Design Survey of Laminated Composite I-Beam

Mrinmoy Saha
Utah State University
DESIGN SURVEY OF LAMINATED COMPOSITE I-BEAM

by

Mrinmoy Saha

A report submitted in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Mechanical Engineering

Approved:

Thomas H. Fronk, Ph.D.  
Major Professor

Steven R. Folkman, Ph.D.  
Committee Member

Robert E. Spall, Ph.D.  
Committee Member

UTAH STATE UNIVERSITY
Logan, Utah

2018
ABSTRACT

Design Survey of a Laminated Composite I-Beam

by

Mrinmoy Saha, Master of Science

Utah State University, 2018

Major Professor: Dr. Thomas H. Fronk
Department: Mechanical and Aerospace Engineering

Composite I-beams are popular for high-strength low-weight applications. Learning the macro-mechanics and designing the composite I-beam properly are necessary. In this report, a design overview of the composite I-beam is discussed which is based on classical lamination theory where it includes the homogenization approach, the plane stress assumption and the Kirchhoff hypothesis. Using these assumptions, a method was developed to come up with the effective material properties of a beam. Formulas to calculate maximum deflection and maximum bending stress and shear stress and the stress concentration at the connection of web-flange are discussed which describe ways for designing and manufacturing the I-beam.

Ideas for manufacturing I-beams are based on unidirectional fibers. Critical design considerations are discussed. The design considerations for stacking sequences, cross section, length of the beam, failure, buckling are discussed to give an insight to achieve intended I-beam design goals more effectively. For stacking sequences, it is found that the sequencing of the layers depends on configuration and loads applied on the beam. To counter shear, 45° layers are needed and to counter tension and compression, cross-ply are
best. Minimizing cross section while having more web height compared to flange width will strengthen the beam. Finally, some recommendations are made for future directions from this project.
Design Survey of a Laminated Composite I-Beam

Mrimoy Saha

Composite I-beams are popular for high strength low weight applications. Understanding the macro-mechanics and the design of composite I-beam is essential. In this report, a design overview of the composite I-beam is discussed which is based on classical lamination theory. A method is developed to calculate the effective material properties of I-Beam. Formulas to calculate maximum deflection, maximum bending stress, shear stress and the stress concentration at the connection of web-flange are discussed which describe ways for designing and manufacturing the I-beam.

Ideas for manufacturing I-beams are also provided and also critical design considerations are discussed. The design consideration for stacking sequences, cross section, length of the beam, failure, buckling has been discussed to give an insight to design the I-beam towards individual intended design goals more effectively. Finally, some recommendations were made for future directions from this project.
ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor Dr. Thomas H. Fronk for the continuous support of my project, for patience, motivation, enthusiasm and immense knowledge. His supervision helped me all the time of project and writing of this report. I could not have imagined a better advisor and mentor for my MS study.

Also, I would especially like to thank my other committee members, Dr. Robert Spall and Dr. Steven Folkman, for their encouragement, support and assistance throughout the entire process. My appreciation also goes to the staff of the MAE department Lindi Brown, Karen Zobell and Sally Yang who helped me during my study. I would also like to thank Mrs. Christine Spall, our former MAE graduate academic advisor to guide me during my academic progression and all the help she did for me during my study.

I give special thanks to my family, friends, colleagues and the Bangladeshi community here for their encouragement, moral support, and patience as I worked my way to this final document. I could not have done it without all of you.

Mrinmoy Saha
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>PUBLIC ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>ix</td>
</tr>
<tr>
<td>1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2 STRESS CALCULATION IN COMPOSITE I-BEAM</td>
<td>7</td>
</tr>
<tr>
<td>3 MANUFACTURING PROCESS OF COMPOSITE I-BEAM</td>
<td>29</td>
</tr>
<tr>
<td>4 DESIGN CONSIDERATIONS OF COMPOSITE I-BEAM</td>
<td>36</td>
</tr>
<tr>
<td>5 CONCLUSION AND RECOMMENDATION</td>
<td>41</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>43</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>45</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>1: Principal Lamina Coordinate system</td>
<td>9</td>
</tr>
<tr>
<td>2: Off-Axis Lamina Coordinate system</td>
<td>10</td>
</tr>
<tr>
<td>3: Geometry of deformation in x-z plane. Mx and the curvature are positive by convention [21]</td>
<td>12</td>
</tr>
<tr>
<td>4: Laminate nomenclature (view from x-z plane) [20]</td>
<td>15</td>
</tr>
<tr>
<td>5: Coordinate system for Composite I-Beam</td>
<td>18</td>
</tr>
<tr>
<td>6: (a) Composite I-beam with actual web dimension, (b) I-beam with pseudo-web</td>
<td>20</td>
</tr>
<tr>
<td>7: Bending and Shear stress calculation in I-beam</td>
<td>24</td>
</tr>
<tr>
<td>8: Formulas for Maximum deflections of beams [21]</td>
<td>26</td>
</tr>
<tr>
<td>9: Composite I-beam made from two C-sections</td>
<td>28</td>
</tr>
<tr>
<td>11: (a) Symmetric Layup (b) Antisymmetric Layup [3]</td>
<td>30</td>
</tr>
<tr>
<td>12: 24 Ply Composite I-Beam with two C-sections [26]</td>
<td>31</td>
</tr>
<tr>
<td>13: Exploded view of Composite I-Beam manufactured by 5 parts [24]</td>
<td>32</td>
</tr>
<tr>
<td>14: Fabrication of Composite I-beam for 2 C-sections with top, bottom flanges [10]</td>
<td>34</td>
</tr>
<tr>
<td>15: Dimensionless laminate modulus $E_x^*$ for Carbon/polymer composites in the family $[0m/\pm 45p/90q]$ r [21]</td>
<td>46</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( \varepsilon^0_x, \varepsilon^0_y, \gamma^0_{xy} \) Strain components at the mid-surface of a laminate

\( \kappa^0_x, \kappa^0_y, \kappa^0_{xy} \) Midplane curvatures due to bending and twisting

\( \sigma_1, \sigma_2 \) Normal stress in Principal Lamina Coordinate system

\( \tau_{12} \) Shear Stress in Principal Lamina Coordinate system

\( \sigma_x, \sigma_y \) Normal stress in Off-Axis Lamina Coordinate system

\( \tau_{xy} \) Shear Stress in Off-Axis Lamina Coordinate system

\( \varepsilon_1, \varepsilon_2 \) Normal strain in Principal Lamina Coordinate system

\( \gamma_{12} \) Shear strain in Principal Lamina Coordinate system

\( \varepsilon_x, \varepsilon_y \) Normal strain in Off-Axis Lamina Coordinate system

\( \gamma_{xy} \) Shear strain in Off-Axis Lamina Coordinate system

\( E_1, E_2 \) Elastic and transverse Modulus in Principal Lamina Coordinate system

\( G_{12} \) Shear Modulus in Principal Lamina Coordinate system

k Lamina number

\( \theta \) Fiber orientation angle

\( u, v, w \) Components of the displacement along the directions x, y, z

\( u_0, v_0, w_0 \) Components of the mid-plane displacements

\( \gamma_{yz}, \gamma_{xz} \) Transverse shear deformation in x-z, y-z plane

\( N_x, N_y, N_{xy} \) Force Resultants

\( M_x, M_y, M_{xy} \) Moment Resultants

H Total thickness of the laminate
\( \varphi_x, \varphi_y \)  
Angles of rotations.

\( F_x, F_y, F_{xy} \)  
Laminate in-plane strength

\[ A \]  
Extension stiffness matrix

\[ B \]  
Bending-extension coupling stiffness matrix and

\[ D \]  
Bending stiffness matrix.

\( \overline{E_x}, \overline{E_y} \)  
Laminate effective modulus in x and y directions

\( G_{xy} \)  
Laminate effective shear modulus

\( \overline{v_{xy}}, \overline{v_{yx}} \)  
Laminate effective Poisson’s ratios

\( E_{px} \)  
Pseudo-web elasticity modulus

\( \alpha_1 \)  
Load factor

\( \phi \)  
Resistance factor

\( \alpha \)  
Shear correction factor

\( \delta \)  
Deflection of the beam

\( \delta_s \)  
Shear deflection

\( \delta_b \)  
Bending deflection

\( Q \)  
First moment of area

\( V \)  
Shear Force

\( I \)  
Second Moment of area

\( A \)  
Total cross-sectional area

\( z \)  
Distance from mid-surface of a laminate

\( F_{xt}^*, F_{xc}^* \)  
Normalized tensile and compressive Laminate Strength

\( F_{xy}^* \)  
Normalized shear Laminate Strength

\( Z \)  
Section Modulus
\( E_x^*, E_y^* \)  Normalized Moduli

\( \nu_{xy}^* \)  Normalized Poisson’s ratios

\( w(x) \)  Deflection of beam at any point \( x \) on the beam span
CHAPTER 1
INTRODUCTION

Structures have existed since the dawn of time. Structural members are also of various types due to the loads applied to them. One of the most common structural members is beams in which a member is subjected to lateral or vertical loads, that is load is applied perpendicular to the longitudinal axis of the member. Beams can be of different types due to their geometry, cross sections, end supports and equilibrium conditions. In this project, an I-section beam is considered due to their high efficiency, that is if an I-beam is subjected to same loading condition as different cross-section beams of same cross-sectional area, they generally deflect less than other beams. [1] Another advantage is the materials of an I-beam are far away from their neutral axis for which the second moment of inertia increases and as a result, the stiffness of the beam increases. [1]

Beams are made of different types of materials e.g. metal, alloys, concrete etc. Anisotropic materials like fiber reinforced composites are also be used as materials for beams due to their high strength and stiffness to weight ratio, long fatigue life, resistance to electrochemical corrosion and the ability to tailor material properties according to the application. [2] Thin walled composite beams are used extensively in truss members, rotor blade spars, columns, stiffeners and many other aerospace, civil, marine engineering applications. [3]

Numerous studies are done on beams manufactured with isotropic materials [4,5,6,7,8]. Compared to the isotropic materials, the number of studies done on beams made of fiber reinforced composite beams are still small in comparison. To analyze the performance of a beam, it is also important to know about the stress limitation of a beam
from macroscopic point of view. Macroscopic stress state means the stress state of an object viewed in larger perspective. By studying the macroscopic view of stress state of the beam, the performance of the beam can be known i.e. the deflection and the limitation for applied loads for specific application and failure of the beam due to applied loads can be predicted easily. That is why, this project is dedicated to the design and finding out the macroscopic stress state of the beam.

1.1 Literature Review

Extensive studies have been done to study the design and analyzing the mechanical properties of the beams to get a clear idea about their performance. Song, Librescu, Jeong [9] discussed an analytical solution for the static response of thin walled Composite I-beam loaded at their free end developed for 2 types of configuration where transverse shear, warping effects, in-plane undeformed cross section contour were taken into account. Layups were considered from the center of a section, not individually for web and flange. They also mentioned that, different types of elastic couplings in structure can be achieved.

Chandra and Chopra [10] studied the structural response of composite I-beams with tip loads and torsional loads and validated theoretical results via experiment. Layups were considered from center of a section, not individually for the web and top and bottom flanges. They also discussed about fabrication of symmetric composite I-beam of different slenderness ratios using autoclave molding technique from prepreg layers. They highlighted that for flanges with symmetry, relative extensional stiffness is less when transverse shear is accounted.
Shin, Kim and Kim [11] developed a numerical method to calculate exact stiffness matrix for composite I-beam with arbitrary layup. They investigated the vertical displacement, twist angle and relative displacement. The laminate itself was not symmetric but the top and bottom flanges along with web were symmetric individually. In another article, Shin and Kim [12] discussed about developing an improved numerical method which reduces discretization error for analyzing coupled deflection for thin walled monosymmetric I-beams and L-shaped beams and validated the results using 3D finite element model. They used same type angle ply with different cross sections and measured the vertical deflection at the midspan. The top flange was shorter in width than the bottom flange and then increased the width of top flange in order to study the vertical deflection with respect to increasing fiber angle. It was found that the vertical deflection increases with the increment of fiber angle. They also found out from their analysis that shear deformation is more significant in I-section beams than its in L-section beams. In their analysis, they used centroidal axis system on the cross section of the beam.

Jung and Lee [13] modeled the I-beam using two C sections with more layers on top and bottom flanges. They discussed symmetry and anti-symmetry configurations and investigated the effects of elastic coupling, shell wall thickness, transverse shear deformation, warping. Desai and N.K. [2] discussed the analytical and finite element procedures for investigating various mechanical properties of composite I-beam which has same stacking sequence for both web and flanges but different materials in each lamina.

Potter and Davies [14] discussed about manufacturing and designing of composite I-beams using adhesive bonding instead of using mechanical fasteners and investigated its performance under a 3 point and 4-point bending tests and mentioned the failure criteria.
Silvestre and Camotim [15] discussed about formulation of generalized beam theory which incorporates shear deformation, warping effects, elastic coupling effect, cross section in plane deformation to analyze the linear buckling behavior of thin walled composite beam of lipped channel with cross ply and angle ply configuration. In another paper [16], they also discussed developing generalized beam theory for vibration of prismatic composite beams using a lipped channel open section beam with cross-ply and angle ply configuration and investigated the fundamental frequency and vibrational mode shapes.

Lee and Kim [17] developed a method for predicting vibrational mode shapes and natural frequency of composite I-beams for various configurations. Khdeir and Reddy [18] developed an exact solution for displacement for both symmetric and antisymmetric Euler-Bernoulli beam and Timoshenko beam using cross-ply laminates only. They used from 2 – 10 layers and found out that the symmetric cross-ply beams give a smaller deflection than antisymmetric ones whereas in antisymmetric cross-ply arrangements, for same thickness, when the number of layers is increased, the deflection will decrease for all boundary conditions.

From all the literature review, it is known that for designing an I-beam, certain factors need to be considered:

a) The stacking sequences
b) Warping effects
c) Shear-extension coupling
d) Bending- torsion coupling
e) Cross section in-plane deformation

f) Shear deformation and buckling of the flanges and web.

It is also seen from the literature review that, most of these studies have been dedicated to numerical and analytical analysis for stiffness of the beam and FEA modeling. None of these studies have given a clear idea for the macroscopic analysis for the stress state of the beam which is made of fiber reinforced composites.

1.2 Cross Section Profile

In the past researches, some authors used different profiles for I-beam. *Shin and Kim* [12] used an I-beam with shorter width of top flange compared to bottom counterpart. *Potter and Davies* [14] designed an I-beam with cornered layups between the web and both flanges to determine the performance of a secondary bonded skin. The purpose of the additional skin layup was to carry the shear loads and they also mentioned about the problem of this profile which is mismatch between Poisson’s ratio between different components. For this profile, they suggested a solution which is the anisotropic layup of the cornered parts. *Lachenal, Daynes and Weaver* [19] discussed a cross-section profile with curved web on the both sides along the height. The purpose of that design was to withstand large twisting deformation on web and having large load carrying capabilities.
1.3 The Motivation

To properly study the stress state of the beam, at first the material properties of the fiber must be known. After that, getting the material properties for whole beam would be the next step. Addressing the stress state in the joints of flanges and web and the corners will be challenging. The problem is that when the beam is manufactured, there can be some gaps between the flanges and the web, for which the fiber properties will not be present there. Also, at the corners of the beam, the stress can be infinite, thus addressing the stress state there will be challenging. Also coming up with an idea for layup for certain application will require stress state testing either via analytical or numerical method in order to design the beam and recommend a manufacturing idea.

1.4 The Objectives

One objective for this project is to come up with a method for predicting the homogenized properties of the beam considering the material properties of fibers in different direction. The next objective is to find the stress state in any point of the beam using the derived material properties. The third objective is to give a manufacturing approach. The final objective is find a way to design best layup for the fiber reinforced I-beam based on application and addressing the stress state of the beam. These objectives are kept in mind for designing an I-beam for the SAMPE annual competition for manufacturing of I-beam. This project will help the students who wish to compete in any category for the competition and also the industries who are looking forward to producing of I-beams for different application.
CHAPTER 2
STRESS CALCULATION IN COMPOSITE I-BEAM

To calculate the stress at any point of the beam, first, we must come up with a method to get the material properties of the beam. As composite beams are anisotropic meaning they have different material properties in different directions, and non-homogeneous, so a method is needed to compute the smeared properties of the whole beam from the individual properties of web and both flanges. After that, these properties can be used to calculate the stiffness matrix and deflection and from that, the stress calculation for each point of the beam can be done. Each section in this chapter will discuss the formulas and equations and methods to come up with the stress calculation of beams.

2.1 Assumptions for Preliminary design

To come up with the design of composite I-beams certain assumptions need to be made. They are as follows:

i) The material used in the I-beam is homogenous. The fibers and the matrix materials are assumed as one single material which has homogenized properties of fiber and matrix. This assumption was made in order to make the analysis of fiber-reinforced composite easier.

ii) The material is orthotropic. The material may be homogenous, but it will have different properties in different directions. Particularly, the fiber direction will be stronger and stiffer compared to the directions perpendicular to it.
iii) Plane stress is assumed for web and flanges individually since the thickness of the laminate is small compared to the length and width. This assumption is made to simplify the calculation of stiffness and compliance matrix. However, this assumption may lead to inaccuracies, specially near the edge of a laminate and when delamination is considered.

iv) As composite I-beams are thin walled laminates, we can assume Kirchhoff hypothesis for good approximation, that is a line originally straight and perpendicular to the geometric mid-surface of the laminate remains straight after the plate is deformed and the length of the line is constant through the thickness. It implies that the transverse shear strains $\gamma_{xz}$ and $\gamma_{yz}$ are constant through the thickness and normal strain in the thickness direction, $\varepsilon_{zz}$, is approximately zero. This assumption is based on experimental observation and it is accurate and good approximation in most cases.

2.2 Formula for Laminate Stiffness and Laminate compliance matrix

Let us assume a single lamina, which has the material properties $E_1$, $E_2$, $G_{12}$ and $v_{12}$. Here 1 and 2-directions are in the principal lamina co-ordinates (Figure 1) where 1-direction is the fiber direction and 2- direction is perpendicular to the fiber direction in plane 1-2. As plane stress is assumed, the properties for thickness directions are neglected.
Based on the assumptions made in section 2.1, from Hooke’s law, the stress-strain relation in reduced stiffness form for each lamina in principal lamina coordinates [20] is:

\[
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{pmatrix}^k =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
^k
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{pmatrix}
(2.1)
\]

And the reduced compliance form [20] is:

\[
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{pmatrix}^k =
\begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix}
^k
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{pmatrix}
(2.2)
\]

Where, \( S_{11} = \frac{1}{E_1} \); \( S_{22} = \frac{1}{E_2} \); \( S_{12} = -\nu_{12} \); \( S_{21} = -\nu_{21} \); \( S_{66} = \frac{1}{G_{12}} \)

\( (2.3) \)

\( k \) = Lamina number

and \( Q_{11} = \frac{S_{22}}{S_{11} - S_{12}^2} \); \( Q_{22} = \frac{S_{11}}{S_{11} - S_{12}^2} \); \( Q_{12} = -\frac{S_{12}}{S_{11} - S_{12}^2} \); \( Q_{66} = \frac{1}{S_{66}} \)

\( (2.4) \)
Let’s assume now that the fibers in lamina are now orientated in $\theta^\circ$ in the off-axis coordinate system (Figure 2). Equation 2.1 and 2.2 is true for fibers with $0^\circ$ angles between the x-direction and fiber direction i.e. 1-direction. When the fibers are orientated in different angles, the reduced compliance and reduced stiffness matrix become transformed reduced compliance and transformed reduced stiffness matrix.

![Figure 2: Off-Axis Lamina Coordinate system](image)

The off-axis stress-strain relationships in teams of transformed reduced stiffness and compliance matrices [20] are as follows:

\[
\begin{align*}
\begin{aligned}
\{\sigma_x \sigma_y \tau_{xy}\}^k &= \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}^k \cdot \begin{bmatrix} \varepsilon_x \varepsilon_y \gamma_{xy} \end{bmatrix}^k \\
\{\varepsilon_x \varepsilon_y \gamma_{xy}\}^k &= \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix}^k \cdot \begin{bmatrix} \sigma_x \sigma_y \tau_{xy} \end{bmatrix}^k
\end{aligned}
\end{align*}
\]
Where,

\( k = \) lamina number

\( m = \cos \theta \); \( n = \sin \theta \) \hfill (2.7)

\( \overline{S_{11}} = S_{11}m^4 + (2S_{12} + S_{66})n^2m^2 + S_{22}n^4 \) \hfill (2.8)

\( \overline{S_{12}} = (S_{11} + S_{22} - S_{66})n^2m^2 + S_{12}(n^4 + m^4) \) \hfill (2.9)

\( \overline{S_{16}} = (2S_{11} - 2S_{12} - S_{66})nm^3 - (2S_{22} - 2S_{12} - S_{66})n^3m \) \hfill (2.10)

\( \overline{S_{22}} = S_{11}n^4 + (2S_{12} + S_{66})n^2m^2 + S_{22}m^4 \) \hfill (2.11)

\( \overline{S_{26}} = (2S_{11} - 2S_{12} - S_{66})mn^3 - (2S_{22} - 2S_{12} - S_{66})m^3n \) \hfill (2.12)

\( \overline{S_{66}} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})n^2m^2 + S_{66}(n^4 + m^4) \) \hfill (2.13)

\( \overline{Q_{11}} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}n^4 \) \hfill (2.14)

\( \overline{Q_{12}} = (Q_{11} + Q_{22} - 4Q_{66})n^2m^2 + Q_{12}(n^4 + m^4) \) \hfill (2.15)

\( \overline{Q_{16}} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m \) \hfill (2.16)

\( \overline{Q_{22}} = Q_{11}n^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}m^4 \) \hfill (2.17)

\( \overline{Q_{26}} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n \) \hfill (2.18)

\( \overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4) \) \hfill (2.19)
Figure 3: Geometry of deformation in x-z plane. M\textsubscript{x} and the curvature are positive by convention [21]

From Kirchhoff hypothesis, the displacement and strains can be expressed in terms of mid-plane displacements and rotations (Figure 3). The lamina displacements in terms of midplane displacements and rotations are [20] as follows:

\[ u(x, y, z) = u_0(x, y) - z\varphi_x(x, y) \quad (2.20) \]
\[ v(x, y, z) = v_0(x, y) - z\varphi_y(x, y) \quad (2.21) \]
\[ w(x, y, z) = w_0(x, y) \quad (2.22) \]

Where, \( u_0(x, y), v_0(x, y) \) and \( w_0(x, y) \) are mid-plane displacements and \( \varphi_x(x, y), \varphi_y(x, y) \) are angles of rotations (Figure 3). The rotations [20] can be expressed as:

\[ \varphi_x(x, y) = \frac{\partial w_0}{\partial x} \quad (2.23) \]
\[ \varphi_y(x, y) = \frac{\partial w_0}{\partial y} \quad (2.24) \]
The strains at any point of the plate [20] can be expressed as follows:

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix} = \begin{pmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{pmatrix} + z \begin{pmatrix}
\kappa_x^0 \\
\kappa_y^0 \\
\kappa_{xy}^0
\end{pmatrix}
\]  

(2.25)

Where, \(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\) are midplane strains representing the stretching and shear of the plate. \(\kappa_x^0, \kappa_y^0, \kappa_{xy}^0\) are the midplane curvatures due to bending and twisting. All of these parameters [20,21] are expressed as:

\[
\varepsilon_x^0(x, y) = \frac{\partial u_0}{\partial x}
\]  

(2.26)

\[
\varepsilon_y^0(x, y) = \frac{\partial v_0}{\partial y}
\]  

(2.27)

\[
\gamma_{xy}^0(x, y) = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}
\]  

(2.28)

\[
\kappa_x^0(x, y) = -\frac{\partial \varphi_x}{\partial x}
\]  

(2.29)

\[
\kappa_y^0(x, y) = -\frac{\partial \varphi_y}{\partial y}
\]  

(2.30)

\[
\kappa_{xy}^0(x, y) = -\left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}\right)
\]  

(2.31)

The transverse shear strains are expressed as:

\[
\gamma_{yz}(x, y, z) = -\varphi_y + \frac{\partial w_0}{\partial y}
\]  

(2.32)

\[
\gamma_{xz}(x, y, z) = -\varphi_x + \frac{\partial w_0}{\partial x}
\]  

(2.33)
If the midplane strains and curvatures are known at every point within the laminate, strains at any point for other laminas within the laminate can be known. These equations imply that the displacements at x and y-direction and strains vary linearly through the thickness. As a result, stresses vary piecewise linearly through the thickness of the laminate and the reduced stiffness is constant through the thickness for a single lamina but different in each lamina [20,21].

Transverse shear strains are laminate strains which act both inside and between laminas. As the plate thickness is much smaller, we can neglect transverse shear strains and thus \( \gamma_{yz} = \gamma_{xz} = 0 \). So, the rotations and midplane curvatures [20,21] from equation (2.29-2.32) can be expressed as:

\[
\varphi_x = \frac{\partial w_0}{\partial x} \tag{2.34}
\]

\[
\varphi_y = \frac{\partial w_0}{\partial y} \tag{2.35}
\]

\[
\kappa_{x}^0 (x, y) = -\frac{\partial^2 w_0}{\partial x^2} \tag{2.36}
\]

\[
\kappa_{y}^0 (x, y) = -\frac{\partial^2 w_0}{\partial y^2} \tag{2.37}
\]

\[
\kappa_{xy}^0 (x, y) = -2 \left( \frac{\partial^2 w_0}{\partial x \partial y} \right) \tag{2.38}
\]

As strains at every point \((x, y, z)\) are replaced in terms of midplane strains and curvatures, we can also replace stress components at every point \((x, y, z)\) as functions of only \(x\) and \(y\). To do these we need to integrate all the stress components which in result give us the force and moment resultants [20,21].
Using Figure 4, the force and moment resultants [20,21]:

\[
N_x = \int_{-H/2}^{H/2} \sigma_x \; dz \quad (2.39)
\]

\[
N_y = \int_{-H/2}^{H/2} \sigma_y \; dz \quad (2.40)
\]

\[
N_{xy} = \int_{-H/2}^{H/2} \tau_{xy} \; dz \quad (2.41)
\]

\[
M_x = \int_{-H/2}^{H/2} \sigma_x \; z \; dz \quad (2.42)
\]

\[
M_y = \int_{-H/2}^{H/2} \sigma_y \; z \; dz \quad (2.43)
\]

\[
M_{xy} = \int_{-H/2}^{H/2} \tau_{xy} \; z \; dz \quad (2.44)
\]
Replacing equation (2.25) into equation (2.5) and then using the formulas for Force and moment resultant equations from equation (2.39) to (2.44), the following laminate stiffness equation [20,21] is obtained:

\[
\begin{pmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy}
\end{pmatrix} =
\begin{pmatrix}
A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\
A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\
A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\
B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\
B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\
B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\varepsilon_{xy}^0 \\
\kappa_x^0 \\
\kappa_y^0 \\
\kappa_{xy}^0
\end{pmatrix}
\]  

(2.45)

This matrix of coefficients \(A_{ij}, B_{ij}, D_{ij}\) is called the [ABD] matrix or laminate stiffness matrix [20,21] where:

\[
A_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k - z_{k-1})
\]

\[
B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\overline{Q}_{ij})_k (Z_k^2 - Z_{k-1}^2)
\]

\[
D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\overline{Q}_{ij})_k (Z_k^3 - Z_{k-1}^3)
\]

[A] matrix is known as extension stiffness matrix, [B] matrix is known as bending-extension coupling stiffness matrix and [D] matrix is known as bending stiffness matrix. The inverse of the [ABD] matrix is known as the laminate compliance matrix and it is expressed as [abd] matrix [20,21].
The laminate compliance equation [20,21] can be written as:

\[
\begin{pmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\varepsilon_{xy}^0 \\
\kappa_x^0 \\
\kappa_y^0 \\
\kappa_{xy}^0
\end{pmatrix}
= 
\begin{pmatrix}
 a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \\
 a_{12} & a_{22} & a_{26} & b_{12} & b_{22} & b_{26} \\
 a_{16} & a_{26} & a_{66} & b_{16} & b_{26} & b_{66} \\
 b_{11} & b_{12} & b_{16} & d_{11} & d_{12} & d_{16} \\
 b_{12} & b_{22} & b_{26} & d_{12} & d_{22} & d_{26} \\
 b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66}
\end{pmatrix}
\begin{pmatrix}
 N_x \\
 N_y \\
 N_{xy} \\
 M_x \\
 M_y \\
 M_{xy}
\end{pmatrix}
\]

(2.46)

Now, we can formulate the equation for laminate effective modulus in x and y directions shear modulus and Poisson’s ratios [20]. They are expressed as follows:

\[
\overline{E_x} = \frac{1}{a_{11}H}
\]

(2.47)

\[
\overline{E_y} = \frac{1}{a_{22}H}
\]

(2.48)

\[
\overline{G_{xy}} = \frac{1}{a_{66}H}
\]

(2.49)

\[
\overline{\nu_{xy}} = -\frac{a_{12}}{a_{11}}
\]

(2.50)

These equations are used to calculate the effective properties for web of the beam and then the web laminate is replaced by a pseudo-web lamina where the thickness of the web is of same width as the flanges and then the we can get the effective properties of the composite I-beam.
2.3 Laminate properties of Web and Flanges together

The averaged or smeared properties of the flanges are found as follows. First, the number of layers in each flange and web should be known. Then the fiber orientation and individual layer material properties based on fiber must be known. Using this information, the laminate properties for the beam can be found. The first step is to find the stiffness and compliance matrix terms and apply transformation accounting the orientation of the fiber in each lamina. Assuming plane stress, the compliance and reduced stiffness matrix terms of each lamina for both flanges and web including the transformation are calculated based on the equations described in section 2.2. Two separate origins were considered individually for web and for top flange in order to calculate the laminate properties (Figure 5). The laminate properties for the web were calculated at first, then the results were used to create a pseudo-web lamina as the same width as top and bottom flanges and connected with them simultaneously.

Figure 5: Coordinate system for Composite I-beam
To come up with the pseudo lamina for the web, the first step is to come up with the \([A], [B], [D]\) matrices from the web laminas and then calculate the smeared properties of the web, especially \(E_x, E_y, G_{xy}\) and \(v_{xy}\). As the laminas for both web and flanges were assumed to be transversely isotropic, the transverse direction properties i.e. \(y\) and \(z\)-direction properties will be same. Now the laminate properties of the web will become the properties of the single pseudo web lamina which has the same width as the flanges.

Let the width of the flanges is \(W\), the laminate thickness of the web is \(b\) and the web laminate width is \(h\), the modulus of elasticity on \(x\)-direction for web laminate is \(E_x\) and the pseudo web modulus of elasticity at \(x\) direction is \(E_{px}\) (Figure 6). As the beam cross section is changed for pseudo-web, so the modification for modulus of elasticity in \(x\) direction is necessary. From beam theory, we know,

\[
E_x I_A = E_{px} I_w
\]  

(2.52)

Where, \(I_A = \frac{bh^3}{12}\) and \(I_w = \frac{Wh^3}{12}\)

From Equation (2.5), we can find the pseudo-web elasticity modulus which is:

\[
E_{px} = \frac{E_x b}{W}
\]  

(2.53)

Using the above equation, we can find the elasticity modulus for \(x\)-direction of the pseudo-web. Then the lamina properties of the web will be 0\(^\circ\) and with thickness \(h\). As we assume the laminas are transversely isotropic, \(E_y\) of the web is equal to \(E_z\) of the web and the transverse direction modulus for the web is \(E_z\). After we calculate the properties of the pseudo-web, the smeared properties for the composite I-beam can be calculated. A
MATLAB program (See Appendix (2)) has been written to calculate the effective properties of the composite I-beam.

![Composite I-beam with actual web dimension, I-beam with pseudo-web](image)

**Figure 6:** (a) Composite I-beam with actual web dimension, (b) I-beam with pseudo-web

The MATLAB program (Appendix (2)) takes the input for the type of materials first. Material properties for three types of materials – Graphite-Epoxy, Glass-Epoxy and Aluminum in SI unit has been set as default, but other materials can be set using as separate file and modifying the code. The code is run from the output file. It asks input for web first, particularly the width, \( h \) of the web, then number of layers and then it asks for material type, fiber orientation and thickness of each lamina from right to left as shown in Figure 6(a). Then it calculates the effective properties of web and calculates the properties of pseudo-web. After that, it takes the input for both top and bottom flanges. For the program, it was assumed that both flanges have same number of layers. Then after taking input for both flanges, it takes the values of effective properties of pseudo-web calculated previously and then it calculates the effective properties for the composite I-beam in SI unit. For displaying output, it will generate a “.doc” format file and it will consist of all the material
properties, stacking sequences with fiber orientation and thickness, the reduced stiffness and compliance matrix, transformed reduced stiffness and compliance matrix, the [ABD] matrix, the [abd] matrix and the laminate smeared properties and also the flange width and thickness as well as the web width and thickness.

### 2.4 Stress Calculation of Composite I-beam from Equilibrium

After finding all the material properties, we can use the constitutive relationship to find the bending stress. The formula for bending stress in case of an isotropic beam [1,21,22] is also sometimes used for preliminary design for I-beam as referenced in [21]:

\[
\sigma_x = \frac{My_1}{I} \quad (2.54)
\]

Where, \( I = \frac{1}{12} [bh^3 - bh_1^3 + tw_h_1^3] \)

For composite I-beams, both the tensile and compressive stress values must be checked as they are different for a laminate. In order to find the shear stress distribution for both web and flanges on certain point on x-axis, the shear stress for web and flanges [1,22] are as follows:

\[
(\tau_{xy})_{web} = \frac{V}{8lt_w} [b(h^2 - h_1^2) + tw(h_1^2 - 4y_1^2)] \quad (2.55)
\]

\[
(\tau_{xy})_{flange} = \frac{V}{8l} [(h^2 - 4y_1^2)] \quad (2.56)
\]

However, using equation (2.54) (2.55) and (2.56), the bending and shear stress distribution will be linear and parabolic consecutively which is not a good approximate for composite I-beam. As laminate properties are already averaged and each lamina has
different properties $E_x$, $E_y$ and $G_{xy}$, for composite I-beam, the bending and shear stress will be piecewise linear and piecewise parabolic. To calculate the actual stress, the following formulation from beam deflection can be used:

$$D_{11} \frac{d^4w(x)}{dx^4} = (EI) \frac{d^4w(x)}{dx^4} = -f(x) \quad (2.57)$$

Where, $w(x) =$ deflection of the beam at any point of the beam and $f(x)$ is the load at any point of the beam or distributed load, $E$ is the effective laminate properties in longitudinal direction. By applying proper boundary conditions, equation (2.57) then can be integrated to calculate the deflection, $w(x)$ at any point of the beam. Here $D_{11} = (EI)$ which is calculated using the bending moment formula:

$$M = \int \sigma_x z dz \quad (2.58)$$

or, $M = \int \overline{Q}_{11} \varepsilon_x z dz \quad (2.59)$

or, $M = \int \overline{Q}_{11} \kappa(x) z^2 dz \quad (2.60)$

Equation (2.60) can be expressed as,

$$M = \left\{ \frac{1}{3} \sum_{k=1}^{N} (\overline{Q}_{11})_k (Z_k^3 - Z_{k-1}^3) \right\} \kappa(x) \quad (2.61)$$

Equation (2.61) is analogous to $M = EI \frac{d^2w}{dx^2}$ which is why we can say $D_{11} = (EI)$. For calculation of bending stress, the following equations are used:

After knowing the deflection of the beam, we can calculate the curvature, $\kappa(x)$

$$\kappa(x) = - \frac{d^2w(x)}{dx^2} \quad (2.62)$$
From there, we can calculate the strains of the laminate using the following formulation:

$$\varepsilon_x = z\kappa_x(x) \quad (2.63)$$

Assuming strains at other directions are zero except in x-direction, the bending stress can be calculated using the constitutive relationship, which is given as follows:

$$\sigma_x = Q_{11} \varepsilon_x \quad (2.64)$$

For every lamina the bending stress will be different, so it will be a piecewise linear curve. For calculating shear stress, we can use the equilibrium equation:

$$\frac{d\sigma_x}{dx} + \frac{d\tau_{xy}}{dy} + \frac{d\tau_{xz}}{dz} = 0 \quad (2.65)$$

In equation (2.60), assuming $$\frac{d\tau_{xz}}{dz} = 0$$, we can find the shear stress distribution as following:

$$\tau_{xy} = \int \frac{d\sigma_x}{dx} dz = \int \frac{d(Q_{11} z\kappa_x(x))}{dx} dz \quad (2.66)$$

It can be noted that the formula mentioned above is accurate enough to calculate the stresses for definite cross section. So, when the cross-section change occurs these formulas will not be accurate, especially calculating the stress at the connection between the web and flanges. That is because stress concentrations occur. Therefore, stress at the point $$y_1 = h_1/2$$ will not be accurate calculated by equation (2.59) and (2.61). To find a value at that point, FEA will be a good approach [22]. By this method, shear stress at any point within web and flange can be calculated accurately. Composite I-beam yield less accurate results in equilibrium than an isotropic homogenous I-beam due to rapid cross
section change because of having two different Poisson’s ratio and different effective modulus in x and y directions.

Figure 7: Bending and Shear stress calculation in I-beam

The section modulus is useful to size the cross section of the composite I-beam for bending.

the section modulus for the beam [21] is defined as:

$$Z = \frac{I}{y_1} = \frac{\alpha_1 M}{\phi F_x}$$  \hspace{1cm} (2.67)

Where $\alpha_1$ = load factor and $\phi$=resistance factor. $F_x$ = the tensile or compressive strength of the laminate.
2.5 Finding deflections for composite I-beams

The deflection of composite I-beam has bending and shear deflection components. The deflection of the beam [21] can be expressed as:

\[ \delta = \delta_b + \delta_s \]  \hspace{1cm} (2.68)

Where, \( \delta_s \) is the shear deflection and \( \delta_b \) is the bending deflection. The bending deflection is dependent on bending stiffness \( (EI) \) and the shear deflection is dependent on shear stiffness \( (GA) \). For preliminary design, following approximation [21] can be used:

\[ (EI) \approx E_x I \]  \hspace{1cm} (2.69)

\[ (GA) \approx G_{xy} A_c \]  \hspace{1cm} (2.70)

Where, \( E_x, G_{xy} \) are effective modulus calculated from section 2.2. and \( A_c = \alpha A \) where \( A \) is the cross-section area of the beam, and \( \alpha \) is the shear correction factor (for wide flange beam, major axis \( \alpha = A_{web}/A \), minor axis, \( \alpha = (5/6) \cdot (A_{flange}/A) \) [17]). The maximum deflection can be calculated using the formulas shown in Figure 8.

The deflection at any point of the beam span can be calculated in dummy load method where a unit fictitious load or moment is applied at the point of interest. After that, two bending and shear moment diagrams are constructed – \( M(x) \), \( V(x) \) for actual load and \( m(x) \), \( v(x) \) for dummy loads. Then the deflection at the point of interest is calculated using the following equation [21]:

\[ \delta = \int_{0}^{L} \left( \frac{M}{EI} m + \frac{V}{GA} v \right) dx \]  \hspace{1cm} (2.71)
Figure 8: Formulas for Maximum deflections of beams [21]
2.6 Formula for Buckling of composite I-beam

As thin-walled composite beams are subjected to bending, compressive loads are experienced by some portion of the beam, mainly one of the flanges. The formulation of buckling load per unit width for each half flange [21] is as follows:

\[ N_x^{CR} = \frac{12D_{66}}{b^2} + \frac{\pi^2D_{11}}{a^2} \]  \hspace{1cm} (2.72)

Where, \( b \) = half-length of flange, \( a \)= unsupported length of flanges or the region over which compressive load acts [21]. In this formulation, transverse shear deformation effects are neglected, thus the equation is not conservative unless the dimension of flange, \( a \) and \( b \) is much larger than thickness of the flange \( (a, b>20t) \) [21]. This formula was derived assuming that each flange was modeled as two plates simply supported by the web with other edge free. The actual buckling load is larger because web provides rotational constraints on flange [21]. Also this formulation gives erroneous results for balanced symmetric laminate with non-zero values \( D_{16}, D_{26} \) and the magnitude of error depends on the value of \( D_{16}, D_{26} \).

2.7 Stress concentration between the connection of web and flanges

As there are stress concentrations occurs between the connection of web and flanges, one way to solve that problem is to manufacture the beam using two C-sections and reinforcing the flanges with more layups on both flanges. As a result, these layups will have fillet like shapes on corners on the connection of web and flanges which can reduce the stress concentrations but still those areas will experience a large amount of stress.
The total flange for the I beam will be the combination of C-section flanges and the reinforced laminas on the top of C-section flange. The web will be comprised of two C-section web (Figure 9). The web should be thin compared to the flanges because flanges will be carrying more loads spanwise. The web will be subjected to shear loads.

2.8 The Type of Configuration for Web and Flanges

There are mainly three types of composite layup configuration – symmetric, antisymmetric and non-symmetric layups. For composite I-beam, it is better to have a symmetric layup because then we can avoid all couplings between in-plane loads/displacements and out of plane loads/displacements as [B] matrix of [ABD] matrix will come to zero due to symmetric layup.

![Composite I-beam made from two C-sections](image)

Figure 9: Composite I-beam made from two C-sections
CHAPTER 3
MANUFACTURING PROCESS OF COMPOSITE I-BEAM

There are various processes for manufacturing a composite I-beam and it depends mainly on type of matrix and fiber, the temperature for curing the parts, cost effectiveness. Savic, Tuttle and Zabinsky [3] mentioned about resin transfer molding, pultrusion and layup of prepreg. The prepreg can be unidirectional or woven. Smith and Bank [23] used a method for unidirectional prepreg where they cured three separate panels and then adhesively bond the web and flanges (Figure 10(b)). The advantage for this method is the fiber orientation of web and the flanges layup will not depend on each other, any desired layups can be done in each of them. The disadvantage for this layup technique is it will be prone to failure on the web flange connection and will also show a stress concentration on the corners [3].

The second technique is more commonly used [3,24] where I-beams are manufactured using two C-sections placed back-to-back and joined together and then additional laminas added to the top and bottom flanges (Figure 10(a)). As a result, the fibers remain continuous in the web and flange connection region and stiffer and stronger than the beam manufactured by previous technique. However, there is a design limitation as web and some of the flanges will be of same fiber orientations and the stacking sequence of the web will not be independent from the flanges. Another problem is, as most practical composite beam includes both symmetric web and symmetric flange laminate, this design technique restricts to either symmetric web or symmetric flange laminate if angle ply laminas are used. [3, 25] Because the fiber orientation of the lamina on the right side of the web or flange will be opposite on the left side of the web or flange and either the web or
flange will be antisymmetric and the other one will be symmetric (Figure 11). To achieve both symmetric web and symmetric flange laminate, the web has to be comprised of cross-ply laminas [3].

Figure 10: Methods of fabricating Composite I-beam [3]

Figure 11: (a) Symmetric Layup (b) Antisymmetric Layup [3]
For manufacturing composite I-beam in this technique, there is another issue that needs to be addressed. By joining two C-sections, there will be a triangular void between the web and the flanges where there will be no fibers, only matrix materials. This can lead to potential failure as there are no fibers present there. To minimize this, the triangular void can be filled up with tows of same fiber material [24,26]. Gilchrist, Kinloch and Matthews [26] put two ropes of same materials which was made from spirally wound prepreg into the triangular void (Figure 12).

![Diagram of 24 Ply Composite I-beam with two C-sections](image)

**Figure 12:** 24 Ply Composite I-beam with two C-sections [26]
The third technique is based on the 2nd technique and techniques described in [3,24,26]. As the symmetric I-beam design is restricted to the either symmetric flange or symmetric web, to achieve symmetry in both web and flange, the web can be made of 3 parts, the 2 C-sections comprised of cross-ply laminates and the middle section is comprised of angle ply laminas (Figure 13). In this technique, the voids will still be present but instead of one big triangular void, it will be two small triangular voids to fill in. The advantage of using a middle section is it will strengthen the web part as well as the fiber orientation design will be less restricted in web to achieve desired symmetry.

Figure 13: Exploded view of Composite I-beam manufactured by 5 parts [24]
3.1 Fabrication Technique for Composite I-beam

Composite I-beam can be fabricated in many ways such as winding, laying and molding depending on the type of matrix and fiber. The process for manufacturing polymer matrix composites [21] are typically as follows-

a) Placing fibers in desired orientation
b) Impregnating fibers with resins
c) Removal of excess resin, air and volatile materials
d) Curing or solidification of the polymer with fibers
e) Extraction from mold and trimming excess parts

Molding is the convenient way for fabricating composite I-beam in Labs. Tape laying is more popular in industries for layup of composites. Using Prepreg saves the steps before molding. There are several ways for curing composites once it is layered. Heated mold technique is one of the ways, but they are expensive and requires more time to manufacture. Hot press and Autoclave moldings are more advantageous in this regard because of void free and denser fiber composition in matrix. [24] The curing for I-beam parts can be done separately [14,24] or together [10,26] in the autoclave.

Chandra and Chopra [10] discussed co-curing autoclave molding technique for fabricating Bending-torsion coupled graphite epoxy or Kevlar epoxy composite I-beams where they used a metal mold consisting of 2 parts. Each part was wrapped with desired number of layers and then they were compacted by using vacuum between mold and layers. Then additional layers were added on the top and bottom of the mold after placing those two molds back to back to provide more layers on top and bottom flanges. After applying peel ply, bleeder and breather layers, the lay-ups were cured in a microprocessor-controlled
autoclave. After curing cycle, the vacuum bag breeder and bleeder layers were removed, the cured part was trimmed to the desired dimension and thus composite I-beam was made [10,24]. Zhou and Hood [25] also described similar technique for fabrication of composite I-beam. This fabrication process is convenient for composite beams having only 2 C-section and top and bottom flange part and it would save manufacturing time.

Figure 14: Fabrication of Composite I-beam for 2 C-sections with top, bottom flanges [10]

3.2 Suitable Mold for fabrication of Composite I-beam

Different molds can be used for fabrication of composite I-beam. The purpose of using molds are to create parts of composite I-beam with desired shapes. Using metal molds will provide a stiff molding parts which doesn’t deform easily in vacuum and provides better heat transfer during curing of the parts. As a result, matrix concentration is low and the parts are stronger and cleaner. According to Rider and Mayta [24], metal molds i.e.
aluminum molds are more effective than wooden molds to create and to shape best quality parts for composite I-beam. They also used wet layup techniques for making all the samples for composite I-beams and for composite I-Beams they used reinforced materials for web and flanges such as aluminum honeycombs and thick piece of foam to increase the strength of the beam. Gilchrist and Matthews [26] discussed the counteract to the thermal strain effects while producing I-beam parts. They produced mold with slightly tapered channel of 1-2° so that when the spring back of the flanges occur, the web/flange angle would be 90°.
CHAPTER 4
DESIGN CONSIDERATIONS OF COMPOSITE I-BEAM

As composite materials are anisotropic, they offer more design possibilities than isotropic materials. Many acceptable designs can be achieved by controlling various parameters. That is why it is important to address the key controlling parameters which can be used to achieve optimum design. Design optimization can be done for various structural properties. To find the best possible properties for specific application, one must know how to choose the parameters so that the design can be done efficiently. In this chapter, various design parameters will be discussed in order to get an idea about how to choose the design parameters.

4.1 Choosing fiber orientation angles for web and flanges

Let us assume that the web and flanges of an I-beam is symmetric in regards of geometric cross section and in case of stacking sequence. Then for both C-sections, stacking sequence must be in 0° and 90° in order to maintain the symmetry. If a middle part is used for the web, it can have angle plies or more cross ply laminas. The lay-ups can be optimized for individual components of I-beam depending on specific loading condition.

For example, to increase the load capacity for tension and compression loads, 0° and 90° fiber orientation is recommended. As web are the vertical load carrying member, the percentage of 90° is higher than that of 0°. Potter, Davies and others [14] designed an I-beam in which the flanges will act as a skin which is why they put 0° and 90° laminas in
flanges. They also accounted for buckling resistance which is why they used carpet plots to come up with the percentage of different fiber orientated layers. For increasing buckling resistance, they used 10% of 90°, 30% of 45 and 60% of 0° in flanges. For web, they used 10% of 0°, 30% of 45 and 60% of 90° so that the web can withstand the vertical compressive load in the testing and also avoid buckling.

The carpet plot (see appendix (1)) is one of the ways to come up with the layers percentage easily. A single carpet plot is sufficient for coming up with the preliminary design of composites made of any materials. Potter used the carpet plots provided by BAE airbus systems. Barbero [21,27] has developed a method for coming up with universal carpet plots. He discussed the carpet plots for carbon/polymer laminates, but it can be derived for other materials as well using the same methodology for stiffness and strength of the laminate.

There are also some commercially available software’s which can be used to design the beam stacking sequence, Such as LAP developed by Anaglyph Ltd. The key design goal was to minimize the number of coupling terms in laminate stiffness matrix and also achieving lowest possible stresses through the thickness. The design goal should be maximizing $E_x$ for flanges and $G_{xy}$ for webs.

4.2 Cross section of the I-beam

Choosing cross section of a beam is important due to cost effectiveness and weight. As composite I-beams are thin walled, they can be easily achieved. To minimize the cost and weight, the cross section of composite I-beam should be as small as possible. To achieve the required bending stiffness, the second moment of area, I should be increased.
This can be done by reducing the thickness of the web and flanges and increasing the web height. Also, a high second moment of area compensates for low modulus of elasticity of fibers.

The section modulus also can be used to size the cross section for bending. Also, to account for shear, it should be resisted by the web laminates. The design also should satisfy the following equation [21]:

\[
\phi F_{xy} > \frac{QV}{It}
\]

(4.1)

Where, \(F_{xy}\) is the shear strength of the laminate. When the shear strength of the laminate is exceeded, there should be increment in thickness of the web or increment of shear strength of the laminate which can be done by adding more \(\pm 45^\circ\) laminas on web.

### 4.3 Shear deformation of the I-beam

Shear deformations are not significant in case of isotropic beams as shear modulus is high for isotropic materials but for composite beams shear modulus is low which is why shear deformations are significant. Shear deformation varies with the span compared to the bending deformation. The larger the span, the lesser shear deformation is experienced compared to bending deformation [21].

### 4.4 Effect of Reducing warping in I-beam

By constraining the warping of I-beam, several things can be achieved. [10] The torsional stiffness of I-beam is increased greatly by constraining warping deformations of
the beam. For a bending torsion beam, constraining warping can reduce the relative induced twisting at tip with respect to bending slopes. Slenderness ratio also plays a role for controlling the effects of constraining warping deformation on torsional stiffness of the beam. Torsional stiffness of the beam is increased for shorter beams than longer beams in case of constraining warping deformations of I-beams. [10]

4.5 Buckling Of I-beam

When the I-beam is under static loading, the buckling of I-beam occurs on the half of length of compressive flange. [24,26] The buckling had maximum crest or trough amplitude at the edges and zero amplitude at the intersection between web and flange region and the flange area was antisymmetric. If there was a crest on one edge, the directly opposite edge had trough. At 50% of failure load buckling had occurred for carbon epoxy beams [26]

Buckling of the flanges can be prevented if the flange is attached to a panel. It is a common practice in civil engineering for designing bridge deck or when a floor system is supported by beam [21].

4.6 Failure of composite I-beams

Composite I-beam failure tends to initiate first at the edge of crest/ trough buckling region of compressive flange due to local tensile mode1 stresses. Then failure propagates through compressive flange and then to the web regions. If the composite I-beams has holes
in web as a design, failure tends to occur at those regions first and fails at lower loads than that of webs with no holes. [26] The tensile and compressive load damage can be seen in form of matrix cracking, whereas fiber fracture and delamination can be seen due to local tensile and compressive stresses within the web region.

4.7 Miscellaneous design considerations of the I-beam

a) Increasing the orthotropy ratio i.e. \( \left( \frac{E_{px}}{E_y} \right) \) will decrease the maximum deflection of the beam [18].

b) Decreasing the web height will increase the deflection of the I-beam [21]. This is because as web height decreases, second moment of inertia, I decreases and maximum stress, deflection increase. Although it decreases the stiffness of the beam, it increases the stability of the beam [21].

c) Transverse shear deformations are negligible for thin walled symmetric I-beams when they are subjected to bending and torsional loads [10].

d) Coupling effects can be reduced by using balanced, biaxial woven or stitched mats. Angle plies (\( \pm \theta \)) should be used as pairs and allocated as close as possible to the middle surface [21].
CHAPTER 5
CONCLUSION AND RECOMMENDATION

Building upon previous studies of composite beam design, a design overview of the composite I-beam has been discussed from classical lamination theory where it includes the homogenization approach, the plane stress assumption and Kirchhoff hypothesis. Ideas to calculate maximum deflection and maximum bending stress and shear stress and also the stress concentration at the connection of web-flange has been discussed which gives insight for designing and manufacturing the I-beam.

Ideas for manufacturing I-beams are also provided and also critical design considerations have been discussed. For manufacturing, it is good to fabricate composite I-beam with 2 C-sections with additional web and flange part, laying the plies in metal molds and curing it in an autoclave to have better fiber volume fraction. The design consideration for stacking sequences, cross section, length of the beam, failure, buckling has been discussed to give an insight to design the I-beam towards individual intended design goals more effectively. For stacking sequences, it is found that the sequencing of the layers depends on configuration and loads applied on the beam. To counter shear, 45° layers are needed and to counter tension and compression, cross-ply are best. Minimizing cross section while having more web height compared to flange width will strengthen the beam.

In the future, the same study can be done for other types of cross section. In this study, only unidirectional fibers are considered, and it can be expanded towards woven
fibers and also natural fibers as well. A good optimization study is also another possibility from this study as well.
REFERENCES


[16] Silvestre N., Camotim D., *Generalized Beam Theory to Analyze the Vibration of Open-Section Thin-Walled Composite Members*, 2013, (ASCE)EM.1943-7889.0000507


1) **Carpet Plots** [21,27]:

Carpet plots are also useful to calculate the laminate Moduli $E_x, G_{xy}$ of each web and flange panels, as a function of percentage of $0^\circ$, $90^\circ$ and $\pm45^\circ$ where the laminate nomenclature for each panel is given by –

$$[0_m/\pm45_p/90_q]_{r,s}$$

Here $m, p, q =$ number of plies of $0^\circ, \pm45^\circ$ and $90^\circ$ simultaneously, $S =$ Symmetric.

The carpet plot uses normalized moduli by the trace. The trace is invariant with respect to rotations and it can be found from materials data. The laminate moduli can be expressed in terms of trace and normalized moduli, which are given as following:

$E_x = E_x^* \times tr(Q); \; E_y = E_y^* \times tr(Q); \; \nu_{xy} = \nu_{xy}^*$

$G_{xy} = G_{12} + G' \frac{x}{100} \text{ where } x = \text{percentage of } \pm45$

$$G' = (Q_{11} + Q_{22} - 2Q_{12})/(4 - G_{12})$$

$$tr(Q) = Q_{11} + Q_{22} + 2Q_{66}$$

The laminate strength values ($F_{xt}, F_{xc}, F_{xy}$) are also another parameter used in carpet plots and thus it can be used to design the strength properties of each web and flange.

$$F_{xt} = F_{xt}^* E_1 \varepsilon_{1t};$$

$$F_{xc} = F_{xc}^* E_1 \varepsilon_{1c};$$

$$F_{xy} = F_{xy}^* E_1 \varepsilon_{1c};$$

$$F_{xy}^* = 0.0884 + 0.3524 \frac{x}{100} \text{ where } x = \text{percentage of } \pm45 \text{ (carbon/polymer)}$$

$$F_{xy}^* = 0.4010 + 0.1741 \frac{x}{100} \text{ where } x = \text{percentage of } \pm45 \text{ (glass/polymer)}$$

Figure 15: Dimensionless laminate modulus $E_x^*$ for Carbon/polymer composites in the family [0m/ ± 45p/90q] r [21]
2) MATLAB Code for finding the Effective properties:

**Material 1 file:**

```matlab
%Properties of Graphite Polymer composite%
E_1 = 155000000000;
E_2 = 12100000000;
u_12 = .248;
u_21= (E_2*nu_12)/E_1;
u_13 = .248;
u_23 = .458;
G_12 = 4400000000;
a_1 = -0.01800*10^-6;
a_2 = 24.3*10^-6;

Sig1T= 1500*10^6;
Sig1C= 1250*10^6;
Sig2T= 50*10^6;
Sig2C= 200*10^6;
Taul12F= 100*10^6;
```

**Material 2 file:**

```matlab
%Properties of Glass Polymer composite%
E_1 = 50000000000; 
E_2 = 15200000000;
u_12 = 0.254;
u_21 = (E_2*nu_12)/E_1;
u_13 = 0.254;
u_23 = 0.428;
G_12 = 4700000000;
a_1 = 6.34*10^-6;
a_2 = 23.3*10^-6;

Sig1T= 1000*10^6;
Sig1C= -600*10^6;
Sig2T= 30*10^6;
Sig2C= -120*10^6;
Taul12F= 70*10^6;
```
**Material 3 file:**

Properties of Aluminum%
E_1 = 72400000000;
E_2 = 72400000000;
nu_12 = 0.3;
nu_21 = (E_2*nu_12)/E_1;
nu_13 = 0.3;
nu_23 = 0.3;
G_12 = E_1/(2*(1+nu_12));

a_1 = 22.5*10^-6;
a_2 = 22.5*10^-6;

**I-beam Web Properties:**

clear all;
close all;
c1c;

h_w = input('Enter the web height h_w = ');
l_b=input('Number of layers for web part of the beam :n');

for j = 1:l_b

material_w(j) = input('Enter Material Type: n Press 1 for Graphite Polymer n Press 2 for Glass Polymer n Press 3 for Aluminum n ');
if material_w(j)==1
material1;
E_1w(j)= E_1;
E_2w(j)=E_2;
nu_12w(j)=nu_12;
G_12w(j)=G_12;
elseif material_w(j)==2
material2;
E_1w(j)= E_1;
E_2w(j)=E_2;
nu_12w(j)=nu_12;
G_12w(j)=G_12;
elseif material_w(j)==3
material3;
E_1w(j)= E_1;
E_2w(j)=E_2;
nu_12w(j)=nu_12;
G_12w(j)=G_12;
else
    while material_w(j)~=1 & & material_w(j)~=2 & & material_w(j)~=3
        fprintf('Invalid Input! n Please press n 1 for graphite polymer n 2 for Glass Polymer n 3 for Aluminum\n');
        material_w(j) = input('');
    end
theta_w(j) = input('Enter Orientation Angle in Degrees \n');
h_k_w(j) = input('Enter Lamina Thickness in SI units \n');

% s matrix elements
s_11w(j) = 1/E_1w(j);
s_22w(j) = 1/E_2w(j);
s_12w(j) = - n_2w(j)/E_1w(j);
s_66w(j) = 1/G_12w(j);

% Q matrix elements
q_11w(j) = s_22w(j)/((s_11w(j)*s_22w(j)) - ((s_12w(j))^2));
q_22w(j) = s_11w(j)/((s_11w(j)*s_22w(j)) - ((s_12w(j))^2));
q_66w(j) = 1/s_66w(j);
q_12w(j) = -(s_12w(j)/((s_11w(j)*s_22w(j)) - ((s_12w(j))^2)));

% s_bar matrix elements
m_w(j)=cosd(theta_w(j));
n_w(j)=sind(theta_w(j));
sb_11w(j) = (s_11w(j)*m_w(j)^4)+((2*s_12w(j))+s_66w(j))*(n_w(j)^2)*(m_w(j)^2)+(s_22w(j)*n_w(j)^4);
sb_22w(j) = (s_22w(j)*m_w(j)^4)+((2*s_12w(j))+s_66w(j))*(n_w(j)^2)*(m_w(j)^2)+(s_11w(j)*n_w(j)^4);
(sb_12w(j)) = ((s_11w(j)+s_22w(j)-s_66w(j))*(n_w(j)^4)+(m_w(j)^4)));
sb_16w(j) = ((2*s_11w(j)-2*s_12w(j)-s_66w(j))^n_w(j)^4)+(m_w(j)^4)));
sb_66w(j) = ((2*s_11w(j)-2*s_12w(j)-s_66w(j))^n_w(j)^4)+(m_w(j)^4)));

% q_bar matrix elements
qb_11w(j) = (q_11w(j)*m_w(j)^4)+((2*q_12w(j))+4*q_66w(j))*(n_w(j)^2)*(m_w(j)^2)+(q_22w(j)*n_w(j)^4);
qb_22w(j) = (q_22w(j)*m_w(j)^4)+((2*q_12w(j))+4*q_66w(j))*(n_w(j)^2)*(m_w(j)^2)+(q_11w(j)*n_w(j)^4);
qb_12w(j) = ((q_11w(j)+q_22w(j)-(4*q_66w(j))^n_w(j)^4)+(m_w(j)^4)));
qb_16w(j) = ((q_11w(j)+q_22w(j)-(4*q_66w(j))^n_w(j)^4)+(m_w(j)^4)));
qb_66w(j) = ((q_11w(j)+q_22w(j)-(4*q_66w(j))^n_w(j)^4)+(m_w(j)^4)));

% matrix construction
s_w(:,:,j)  = [s_11w(j),s_12w(j),0; s_12w(j),s_22w(j),0; 0,0,s_66w(j)];
q_w(:,:,j)  = [q_11w(j),q_12w(j),0; q_12w(j),q_22w(j),0; 0,0,q_66w(j)];
sb_w(:,:,j)  = [sb_11w(j),sb_12w(j),sb_16w(j); sb_12w(j),sb_22w(j),sb_26w(j); sb_16w(j),sb_26w(j),sb_66w(j)];
qb_w(:,:,j) = [qb_11w(j),qb_12w(j),qb_16w(j); qb_12w(j),qb_22w(j),qb_26w(j); qb_16w(j),qb_26w(j),qb_66w(j)];
end

% total thickness of the layer and z values%
b_w= sum(h_k_w);

% here I defined z(0) as z(1) as Matlab does not accept zero index %
z_w = - b_w/2;
for j=1:l_b
z_w(j)=z_w+h_k_w(j);
for j=2:l_b
z_w(j)=z_w(j-1)+h_k_w(j);
end
end

% A matrix%
A11w = [qb_11w]*[h_k_w].';
A12w = [qb_12w]*[h_k_w].';
A16w = [qb_16w]*[h_k_w].';
A22w = [qb_22w]*[h_k_w].';
A26w = [qb_26w]*[h_k_w].';
A66w = [qb_66w]*[h_k_w].';
A_W = [A11w,A12w,A16w,A22w,A26w,A66w];

% B matrix%
for j = 1
Bs11w(j) = 0.5 * qb_11w(j) * ((z_w(j)^2)- (- b_w/2)^2);
Bs12w(j) = 0.5 * qb_12w(j) * ((z_w(j)^2)- (- b_w/2)^2);
Bs16w(j) = 0.5 * qb_16w(j) * ((z_w(j)^2)- (- b_w/2)^2);
Bs22w(j) = 0.5 * qb_22w(j) * ((z_w(j)^2)- (- b_w/2)^2);
Bs26w(j) = 0.5 * qb_26w(j) * ((z_w(j)^2)- (- b_w/2)^2);
Bs66w(j) = 0.5 * qb_66w(j) * ((z_w(j)^2)- (- b_w/2)^2);
for j=2:l_b
Bs11w(j) = 0.5 * qb_11w(j) * ((z_w(j)^2)- (z_w(j-1))^2);
Bs12w(j) = 0.5 * qb_12w(j) * ((z_w(j)^2)- (z_w(j-1))^2);
Bs16w(j) = 0.5 * qb_16w(j) * ((z_w(j)^2)- (z_w(j-1))^2);
Bs22w(j) = 0.5 * qb_22w(j) * ((z_w(j)^2)- (z_w(j-1))^2);
Bs26w(j) = 0.5 * qb_26w(j) * ((z_w(j)^2)- (z_w(j-1))^2);
Bs66w(j) = 0.5 * qb_66w(j) * ((z_w(j)^2)- (z_w(j-1))^2);
end
end
\[ B_{11w} = \text{sum}(B_{s11w}); \]
\[ B_{12w} = \text{sum}(B_{s12w}); \]
\[ B_{16w} = \text{sum}(B_{s16w}); \]
\[ B_{22w} = \text{sum}(B_{s22w}); \]
\[ B_{26w} = \text{sum}(B_{s26w}); \]
\[ B_{66w} = \text{sum}(B_{s66w}); \]

\[ B_W = \begin{bmatrix} B_{11w}, B_{12w}, B_{16w}; & B_{12w}, B_{22w}, B_{26w}; & B_{16w}, B_{26w}, B_{66w} \end{bmatrix}; \]

\text{% D matrix%}  
\text{for } j = 1 
\hspace{1em} D_{s11w}(j) = \frac{1}{3} \cdot q_b_{11w}(j) \cdot (z_w(j)^3 - (-b_w/2)^3); 
\hspace{1em} D_{s12w}(j) = \frac{1}{3} \cdot q_b_{12w}(j) \cdot (z_w(j)^3 - (-b_w/2)^3); 
\hspace{1em} D_{s16w}(j) = \frac{1}{3} \cdot q_b_{16w}(j) \cdot (z_w(j)^3 - (-b_w/2)^3); 
\hspace{1em} D_{s22w}(j) = \frac{1}{3} \cdot q_b_{22w}(j) \cdot (z_w(j)^3 - (-b_w/2)^3); 
\hspace{1em} D_{s26w}(j) = \frac{1}{3} \cdot q_b_{26w}(j) \cdot (z_w(j)^3 - (-b_w/2)^3); 
\hspace{1em} D_{s66w}(j) = \frac{1}{3} \cdot q_b_{66w}(j) \cdot (z_w(j)^3 - (-b_w/2)^3); 
\text{for } j = 2:1:b 
\hspace{1em} D_{s11w}(j) = \frac{1}{3} \cdot q_b_{11w}(j) \cdot (z_w(j)^3 - (z_w(j-1))^3); 
\hspace{1em} D_{s12w}(j) = \frac{1}{3} \cdot q_b_{12w}(j) \cdot (z_w(j)^3 - (z_w(j-1))^3); 
\hspace{1em} D_{s16w}(j) = \frac{1}{3} \cdot q_b_{16w}(j) \cdot (z_w(j)^3 - (z_w(j-1))^3); 
\hspace{1em} D_{s22w}(j) = \frac{1}{3} \cdot q_b_{22w}(j) \cdot (z_w(j)^3 - (z_w(j-1))^3); 
\hspace{1em} D_{s26w}(j) = \frac{1}{3} \cdot q_b_{26w}(j) \cdot (z_w(j)^3 - (z_w(j-1))^3); 
\hspace{1em} D_{s66w}(j) = \frac{1}{3} \cdot q_b_{66w}(j) \cdot (z_w(j)^3 - (z_w(j-1))^3); 
\text{end} 
\text{end} 

\[ D_{11w} = \text{sum}(D_{s11w}); \]
\[ D_{12w} = \text{sum}(D_{s12w}); \]
\[ D_{16w} = \text{sum}(D_{s16w}); \]
\[ D_{22w} = \text{sum}(D_{s22w}); \]
\[ D_{26w} = \text{sum}(D_{s26w}); \]
\[ D_{66w} = \text{sum}(D_{s66w}); \]

\[ D_W = \begin{bmatrix} D_{11w}, D_{12w}, D_{16w}; & D_{12w}, D_{22w}, D_{26w}; & D_{16w}, D_{26w}, D_{66w} \end{bmatrix}; \]

\text{% ABD matrix and abd matrix%}  
\[ ABD_w = [A_W, B_W; B_W, D_W]; \]
\[ \text{abd}_w = \text{inv}(ABD_w); \]
\[ a_W = [\text{abd}_w(1,1), \text{abd}_w(1,2), \text{abd}_w(1,3); \text{abd}_w(2,1), \text{abd}_w(2,2), \text{abd}_w(2,3); \text{abd}_w(3,1), \text{abd}_w(3,2), \text{abd}_w(3,3)]; \]
\[ b_W = [\text{abd}_w(1,4), \text{abd}_w(1,5), \text{abd}_w(1,6); \text{abd}_w(2,4), \text{abd}_w(2,5), \text{abd}_w(2,6); \text{abd}_w(3,4), \text{abd}_w(3,5), \text{abd}_w(3,6)]; \]
\[ d_W = [\text{abd}_w(4,4), \text{abd}_w(4,5), \text{abd}_w(4,6); \text{abd}_w(5,4), \text{abd}_w(5,5), \text{abd}_w(5,6); \text{abd}_w(6,4), \text{abd}_w(6,5), \text{abd}_w(6,6)]; \]
%Web smeared Properties

\[ Eb_x_w = \frac{1}{a_W(1,1) \cdot b_w} \]
\[ Eb_y_w = \frac{1}{a_W(2,2) \cdot b_w} \]
\[ G_{b_{xy}}_w = \frac{1}{a_W(3,3) \cdot b_w} \]
\[ nub_{xy}_w = -\frac{a_W(1,2)}{a_W(1,1)} \]
\[ nub_{yx}_w = -\frac{a_W(1,2)}{a_W(2,2)} \]
\[ etab_{xy_x}_w = \frac{a_W(1,3)}{a_W(1,1)} \]
\[ etab_{xy_y}_w = \frac{a_W(2,3)}{a_W(2,2)} \]
\[ etab_{x_{xy}}_w = \frac{a_W(1,3)}{a_W(3,3)} \]
\[ etab_{y_{xy}}_w = \frac{a_W(2,3)}{a_W(3,3)} \]

I-beam Effective properties:

Ibeamweb

\[ W = \text{input('Width of the flange in meters = \n')} \]
\[ l = \text{input('Number of layers:\n')} \]

\textbf{for} i = 1:l

\textbf{if} i<((l-1)/2)+1 || i>((l-1)/2)+1

\[ \text{material}(i) = \text{input('Enter Material Type: \n Press 1 for Graphite Polymer \n Press 2 for Glass Polymer \n Press 3 for Aluminum \n ')} \]

\textbf{if} material(i)==1

\[ \text{material1} \]
\[ E_{1m}(i) = E_1 \]
\[ E_{2m}(i) = E_2 \]
\[ nu_{12m}(i) = nu_{12} \]
\[ G_{12m}(i) = G_{12} \]
\textbf{elseif} material(i)==2

\[ \text{material2} \]
\[ E_{1m}(i) = E_1 \]
\[ E_{2m}(i) = E_2 \]
\[ nu_{12m}(i) = nu_{12} \]
\[ G_{12m}(i) = G_{12} \]
\textbf{elseif} material(i)==3

\[ \text{material3} \]
\[ E_{1m}(i) = E_1 \]
\[ E_{2m}(i) = E_2 \]
\[ nu_{12m}(i) = nu_{12} \]
\[ G_{12m}(i) = G_{12} \]
\textbf{else}

\textbf{while} material(i)~=1 && material(i)~=2 && material(i)~=3

\[ \text{fprintf('Invalid Input! \n Please press \n 1 for graphite polymer \n 2 for Glass Polymer \n 3 for Aluminum\n')} \]
\[ \text{material}(i) = \text{input('')} \]
theta(i) = input('Enter Orientation Angle in Degrees \n');
h_k(i) = input('Enter Lamina Thickness in SI units \n');

elseif i == ((l-1)/2)+1
    E_1m(i) = (Eb_x_w*b_w)/W;
    E_2m(i) = Eb_y_w;
    nu_12m(i) = nub_xy_w;
    G_12m(i) = Gb_xy_w;
    h_k(i) = h_w;
    theta(i) = 0;
end

for i= 1:l
    % s matrix elements
    s_11(i) = 1/E_1m(i);
    s_22(i) = 1/E_2m(i);
    s_12(i) = -nu_12m(i)/E_1m(i);
    s_66(i) = 1/G_12m(i);

    % Q matrix elements
    q_11(i) = s_22(i)/((s_11(i)*s_22(i)) - ((s_12(i))^2));
    q_22(i) = s_11(i)/((s_11(i)*s_22(i)) - ((s_12(i))^2));
    q_66(i) = 1/s_66(i);
    q_12(i) = -s_12(i)/((s_11(i)*s_22(i)) - ((s_12(i))^2));

    % s_bar matrix elements
    m(i) = cosd(theta(i));
    n(i) = sind(theta(i));
    sb_11(i) = (s_11(i)*m(i)^4) + ((2*s_12(i)) + s_66(i))*(n(i)^2)*(m(i)^2) + (s_22(i)*n(i)^4);
    sb_22(i) = (s_22(i)*m(i)^4) + ((2*s_12(i)) + s_66(i))*(n(i)^2)*(m(i)^2) + (s_11(i)*n(i)^4);
    sb_12(i) = ((s_11(i)+s_22(i)-s_66(i))*(n(i)^2)*(m(i)^2)) + (s_12(i)*((n(i)^4)+(m(i)^4)));
    sb_16(i) = (((2*s_11(i)) - (2*s_12(i)) - s_66(i)) * n(i) * m(i)^3) - ((2*s_22(i)) - (2*s_12(i)) - s_66(i)) * m(i) * n(i)^3;
    sb_26(i) = (((2*s_11(i)) - (2*s_12(i)) - s_66(i)) * m(i) * n(i)^3) - ((2*s_22(i)) - (2*s_12(i)) - s_66(i)) * n(i) * m(i)^3;
    sb_66(i) = 2*((s_11(i)+s_22(i)-s_66(i))*(n(i)^2)*(m(i)^2)) + (s_12(i)*((n(i)^4)+(m(i)^4)));

    % q_bar matrix elements
\[
\begin{align*}
\text{qb}_{11}(i) &= (q_{11}(i) * m(i)^4) + ((2* q_{12}(i)) + (4* q_{66}(i))) * (n(i)^2) * (m(i)^2) + (q_{22}(i) * n(i)^4), \\
\text{qb}_{22}(i) &= (q_{22}(i) * m(i)^4) + ((2* q_{12}(i)) + (4* q_{66}(i))) * (n(i)^2) * (m(i)^2) + (q_{11}(i) * n(i)^4), \\
\text{qb}_{12}(i) &= ((q_{11}(i) + q_{22}(i) - (4* q_{66}(i))) * (n(i)^2) * (m(i)^2)) + (q_{12}(i) * (n(i)^4) + (m(i)^4)), \\
\text{qb}_{16}(i) &= ((q_{11}(i) - q_{12}(i) - (2* q_{66}(i))) * n(i) * m(i)^3) - ((q_{12}(i) - q_{22}(i) + (2* q_{66}(i))) * m(i) * n(i)^3), \\
\text{qb}_{26}(i) &= ((q_{11}(i) - q_{12}(i) - (2* q_{66}(i))) * m(i) * n(i)^3) - ((q_{12}(i) - q_{22}(i) + (2* q_{66}(i))) * n(i) * m(i)^3), \\
\text{qb}_{66}(i) &= ((q_{11}(i) + q_{22}(i) - (2* q_{12}(i)) - (2* q_{66}(i))) * (n(i)^2) * (m(i)^2)) + (q_{66}(i) * (n(i)^4) + (m(i)^4)).
\end{align*}
\]

% matrix construction

\[
\begin{align*}
s(:,:,i) &= [s_{11}(i), s_{12}(i), 0; s_{12}(i), s_{22}(i), 0; 0, 0, s_{66}(i)]; \\
q(:,:,i) &= [q_{11}(i), q_{12}(i), 0; q_{12}(i), q_{22}(i), 0; 0, 0, q_{66}(i)]; \\
sb(:,:,i) &= [sb_{11}(i), sb_{12}(i), sb_{16}(i); sb_{12}(i), sb_{22}(i), sb_{26}(i); sb_{16}(i), sb_{26}(i), sb_{66}(i)]; \\
qb(:,:,i) &= [qb_{11}(i), qb_{12}(i), qb_{16}(i); qb_{12}(i), qb_{22}(i), qb_{26}(i); qb_{16}(i), qb_{26}(i), qb_{66}(i)].
\end{align*}
\]

% total thickness of the layer and z values%

\[
\text{TT} = \text{sum}(h_k);
\]

% here I defined z(0) as z(1) as Matlab does not accept zero index %

\[
\begin{align*}
z &= -\text{TT}/2; \\
\text{for } i &= 1 \\
z(i) &= z + h_k(i); \\
\text{for } i &= 2:1 \\
z(i) &= z(i-1) + h_k(i); \\
\end{align*}
\]

% A matrix%

\[
\begin{align*}
\text{A11} &= [\text{qb}_{11}][h_k].'; \\
\text{A12} &= [\text{qb}_{12}][h_k].'; \\
\text{A16} &= [\text{qb}_{16}][h_k].'; \\
\text{A22} &= [\text{qb}_{22}][h_k].'; \\
\text{A26} &= [\text{qb}_{26}][h_k].'; \\
\text{A66} &= [\text{qb}_{66}][h_k].';
\end{align*}
\]

\[
\begin{align*}
\text{A} &= [\text{A11}, \text{A12}, \text{A16}, \text{A12}, \text{A22}, \text{A26}, \text{A16}, \text{A26}, \text{A66}];
\end{align*}
\]

% B matrix%

\[
\begin{align*}
\text{Bs11}(i) &= 0.5 * \text{qb}_{11}(i) * ((z(i)^2) - (-\text{TT}/2)^2);
\end{align*}
\]
Bs_{12}(i) = 0.5 \cdot q_{b_{12}}(i) \cdot ((z(i)^2) - (-TT/2)^2);
Bs_{16}(i) = 0.5 \cdot q_{b_{16}}(i) \cdot ((z(i)^2) - (-TT/2)^2);
Bs_{22}(i) = 0.5 \cdot q_{b_{22}}(i) \cdot ((z(i)^2) - (-TT/2)^2);
Bs_{26}(i) = 0.5 \cdot q_{b_{26}}(i) \cdot ((z(i)^2) - (-TT/2)^2);
Bs_{66}(i) = 0.5 \cdot q_{b_{66}}(i) \cdot ((z(i)^2) - (-TT/2)^2);
for i=2:l
Bs_{11}(i) = 0.5 \cdot q_{b_{11}}(i) \cdot ((z(i)^2) - (z(i-1))^2);
Bs_{12}(i) = 0.5 \cdot q_{b_{12}}(i) \cdot ((z(i)^2) - (z(i-1))^2);
Bs_{16}(i) = 0.5 \cdot q_{b_{16}}(i) \cdot ((z(i)^2) - (z(i-1))^2);
Bs_{22}(i) = 0.5 \cdot q_{b_{22}}(i) \cdot ((z(i)^2) - (z(i-1))^2);
Bs_{26}(i) = 0.5 \cdot q_{b_{26}}(i) \cdot ((z(i)^2) - (z(i-1))^2);
Bs_{66}(i) = 0.5 \cdot q_{b_{66}}(i) \cdot ((z(i)^2) - (z(i-1))^2);
end

B_{11} = \text{sum}(Bs_{11});
B_{12} = \text{sum}(Bs_{12});
B_{16} = \text{sum}(Bs_{16});
B_{22} = \text{sum}(Bs_{22});
B_{26} = \text{sum}(Bs_{26});
B_{66} = \text{sum}(Bs_{66});

B = [B_{11}, B_{12}, B_{16}; B_{12}, B_{22}, B_{26}; B_{16}, B_{26}, B_{66}];

% D matrix %
for i = 1
Ds_{11}(i) = (1/3) \cdot q_{b_{11}}(i) \cdot ((z(i)^3) - (-TT/2)^3);
Ds_{12}(i) = (1/3) \cdot q_{b_{12}}(i) \cdot ((z(i)^3) - (-TT/2)^3);
Ds_{16}(i) = (1/3) \cdot q_{b_{16}}(i) \cdot ((z(i)^3) - (-TT/2)^3);
Ds_{22}(i) = (1/3) \cdot q_{b_{22}}(i) \cdot ((z(i)^3) - (-TT/2)^3);
Ds_{26}(i) = (1/3) \cdot q_{b_{26}}(i) \cdot ((z(i)^3) - (-TT/2)^3);
Ds_{66}(i) = (1/3) \cdot q_{b_{66}}(i) \cdot ((z(i)^3) - (-TT/2)^3);
for i=2:l
Ds_{11}(i) = (1/3) \cdot q_{b_{11}}(i) \cdot ((z(i)^3) - (z(i-1))^3);
Ds_{12}(i) = (1/3) \cdot q_{b_{12}}(i) \cdot ((z(i)^3) - (z(i-1))^3);
Ds_{16}(i) = (1/3) \cdot q_{b_{16}}(i) \cdot ((z(i)^3) - (z(i-1))^3);
Ds_{22}(i) = (1/3) \cdot q_{b_{22}}(i) \cdot ((z(i)^3) - (z(i-1))^3);
Ds_{26}(i) = (1/3) \cdot q_{b_{26}}(i) \cdot ((z(i)^3) - (z(i-1))^3);
Ds_{66}(i) = (1/3) \cdot q_{b_{66}}(i) \cdot ((z(i)^3) - (z(i-1))^3);
end
end

D_{11} = \text{sum}(Ds_{11});
D_{12} = \text{sum}(Ds_{12});
D_{16} = \text{sum}(Ds_{16});
D_{22} = \text{sum}(Ds_{22});
D_{26} = \text{sum}(Ds_{26});
D_{66} = \text{sum}(Ds_{66});

D = [D_{11}, D_{12}, D_{16}; D_{12}, D_{22}, D_{26}; D_{16}, D_{26}, D_{66}];

% ABD matrix and abd matrix %
ABD = [A, B; B, D];
abd = inv(ABD);
a = [abd(1,1), abd(1,2), abd(1,3); abd(2,1), abd(2,2), abd(2,3); abd(3,1), abd(3,2), abd(3,3)];
b = [abd(1,4), abd(1,5), abd(1,6); abd(2,4), abd(2,5), abd(2,6); abd(3,4), abd(3,5), abd(3,6)];
d = [abd(4,4), abd(4,5), abd(4,6); abd(5,4), abd(5,5), abd(5,6); abd(6,4), abd(6,5), abd(6,6)];

% Beam smeared properties

Eb_x = 1/(a(1,1)*TT);
Eb_y = 1/(a(2,2)*TT);
Gb_xy = 1/(a(3,3)*TT);
nub_xy = -a(1,2)/a(1,1);
nub_yx = -a(1,2)/a(2,2);
etab_xy_x = a(1,3)/a(1,1);
etab_xy_y = a(2,3)/a(2,2);
etab_x_xy = a(1,3)/a(3,3);
etab_y_xy = a(2,3)/a(3,3);

Output file:

Ibeamflange;

% Output representations%
f=fopen('C:\Users\Mrinmoy\Documents\MATLAB\Trial 1.doc','w');
% Title
fprintf(f,'                          Composite I Beam
\n');
fprintf(f,'-----------------------------------------------
-----\n
');
% Material properties
fprintf(f,'Material 1 - Graphite polymer properties in SI units
\n');
fprintf(f,'-----------------------------------------------
-----\n');
material1
fprintf(f,'E1 = %.2e Pa E2 = %.2e Pa Nu12 = %.3f Nu21 = %.3f G12 = %.2e Pa\n\n',E_1,E_2,nu_12,nu_21,G_12);

fprintf(f,'

');
 fprintf(f,'Material 2 - Glass polymer properties in SI units\n');
fprintf(f,'-----------------------------------------------
-----\n');
material2
fprintf(f,'E1 = %.2e Pa E2 = %.2e Pa Nu12 = %.3f Nu21 = %.3f G12 = %.2e Pa\n\n',E_1,E_2,nu_12,nu_21,G_12);
Material 3 - Aluminum properties in SI units

\[ E_1 = 2.0 \text{e} \text{ Pa}, \quad E_2 = 0.2 \text{e} \text{ Pa}, \quad \nu_{12} = 0.3, \quad \nu_{21} = 0.3, \quad G_{12} = \text{0.2e Pa} \]

%web lamina inputs

Web Lamina Input (from left to right) in SI Units

Lamina=[1:l_b];
MaterialType=[material_w];
Thickness=[h_k_w];
Orientation=[theta_w];
fprintf(f,'Lamina\tMaterial Type\tThickness\tOrientation\n');
fprintf(f,'---------------------------------------------------
');
for j = 1:l_b
fprintf(f,'%d\t%g\t%2.11gm\t%0f°',j,material_w(j),h_k_w(j),theta_w(j));
end

%web Reduced Stiffness Matrix output

The Reduced Stiffness Matrix for Each Lamina of Web

\[ \begin{bmatrix} q_{11w} & q_{12w} & 0 \\ q_{12w} & q_{22w} & 0 \\ 0 & 0 & q_{66w} \end{bmatrix} (\text{Pa}) \]

% web Transformed Reduced Stiffness Matrix output

The Transformed Reduced Stiffness Matrix for Each Lamina of Web

\[ \begin{bmatrix} q_{11w} & q_{12w} & 0 \\ q_{12w} & q_{22w} & 0 \\ 0 & 0 & q_{66w} \end{bmatrix} \]
fprintf(f,'**Thickness = %2.11gm ',h_k_w(j));
fprintf(f,'**Orientation = %.0f* ',theta_w(j));
fprintf(f,'**Material Type = %2g \n',material_w(j));

fprintf(f,'| \t| \t\t| %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | (Pa) \\
| \n',qb_11w(j),qb_12w(j),qb_16w(j));
fprintf(f,'| [Qb] = \t| %6.3e \t| %6.3e \t| %6.3e \t| (Pa) \\
| \n',qb_12w(j),qb_22w(j),qb_26w(j));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| (Pa) \\
| \n',qb_16w(j),qb_26w(j),qb_66w(j));

\nend

% web Output of ABD and abc Matrix
fprintf(f,'The Full [ABD] Matrices for Web 
\n');
fprintf(f,'-----------------------------------
---------
\n');
fprintf(f,'| A | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
| \n',A_W(1,1),A_W(1,2),A_W(1,3));
fprintf(f,'| [A] = \t| %6.3e \t| %6.3e \t| %6.3e \t| (N/m) \\
| \n',A_W(2,1),A_W(2,2),A_W(2,3));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| (N/m) \\
| \n',A_W(3,1),A_W(3,2),A_W(3,3));

fprintf(f,'| B | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
| \n',B_W(1,1),B_W(1,2),B_W(1,3));
fprintf(f,'| [B] = \t| %6.3e \t| %6.3e \t| %6.3e \t| (N) \\
| \n',B_W(2,1),B_W(2,2),B_W(2,3));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| (N) \\
| \n',B_W(3,1),B_W(3,2),B_W(3,3));

fprintf(f,'| D | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
| \n',D_W(1,1),D_W(1,2),D_W(1,3));
fprintf(f,'| [D] = \t| %6.3e \t| %6.3e \t| %6.3e \t| (N.m) \\
| \n',D_W(2,1),D_W(2,2),D_W(2,3));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| (N.m) \\
| \n',D_W(3,1),D_W(3,2),D_W(3,3));

fprintf(f,'| ABD | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
| \n',ABD_w(1,1),ABD_w(1,2),ABD_w(1,3),ABD_w(1,4),ABD_w(1,5),ABD_w(1,6));
fprintf(f,'| [ABD] = \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e |
| \n',ABD_w(2,1),ABD_w(2,2),ABD_w(2,3),ABD_w(2,4),ABD_w(2,5),ABD_w(2,6));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e |
| \n',ABD_w(3,1),ABD_w(3,2),ABD_w(3,3),ABD_w(3,4),ABD_w(3,5),ABD_w(3,6));
fprintf(f,'| [ABD] = \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e |
| \n',ABD_w(4,1),ABD_w(4,2),ABD_w(4,3),ABD_w(4,4),ABD_w(4,5),ABD_w(4,6));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e |
| \n',ABD_w(5,1),ABD_w(5,2),ABD_w(5,3),ABD_w(5,4),ABD_w(5,5),ABD_w(5,6));
fprintf(f,'| \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e |
| \n',ABD_w(6,1),ABD_w(6,2),ABD_w(6,3),ABD_w(6,4),ABD_w(6,5),ABD_w(6,6));
fprintf(f,'\n');
fprintf(f, 'The inverted [ABD] Matrices
\n');
fprintf(f, '------------------------------------------------------------
\n\');
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e | \n', a_W(1,1), a_W(1,2), a_W(1,3));
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e | (m/N) \n', a_W(2,1), a_W(2,2), a_W(2,3));
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e | \n', a_W(3,1), a_W(3,2), a_W(3,3));
fprintf(f, '\n\n');
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e | \n', abd_w(1,1), abd_w(1,2), abd_w(1,3), abd_w(1,4), abd_w(1,5), abd_w(1,6));
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e | \n', abd_w(2,1), abd_w(2,2), abd_w(2,3), abd_w(2,4), abd_w(2,5), abd_w(2,6));
fprintf(f, '\t| %6.3e | \n', abd_w(3,1), abd_w(3,2), abd_w(3,3), abd_w(3,4), abd_w(3,5), abd_w(3,6));
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e | \n', abd_w(4,1), abd_w(4,2), abd_w(4,3), abd_w(4,4), abd_w(4,5), abd_w(4,6));
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e \t| %6.3e | \n', abd_w(5,1), abd_w(5,2), abd_w(5,3), abd_w(5,4), abd_w(5,5), abd_w(5,6));
fprintf(f, '\t| %6.3e \t| %6.3e \t| %6.3e | \n', abd_w(6,1), abd_w(6,2), abd_w(6,3), abd_w(6,4), abd_w(6,5), abd_w(6,6));
fprintf(f, '\n\n');
fprintf(f, 'The Web Laminate Smeared Properties
\n');
fprintf(f, '------------------------------------------------------------
\n\n');
fprintf(f,'Effective Modulus in x , y and Effective shear Modulus
\n');
fprintf(f,'E bar_x = %.4f \ t \ \ E bar_y = %.4f \ t \ \ G bar_xy = %.4f \n\n',Eb_x_w,Eb_y_w,Gb_xy_w);
fprintf(f,'Effective poissons ratios \n');
fprintf(f,'nu bar_xy = %.4f \ t \ nu bar_yx = %.4f \n\n',nub_xy_w,nub_yx_w);
fprintf(f,'Effective Coefficient of Mutual influences of 1st kind and 2nd kind \n');
fprintf(f,'eta bar_x_xy = %.4f \ t \ \ eta bar_y_xy = %.4f \n\n',etab_x_xy_w,etab_y_xy_w);
fprintf(f,'eta bar_xy_x = %.4f \ t \ \ eta bar_xy_y = %.4f \n\n',etab_xy_x_w,etab_xy_y_w);
fprintf(f,'The Web cross section Dimension
\n');
fprintf(f,'Web Height = %.5f m \n',h_w);
fprintf(f,'Web Width(Total lamina Thickness) = %.5f m \n',b_w);
fprintf(f,'Lamina\tMaterial Type\tThickness\tOrientation
\n\n');
for i = 1:l
fprintf(f,'%d \t\t\t%d \t\t%2.11gm \t\t%.0f° \n',i,material(i),h_k(i),theta(i));
end
fprintf(f,'\n[Q] = [%6.3e \t %6.3e | %6.3e | %6.3e | (Pa)
\n\n',q_12(i),q_22(i),0,q_66(i));
end

%Transformed Reduced Stiffness Matrix output
fprintf(f,' The Transformed Reduced Stiffness Matrix for Each Lamina of Flange
');
fprintf(f,'---------------------------------------------------------
');
for i=1:l
fprintf(f,'lamina = %d ',i);
fprintf(f,'Thickness = %2.11gm , h_k(i));
fprintf(f,'Orientation = %.0f° , theta(i));
fprintf(f,'Material Type = %2g
',material(i));
fprintf(f,'| %6.3e | %6.3e | %6.3e |
',qb_11(i),qb_12(i),qb_16(i));
fprintf(f,'[Qb] = | %6.3e | %6.3e | %6.3e | (Pa)
',qb_12(i),qb_22(i),qb_26(i));
fprintf(f,'| %6.3e | %6.3e | %6.3e |
',qb_16(i),qb_26(i),qb_66(i));
end

%Output of ABD and abc Matrix
fprintf(f,' The Full [ABD] Matrices
');
fprintf(f,'---------------------------------------------------------
');
fprintf(f,'| %6.3e | %6.3e | %6.3e |
',A(1,1),A(1,2),A(1,3));
fprintf(f,'[A] = | %6.3e | %6.3e | %6.3e | (N/m)
',A(2,1),A(2,2),A(2,3));
fprintf(f,'| %6.3e | %6.3e | %6.3e |
',A(3,1),A(3,2),A(3,3));
fprintf(f,'| %6.3e | %6.3e | %6.3e | (N)
',B(1,1),B(1,2),B(1,3));
fprintf(f,'[B] = | %6.3e | %6.3e | %6.3e | (N)
',B(2,1),B(2,2),B(2,3));
fprintf(f,'| %6.3e | %6.3e | %6.3e |
',B(3,1),B(3,2),B(3,3));
fprintf(f,'| %6.3e | %6.3e | %6.3e | (N.m)
',D(1,1),D(1,2),D(1,3));
fprintf(f,'[D] = | %6.3e | %6.3e | %6.3e | (N.m)
',D(2,1),D(2,2),D(2,3));
fprintf(f,'| %6.3e | %6.3e | %6.3e |
',D(3,1),D(3,2),D(3,3));
fprintf(f,'| %6.3e | %6.3e | %6.3e | (N.m)
',ABD(1,1),ABD(1,2),ABD(1,3),ABD(1,4),ABD(1,5),ABD(1,6));
fprintf(f,'| %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
',ABD(2,1),ABD(2,2),ABD(2,3),ABD(2,4),ABD(2,5),ABD(2,6));
fprintf(f,'| %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
',ABD(3,1),ABD(3,2),ABD(3,3),ABD(3,4),ABD(3,5),ABD(3,6));
fprintf(f,'| %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
',ABD(4,1),ABD(4,2),ABD(4,3),ABD(4,4),ABD(4,5),ABD(4,6));
fprintf(f,'| %6.3e | %6.3e | %6.3e | %6.3e | %6.3e | %6.3e |
',ABD(5,1),ABD(5,2),ABD(5,3),ABD(5,4),ABD(5,5),ABD(5,6));
fprintf(f,'\t\t\t\t%6.3e \t%6.3e \t%6.3e \t%6.3e \t%6.3e \t%6.3e \t%6.3e \t%6.3e
',ABD(6,1),ABD(6,2),ABD(6,3),ABD(6,4),ABD(6,5),ABD(6,6));

fprintf(f,'\n');
fprintf(f,'---------------------------------

\n');
fprintf(f,'The inverted [ABD] Matrices
\n');
fprintf(f,'---------------------------------

\n');
fprintf(f,'The Beam Laminate Smeared Properties
\n');
fprintf(f,'---------------------------------

\n');
fprintf(f,'------------------------------------

\n');
fprintf(f,'Effective Modulus in x, y and Effective shear Modulus
\n');
fprintf(f,'------------------------------------

\n');
fprintf(f,'Effective poissons ratios
\n');
fprintf(f,'------------------------------------

\n');
fprintf(f,'Effective Coefficient of Mutual influences of 1st kind and
2nd kind
\n');
fprintf(f,'------------------------------------

\n');
fprintf(f,'eta bar_x_xy = %.4f\t eta bar_y_xy = %.4f \n',etab_x_xy,etab_y_xy);
fprintf(f,'eta bar_xy_x = %.4f\t eta bar_xy_y = %.4f \n\n',etab_xy_x,etab_xy_y);
fprintf(f,' The Flange cross section Dimension
\n');
fprintf(f,'-----------------------------------------------
\n');
fprintf(f,'Flange Height (Total lamina Thickness on one flange) = %.5f m \n',(TT-h_w)/2);
fprintf(f,'Flange Width = %.5f m \n',W);