Electromagnetic Scattering from Periodic Conducting Arrays

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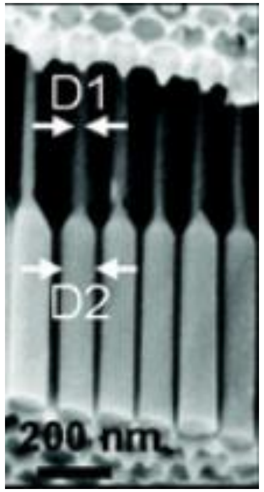
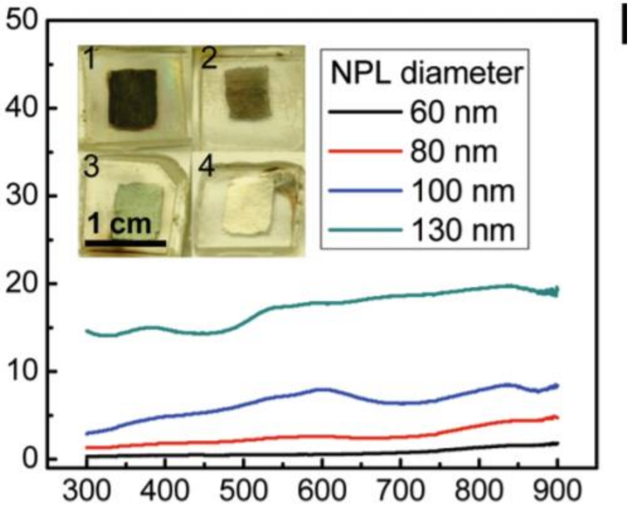
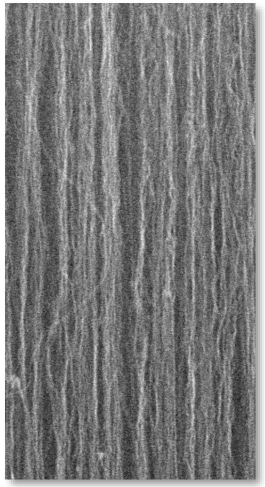
- Previous work used scalar wave approximation.
- Electromagnetic scattering theory can give more physical understanding.



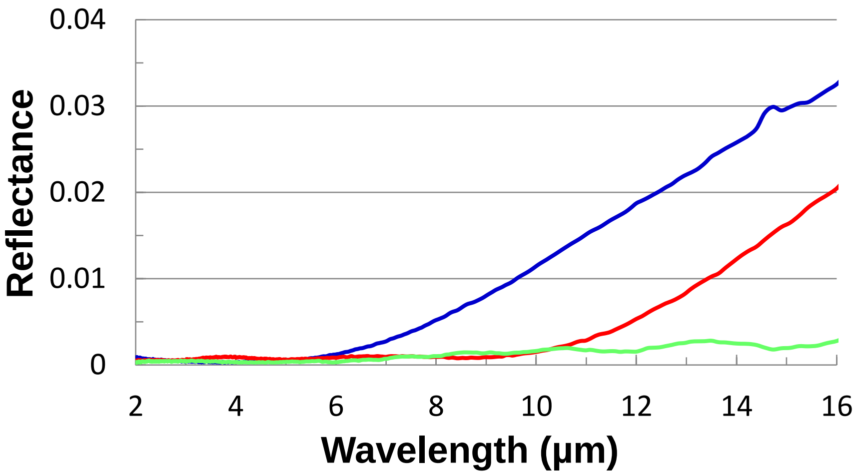
Dual-diameter Germanium nanopillars



Nano Lett. **10**, 3823 (2010)

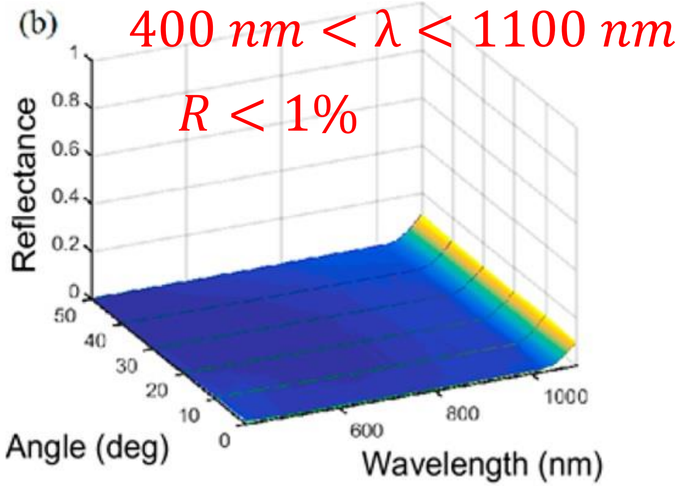


Carbon-nanotube forest



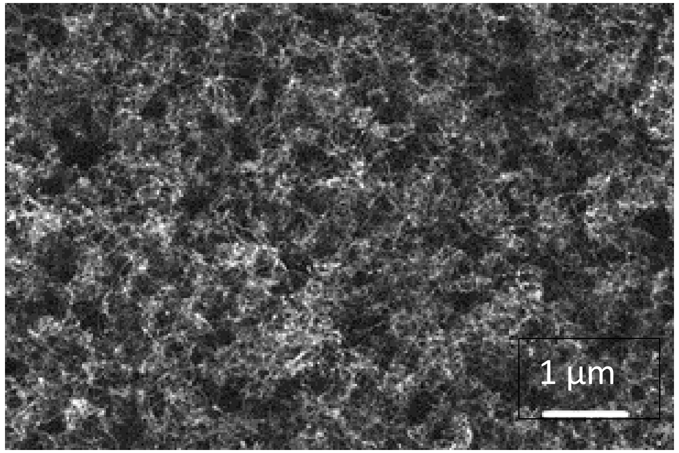
J. Appl. Phy. **118**, 013106 (2015).

Silicon nanocones



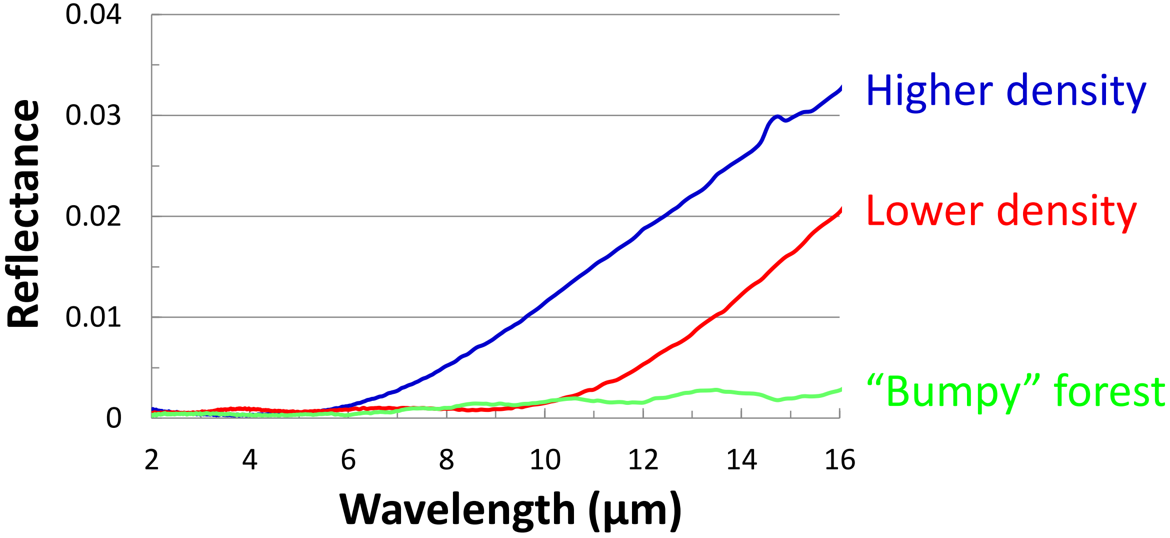
ACS Photonics **3**, 1854 (2016).

The Role of the Interface in Reflectance

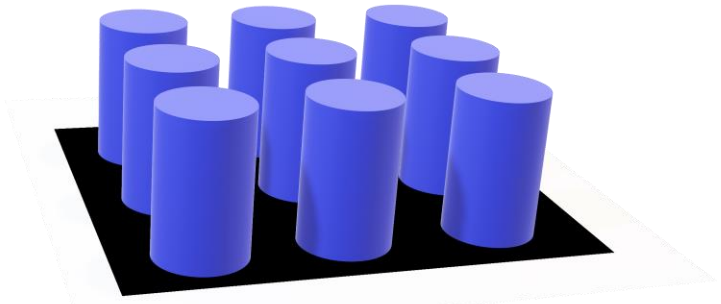


CNT forest crust

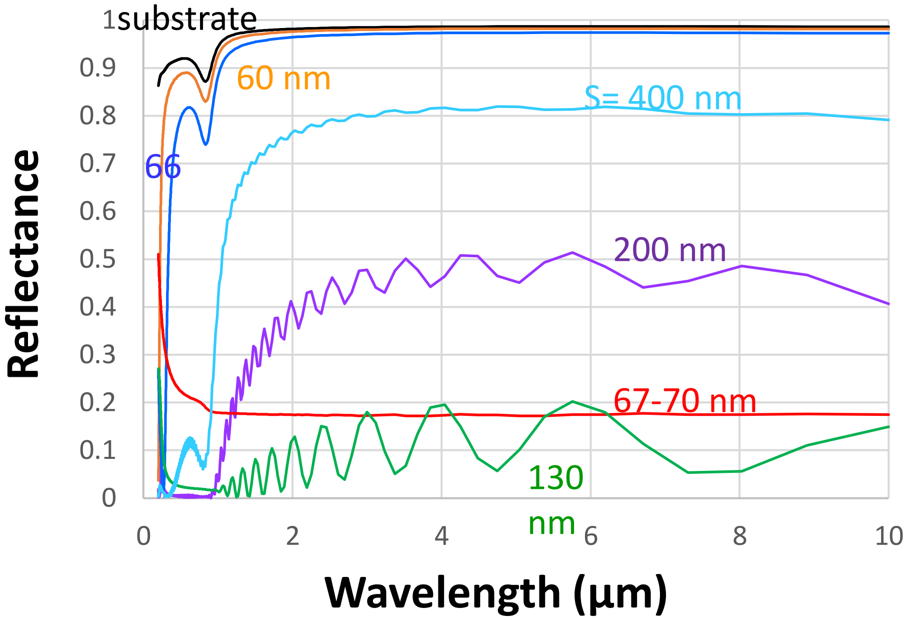
- CNT forest crust has many voids.
- Random height modulation can add larger scale voids in the crust.



- Finite-difference-time-domain simulation

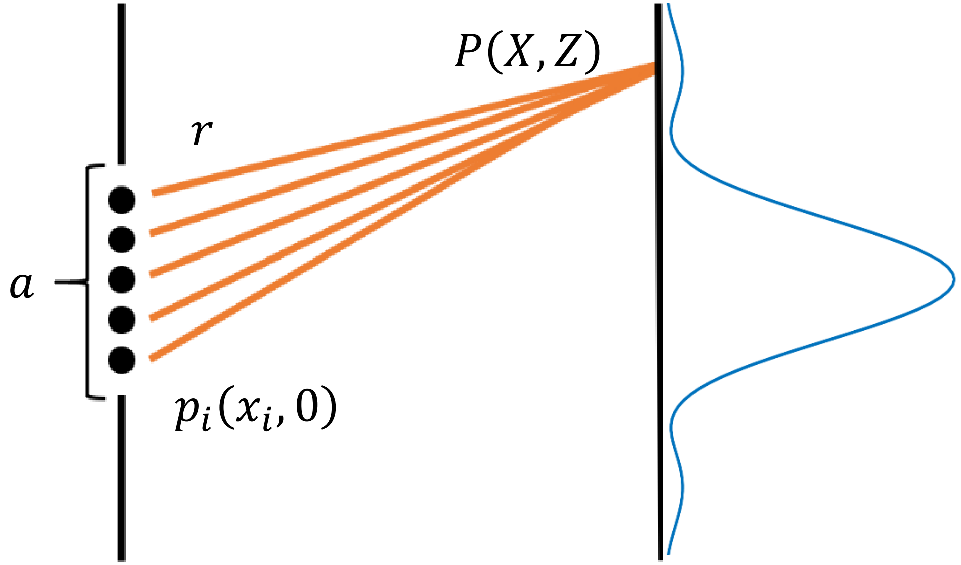


- Oscillations indicate interference plays a role

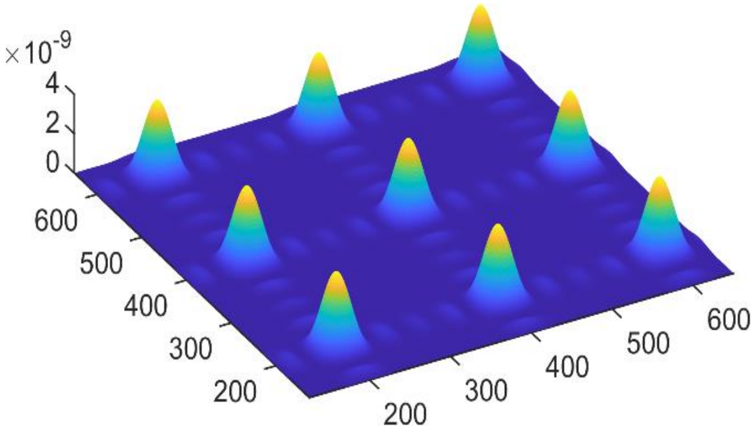


Scalar Wave Model

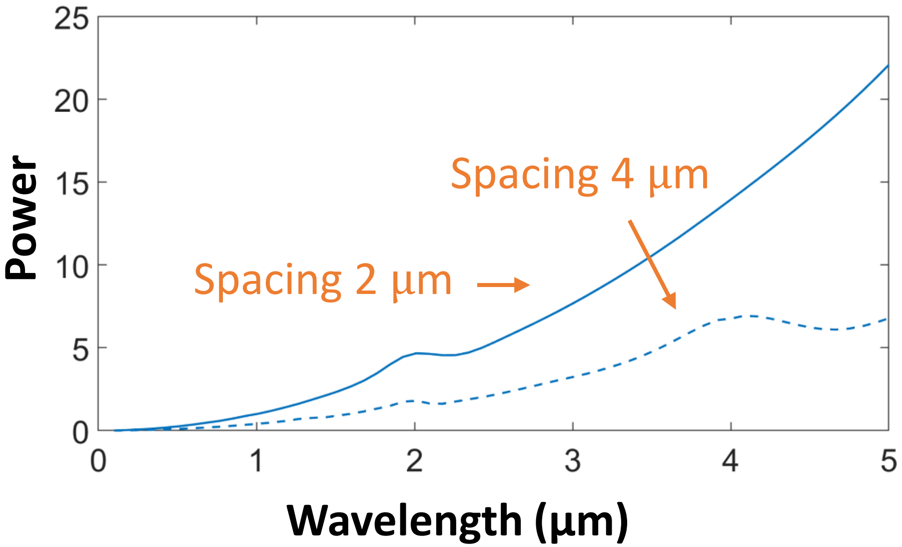
- Reflected power is proportional to the area under the diffraction pattern.
- Reflected power increases with wavelength.
- An increase in aperture spacing decreases reflected power.

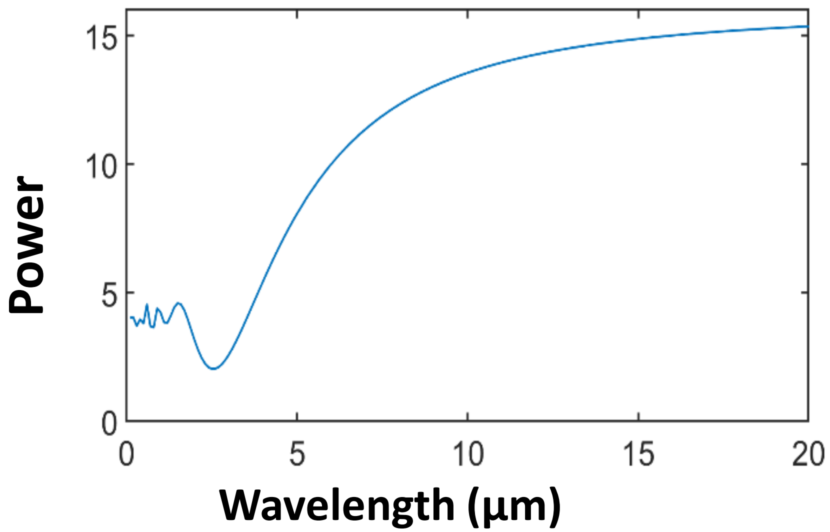


Huygens-Fresnel principle

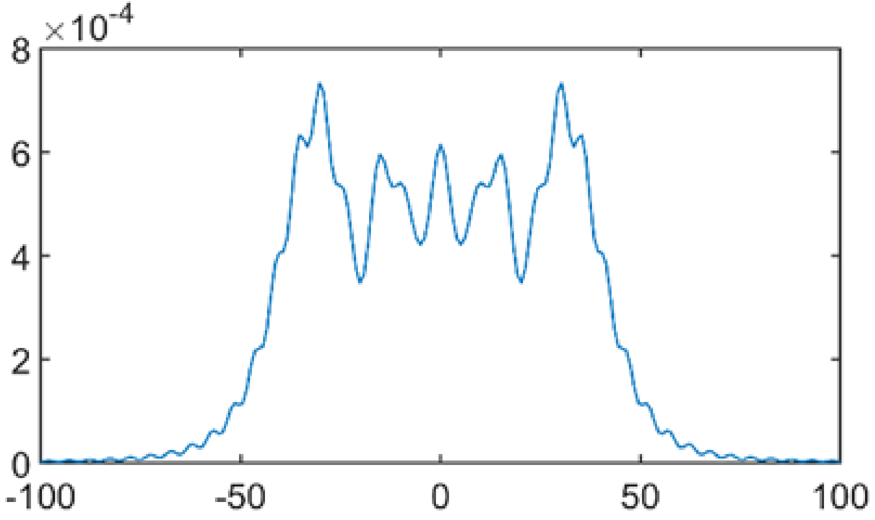


- Point sources have oscillations like FDTD simulation

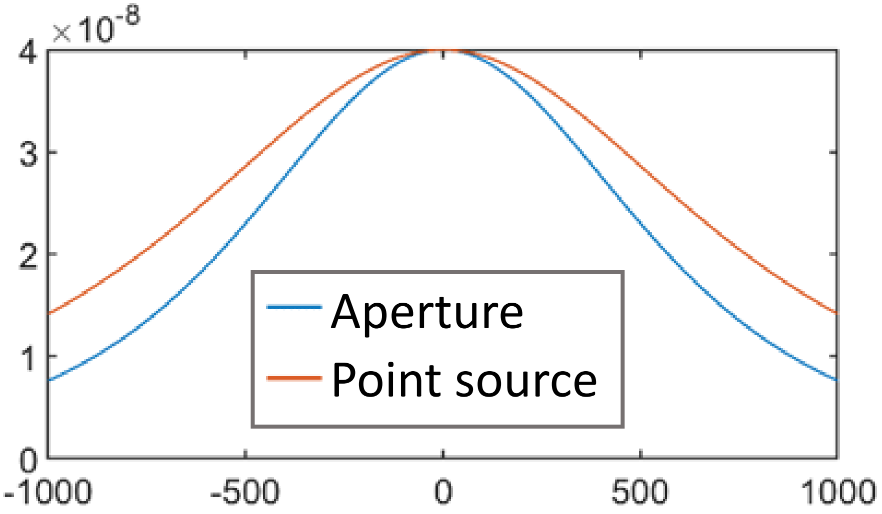


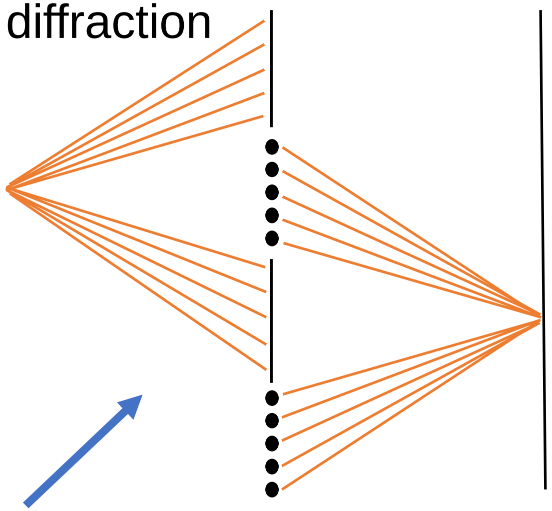


Issues with the Scalar Wave Model



Increase in wavelength decreases destructive interference.



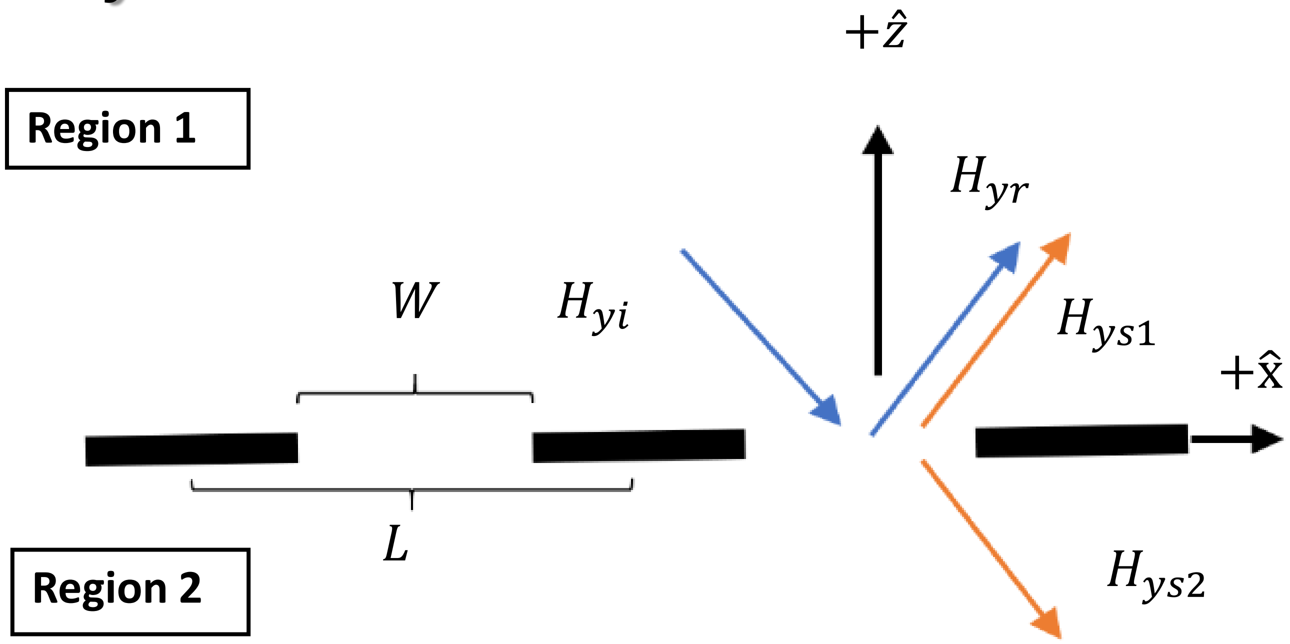


- Interference decreases reflected power.
- Similarly, interference should decrease transmitted power.

- Conservation of energy says that they can't both decrease.
- The $\frac{1}{r^2}$ dependence of power goes to infinity as the screen approaches the aperture.

Electromagnetic Scattering Theory

- Incident and reflected waves are plane waves.
- Scattered must have periodicity L.
- Integral equation contains boundary conditions, but no information about the current in the conducting strips.
- We assume form of electric field and solve for a free parameter.



Boundary condition of electric field

$E_x(x, 0) = 0$ at $x = \pm W/2$

Continuity of magnetic field

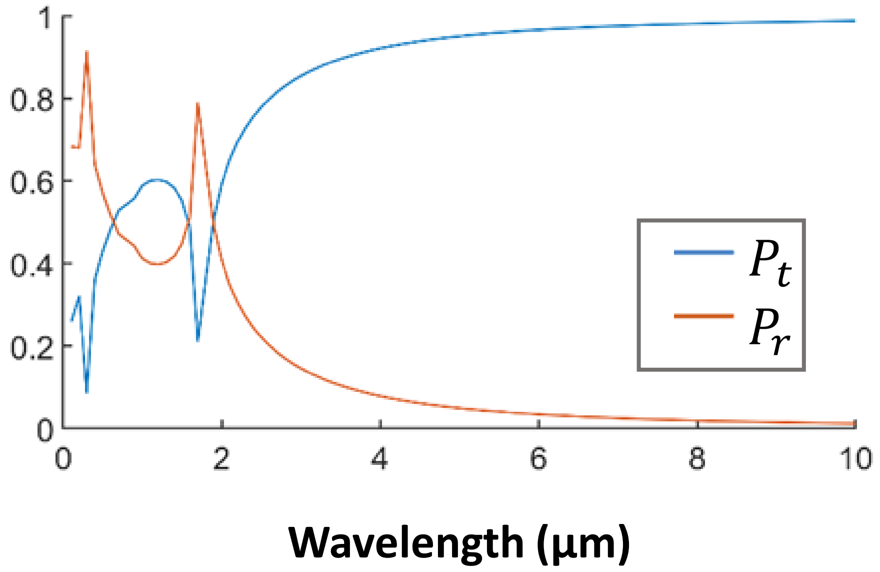
$H_{yi} + H_{yr} + H_{ys1} = H_{ys2}$ at $z = 0$

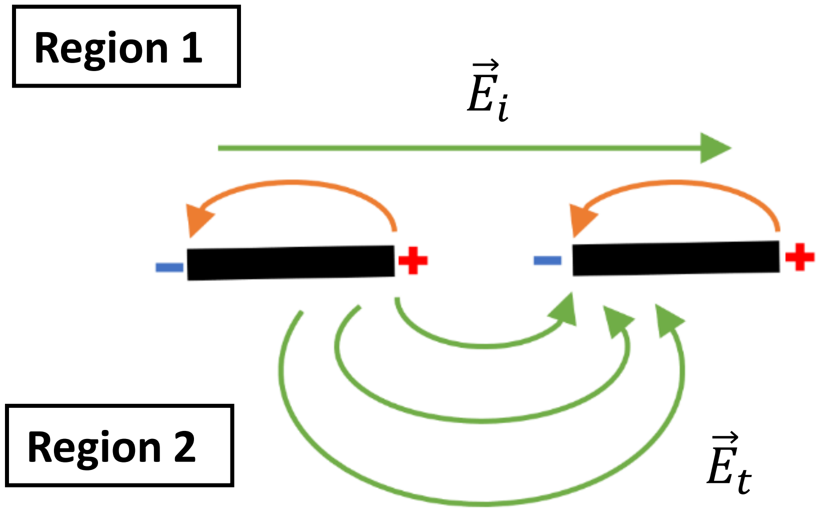
$$A_0 e^{-i\beta_0 x} = -i\omega\epsilon \int_{-W/2}^{W/2} E_x(x', 0) G(x, x') dx'$$

$$G(x, x') = \sum_{n=-\infty}^{\infty} \frac{e^{-i\beta_n (x - x')}}{i q_n L}$$

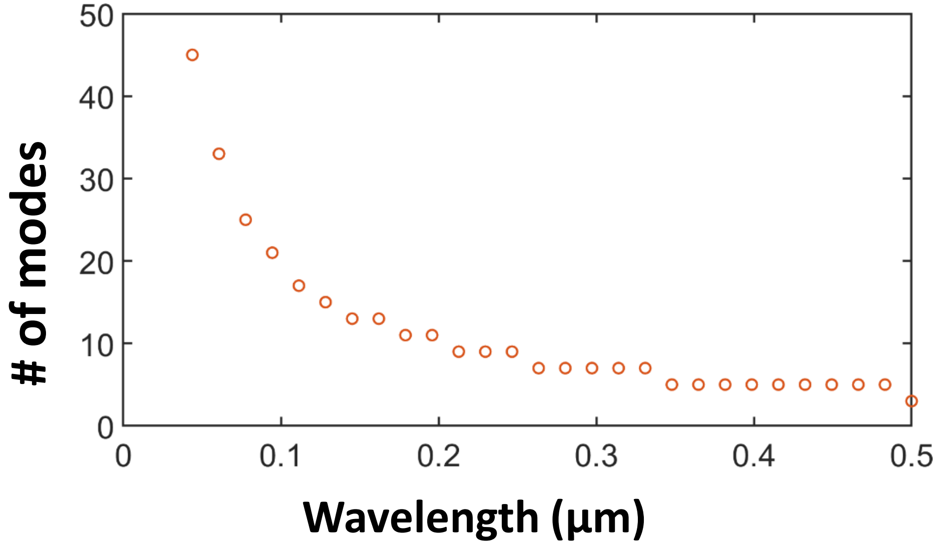
Dependence of Power on Wavelength

- Scattering theory satisfies conservation of energy.
- Reflectance decreases with wavelength, contrary to scalar wave prediction.
- A very long wavelength is like a static field, which may be able to transmit by inducing dipoles.





- Instead of oscillations, the reflectance has sharp peaks in short wavelength region.
- As wavelength increases, the number of propagating modes decreases.



Dependence of Power on Aperture Width

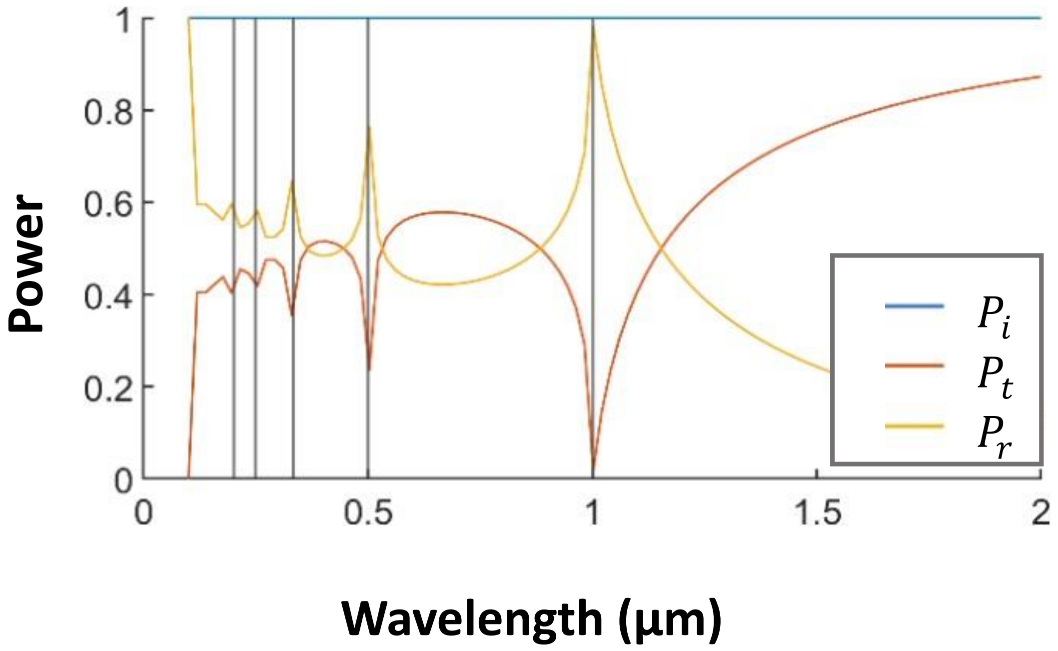
- Resonances in lattice occur whenever $\lambda = L/n$ for some integer n
- Transmission drops to zero and reflectance goes to one.
- Increase L leads to more resonances at long wavelengths.
- In the free-space limit, the integral equation is analytically solvable.

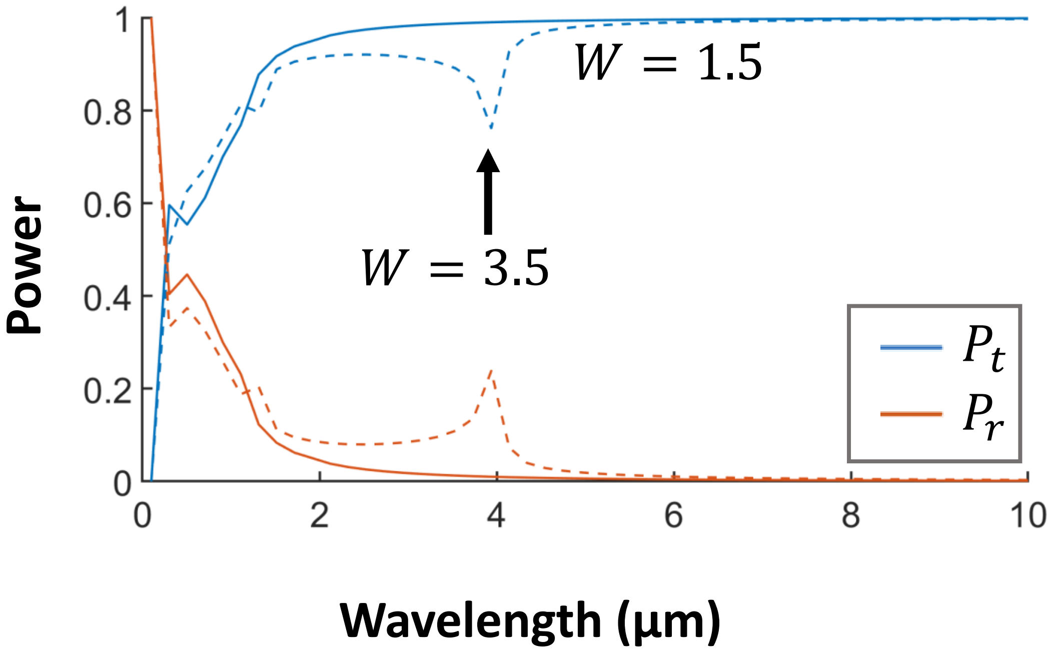
Free-Space Limit

$$A_0 e^{-i\beta_0 x} = -i\omega\epsilon \int_{-\infty}^{\infty} E_x(x,0) G(x,x')\,dx'$$

➡ $$E_x(x,0) = -\frac{A_0}{\omega\epsilon} e^{-i\beta_0 x} \sqrt{k^2 - \beta^2}$$

➡ $$H_{ys2} = H_{yi}$$





Conclusions

- This scattering solution predicts a decrease in reflectance at long wavelengths, which disagrees with the scalar wave model.

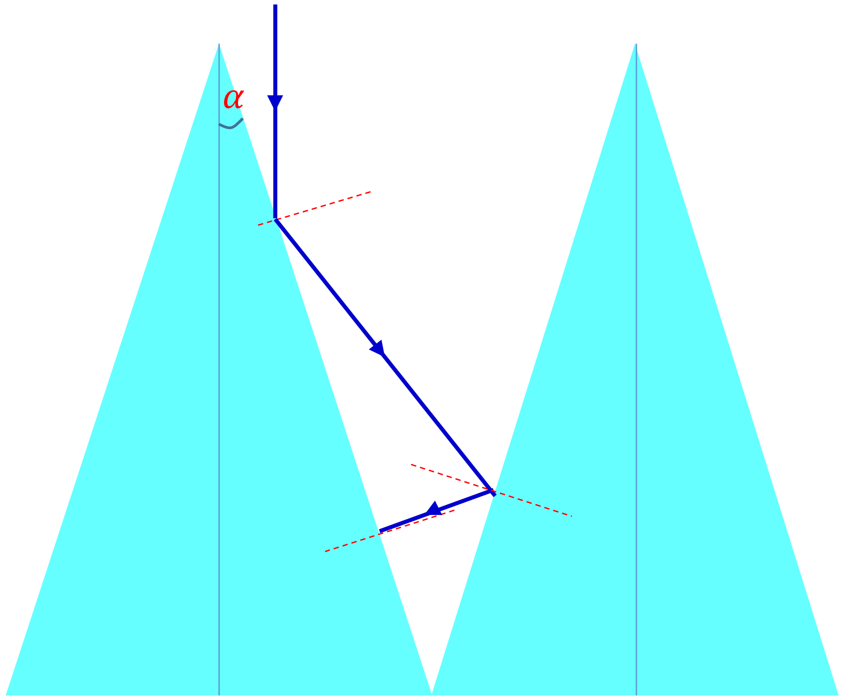
- This seems to indicate that the substrate reflection is more important than the interface at long wavelengths, but this would not be able to account for the experimental observation of extremely low reflectance for long wavelength light.

- This solution predicts resonances of incident light with the interface, which decrease reflectance.

- An analysis of surfaces with multiple periodicities, or an extension of the theory to two dimensional surfaces, could shed light on the effect of resonance on the reflectance.

Additional Material

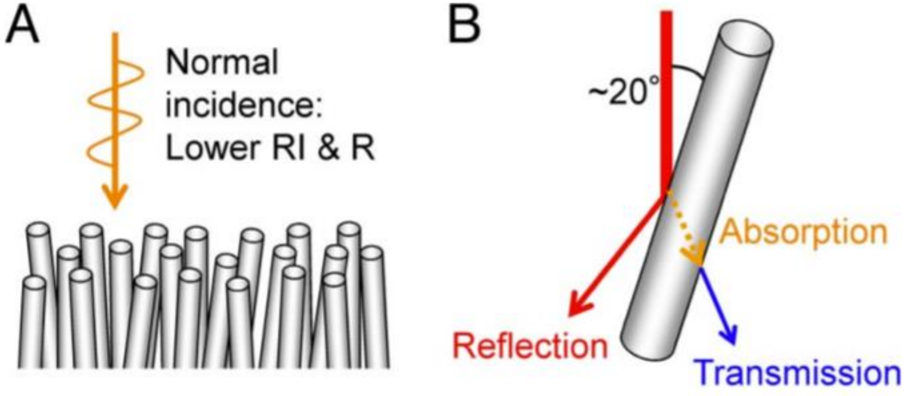
Ray optics explanation



# of reflection before going backward

$$n < \frac{\pi}{4\alpha} - \frac{1}{2}$$

- Smaller α provides more downward reflection.
- Height, density, and α dictate absorption.



- Thin rods may minimize upward reflection.
- But this fails to explain why reflectance increases at long wavelengths.
- In the case of CNT forest, wavelength is much greater than spacing, so we must consider the wave nature of light.

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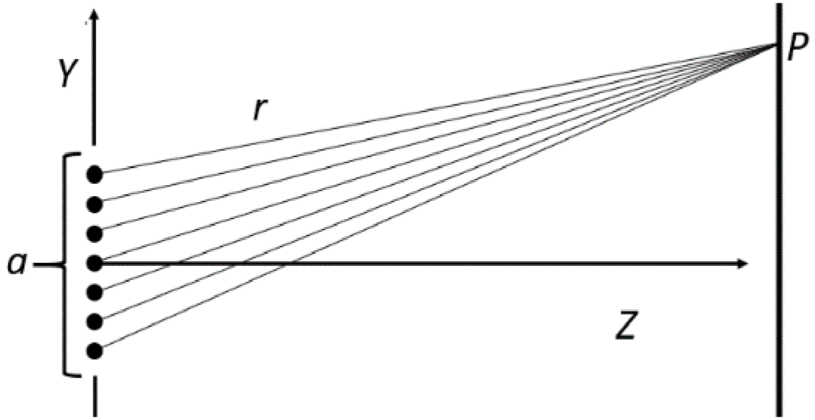
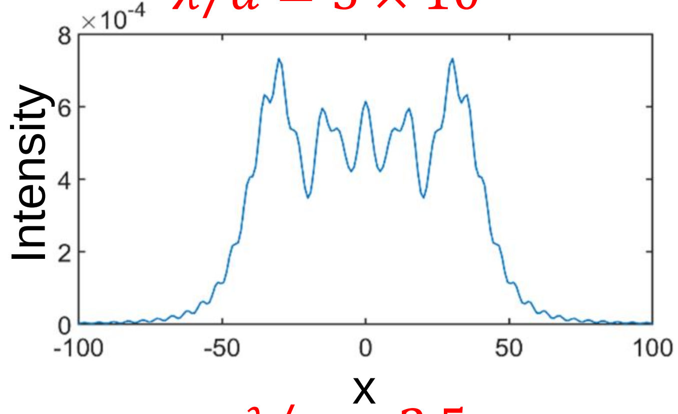
One Dimensional Aperture

- Single slit diffraction is the simplest phenomenon of wave interference.
- The Huygens-Fresnel principle gives an intuitive explanation for interference phenomena.

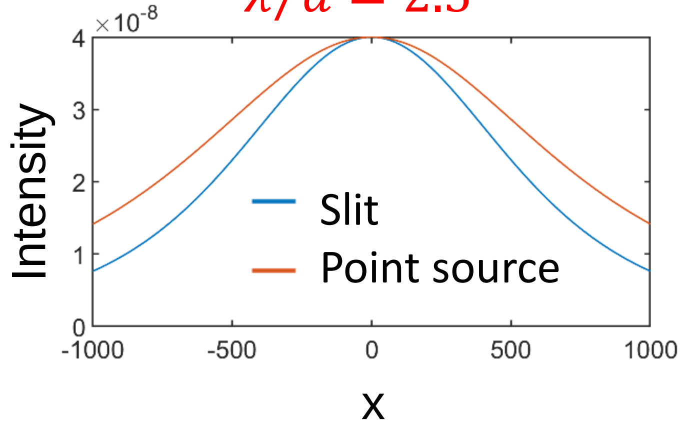


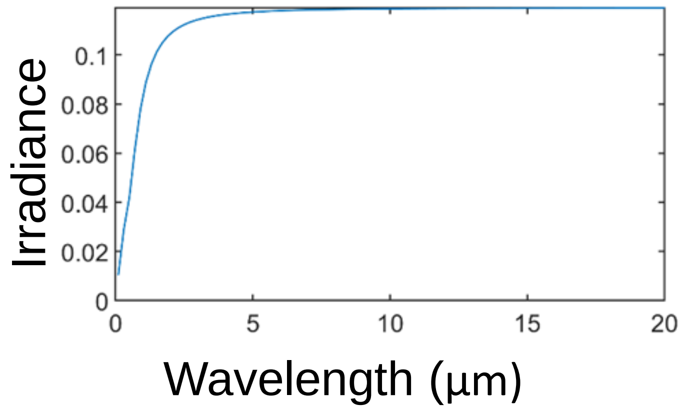
- The light intensities (diffraction patterns) are distinctively different for small λ/a and large λ/a at a distance $r \gg a$.
- The total hemispherical irradiance depends on wavelength.

$\lambda/a = 5 \times 10^{-3}$



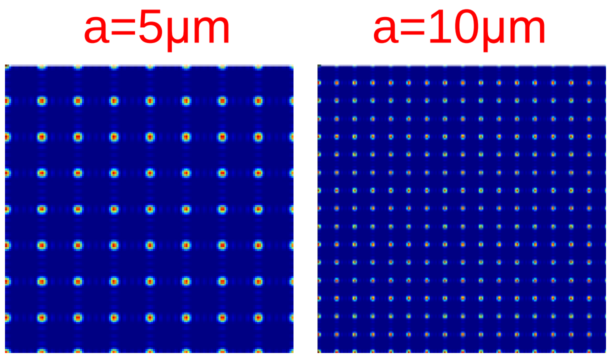
$\lambda/a = 2.5$



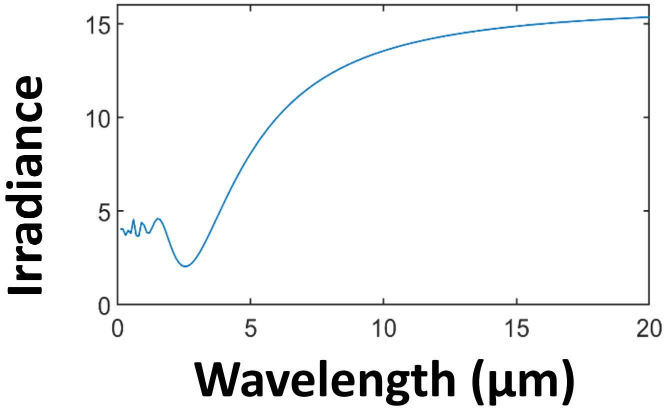


Hemispherical Irradiance of Point Source Arrays

- Diffraction lattice constant is inversely proportional to the real space lattice constant.
- Consistent with Bragg diffraction
- Oscillates around N and increases monotonically to N^2 due to less destructive interference.

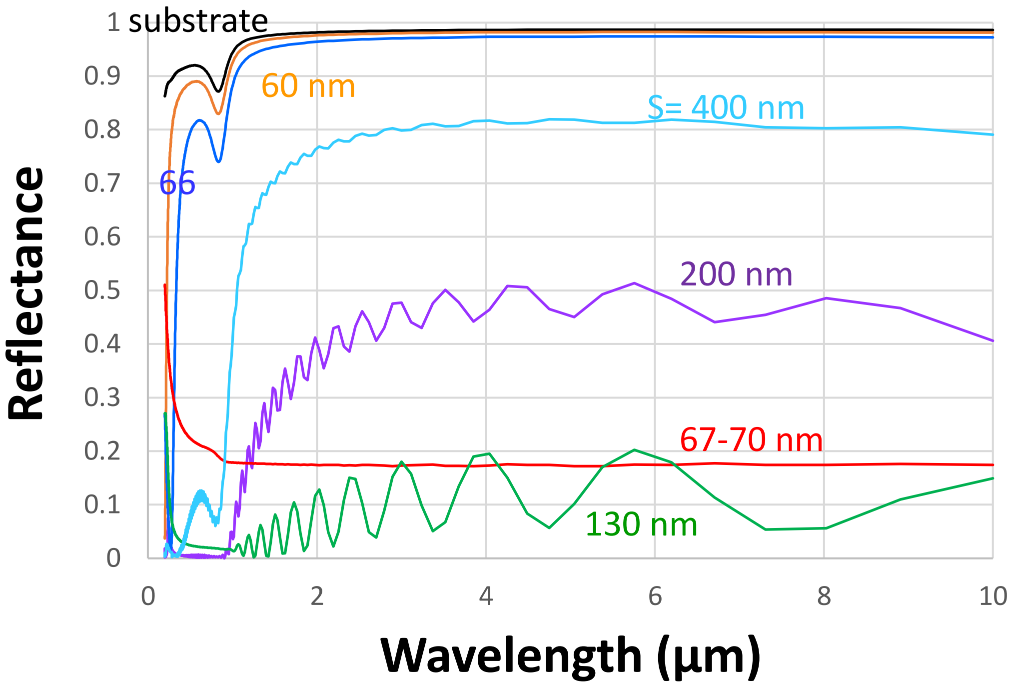


Diffraction pattern from 5x5 point source lattice with lattice constant a.



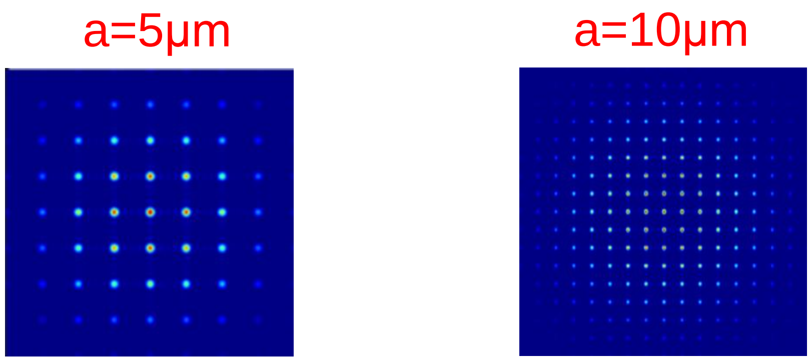
Irradiance from 2x2 points sources

- Oscillations and rising reflectance were observed in nanorods and CNT forest reflectance spectra.

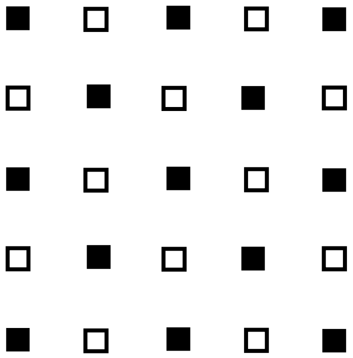


Reflectance spectra from FDTD simulation of 5x5 nanorod arrays.

Model an Array of Nanorods with 5x5 Apertures



Diffraction pattern from 5x5 aperture lattice with lattice constant a.

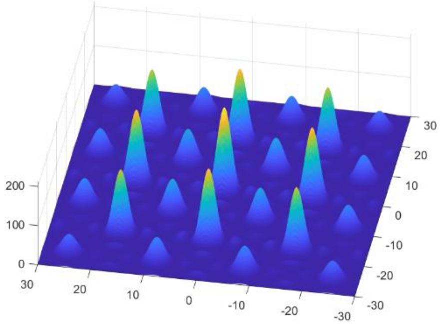


- If light is reflected from the tops of nanorods and the substrate between rods, it may be like light coming through an array of apertures.

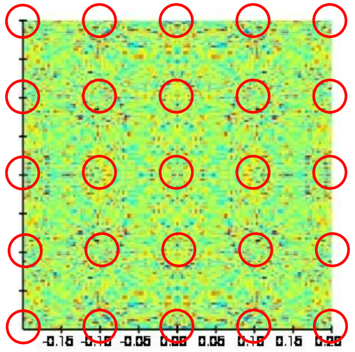
- Closed squares represent rods; open squares represent substrate.

- Substrate reflection is attenuated by a factor *f* and has different phase δ determined by the rod height *h* and wavelength λ. $\delta = 4\pi h/\lambda$

- The diffraction patterns reflect the symmetry of the rods.

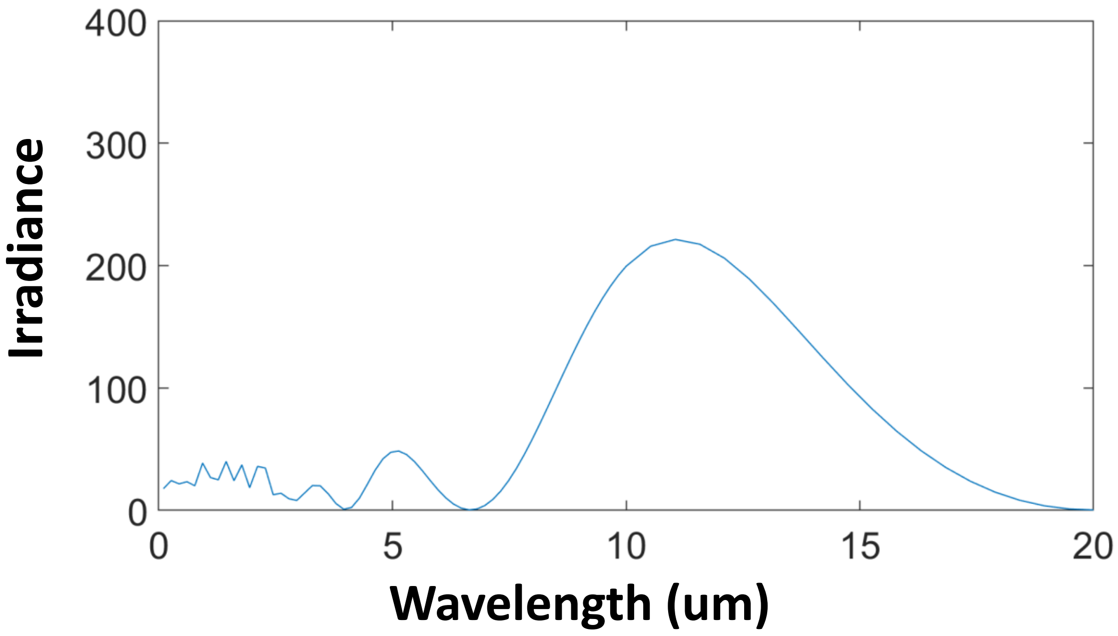


The diffraction pattern of a 5x5 array of apertures.

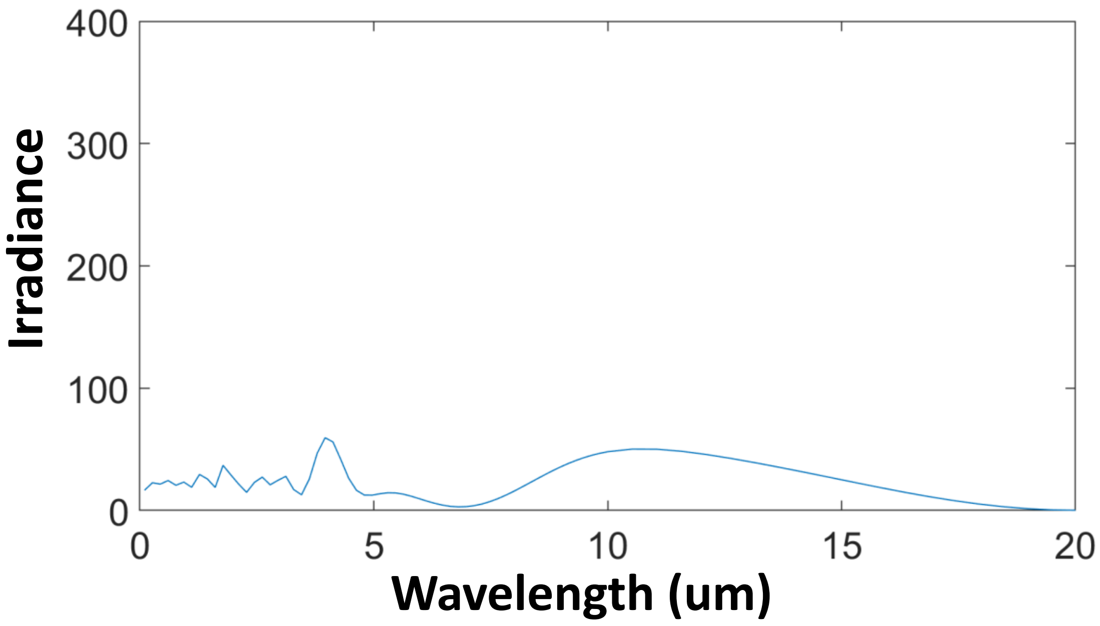


FDTD simulation of 5x5 nanorod array reflection.

Lattice Spacing



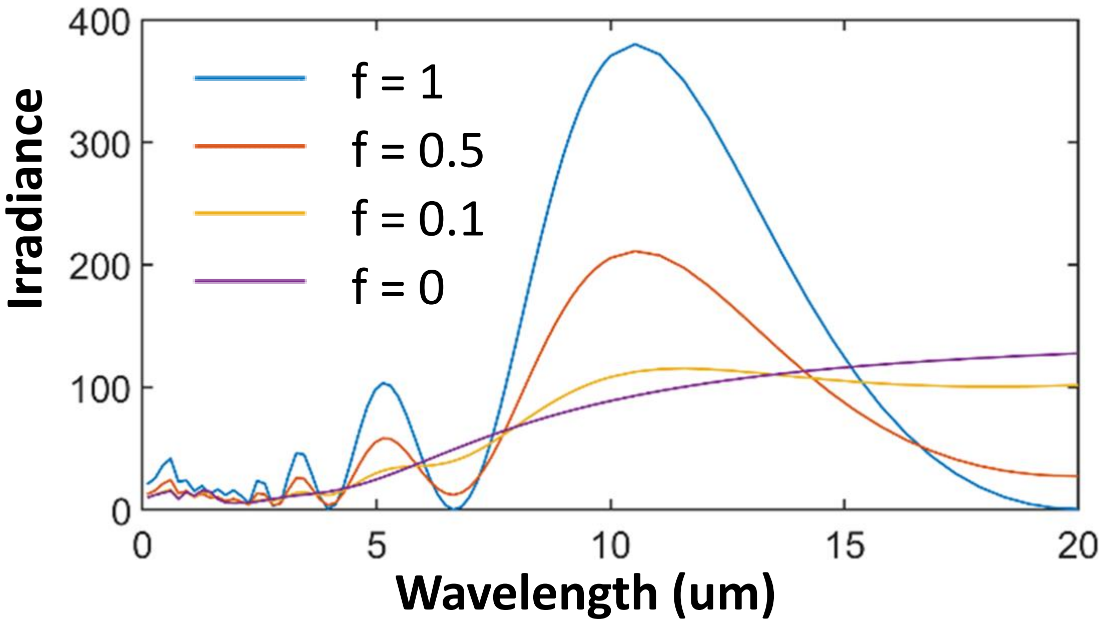
h=5, f=1, a=1.5 μm



h=5, f=1, a=3 μm

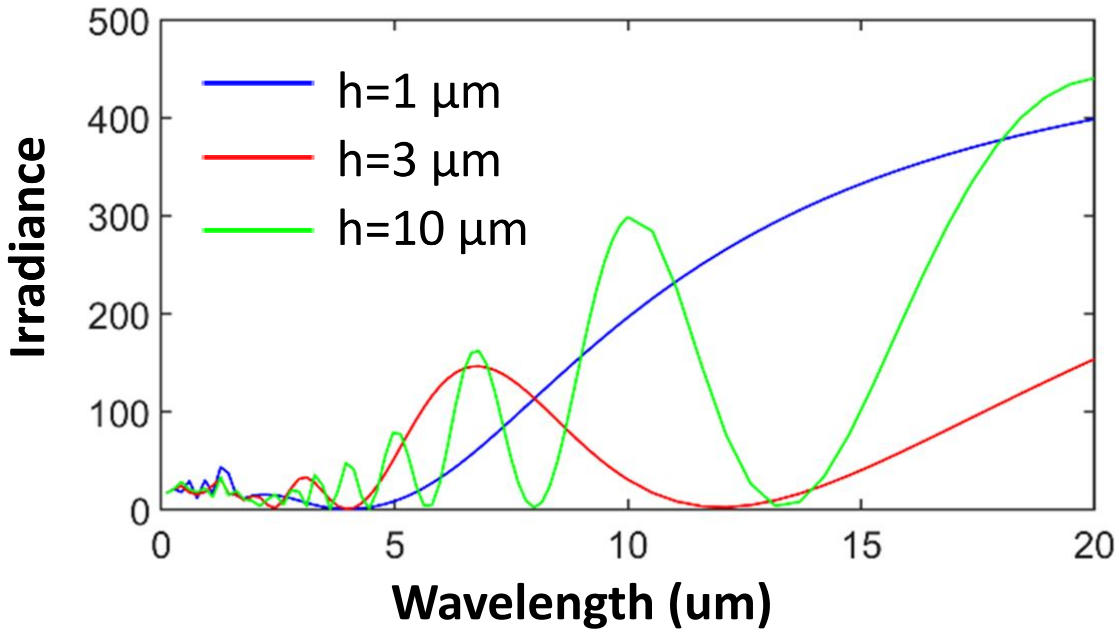
- Increasing the lattice spacing decreased the irradiance at longer wavelengths
- This agrees with experimental observation that lower density CNT forests are darker at longer wavelengths.

Substrate Reflectance



5x5 aperture array irradiance with $a = 1$, $h = 5$

Rod Height



5x5 aperture array with $a = 1$, $f = 0.8$

- Increasing the height introduced more minima but did not decrease the envelope of oscillation.
- The strength of substrate reflection is difficult to predict but may be proportional to the height of the array.