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FEASIBILITY OF MULTI-YEAR FORECAST FOR THE COLORADO RIVER
WATER SUPPLY: TIME SERIES MODELING

by

Brian J. Plucinski

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Statistics

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2019

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ABSTRACT

Feasibility of Multi-Year Forecast for the Colorado River Water Supply: Time Series

Modeling

by

Brian J. Plucinski, Master of Science

Utah State University, 2019

Major Professor: Dr. Yan Sun

Department: Statistics

The future of the Colorado River water supply (WS) affects millions of people and the U.S. economy. A recent study suggested a cross-basin correlation between the Colorado River and its neighboring Great Salt Lake (GSL). Following that study, the feasibility of using the previously developed multi-year prediction of the GSL water level to forecast the Colorado River WS was tested. Time-series models were developed to predict the changes in WS out to 10 years. Regressive methods and the GSL water level data were used for the depiction of decadal variability of the Colorado River WS. Various time-series models suggest a decline in the 10-year-averaged WS since 2013 before starting to increase around 2020. Comparison between this WS prediction and the WS projection published in a 2012 government report (derived from climate models) reveals a widened imbalance between supply and demand by 2020. Further research to update similar multi-year prediction of the Colorado River WS is needed. Such information could aid in management decision making in the face of future water shortages.

(50 pages)

PUBLIC ABSTRACT

Feasibility of Multi-Year Forecast for the Colorado River Water Supply: Time Series Modeling

Brian J. Plucinski

The Colorado River is one of the largest resources for water in the United States, as well as being an important asset to the economy. Previous studies have shown a connection between the Great Salt Lake and the Colorado River. This study used time series analysis to build models to predict the water supply of the Colorado River ten years out. These models used data from the Colorado River in addition to Great Salt Lake water elevation. Several models suggest a decline in water supply from 2013 – 2020, before starting to increase. These predictions differ from predictions published by a 2012 government report that came from climate modeling. Comparing this study's predictions with the 2012 government report's predictions state a large difference between water supply and water demand around the year 2020. Further research is needed to update similar models to predict water supply. This information could be helpful in management's decisions to influence their water saving plans.

ACKNOWLEDGMENTS

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Special thanks to friends and family, including Greta and Betsie.

Brian J. Plucinski

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1. Introduction:

The Colorado River is one of the largest water resources in the United States, supplying water to about forty million people in the southeast and intermountain states. A 2018 study published by Arizona State reported that the river provides over \$1.4 trillion in annual economic benefits as well as jobs to about 16 million people. [*Audubon Arizona*, 2018]. It is clear that the river is an important asset to the country and any negative changes to the river could be detrimental. As recently reported by the Colorado River Research Group [2018], water supply of the Colorado River (hereafter “WS”) has decreased in recent decades while the river basins are facing the biggest drought in history (<https://www.coloradoriverresearchgroup.org/>). These changes have largely been attributed to increasing temperatures that led to less snow and more water being evaporated than normal [*Barnett and Pierce*, 2008; *McCabe and Wolock*, 2007; *Udall and Overpeck*, 2017]. However, the long-term future of the WS made by climate model projections showed an upturn in the Colorado River WS into 2020 [*BOR*, 2012] (Figure

1, shown below); this is in contradiction to an observed decrease through 2018.

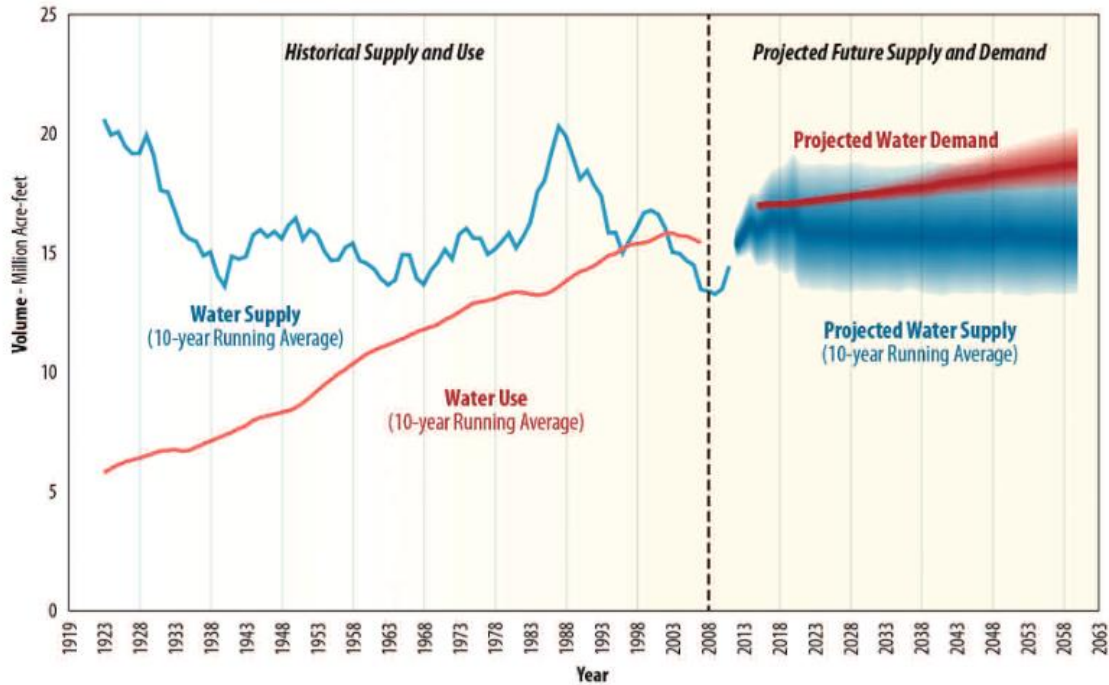


Figure 1. Time series of the projected water supply versus projected water demand of the Colorado River from the 2012 BOR report.

Notwithstanding future water supply projections, *BOR* [2012] anticipates water demand to surpass such by as much as 3.2 million acre-feet through 2050. Such growing imbalance between supply and demand advocates for a more accurate and longer-term predictions of the Colorado River WS given projected water demand.

In addition to the long-term trend, western water managers are concerned about multi-year droughts, even more so than for a single dry year, since the shifts between persistently dry and wet regimes have important implications for managing the limited water resources [*Gangopadhyay and McCabe, 2010*]. Multi-year drought is reflected in the pronounced quasi-decadal oscillation (QDO) of 10–20 years featured in the Colorado River WS’s historical record, as noted by a recent study [*Wang et al., 2018*]; however,

this QDO feature is unaccounted for by the climate model projections published in the government report [BOR, 2012]. The neighboring watershed of the Upper Colorado River Basin – the Great Salt Lake (GSL) of Utah – also shows a marked QDO in its water level variation [Lall and Mann, 1995; Wang *et al.*, 2010]. The GSL water level variation is coherent with the low-frequency variation of the Colorado River WS [Wang *et al.*, 2018]. Meanwhile, past studies [Gillies *et al.*, 2011; Gillies *et al.*, 2015] have developed prediction methods to anticipate the GSL water level out to 5-10 years. Therefore, a compelling case can be made that the same multi-year forecast developed for the GSL water level could apply to the Colorado River WS.

Given the challenges in anticipating the near-term (decadal) variation of the Colorado River WS imposed by certain limitations in climate models, an empirical prediction model for the Colorado River WS offers a provisional solution. The goal therefore, was to explore the role of the aforementioned decadal-scale climate oscillations and their interconnection to water supply for the near future, beyond seasonal streamflow prediction. The concept is to examine multi-year predictability in water supply by conducting time series modeling that uses observational data of the GSL water level and the Colorado River WS. The work presented here does not analyze the mean-state hydroclimate changes towards the end of the century, which has been already done [Barnett and Pierce, 2008; Vano *et al.*, 2014], nor does it apply climate indices to enhance seasonal prediction for the Colorado River WS: To this end, data and the time series modeling are introduced in Section 2. Results and validation of the two modeling approaches are presented in Sections 3 and 4, respectively. Discussion of the results, the physical explanation and the implications are offered in Section 5. Section 6 provides

some concluding remarks.

2. Data and Methodology:

We used the available WS data from the BOR study up to 2012 and subjected it to a backward 10-year moving average (as was done in the BOR report). The GSL water level data was obtained from the U.S. Geological Survey (<http://waterdata.usgs.gov/>) up to June 2016. To build competing models to assess the multi-year predictability of the WS, we used the software RStudio.

Figure 2 below shows the observed time series of the annual Colorado River WS data (denoted by X_t) from 1906 to 2012, as well as the observed time series of the Great

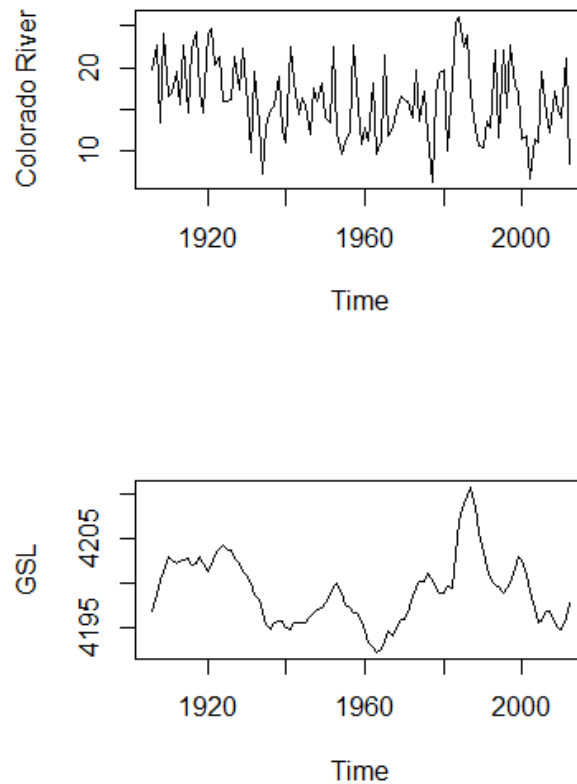


Figure 2. Time series of the annual basin-wide water supply of the Colorado River (top) and annual Great Salt Lake water elevation (bottom).

Salt Lake water elevation. We can see from these time series how the Great Salt Lake could be a useful predictor for the Colorado River. It appears at first glance that how the river trends follows how the lake has been trending. This analysis will be looked into further later on in this study. An additional note: the river's time series does not appear stationary since the mean seems to fluctuate.

To assume stationarity, we differenced the time series before conducting the time-series modeling. Next, the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the first difference were conducted (Figure 3, below).

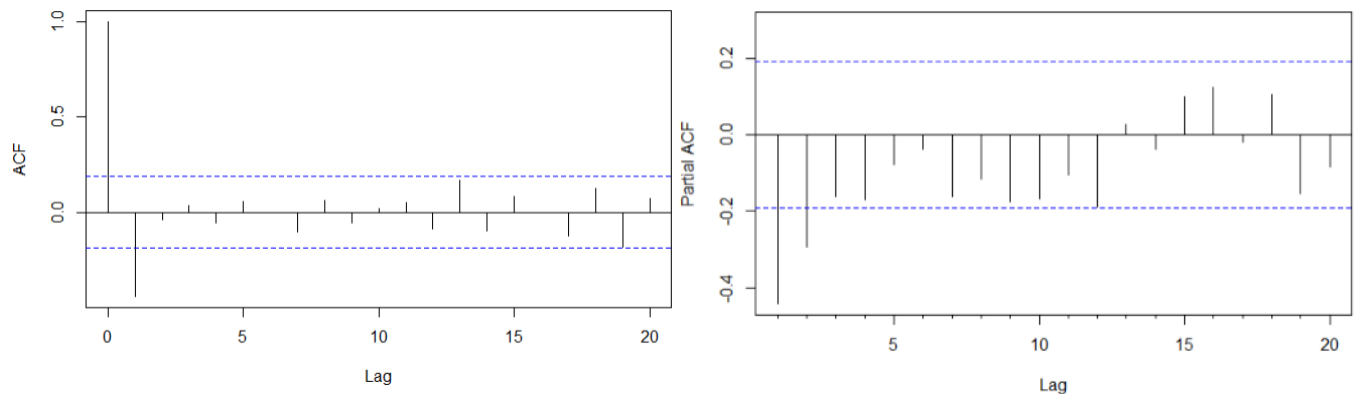


Figure 3. The autocorrelation function (ACF) and partial autocorrelation function (PACF) of the first difference of the Colorado River time series.

There appear to be a few significant lags, in the ACF at lag 1 and in the PACF at lags 1 and 2. The correlations at lags 13 and 19 are marginally significant in the ACF,

and so is the correlation at lag 12 in the PACF. These significant lags suggest that an autocorrelation structure in the WS data is appropriate, one that we utilized to build univariate time series models for the prediction of WS.

The cross-autocorrelation function (CACF) of the Colorado River WS time series and the Great Salt Lake/GSL water level is shown next in Figure 4.

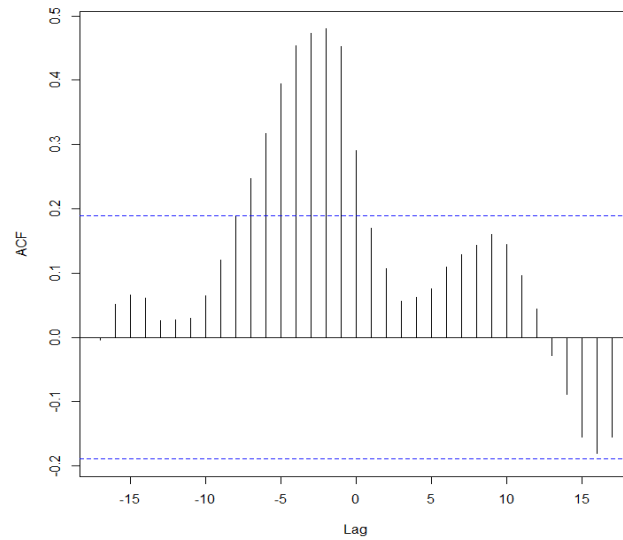


Figure 4. The cross-autocorrelation function (CACF) of the Colorado River time series and Great Salt Lake time series. Note the significant cross-autocorrelations from lags -7 to 0.

There is a significant cross-autocorrelation structure, which leads us to use the GSL time series as an exogenous variable to further improve the univariate model. This approach gave us an ARMAX model, which stands for an ARMA model with exogenous variables. Each of the ensuing models, which were tested to fit the WS, is discussed in the following sections.

3. Raw-Data Modeling:

Hereafter, the differenced Colorado River time series is denoted by X_t and the exogenous GSL time series is denoted by S_t .

Sparse AR (19) Model:

Through the analysis based on ACF and PACF plots, the best univariate model chosen was a sparse autoregressive (AR) (19) time series model. This model included autoregressive terms at lags 1-12, as well as 17 and 19. The fitted model is:

$$\begin{aligned} X_t = & -0.7649X_{t-1} - 0.6941X_{t-2} - 0.6056X_{t-3} - 0.5008X_{t-4} - \\ & 0.4875X_{t-5} - 0.4043X_{t-6} - 0.5403X_{t-7} - 0.4468X_{t-8} - 0.456X_{t-9} - \\ & 0.3866X_{t-10} - 0.2442X_{t-11} - 0.2442X_{t-12} - 0.135X_{t-17} - 0.2315X_{t-19} + Z_t, \end{aligned}$$

where Z_t represents a white-noise process with an estimated variance of 16.47. The residual plots and their ACF of this model are shown below in Figure 5. Notice that the residuals resemble a white noise with no significant autocorrelations, which indicates sufficiency of the model.

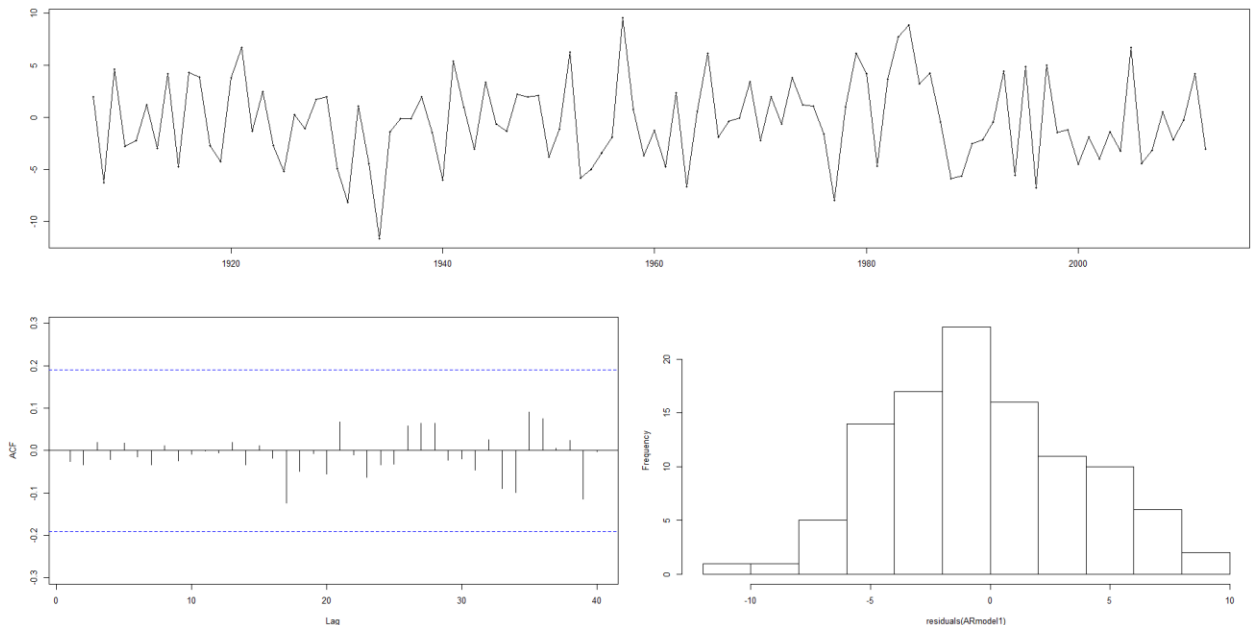


Figure 5. Plot (top) of the residuals over time, ACF of the residuals (bottom left) and histogram (bottom right) of the residuals of the sparse AR (19) model.

For the illustration of the actual time series, Figure 6 shows the model fitted values (red) plotted along with the 95% confidence intervals (grey) in comparison to the observed values (black).

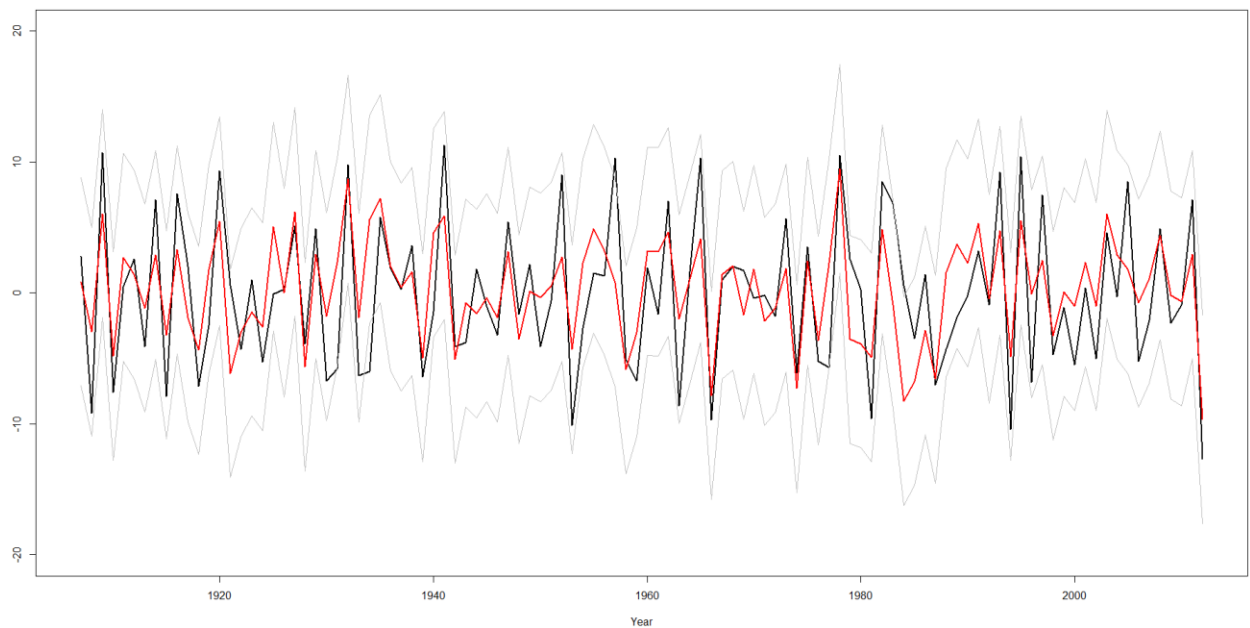


Figure 6. Observed values (black), shown with the model fitted values (red), along with 95% confidence intervals of the sparse AR (19) model.

ARMAX (19, 0, 1, 0):

The first ARMAX model selected is ARMAX (19, 0, 1, 0), meaning that the WS data has an AR (19) fitted to the base model, with the first lag of GSL as the exogenous variable. The model equation is expressed as:

$$\begin{aligned}
X_t = & 0.8792 * (S_t - 0.9661S_{t-1}) - 0.8349X_{t-1} - 0.7962X_{t-2} - 0.7299X_{t-3} \\
& - 0.6238X_{t-4} - 0.6297X_{t-5} - 0.5177X_{t-6} - 0.6887X_{t-7} \\
& - 0.5986X_{t-8} - 0.6169X_{t-9} - 0.5325X_{t-10} - 0.3636X_{t-11} \\
& - 0.3222X_{t-12} - 0.1264X_{t-17} - 0.2573X_{t-19} + Z_t
\end{aligned}$$

and the estimated variance of the white noise is 14.07. Figure 7 shown below displays the residual plots and the ACF of the residuals, which appear to be white noise.

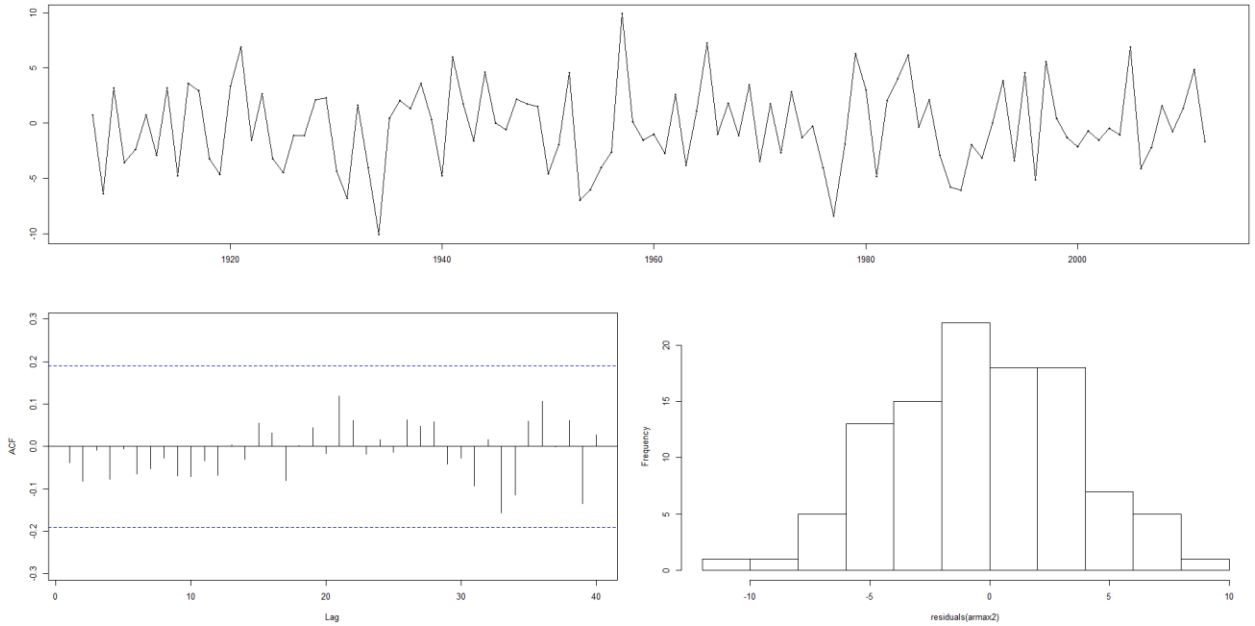


Figure 7. Plot (top) of the residuals over time, ACF of the residuals (bottom left) and histogram (bottom right) of the residuals of the ARMAX (19, 0, 1, 0) model.

Figure 8 (below) shows the model fitted values (blue) with the observed values (black), along with the 95% CI (grey).

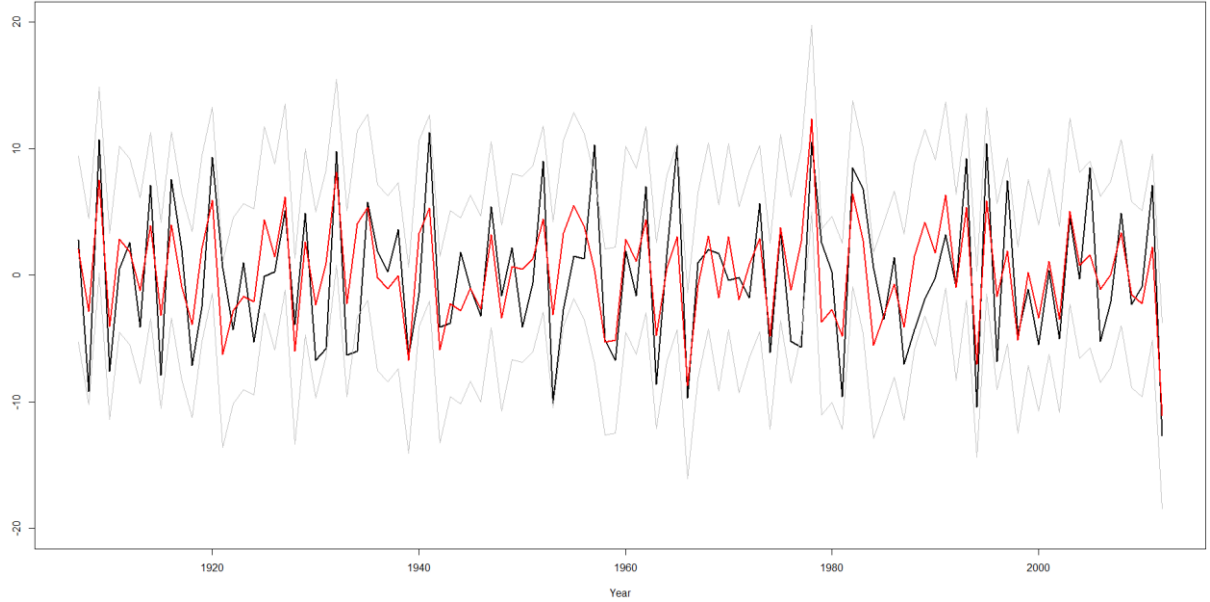


Figure 8. Observed values (black), shown with the model fitted values (red), along with 95% confidence intervals of the ARMAX (19, 0, 1, 0) model.

ARMAX (19, 1, 2, 0):

The second ARMAX model selected is ARMAX (19, 1, 2, 0). The estimated model is:

$$\begin{aligned}
 X_t = & 1.4374 * (S_t - 1.1914S_{t-1} - 0.9204S_{t-2}) - 0.2543X_{t-1} - 0.1875X_{t-2} \\
 & - 0.2494X_{t-3} - 0.2458X_{t-4} - 0.1542X_{t-5} - 0.2171X_{t-6} \\
 & - 0.3968X_{t-7} - 0.1841X_{t-8} - 0.2702X_{t-9} \\
 & - 0.2321X_{t-10} - 0.0992X_{t-11} - 0.1948X_{t-12} - 0.1743X_{t-17} \\
 & - 0.2719X_{t-19} - 0.7901Z_{t-1} + Z_t
 \end{aligned}$$

and the estimated variance of white noise is further reduced to 13.06. Figure 9 shows the residual plots and the ACF of residuals, which again appear to be white

noise.

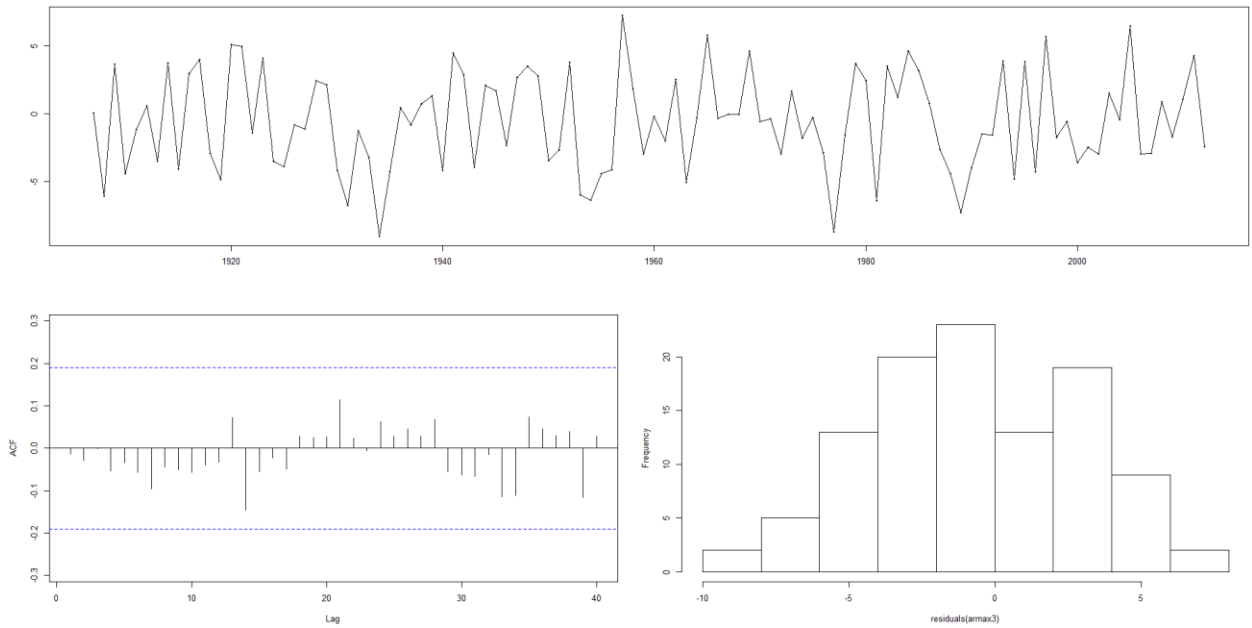


Figure 9. Plot (left) of the residuals and ACF of the residuals (right) of the ARMAX (19, 1, 2, 0) model.

Figure 10 below displays the observed values, model fitted values and the 95% CI.

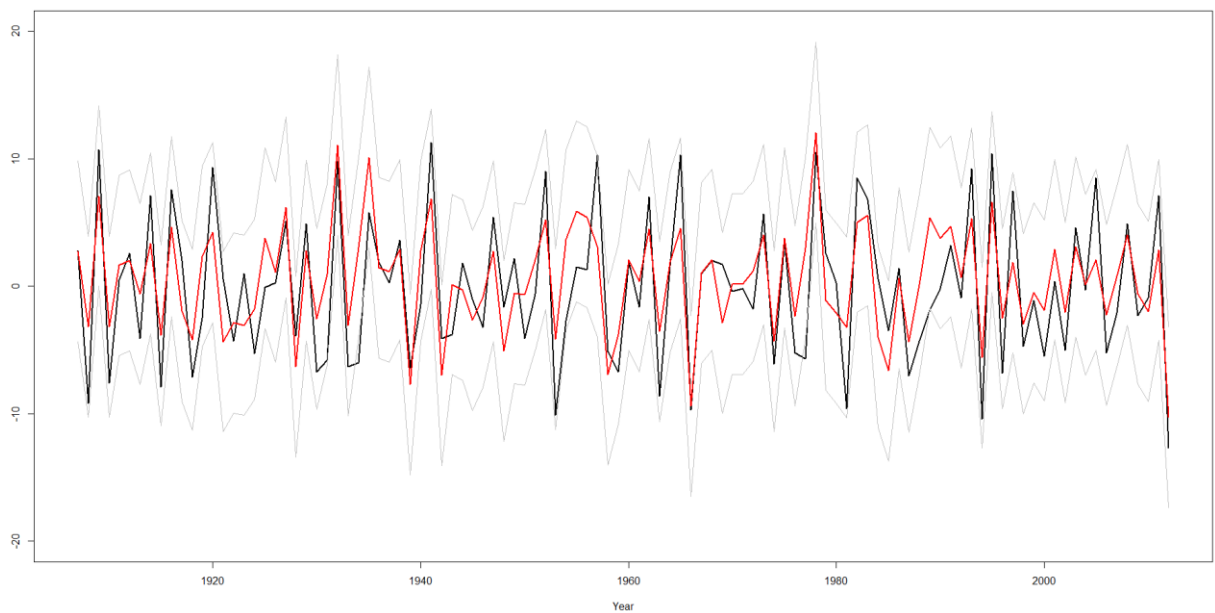


Figure 10. Observed values (black), shown with the model fitted values (red), along with 95% confidence intervals of the ARMAX (19, 1, 2, 0) model.

Next, cross-validation was performed to assess the forecasts of the models. First, the Colorado River WS was divided into a training and test set, with the first 101 observations being the training set while the last five observations being the test data. We used 10 forecasting horizons ($h = 1, 2, \dots, 10$, i.e. length of the forecast) and the coefficients of the model were updated at every step. For $h = 1$, (one-step-ahead prediction), the model was fitted on observations 1-101 to predict observation 102, and then the model was fitted using observations 1-102 to predict observation 103, and so on. For $h = 2$, (two-step-ahead prediction) the model was fitted using observations 1-100 to predict observation 102, then the model was fitted using observations 1-101 to predict observation 103, and so on. The results of the ten forecasting horizons are shown below in Table 1, along with the corresponding root mean squared error and the skill score of each model.

RMSE	1	2	3	4	5	6	7	8	9	10
Sparse AR(19)	2.8673	4.4851	4.5344	4.3190	4.1826	4.1037	4.3180	5.1207	5.2668	5.2246
ARMAX1	3.1274	4.3809	4.6646	4.1570	3.7697	3.7116	4.1295	5.0421	5.1573	5.1757
ARMAX2	2.8770	4.3947	4.9787	5.1741	4.0544	3.9955	3.7328	4.4858	4.9159	6.2473
SS										
Sparse AR(19)	0.8311	0.5830	0.5727	0.6162	0.6373	0.6510	0.6165	0.4562	0.4252	0.4301
ARMAX1	0.7991	0.6022	0.5479	0.6444	0.7054	0.7145	0.6493	0.4727	0.4489	0.4407
ARMAX2	0.8300	0.5997	0.4849	0.4491	0.6592	0.6692	0.7134	0.5827	0.4993	0.1852

Table 1. Ten-year cross-validation results for the three annual models discussed.

The column names are the ten forecasting horizons ($h = 1, 2, \dots, 10$). Root mean squared

error is shown on top while the skill score is shown on the bottom. The column names are the ten forecasting horizons ($h = 1, 2, \dots, 10$). Highlighted values are the best statistic for each forecasting horizon.

Here in Table 1, ‘ARMAX2’ represents the ARMAX (19, 0, 1, 0) model and ‘ARMAX3’ represents the ARMAX (19, 1, 2, 0) model.

The sparse AR (19) model has the lowest root mean squared error for one-step-ahead prediction and three-step-ahead prediction. For $h = 2, 4, 5$ and 6 , the ARMAX (19, 0, 1, 0) model is the lowest. This model also has the lowest error when $h = 10$. For $h = 7$ to 9 , the model that has the lowest root mean squared error is ARMAX (19, 1, 2, 0). These results are shown in Table 1, while the skill scores in Table 1 are the same as the Root Mean Square Error (RMSE) table.

In conclusion of the cross-validation presented here, the best model appears to be ARMAX (19, 0, 1, 0), as this model can achieve the forecasting horizons up to six years.

4. Benchmark Modeling using Moving-Average Data:

Recall that, in the government study of *BOR* [2012], the WS data was presented in the form of the (backward) 10-year moving average in order to highlight its marked decadal-scale variability. Thus, it is prudent to conduct a similar time series analysis using the 10-year moving-average of both the Colorado River WS and the Great Salt Lake water level. Below is a general formula that was used to find the moving average:

$$\dot{X}_t = \frac{\sum_{i=0}^9 X_{t-i}}{10}, \quad \text{for } t = 10, 11, 12, \dots$$

where \dot{X}_t is the new ten-year moving-average time series and X_t represents the raw time

series. (Note: the first 9 observations of each data set were removed.) Figure 11 below shows the time series of the 10-year moving-average WS and GSL.

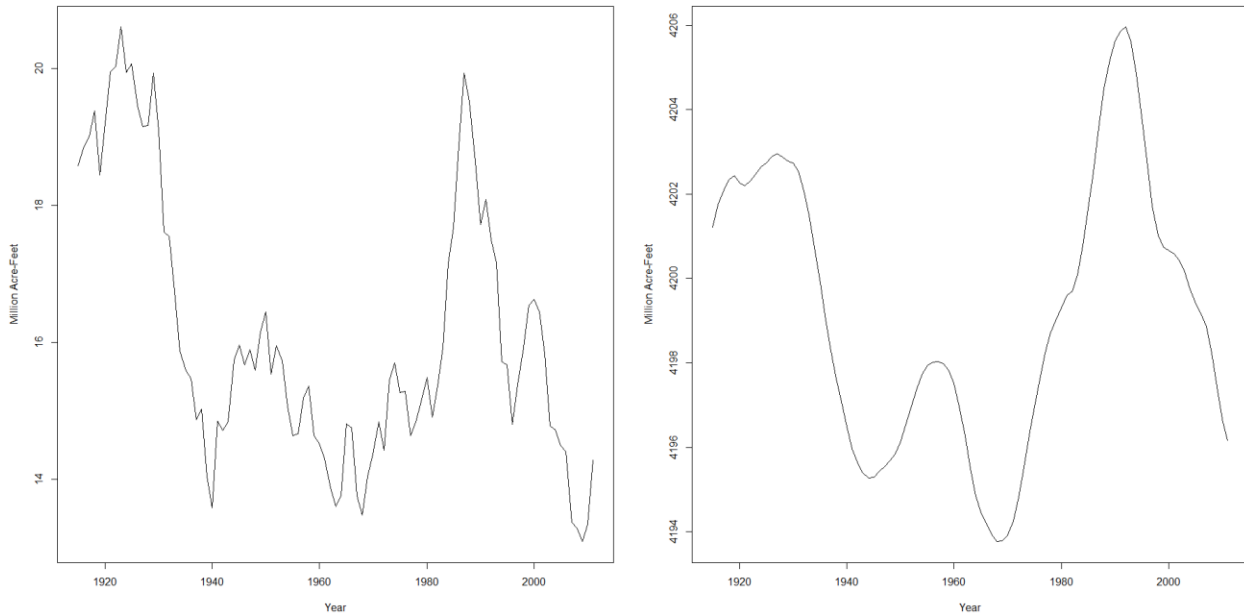


Figure 11. Time series of the ten-year moving-average water supply of the Colorado River (left) and time series of the ten-year moving average of Great Salt Lake water level (right).

The GSL time series is very smooth due to the large and shallow lake minimizing the interannual variability and thereby enhancing the low-frequency, climate-driven variations [Lall and Mann, 1995; Wang *et al.*, 2010, 2012]). The final models selected are:

Ten-Year Moving-Average Univariate Model:

The best univariate model chosen for the 10-year moving-average WS is an AR (19) model; this is consistent with the raw-data model in Section 3. The fitted model is:

$$\begin{aligned}
X_t = & 0.1992X_{t-1} - 0.0514X_{t-2} + 0.1731X_{t-3} + 0.0887X_{t-4} - 0.0282X_{t-5} \\
& + 0.0334X_{t-6} - 0.1966X_{t-7} + 0.1174X_{t-8} - 0.0532X_{t-9} \\
& - 0.5161X_{t-10} + 0.1626X_{t-11} \\
& - 0.1017X_{t-12} + 0.3447X_{t-13} + 0.0144X_{t-14} + 0.1150X_{t-15} \\
& - 0.1354X_{t-16} - 0.2810X_{t-17} + 0.0375X_{t-18} - 0.2447X_{t-19} + Z_t
\end{aligned}$$

where Z_t is a white-noise process with an estimated variance of 0.1946. Next, two competitive ARMAX models were chosen using a slightly different method than previously mentioned. First, multiple linear regression was run among the Colorado River water supply and the Great Salt Lake elevation. The equation is as follows:

$$\begin{aligned}
X_t = & \beta_0 + \sum_{i=1}^n \beta_i S_{t-i} + e_t \\
= & -0.0493 + 1.7720 * S_{t-1} - 2.4889 * S_{t-2} + 1.1278 * S_{t-3} - 0.6349 \\
& * S_{t-4} - 0.4446 * S_{t-5} + 1.4953 * S_{t-6} - 0.7594 * S_{t-7} + e_t
\end{aligned}$$

Where X_t denotes the Colorado River water supply, S_{t-i} denotes the number of lags (n) of Great Salt Lake terms and e_t denotes the error term at each t . The chosen models are shown below:

Ten-Year Moving-Average ARMAX (13, 10, 7, 0):

One of the ARMAX models chosen had an ARMA (13, 10) base, and it includes 7 GSL elevation lags. The model equation is:

$$\begin{aligned}
X_t = & -.0493 + 1.7720 * S_{t-1} - 2.4889 * S_{t-2} + 1.1278 * S_{t-3} - 0.6349 * S_{t-4} \\
& - 0.4446 * S_{t-5} + 1.4953 * S_{t-6} - 0.7594 * S_{t-7} + 0.1840 * X_{t-3} \\
& + 0.2239 * X_{t-13} - 0.6748 * Z_{t-10} + Z_t
\end{aligned}$$

where Z_t is a white noise process with an estimated variance of 0.2051.

Ten-Year Moving-Average ARMAX (19, 10, 7, 0):

The other competitive ARMAX model for the ten-year moving-average WS also included 7 GSL lags and had an ARMA (19, 10) base. The equation looks like:

$$\begin{aligned} X_t = & -0.0493 + 1.7720 * S_{t-1} - 2.4889 * S_{t-2} + 1.1278 * S_{t-3} - 0.6349 * S_{t-4} \\ & - 0.4446 * S_{t-5} + 1.4953 * S_{t-6} - 0.7594 * S_{t-7} + 0.1837 * X_{t-3} \\ & + 0.2223 * X_{t-13} + 0.0153 * X_{t-18} - 0.1264 * X_{t-19} - 0.6690 * Z_{t-10} \\ & + Z_t \end{aligned}$$

where Z_t is a white noise process with an estimated variance of 0.2018. The residuals for this model are shown below in Figure 12.

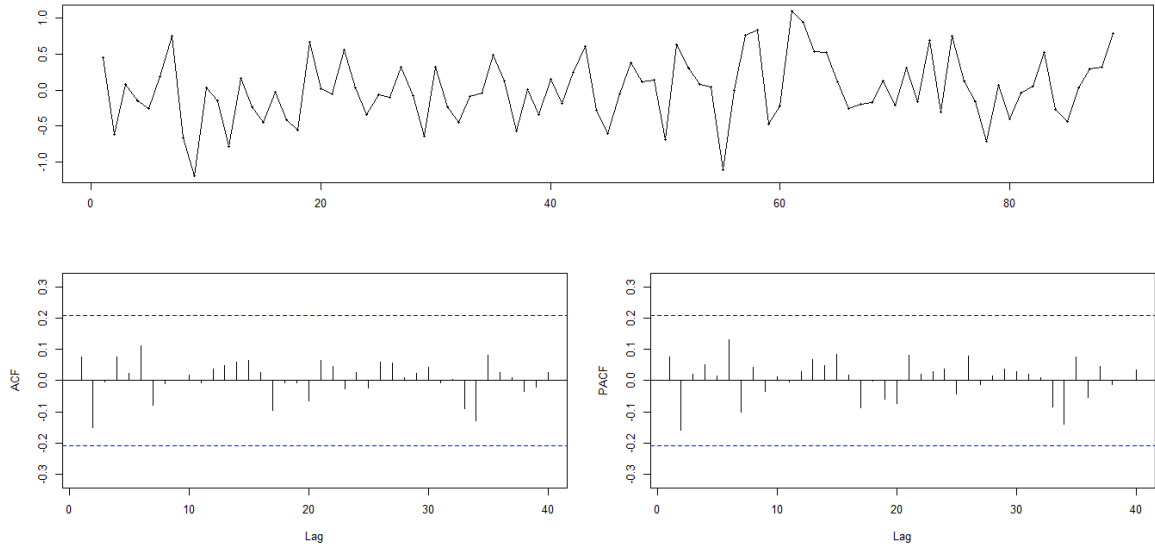


Figure 12. Residuals for the ARMA (19, 10) on the error terms.

The formula for finding the prediction variances is shown below, where \hat{X}_{t+h} is the h-step prediction for $i = 1, 2, \dots, 10$ and \tilde{X}_{t+h} is the h-step 10-year average

prediction:

$$\tilde{X}_{t+i} = \sum_{j=1}^i \hat{X}_{t+j} + \sum_{j=0}^{10-i-1} X_{t-j}, \text{ for } i = 1, 2, \dots, 10$$

$$Var(\tilde{X}_{t+i}) = Var\left(\sum_{i=1}^i \hat{X}_i\right) = \left(\frac{1}{100}\right) * \left[\sum_{l=1}^i \sum_{m=1}^i Cov(X_l X_m)\right]$$

Cross-validation was subsequently applied by dividing the ten-year moving-average WS time series into a training set and a test set. The training set included the first 81 observations, while the test data set included the last 15 observations. Ten forecasting horizons were used ($h = 1, 2, \dots, 10$) as was previously done. The results are shown below in Table 2, along with the corresponding RMSE and the skill score of each model.

RMSE	1	2	3	4	5	6	7	8	9	10
AR(19)	0.5522	0.5374	0.5470	0.4876	0.5062	0.4872	0.4805	0.4951	0.5087	0.4506
ARMAX1	0.4152	0.4123	0.4079	0.4268	0.4297	0.4266	0.4316	0.4369	0.4315	0.4312
ARMAX2	0.4153	0.4190	0.4125	0.4197	0.4208	0.4181	0.4185	0.4231	0.4243	0.4265
SS										
AR(19)	0.0181	0.0802	0.0541	0.2458	0.1811	0.2350	0.2537	0.2091	0.1605	0.3391
ARMAX1	0.4450	0.4585	0.4739	0.4221	0.4099	0.4135	0.3977	0.3841	0.3959	0.3949
ARMAX2	0.4448	0.4409	0.4621	0.4413	0.4340	0.4367	0.4337	0.4223	0.4160	0.4081

Table 2. Ten-year cross-validation results for the three models discussed. Root mean squared error is shown on top while the skill score is shown on the bottom. The column names are the ten forecasting horizons ($h = 1, 2, \dots, 10$). Highlighted values are the best statistic for each forecasting horizon.

The two ARMAX models had a similar performance in terms of the MSE of

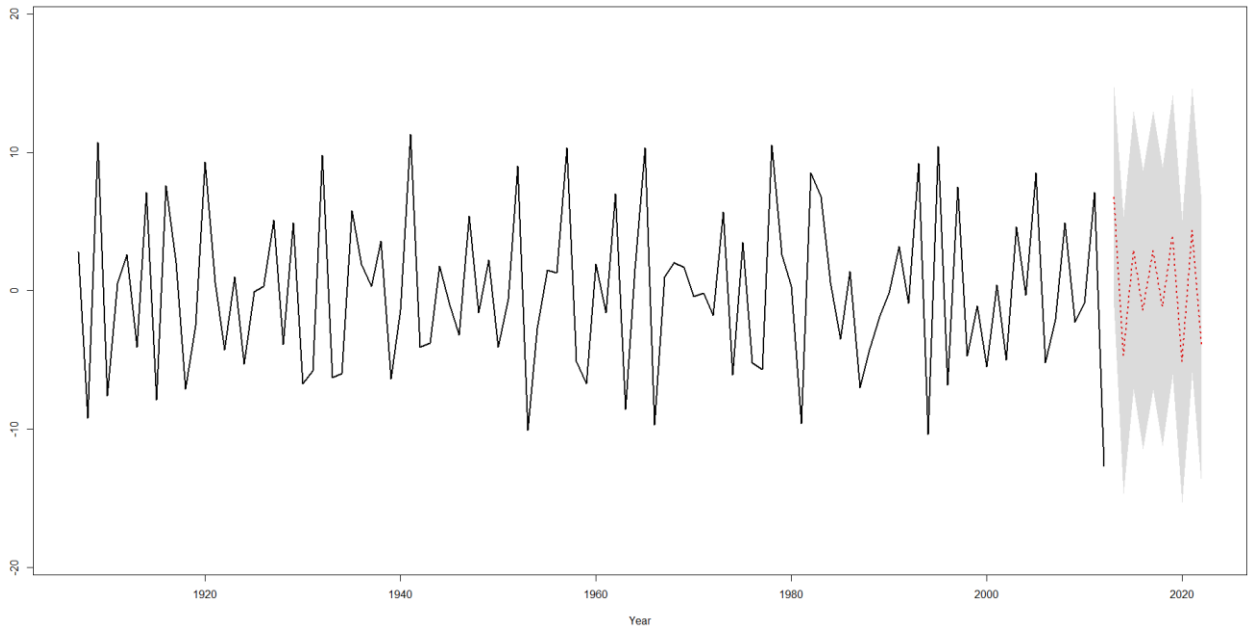
fitting, i.e., the estimated white noise variance. More importantly, they significantly outperformed the univariate model in terms of forecasting, as shown in Table 2 (in which the lowest RMSE at each horizon is bold). Table 2 also displays the results of the skill score, where the model with the highest skill score is indicated.

In conclusion of this cross-validation study, the best model for $h = 4$ and beyond (i.e. forecast out to at least five years) is the ARMAX (19, 10, 7, 0) model. The prediction of these models is discussed next.

5. Results and Discussions

5.1 The predictions:

Figure 13(a) shows the sparse AR (19) model of WS that was first presented, its 10-year prediction and the 95% confidence intervals. Figure 13(b) displays the first ARMAX (19, 0, 1, 0) model in the same manner, while Figure 13(c) shows the last model, ARMAX (19, 1, 2, 0).



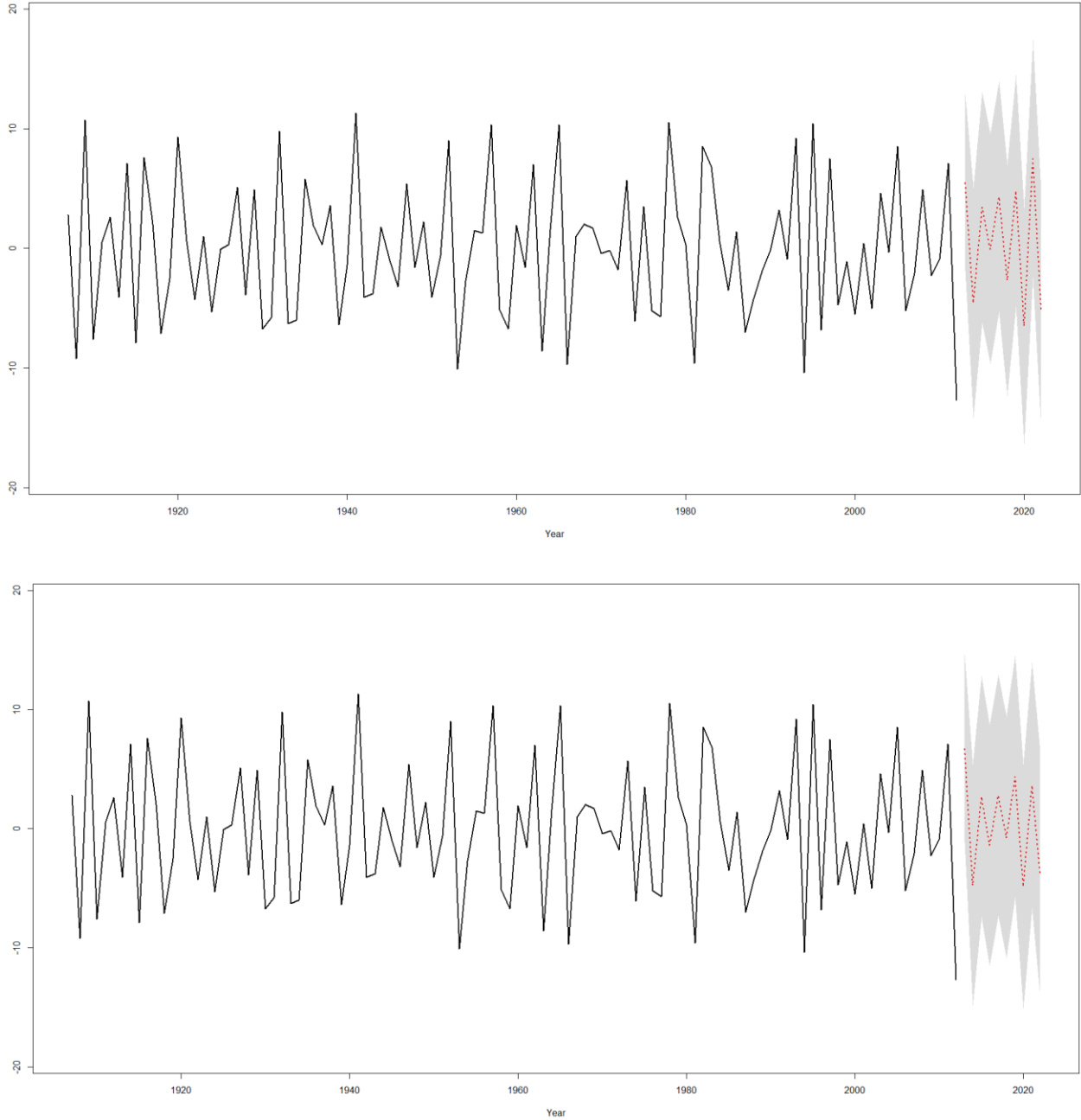


Figure 13. (a) The observed values (black) shown with the predictions (red) ten years out for the sparse AR (19) model. (b) The observed values (black) shown with the predictions (red) ten years out for the ARMAX (19, 0, 1, 0) model. (c) The observed values (black) shown with the predictions (red) ten years out for the ARMAX (19, 1, 2, 0) model.

Despite the cross validations that seem to justify their forecast performance, the

predicted WS in all these models seemed noisy and the confidence intervals appear to be too big to be practical. In other words, these models are not suitable for predicting individual years' value. Subsequently and to be comparable with the BOR study, i.e. to focus on the low-frequency variation of WS, we display in Figure 14 (below) the 10-year moving-average of WS along with both predictions using the best models of the annual data and ten-year moving average data.

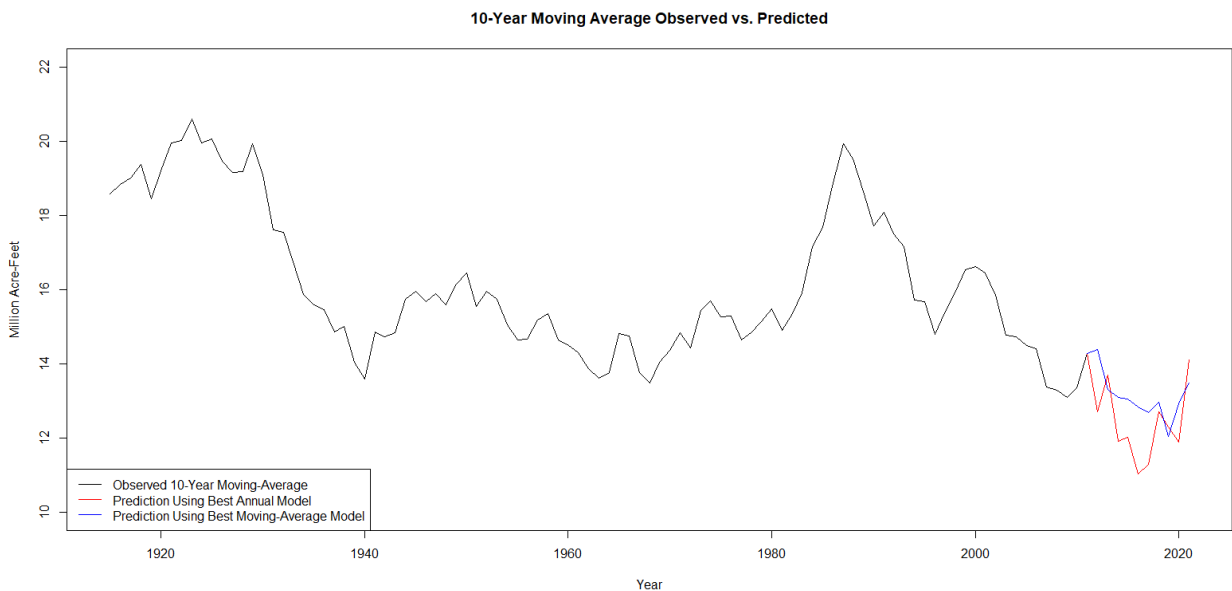


Figure 14. The observed values for the ten-year moving average (black) shown with the predictions (red) ten years out for the best annual model and the predictions (blue) ten years out for the best moving-average model.

The predictions here are shown in terms of their 10-year moving average. Despite the slightly more drastic variation predicted by the annual data model, forecasts of the smoothed WS by both models show an overall decreasing trend in water supply for about

seven years and then an increase for the next three. This trend appears to be reflecting the recent climate anomalies: The 2018 water year saw four consecutive months with below average precipitation in the Colorado River Basin (<https://www.cbrfc.noaa.gov/wsup/pub2/discussion/cbrfc.2018.2.1.pdf>) and many states reported their lowest snowpack levels on record. The predicted WS (with 10-year MA) did track the observed values through 2018 (cf., <https://www.usbr.gov/rsvrWater/HistoricalApp.html>). This means that the water levels will not be higher than they are now for at least a decade, at least compared to what was previously projected in *BOR* [2012].

5.2 Discussion:

The various time lags adopted in the models present somewhat different physical meanings. For instance, the 19-year lag in GSL reflects one half-cycle of its multi-decadal variability of ~35 years [Wang *et al.*, 2012] associated with the Interdecadal Pacific Oscillation [Dai, 2012]. The 3-year lag in GSL echoes a quarter-phase of a ~12-year cycle that characterizes the Great Basin precipitation [Smith *et al.*, 2015; Wang *et al.*, 2009]. Likewise, the 6-year lag in GSL reflects a half-phase of the same climate cycle. The different time lags linking the Colorado River WS and the GSL water level reflect the hydrologic buffering shared by the two basins [Wang *et al.*, 2018], since a majority fraction of the surface water is contributed by groundwater. Fang and Shen [2017] showed that the annual streamflow-storage correlation in this region is medium-high, depicting a high baseflow fraction that normally leads to a high annual correlation between streamflow-storage correlation. As a result, the GSL water level and the low-

frequency variation of the Colorado River WS are highly coherent; this, in turn, enables the former to be a predictor for the latter.

Compared to the seasonal statistical water supply forecasts for the Western U.S. [Pagano *et al.*, 2009], which uses the Z-score regression and has been in operation, the work presented here employs time series forecasting to predict the multi-year tendency (instead of individual years' value). The prediction presented here is not without caveats and one potential problem is in the use of a moving average in the predictand (i.e. the WS) and the predictor (the GSL); this is because a MA operator can alter the time structure. When interpreting the forecast of these moving-average model(s), there could be missing information at the interannual timescale making the year-by-year validation impractical. Earlier in the analysis when coming up with ARMAX models, several MA terms were used. Cross-validation was performed and the results were less than desirable. This was because the models were built with such high-ordered MA terms, which made the predictions very unstable. Models that performed similarly in terms of AIC with fewer MA terms were eventually chosen. Nevertheless, the 170-year-long record of the GSL water level makes it a useful indicator of the Colorado River WS variation at its decadal frequency [Wang *et al.*, 2018]. The analysis presented here demonstrates that decadal-scale predictability is feasible through common time-series modeling approaches.

It should be noted that the GSL undergoes considerable diversions leading to a long-term decline in its water level [Bedford, 2009; Mohammed and Tarboton, 2012]. Quantification of the GSL diversions is difficult due to the complexity of the Bear River that runs through three states prior to entry into the GSL, as well as the effect of

groundwater withdrawal that reduces recharge to the lake [*Hakala, 2014; Masbruch et al., 2016*]. While GSL diversions create a slow, monotonic downward trend in the lake level, such a trend produces autocorrelation and thereby should be corrected via the inclusion of autoregressive terms in the model.

As of this writing, the Upper Colorado River snowpack hovers slightly above average (e.g. the February 18, 2019 BOR data puts the Upper Colorado River Basin headwaters at 110% of the historic average). This current condition may seem contradictory to the downtrend predicted in Figure 14, but again the prediction is not made for the year-to-year comparison. The present prediction depicts the 10-year MA and it should not be a quantitative indication for any individual year. Given the recent succession in drought years rendering the soil's capability of moisture absorption, the bulk of the 2019 WS might be lower than the robust snowpack may suggest. Nevertheless, the outcome of this study highlights the need to reexamine the water supply of the Colorado River called for by *Ficklin et al. [2016]*, based upon the profound inconsistency exhibited by the hydrological projections between the Coupled Model Intercomparison Project Phase 5 (CMIP5) and CMIP3.

6. Conclusions:

The temporal coherence between the low-frequency variations of the Colorado River and GSL storage systems, as documented by *Wang et al. [2018]*, provides the basis for the time series modeling undertaken here and is remarkable. The models were used to assess the forecast potential and the results point to the feasibility of using the GSL water level to assist in the forecast of the Colorado River WS, beyond seasonal timescales.

Multiple datasets and time series models were used to predict the WS. Predictions by these models were compared in terms of annual data versus ten-year moving-averages. The predictive potential of the GSL elevation, as was demonstrated in previous studies, is one of conceivable application and adoption as a forecast method useful for the Colorado River WS, especially in the face of prolonged drought as has been observed in the recent decade.

The multi-year prediction of the Colorado River (Figure 14) is particularly striking in that it suggests a decrease through 2020 rather than an increase as shown in BOR [2012]. In other words, the prediction implies a widened imbalance between supply and demand by as much as 2.5 maf in 2060; this imbalance is considerably greater than the 2012 estimation of the BOR study. While water-saving plans can be tailored more to scenarios where water supplies decrease further in the future, this paper presents possible prediction methods of WS that can be useful in the implementation of these plans. We conclude that Colorado River managers would be pragmatic to periodically update and assess future WS through the adaptation and consideration of a similar decadal prediction scheme.

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APPENDIX

LIST OF R CODE

```
# Reading in data and taking the difference

# colorado river water data

mydata <- read.csv("C:/Users/Brian/Documents/Grad School/Grad School/Thesis
Project/thesisData.csv", skip = 3)

mydataTS <- ts(mydata[,-1], frequency = 1, start = 1906)

mydataTS <- mydataTS[,-1]

colorado.river.diff.ts <- diff(mydataTS)

# great salt lake data

gsl <- read.csv(file = "C:/Users/Brian/Documents/Grad School/Grad School/Thesis
Project/GSL.csv", header = TRUE)

#make the same time length as mydataTS - from 1906 to 2012

gsl <- gsl[c(-1:-31),]

gsl <- gsl[c(1:107),]

#turn into a time series object

gslTS <- ts(gsl[,c(-1,-3:-5)], frequency = 1, start = 1906)

diff.gsl.timeseries <- diff(gslTS)
```



```

#acf/pacf

acf(mydataTS)

pacf(Colorado.river.diff.ts)


# cross correlation of two time series

ccf(diff.ws.timeseries, diff.gsl.timeseries, lag.max = 30)


# Checking residuals

tsdisplay(residuals(residual.ar13), lag.max = 30)


# building armax model

arimax2 <- arimax(x = colorado.river.diff.ts, order = c(19,0,0), include.mean = FALSE,
method = "ML",
               fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA, NA), transform.pars =
FALSE, xtransf = diff(gslTS), transfer = list(c(1,0)))


# armax cross-validation

arimax2.function <- function(h, val.length){

```

```

# create training and test data sets

train <- colorado.river.diff.ts[1:(106-val.length)]

test <- colorado.river.diff.ts[(106-val.length+1):106]

# initialize prediction and se vectors

pred <- rep(0,length(test))

se <- rep(0,length(test))

for(i in 1:length(test)){

  # fit the model

  armax <- arima(x = colorado.river.diff.ts[1:(106-val.length+i-h)], order = c(19,0,0),

    fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA, NA), transform.pars =

FALSE,

    include.mean = FALSE, method = "ML", xtransf = diff(gslTS)[1:(106-

val.length+i-h)], transfer = list(c(1, 0))) #21

    y_tf <- filter(diff(gslTS), filter = armax$coef[(20):(20)], method='recursive', side=1)

* armax$coef[(21)]

    armax_forc <- arimax(x=colorado.river.diff.ts[1:(106-val.length+i-h)], order =

c(19,0,0), transform.pars = FALSE,

    fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA),

    include.mean = FALSE, method = "ML", xreg=y_tf[1:(106-val.length+i-

h)]) #20

  # start making predictions

  armax_pred <- predict(armax_forc, newxreg=y_tf[(106-val.length+i-h+1):(106-

val.length+i)], n.ahead=h, se.fit=TRUE)

```

```

pred[i] <- armax_pred$pred[h]

se[i] <- armax_pred$se[h]

}

df <- data.frame("Step" = c((106-val.length+1):106),

                 "Predicted Value" = pred,

                 "Actual Value" = test,

                 "Difference" = (pred-test),

                 "Diff Sq." = (pred-test)^2,

                 "se" = se)

# root mean squared error

rmse <- sqrt((sum(df$Diff.Sq.) / val.length))

# find mse of using mean to predict the future

average.value <- rep(0, length(test))

for(j in 1:length(test)){

  average.value[j] <- mean(colorado.river.diff.ts[1:(106-val.length+j-h)])

}

diff.squared <- (test - average.value)^2

mse.mean <- sum(diff.squared) / length(test)

ss <- 1 - ((rmse^2)/mse.mean)

ase <- sum(df$se) / length(test)

output <- data.frame("rmse" = rmse, "ss" = ss, "average standard error" = ase)

output

```

```

}

# table to generate rmse and ss

new.rmse.ss.Function <- function(h, val.length){

  # create the rmse matrix

  rmseMatrix <- matrix(ncol = h, nrow = 3)

  for(i in 1:h){

    rmseMatrix[1,i] <- ARmodel1.function(i,val.length)[1,1]

  }

  for(j in 1:h){

    rmseMatrix[2,j] <- armax2.function(j,val.length)[1,1]

  }

  for(k in 1:h){

    rmseMatrix[3,k] <- armax3.function(k,val.length)[1,1]

  }

  # create the ss matrix

  ssMatrix <- matrix(ncol = h, nrow = 3)

  for(i in 1:h){

    ssMatrix[1,i] <- ARmodel1.function(i,val.length)[1,2]

  }

  for(j in 1:h){

    ssMatrix[2,j] <- armax2.function(j,val.length)[1,2]

```

```

}

for(k in 1:h){

  ssMatrix[3,k] <- armax3.function(k,val.length)[1,2]

}

colnames(rmseMatrix) <- seq(1:h)

rownames(rmseMatrix) <- c("ARmodel1", "armax2", "armax3")

colnames(ssMatrix) <- seq(1:h)

rownames(ssMatrix) <- c("ARmodel1", "armax2", "armax3")

list("rmse" = rmseMatrix, "ss" = ssMatrix)

}

```

Observed vs fit

```

plot(colorado.river.diff.ts,type="l",col="black",xlab="Year")

lines(fitted(armax2),col="red", lwd = 2)

ub <- fitted(armax2) + 1.96 * sqrt(armax2$sigma2)

lb <- fitted(armax2) - 1.96 * sqrt(armax2$sigma2)

lines(ub, col = "gray")

lines(lb, col = "gray")

```

creating 10 year moving average data

read in colorado river water supply (WS) data

```

ws <- read.csv("C:/Users/Brian/Documents/Grad School/Grad School/Thesis
Project/thesisData.csv", skip = 3) # 1906 - 2012

# add a column in ws for the 10 year moving average

ws$ten.year.ma <- NA

# calculate 10 year moving-average

for(i in 10:107){

  ws$ten.year.ma[i] <- sum(ws$wSupply[(i-9):i]) / 10

}

# 10 year moving-average of WS goes from 1915 - 2012

# make ws into a time series from 1915 - 2011

ws.timeseries <- ts(ws[c(-1:-9,-107),c(-1:-3)], start = 1915, frequency = 1)


# GSL data

# read in gsl data and create a time series

gsl <- read.csv(file = "C:/Users/Brian/Documents/Grad School/Grad School/Thesis
Project/GSL.csv", header = TRUE) # 1875 - 2016


# add a column in gsl for the 10 year moving average

gsl$ten.year.ma <- NA


# calculate 10 year moving-average

for(i in 10:149){

  gsl$ten.year.ma[i] <- sum(gsl$GSL[(i-9):i]) / 10

```

```

}

# 10 year moving average of gsl goes from 1884 - 2016

# make gsl into a time series from 1915 - 2011

gsl.timeseries <- ts(gsl[c(-1:-40,-138:-149),c(-1:-5)], start = 1915, frequency = 1)


# predictions

#armax2

# fit the armax model

armax <- arima(x = colorado.river.diff.ts[1:(106)], order = c(19,0,0),

              fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA, NA), transform.pars =

FALSE,

              include.mean = FALSE, method = "ML", xtransf = diff(gslTS)[1:(106)],

transfer = list(c(1, 0)))

y_tf <- filter(diff(gslTS), filter = armax$coef[(20):(20)], method='recursive', side=1) *

armax$coef[(21)]

armax_forc <- arimax(x=colorado.river.diff.ts, order = c(19,0,0), transform.pars =

FALSE,

                    fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA),

                    include.mean = FALSE, method = "ML", xreg=y_tf[1:(106)])

```

```

# start making predictions

pred2 <- predict(armax_forc, newxreg=y_tf[(102):(106)], n.ahead=10, se.fit=TRUE)

ts.plot(colorado.river.diff.ts, pred2$pred, type = "l", col = c("black", "red"), lty = c(1,3),
xlab = "Year")

ub <- pred2$pred + 1.96 * pred2$se

lb <- pred2$pred - 1.96 * pred2$se

polygon(x=c((2013:2022),rev(2013:2022)), y=c(ub, rev(lb)), col = gray(level = 0.3,alpha
= 0.2), border = NA)

```

```

# plot of 10-year moving-average side by side

# plot of the two time series side-by-side

par(mfrow=c(1,2))

plot(ws.timeseries, xlab = "Year", ylab = "Million Acre-Feet")

plot(gsl.timeseries, xlab = "Year", ylab = "Million Acre-Feet")

```

```

#plot of best annual model in terms of 10-year-moving average

# the best annual model

armax2 <- arimax(x = colorado.river.diff.ts, order = c(19,0,0), include.mean = FALSE,
method = "ML",

fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA, NA), transform.pars =
FALSE,

```



```

xtransf = diff(gslTS), transfer = list(c(1,0)))

# setting up to make predictions

y_tf <- filter(diff(gslTS), filter = armax$coef[(20):(20)], method='recursive', side=1) *
armax$coef[(21)]

armax_forc <- arimax(x=colorado.river.diff.ts, order = c(19,0,0), transform.pars =
FALSE,

fixed = c(rep(NA,12), 0, 0, 0, 0, NA, 0, NA, NA),

include.mean = FALSE, method = "ML", xreg=y_tf)

# start making predictions

armax_pred <- predict(armax_forc, newxreg=y_tf[(106-10+1):(106)], n.ahead=10,
se.fit=TRUE)

# actual colorado river water supply

colorado.ws <- mydataTS

# undo the differencing of your prediction

colorado.ws.pred <- diffinv(armax_pred$pred, xi = colorado.ws[length(colorado.ws)])

# actual prediction

colorado.ws.pred <- ts(colorado.ws.pred, frequency = 1, start = 2012)

# plot of actual water supply and my prediction 10 years out

ts.plot(colorado.ws, colorado.ws.pred, col = c("black", "red"), xlab = "Year", ylab =
"Million Acre-Feet")

```

```

# get everything in 10-year moving average

# combine actual vs prediction

combined.ts <- c(colorado.ws, colorado.ws.pred)


# find 10-year moving average

ty.ma.combined <- rep(NA,(length(combined.ts)-9))

for (i in 10:(length(combined.ts))) {

  ty.ma.combined[i] <- sum(combined.ts[(i-9):i]) / 10

}

### testing

# find 10-year moving average of colorado.ws

colorado.ws.ma <- rep(NA,(length(colorado.ws)-9))

for (i in 1:(length(colorado.ws.ma)-9)) {

  colorado.ws.ma[i] <- sum(colorado.ws[(i):(i+9)]) / 10

}


# find 10-year moving average of colorado.ws.pred

colorado.ws.pred.ma <- rep(NA,10)

for (i in 1:(length(colorado.ws.pred.ma))) {

  colorado.ws.pred.ma[i] <- sum(combined.ts[(108+i-9):(108+i)]) / 10

}

```

```
# turn both moving-averages into time series

colorado.ws.ma <- ts(colorado.ws.ma, frequency = 1, start = 1915, end = 2011)

colorado.ws.pred.ma <- ts(colorado.ws.pred.ma, frequency = 1, start = 2012, end = 2021)

# plot of actual vs prediction

ts.plot(colorado.ws.ma, colorado.ws.pred.ma, col = c("Black", "Red"))

annual.lb <- colorado.ws.pred.ma - 1.96 * armax_pred$se

annual.ub <- colorado.ws.pred.ma + 1.96 * armax_pred$se
```