Single-station Tracking for Orbit Determination of Small Satellites

Roger Hart

Globesat, Inc., Logan, Utah

Available methods of orbit determination and satellite observation are briefly reviewed and evaluated for suitability in tracking small satellites from a single ground station. Radio interferometer and Doppler tracking appear useful and are discussed in further detail. Distinctions in accuracy and application between the two methods are made.

INTRODUCTION

The economy and independence of operation anticipated with the use of small satellites cannot be fully realized unless comparably simplified means of tracking and orbit determination are available to the user. By locally tracking a satellite, direct observational data are immediately available, giving the user more flexibility in orbit determination than by relying only on predicted values from outside sources. Military tracking centers, designed for identifying, cataloging, and tracking large numbers of objects in earth orbit, use large antennas and high power transmitters at costs prohibitive to many expected users of small satellites. However, power and size requirements are significantly reduced when the object to be tracked is actively transmitting, is controlled by the observer, and is in a predetermined orbit. The best orbit determinations come from observations made from several tracking stations distributed around the earth, but for the present objectives of simplicity and cost, a single-site tracking system is desirable. Many methods of tracking satellites are currently employed, but two, Doppler tracking and radio interferometry, stand out as being most suitable for single-site tracking of small satellites. Several methods of tracking and orbit determination will be discussed briefly with more direct attention being given to Doppler and interferometric tracking.

METHODS OF ORBIT DETERMINATION

Six independent parameters, three in position, and three in velocity completely describe the motion of the center of mass of a satellite. In the simplest sense, the task of a tracking station is to make observations from which these six elements of motion can be deduced and an orbit computed. Early schemes of orbit determination were developed for planetary bodies in the solar system; calculations were done by hand, so the methods emphasize use of the minimum data sufficient for orbit determination. Other methods allow for redundant data with which new orbits may be computed or preliminary orbits improved.

Gauss' method of orbit determination is noted for its roll in the rediscovery of the minor planet, Ceres, after it was lost near the sun soon after its discovery in 1801. Gaussian orbit determination requires two positions and the corresponding times of observation for a complete data set. Three positional degrees of freedom are explicit in the observations. Velocities to complete the determination are found by computing the one ellipse that passes through the two points in the observed time of flight. In practice, Gauss' method has found limited application to artificial satellites.
Gibbs' method utilizes three observations of position and is purely geometrical. Because only one ellipse can pass through the three points, the shape and orientation of the ellipse is exactly determined. Time enters the equations only to specify the satellite's position within the ellipse. Variations of Gibb's method will be applicable to satellites utilizing the Global Positioning System to determine location.

Laplacian orbit determination is iterative like Gauss' method but uses observations of right ascension, declination, and time instead of positions. Three pointing angles and respective times of observation completely characterize the motion. Laplace's method has been widely used because of its rapid convergence and stability.

Once a preliminary orbit has been established, further data serve to update and refine the orbit, whose size and orientation in space continually change due to the influence of the earth's oblateness, atmospheric drag, and other perturbations. If an adequate number of observations is available, the orbit may simply be redetermined, but if a complete set of observations is not available, deviations from the expected path provide information for rectification of the preliminary orbit by use of differential correction. Many differential correction procedures exist for orbit improvement; a basic application will be discussed in connection with Doppler tracking.

METHODS OF OBSERVATION

Means of obtaining the six parameters necessary for orbit determination are as numerous as the mathematical methods for processing the observations, and as might be expected, size, cost, and accuracy differ appreciably among available tracking systems. Observation of satellites is accomplished in both optical and radio regions of the spectrum. Optical observation includes visual, photographic, and laser ranging measurements. Radio frequency measurements include radar, interferometry, radio ranging, and Doppler tracking. All of these methods are in current use, but the present constraints of low cost, moderate size, and operation from a single site limit the applications to small satellites.

Optical observations

In general, optical tracking achieves more precise results than radio tracking because of the shorter wavelengths used. Accuracies of better than 10 m are attainable with the large cameras designed for satellite tracking, and laser ranging can give positions good to a few centimeters. Optical tracking, while yielding excellent data, cannot be relied upon as the primary method of tracking from a single station because observation is possible ordinarily at night and only when skies are clear, making the availability of data unreliable and infrequent. Excluding visual observing, which requires only binoculars and stopwatch, equipment and operating costs for optical tracking are high.

Radio observations

Tracking at radio wavelengths is attractive because the weather and seasonal constraints affecting optical tracking do not inhibit radio frequency observations, so a satellite is observable at least as often as it rises above the horizon. Radio observations may be of two types: active, with the satellite transmitting the tracking signal, or passive, using a reflected signal from a ground-based transmitter. Passive satellite systems have the advantage that they need to operate at only one frequency; however, they must radite costly amounts of power in order to "see" a small satellite. Radar tracking works well in practice, but again, the physically large antennas and high power needed conflict with the small satellite objectives. Active-satellite systems use little power, are generally smaller, and are more consistent with the anticipated functions of small satellites. Two active-satellite systems, radio interferometers and Doppler tracking are examined more closely in the following sections.
Radio interferometry is a proven and reliable means of tracking satellites; the first Sputniks were successfully tracked using this technique, and it now forms the core of the U.S. Navspasur system. Interferometers make use of the effect that there will be a phase difference between the signals received by two separated antennas. The phase difference, \( \Delta \varphi \), is a function of the zenith angle, \( \theta \), of the satellite and separation of the antennas, \( D \),

\[
\Delta \varphi = \frac{2\pi D}{\lambda} \sin \theta
\]

as shown in Figure 1. The solution of Equation (1) for zenith angle is ambiguous, so a second pair of antennas with a different spacing is used to obtain a unique solution in one dimension. Two more pairs of antennas, at right angles to the first, are used to fix the azimuth and elevation of the line of sight to the satellite.

**Orbit Computation**

Methods used to reduce the data are variations of Laplace's method, for which three measurements of time, azimuth, and elevation are sufficient for determination of an orbit. In an illustrative solution, the position of a satellite can be expressed as \( r = R + pL \) where \( R \) is the observation site position, \( p \) is the slant range, and \( L \) is a unit vector along the line of sight as shown in Figure 2. The position, \( r \), may also be expressed in terms of an epochal position, \( r_0 \), and velocity \( v_0 \), as \( r = f r_0 + g v_0 \), where \( f \) and \( g \) are series expressions that can be found in most texts on celestial mechanics. A set of linear equations, in which \( r_0 \) is the only unknown, is formed from the combination of the equations for \( r \) and the three observations of angle. A good estimate of \( r_0 \) is adequate to start an iterative solution that converges to give \( r_0 \) and \( v_0 \), defining the orbit.
Equipment

The interferometer requires four pairs of antennas, but only five, not eight, antennas are required since one antenna can be common to each of the four pairs. An interferometer proposed for tracking small satellites has antenna pairs spaced 0.5\(\lambda\) for coarse resolution and 4\(\lambda\) for the fine resolution pair (60-135 m for satellite transmitter frequencies between 100-450 MHz) to obtain pointing accuracies of \(\pm 0.02\) degrees. At 1500 km this gives along and cross track errors of about 500 m. Required equipment includes a receiver with a channel for each antenna, a clock, phase comparators for the antenna pairs, and means to record data. To maintain accuracy, care must be taken to keep long cables at a constant temperature, and the system must be calibrated often. Calibration is accomplished by measuring the position of a transmitter in a known location, such as that on a navigation satellite or on an airplane photographed at night against a background of stars.

Application to small satellites

A degree of flexibility is sacrificed by using an active-satellite system since the antennas are designed for optimal performance at a specific wavelength, complicating the problem of tracking several satellites that may not be transmitting at the same frequency. North-south, east-west alignment of the antennas is also a concern for reasons of accuracy and expense of surveying. With proper calibration, the accuracy of the interferometer, as described, allows a good determination of an orbit in a single pass. Calculation is rapid and straightforward, and automatic control by a micro-computer possible. Finally, cost of the interferometer will be in proportion to that of a small satellite; this, in combination with the predicted performance of the system, suggests the interferometer as being an acceptable means of tracking from a single station.

DOPPLER TRACKING

Observation of the Doppler frequency shift during a satellite pass can also provide sufficient information for accurate orbit determination. During the transit of a satellite in low earth orbit, the received signal frequency will typically vary from several thousand Hz above, to several thousand Hz below the actual transmitter frequency. Figure 3 shows a representative variation of frequency with time for a high inclination orbit. The Doppler curve is a function of the relative motion between the receiving site and the satellite. Given that the ground station coordinates are well determined, the only unknown components of the relative motion are due to the motion of the satellite, thus the curve is soluble for the orbit. Con-
versely, if the satellite orbit is well determined, location of the receiving station can be computed, and
the satellite becomes useful for navigation. The Transit system of navigational satellites employs both
tracking and navigational aspects of Doppler tracking to achieve positional accuracies on the order of 1
m.

Computation
Orbits may be directly fitted to the Doppler data in a trial and error fashion, but a much less laborious
way of computing is to differentially correct an initial approximation of the orbit\(^7\). The relative velocity
between satellite and observer is computed from the Doppler shift by \( \dot{p} = c(1-v'/v) \), where \( v' \) is the
frequency received at the ground station, and \( v \) is the nominal transmitter frequency. \( \dot{p} \) is defined as being
positive when the relative separation is increasing. Since measurement of frequency is one dimensional,
at least six observations of the Doppler shift are necessary for a complete determination. Relative
velocity is a function of the geometry at some epoch, i.e., \( \dot{p} = f(r_0,v_0,R_0,V_0) \). Due to uncertainties in the
preliminary orbit or to errors caused by atmospheric drag and other perturbations, observed frequency
shifts will differ, somewhat, from those predicted. The differences are called residuals and are com-
puted from the differential of \( \dot{p} \). The differential of \( \dot{p} \) is expressed

\[
d\dot{p} = \frac{\partial \dot{p}}{\partial r_{zo}} dr_{zo} + \frac{\partial \dot{p}}{\partial r_{yo}} dr_{yo} + \frac{\partial \dot{p}}{\partial r_{zo}} dv_{zo} + \frac{\partial \dot{p}}{\partial v_{yo}} dv_{yo} + \frac{\partial \dot{p}}{\partial v_{zo}} dv_{zo}
\]

If all differentials are small, Equation (2) may be rewritten

\[
\Delta \dot{p} = \frac{\partial \dot{p}}{\partial r_{zo}} \Delta r_{zo} + \frac{\partial \dot{p}}{\partial r_{yo}} \Delta r_{yo} + \frac{\partial \dot{p}}{\partial r_{zo}} \Delta v_{zo} + \frac{\partial \dot{p}}{\partial v_{yo}} \Delta v_{yo} + \frac{\partial \dot{p}}{\partial v_{zo}} \Delta v_{zo}
\]

\[
+ \frac{\partial \dot{p}}{\partial R_{zo}} \Delta R_{zo} + \frac{\partial \dot{p}}{\partial R_{yo}} \Delta R_{yo} + \frac{\partial \dot{p}}{\partial R_{zo}} \Delta V_{zo} + \frac{\partial \dot{p}}{\partial V_{yo}} \Delta V_{yo} + \frac{\partial \dot{p}}{\partial V_{zo}} \Delta V_{zo}
\]

\( \Delta \dot{p} \) represents the residual of observed and computed relative velocities; \( \Delta r, \Delta v, \Delta R, \) and \( \Delta V \) represent
the corrections to \( r, v, R, \) and \( V \). For \( n \) observations we obtain the system of equations,
which may be solved for the corrections to the original orbit. If the location of the tracking station is known, the corrections for $R$ and $V$ are zero, and the second term becomes zero, leaving the residuals dependent only on the uncertainties in the orbit parameters. Similarly, an unknown receiving site can be located if the orbit is well determined, in which case the first term drops out. Partial derivatives are computed analytically or numerically by introducing small variations in the orbital elements and evaluating

$$\frac{\partial \tilde{\rho}}{\partial \xi} \approx \frac{\tilde{\rho}(r_{z0}, \ldots, \xi + \delta \xi, \ldots, v_{z0}) - \tilde{\rho}(r_{z0}, \ldots, \xi, \ldots, v_{z0})}{\delta \xi}$$

where $\xi$ is the element of differentiation. The corrections are added to the epochal orbital elements and, the residuals are recomputed at the six times of observation; this produces a new set of corrections, and the iteration is continued until the RMS of the residuals reaches a minimum.

A set of six observations is the minimum information needed for complete determination, but an automated system can clearly provide more than six measurements during a single pass. A strength of the differential correction method is that it readily accommodates redundant data, giving more accurate results than the exactly determined case because of limitations in measuring the Doppler frequency shift. A weighted least squares solution is expressed in matrix form as, $z = (A^T W A)^{-1} A^T W b$. $z$ is the correction to the original orbit, $A$ holds the partial derivatives, $b$ is the set of residuals, and $W$ is the covariance of the uncertainty of the orbital parameters.

**Implementation of a Doppler Tracking System**

Tracking may be accomplished manually or automatically. In either case, the basic elements of a Doppler tracking station consist of a receiver, a reference oscillator, and a clock. Automation requires extra hardware for control, but the procedure for data acquisition is the essentially the same for both methods of operation. The received frequency will be shifted above the nominal transmitter frequency as a satellite comes into view and will decrease with time, passing through the true transmitter frequency near the time of closest approach. With a manual system, the frequency of the local oscillator is set just below that anticipated as the satellite rises and is mixed with the received frequency. The mixed signal is amplified to produce either audible beats over a speaker or a Lissajous figure on an oscilloscope. Time and frequency are recorded when the beat frequency goes to zero or when the Lissajous figure becomes stationary. Further measurements (at least five more for a complete determination) are made by decreasing the local oscillator frequency in steps and zero beating the received signal as before.

The inflection point of the curve, obtained by plotting the received frequencies against time, represents the true transmitter frequency, or time of closest approach. This can be used to verify the stability of a known satellite transmitter or to determine the transmitter frequency of an unknown vehicle. Once the transmitter frequency is ascertained, relative velocities and residuals may be computed for use in the differential correction scheme.

**Accuracy and Limitations**

Accuracy of orbit determination using the Doppler shift is dependent on the stability of the frequency sources, both transmitter and reference oscillator. Frequency stability of 1 part in $10^7$ during a pass is sufficient for tracking, but stability on the order of 1 part in $10^9$ is necessary for the accuracy attained by the Transit system. Short term stability of 1 in $10^7$ is not difficult to achieve, and 1 in $10^8$ is feasible without incurring excessive costs.

Ionospheric refraction of the signal path causes some error, but the effects are small for frequencies above 100 MHz. A second transmitter, operating at a different frequency, makes correction for ionospheric refraction possible for higher precision applications.
When the initial orbit approximation is within the range of convergence of the differential corrector, data from one ground station and a single pass yield an excellent determination of the orbital plane and a good determination of the shape of the ellipse\(^4\). Determination of the orientation of the ellipse in the orbital plane is poor. Computer simulation suggests that with a good prediction of the orbit, positions can be determined to a few tens of kilometers from a single observing station on a single pass, with improvement over subsequent transits.

The need of a starter for the differential corrector suggests that single-site Doppler tracking might be better used as a secondary tracking system; once an orbit has been established by a central tracking station (probably using a radio interferometer) control is transferred to the Doppler tracking station.

**Application to Small Satellites**

Several aspects of Doppler tracking make it well suited for use with small satellites. Cost of the tracking system is kept low because equipment needs beyond the essential receiver are small, at a minimum consisting of an amplifier and a variable oscillator. Large antenna arrays and concerns of temperature stabilization are overcome with the Doppler tracking system, which needs only one antenna and no outdoor cables. Furthermore, calibration may be done in the lab, eliminating the use of satellites for alignment. Precise surveying for antenna alignment is avoided because only an accurate knowledge of the receiver location is required for correct orbit determination.

Potential for expanded application of Doppler tracking is also evident. Greater accuracy and a greater range of convergence can be gained when several widely separated tracking stations are employed. Data from more than one tracking site is readily accepted by the differential correction scheme, so it is feasible that remote data stations could send Doppler information to the satellite at the end of a pass for later orbit improvement. Finally, with portable receivers, the possibility of using satellites for navigation is inherent in the differential correction method.

**CONCLUSION**

From the viewpoint of economy and independence, the two methods of Doppler and interferometer tracking stand as the most suitable means of observing small satellites for orbit determination. For single-station tracking, the different accuracies of the two systems form a criterion for selection of the appropriate one for a given use; satellites used for communication or data relay can be tracked adequately with the lower precision, lower cost Doppler tracking, while applications necessitating higher accuracy dictate use of the more costly radio interferometer. Both means alleviate dependence on outside sources for orbit information, allowing the ground station and satellite to work together as a complete system.

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**REFERENCES**


10. Escobal, p. 442.


