DESIGN AND TESTING OF DIGITALLY MANUFACTURED PARAFFIN ACRYLONITRILE-BUTADIENE-STYRENE HYBRID ROCKET MOTORS

by

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A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace Engineering

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2012
Abstract

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This research investigates the application of additive manufacturing techniques for fabricating hybrid rocket fuel grains composed of porous Acrylonitrile-butadiene-styrene impregnated with paraffin wax. The digitally manufactured ABS substrate provides mechanical support for the paraffin fuel material and serves as an additional fuel component. The embedded paraffin provides an enhanced fuel regression rate while having no detrimental effect on the thermodynamic burn properties of the fuel grain. Multiple fuel grains with various ABS-to-Paraffin mass ratios were fabricated and burned with nitrous oxide. Analytical predictions for end-to-end motor performance and fuel regression are compared against static test results. Baseline fuel grain regression calculations use an enthalpy balance energy analysis with the material and thermodynamic properties based on the mean paraffin/ABS mass fractions within the fuel grain. In support of these analytical comparisons, a novel method for propagating the fuel port burn surface was developed. In this modeling approach the fuel cross section grid is modeled as an image with white pixels representing the fuel and black pixels representing empty or burned grid cells.
Public Abstract

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Hybrid motors that employ non-toxic, non-explosive components with a liquid oxidizer and a solid hydrocarbon fuel grain have inherently safe operating characteristics. The oxidizer is blown though the solid fuel where it is combusted through a nozzle to produce thrust. This research investigated the combination of Acrylonitrile-butadiene-styrene impregnated with paraffin wax as the solid fuel component burned with nitrous oxide. The paraffin provides an enhanced regression rate over ABS; however, it lacks structural integrity and combustion efficiency. Multiple fuel grains with various ABS-to-Paraffin mass ratios were fabricated and burned with nitrous oxide. Analytical predictions for end-to-end motor performance and fuel regression are compared against static test results. In support of these analytical comparisons, a novel method for propagating the fuel port burn surface was developed. In this modeling approach the fuel cross section grid is modeled as an image with white pixels representing the fuel and black pixels representing empty or burned grid cells.
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Jonathan Martin McCulley
Contents

Abstract ................................................................. iii
Public Abstract ......................................................... iv
Acknowledgments ....................................................... v
List of Tables ........................................................... viii
List of Figures .......................................................... ix
Nomenclature .......................................................... xi
1 Introduction .......................................................... 1
  1.1 Advantages of Hybrid Rocket Systems ......................... 2
  1.2 Technical Limitations of Hybrid Rocket Systems .............. 3
2 Literature Review ..................................................... 5
3 Hybrid Rocket Regression Rate Modeling Theory .................. 7
  3.1 Regression Rate ................................................. 7
  3.2 St. Robert’s Law for Solid Propellant Regression Modeling and Limitations . 7
  3.3 Marxman Regression Rate Modeling ................................ 8
  3.4 Hybrid Rocket Regression Rate Enhancement Techniques ........ 15
  3.5 Paraffin Wax Formulations as High Regression Rate Hybrid Fuel Material ........ 15
  3.6 Additive Manufacturing as a Regression Rate Enhancement Technique .... 20
4 Research Objectives .................................................. 26
5 Experimental Setup, Fuel Grain Fabrication, and Test Procedures . 27
  5.1 Experimental Setup .............................................. 27
  5.2 Test Grain Fabrication ............................................ 30
6 Results and Discussion ............................................... 33
  6.1 Modeling and Chemical Analysis .................................. 33
  6.2 Regression Rate Modeling for Fuel Grain Design ............... 40
  6.3 Analytical Modeling for Fuel Regression Propagation ........... 42
  6.4 Comparison of Analytical and Experimental Results ............ 50
  6.5 Conclusion ....................................................... 59
References ................................................................. 60
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Comparison of chemical rocket motor characteristics.</td>
<td>1</td>
</tr>
<tr>
<td>3.1 Additive manufacturing processes</td>
<td>24</td>
</tr>
<tr>
<td>6.1 Test fire data for ABS-paraffin grains.</td>
<td>49</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Hybrid rocket motor schematic</td>
<td>2</td>
</tr>
<tr>
<td>3.1 Linear Fuel Port Regression</td>
<td>7</td>
</tr>
<tr>
<td>3.2 Classical regression model block diagram</td>
<td>12</td>
</tr>
<tr>
<td>3.3 Longitudinal boundary layer development within the fuel port.</td>
<td>13</td>
</tr>
<tr>
<td>3.4 Burned HTPB and ABS fuel grains.</td>
<td>14</td>
</tr>
<tr>
<td>3.5 Entrainment model for paraffin based fuels.</td>
<td>16</td>
</tr>
<tr>
<td>3.6 Predicted theoretical entrainment, vaporization and total regression rate of paraffin for various mass fluxes.</td>
<td>19</td>
</tr>
<tr>
<td>3.7 HTPB and ABS fuel regression rates for various oxidizer mass fluxes</td>
<td>22</td>
</tr>
<tr>
<td>3.8 Initial grain geometry for hidden oxidizer flow paths in a rapid prototyped ABS grain</td>
<td>23</td>
</tr>
<tr>
<td>5.1 Motor test stand with 78mm motor.</td>
<td>28</td>
</tr>
<tr>
<td>5.2 MoNSTeR cart piping diagram.</td>
<td>29</td>
</tr>
<tr>
<td>5.3 Exploded view of motor configuration.</td>
<td>30</td>
</tr>
<tr>
<td>5.4 Melted paraffin with carbon black ready for casting</td>
<td>31</td>
</tr>
<tr>
<td>5.5 Oven used to thermally soak ABS grain before paraffin impregnation.</td>
<td>31</td>
</tr>
<tr>
<td>5.6 Constant mixture-ratio grain filled with paraffin.</td>
<td>32</td>
</tr>
<tr>
<td>6.1 Chemical structure of N=23 polymerization of paraffin</td>
<td>34</td>
</tr>
<tr>
<td>6.2 Thermodynamic and transport properties of $N_2O/Paraffin$ combustion products.</td>
<td>36</td>
</tr>
<tr>
<td>6.3 Thermodynamic and transport properties of $N_2O/ABS$ combustion products.</td>
<td>38</td>
</tr>
<tr>
<td>6.4 Thermodynamic and transport properties of $N_2O/ABS-Paraffin$ combustion products at 75 bars.</td>
<td>39</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>6.5</td>
<td>Constant mixture ratio grain geometry</td>
</tr>
<tr>
<td>6.6</td>
<td>Regressed constant mixture ratio fuel grain</td>
</tr>
<tr>
<td>6.7</td>
<td>Geometric regression via blurring</td>
</tr>
<tr>
<td>6.8</td>
<td>Single fuel software block diagram</td>
</tr>
<tr>
<td>6.9</td>
<td>Masks used in grain regression model</td>
</tr>
<tr>
<td>6.10</td>
<td>Grain regression process using ABS and paraffin masks, then combining the fuels back together.</td>
</tr>
<tr>
<td>6.11</td>
<td>Multiple fuel software block diagram</td>
</tr>
<tr>
<td>6.12</td>
<td>Images of fuel grains where black is ABS and red is paraffin</td>
</tr>
<tr>
<td>6.13</td>
<td>Image processing techniques to import final grain geometry into MATLAB</td>
</tr>
<tr>
<td>6.14</td>
<td>Image processing techniques for determining final grain geometry</td>
</tr>
<tr>
<td>6.15</td>
<td>Burned fuel cross section overlaid on analytical model</td>
</tr>
<tr>
<td>6.16</td>
<td>Initial and final ports for constant mixture ratio grains.</td>
</tr>
<tr>
<td>6.17</td>
<td>Experimental and analytical predictions for constant mixture ratio thrust and pressure</td>
</tr>
<tr>
<td>6.18</td>
<td>Un-combusted paraffin being ejected from 25% paraffin grain.</td>
</tr>
<tr>
<td>6.19</td>
<td></td>
</tr>
<tr>
<td>6.20</td>
<td>Grain geometry time lapse for 25% paraffin grain.</td>
</tr>
<tr>
<td>6.21</td>
<td>Experimental and analytical predictions for 25% paraffin thrust and pressure.</td>
</tr>
<tr>
<td>6.22</td>
<td>Regression rate comparison for constant mixture ratio and 25% paraffin grains</td>
</tr>
<tr>
<td>6.23</td>
<td>Hilecial fuel grain pre and post burn</td>
</tr>
<tr>
<td>6.24</td>
<td>Plume from constant mixture ratio grain with carbon black</td>
</tr>
</tbody>
</table>
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Absorption coefficient</td>
</tr>
<tr>
<td>$ABS_{Filter}$</td>
<td>Regression rate using pixelated disk filter</td>
</tr>
<tr>
<td>$A_{chamber}$</td>
<td>Chamber area</td>
</tr>
<tr>
<td>$A_{ox}$</td>
<td>Injector discharge area</td>
</tr>
<tr>
<td>$A_{P2A}$</td>
<td>Area of triangle formed from paraffin radius to ABS radius</td>
</tr>
<tr>
<td>$A_{port}$</td>
<td>Port area</td>
</tr>
<tr>
<td>$B$</td>
<td>Blowing parameter</td>
</tr>
<tr>
<td>$C$</td>
<td>Carbon</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Injector discharge coefficient</td>
</tr>
<tr>
<td>$\text{ceiling}(x)$</td>
<td>Smallest integer not greater than $x+1$</td>
</tr>
<tr>
<td>$C_{H}, C_{H0}$</td>
<td>Stanton number with and without blowing</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat</td>
</tr>
<tr>
<td>$dt$</td>
<td>Timestep</td>
</tr>
<tr>
<td>$dx$</td>
<td>Horizontal image conversion factor</td>
</tr>
<tr>
<td>$dy$</td>
<td>Vertical image conversion factor</td>
</tr>
<tr>
<td>$\text{floor}(x)$</td>
<td>Largest integer not greater than $x$</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Froude number</td>
</tr>
<tr>
<td>$G$</td>
<td>Instantaneous mass flux</td>
</tr>
<tr>
<td>$H$</td>
<td>Image height</td>
</tr>
<tr>
<td>$h$</td>
<td>Melt layer thickness</td>
</tr>
<tr>
<td>$h_{m}, h_{e}$</td>
<td>Effective heats</td>
</tr>
<tr>
<td>$H$</td>
<td>Hydrogen</td>
</tr>
<tr>
<td>$h_v$</td>
<td>Heat of vaporization</td>
</tr>
<tr>
<td>$\bar{I}$</td>
<td>Image</td>
</tr>
<tr>
<td>$I_{ABS}$</td>
<td>Filtered ABS port image</td>
</tr>
<tr>
<td>$I_{ABS\text{Mask}}$</td>
<td>ABS location in the fuel grain</td>
</tr>
<tr>
<td>$I_c$</td>
<td>New port image</td>
</tr>
<tr>
<td>$I_{Para}$</td>
<td>Filtered paraffin fort image</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$I_{ParaMask}$</td>
<td>Paraffin location in the fuel grain</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>Specific impulse</td>
</tr>
<tr>
<td>$L$</td>
<td>Motor length</td>
</tr>
<tr>
<td>$L_v$</td>
<td>Latent heat of vaporization</td>
</tr>
<tr>
<td>$m$</td>
<td>Columns of $\bar{I}$</td>
</tr>
<tr>
<td>$m_{ABS}$</td>
<td>Actual mass of ABS burned</td>
</tr>
<tr>
<td>$\dot{m}_{ABS}$</td>
<td>Mass flow rate of ABS</td>
</tr>
<tr>
<td>$\dot{m}_{ox}$</td>
<td>Oxidizer mass flow rate</td>
</tr>
<tr>
<td>$\dot{m}_{Para}$</td>
<td>Mass flow rate of paraffin</td>
</tr>
<tr>
<td>$m_{Para}$</td>
<td>Actual mass of paraffin burned</td>
</tr>
<tr>
<td>$mtot_{ABS}$</td>
<td>Total ABS mass burned using floor function</td>
</tr>
<tr>
<td>$mtot_{ABS2}$</td>
<td>Total ABS mass burned using ceiling function</td>
</tr>
<tr>
<td>$mtot_{Para}$</td>
<td>Total paraffin mass burned using floor function</td>
</tr>
<tr>
<td>$mtot_{Para2}$</td>
<td>Total paraffin mass burned using ceiling function</td>
</tr>
<tr>
<td>$n$</td>
<td>Rows of $\bar{I}$</td>
</tr>
<tr>
<td>$N$</td>
<td>Nitrogen</td>
</tr>
<tr>
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<td>Number of pixels in the x-direction of the image</td>
</tr>
<tr>
<td>$n_{py}$</td>
<td>Number of pixels in the y-direction of the image</td>
</tr>
<tr>
<td>$N_{Slots}$</td>
<td>Number of paraffin slots in fuel port</td>
</tr>
<tr>
<td>$P$</td>
<td>Port perimeter</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Combustion pressure</td>
</tr>
<tr>
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<td>Regression rate using pixelated disk filter</td>
</tr>
<tr>
<td>$P_{dyn}$</td>
<td>Dynamic pressure in the port</td>
</tr>
<tr>
<td>$I_{PixPara}$</td>
<td>Image containing only paraffin pixels</td>
</tr>
<tr>
<td>$P_{ox}$</td>
<td>Oxidizer pressure</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$Q_{r}, Q_{c}$</td>
<td>Radiative and convective heat flux a the surface</td>
</tr>
<tr>
<td>$Q_w$</td>
<td>Heat transfer to the wall</td>
</tr>
<tr>
<td>$\dot{r}$</td>
<td>Regression rate</td>
</tr>
<tr>
<td>$R_{he}, R_{hv}$</td>
<td>Ratio of heat of gasification for entrainment and vaporization</td>
</tr>
<tr>
<td>$r_{ABS}$</td>
<td>Radius of ABS in fuel port</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>rem</td>
<td>Disk filter remainder</td>
</tr>
<tr>
<td>$r_{para}$</td>
<td>Radius of paraffin in fuel por</td>
</tr>
<tr>
<td>$r_{p2a}$</td>
<td>Distance from paraffin to ABS</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Combustion flame temperature</td>
</tr>
<tr>
<td>$T_{fuel}$</td>
<td>Fuel temperature</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Fuel port volume</td>
</tr>
<tr>
<td>$W$</td>
<td>Image width</td>
</tr>
<tr>
<td>$\tilde{X}$</td>
<td>X boundary coordinates</td>
</tr>
<tr>
<td>$\tilde{Y}$</td>
<td>Y boundary coordinates</td>
</tr>
<tr>
<td>$z$</td>
<td>Axial distance along the port</td>
</tr>
<tr>
<td>$\dot{\alpha}, \dot{\beta}$</td>
<td>Dynamic pressure and thickness exponents</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>Effective heat of gassification</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\rho_{fuel}$</td>
<td>Fuel density</td>
</tr>
<tr>
<td>$\rho_{ox}$</td>
<td>Oxidizer density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Surface tension</td>
</tr>
<tr>
<td>$\theta_{ABS}$</td>
<td>Angle of ABS</td>
</tr>
<tr>
<td>$\theta_{para}$</td>
<td>Angle of paraffin slot</td>
</tr>
<tr>
<td>$\Theta_{space}$</td>
<td>Angle between paraffin slot</td>
</tr>
<tr>
<td>$\Theta_{start}$</td>
<td>Half angle from 90 degrees to edge of paraffin slot</td>
</tr>
<tr>
<td>$c$</td>
<td>Combustion zone</td>
</tr>
<tr>
<td>$e$</td>
<td>Edge of boundary layer</td>
</tr>
<tr>
<td>$ent$</td>
<td>Entrainment</td>
</tr>
<tr>
<td>$g$</td>
<td>Gas</td>
</tr>
<tr>
<td>$l$</td>
<td>Liquid</td>
</tr>
<tr>
<td>$s$</td>
<td>surface</td>
</tr>
<tr>
<td>$v$</td>
<td>Vaporization</td>
</tr>
<tr>
<td>$w$</td>
<td>Wall</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

During the past 50 years conventional launch systems have been developed to a high level of capability; however, for a variety of reasons these vehicles have become increasingly expensive to operate. Some of these reasons include manufacturing and operational complexity, safety and environmental regulations for dealing with hazardous materials, and the generally large “support army” required for flight preparations. Because of high launch performance demands, including specific impulse ($I_{sp}$) and thrust-to-weight ratio, conventional liquid and solid-propelled rocket stages that employ highly-energetic, explosive, or toxic propellants will likely remain the systems of choice for large military-class payloads or for human spaceflight. However, there exists an emerging commercial market that is willing to accept a lower system performance in exchange for reduced operational costs and lower environmental impact. Hybrid rockets, powered by safe, non-toxic propellants, have the ability to fill this growing niche market.

There are three types of chemically-propelled rockets: liquid, solid and hybrid. Table 1.1 compares the characteristics of these three types of chemical rockets. Liquid propellant

<table>
<thead>
<tr>
<th>Table 1.1: Comparison of chemical rocket motor characteristics.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factor</strong></td>
</tr>
<tr>
<td>Command Shutdown &amp; Throttle Capability</td>
</tr>
<tr>
<td>Non-Toxic Combustion Exhaust</td>
</tr>
<tr>
<td>Ease of Transport, Storage, &amp; Handling</td>
</tr>
<tr>
<td>Maintenance &amp; Launch Processing Cost</td>
</tr>
<tr>
<td>Manufacturing Cost</td>
</tr>
<tr>
<td>Readily Scalable</td>
</tr>
<tr>
<td>$I_{sp}$</td>
</tr>
<tr>
<td>Propellant Mass Fraction</td>
</tr>
<tr>
<td>Safe, Non-Explosive Propellants</td>
</tr>
</tbody>
</table>
rockets use highly volatile liquid oxidizer and liquid fuel components that are mixed and burned in the combustion chamber. The vast majority of liquid chemical propellants are both environmentally toxic, and highly explosive. Solid rocket motors use a solid propellant grain that mixes both the oxidizer and fuel in a hydrocarbon binder. All solid propellants are composed of highly energetic and explosive materials, and require extreme caution in storage and handling. Both liquid rocket engines (due to combustion instability) and solid rocket motors (due to the extreme volatility and energy levels of the combined propellants) have a potential for catastrophic failure. Multiple catastrophic ordnance explosion events have occurred [1]. By contrast, hybrid rocket motors separate the oxidizer (typically benign oxidizers like nitrous oxide) and fuel (usually inert solid hydrocarbon fuel grains), and thus present little risk of explosion. Figure 1.1 shows a conventional hybrid rocket with liquid oxidizer and solid fuel [2].

![Fig. 1.1: Hybrid rocket motor schematic](image)

### 1.1 Advantages of Hybrid Rocket Systems

Hybrid motors that employ non-toxic, non-explosive propellants have the potential to fulfill the previously described “market niche.” The physical regression process on hybrid fuel grains differs significantly from solid propellant grains. Solid fuel grains burn via pressure coupling and the higher the chamber pressure, the faster the fuel burn rate. This property makes solid propellant fuel grains potentially explosive, and very susceptible to fuel grain
flaws. These grain flaws can produce a burn rate pressure coupling that presents a significant safety issue [3].

Unlike solid-propelled rockets, hybrid fuel regression rates are driven primarily by the oxidizer flux rate, which can be independently controlled. This property allows hybrid rockets to exhibit a relative insusceptibility to grain flaws. For the majority of hybrid design geometries, no pressure coupling feedback exists. Heterogeneous reactions between oxidizer and fuel are shielded by a flame zone, so imperfections may increase surface area, but are not detrimental to the internal ballistics [4]. Because the propellant components remain inert until ignited within the motor chamber, hybrid rockets are inherently safer to transport, load, store and operate [5]. This inherent safety greatly reduces ground handling and transportation costs, and can potentially lead to an overall reduction in system operating costs.

In 2003 a study performed by the European Space Agency (ESA) showed the potential for considerable operational cost savings by simplifying propellant ground handling procedures [6]. Unlike solid-propelled rockets, where fuel grain flaws and age-induced cracks present a significant safety issue, hybrid rockets exhibit a relative insusceptibility to grain flaws. Other advantages of hybrid rockets that can potentially offset the lower performance level include the ability to be restarted in flight and demonstrated the ability to be throttled [7] over a significantly wider range of thrust levels compared to conventional liquid bi-propellant systems.

1.2 Technical Limitations of Hybrid Rocket Systems

Considering the above listed advantages, hybrid motors are not without technical difficulties and operational shortcomings. Hybrid rocket motors have traditionally suffered from two primary insufficiencies; 1) lower $I_{sp}$ than conventional bi propellant liquid or lower volumetric efficiency than solid rockets of the same thrust level, and 2) low fuel regression rates. These low regression rates result in low fuel mass flow rates for a given oxidizer flux level. To achieve oxidizer-to-fuel (O/F) ratios that produce acceptable combustion characteristics traditional cylindrical fuel ports must have a very long length-to-diameter ratio. This
high aspect ratio results in poor volumetric efficiency and can result in substantial unused residual fuel.

Of primary concern is the low fuel regression rate typically seen in hybrid rocket motors. A popular fuel for hybrids is Hydroxyl-Terminated Polybutadiene (HTPB), which is a legacy thermosetting polymer material that is mixed from its liquid base-components, degassed under vacuum, and then cast and cured in a fuel grain mold. HTPB does not melt in the presence of heat, but instead chars and ablates. The well-known “blowing effect” induced by the radial flow of ablated fuel generally results in low overall fuel regression rates [8]. Hybrid motors typically produce regression rates that are significantly lower than solid fuel motors in the same thrust and impulse class. Increasing the oxidizer mass flux increases fuel regression rates; unfortunately, the resulting combustion instabilities at high flux rates limit the effectiveness of this option [9].
Chapter 2
Literature Review

The first successful hybrid rocket GIRD-09 was created by the Soviet Union in 1933 using LOX (Liquid Oxygen) and gellified gasoline reaching an altitude of about one mile [10]. The first testing of hybrids done in the United States were performed by the Californian Rocket Society using coal and GOX (Gaseous Oxygen). During the same period in Germany, research included a LOX-graphite rocket, which due to the high heat of sublimation of graphite makes it a very poor fuel, but makes it an ideal material for nozzles and insulating surfaces [3]. During the 1940’s the Pacific Rocket Society used wood as a fuel including the nozzle which eroded during the burn. After many revisions the Society successfully flew a Lox-rubber based fuel in 1951, which reached an altitude of 30,000 feet. In 1952, The Applied Physics Laboratory invented the reverse hybrid motor, which uses a liquid fuel and solid oxidizer [11]. Reverse hybrids have been abandoned due to lackluster performance and combustion instabilities. In 1964 ONERA, a French organization, demonstrated a throttleable motor to optimize flight performance. This motor was used in the first hybrid sounding rocket reaching in excess of 100km.

During the 1980’s and 1990’s the American Rocket Company (AMROC) worked towards developing large hybrid boosters [12–16]. The motivation for developing these large boosters came from the Space Shuttle Challenger disaster [17] and a Titan III failure. During this period, AMROC developed and tested motors up to 250,000 lbf thrust range. Unfortunately, AMROC’s attempts to prove the flight worthiness of their large hybrid designs stalled during the development of the Hybrid Technology Option Project, which experienced low frequency combustion instabilities. The financial burden of these problems and their proposed fixes eventually drove AMROC out of the project and large scale hybrid motor research declined shortly thereafter.
In 2004, hybrid motors enjoyed a flare of attention after SpaceShipOne, a rocket plane built by Scaled Composites and propelled by a hybrid rocket motor designed by SpaceDev, won the Ansari X Prize after launching a commercial vehicle to 103 km altitude [18]. SpaceDev acquired all patents pertaining to the AMROC hybrid motors, which they based their design on. The inherent safety and low cost of hybrid motors was demonstrated and continues to make hybrids an attractive choice for space tourism.

Leveraging the heritage of the successful SpaceShipOne motor, the Sierra Nevada Corporation (which acquired SpaceDev in 2008) is designing and testing a hybrid rocket propelled “Dream Chaser” vehicle as part of the Commercial Crew Development program (now called the Commercial Crew Integrated Capability initiative) [19] [20]. The Dream Chaser vehicle is a lifting-body design propelled by two 12,000 pound-force thrust nitrous oxide and HTPB motors. Dream Chaser is designed to be launched on top of an Atlas V launch and supply crew and cargo to low earth orbit, especially to the International Space Station. The Dream Chaser Program aims to have an orbital flight by 2014, an accomplishment which would undoubtedly create a surge of renewed interest in hybrids for crew and space applications.

Recently, efforts at Stanford University, NASA Ames Research Center, and the Space Propulsion Group (SPG) have significantly advanced the design and understanding of hybrid rocket motors using paraffin-based fuels. Stanford University and NASA Ames have been developing a nitrous oxide paraffin 100 km max altitude sounding rocket [21–23]. The SPG, under a contract from the Air Force Research Labs, has contributed a great deal to the understanding of regression mechanisms for liquifying fuels, such as paraffin [24–27]. Their tests have shown regression rates for paraffin fuels several times those seen with conventional thermoset hybrid rocket fuels.
Chapter 3
Hybrid Rocket Regression Rate Modeling Theory

3.1 Regression Rate

Regression rate is the burn rate of the solid fuel, which is modeled linearly and normal to the local surface for a given fuel grain cross section. Figure 3.1 demonstrates this concept.

![Fig. 3.1: Linear Fuel Port Regression](image)

3.2 St. Robert’s Law for Solid Propellant Regression Modeling and Limitations

Tailoring the hybrid grain geometry to achieve a prescribed thrust profile is significantly more difficult than with solid propellants where the combustion chemistry can be precisely controlled by a-priori formulations. The St. Robert’s law [28],
\[ \dot{r} = aP_0^n \] (3.1)
typically used for modeling regression rates on solid motors has been demonstrated to be inaccurate in hybrid motors.

Conventional hybrid motor designs have very low levels of pressure coupling. The ratio of the propellant grain surface area to chamber volume has an influence on the evolving chamber pressure as with a solid motor, the oxidizer feed mass flux also has a very significant effect and the burn profile is a function of a whole suite of control variables. A motor with a particular fuel grain pattern that behaves in one manner for a given propellant combination and initial mixture ratios will perform significantly differently for a different combination of propellants.

In contrast to solid rocket motors, the combustion process for hybrid motors is significantly more complex. With hybrid rocket motors as the fuel grain burns and the surface geometry changes, the oxidizer mass-flux also changes. This changing mass flux in turn changes the solid fuel regression rate and alters the thermodynamic and transport properties of the combustion products. The O/F ratio varies continuously throughout the motor burn. The primary consequence of the hybrid flow physics is that regression rate-models based on St. Robert’s law are inaccurate. Several studies have demonstrated that hybrid fuel regression rates have little or no dependence on chamber pressure [28].

3.3 Marxman Regression Rate Modeling

Marxman and Gilbert first proposed an enthalpy-based fuel regression model for hybrid rocket motors in the early 1960’s [29]. The fundamental assumption made by Marxman and his colleagues was that regression rates in a hybrid rocket are dominated by thermal diffusion and not chemical kinetics [8]. Consequently the fuel surface regression is strongly a function of turbulent boundary-layer heat transfer. Boundary layer mixing creates a region where oxidizer flow from the center of the motor combustion port mixes with vaporizing solid fuel leaving the fuel wall. Close to the fuel wall is the flame zone where the combustion
of fuel and oxidizer primarily takes place. Heat transfer from this zone to the solid fuel grain drives the regression rate behavior of hybrid rocket motors.

The regression rate is proportional to the heat flux from the flame to the wall given as

$$\rho_f \dot{r} = \frac{\dot{Q}_w}{\Delta H}$$ (3.2)

For a turbulent boundary layer the regression rate can be parameterized in terms of the Stanton number and mass flux as

$$\dot{r} = \frac{0.03 G R e_x^{-0.2}}{\rho_f} \frac{C_H}{C_{H0}} \frac{\mu_e}{\mu_c} \frac{(h_{cs} - h_{wg})}{\Delta H}$$ (3.3)

This assumes the radiation heat transfer is negligible, for most non metalized fuels, this is an appropriate assumption. They also characterized the Stanton number in terms of mass addition or a blowing parameter given as

$$\frac{C_H}{C_{H0}} = 1.2 B^{-0.77}$$ (3.4)

where,

$$B = \frac{\mu_e}{\mu_c} \frac{(h_{cs} - h_{wg})}{\Delta H}$$ (3.5)

A simplified regression rate formula is developed from the combination of Eq. (3.3),(3.4), and (3.5) is

$$\dot{r} = 0.036 \frac{G}{\rho_f} R e_x^{-0.2} \left( \frac{\mu_e}{\mu_c} \frac{(h_{cs} - h_{wg})}{\Delta H} \right)^{0.23}$$ (3.6)

Marxman and Muzzy later determined that regression rate is limited by the heat and mass transfer to the fuel surface [8]. Therefore an increasing $h_{cs} - h_{wg}$ increases $\dot{r}$, which consequently strengthens the blowing parameter which reduces $\dot{r}$. The regression rate is dependent on mass flux through the system rather than changes in enthalpy.

Later studies performed by Strand et al. [30] and later Chiaverini et al. [31] showed that
the experimental coefficients predicted by Marxman, specifically the exponents on mass flux and the surface blowing coefficient, were substantially different from the theoretical values derived in the classical relation. Due these deviations from the experimental data, the original form of the model derived by Marxman model is not often used in modern hybrid rocket performance analysis. Additionally, the Marxman model relates the fuel regression rate to the surface skin friction, but does not close sufficiently to allow a priori regression rate prediction [32].

A closed-form regression rate model based on flat-plate flow theory was developed by Eilers and Whitmore [33] and corrected by Whitmore and Chandler [34] for non-unity Prandtl number

\[ \dot{r} = \frac{0.047}{Pr^{0.153} \rho_{fuel}} \left( \frac{c_p [T_0 - T_{fuel}]}{h_v_{fuel}} \right)^{0.23} \left( \frac{\dot{m}_{ox}}{A_{chamber}} \right)^{\frac{4}{5}} \left( \frac{\mu}{L} \right)^{\frac{1}{5}} \]  

(3.7)

In Eq. (3.7) the parameters \( \mu \) and \( Pr \) refer to the combustion product gas properties, the parameters \( P_{ox} \) and \( \rho_{ox} \) refer to the incompressible oxidizer liquid properties upstream of the injector, and \( c_p, \rho_{fuel}, T_{fuel}, \) and \( h_v \) refer to the properties of the solid fuel grain. The parameters \( A_{ox}, C_d, A_{chamber}, \) and \( L \) are the injector discharge area, fuel port cross sectional area, and fuel grain length, respectively. Equation (3.7) predicts rate of regression for the entire motor averaged longitudinally along the length of the motor.

The model of Eq. (3.7) was developed from an enthalpy balance between the latent heat of the burning fuel and the heat convection into the combustion flame zone. Applying the generalized (non-unity Prandtl number) form of the Reynold’s analogy between the Stanton number and the surface skin friction coefficient allows the heat transfer coefficient to be calculated. The model uses the Reynold’s-Colburn analogy to relate the heat transfer at the surface of the fuel grain to the local boundary layer heat transfer, and overcomes the shortcoming of Marxman’s original model.

In Eq. (3.7) the oxidizer mass flow rate of \( N_2O \) is modeled by the incompressible discharge coefficient formula:
Equation (3.8) is reasonably accurate as long as the motor is burned using a top pressure that is higher than the saturation pressure of the \( \text{N}_2\text{O} \) at the injector temperature. For blow down systems that use only the natural vapor pressure of the oxidizer, a more complicated two-phase model is required to accurately model the injector mass flow [34]. For purely compressible gaseous oxidizer flows, the oxidizer mass flow rate becomes

\[
\dot{m}_{ox} = A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox}(P_{ox} - P_0)}
\]  

(3.8)

Observing both equations (3.7) and (3.8), it can be noted that the third term in Eq. (3.7) is actually the mean oxidizer mass flux through the port, where oxidizer mass flux is defined as:

\[
G = \frac{\dot{m}_{ox}}{A_{chamber}}
\]  

(3.10)

This comparison supports Marxman’s original assertion that oxidizer mass flux is a major driving factor in hybrid fuel grain regression rates. The total fuel mass flow rate can be calculated from the regression rate model by:

\[
\dot{m}_{fuel} = A_{burn} \rho_{fuel} \dot{r}
\]  

(3.11)

In Eq. (3.11), \( A_{burn} \) is the total fuel port surface area. The oxidizer to fuel ratio for an incompressible fluid is therefore given by:

\[
O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox}(P_{ox} - P_0)}}{A_{burn} \rho_{fuel} \dot{r}}
\]  

(3.12)

Clearly, examining Eq. (3.7) and (3.12) show that as the fuel grain burns and the surface burn area changes, O/F ratio will vary significantly. Since the O/F ratio is highly dependent on the mean oxidizer mass flux, the chamber pressure will be a major driver in
the overall mean regression rate.

Assuming the nozzle throat chokes immediately, a balance between the gases coming into the fuel port and the gases leaving through the choked throat determines the time response of this chamber pressure growth. Here the equation that describes the time evolution of the chamber pressure is:

\[
\frac{\delta P_o}{\delta t} = \frac{A_{\text{burn}} \dot{V}}{V_c} \left[ \rho_{\text{fuel}} R_g T_0 - P_0 \right] - P_0 \left[ \frac{A^*}{V_c} \sqrt{\frac{\gamma R_g T_0}{\gamma + 1}} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}} \right] + \frac{R_g T_0}{V_c} \dot{m}_{\text{ox}} \quad (3.13)
\]

In Eq. (3.13), \( T_0 \) is the combustion flame temperature at the current O/F ratio and \( V_c \) is the total fuel port volume, including both pre and post-combustion chambers. Figure 3.2 shows a block diagram of the total algorithm. Equation (3.7) is derived based on the assumption that typical hybrid motors have very long aspect ratios with length to diameter ratios greater than 20.

Fig. 3.2: Classical regression model block diagram
Along the entire length of the motor, fuel is being dumped into the core oxidizer flow. This process does not allow fully developed channel flow to develop until far downstream in the fuel port. For this analysis, a simple empirical skin friction model based on 2-dimensional boundary layer theory was used in lieu of a fully developed model for pipe-flow skin-friction. Figure 3.3 depicts the proposed boundary layer growth process.

![Diagram of Boundary Layer Development](image)

**Fig. 3.3: Longitudinal boundary layer development within the fuel port.**

Experimental tests performed by Whitmore et al. [35] with both HTPB and Acrylonitrile-Butadiene-Styrene (ABS) fuel grains support the accuracy of this “undeveloped” flow assumption. Figure 3.4 shows side-by-side comparisons of post 10-second burn HTPB and ABS fuel grains. The regression measurement stations are marked on each grain. For both the HTPB and ABS grains, fossilized surface flow patterns are visible, and the transition from laminar to turbulent flow patterns is clearly visible. The surface burn patterns transition from laminar to turbulent moving aft along the motor flow channel. The flow patterns are very similar to the classical “flat plate” flow transition pattern.

A popular fuel for hybrids is HTPB, which is a legacy thermosetting polymer material that is mixed from its liquid base components, degassed under vacuum, and then cast and cured in a fuel grain mold. HTPB does not melt in the presence of heat, but instead chars and ablates. The well-known blowing effect induced by the radial flow of this ablated fuel
material generally results in low overall fuel regression rates [9]. Hybrid motors which are based on ablating fuel grains typically produce regression rates that are significantly lower than solid fuel motors in the same thrust and impulse class. Increasing the oxidizer mass flux increases fuel regression rates; unfortunately, the resulting combustion instabilities at high flux rates limit the effectiveness of this option.

To achieve enhanced fuel mass flows for a given oxidizer flux, hybrid fuel grain designers typically resorted to increasing the fuel grain surface burn area by casting multiple fuel ports with complex internal geometries [36]. These complex geometries require the development of extensive tooling, and present an unavoidable difficulty with removing the tooling once the grain material is set. There is often a requirement for an embedded structure to support the fuel port as it regresses. This support structure results in excessive unburned mass fractions, typically in the 5% to 10% range. Multiple fuel ports require a large pre-combustion chamber or individual injectors for each port. This design feature often produces uneven burning within the individual ports. Finally, multiple port designs present an increased risk of instabilities related to dynamic flow interactions between ports and/or the presence of a large pre-combustion chamber.
3.4 Hybrid Rocket Regression Rate Enhancement Techniques

The main disadvantage of hybrid rockets is the low regression rate of the solid fuel. Several methods have been investigated to enhance fuel regression rates. Addition of oxidizing agents such as ammonium perchlorate [37], however this makes the fuel more of a solid propellant losing the safety appeal of hybrids as well as making the exhaust products harmful. While adding metals such as aluminum, iron oxide, and copper chromite can increase the regression, there is a negative effect on the environment [38]. The process of adding metal to the fuel grain increases fuel density and \( I_{sp} \) of the system. Vortex injection at the aft end of the motor produced regression up to 7 times as high as a normal hybrid [39]. This regression rate enhancement results from the oxidizer being injected directly impinging on the fuel grain. The complexity of this system and unknown scalability make it an undesirable solution.

3.5 Paraffin Wax Formulations as High Regression Rate Hybrid Fuel Material

Karabeyoglu et al. [25, 40, 41] have recently investigated a class of fast burning hybrid fuel grain materials based on paraffin wax formulations. These paraffin-based fuels melt before vaporizing, and a properly formulated mix produces a melt layer with a low viscosity and high surface tension. When the oxidizer flows at high speed over the upper side of the melting fuel surface, the liquid layer becomes unstable and minute surface waves are formed [26]. The resulting fluid boundary layer is hydrodynamically unstable and allows fuel droplets to be entrained into the core flow. Figure 3.5 shows this entrainment process.

The entrained fluid droplets significantly increase the massflow generated by regressing fuel, but does not increase the blowing-effect regression rate suppression resulting from mass flow ablating normal to the surface. For stable oxidizer flux levels droplet entrainment massflow is significantly greater than massflow resulting from direct gasification [25]. The massflow of an entraining fuel is given empirically as

\[
\dot{m}_{\text{ent}} \propto \frac{P_{\text{dyn}}^{\alpha} \mu^{\beta}}{\sigma^{\gamma} \mu^{\pi}}
\]  

(3.14)
where $P_{dyn}$ is the dynamic pressure, $h$ is the melt layer thickness, $\sigma$ is the surface tension, and $\mu$ is the viscosity. In Eq. (3.14) the superscripts $\alpha$ and $\beta$ range from 1-1.5, and experimental results show that $\gamma > \pi$ [25]. The onset of entrainment for $Re \leq 300$ and in terms of practical motor conditions [42] is given as

$$G^{1.6}h^{0.6} \geq 2.5 \times 10^{-3} \frac{1}{G_{f}^{0.8} G_{l}^{0.3}} \frac{\rho_{l}^{1.3} \mu_{l}^{0.6}}{\mu_{g}}$$  \hspace{1cm} (3.15)$$

Of particular importance in Eq (3.14) and (3.15) is that entrainment will only occur when the melt layer has a small viscosity. For example, the viscosity of Acrylonitrile-Butadiene-Styrene and paraffin wax, are respectively 195 $Pa \cdot s$ and $4.6 \times 10^{-4} Pa \cdot s$ [43]. ABS has a viscosity 7 orders of magnitude higher than paraffin, causing entrainment to have no significant impact upon regression rate.

Paraffin-based fuels have been developed that burn at surface regression rates three to four times that of conventional hybrid fuels [24]. The high regression rate hybrid fuels remove the need for a complex multiport grain, and most applications up to large boosters can be designed with a single port configuration. Space Propulsion Group Inc. has developed a motor capable of replacing the Orion 38 upper stage motor [44]. Their motor design, using liquid oxygen and a paraffin based fuel, shows significantly increased performance over the solid motor system it is designed to replace. This motor is 15% lighter, which leads to 40%
increase in payload capacity.

### 3.5.1 Droplet Entrainment Regression Rate Theory

Classical hybrid theory fails to predict the regression rate for fuels that have entrainment of liquid droplets into the flow. This is due to the fuels having low heats of vaporization which causes the entrainment to have a dominant mass transfer mechanism, rather than conventional vaporization of the fuel.

A new energy balance was derived by Karabeyoglu and is summarized as follows [27]. The total regression rate of a hybrid is the sum of the vaporization and entrainment regression rates.

\[ \dot{r} = \dot{r}_v + \dot{r}_{ent} \]  

The energy balance at the liquid gas interface for the combination of entrainment and evaporative mass transfer is

\[ \dot{r}_v + \left[ R_{he} + R_{hv} \left( \frac{\dot{r}_v}{\dot{r}} \right) \right] \dot{r}_{ent} = Fr \frac{0.03 \mu^0.2}{\rho_f} \left( 1 + \frac{\dot{Q}_r}{\dot{Q}_c} \right) B \frac{C_H}{C_H 0} G^{-0.2} \]  

where

\[ R_{hv} = \frac{C_l \Delta T_i}{h_e + L_v} \quad \text{and} \quad R_{he} = \frac{h_m}{h_e + L_v} \]  

These terms are added because of entrainment rather than conventional vaporization of fuel. A roughness parameter, \( F_r \) is introduced to account for increased heat transfer of the liquid surface due to wrinkling.

\[ F_r = 1 + \frac{14.1 \rho_g^0.4}{G^0.8 \left( \frac{T_s}{T_v} \right)^{0.2}} \]
Due to high predicted blocking factors, a new curve fit was needed for the Stanton number given as

\[
\frac{C_H}{C_{H0}} = \frac{C_{B1}}{C_{B1} + C_{B2} \left( \frac{\dot{r}_{\text{v}}}{\dot{r}_{\text{cl}}} \right)^{0.75}} \tag{3.20}
\]

where

\[
C_{B1} = \frac{2}{2+1.25B^{0.75}} \quad \text{and} \quad C_{B2} = \frac{1.25B^{0.75}}{2+1.25B^{0.75}} \tag{3.21}
\]

The \( \dot{r}_{\text{cl}} \) defined in Eq. (3.20) is the classical regression rate formula developed by Marxman in Eq. (3.6). The regression rate for entrainment can be expressed as

\[
\dot{r}_{\text{ent}} = a_{\text{ent}} \frac{G^{2\alpha}}{T^B} \tag{3.22}
\]

Figure (3.6) shows the total regression rate, as the sum of entrainment and vaporization of the fuel. As the mass flux is increased the entrainment mechanism becomes dominant, whereas non entraining fuels are limited by the heat transfer from the fuel.

### 3.5.2 Technical Limitations of Paraffin

Unfortunately, paraffin-based fuels introduce mechanical and structural problems that reduce the fuel grain integrity as the propellant burns. Solid phase paraffin is rather brittle and is easily cracked when subjected to launch vibration loads. As the paraffin melts it has the potential to soften and “slough” under axial launch loads. Thus, paraffin-based fuels require either special additives or a support lattice to keep the grain structure intact under launch loads.

Several strengthening materials have been tested in hybrid motors. Galfetti et al. have tested Polyurethane foam (PUF) strengthening structure shows promising results, but leads to heterogeneous fuel formulations [40]. These heterogeneous grain structures are difficult to manufacture. To avoid this problem and ensure paraffin-based formulations with sufficient
elasticity to survive launch vibration levels, Galfetti et al. also tested a miscible thermoplastic elastomer Styrene-Ethylene-Butylene-Styrene (SEBS) as a strengthening alternative to PUF. Mixing SEBS into the paraffin fuel produces a homogenous fuel grain and offers significantly lower manufacturing costs. During the combustion of the homogeneous material the material melts; when using heterogeneous materials only the paraffin melts. In both cases SEBS fuel additive and PUF structural support materials reduced the burn effectiveness and performance of the hybrid motor.

Aluminum and carbon black are common additives to many hybrid fuels which reduce thermal radiation from propagating throughout the fuel [45]. These materials help improve combustion efficiency and mechanical properties, however they reduce the regression rate of the fuel.

Another technique involves using a combustible diaphragm to promote mixing, and thus improving combustion efficiency [46]. Kim et al. showed an increase in combustion efficiency of 15%, however, this led to an effective efficiency of 85%. Diaphragms show an improvement in efficiency, however offer no structural support. Similar research was
conducted by Ishiguro, et al. which used a baffle plate increasing the combustion efficiency to 96% while causing a pressure drop of 0.5% [47]. While both of these techniques increase combustion efficiency, they were performed on laboratory scale hybrid motors, and are not feasibly scalable.

3.6 Additive Manufacturing as a Regression Rate Enhancement Technique

This research investigates techniques for increasing the volumetric efficiency of hybrid rockets by embedding fast burning paraffin fuels into a substrate composed of porous Acrylonitrile butadiene styrene. The ABS sub-strate provides mechanical support for the paraffin fuel material and serves as an additional fuel component. The embedded paraffin provides an enhanced regression rate while having no detrimental effect on the thermodynamic burn properties of the fuel grain. This processes is enabled by employing additive manufacturing techniques to fabricate the ABS shell material. This approach allows multiple support structure geometries to be rapidly designed, fabricated, and tested.

3.6.1 Acrylonitrile-Butadiene-Styrene as a Hybrid Rocket Fuel

Acrylonitrile-Butadiene-Styrene has several mechanical properties that make it very attractive as a hybrid rocket fuel. This material is widely mass-produced for a variety of non-combustion applications including household plumbing and structural materials. More than 1.4 billion kilograms of ABS material were produced by petrochemical industries worldwide in 2010 [48]. ABS is an inexpensive, recyclable, thermoplastic that melts at a relatively low temperature and can be reshaped and recycled multiple times with little or no degradation of the material properties. Because ABS has a much higher heat of gasification and thermal capacitance, very little heat is transferred, and allows the external motor case to remain cool during the burn. This self-cooling property of ABS presents a very significant advantage for in space applications where thermal management becomes a big issue. Finally, ABS has a very high structural modulus (2.3 GPa) and tensile yield strength (40 MPa).

A major result of research just recently completed by Whitmore et al. at Utah State University [35] (USU) was the demonstrated thermodynamic equivalence of ABS to the most
commonly used hybrid rocket fuel, HTPB. This research demonstrated that when ABS is burned with nitrous oxide ($N_2O$) the combustion flame temperature is slightly cooler than HTPB, but the products of combustion have a lower molecular weight. Thus ABS achieves $I_{sp}$ and characteristic velocity ($c^*$) that are nearly equivalent to HTPB. ABS and HTPB fuel regression rates were measured to be nearly identical.

Figure 3.7 plots the longitudinally averaged regression-rate measurements of the HTPB and ABS burns performed by Whitmore et al. [35] against the mean oxidizer mass flux for the burn. These data are compared with the analytical model predictions of Eq. (3.7). Following the end of each static test, the motor was quenched and then split longitudinally to expose the burned grain pattern. The final regression dimensions were measured at multiple points along the fuel grain, and the mean end-to-end longitudinal fuel regression was calculated. The mean regression rates were calculated using the two of the methods developed by Karabeyoglu et al. [49] based on the mean longitudinal change in diameter divided by one-half of the burn time and the overall change in propellant mass divided by the burn time.

The mean oxidizer mass flux is calculated using the mean of the initial and final port diameters. These comparisons verify the ability of Eq. (3.7) to accurately predict the mean longitudinal rate of regression for hybrid fuel grains, based on a priori knowledge.

With the advancement of rapid prototyping of ABS, complex grain geometries can be made with low cost and reasonable timetable. Grain geometries can be made without mandrels or the need for curing time associated with HTPB and other solid fuels. These grain geometries can have hidden voids so as the grain regresses out, new areas or oxidizer flow paths are connected as shown in figure 3.8 [50]. The ABS material is quite strong and can be used as its own pressure vessel, thus alleviating the need for a case and other insulating materials.

### 3.6.2 Background on Digital Manufacturing

Digital manufacturing (DM), rapid manufacturing (RM), layered manufacturing (LM),
Fig. 3.7: HTPB and ABS fuel regression rates for various oxidizer mass fluxes

and solid free-form fabrication (SFF) are all names given to the evolution of the now mature rapid prototyping (RP) technologies. More recently, many of these technologies are used to produce parts for the final consumer, contrary to RP that had only design purposes.

In the late 1960s, Herbert Voelcker—then an engineering professor at the University of Rochester—asked himself how to do "interesting things" with the automatic, computer-controlled machine tools that were just beginning to appear on factory floors. With funding from the National Science Foundation (NSF), Voelcker first by developed the basic mathematical tools needed to unambiguously describe three-dimensional parts [51]. Thus, a computer-controlled machine tool would cut away at a hunk of metal until what remained was the required part.

In 1968 Charles Hull patented a process he coined “Stereolithography” (SLA) for automated manufacture of plastic 3D objects directly from CAD models by adding material layer-by-layer using an ultraviolet laser and photo-curable liquid polymers.

Similarly, in 1987, University of Texas researcher Carl Deckard came up with the idea of building up parts layer by layer using a laser and powders. Deckard took his idea to NSF, which gave him support to pursue what he called "selective laser sintering." Deckard’s
Fig. 3.8: Initial grain geometry for hidden oxidizer flow paths in a rapid prototyped ABS grain

initial results were promising and in the late 1980s his team was awarded one of NSF’s first Strategic Manufacturing (STRATMAN) awards. The result of Voelcker’s, Deckard’s, and Hull’s efforts helped launch the additive manufacturing industry, which has revolutionized how products are designed and manufactured [52].

The similarity of a prototype to the “real product” is determined by its form, fit and function. Advantages of creating prototypes are: improve the ability to visualize the part geometry, due to its physical existence, enables earlier detection and reduction of design errors, and increases the capability to compute mass properties of components and assemblies. Preparing prototypes will help you describe your product more effectively with your team and customers contributing to the elimination of waste and costly late design changes.

In the last decades globalization has made the world a more competitive environment, especially in the industrial market. The bar has been raised for all companies that offer any product or service. Customers now require products with better quality, at lower prices and decreased lead times. Rapid prototyping, now known as additive manufacturing (AM), arose as a tool for designers and developers to reduce their product design cycle; as a result, launching products faster and cheaper. Objects that have traditionally been impossible to build because of the complex shapes or variety in materials can now be built by additive
Table 3.1: Additive manufacturing processes

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<thead>
<tr>
<th>Category</th>
<th>Rapid Prototyping System</th>
<th>Manufacturer</th>
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<td>3D System</td>
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<td>Solid Creation System (SCS)</td>
<td>D-MEC</td>
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<td>Stratasys</td>
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<td>3D System</td>
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<td>Solidica</td>
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<td>3D Systems</td>
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<td></td>
<td>Direct Shell Production Casting (DSPC)</td>
<td>Soligen</td>
</tr>
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<td></td>
<td>Multiphase Jet Solidification (MJS)</td>
<td>Fraunhofer</td>
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<td></td>
<td>3D Printing (3DP)</td>
<td>MIT</td>
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<td></td>
<td>Laser Sintering</td>
<td>EOS</td>
</tr>
</tbody>
</table>

manufacturing.

First a solid model is designed in a conventional CAD system; it is usually saved in the
STL file format for it to be processed by the AM process planner, which inputs the data
to the automated AM machine for it to build the physical object layer by layer. Additive
Manufacturing Technologies are often labeled as Non-Traditional processes because they use
techniques not commonly used previously to fabricate parts. Table 3.1 shows some of the
existing additive manufacturing processes and techniques [53].

Some of the most important technologies due to their market presence are: 3D printing
systems (3DP), Selective Laser Sintering (SLS) of metals and plastics, Stereo-lithography
(SL), and Fused Deposition Modeling (FDM) [54].

Recent advancements in Digital Manufacturing techniques provide an opportunity to
revolutionize the current approach to building launch vehicles for small satellites. These
mass-production technologies offer the potential to dramatically increase NanoSat launch
rates and may reduce launch costs by as much as an order of magnitude. Multiple small
businesses are currently thriving in the additive manufacturing market. The overhead of manufacturing a rocket no longer has to be carried by the aerospace industry alone. The ability to order a rocket and have it manufactured and delivered in days to weeks versus months to years will empower the Nanosat applications market. This approach offers the potential to revolutionize methods used to fabricate hybrid rocket fuel grains. If matured and commercialized, this technology will have a transformational effect on hybrid rocket motor production by improving quality, consistency, and performance, while reducing development and production costs.

Fused Deposition Modeling, developed by Stratasys, Inc. Eden Prairie, MN is the most common technique used for “Rapid Prototyping.” Using FDM methods allows precision fabrication of high-density, consistent-quality, solid-structures from a variety of polymeric materials. FDM manufacturing uses additive fabrication principles by depositing materials in layers to build up a structure. A thermoplastic laminate is supplied to an extrusion nozzle, which heats the material to near its melting point and extruded. The nozzle is then moved in both horizontal and vertical directions by a computer numerically controlled (CNC) mechanism. This manufacturing method can support high production rates, and offers the potential to improve hybrid fuel grain quality, consistency, and performance, while reducing development and production costs. The material most commonly used for FDM manufacturing is ABS. ABS is an inexpensive, recyclable thermoplastic with a relatively low melting point. ABS can also be reshaped and recycled multiple times with little or no degradation of material properties. This material is widely produced for a variety of non-combustion applications including household plumbing, structural materials, and children’s toys.

FDM processes have the potential to revolutionize the manufacture of hybrid rocket fuel grains. This process allows very complex grain shapes to be mass fabricated from a monolithic piece of plastic with very low level of grain-to-grain variability. The FDM process is an enabling technology for the dual-material fuel grains to be tested during this research campaign.
Chapter 4
Research Objectives

- Compare analytical results with experimental static test firings to show ABS-Paraffin can be a competitive fuel compared to HTPB, ABS, and Paraffin.

  - Use previous data on regression rates and thrust to compare fuel grains. Fuel grains with varying percentages of paraffin will be rapid prototyped using ABS and filled with paraffin, then the grain will be burned using Nitrous Oxide and thrust levels and regression rates will be measured.

  - Create a 2-D analytical model capable of multiple fuels with varying geometric shapes

- Design a fuel grain with a constant mixture ratio over the course of the burn.

  - Simulate grain regression analytically to create a fuel that maintains a constant fuel mass flow. Using Chemical Equilibrium with Applications, propellant properties for the different percentages of paraffin will be computed.

- Increase combustion efficiency over neat paraffin which is around 70%.

  - Utilize the ability to print complex shapes to enhance mixing in the post combustion chamber.
Chapter 5
Experimental Setup, Fuel Grain Fabrication, and Test Procedures

5.1 Experimental Setup

Motor static ground testing was performed on the USU campus using a legacy propulsion systems test cell that has been retrofitted for rocket motor testing. The propulsion test facilities used for this project leveraged prior USU hardware development activities. This existing hardware formed the basis of the test facility for developing and evaluating the proposed hybrid motor configurations. To date, more than 65 hybrid and 15 solid rocket motor tests have been performed in this test facility.

The test cell is fully instrumented and has expansion capability necessary to support all phases of this characterization testing. Available measurements obtained include chamber pressure, 1-degree of freedom (1-DOF) thrust, total impulse, motor case temperatures, exhaust plume temperatures, specific impulse, mass flow rate, consumed propellant mass, and propellant regression rate.

5.1.1 Mobile Nitrous Oxide Supply and Testing Resource (MoNSTeR) cart oxidizer delivery system.

The test cell has been specially adapted for hybrid rocket testing using a mobile test cart. Figure 5.1 shows a 78mm diameter motor mounted on the 1-DOF test stand. To allow sufficient mass flow rates with minimal line losses, a predetermined mass of N2O Oxidizer is delivered to a closely coupled “run tank” from a series of “K” sized industrial pressure cylinders. The Helium top pressure is set by a manual regulator, and is typically maintained near 5650 kPa (820 psi) for these tests. The top pressure keeps the N2O above saturation
pressure for the entire run and insures a single-phase liquid flow through the injector. The pneumatic run valve is triggered by an electronic relay and is automatically controlled by the instrumentation software. Oxidizer mass flow is sensed by vertical load cells mounted on the run tank and by an inline venturi flow meter mounted in the oxidizer feed-line just ahead of the injector. Figure 5.2 shows a piping and instrumentation diagram (P&ID) for the MoNSTeR cart.

5.1.2 Data Acquisition

Two National Instruments data acquisition and control devices manage motor fire control, and log test data. An NI-compact DAQ® 4-slot bus controller with multiple analog input (16-bit), analog output, digital output, and thermocouple modules (24-bit) bus-cards manage the majority of the measurements and valve control. The digital outputs from a separate NI USB-6009® module are used to trigger the relays that fire the igniter e-matches. Operators and experimenters are remotely located in a secure control room separated from the test area. Communications to the test stand are managed by an operator-controlled
laptop via universal serial bus (USB) using amplified extension cables. All control and measurement functions are controlled by a LABview® program hosted on the control laptop.

5.1.3 75mm Test Motor

The lab-scale motor test hardware consists of an off-the-shelf Cesaroni Pro75 [55] aluminum case with custom-designed nozzle and forward endcap sections. The case houses a 14” grain section, which was modified to accommodate grains as short as 6”, with an outer radius of 75mm (2.95”). The injector is a full-cone spray nozzle with an orifice diameter of 0.1”, which is attached to the oxidizer supply line of the test stand cart. The system has a nominal chamber pressure of about 450 psi. A graphite nozzle with an expansion ratio of 4.5. The motor incorporates a 1” long post combustion chamber. For a test burn, the motor is ignited by two 1/2A3-4T Estes® solid motors integrated into the injector-end of the motor. Figure 5.3 shows a exploded view of the motor configuration.
5.2 Test Grain Fabrication

Fuel grain fabrication for the ABS shell is done with Dimension 1200ES FDM system [56]. Typically, these systems are used for 3-dimensional modeling and rapid-prototyping. The Dimension FDM systems use production-grade thermoplastic that is stable and has no appreciable warping, shrinkage, or moisture absorption. The ABS stock material used for these tests is approximately 50% (mole fraction) butadiene, 43% acrylonitrile and 7% styrene. The mean density of the stock material used for these test was approximately $915 \ \text{kg/m}^3$. This density was considered to be sufficient to insure structural integrity of the fuel grain during static test firings. The paraffin used for testing was IGI 1250. The mean density of the paraffin used for these test was approximately $960 \ \text{kg/m}^3$.

Using the Dimension 1200ES, fuel grains were manufactured with “hollow” sections which could be filled with paraffin. To ensure complete infill of paraffin into the ABS voids, the grain is heated to $165^\circ F$, which is past the melting point of paraffin, but below the melting point of ABS. Figure 5.4 shows paraffin heated separately from the ABS grain to ensure complete mixing of 1% carbon black by mass. Figure 5.5 shows a vacuum chamber inside the oven used to heat the ABS grain.
The ABS grain and vacuum chamber become thermal masses allowing the paraffin to cool from the bottom up. The grain is cooled from the bottom to account for the decrease in volume as the paraffin cools. The grain is cooled under a vacuum to remove air bubbles.
in the paraffin. Figure 5.6 shows the final stage of the grain before it is burned.

Fig. 5.6: Constant mixture-ratio grain filled with paraffin.
Chapter 6
Results and Discussion

6.1 Modeling and Chemical Analysis

Because there are no industry standard for the enthalpy of formation of ABS and Paraffin; this study employs a systematic approach for calculating $\Delta H_f^0$ using the “Group Addition” methods developed by Van Krevelen and Chermin [57,58]. The enthalpy of formation is required to calculate combustion products of ABS, paraffin, and a combination of ABS and paraffin when burned with nitrous oxide at varying O/F ratios and pressures.

6.1.1 Group Addition Method

Krevelen modified Franklin’s method for calculating the Gibbs free energy, which he considered molecules to be built of groups. These groups provided individual contributions to the heat of formation, heat content, free enthalpy function and free enthalpy of formation. Franklin’s method works well for a paraffin hydrocarbon for which he assumed it worked well for all hydrocarbons. He defined the Gibbs free enthalpy of formation as

$$\Delta G_{f_{h.c.}} = \sum \text{contributions of composing groups} + R \cdot T \cdot \ln(\sigma) \quad (6.1)$$

Equation (6.1) was later found to be inconsistent, and a correction was made, however this only worked at the temperatures the corrections were made. Krevelen modified Franklin’s work and linearized the group contributions as a function of temperature.

$$\Delta G_{f_{\text{group.}}} = A + \frac{B}{100} \cdot T \quad (6.2)$$

The general equation for Gibbs free enthalpy is given as
\[ \Delta G = \Delta H - T \cdot \Delta S \]  

Comparing Eq. (6.2) and (6.3) reveals that \( \Delta A \) is the heat of formation and \( \frac{B}{100} \) is the entropy of formation. These values are assumed to be in an ideal gaseous state at 1 atm.

### 6.1.2 Heat of Formation for Paraffin

For this analysis a middle of the road polymer ratio for paraffin wax was chosen, having a chemical formula of \( C_{25}H_{52} \). Figure 6.1 corresponds to the chemical structure for paraffin wax consisting of two main chemical bonds.

![Chemical structure of N=23 polymerization of paraffin](image)

The chemical formula becomes

\[ (-CH_3 - CH_2 - CH_3-) \]

Using the group addition method the Heat of Formation is calculated as

\[
\begin{align*}
2 \cdot (-CH_3-) & \quad 2 \cdot \left(-44 \frac{kJ}{g-mol}\right) \\
23 \cdot (-CH_2-) & \quad 23 \cdot \left(-22 \frac{kJ}{g-mol}\right) \\
\Delta H_{f_{paraffin}} & = -598 \frac{kJ}{g-mol}
\end{align*}
\]

This can also be expressed in terms of energy per mass by using the molecular weights of the chemical formula yields \( \Delta H_{f_{paraffin}} \) of \(-1698.86 \text{kJ/kg}\)
6.1.3 Heat of Formation for ABS

Previous work done by Peterson and Whitmore employed the same group addition method in a comparison of HTPB and ABS. ABS was evaluated using 3 monomers: acrylonitrile, butadiene, and styrene. The typical formulation of readily available ABS consists of approximately 50% butadiene (mole fraction), 43% acrylonitrile, and 7% styrene. Using the three monomers the chemical formulation and corresponding Heat of Formation are given as [59]

\[
\begin{bmatrix}
\text{butadiene} \\
\text{acrylonitrile} \\
\text{styrene}
\end{bmatrix} = \begin{bmatrix}
CH_3 = CH - C = N \\
C_4H_6 \\
C_6H_5CH = CH_2
\end{bmatrix} = \begin{bmatrix}
42.27 \frac{kJ}{g\text{-mol}} \\
16.00 \frac{kJ}{g\text{-mol}} \\
4.36 \frac{kJ}{g\text{-mol}}
\end{bmatrix}
\] (6.6)

Combining the individual \( \Delta H_f \) for each monomer ratio with the corresponding mole fraction yields a net Heat of Formation \( \Delta H_{f,ABS} \), of 62.63 kJ/g - mol or 1097.42 kJ/kg [35].

6.1.4 Thermodynamic and Transport Properties of Paraffin/Nitrous Oxide Combustion.

The value for \( \Delta H_{f,\text{paraffin}} \) calculated by Eq. (6.5) and the molecular formula given by Eq. (6.4) were directly input into the NASA program “Chemical Equilibrium with Applications” (CEA) [60, 61]. The CEA program was configured to calculate the thermodynamic and transport properties of the motor. The thermodynamic and transport properties obtained were functions of combustion pressure \( P_0 \), and O/F ratio. Calculated motor properties include: ratio of specific heats \( \gamma \), molecular weight \( M_w \), combustion efficiency \( c^* \), adiabatic flame temperature \( T_0 \), viscosity \( \mu \), and Prandtl number \( P_r \). Figure 6.4 shows these properties as a function of O/F ratio and chamber pressure.

6.1.5 Thermodynamic and Transport Properties of ABS/Nitrous Oxide Combustion.

The reduced chemical formula for \( \Delta H_{f,ABS} \) used in CEA is
Fig. 6.2: Thermodynamic and transport properties of N$_2$O/Paraffin combustion products.
Figure 6.3 shows the thermodynamic and transport properties as a function of O/F ratio and chamber pressure.

6.1.6 Thermodynamic and Transport Properties for ABS impregnated with Paraffin/Nitrous Oxide Combustion

The previous sections outline calculating thermodynamic and transport properties for ABS and paraffin. However, during combustion these fuels will mix and simply interpolating based on mass between homogeneous ABS and paraffin will not provide accurate results. To calculate $\Delta H_{fPA}^0$ of the mixture, the group addition method is used on a per mass basis.

\[
\Delta H_{fPA}^0 = \Delta H_{fABS}^0 \cdot \%ABS + \Delta H_{fParaffin}^0 (1 - \%ABS) \\
\Delta H_{fPA}^0 = 660.63 \cdot \frac{kJ}{g-mol} \cdot \%ABS - 598 \cdot \frac{kJ}{g-mol}
\]  

(6.8)

Using the chemical formula given in Eq. (6.7) for ABS and a polymer ratio of $C_{25}H_{52}$ for paraffin, the molecular formula for the polymer based on the percentage of ABS in the mixture becomes

\[
C = 3.85 \cdot \%ABS + 25 \cdot (1 - \%ABS) \\
H = 4.85 \cdot \%ABS + 52 \cdot (1 - \%ABS) \\
N = 0.43 \cdot \%ABS
\]  

(6.9)

The percentage of ABS was varied from 0 to 100%, and using CEA tables were created for the thermodynamic and transport properties. These properties not only varied in ABS and paraffin content, but included changes in chamber pressure and O/F ratios when combined with Nitrous Oxide. Figure 6.4 shows these properties at a constant pressure of 75 bars for varying percentages of ABS as well as different O/F ratios. Even at small percentages of paraffin the ideal O/F begins to shift rapidly from 5 to 7.
Fig. 6.3: Thermodynamic and transport properties of $N_2/O_2$ combustion products.
Fig. 6.4: Thermodynamic and transport properties of $N_2O/ABS – Paraffin$ combustion products at 75 bars.
6.2 Regression Rate Modeling for Fuel Grain Design

Fuel grains with multiple fuels can be designed to achieve various O/F ratios during the course of a burn. A grain can be tailored to achieve a specific O/F by using varying regression rates to open up new surface areas as the burn progresses. This applies directly to paraffin, which has a much higher regression rate than ABS. The paraffin fuel regresses, opening up more ABS surface area, which adds more mass flow to the system. This could also be used to create specific thrust profiles without the need for throttling the oxidizer flow. Baseline fuel grain regression calculations use the model developed by Eilers and Whitmore [33] with the thermodynamic and transport properties based on the mean paraffin/ABS mass factions within the fuel grain.

6.2.1 Constant Mixture Ratio Fuel Grain Design

Using the regression theory outlined in sections 3 and 3.5.1 a constant mixture ratio grain was designed. This was achieved by using the high regression rate of paraffin to open voids in the grain, which allows for a greater massflow of the supporting ABS structure. Figure 6.5 shows the constant mixture ratio fuel grain where red outlines the location of paraffin within the grain.

This grain was designed to maintain a constant mixture ratio after 0.5 seconds to allow for start up transients in the grain to be avoided. This was achieved by changing the value for $\theta_{para}$ as a function of $r_{para}$ during each iteration of the hybrid simulation outlined in Figure 3.2. This grain was to burn for 0.5 then maintain a mixture ratio of 4.2, which was set by an initial $\theta_{para}$ of 10deg, for 5.5 seconds.

It is important to note that the fuel typically regresses perpendicular at all points along the burning surface. This is difficult to model accurately, thus for initial grain design the perpendicular criteria was only applied at the intersection of the paraffin slot and the radius of the abs. This allowed for the surface and chamber areas to be calculated as
Fig. 6.5: Constant mixture ratio grain geometry

\[ A_{surf} = (N_{slots} \cdot \theta_{para} \cdot r_{para}) + 2 \cdot N_{slots} \cdot r_{p2a} + (\theta_{space} - 2 \cdot (\theta_{start} - \theta_{ABS})) \cdot r_{ABS} \cdot N_{slots} \]  
(6.10)

where \( r_{p2a} \) is the distance from the paraffin radius to the ABS radius.

\[ r_{p2a} = \sqrt{(r_{para} \cdot \cos(\theta_{start}) - r_{ABS} \cdot \cos(\theta_{ABS}))^2 + (r_{para} \cdot \sin(\theta_{start}) - r_{ABS} \cdot \sin(\theta_{ABS}))^2} \]  
(6.11)

The chamber area is given as

\[ A_{chamber} = \pi r_{ABS}^2 + N_{slots} \cdot [\theta_{para} \cdot (r_{para}^2 - r_{ABS}^2) + 2 \cdot A_{P2A}] \]  
(6.12)

where \( A_{P2A} \) is the area of a triangle formed from the paraffin to the ABS.
\[ P_x = r_{\text{para}} \cdot \cos(\theta_{\text{start}}) \quad P_y = r_{\text{para}} \cdot \sin(\theta_{\text{start}}) \]
\[ A_{x1} = r_{\text{ABS}} \cdot \cos(\theta_{\text{start}}) \quad A_{y1} = r_{\text{ABS}} \cdot \sin(\theta_{\text{start}}) \]
\[ A_{x2} = r_{\text{ABS}} \cdot \cos(\theta_{\text{start}} - \theta_{\text{ABS}}) \quad A_{y2} = r_{\text{ABS}} \cdot \sin(\theta_{\text{start}} - \theta_{\text{ABS}}) \]
\[ A_{P2A} = \left| \frac{P_x (A_{y1} - A_{y2}) + A_{x1} (A_{y2} - P_y) + A_{x2} (P_y - A_{y1})}{2} \right| \quad (6.13) \]

Figure 6.6 shows the time propagated burning surface for the fuel grain.

**Fig. 6.6: Regressed constant mixture ratio fuel grain**

### 6.3 Analytical Modeling for Fuel Regression Propagation

There have been several investigations on modeling geometric regression of complex fuel grains, but many of them are difficult to implement, are simply extremely slow, and all were intended for solid rocket motors [62–71].
6.3.1 Rasterized Technique for Fuel Boundary Propagation

A novel method whereby the fuel cross section will be modeled as an array of grayscale pixels and image-processing techniques will be used to regress the fuel grain geometry.

Figure 6.7 shows an example of this process. If a binary image of the grain is used Figure 6.7a, its edges can be blurred using an image filter Figure 6.7b, and then all non-binary pixels in the image are removed Figure 6.7c. The fuel is then regressed by the radius of the blur filter Figure 6.7d.

![Image](image_url)

(a) Initial grain, (900x900 pixels)  (b) Blurred with a 40 pixel radius disk filter.

(c) Blurred Image with non-binary pixels removed.  (d) Borders regressed via blur filter.

Fig. 6.7: Geometric regression via blurring.

This technique produces comparable results to numerical propagation of the surface boundary. Additionally, blurring filters round sharp edges, which is difficult to do using
geometric propagation. Sharp edges are rounded in hybrid motor grains during burns due to including boundary layer effects and heat transfer concentrations. This method also works for any grain geometry, not just simple geometric shapes.

6.3.2 Single Fuel Regression Model

The geometric regression uses a 2D, binary image of a fuel port for the initial, unburnt grain. The edges of the picture are assumed to be the case radius.

The picture is stored as a matrix \( \bar{I} \), where the index of the matrix corresponds to the \((x,y)\) position, and value in each matrix cell is a grayscale value (between 0 and 1).

The conversion factor between pixels and any arbitrary unit is then simply:

\[
dx = \frac{W}{n_{px}}, \quad dy = \frac{H}{n_{py}} \tag{6.14}\]

Where \( W \) is the width of the image in meters, \( H \) is the height of the image in meters, and \( n_{px} \) and \( n_{py} \) are the number of pixels in the \( x \) and \( y \) dimensions of the picture.

Before the image is regressed, important geometric properties of the grain can be easily extracted. Using the “bwboundaries” function in MATLAB, the \( \bar{X} \) and \( \bar{Y} \) vectors of the boundary between the black and white edges of the picture are returned. The perimeter of the port is then:

\[
P = \sum \sqrt{(\delta \bar{X} \cdot dx)^2 + (\delta \bar{Y} \cdot dy)^2} \tag{6.15}\]

Where \( \delta \bar{X} \) and \( \delta \bar{Y} \) are the difference vectors of \( \bar{X} \) and \( \bar{Y} \) respectively, such that

\[
\delta \bar{X} = \begin{bmatrix} \bar{X}(2) - \bar{X}(1) \\ \bar{X}(3) - \bar{X}(2) \\ \vdots \\ \bar{X}(n) - \bar{X}(n-1) \end{bmatrix} \tag{6.16}\]

The port area calculation is even simpler

\[
A_{port} = \sum_{i=1}^{n} \left[ \sum_{k=i}^{m} [I(i,k)] \right] \cdot dx \cdot dy \tag{6.17}\]
where \( n \) and \( m \) are the dimensions of \( \bar{I} \).

Using the equations from the previous section, oxidizer mass flux \( G \) and \( \dot{r} \) can be calculated. With these values known, the image can now be regressed. To regress the image, a disk filter of radius \( \dot{rdt} \) (converted to pixels) is applied to the image. This disk filter moves through each pixel in the image and takes a spacial average of all of the pixels within the radius of the filter. This average is then the new value of the pixel.

The image is then thresholded such that any pixels that are not perfectly black (zero) are regressed and turned white (one). Thresholding creates a new binary image of the fuel grain that has regressed by \( rdt \) pixels. The port perimeter and area are calculated again, which again yields a new \( \dot{r} \), and a new disk filter of size \( \dot{rdt} \) is created. This process is then looped through the desired burn time. Figure 6.8 shows a block diagram of the total algorithm.

![Fig. 6.8: Single fuel software block diagram.](image)

### 6.3.3 Multiple Fuel Regression Model

Using the single fuel regression as a foundation for expanding to multiple fuels simply requires an image of where the fuel is located within the grain. Each fuel must have a unique
“mask” where white pixels represent the fuel location in the grain. Three images are needed to fully define a fuel grain cross section with two fuels. Figure 6.9 shows example images of the initial port and two images representing the fuels being modeled. In this paper this algorithm is applied to fuel grains consisting of ABS and paraffin.

![Initial Port, Paraffin Mask, ABS Mask](image)

Fig. 6.9: Masks used in grain regression model.

After finding regression rate, a disk filter is created for each fuel using the floor of $ABS_{\text{Filter}}$ and $Para_{\text{Filter}}$. Since the image can only be processed using integer values of $\dot{r}$ a remainder is created to keep track of partial pixels,

$$ABS_{\text{Filter}} = \dot{r}_{ABS} \cdot \frac{dt}{dx} + \text{rem}(1)$$

$$Para_{\text{Filter}} = \dot{r}_{Para} \cdot \frac{dt}{dx} + \text{rem}(2)$$

where the remainder is given as

$$\text{rem}(1) = ABS_{\text{Filter}} - \text{floor}(ABS_{\text{Filter}})$$

$$\text{rem}(2) = Para_{\text{Filter}} - \text{floor}(Para_{\text{Filter}})$$

The port, $\bar{I}$ is then regressed by applying the two filters to the original image,

$$\bar{I}_{ABS} = \text{imfilter}(\bar{I}, \text{floor}(ABS_{\text{Filter}}))$$

$$\bar{I}_{Para} = \text{imfilter}(\bar{I}, \text{floor}(Para_{\text{Filter}}))$$

Figure 6.10 shows $\bar{I}_{ABS}$ and $\bar{I}_{Para}$ as ABS Regression and Para Regression. A new image $\bar{I}_c$ is created by applying the logic, if the filtered image and the mask image both contain white pixels, that pixel is kept white,
Fig. 6.10: Grain regression process using ABS and paraffin masks, then combining the fuels back together.

\[
\bar{I}_c(I_{\text{ABSMASK}} == 1) = \bar{I}_\text{ABS}(I_{\text{ABSMask}} == 1)
\]

\[
\bar{I}_c(I_{\text{ParaMASK}} == 1) = \bar{I}_\text{Para}(I_{\text{ParaMask}} == 1)
\]  

(6.21)

Figure 6.10 shows $\bar{I}_c$ as the masked regression. Notice that the pixels kept for the ABS regression and paraffin regression correspond to the location of white pixels in the corresponding mask.

At this point, stray pixels are removed from the image that aren’t directly connected to the port using the “bwareaopen” command in MATLAB. This occurs when the paraffin filter regresses far enough to reach its mask boundary but the ABS filter has not reached the paraffin yet.

Now that the image has been regressed for both fuels, the mass in the system can be calculated. The total mass of paraffin that has been used is determined such that if the port and the paraffin mask both contain white pixels the mass of that pixel is allocated to paraffin.

\[
\bar{I}_\text{ParaPix}(I_{\text{ParaMASK}} == 1) = \bar{I}_c(I_{\text{ParaMask}} == 1)
\]  

(6.22)
\[ m_{\text{tot} \text{Para}} = \sum_{i=1}^{n} \sum_{k=1}^{m} I_{\text{ParaPix}} \cdot (dx \cdot dy \cdot L \cdot \rho_{\text{Para}}) \] (6.23)

The total mass of ABS is the sum of white pixels in \( I_c \) minus the paraffin pixels \( \bar{I}_{\text{ParaPix}} \).

\[ m_{\text{tot} \text{ABS}} = \sum_{i=1}^{n} \sum_{k=1}^{m} I_{c} - \sum_{i=1}^{n} \sum_{k=1}^{m} \bar{I}_{\text{PixPara}} (dx \cdot dy \cdot L \cdot \rho_{\text{ABS}}) \] (6.24)

The image processing is repeated using the ceiling function, where the actual mass of ABS for a given time step is given as:

\[
m_{\text{ABS}} = \text{rem}(1) \cdot m_{\text{tot} \text{ABS}} + (1 - \text{rem}(1)) \cdot m_{\text{tot} \text{ABS}2} \\
m_{\text{Para}} = \text{rem}(2) \cdot m_{\text{tot} \text{Para}} + (1 - \text{rem}(2)) \cdot m_{\text{tot} \text{Para}2} \] (6.25)

The massflows are then determined based upon the current mass, and the previous time steps mass.

\[
\dot{m}_{\text{ABS}} = \frac{m_{\text{ABS}(i)} - m_{\text{ABS}(i-1)}}{dt} \\
\dot{m}_{\text{Para}} = \frac{m_{\text{Para}(i)} - m_{\text{Para}(i-1)}}{dt} \] (6.26)

The massflows are then used to determine new properties of combustion using a CEA lookup table based on the percentage of paraffin in the system. These properties are then used to calculate the motor properties, which are then iteratively fed back to compute \( \dot{r} \). Figure 6.11 shows a block diagram of the total algorithm.

### 6.3.4 Experimental Results

A series of experimental tests were performed at Utah State University using ABS/Paraffin fuel grains with nitrous oxide \( N_2O \). Table 6.1 summarizes these tests.
Table 6.1: Test data for ABS-paraffin grains.

<table>
<thead>
<tr>
<th>Grain Type</th>
<th>Grain Length</th>
<th>O/F</th>
<th>$P_0$</th>
<th>$\Delta m$</th>
<th>Thrust</th>
<th>$e^*$ efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 25% Paraffin</td>
<td>0.30</td>
<td>3.9</td>
<td>3620</td>
<td>0.21</td>
<td>450</td>
<td>70</td>
</tr>
<tr>
<td>2 Constant MR</td>
<td>0.15</td>
<td>5.6</td>
<td>3030</td>
<td>0.16</td>
<td>390</td>
<td>70</td>
</tr>
<tr>
<td>3 Constant MR with Carbon Black</td>
<td>0.15</td>
<td>6.15</td>
<td>3030</td>
<td>0.13</td>
<td>370</td>
<td>76</td>
</tr>
<tr>
<td>4 Helical Constant MR</td>
<td>0.15</td>
<td>5.34</td>
<td>2965</td>
<td>0.15</td>
<td>350</td>
<td>77</td>
</tr>
</tbody>
</table>

The first grain was designed to have 25 percent paraffin by volume, and is shown in Figure 6.12a. The other three grains were designed to maintain a constant mixture ratio after burning through 0.1 inches of ABS shown in Figure 6.12b.
6.4 Comparison of Analytical and Experimental Results

The fuel grains were burned with $N_2O$ for 4 seconds to account for start up transients, and to avoid any problems with the grain burning out to the case. Typically regression rate is determined by comparing the final and initial radius over the duration of the burn given as

$$\dot{r} = \frac{r_{\text{final}} - r_{\text{initial}}}{t_{\text{burn}}} \quad (6.27)$$

This is not applicable to using multiple fuels or complex geometries due to a non-symmetrical radius and densities. After each burn the fuel grain was cut longitudinally so the cross section of the grain could be imported into MATLAB. Figure 6.13a shows a burned grain cross section cut in half. The grain was then painted white to enhance contrast shown in Figure 6.13b. This paint scheme allowed the grain to be scanned in black and white to create the image of Figure 6.14a. The scanned image contains noise due to contrast, which
is easily fixed by removing small groups of black and white pixels. This process eliminates stray pixels and produces the result shown in Figure 6.14b. This image clearly has the port separated from the rest of the grain.

Fig. 6.13: Image processing techniques to import final grain geometry into MATLAB

Fig. 6.14: Image processing techniques for determining final grain geometry
Boundary detection is used to find the outer and inner profiles, which are used to scale the image. Figure 6.15 shows the burned cross section overlaid on the analytical model. Clearly the analytical model doesn’t predict accurately the grain regression, therefore comparing motor performance produces no relevant data. Typical methods lead to sharp corners which are not evident in actual experimental tests.

Fig. 6.15: Burned fuel cross section overlaid on analytical model

6.4.1 Results for Constant Mixture Ratio Grain

Using the pixelated regression model, end to end motor performance was modeled and compared to the experimental results, which are found in Table 6.1. Figure 6.16a shows the initial port for the constant mixture ratio burns, while Figure 6.16b shows the experimental results overlaid on analytical predictions.

Figure 6.17 compares the analytical and experimental thrust and pressure profiles. It is important to note that the simulation predicts the spike in thrust when the fuel boundary layer hits the paraffin section.
Fig. 6.16: Initial and final ports for constant mixture ratio grains.

Fig. 6.17: Experimental and analytical predictions for constant mixture ratio thrust and pressure

Clearly the measured thrust is lower than predicted. This result is likely due to poor combustion efficiency for the real motor. Considering that the simulation model assumes perfect combustion efficiency, the over estimated combustion performance is not surprising. Figure 6.18 shows large amounts of unburned paraffin were expelled from the supporting ABS mesh.
6.4.2 Results for 25% Paraffin Grain

The pixelated model was also applied to a 25% paraffin grain. This grain was the first attempt to combine paraffin with the ABS mesh, and was not modeled initially due to the complex geometries that evolved as the grain regressed. The algorithm was propagated from the initial port shown in Figure 6.9. Both experimental and analytical results of a 3 second burn are shown in Figure 6.19b.

The final experimental boundary closely matches the predicted, regressed fuel boundary. Figure 6.20 shows a time lapse of the analytically predicted port over the course of the burn. The number of different geometries present throughout the burn would require numerous if statements. These statements not only slow the model down, they are also difficult to predict without simplifying geometries. The pixelated model takes care of complex geometries, and provides sufficient results.
Figure 6.19 compares the analytically predicted thrust and pressure profiles against experimental data for the 25% paraffin grain.
6.4.3 Regression Rate Comparison

The regression rate for the combined fuels was determined based upon the percentage of mass being combusted given as [49]

\[ \dot{r}_{comb} = \frac{\dot{m}_{ABS}}{\dot{m}_{total}} \dot{r}_{ABS} + \frac{\dot{m}_{para}}{\dot{m}_{total}} \dot{r}_{para} \] (6.28)

Figure 6.22 shows theoretical regression rate for paraffin and ABS as well as the constant mixture ratio and 25% paraffin grain. The two grains had similar regression rates but drastically different mass fluxes. This is due to the burn time of the grains as well as the different geometries as the grains burned out. The addition of paraffin in the ABS structure increased the effective regression rate three times that of pure ABS.

6.4.4 Combustion Efficiency

Due to the lackluster performance of “neat” or paraffin with no additives, several methods were tried to improve the baseline combustion efficiency. The first attempt was to add carbon black to the mixture of paraffin to reduce radiative heat transfer. This provided a successful improvement of 6% over standard paraffin, however the total efficiency was 75%. Figure 6.23 shows the second attempt which used the advantages of rapid prototyping to create helical slots for the paraffin as well as a helical post combustion chamber.
Fig. 6.22: Regression rate comparison for constant mixture ratio and 25% paraffin grains

(a) Pre-burn where blue is the center port, red represents the paraffin slots, and green is the helical post combustion chamber.

(b) Post-burn fuel grain with helical flow paths

Fig. 6.23: Hilical fuel grain pre and post burn
The helical slots were to help prevent sloughing of the paraffin, and to reduce radiative heat transfer. The idea behind the post combustion chamber being a helix was to keep the entraining droplets to the outside by spinning the flow. This only provided an increase of 1% over standard carbon black, this also lowered the average O/F ratio of the grain due to the quick disintegration of the post combustion chamber. Figure 6.24 shows the plume from the third fuel grain which has no paraffin being ejected and a very tight form factor. Comparing the plumes from the non carbon black grains to those with carbon black reveal that very little paraffin is actually expelled from the nozzle uncombusted, thus having an efficiency of 77% is directly related to the motor design.

A main concern is the atomization of the oxidizer through the injector. The injector chosen for this research was an off the shelf spray nozzle from McMaster Carr. This spray nozzle was not made to use inside a rocket motor or with nitrous oxide, and thus had no need
to have really fine atomization. The efficiency of the motor could be drastically improved by changing the injector to something that would atomize the flow better.

6.5 Conclusion

This research has investigated techniques for fabricating and modeling of hybrid rocket fuel grains composed of porous ABS impregnated with paraffin wax. FDM additive manufacturing techniques were used to fabricate the ABS substrate that supports a paraffin fuel. The substrate was then impregnated with paraffin wax which provides an enhanced regression rate while having no detrimental effect on the thermodynamic burn properties of the fuel grain. Two unique fuel grains were designed, experimentally tested, and successfully compared to analytical models. These grains differed in paraffin content, grain geometries, and fuel additives. Static test fires showed that complex grain geometries can be created easily using FDM techniques, and combined with another fuel in a non-homogeneous mixture while maintaining structural integrity.

In traditional surface-regression calculations, analytical equations are used to describe the grain surface, and those equations are linearly regressed to allow the port surface area and volume to be calculated at all times. This curve-propagation method becomes difficult to implement complex geometries, and does not accurately round corners. A novel method was created that uses images to model the fuel grain port, along with the two fuel locations throughout the grain. In this modeling approach, the fuel cross section grid is modeled using pixelated images, where black pixels represent fuel and white pixels represent the fuel grain port.

The first fuel grain manufactured was 25% paraffin by mass, and was created to prove feasibility of combining the two fuels. This grain was later compared using the dual fuel rationalization model fore end to end motor performance. It was found that the regression of the fuels was quite accurate, however the thrust level profile was not predicted at all. Due to poor combustion of the paraffin fuel, which had no additives, it was not surprising the thrust level was off.

Using FDM techniques a specific O/F was achieved using multiple fuels and complex
geometries to open up new surface areas as the burn progressed. Three grains were made to show increases in combustion efficiency using carbon black and a helical post combustion chamber. For the fuel-grain geometries tested, the analytical model very accurately predicts surface regression, but over predicts the mean thrust level by more than 15%. This result is likely produced by poor combustion efficiency for the real motor.

Future research activities will attempt to assess the source of this low combustion efficiency, and to construct supporting ABS meshes that allow for more complete combustion of embedded paraffin materials. Another interesting research topic would be to create varying thrust profiles using different geometries and fuels without the need for oxidizer throttling.
References


Grain Design and Burnback Simulation using a Minimum Distance Function,” 41st