 1: Review angle measurements, initial side, terminal side and trigonometric functions (13-2).
Day 2: Introduction to radians: students come up with an equation to convert between radians and degrees after completing an activity in Geometer’s Sketchpad where they construct the concept of a radian (13-3).

Day 3: Students measure angles, radii, and arc lengths using Geometer’s Sketchpad and discover a formula to find one of these measures based on the other two. Students discover a formula to find the area of a sector given the measure of the central angle and arc length (13-3).

Day 4: Students construct unit circles and discover the relationship between the x- and y-coordinates and the sine and cosine (also slope and tangent). Trigonometric functions are defined using the unit circle (using Geometer’s Sketchpad).

Day 5: Students determine the exact values of trigonometric functions for special angles of the unit circle using reference angles (13-2).


Day 7: Students find angle measures in degrees using inverse trigonometric functions (14-2).

Day 8: Students find angle measures in radians using inverse trigonometric functions (14-3).

Day 9: Students explore real world problems involving trigonometric functions.

Day 10: Students explore graphs of sine and cosine, including domain and range (13-4 and 13-5).

Day 11: Students investigate translations of sine and cosine through Geometer’s Sketchpad (activity from Mathbits.com).

Day 12: Students continue graphing sine and cosine functions identifying domain and range, including translations of sine and cosine (13-7).
Day 13: Students review concepts learned and write real world problems that use trigonometric functions, write equations that model these problems and explain what each term in the function represents.

Day 14: Review.

Day 15: Unit Test.

The activities in this unit rely heavily on the use of Geometer's Sketchpad. I created all of the activities except for the activity to be done on the eleventh day, which comes from an activity I found online at mathbits.com and received permission to use. These activities allow students to manipulate their constructions to gain deeper understandings of radians, the construction of the unit circle, trigonometric functions in the unit circle, and translations of trigonometric functions on the coordinate plane.
Introduction to Radians

1) Using Geometer's Sketchpad create a circle (this should fill most of the screen).
2) Draw in a horizontal radius.
3) Measure the length of the radius and record the result.
4) Construct an arc that has the same length as the radius.
   - Create two points on the circle
   - Select the point from the end of the radius and the two points created (order is important)
   - Select \textbf{CONSTRUCT – ARC THROUGH 3 POINTS}
   - Select \textbf{MEASURE – ARC LENGTH}
   - Record the length of the arc to nearest hundredth.

5) Draw in another radius to create a central angle whose terminal side intersects the arc that has the same length as the radius. This angle is one radian. (Definition: \textbf{one radian} – the measure of an angle whose initial and terminal sides intercepts an arc whose length is equal to the length of the radius).

6) Repeat the process to draw in consecutive angles each measuring one radian until there is not room to draw in more.
7) Print out a copy (make sure it fits on one page) of your drawing. This should look similar to the figure to the right. You will turn this in.

8) Approximately how many radians does it take to circumscribe (go completely around) the circle? Explain your answer.

9) Recall the formula for the circumference of a circle is $C = 2\pi r$ (Where $C$ is circumference and $r$ is the length of the radius). Use this formula to determine exactly how many radians are in a circle (to start and end at the same point). Show your work.
10) Fill in the table to the right (how many degrees are in a complete circle, ...)

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td></td>
</tr>
<tr>
<td>Semi-circle</td>
<td></td>
</tr>
</tbody>
</table>

11) How many radians are in 90°? Explain your answer.

12) Explain how you could convert any value given in degrees to radians.

13) Write a formula for converting degrees to radians.

14) Explain how you could convert any value given in radians to degrees.

15) Write a formula for converting degrees to radians.

16) Fill in the missing values in the chart (convert between radians and degrees). Leave radian answers in exact form (fractions involving $\pi$). Show your work to the right.

<table>
<thead>
<tr>
<th>DEGREES</th>
<th>RADIANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 30°</td>
<td>$\frac{\pi}{6}$</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{\pi}{2}$</td>
</tr>
<tr>
<td>C 240°</td>
<td>$\frac{7\pi}{4}$</td>
</tr>
<tr>
<td>D</td>
<td>$\frac{3\pi}{2}$</td>
</tr>
<tr>
<td>E 135°</td>
<td>$\frac{3\pi}{4}$</td>
</tr>
<tr>
<td>F</td>
<td>$\frac{2\pi}{3}$</td>
</tr>
<tr>
<td>G 330°</td>
<td>$\frac{\pi}{2}$</td>
</tr>
<tr>
<td>H 120°</td>
<td>$\frac{\pi}{3}$</td>
</tr>
<tr>
<td>I 270°</td>
<td>$\frac{\pi}{2}$</td>
</tr>
<tr>
<td>J</td>
<td>$\frac{\pi}{12}$</td>
</tr>
</tbody>
</table>
ARC LENGTH AND AREA OF A SECTOR

Use Geometer’s Sketchpad to fill in the table below with a partner. Create a circle with a central angle. Measure the radius, the central angle and the length of the arc the central angle intercepts. You need at least 5 different radii with different central angles. Make sure you are measuring the central angles in radians and not degrees (Edit – Preferences – unit – set angle to radian).

<table>
<thead>
<tr>
<th>Length of Radius (cm)</th>
<th>Measure of Central Angle (radians)</th>
<th>Length of Arc (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>4</td>
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<tr>
<td>10</td>
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</tbody>
</table>

Look at each row individually. How do these values relate to each other? Compare them to each other to come up with a formula to find one of the values based on the other two values. Check to see if this formula holds true for the remaining rows. Explain what you did, how you came up with this formula and show that it holds true for the other rows.
Use the circles shown below in the left hand column of the table to fill in the rest of the table. Change the measure of the central angle from degrees to radians. Find the area of the shaded portion of the circle.

<table>
<thead>
<tr>
<th>Circle</th>
<th>Portion of circle shaded</th>
<th>Length of Radius (cm)</th>
<th>Measure of Central Angle (radians)</th>
<th>Area of Sector (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>1</td>
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<td>4</td>
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<td>12</td>
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</table>

Based on the answers in the table, come up with a formula to find the area of a sector in a circle based on radius and measure of central angle (in radians).
The Unit Circle and Trigonometric Functions

1) Open Geometer’s Sketchpad to create a new sketch. Set the angle units to radians, set the precision for slopes and ratios to thousandths. (Edit – Preferences)

2) Choose Graph – Show Grid and resize the axes so that the x- and y- values range from -1 to 1. (Center the grid on your screen)

3) Label the Origin A and the unit point B (right click with the mouse and select Show Label).

4) Construct a unit circle: first select point A then B. Choose Construct – Circle by Center + Point (depending on the shape of your screen your circle may look like an oval – it is a circle but it is distorted by the screen’s shape).

5) Construct a point on the circle and label it C (make sure you don’t construct the point on one of the axes).

6) Measure the x- and y-coordinates of this new point (with the point selected, choose Measure – Coordinates)

7) Construct \( \overline{BC} \) (notation read “arc BC”, meaning the arc that starts at point B and ends at point C) Select the circle, point B, then point C (order is important). Choose Construct – Arc On Circle.

8) With \( \overline{BC} \) selected, choose Measure – Arc Angle (recall the measure of the central angle is the same as the arc it intercepts when measured in radians)

9) Construct a line through the center of the circle (point A) and point C. Measure the slope of this line by choosing Measure – Slope.

10) Use your construction to fill in the table on the next page. For the 12 arc angle measures, 2 need to come from each quadrant and 4 for each axis. Your current constructions should look similar to the figure shown at the top right of the next page.
### Measure of Arc Angle

<table>
<thead>
<tr>
<th>Measure of Arc Angle</th>
<th>x-value</th>
<th>y-value</th>
<th>Slope of AC</th>
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</tbody>
</table>
11) What is the greatest value of
   a) \(\overrightarrow{BC}\)  
   b) \(x\)  
   c) \(y\)  
   d) the slope of \(\overrightarrow{AC}\) 

12) What is the least value of:
   a) \(\overrightarrow{BC}\)  
   b) \(x\)  
   c) \(y\)  
   d) the slope of \(\overrightarrow{AC}\) 

13) Fill in the heading of the three blank columns in the table as \(\sin \theta\), \(\cos \theta\), and \(\tan \theta\). Use a calculator to calculate each of these values for \(\theta\) (where \(\theta\) is the measure of the arc angle)

14) How do these values compare to the values already in the table?

15) Make a conjecture on how the values in the table relate to each other.

16) Can you prove this conjecture? If so, explain how. If not, explain why not.
Investigating Trigonometric Functions

The Basic Functions

I. Click on the section titled “Sine Function” (or click the tab at the bottom of the screen). Answer the following questions:
1. What is the amplitude of the function \( f(x) = \sin(x) \)?
2. What is the approximate decimal length of the period of the function \( f(x) = \sin(x) \)?
3. Click on “Show Full Function”.
4. The sliders on the left side of the screen are controlled by moving the right end of the segment slider. As closely as possible, set the following values: \( a = 1 \); \( b = 1 \); \( c = 0 \); \( d = 0 \). What function do you have with these settings?

5. Set \( a = 1 \); \( b = 1 \); \( c = 0 \); \( d = 2 \). Describe what happens to the graph when \( d \) changes.

6. Set \( a = 1 \); \( b = 1 \); \( c = 2 \); \( d = 0 \). Describe what happens to the graph when \( c \) changes.

7. Set \( a = 1 \); \( b = 3 \); \( c = 0 \); \( d = 0 \). Describe what happens to the graph when \( b \) changes.

8. Set \( a = 3 \); \( b = 1 \); \( c = 0 \); \( d = 0 \). Describe what happens to the graph when \( a \) changes.

This “Full Function” equation, \( g(x) = a \sin(b(x - c)) + d \), is called a sinusoidal function. Sinusoidal functions are used to model real life data such as AC circuits, ocean tides, earthquakes, measured daylight over time, temperatures over time, drought patterns, tsunamis, weights of animals over time, and numerous other examples.

9. Using the grid below, graph the sinusoidal function \( f(x) = -2 \sin(2x - 4) - 1 \).

   Look carefully at this function before you start adjusting the sliders!!
   (As you set each slider, look carefully at the graph to see what is happening to it.)

   For this new sinusoidal function:
   a.) what is the amplitude?
   b.) what is the period?
   c.) what is the vertical shift?
   d.) what is the phase shift?
II. Click on the section titled “Cosine Function” (or click the tab at the bottom of the screen). Answer the following questions:

1. What is the amplitude of the function \( f(x) = \cos(x) \)?
2. What is the approximate decimal length of the period of the function \( f(x) = \cos(x) \)?
3. Click on “Show Full Function”.
4. As closely as possible, set the sliders so that the "Full Function" and the "Parent Function" are the same. What are your values for: \( a = \), \( b = \), \( c = \), \( d = \).
5. Using the grid below, graph the function \( f(x) = \cos(x + 2) + 1 \). Look carefully!
   (As you set each slider, look carefully at the graph to see what is happening to it.)

   For this new function:
   a.) what is the amplitude?
   b.) what is the period?
   c.) what is the vertical shift?
   d.) what is the phase shift?

III. Click on the section titled “Tangent Function” (or click the tab at the bottom of the screen). Answer the following questions:

1. In the \( x \)-interval 0 to 3, what is the decimal approximation of the asymptote of \( f(x) = \tan(x) \)?
2. Click on “Show Full Function”.
3. As closely as possible, set the sliders so that the "Full Function" and the "Parent Function" are the same. What are your values for: \( a = \), \( b = \), \( c = \), \( d = \).
4. Using the grid at the left, graph the function \( f(x) = -\tan(2(x - 2)) \). Look carefully!

   For this new function:
   a.) what is the decimal approximation of the asymptote on the \( x \)-interval 0 to 2.5?
   b.) what appears to happen to the graph as “\( d \)” approaches the value of 10?
   Explain why this appears to be happening?

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PROBABILITY, COMBINATIONS AND PERMUTATIONS UNIT: The state standards and objectives that the activities for this unit cover:

**Standard IV:** Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

**Objective 1:** Apply basic concepts of probability.

a. Distinguish between permutations and combinations and identify situations in which each is appropriate.

b. Calculate probabilities using permutations and combinations to count events.

c. Compute conditional and unconditional probabilities in various ways, including by definitions, the general multiplication rule, and probability trees.

d. Define simple discrete random variables.

Except for the first two days of this unit, all activities are taken from *SIMMS Integrated Mathematics Level 2: a modeling approach using technology* pp. 303-321 (Souhrada et al., 2006). The majority of this chapter consists of students working together in assigned groups with investigations, explorations, and much discussion involved.

**Scope and Sequence of Unit:**

Day 1: Review simple probabilities and tree diagrams. Define simple discrete random variables. All work will be done with whiteboards.

Day 2: Distinguish between conditional and unconditional probabilities and solve probabilities in various ways (12-2).

Day 3-5: Students will be introduced to the lesson by Ms. Watterson bringing in a combination lock and locking an item students would be interested in obtaining in a safe (such as an ipod, etc). Students then can write down a three digit combination and if their combination opens the safe
they can keep the item inside (since there is a $\frac{1}{64,000}$ chance of this happening it is very unlikely any student will be able to open the safe with only one attempt). This leads to a class discussion on how many possibilities there are for the combination lock and the probability of any one combination being able to open the lock. The students then begin the introduction (pg 304) through which they will encounter counting problems that could arise in the development of a mall. Students use the fundamental counting principle and apply their knowledge of the fundamental counting principle to determine simple probabilities. Students complete an exploration, discussion and assignment found on pages 305-310 in SIMMs textbook. Students complete Flashback Activity 1.

Day 6-8: Students explore counting situations in which repetition is not allowed. The formula for permutations is developed using the fundamental counting principal. Students write and interpret factorial notation, apply a formula for permutations, and apply their knowledge of permutations to determine simple probabilities. Students complete an exploration, discussion and assignment found on pages 310-314 in SIMMs textbook. While going through the exploration and discussion, there are many times that the activities are teacher led. Students complete Flashback Activity 2.

Day 9: Review of concepts learned in class and clarification as needed. Quiz (Assessment 1).

Day 10-11: Students explore counting situations in which repetition is not allowed and order does not matter. Students develop the formula for combinations while investigating how to count numbers of possible committees that can be formed from a group. This is compared to forming a presidency with specific roles to distinguish between combinations and permutations. Students apply their knowledge of combinations to calculate simple probabilities. As part of the activity students list all possible arrangements of 3 people chosen
from 5 (template A assists in this). Students then decide how many of these groups consist of the same people. Students complete an exploration, discussion and assignment found on pages 315-320 in SIMMs textbook. Students complete Flashback Activity 3.

Day 12: Students work on problems involving permutations and combinations, determining what type each problem is when solving (6-7).

Day 13: Students create problems with real-world contexts to apply their understanding or combinations and permutations, explaining why each type is used for a specific problem and review.

Day 14: Review.

Day 15: Unit test.

All details for this unit can be found in the SIMMS integrated textbook, except for day 12. The book is very precise in leading students to discover formulas and construct concepts. In class problems will be assigned during the first and last few minutes of class as a brief introduction each day and to reinforce the concepts learned that day. These problems will be based on the concepts learned and worked on the whiteboards.
STATISTICS UNIT: The state standards and objectives that the activities for this unit cover:

Standard IV: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

Objective 2: Use percentiles and measures of variability to analyze data.

a. Compute different measures of spread including range, standard deviation and interquartile range.

b. Compare the effectiveness of different measures of spread, including the range, standard deviation, and interquartile range in specific situations.

c. Use percentiles to summarize the distribution of a numerical variable.

d. Use histograms to obtain percentiles.

This unit will begin with a class discussion on data investigations. Through the discussion we will talk about ways to analyze data, and find information that students are interested in. Based on what the class decides, the unit will focus around that. Questions that will be focused on during the discussion:

- What do we want to find out about?
- What can we find out about?
- What can we observe?
- What can we measure?
- Can we measure what we want?
- Are there other things we should record besides what we want to measure?
- How can we gather the information?
- Is this information we can gather by the beginning of class tomorrow?

Based on the students' discussion, we will use the information they chose to focus on for our statistics unit. The unit will be divided into stages to teach students the standards that they need to know based on the Utah State Core Curriculum. As this unit is more of a discovery unit, it will be mostly student led; therefore, the scope and sequence could greatly vary based upon class discussion and students' interest. The students will learn all of the objectives within the state core curriculum stated above for
this unit. Days six through twelve will include discussion, comparisons to the class statistics found, and practicing problems on the whiteboards in addition to being assigned problems from the specified section in the textbook.

**Preliminary Scope and Sequence of Unit:**

**Day 1:** Discuss various types of data, ways of collecting data, reasons for collecting data, accuracy of various techniques of collecting data, etc. As a class, decide what data we want to collect and analyze how we can achieve this. I will guide the discussion to include how samples are taken, how accurately a sample reflects on the whole population, and errors that might occur due to sampling. I will also teach students about discrete vs. continuous data.

**Day 2-3:** Students either return to class having collected data or as a class we will collect data within our class at this time (such as a specific trait of students). As a class we will review mean, median, and mode. Discuss ways to display the data (box-and-whisker plots, circle graphs, bar graphs/histograms, line graphs, stem-and-leaf plots). Students will be divided into five groups to organize the class data according to one of the displays listed. Each group will have a worksheet giving an explanation and example of the type of chart they are assigned to (information on these worksheets are based on Cecire, 2002 and Statistics Canada, 2007). Each group will create a poster to display in the class and teach the class how to produce that specific type of chart. As a class, we will compare and contrast the different ways to display data, discussing the advantages and disadvantages of each one (both with our data and other types of data). We will decide which graph(s) would be the most appropriate for our data. Through the class discussion, the concepts of mean, median, mode, range, outliers, and interquartile range will be discussed. Students will find each of these statistics for the data and discuss which statistics they think are more informative and which ones
they think are less informative. As an assignment, students will find charts/graphics to bring to school the next day (newspaper, magazines, internet, etc) along with an interpretation of the graph.

Day 4: Students will be introduced to standard deviation, talk about how it applies and discuss what it tells us about the data. Students will calculate the standard deviation for the data set and explain how the data would vary if the standard deviation had been greater or less than that calculated for the data (In class: calculate standard deviation of various sets – to be completed at home if not completed in class).

Day 5: Students discuss the results from their data and the various statistics. The class then reviews topics for the next few days, doing work from Prentice Hall chapter 12 to reinforce concepts learned the past few days. On day 5 students will receive the assignment to complete their own project (gather and analyze data based on methods they have learned). Day 5 will be Probability Distributions (12-1).

Day 6: Analyzing Data: Measures of Central Tendency (12-3).

Day 7: Standard Deviation (12-4).

Day 8: Working with Samples (12-5).

Day 9-10: Binomial Distributions (12-6).

Day 11-12: Normal Distributions (12-7).


Day 14-15: Students present projects to class.

Day 16: Paper for project due, discuss concepts learned during the unit.
Frequency Histograms

The histogram is a popular graphing tool. It is used to summarize discrete or continuous data that are measured on an interval scale. It is often used to illustrate the major features of the distribution of the data in a convenient form. A histogram divides up the range of possible values in a data set into classes or groups. For each group, a rectangle is constructed with a base length equal to the range of values in that specific group, and an area equal to the number of observations falling into that group. A histogram has an appearance similar to a vertical bar graph, but when the variables are continuous, there are no gaps between the bars. When the variables are discrete, however, gaps should be left between the bars. Figure 1 is a good example of a histogram

Histogram characteristics: Generally, a histogram will have bars of equal width. Choosing the appropriate width of the bars for a histogram is very important. The histogram is used for variables whose values are numerical and measured on an interval scale. It is generally used when dealing with large data sets (greater than 100 observations). A histogram can also help detect any unusual observations (outliers) or any gaps in the data.

Sounds complicated but the concept really is pretty simple. We graph groups of numbers according to how often they appear. Thus if we have the set \( \{1,2,2,3,3,3,4,4,5,6\} \), we can graph them like the graph shown to the right.

This graph is pretty easy to make and gives us some useful data about the set. For example, the graph peaks at 3, which tells us that the number 3 occurred the most (called the mode). The area of each rectangle informs us of how many in that range occurred. Since each rectangle has a base length of 1, the area (base times height) is 1 times the height which will equal the height. Therefore by looking at the graph I can easily tell how many of each value there are in the data set.

How is a Real Histogram Made?
The example above is very simple. In most real data sets almost all numbers will be unique. Consider the set \( \{3, 11, 12, 19, 22, 23, 24, 25, 27, 29, 35, 36, 37, 45, 49\} \). A graph which shows how many ones, how many twos, how many threes, etc. would be meaningless. Instead we group the data into convenient ranges. In this case we group the data within a range of 10, by looking at how many value fall in the range 0-10, 10-20, 20-30, etc.

Note: Changing the size of the range to group the data in changes the appearance of the graph and the conclusions you may draw from it.

Now, let's make a histogram from the data given: \( \{3, 11, 12, 19, 22, 23, 24, 25, 27, 29, 35, 36, 37, 45, 49\} \).
Steps to make a histogram:

1) Arrange the data from least to greatest. (this data is already arranged in that way)
2) Determine how to group the data (this will depend on how many values there are and what ranges they fall in, if the range is too large too many values will be grouped together and the graph won’t be as informative, if the range is too small the graph would be meaningless). To determine the range to group the data in, look at the data once ordered from least to greatest and see how different groupings would work (group by 5’s, 10’s, 20’s, etc). Here grouping by 10’s seems to work well.
3) Record how many values fall in each range (frequency): (see table below to the left)
4) Draw the histogram by making the base of each rectangle the same and labeling the range, making the height equal to the frequency of the data in that range.

<table>
<thead>
<tr>
<th>Data Range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>1</td>
</tr>
<tr>
<td>10-20</td>
<td>3</td>
</tr>
<tr>
<td>20-30</td>
<td>6</td>
</tr>
<tr>
<td>30-40</td>
<td>4</td>
</tr>
<tr>
<td>40-50</td>
<td>2</td>
</tr>
</tbody>
</table>

ASSIGNMENT:

Use the data collected by the class to make a histogram with your group. List the data below and make a histogram, then when completed accurately, draw a larger histogram to display to the class on the provided poster board (meter sticks, rulers and markers are available to make the display). As a group you will display the data and teach the class how to make a histogram and the advantages of using a histogram. Every group member needs to participate in the class presentation so decide who will teach what.
Box and Whisker Plot

A box and whisker plot (sometimes called a boxplot) is a graph that presents information from a five-number summary. It does not show a distribution in as much detail as a stem and leaf plot or histogram does, but is especially useful for indicating whether a distribution is skewed and whether there are potential unusual observations (outliers) in the data set. Box and whisker plots are also very useful when large numbers of observations are involved and when two or more data sets are being compared. Box and whisker plots are ideal for comparing distributions because the centre, spread and overall range are immediately apparent. A box and whisker plot is a way of summarizing a set of data measured on an interval scale. It is often used in explanatory data analysis. This type of graph is used to show the shape of the distribution, its central value, and its variability.

In a box and whisker plot:

- The ends of the box are the upper and lower quartiles, so the box spans the interquartile range
- The median is marked by a vertical line inside the box
- The whiskers are the two lines outside the box that extend to the highest and lowest observations.

Making a Box and Whiskers Plot

1. The numbers you are working with need to be in order from lowest to highest value.
2. Find the median (the number in the middle of the data).
3. Find the median of the lower half of your numbers. This is the lower quartile.
4. Find the median of the upper half of your numbers. This is your upper quartile.
5. Now draw a small box above or below a number line that extends to all values in your data set. One end of the box corresponds with the upper quartile, the other with the lower quartile. Draw a vertical line in the box which corresponds to the median.
6. Find your highest number and your lowest number. These are the upper and lower extremes.
7. Draw a line (whisker) from the upper end of your box to a point corresponding with the upper extreme. Do the same for the lower extreme.

A common error is to count scores wrong when finding the upper and lower quartiles. On an even number of scores, the two middle scores become a part of the upper and lower halves of the scores, one in the upper half, one in the lower half, and are counted again when finding the upper and lower quartiles.
EXAMPLE
Ten scores from the last math test are as follows: 80, 75, 90, 95, 65, 65, 80, 85, 70, 100. Make a box and whisker plot of this data

1) Order from least to greatest: 65, 65, 70, 75, 80, 80, 85, 90, 95, 100

2) Find the median (count the number of data add 1 and divide by 2) In this data set there are 10 values: \[
\frac{10+1}{2} = \frac{11}{2} = 5.5
\]
   This number tells us how far to move over. Since there is not a 5.5\(^{th}\) number I will find the average of the 5\(^{th}\) and 6\(^{th}\) number (80 and 80 which averages to 80).

3) Find the median of the lower half of the numbers. The lower half are all those numbers below the median 65, 65, 70, 75, 80. Find the median the same way as in step 2. Here there are 5 numbers so I add 1 to five and divide by 2: \[
\frac{5+1}{2} = \frac{6}{2} = 3
\]
   So the lower quartile is the 3\(^{rd}\) number over: 70

4) Find the median of the upper half of the numbers: 80, 85, 90, 95, 100 (similar to step 3) 90

5) Now draw a small box above or below a number line that extends to all values in your data set. One end of the box corresponds with the upper quartile, the other with the lower quartile. Draw a vertical line in the box which corresponds to the median.

6) Find your highest number and your lowest number. These are the upper and lower extremes.

7) Draw a line (whisker) from the upper end of your box to a point corresponding with the upper extreme. Do the same for the lower extreme.

ASSIGNMENT:

Use the data collected by the class to make a histogram with your group. List the data on a separate piece of paper and make a box and whisker plot. When completed accurately, draw a larger box and whisker plot to display to the class on the provided poster board (meter sticks, rulers and markers are available to make the display). As a group you will display the data and teach the class how to make a box and whisker plot and the advantages of using a box and whisker plot. Every group member needs to participate in the class presentation so decide who will teach what.
Line graphs

Line graphs have visual characteristics that reveal data trends clearly and these graphs are easy to create. Line graphs are one of the most common tools used to present data. A line graph is a visual comparison of how two variables—shown on the x- and y-axes—are related or vary with each other. It shows related information by drawing a continuous line between all the points on a grid. The y-axis in a line graph usually indicates quantity (how much) or percentage, while the horizontal x-axis often measures units of time. Although line graphs do not present specific data as well as tables do, they are able to show relationships more clearly than tables do.

In summary, line graphs

- show specific values of data well
- reveal trends and relationships between data
- compare trends in different groups of a variable

Example 1 – Comparing two related variables

Figure 1 is a single line graph comparing two items; in this instance, time is not a factor. The graph compares the number of dollars donated by the age of the donors. According to the trend in the graph, the older the donor, the more money he or she donates. The 17-year-old donors donate, on average, $84. For the 19-year-olds, the average donation increased by $26 to make the average donation of that age group $110.

Example 2 – Using correct scale

When drawing a line, it is important that you use the correct scale. Otherwise, the line's shape can give readers the wrong impression about the data. Compare Figure 2 with Figure 3:
Using a scale of 350 to 430 (Figure 2) focuses on a small range of values. It does not accurately depict the trend in guilty crime offenders between January and May since it exaggerates that trend and does not relate it to the bigger picture. However, choosing a scale of 0 to 450 (Figure 3) better displays how small the decline in the number of guilty crime offenders really was.

Example 3 – Multiple line graphs

A multiple line graph can effectively compare similar items over the same period of time (Figure 5).

In Figure 4, the message is clearly stated in the title, and each of the line graphs is properly labeled. It is easy to see from this graph that the total cell phone use has been rising steadily since 1996, except for a two-year period (1999 and 2000) where the numbers drop slightly. The pattern of use for women and men seems to be quite similar with very small discrepancies between them.

**ASSIGNMENT:**

Use the data collected by the class to make a line graph with your group. List the data below and make a line graph, then when completed accurately, draw a larger line graph to display to the class on the provided poster board (meter sticks, rulers and markers are available to make the display). As a group you will display the data and teach the class how to make a line graph and the advantages of using a line graph. Every group member needs to participate in the class presentation so decide who will teach what.
Stem and Leaf Plots

A stem and leaf plot, or stem plot, is a technique used to classify either discrete or continuous variables. A stem and leaf plot is used to organize data as they are collected.

A stem and leaf plot looks something like a bar graph. Each number in the data is broken down into a stem and a leaf, thus the name. The stem of the number includes all but the last digit. The leaf of the number will always be a single digit.

Elements of a good stem and leaf plot

- Shows the first digits of the number (thousands, hundreds or tens) as the stem and shows the last digit (ones) as the leaf.
- Usually uses whole numbers. Anything that has a decimal point is rounded to the nearest whole number. For example, test results, speeds, heights, weights, etc.
- Looks like a bar graph when it is turned on its side.
- Shows how the data are spread—that is, highest number, lowest number, most common number and outliers (a number that lies outside the main group of numbers).

Draw a stem and leaf plot as follows:

- Order the data from least to greatest.
- On the left hand side of the page, write down the thousands, hundreds or tens (all digits but the last one). These will be your stems.
- Draw a line to the right of these stems.
- On the other side of the line, write down the ones (the last digit of a number). These will be your leaves.

For example, if the observed value is 25, then the stem is 2 and the leaf is the 5. If the observed value is 369, then the stem is 36 and the leaf is 9. Where observations are accurate to one or more decimal places, such as 23.7, the stem is 23 and the leaf is 7. If the range of values is too great, the number 23.7 can be rounded up to 24 to limit the number of stems.

The main advantage of a stem and leaf plot is that the data are grouped and all the original data are shown, too.

Example – Making a stem and leaf plot

A teacher asked 15 of her students how many books they had read in the last 12 months. Their answers were as follows:

12, 23, 19, 6, 10, 33, 7, 15, 25, 21, 12, 2, 31, 28, 18

Prepare a stem and leaf plot for these data.
Step 1: Organize the data from least to greatest value. (2, 6, 7, 10, 12, 12, 15, 18, 19, 21, 23, 25, 28, 31, 33)

Step 2: Determine what the stems will be. In this example the largest value has 2 digits, so the stem will be the digit in the tens place and the leaf will be the digit in the ones place. For the values 6 and 7 the stem will be 0. (Note: stem 0 represents the interval 0 to 9; stem 1 represents the interval 10 to 19; and stem 2 represents the interval 20 to 29.)

Step 3: Create a stem and leaf plot by writing the stems in a column from least to greatest, draw a line to the right of the stems, then write the corresponding leaves to the right of the line.

Using the data from Table 2, we made the ordered stem and leaf plot shown below:

**Books read in a year by 15 students**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2 6 7</td>
</tr>
<tr>
<td>1</td>
<td>0 2 2 5 8 9</td>
</tr>
<tr>
<td>2</td>
<td>1 3 5 8</td>
</tr>
<tr>
<td>3</td>
<td>1 3</td>
</tr>
</tbody>
</table>

**ASSIGNMENT:**

Use the data collected by the class to make a stem and leaf plot with your group. List the data below and make an initial stem and leaf plot, then when completed accurately, draw a larger stem and leaf plot to display to the class on the provided poster board (meter sticks, rulers and markers are available to make the display). As a group you will display the data and teach the class how to make a stem and leaf plot and the advantages of using a stem and leaf plot. Every group member needs to participate in the class presentation so decide who will teach what.
Circle Graphs/Pie Charts

A circle graph or pie chart is a way of summarizing a set of categorical data or displaying the different values of a given variable (e.g., percentage distribution). This type of chart is a circle divided into a series of segments. Each segment represents a particular category. The area of each segment is the same proportion of a circle as the category is of the total data set. The use of the pie chart is quite popular, as the circle provides a visual concept of the whole (100%). Despite its popularity, pie charts should be used sparingly for two reasons. First, they are best used for displaying statistical information when there are no more than six components—otherwise, the resulting picture will be too complex to understand. Second, pie charts are not useful when the values of each component are similar because it is difficult to see the differences between slice sizes.

A pie chart uses percentages to compare information. Percentages are used because they are the easiest way to represent a whole. The whole is equal to 100%. For example, if you spend 7 hours at school and 55 minutes of that time is spent eating lunch, then 13.1% of your school day was spent eating lunch. To present this in a pie chart, you would need to find out how many degrees represent 13.1%. This calculation is done by developing the equation:

\[ \text{percent} \times 360 \text{ degrees} = \text{the number of degrees} \]

Constructing a pie chart

A pie chart is constructed by converting the share of each component into a percentage of 360 degrees. In the figure to the right, music preferences in 14 to 19-year-olds are clearly shown. This pie chart quickly tells you that the majority of students like rap best (50%), and the remaining students prefer alternative (25%), rock and roll (13%), country (10%) and classical (2%).

Tip! When drawing a pie chart, ensure that the segments are ordered by size (largest to smallest) and in a clockwise direction.

In order to reproduce this pie chart, follow this step-by-step approach:

1. Draw a circle with your protractor.
2. Starting from the 12 o'clock position on the circle, measure an angle of 180 degrees with your protractor (50% of 360 degrees). The rap component should make up half of your circle. Mark this radius off with your ruler.
3. Repeat the process for each remaining music category, drawing in the radius according to its percentage of 360 degrees. The final category need not be measured as its radius is already in position.
Labeling the segments with percentage values often makes it easier to tell quickly which segment is bigger. Whenever possible, the percentage and the category label should be indicated beside their corresponding segments. This way, users do not have to constantly look back at the legend in order to identify what category each color represents.

In the pie chart to the right, the legend is formatted properly and the percentages are included for each of the pie segments. However, there are too many items in the pie chart to quickly give a clear picture of the distribution of movie genres. If there are more than five or six categories, consider using another graph to display the information. Figure 5 would certainly be easier to read as a bar graph.

ASSIGNMENT:

Use the data collected by the class to make a pie chart with your group. List the data below and make a pie chart, then when completed accurately, draw a larger pie chart to display to the class on the provided poster board (meter sticks, rulers and markers are available to make the display). As a group you will display the data and teach the class how to make a pie chart and the advantages of using a pie chart. Every group member needs to participate in the class presentation so decide who will teach what.
For the unit project, you will gather data on your own similarly to the data gathered as a class. Answer these questions:

- What do you want to find out about?
- What can you find out about?
- What can you observe?
- What can you measure?
- Can you measure what you want?
- Are there other things you should record besides what you want to measure?
- How can you gather the information?
- Why is this useful to know about?

All projects must be cleared with Ms. Watterson first. For your project you will need to:

1.) Create a method to gather data.
2.) Gather data (survey, measure, etc).
3.) Record results for each measurement/survey.
4.) Compile data to share with the class.
5.) Create a histogram to display to the class and any other graph/chart that is needed to display your project.
6.) Obtain 5 different percentiles from your histogram.
7.) Compute the different measures of spread (range, standard deviation and interquartile range)
8.) Compare the effectiveness of the different measures of spread for your data.
9.) Calculate the mean, median, and mode for the data.
10.) Discuss what each of these measures indicate (range, standard deviation, interquartile range, mean, median, and mode) and how informative they are based on the data you’ve gathered
11.) Discuss sampling issues – what was done to make your sample accurate, do you think your sample accurately reflects the population?

You will present your project to the class and turn in a written report summarizing the project (make sure your paper covers all of the requirements listed above).
References


