Projectile Motion on a Rotating Earth

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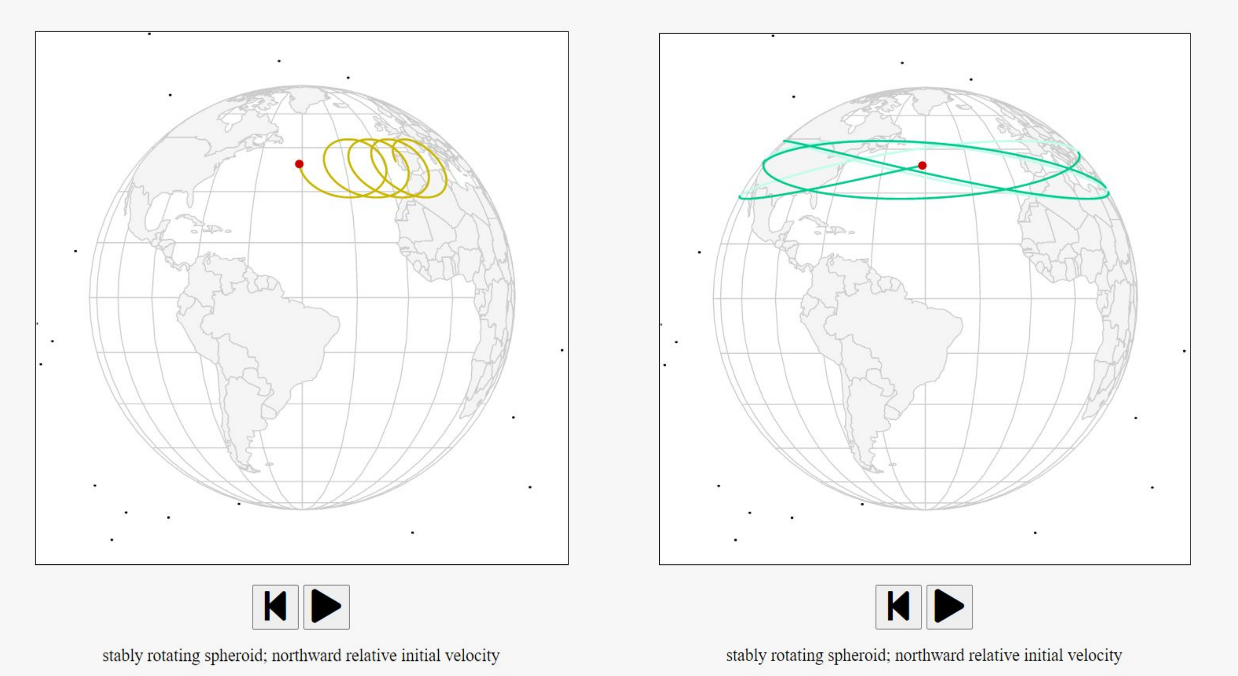
Joining the Coriolis Group

Group's focus on motion of a hockey puck on a smooth frictionless earth

Branching off to projectile motion



An example of the visualizations the group is creating to demonstrate the Coriolis force. For the same initial conditions, the left is the motion as seen in a rotating frame while the right is the motion seen in an inertial frame of reference [1]

Preview

Set up and assumptions

Using Kepler's equation and initial conditions to find constants

Using constants to find initial and final positions in terms of phi

Together these describe projectile motion in an inertial frame

Transforming Kepler's equation from an inertial to rotating frame using Newton's laws

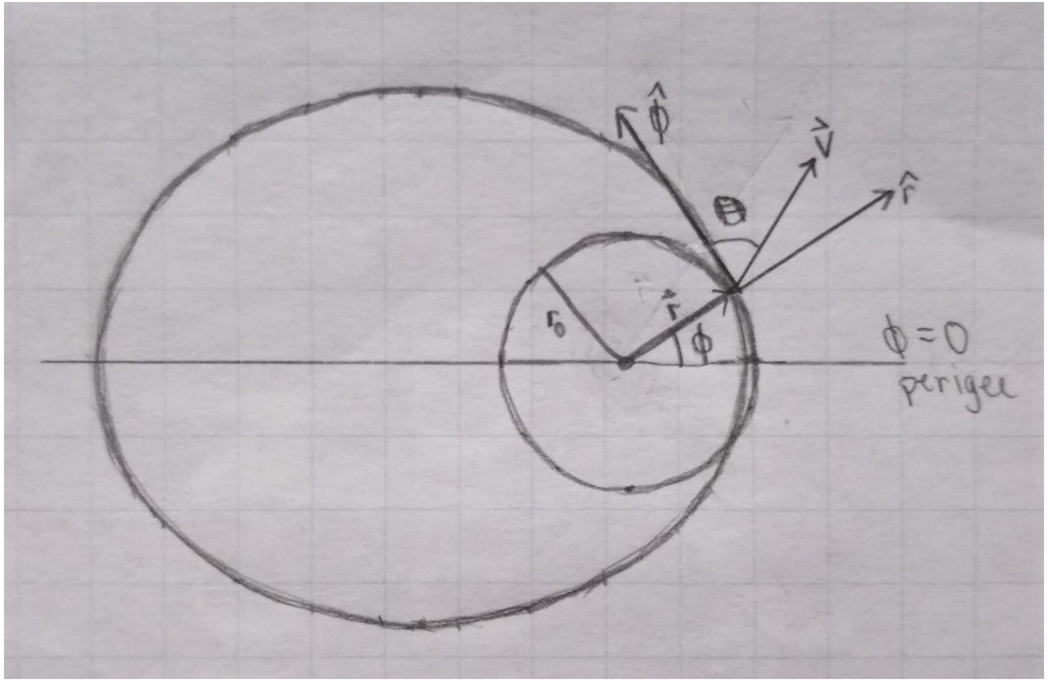
Examining results

Set Up and Assumptions

The earth is spherical

No air resistance

Stay in the equatorial plane



Kepler's Equation

Using $r(\phi) = \frac{c}{1 + \epsilon \cos \phi}$ and $c = \frac{l^2}{GMm^2}$. [2]

We find $l = r_0 v_0 m \cos(\Theta)$ and $\epsilon = \sqrt{(\frac{c}{r_0} - 1)^2 + \frac{c}{r_0}^2 \tan^2 \Theta}$

From these we get $\phi_0 = \tan^{-1}(\frac{\tan \Theta}{1 - \frac{r_0}{c}})$ and $\phi_0 = -\phi_f$

Inertial and Rotating Reference Frames

Going from an inertial reference frame to a rotating reference frame

[2]

$$\mathbf{F} = m\mathbf{a} \rightarrow m\mathbf{a} = \mathbf{F} + 2m\mathbf{v} \times \boldsymbol{\Omega} + m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}$$

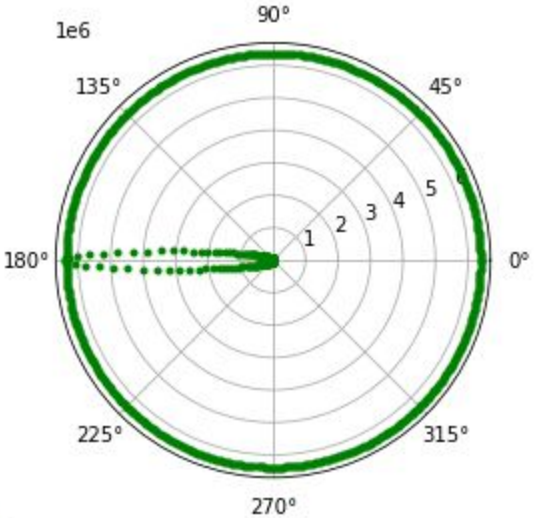
Newton’s Second Law

Net force as seen in the inertial frame

Coriolis force term

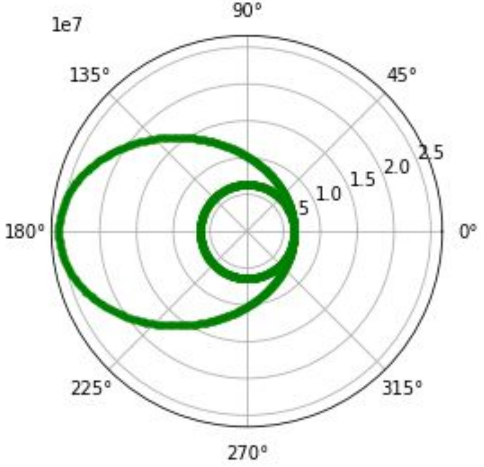
Centrifugal force term

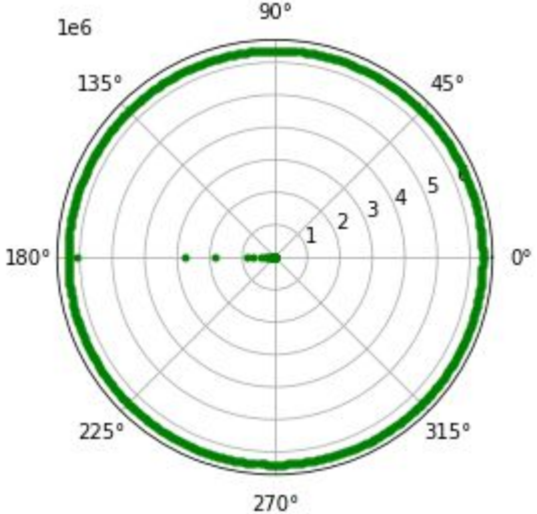
Results



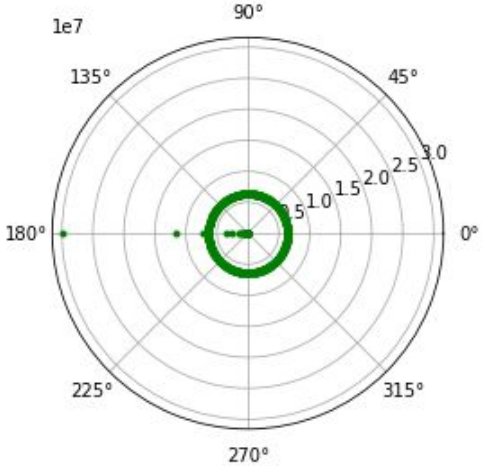
Small v_0

$\Theta \to 0$



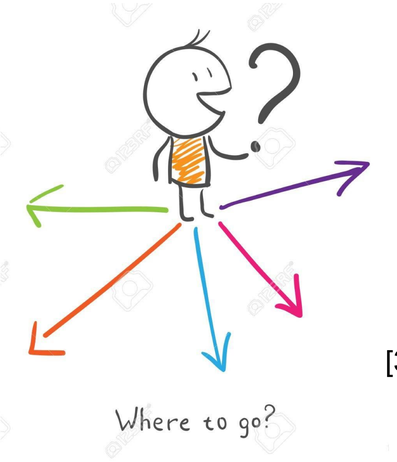


$\Theta \to \frac{\pi}{2}$



Large v_0

Where To Go From Here



[3]

Go from 2D to 3D

Go from spherical coordinates to geodetic

Include the oblateness of the Earth

Extend the current visualizations of CorioVis to include projectile motion

References

[1] John Edwards, CorioVis, Demo 2, Accessed Apr 5 2021 https://edwardsjohnmartin.github.io/coriolis/

[2] John Taylor, Classical Mechanics, University Science Books, 2005, Sections 8.6 and 9.5

[3] Where to go, 123RF, https://www.123rf.com/photo_14579896_where-to-go-man-chooses-where-to-go-.html

Questions?