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Applications of Bayesian Statistics in Fluvial Bed Load Transport

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APPLICATIONS OF BAYESIAN STATISTICS
IN FLUVIAL BED LOAD TRANSPORT

by

Mark Schmelter

A dissertation submitted in partial fulfillment
of the requirements for the degree
of
DOCTOR OF PHILOSOPHY
in
Civil and Environmental Engineering

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2013
ABSTRACT

Applications of Bayesian Statistics in Fluvial Bed Load Transport

by

Mark Schmelter, Doctor of Philosophy
Utah State University, 2013

Major Professor: Dr. David Stevens
Department: Civil and Environmental Engineering

Fluvial sediment transport is a process that has long been important in managing water resources. While we intuitively recognize that increased flow amounts to increased sediment discharge, there is still significant uncertainty in the details. Because sediment transport—and in the context of this dissertation, bed load transport—is a strongly nonlinear process that is usually modeled using empirical or semi-empirical equations, there exists a large amount of uncertainty around model parameters, predictions, and model suitability. The focus of this dissertation is to develop and demonstrate a series of physically- and statistically-based sediment transport models that build on the scientific knowledge of the physics of sediment transport while evaluating the phenomenon in an environment that leads us to robust estimates of parametric, predictive, and model selection uncertainty. The success of these models permits us to put theoretically and procedurally sound uncertainty estimates to a process that is widely acknowledged to be variable and uncertain but has, to date, not developed robust statistical tools to quantify this uncertainty. This dissertation comprises four
individual papers that methodically develop and prove the concept of Bayesian statistical sediment transport models. A simple pedagogical model is developed using synthetic and laboratory flume data—this model is then compared to traditional statistical approaches that are more familiar to the discipline. A single-fraction sediment transport model is developed on the Snake River to develop a probabilistic sediment budget whose results are compared to a sediment budget developed through an ad hoc uncertainty analysis. Lastly, a multi-fraction sediment transport model is developed in which multiple fractions of laboratory flume experiments are modeled and the results are compared to the standard theory that has been already published. The results of these models demonstrate that a Bayesian approach to sediment transport has much to offer the discipline as it is able to 1) accurately provide estimates of model parameters, 2) quantify parametric uncertainty of the models, 3) provide a means to evaluate relative model fit between different deterministic equations, 4) provide predictive uncertainty of sediment transport, 5) propagate uncertainty from the root causes into secondary and tertiary dependent functions, and 6) provide a means by which testing of established theory can be performed.
PUBLIC ABSTRACT

Applications of Bayesian Statistics in Fluvial Bed Load Transport

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The science of fluvial sediment transport studies the processes involved in the movement of river sediments. It is commonly understood that when rivers flood they have a great capacity to move sand, gravel, and even larger cobbles and boulders. This process is not only limited to the big floods that usually attract so much attention, but also the more common river flows play a very important role in forming a river. As engineers and scientists, we like to be able to develop equations and relationships that describe some natural phenomenon—in this case, fluvial sediment transport. While we are able to generally characterize these processes using these equations it is not too difficult to understand why there is usually very high uncertainty surrounding the scientific description of this process. The focus of the research in this dissertation is to use the knowledge we have gained from the equations and relationships and try to marry those concepts with statistics so that we can not only generally describe the process of rocks moving in a river, but also have an idea of how certain we are of our predictions as well as the value of our equations. Naturally, we try to use our knowledge to make predictions of what
could happen in rivers if they flood so that we can design facilities—recreational, commercial, and protective—to some level of risk tolerance. This research uses a statistical framework called Bayesian statistics that allows us to use the equations and relationships that focus on the approximate physics of sediment transport in a statistical framework to realize predictions with estimates of certainty (or uncertainty, as the case may be). The chapters that follow use simulated (fake) data as well as laboratory flume and data collected from the Snake River to develop an equation-based statistical sediment transport model. This model is tested and evaluated throughout the dissertation and it is concluded that this methodology shows great promise in solving the matter of measuring/estimating uncertainty in the processes of sediment movement in rivers and streams.
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for my dissertation and was able to provide useful advice and distill practical
context. Gilberto Urroz helped me have a better appreciation for fluid dynamics
and its role in the movement of eroded sediments in rivers and streams.

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Bureau of Reclamation Trinity River Restoration Program (TRRP) provided
project funding thereafter. Further, my work with TRRP helped me better
understand the needs of river restoration programs and how applied fluvial
geomorphology and engineering can tackle extremely challenging problems related
to rivers and sediment transport.

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Being in graduate school made it possible for me to spend lots of time in the home with my children during their early years—this made for fun summer vacations and many fond memories. It is my hope that my children were blissfully unaware of the challenges and inherent grumpiness that graduate school can bring. It is my suspicion, however, that occasionally they found me out, so I am very grateful for my kids’ good attitudes, their imperfect (and hopefully conveniently incomplete) memories of childhood, and that they were forgiving and kind to their occasionally out-of-sorts father.

Mark Schmelter
To Keely.
CONTENTS

ABSTRACT ................................................................. iii
PUBLIC ABSTRACT ....................................................... v
ACKNOWLEDGMENTS ..................................................... vii
LIST OF TABLES .......................................................... xiii
LIST OF FIGURES .......................................................... xv

CHAPTER

1 INTRODUCTION ......................................................... 1

1.1 Sediment Transport ................................................. 1
1.2 Uncertainty .......................................................... 3
1.3 Objectives ............................................................ 4
References ................................................................. 5

2 LITERATURE REVIEW .................................................. 8

2.1 Background and Literature Review ................................. 8
  2.1.1 Sediment Transport ............................................. 8
  2.1.2 Approaches to Sediment Transport Modeling ................. 10
  2.1.3 Bayesian Statistical Models and Sediment Transport ....... 14
2.2 Methods ............................................................... 24
  2.2.1 Bayesian Statistical Model Fundamentals .................... 24
  2.2.2 Numerical Methods ............................................. 26
References ................................................................. 27

3 BAYESIAN SEDIMENT TRANSPORT MODEL FOR UNI-SIZE BED LOAD ................................. 33

Abstract ................................................................. 33
3.1 Introduction .......................................................... 34
  3.1.1 Determinism and Statistics ..................................... 35
  3.1.2 Challenges in Sediment Transport .............................. 36
3.2 Purpose and Objectives ............................................. 38
3.3 Formulation of Bayesian Model ................................... 39
  3.3.1 Governing Sediment Transport Relations ..................... 39
  3.3.2 Specification of Bayesian Transport Model ................. 42
3.4 Computational Methods ............................................ 46
## 3.4.1 Markov Chain Monte Carlo

- Page 46

## 3.4.2 Sampling Algorithm for Transport Model

- Page 47

## 3.5 Model Evaluation

- Page 48
  - 3.5.1 Sensitivity to Prior Specification
    - Page 49
  - 3.5.2 Posterior Prediction
    - Page 49
  - 3.5.3 Model Discrimination
    - Page 50

## 3.6 Experimental Setup

- Page 50

## 3.7 Results and Discussion

- Page 51
  - 3.7.1 Simulation and Sensitivity Studies
    - Page 51
  - 3.7.2 Flume Observations
    - Page 58

## 3.8 Implications and Conclusions

- Page 63
  - 3.8.1 General
    - Page 63
  - 3.8.2 Model Extensions for Future Implementations
    - Page 66

## References

- Page 66

## 4 TRADITIONAL AND BAYESIAN STATISTICAL MODELS IN FLUVIAL SEDIMENT TRANSPORT

- Page 79

## Abstract

- Page 79

## 4.1 Introduction

- Page 80

## 4.2 Methods

- Page 83
  - 4.2.1 Model Formulation
    - Page 83
  - 4.2.2 Method Testing
    - Page 85

## 4.3 Results and Discussion

- Page 86

## 4.4 Conclusions

- Page 89

## References

- Page 90

## 5 ACCOUNTING FOR UNCERTAINTY IN CUMULATIVE SEDIMENT TRANSPORT USING BAYESIAN STATISTICS

- Page 96

## Abstract

- Page 96

## 5.1 Introduction

- Page 97
  - 5.1.1 Sediment budgets
    - Page 98
  - 5.1.2 Bayesian models
    - Page 100

## 5.2 Study area

- Page 104

## 5.3 Methods

- Page 106
  - 5.3.1 Sediment transport observations
    - Page 107
  - 5.3.2 Bayesian sediment transport model
    - Page 107
  - 5.3.3 Sediment budget calculation
    - Page 114

## 5.4 Results

- Page 115

## 5.5 Discussion

- Page 117
  - 5.5.1 Bayesian sediment transport model
    - Page 117
  - 5.5.2 Implications for sediment budgets
    - Page 121

## 5.6 Conclusions

- Page 124
LIST OF TABLES

Table | Page
---|---
3.1 Values for fixed parameters | 73
3.2 Priors for $\tau_c$. Critical Shields Number in parentheses | 73
3.3 Vague priors for $\sigma^2$ | 73
3.4 Posterior inference results for $\tau_c$. Critical Shields Number in parentheses. For each data scenario the true value of $\tau_c$ is 6.72 Pa ($\tau^*_c = 0.052$). | 73
3.5 Comparison of priors for data scenario 4. Critical Shields Number in parentheses. True value of $\tau_c$ is 6.72 Pa ($\tau^*_c = 0.052$). | 74
3.6 Posterior inference on model parameters using priors 1A–1C for $\sigma^2$ and prior 1 (diffuse) for $\tau_c$. Expected value of posterior distribution—values in parentheses denote the 95% credible interval. True critical shear value is 6.72 Pa; true variance for data scenario 1 is 0.05 and 1.1 for data scenario 3. | 74
3.7 Deviance Information Criterion results for competing simulation models. Critical Shields Number in parentheses. True value of $\tau_c$ is 6.72 Pa ($\tau^*_c = 0.052$). | 74
3.8 Comparison of process model performance on the Smart (1984) 10.5 mm flume observations—$\tau_c$. Critical Shields Number in parentheses. | 74
3.9 Comparison of process model performance on the Smart (1984) 10.5 mm flume observations—$\sigma^2$. | 75
4.1 Inferred critical shear. BM-D = Bayesian model, diffuse prior; BM-I = Bayesian model, informative prior; NLS = Nonlinear least squares. “C.I.” denotes the credible interval for BM-D and BM-I and confidence interval for NLS | 92
5.1 Prior distributions for $\tau_r$ and $\sigma^2$. The truncated normal distributions are specified as $T.N.(\mu_{\tau_r}, \sigma_{\tau_r}, \tilde{a}, \tilde{b})$. The inverse gamma distribution is specified as $I.G.(\mu_{\sigma^2}, \sigma_{\sigma^2})$. | 132
5.2 Inferred and fixed parameters for bed load transport function. Top section displays results from the posterior distributions. Middle section shows selected parameters from [Erwin et al. (2011)]. Bottom section shows channel properties for each site. Point estimate is the expected value of the posterior distribution and values in parentheses comprise the 95% posterior credible interval for the Bayesian estimates. Values without credible intervals are assumed fixed and known.

6.1 Data summary for sediment mixtures. Inferred bulk transport posterior means for $\tau_r$, in Pa and $\sigma$, in $m^3/m/s$, number of observational runs for each sediment, and the proportion sand in both the surface and total mixture.

6.2 Inferred fractional transport posterior means for $\tau_{r,j}$, in Pa.

6.3 Number of observations of fractional transport.

6.4 Inferred fractional transport posterior means for $\sigma_j$, in $m^3/m/s$. 

# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Schematic of bed load transport</td>
<td>32</td>
</tr>
<tr>
<td>3.1</td>
<td>Plot of simulated observation scenarios. Each data scenario is a distinct</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>combination of high/low variance and many/few observations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Data scenario 1: $N = 21$, $\sigma^2 = 0.05$, (b) data scenario 2:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N = 6$, $\sigma^2 = 0.05$, (c) data scenario 3: $N = 21$, $\sigma^2 = 1.10$,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) data scenario 4: $N = 6$, $\sigma^2 = 1.10$.</td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>Four prior scenarios. Vertical line (red) denotes true $\tau_c$ in simulated</td>
<td></td>
</tr>
<tr>
<td></td>
<td>data scenarios. Parameterization follows: $T.N.(\mu, \sigma, \tilde{a}, \tilde{b})$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Prior scenario 1, (b) prior scenario 2, (c) prior scenario 3, and (d)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>prior scenario 4.</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td>Progressively informative priors for data scenario 4</td>
<td>77</td>
</tr>
<tr>
<td>3.4</td>
<td>Prior distributions for Smart (1984) observations. Vertical line (red)</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>denotes the minimum shear that produced transport.</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>Posterior predictive intervals for Smart (1984) observations. Plots (a) and</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>(b) use the MP-M bed load relation. Plots (c) and (d) use the W-P relation.</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Data scenarios. (a) data scenario 1, low variance, many observations;</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>(b) data scenario 2, low variance, few observations; (c) data scenario 3,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>high variance, many observations; (d) data scenario 4, high variance,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>few observations; (e) Smart (1984) 10 mm observations.</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>Prior distributions on $\tau_c$: (a) diffuse prior (BM-D), (b) informative</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>prior (BM-I). Vertical line denotes the realized value of $\tau_c$ at 6.72 Pa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>in simulations. Truncated normal distribution is parameterized as $T.N.(\mu, \sigma, \tilde{a}, \tilde{b})$.</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Predictions for Smart (1984) observations. Shaded regions in the main plot</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>represent the 68%, 90%, and 95% credible intervals on the predictive</td>
<td></td>
</tr>
<tr>
<td></td>
<td>distribution. Black line (dash-dot linetype) is the NLS prediction. Inset</td>
<td></td>
</tr>
<tr>
<td></td>
<td>plots (a) through (c) correspond to cross-sections of the predictive</td>
<td></td>
</tr>
<tr>
<td></td>
<td>distribution at the observations denoted by black triangles and</td>
<td></td>
</tr>
<tr>
<td></td>
<td>labels 'a'), '(b)', or '(c)'. Inset plots show the predictive distribution</td>
<td></td>
</tr>
<tr>
<td></td>
<td>as a density function, the 0.025 and 0.975 quantiles as black dashed lines,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>and the observed transport value as a solid vertical line.</td>
<td></td>
</tr>
</tbody>
</table>
5.1 The Snake River in Grand Teton National Park. The study area extends from the confluence of the Snake River with Pacific Creek downstream to Deadman’s Bar. Locations of bedload sampling sites are indicated by stars. Modified from Erwin et al. (2011).

5.2 Sediment transport sampling on the Snake River at Deadman’s Bar.

5.3 Potential prior distributions for reference shear. Prior (A) specifies a mean value for reference shear centered on 10 Pa, with some variance; prior (B) assumes that every value between zero and the upper-bound is equally likely; prior (C) gives higher probability to values near the upper limit of the prior distribution. Priors are parameterized according to $T.N(\mu_\tau_r, \sigma_\tau_r, \tilde{a}, \tilde{b})$.

5.4 Prior (dash-dot) and posterior (solid) distribution densities for reference shear and variance at Deadman’s Bar on the Snake River. (A) Reference shear; (B) variance.

5.5 Probabilistic sediment rating curves. (A) Snake River at Deadman’s Bar, (B) Buffalo Fork, and (C) Pacific Creek. The shading (lightest to darkest) represents the 68%, 90%, and 95% credible intervals.

5.6 Flowchart comparing workflows of a forward stochastic model and a Bayesian model for sediment transport. Forward stochastic model workflow goes from (A) to (C) to (E) to (G). The Bayesian model goes from (B) to (C) to (D) to (F) to (H). The ensemble predictions in (G) assume a uniform distribution of $\tau_r$ with upper and lower limits defined by Erwin et al. (2011) shown in Table 5.2 at Deadman’s Bar. (H) PPD for Deadman’s Bar. The density curves shown on the right of (G) and (H) are the distributions of sediment transport at the water discharges denoted by the vertical line (red) on the rating curves. The thin horizontal lines correspond to the 68%, 90%, and 95% prediction intervals.

5.7 Sediment budget using 95% credible interval. (A) Annual sediment yield. Shaded regions represent the 95% credible interval on the posterior predictive distribution of total annual sediment flux. A ‘significant’ difference between the influx and outflux occurs where the regions do not overlap. (B) Cumulative sediment yield starting in 1950. (C) Net change in sediment storage over period of record.
5.8 Sediment budget using 68% credible interval. (A) Annual sediment yield. Shaded regions represent the 68% credible interval on the posterior predictive distribution of total annual sediment flux. A ‘significant’ difference between the influx and outflux occurs where the regions do not overlap. (B) Cumulative sediment yield starting in 1950. (C) Net change in sediment storage over period of record.

5.9 Posterior predictive distributions of annual transport for 1960 and 1984. Dashed lines (red) denote sediment influx, solid lines (black) denote outflux. Vertical lines correspond to the 0.025 and 0.975 quantiles for each distribution.

6.1 Particle size distribution. Compare to Fig. 1 of Wilcock and Crowe (2003).

6.2 Bulk transport. Compare to Fig. 5a of WKC.

6.3 Prior distributions for parameters in transport models.

6.4 Trace plots for J06 sediment. Results are representative of all bulk sediments. Red line represents the prior distribution as specified in Figure 6.3.

6.5 Bulk similarity collapse.

6.6 Bulk rating curves. Red line shows the NLS prediction. Color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.

6.7 Fractional trace plot of $\tau_{r,j}$ for J14.

6.8 Fractional trace plot of $\sigma_{tune,\tau,j}$ for J14.

6.9 Fractional posterior distributions of $\tau_{r,j}$ for J14. Red line represents the prior distribution as specified in Figure 6.3.

6.10 Fractional trace plot of $\sigma_j$ for J14.

6.11 Fractional trace plot of $\sigma_{tune,\sigma,j}$ for J14.

6.12 Fractional posterior distributions of $\sigma_j$ for J14. Red line represents the prior distribution as specified in Figure 6.3.

6.13 Inferred reference stress densities.

6.15 Individual sediment comparisons of original reference stress curves from *Wilcock and Crowe* (2003) to inferred curves from Bayesian Model. Error bars are those from *Wilcock and Crowe* (2003). Credible intervals are color coded from darkest to lightest for 95%, 90%, and 68%. The 0.707 mm value was jittered so as to not overplot for J06. ........................................... 193

6.16 Inferred reference stresses at 68% credible interval. ........................................ 194

6.17 Inferred reference stresses at 90% credible interval. ........................................ 195

6.18 Inferred reference stresses at 95% credible interval. ........................................ 196

6.19 Posterior predictive distribution for J06, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals. ........................................... 197

6.20 Posterior predictive distribution for J14, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals. ........................................... 198

6.21 Posterior predictive distribution for J21, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals. ........................................... 199

6.22 Posterior predictive distribution for J27, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals. ........................................... 200

6.23 Posterior predictive distribution for BOMC, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals. ........................................... 201

6.24 Hiding function using inferred mean surface size reference stresses. .... 202

6.25 Shields stresses plotted against $\tau_{r,sm}$. ........................................... 202

6.26 Shields stresses plotted against $\tau_{r,sm}$ with credible intervals, color coded darkest to lightest for 95%, 90%, and 68%. ........................................... 203

6.27 Fractional similarity collapse. ........................................... 204
1.1 Sediment Transport

The discipline of fluvial sediment transport has been the focus of research for many hundreds of years. Leonardo da Vinci (1452–1519) was likely not the first individual to have been interested in sediment transport, though his words are some of the earliest expressions regarding the phenomenon: “Where the water has least movement, the bottom will be of the finest mud or sand, where the water has a stronger current the shingle is larger” (Simons and Senturk, 1992). Several hundred years later, Pierre duBuat (1734–1809) noted: “The way running water attacks the bed which cannot resist any more, and how the sand is shoved is fascinating indeed, and deserves description” (Dubuat-Nançay, 1786). Indeed, by duBuat’s time, the evolution of sediment transport and river hydraulics had already long begun its transition from art to science. Building on the framework provided by the French and Italian schools of hydraulics, researchers during the 20th century were the first to systematically collect and publish the results of sediment transport experiments. Grove K. Gilbert’s early experiments (Gilbert, 1914), to name one such study, formed the basis for many of the modern mathematical models of sediment transport (Graf, 1971).

Certainly, the motivation for sediment transport research is hardly esoteric. As humans, we have always been and will continue to be reliant on freshwater systems for survival (Vitousek, 1997; Nilsson et al., 2005; Norris et al., 2007). As such, the development of water resources, as well as other natural resources, will continue to not only influence the amount of water flowing in rivers, but will change the
landscape, alter ecosystems, and be the source of contention in a growing world of competing interests. Sediment transport’s strong coupling with water makes it an issue by association. Luna Leopold stated: “Rivers are both the means and the routes by which the products of continental erosion are carried to the oceans of the world” (Leopold 1994) underscoring the fact that rivers are composed of sediment as much as they are of water and in natural systems, where water flows, so will sediment.

Sediment transport is significant for a number of reasons—the sedimentation of dams, accumulation in pipe systems, bridge scour, bank protection, soil conservation, mobilization of contaminated or exogenous sediments, and river restoration all, in one way or another, involve sediment transport. Take, for instance, river restoration. Many of the human-induced disturbances in river systems were intentional changes to the physical conditions of the river, including dams, channelization, riparian corridor development and modification, resource extraction, and introduction of exotic species to the ecosystem. The majority of the significant disturbances, however, were brought about by the inadvertent changes to river systems’ water and sediment regimes caused by imperfect understanding of management consequences (Bisson and Wissmar 2003; Brierley and Fryirs 2005). As a result, significant monetary resources are presently being committed to river restoration—in which sediment transport plays a significant role—in the U.S. and abroad (Bernhardt et al. 2005; Wheaton et al. 2008). While the wide-spread increase in attention in river restoration is certainly encouraging, the restoration community is beset by extensive uncertainty (Graf 2008) which, in many instances, either goes unquantified, unacknowledged, or purposefully ignored (Bisson and Wissmar 2003; Wheaton et al. 2008).
1.2 Uncertainty

The scope of uncertainty in sediment transport is broad. First, there is significant difficulty in obtaining reliable observations of sediment transport due to a number of factors, not the least of which is that sediment transport occurs when the river is flooding thereby necessitating robust measurement techniques and substantial protection for personal safety. Further, the measurement implements themselves may impose bias—for example, operator error or truncation of sample populations—thereby introducing uncertainty (McLean et al., 1999; Diplas et al., 2008). Second, sediment transport is spatially and temporally variable due to constantly changing hydraulic conditions, including changing bed topography, turbulence, varying supplies of bed material from upstream processes, and lastly irreducible noise brought about by stochasticity (Grass, 1970; Kirchner et al., 1990; Gomez and Phillips, 1999; McLean et al., 1999; Bunte and Abt, 2005; Diplas et al., 2008). Third, there is no unified agreement on conceptual models for sediment transport and, as a result, there has been a proliferation of sediment transport relations and equations (Gomez and Church, 1989; Garcia, 2008). Because of this, there is a tyranny of choice when it comes to model selection for the characterization of sediment transport. Further, even after a model is selected, any uncertainty in the model parameters may result in unacceptable uncertainty in predictions due to the inherent nonlinearities of the system (Mueller et al., 2005).

Within this context, there is a demonstrated need to develop a modeling framework for sediment transport that makes it possible to account for the varying sources of uncertainty and provide probabilistic predictions of future transport events. In recent years, Bayesian statistical models have been employed to address these challenges in a wide-variety of disciplines. Researchers in ecology, hydrology, and atmospheric and environmental science have increasingly used the Bayesian
framework to model complex phenomena—for example, distributed rainfall-runoff models, species invasion dynamics, and uncertainty estimation in climate models (Hooten and Wikle, 2007; Vrugt et al., 2008; Cressie et al., 2009; Smith et al., 2009; Renard et al., 2010; Wikle and Hooten, 2010). These methods are also amenable to the process of sediment transport and provide significant benefits over a purely deterministic approach.

1.3 Objectives

The underlying hypothesis of this dissertation is that Bayesian statistical methods present a new opportunity to advance sediment transport modeling, especially in the area of quantifying uncertainty; a problem on which the discipline has been working for some time now. The goal of this research is to incorporate and address many of the elements introduced in the previous section and develop them into several fluvial sediment transport models. In particular, the proposed objectives are:

1. Develop and implement a uni-size Bayesian sediment transport model that, when given transport observations, makes it possible to estimate model parameters as random variables, compare competing deterministic models, and provide probabilistically-based predictions.

2. Compare the developed Bayesian sediment transport model to traditional nonlinear regression techniques which may be used in parameter estimation and prediction. This will discuss the statistical considerations and underlying philosophical considerations that are addressed by the two methods.

3. Apply the uni-size sediment transport model developed previously to make inference and prediction using observations collected by Erwin et al. (2011) on
the Snake River in Idaho. Compare predictive distribution obtained through the Bayesian model to the uncertainty envelopes—which will have used the same data as for the Bayesian model—developed in Erwin et al. (2011). Then, construct a probabilistic sediment budget that recognizes the uncertainty communicated in the posterior predictive distributions.

4. Develop, implement, and test a multi-fraction Bayesian sediment transport model that, when given transport observations, makes it possible to estimate model parameters as random variables and compare the Bayesian model results to current sediment transport theory.

References


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2.1 Background and Literature Review

2.1.1 Sediment Transport

For the purposes of this research, the type of sediment transport that will be addressed is that of the non-cohesive grains that are transported as bed load in laboratory flumes and river systems. Figure 2.1 is provided as a schematic of this process and illustrates the velocity profile of the overlying water, along with the saltation of individual particles along the bed surface.

The mathematical formulations that we use to describe the process of sediment transport are, in varying degrees, simplifications of reality. Such simplifications include models that are uni-size (grains which are uniform in size), single-fraction (a distribution of grain sizes is modeled as a single, aggregate size distribution), two-fraction (a distribution of grains is binned into fine or coarse fractions), and multi-fraction (a grain distribution is binned into more than two size fractions). Often, the overlying flow conditions are determined at steady-state conditions using time- and depth-averaged observations of velocity. Further, many of the models used to predict or explain sediment transport are comprised of latent parameters—values that cannot be measured directly but must be inferred from observation. Further, these models are generally semi-empirical—that is, they have components from both physics and experimentation to arrive at the final formulation. Thus, when it comes time to model sediment transport, one must first decide which complexities are insignificant—that is, to say which model simplifications will yield acceptable
approximations of reality—and then select reasonable parameter values as required by the preferred model. This process exemplifies two challenges related to any branch of mathematical modeling, sediment transport included; one must determine, first, how appropriate the selected model is and second, determine appropriate values for model parameters. A third challenge, relating to latent model parameters, is that these must be inferred from imperfect observations. This is further compounded because sediment transport is a highly nonlinear system, therefore small errors in parameter estimates can lead to large predictive errors.

The field of sediment transport has historically been focused on explaining transport events using physics-based arguments. As a result, the use of statistical methods in this discipline has lagged behind other scientific areas (Clement and Piegay, 2005). This emphasis on deterministic relationships has resulted in a literature that is full of competing conceptual models, not one of which works well in all situations. In fact, the proliferation of conceptual models for sediment transport is linked to this issue—when given a scenario to model, if the existing relationships do not provide a satisfactory fit, then new relationships that fit the current problem are developed for the situation at hand. The result is a discipline that has no uniformly accepted governing relationship (Gomez and Church, 1989).

One of the challenges in developing purely deterministic models for complex systems, like sediment transport, is that deterministic approaches generally do not provide avenues to account for uncertainty and variability. Thus, it is not inconceivable that some of the models that are developed are too concerned with fitting to the noise rather than developing methods to account for deterministic governing relationships in the presence of stochasticity. Regarding the notion of strict determinism in sediment transport, Gomez (1991) noted that “reluctance to acknowledge that bedload transport is inherently unstable, rather than a lack of
fundamental knowledge per se, is adduced to be a principal factor in limiting progress.” It has long been known that sediment transport is highly-variable, that deriving from the multiplicity of different bed configurations and sediment packing conditions \cite{Kirchner et al. 1990}. Thus, if the conditions that give rise to sediment transport are stochastic and random in nature, then the models that describe transport should make in-kind accommodations.

In most applied settings, sediment transport models are employed using fixed parameters—that is, strictly deterministic functions that have fixed parameter values are used to make predictions. The result of these predictions is that, for a given discharge, there is one, and only one, predicted value. As will be discussed in a later chapter, the notion of a point prediction is analogous to a 0% confidence or credible interval. There is a significant literature documenting the distribution of sediment transport rates for fixed conditions \cite{Grass 1970, Gessler 1971, Paintal 1971, Kirchner et al. 1990, Gomez and Phillips 1999} so the continued use of purely deterministic functions and fixed parameter values may simply be a symptom of insufficient computational frameworks in which sediment transport predictions can be carried out.

### 2.1.2 Approaches to Sediment Transport Modeling

There have been numerous approaches offered over the last 60 years whose aims were to capture the stochasticity of the sediment transport process. Most notable was that of \cite{Einstein 1950} who described bed load as discrete hops that occur with some probability in relation to the instantaneous lift forces imposed by the overlying flow. The instantaneous lift force—a function of the fluid turbulence—was given a normal distribution about some mean lift force, and this was incorporated into the bed load function to calculate the probability of entrainment. This work gave way
to much research in the field of stochastic sediment transport. \cite{Shen1980} provided a review of the work done up to that point on stochastic transport modeling and mention probabilistic approaches for suspended sediment concentration distributions, homogeneous Poisson process models for particle entrainment, exponential distributions used to describe step lengths and resting periods between entrainment events, and others.

While a majority of the approaches to sediment transport have been purely deterministic, there are several approaches whose goal was to analytically determine a probability density function for sediment transport. \cite{Hamamori1962}, for instance, developed a probability distribution for bed load transport in bedform-dominated systems. Recognizing that, even at steady conditions, bed load transport fluctuates widely, Hamamori derived a distribution for bed load using physical and probabilistic arguments. \cite{Turowski2011} revisited Hamamori’s work, re-deriving the distribution under slightly different assumptions and arrived at a distribution that was tested against gamma, exponential, and a distribution function derived using Rayleigh-distributed ripple heights. \cite{Turowski2010} discussed another approach in which the distribution of bed load was constructed using statistical arguments. Instead of appealing to a physics-based approach, Turowski considered a distribution of waiting times between particle arrivals and used this to derive a probability density for bed load transport rates at a cross-section. Depending on the assumed structure of the waiting times, the final distribution was either a Poisson, exponential, or Birnbaum-Saunders distribution.

Sediment transport models have also been constructed using machine learning algorithms (\cite{Bhattacharya2007, Sasal2009}). These algorithms differ from the approaches mentioned to this point in that they seek to obtain the best possible fit to the observed transport rates without respect to any physical or
probabilistic arguments. Given a set of independent variables (such as discharge, velocity, grain size, etc.) machine learning algorithms try on different combinations that result in the lowest objective function value. In effect, machine learning algorithms estimate the underlying process. While machine learning algorithms often result in good agreement between the model and observations, it is at the cost of parameter interpretability. Because the inferred process is derived empirically from the independent variables and observations, learning about parameters, such as critical shear for example, is confounded. Further, Gomez and Church (1989) noted that most of the “independent” variables used in sediment transport relations are highly-correlated, thus machine learning algorithms may employ orthogonal transformations—similar to principal components analysis (PCA)—to create a set of uncorrelated forcing variables. In the case of orthogonal transformations, the resulting model has lost nearly all parameter interpretability.

Asselman (2000) noted that most approaches to fitting sediment rating curves involve power laws—\( Y = \theta_1 Q^{\theta_2} \), for example—where the response, \( Y \) is a nonlinear function of discharge, \( Q \), and some latent parameters, \( \theta_1 \) and \( \theta_2 \), whose values generally hold no physical meaning and whose estimation usually involve some type of least squares criterion. Despite demonstrating good agreement with observed values, statistical approaches to sediment transport have been criticized as being black-box methods (Asselman, 2000; Clement and Piegay, 2005) because model parameters, such as those in a power-law relationship or machine learning approaches, have—in a physical sense—uninterpretable parameters. Linear and nonlinear regression, however, have been employed in sediment transport frequently. For example, Weber et al. (1976) compared four different linear regression models for sediment transport and discussed the validity of such models based on how well the model residuals satisfy the underlying methodological requirements; Vericat and
Batalla (2005) used a power-law relationship as shown above to predict suspended sediment concentrations in the Ebro River in Spain; and Vericat and Batalla (2006) used ordinary least squares linear regression to develop their rating curves. The relationships proposed in Vericat and Batalla (2006) and the associated parameter inference regions, however, are questionable since after inspection of the regression equations and fitted models, it is very likely that the underlying assumptions and requirements model residuals for OLS regression are not met—a general problem that was noted by Weber et al. (1976).

While other statistical methods (such as those mentioned in the preceding paragraphs) have been explored, Bayesian statistical models have been employed in only a few instances. Griffiths (1982) first proposed using Bayesian inference models for empirical modeling problems in geomorphology—including sediment transport—but only discussed fully conjugate models in which the posterior distributions could be derived analytically through arguments of calculus. Indeed, more complex models were unattainable due to the unavailability of numerical methods to perform the calculus. It was not until the papers of Geman and Geman (1984) and Gelfand and Smith (1990) that Markov Chain Monte Carlo (MCMC) methods became widely available and useful to Bayesian statistical analysis. Since these computational advances, a wide variety of problems have been modeled using Bayesian methods.

To date, there are only a few instances in which Bayesian statistical models have been used in sediment transport applications. Kanso et al. (2005) employed Bayesian statistics in parameter estimation related to sediment erosion in Parisian sewers, and Wu and Chen (2009) provided an example of how Bayesian methods can be utilized in sediment entrainment problems. Wu and Chen (2009) also noted the sparse use of Bayesian and MCMC applications in hydraulic engineering and
conjectured that this is due, in large part, to a lack of demonstration as well as the foreign resemblance it bears to traditional deterministic approaches commonly employed by engineers in parameter estimation problems. Another paper (Ruark et al., 2011) passively mentioned the possibility of using a Bayesian approach to sediment transport but did not cite to any work done in this area by either themselves or others. Henderson and Bui (2005) generally discusses the problems of sediment transport modeling and uncertainty and then discuss several potential methods to address these issues—one of these is a Bayesian model—though they do not develop an actual model. Sabatine (2011) evaluated a Bayesian approach for multi-objective optimization problems by using the same dataset as Ruark et al. (2011) and found the Bayesian approaches to be superior for more accurate predictions attributable to the model averaging code they applied, something they evaluated using the Bureau of Reclamations SRH-1D code. Sabatine (2011) is now under review in River Research and Applications for which I served as a peer reviewer. In a closely related field, there are instances of Bayesian statistical models being used for watershed-scale landscape erosion models, Fox and Papanicolaou (2008) is one such example, but these landscape process models are fundamentally different than fluvial sediment transport processes in that they characterize a different transport process.

### 2.1.3 Bayesian Statistical Models and Sediment Transport

In considering the possibility of using Bayesian statistics in sediment transport modeling, one could justify this in a number of ways. The first justification is that the literature indicates very few people have considered the possibility and fewer have actually implemented a model. Thus, exploring this possibility is simply new territory. A second and more satisfying justification for exploring Bayesian sediment
transport models is that they have numerous desirable properties that may make them very useful to the disciplines of geomorphology and fluvial engineering because this approach addresses a number of problems and issues that have been identified in the literature over the years.

To begin, sediment transport modeling involves the estimation of model parameters. Excess shear models, for example, require the specification of the critical Shields number or reference shear—quantities that cannot be directly measured (Wilcock, 1988). Traditional approaches to modeling transport entail the point-estimation of model parameters; that is, each parameter has a fixed unknown value and the point estimate is a data-based approximation of this value. Mueller et al. (2005), however, noted that uncertainties in specifying a single critical shear can lead to large errors because the process is nonlinear. Further, Buffington and Montgomery (1997) discussed the problems with specifying a single value for critical shear. In their paper, Buffington and Montgomery (1997) compiled data from eight decades of incipient motion studies and created lists of reported critical shear values. They noted that the earlier researchers in the field (Grass, 1970; Gessler, 1971; Paintal, 1971; Lavelle and Mofjeld, 1987) acknowledged that the threshold at which sediments move is “inherently a statistical problem” and asserted that a frequency distribution of critical shear values for any given grain size is more appropriate than any single value. They suggested that more emphasis should be placed on choosing defendable parameter values for particular applications, given the observed data and known sources of uncertainty. Further, the idea of critical shear has been the subject of criticism in the past. Lavelle and Mofjeld (1987) questioned the idea of whether or not a single threshold exists at all citing to the fact that stochastic aspects of transport lead us to reason that the critical shear concept will not be sharply defined and that it is most appropriately described using
statistical distributions. A Bayesian statistical framework for sediment transport not only provides a formal and robust method to estimate model parameters, it treats these parameters as random variables arising from distributions, thereby satisfying a very fundamental philosophical consideration of sediment transport.

The Bayesian estimation of these parameters involves specifying prior information for each parameter. Prior distributions, as will be discussed in the Methods section of this chapter, can range from being highly-informative to vague or diffuse, depending on the expert opinion of the modeler. Prior information on these parameters is incorporated into the parameter estimation procedure and is weighed against the information available in the observations—the result is a posterior distribution (the “answer” in the Bayesian formulation) for these parameters. The posterior distribution is a weighted combination of the parameter information provided by the prior distributions and the information available in the observed values. In words, the posterior distribution represents the most likely parameter values given the observations of the process. Because the posterior distribution describes the joint distribution of all the model parameters, it is possible to “partition-out” variability contributions to multiple sources by viewing the marginal distributions—thus, the variability in predictions due to inferred distribution of critical shear can be separated from the variability in predictions due to other sources, such as measurement error, model misspecification, and random noise.

Next, measurements of sediment transport are not only difficult and expensive to make, but are also prone to substantial uncertainty and variability\textsuperscript{1} (Wilcock, 2001; Bunte and Abt, 2005; Gaeuman et al., 2009). In addition to measurement error, stochasticity is inherent in natural systems.\textsuperscript{[1]} Poff et al. (1997) discussed how natural variability is crucial in the proper functioning of ecosystems and cited the

\textsuperscript{1}Care should be taken to make a distinction between variability and uncertainty—this will be discussed in the following paragraph.
importance of natural variability of streamflow on river ecosystem health. Thus, variability is an intrinsically important characteristic and should be considered when modeling natural systems (Clement and Piegay, 2005). Bayesian approaches allow for the latent process and observations to each have quantifiable variability, thereby providing an idea of how much variability comes from each component. In natural river systems, sampling bed load transport is difficult and likely to have considerable spread, thus methods used for making inference from these observations should account for this variability and uncertainty.

Much of the work in the hydrologic and sediment transport literature often refer to process and parameter uncertainty. While viewing sediment transport through the lens of “uncertainty” captures the net effect, it is not entirely accurate. For instance, previously-cited literature indicate that model parameters, critical shear for example, are best described as random variables and not as fixed values. Thus, the spread of possible critical shear values does not represent parameter uncertainty, it represents parameter variability. In similar fashion, variability in sediment transport observations is known to exist in flume experiments, even under steady conditions (Knighton, 1988; Hicks and Gomez, 2005; Turowski, 2010). As practitioners, we recognize our inability to perfectly measure sediment transport, but even if we were able to make perfect observations, variability (and not uncertainty) would still exist due to the nature of the process. Because of this, it is easy to see why traditional approaches to hydrology and sediment transport explain variability under the name of uncertainty—traditional approaches, philosophically, assume that the underlying process parameters are fixed, thus any variation from this fixed value is not a measure of the intrinsic variability of the parameter, but of the uncertainty of the parameter estimate. In this way, a Bayesian approach to sediment transport is much more philosophically sound than traditional fixed-value
methods.

Another consideration that reflects favorably on a Bayesian approach is that traditional curve-fitting approaches do not necessarily discriminate errors due to measurement and those due to inaccurate process or conceptual models (Cressie et al. 2009). In the statistical and hydrologic modeling literature, there is a distinction between structural, parameter, and input uncertainty (Cressie et al. 2009; Freni and Mannina 2010; Renard et al. 2010). The structural, or process-model, uncertainties involve the conceptual and deterministic models that are used to calibrate and predict transport. Stedinger et al. (2008) stated that “structural errors (or equivalently model errors) describe the inability, of even the best model with optimal parameters, to exactly reproduce the target output.” Kuczera et al. (2006) noted that predictive uncertainty in rainfall-runoff models is often dominated not by parameter uncertainty, but by model misspecification. In sediment transport, numerous bed load relations have been suggested [see, for example, Gomez and Church (1989) or Garcia (2008)], thus there is a need to select one relationship over another, and therefore some degree of uncertainty and judgment are associated with this selection. Sediment transport implemented in a Bayesian and hierarchical Bayesian framework provides a means to rigorously discriminate between competing process models and make a distinction between parameter variability and process model misspecification. One such method is to compute the Deviance Information Criterion (DIC) (Spiegelhalter et al. 1998). Similar in spirit to Akaike’s Information Criterion (AIC) and Bayesian Information Criterion (BIC) for classical statistical models, DIC can be used as a relative metric of the goodness of model fit for the number of parameters required to obtain that fit. Thus, given a set of observations, one can calculate DIC values for each of the competing process models to get a quantification of which model provides the best
fit to the observations. Other methods also include hierarchical Bayesian models 
(Gelman et al., 2004) (which provide a means to estimate model variance separately 
from observation variance) and mixture models (Nippert et al., 2010).

In making model predictions, Wilcock (2001) noted that sediment transport 
predictions based solely on formulae are notoriously inaccurate. This predictive 
uncertainty problem is related to the first three issues mentioned above, namely, 
estimation of parameter values (which, in the absence of data is entirely based on 
expert judgment), measurement error, and model misspecification. As an alternative 
to traditional deterministic prediction—which generally only yield a line through a 
cloud of observations—predictive probability distributions can be calculated using 
the posterior distribution obtained in a Bayesian model. The predictive 
distributions differ from traditional deterministic predictions in that for a given set 
of forcing conditions, there is a distribution of transport rates; this predictive 
behavior is consistent with sediment transport observations even under steady-state 
conditions (Knighton, 1988; Hicks and Gomez, 2005). This posterior predictive 
distribution (PPD) is, in essence, analogous to the analytically-derived distributions 
proposed by Hamamori (1962) and Turowski (2010), but arises naturally from the 
prior parameter distributions, the model likelihood, and the transport observations.

In addition to making robust predictions that incorporate uncertainty and 
variability, the Bayesian approach being proposed for this dissertation makes it 
possible for us to learn about meaningful model parameters. Recall the brief 
discussion on machine learning algorithms in which a vector of observations are 
specified in conjunction with a matrix of independent variables. The goal of a 
machine learning algorithm is to, using the independent variables, divine some sort 
of relationship that results in good agreement between predictions and observations. 
The combination of variables, and the parameters associated with these variables,
do not necessarily hold any physically-relevant meaning. One advantage of using the Bayesian framework proposed herein is that physically-based (i.e., deterministic) models can be incorporated into the probabilistic framework. So, if it is believed that the equation \( y = \theta_1(\theta_2 - \theta_3)^{\theta_4} \) describes the process being modeled, it can be incorporated into the analysis and inference on the important parameters may be obtained.

Upon building a model for some process, the question of how well the model fits the observations as well as how reasonable the inferred parameters are often arises. In classical inferential systems (nonlinear least squares, linear regression, etc.) model fits and inferences must always be checked against underlying statistical assumptions, usually transformable normality of residuals, residual mean of zero, and residual homoscedasticity. It is entirely possible that a seemingly “good” fitting relationship has invalid parameter inference regions due to underlying statistical model assumptions not being satisfied. In classical statistical methods, this can be an insidious problem. While Bayesian analysis does not guarantee inferential and predictive success, failure of a model is due to the inadequacy of the model itself and not of the underlying method. For example, a model that infers parameter regions that are unreasonable may be caused by the incorporation of an inappropriate deterministic relationship or likelihood function. Regarding this, Box and Tiao (1992) stated that, in Bayesian models, “...inferences that are unacceptable must come from inappropriate assumption and not from inadequacies of the inferential system. Thus all the parts of the model, including the prior distribution, are exposed to appropriate criticism.”

Historically, classical nonlinear models have been calibrated using nonlinear regression tools. These approaches assume that model parameters—or some

\[ \text{It is not inconceivable, however, that such methods may force us to consider previously unrecognized relationships.} \]
transformation thereof—are normally-distributed and seldom does one see the possibility to use another type of distribution. Further, these models also assume that the underlying parameter values are fixed—as was shown above, this assumption is inappropriate for critical shear. Bayesian methods make it possible to relax the normally-distributed assumption on the parameters as well as to provide a formal mechanism to incorporate prior information and expert knowledge into the model. Box and Tiao (1992) noted that using statistics derived from classical sampling theory assumes “that the probability distribution of the data was exactly Normal, and that each observation had exactly the same variance, and was distributed exactly independently of every other observation.” For this reason, residual diagnostic checks are vital to assessing model validity. They continued: “If non-Normality was suspected, for example, it might be sensible to postulate that the sample came from a wider class of parent distributions of which the Normal was a member. The consequential analysis could be difficult via sampling theory but is readily accomplished in a Bayesian framework.” Bayesian methods require prior information on the parameters in order to make inference. Some view the incorporation of prior information into an inferential model as a source of bias because the model parameters are assigned a distribution at the discretion of the modeler. But, as was mentioned in the quotations above, classical approaches to parameter estimation have assumptions as well, often directly related to the assumed parameter distributions. Unlike a traditional approach where the parameters are assumed to be normal, the parameter values can be assigned a distribution by scientific considerations—Box and Tiao (1992) stated: “Because this system of inference may be readily applied to any probability model, much less attention need be given to the mathematical convenience of the models considered and more to their scientific merit.”
Modeling sediment transport using Bayesian statistics also makes it possible to evaluate the optimality of decisions. Decision theory is a branch of statistics that addresses how to make optimal decisions in the face of uncertainty or risk, while recognizing the cost of decisions. Because Bayesian methods have a strong decision theoretic basis (Robert, 2007) it is entirely reasonable to expect that decision theory can help inform river management decision and engineering design. For example, Lopez and Garcia (2001) illustrated that, if the critical shear stress distribution, along with the the flow-induced wall shear stress distribution are known, that a risk function can be written for erosion. In decision theory, risk functions are a quantification of the average loss of a given decision (Casella and Berger, 2002). Davis et al. (1972) defined risk as the “...consequential effect of possible uncertain outcomes.” In this context provided a quantification of this consequential effect on erosion predictions given the uncertainty of parameter values of incipient motion. While demonstrated the utility of such an approach, they assumed that the underlying probability functions were known. A Bayesian approach to sediment transport would allow those distributions to be accurately characterized based on expert opinion and observation and would provide the information needed to carry out the risk-based approach described in Lopez and Garcia (2001).

Davis et al. (1972) provided an example of how Bayesian decision theory can be applied to design problems in hydrology. They outlined a procedure for designing a flood levee that accounted for the cost of over-design as well as for the cost of a system failure in the case of an inadequate design. Their reasoning for adopting a decision theoretic approach to engineering design was in part due to the structural and parameter uncertainty that is present in the predictive models, as well as, what the authors referred to as “the vagaries of nature.” The design in Davis et al. (1972) minimized a goal function, which represented the expected cost of flood
damage, plus the annual costs of flood protection, minus the benefits associated with designing the structure. From this goal function, they made the decision that minimized the Bayes risk. Duckstein and Szidarovszky (1977) also used Bayesian decision theory to design a reservoir under random sediment yield. Using Bayesian methods to model the rainfall-runoff relationships, and using a universal soil loss equation, the authors illustrated, in a similar way to Davis et al. (1972), how a Bayesian approach can help inform the optimal decision in reservoir design.

The analyses illustrated by Davis et al. (1972) and Duckstein and Szidarovszky (1977) could be applied to the field of sediment transport in a number of ways. For instance, in river restoration design, it is often the case that the river reaches to be restored are located downstream of a dam. Dams cause a loss of sediment supply to the downstream reaches, so in order to maintain wildlife habitat and avoid system degradation, the sediment supply is augmented by river managers. Because sediment augmentation involves estimating how much sediment will be mobilized at a given flow (which the literature tells us is a distribution of values, and not a fixed value) a Bayesian sediment transport model has the potential to be the foundation upon which loss and risk functions are constructed and optimal decisions regarding flood releases and sediment quantities are based. It has been illustrated on the lower Colorado River in Arizona, as well as on the Trinity River in California, that sediment augmentation, which is costly due to (1) the cost of stockpiling sediment for injection during flood flows, and (2) the revenue lost to power companies operating the dams where spring flood releases bypass the power generation facilities, is an important management strategy in impaired river systems. Thus, being able to provide an optimal decision that factors in the “vagaries of nature”, the uncertainties of sediment transport theory, and the costs associated with varied management decisions has the potential to be very valuable.
2.2 Methods

A brief overview of Bayesian statistics is provided in what follows as a general description of the methods that will be used to accomplish the stated objectives.

2.2.1 Bayesian Statistical Model Fundamentals

Bayesian statistical modeling is derived from Bayes’ rule of conditional probability, which is:

\[
P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}. \tag{2.1}
\]

The events \(B_1,...B_k\), are partitions of the sample space. The posterior probability of an arbitrary event, \(B_j\), is conditional on some event \(A\). The expression found in (2.1) is often referred to as Bayes’ Theorem. The notation ‘\(P(B_j|A)\)’ denotes the probability of the event, \(B_j\) given the event \(A\). Over a continuous joint distribution, Bayes’ Theorem becomes:

\[
[x|y] = \frac{[y|x][x]}{\int[y|x][x]dx} = \frac{[y|x][x]}{[y]}, \tag{2.2}
\]

where the denominator is a normalizing constant. For the continuous case, the bracket notation ‘[ ]’ denotes a probability distribution—\([x|y]\) is the conditional probability distribution of \(x\) given \(y\). Alternately, (2.2) can be written:

\[
[x|y] = c[y|x][x], \tag{2.3}
\]

where \(c = (\int[y|x][x]dx)^{-1}\). The normalizing constant, \(c\), can be dropped altogether since \(c = \frac{1}{[y]}\) which, if \(y\) is known (which is a valid assumption since, as will be explained shortly, \(y\) represents observations), is fixed and the relationship can be
expressed as a proportionality:

\[ [x|y] \propto [y|x][x]. \]  \hspace{1cm} (2.4)

To illustrate the utility in the context of applied problems, let us assume process \( z \) is a function of some parameter \( \theta \). A Bayesian model can be written as:

\[
\begin{vmatrix}
\theta \\
\end{vmatrix}
\propto
\begin{vmatrix}
\theta \\
\end{vmatrix}
\begin{vmatrix}
z \\
\end{vmatrix}
\begin{vmatrix}
\theta \\
\end{vmatrix}.
\]

Were we to observe, or collect data, on the process \( z \), the model in (5.1) tells us that it would be controlled by some underlying parameter \( \theta \). The posterior distribution, \( [\theta|z] \), describes the distribution of values that \( \theta \) can assume, given the observations of \( z \). It is the posterior distribution that allows us to make inference on the model parameters and thereby establish credible intervals of values for \( \theta \). The right-hand side of (5.1) is comprised of two parts: the likelihood and the prior. The likelihood, \( [z|\theta] \)—also referred to as the data model—describes the distribution of the observations, given \( \theta \). The likelihood should describe the structure of the process, \( z \), given the fact that it is controlled by a parameter \( \theta \). Lastly, the prior distribution, \( [\theta] \), encompasses what is known about \( \theta \) before considering our new observations—it can be the summary of all previous research in the literature on \( \theta \), it can be an educated guess, or it can simply limit possible ranges of values for \( \theta \). For example, if we know that the parameter \( \theta \) cannot take on values less than or equal to zero or greater than some value, \( \theta_{\text{lim}} \) then we can impose these constraints in our prior. Alternately, if the literature indicates that \( \theta \) can take on any value (positive or negative), then perhaps we give it a normal distribution centered around what we think the mean should be. In specifying a prior distribution, the scientist/engineer
has the ability to inform the model—indeed of the newly gathered observations—with some degree of specificity.

This basic example can be extended to include a vector of parameters, $\theta$, such that:

$$[\theta|z] \propto [z|\theta][\theta], \quad (2.6)$$

provided that either an appropriate multivariate prior, or individual prior distributions are specified for each element in $\theta$.

2.2.2 Numerical Methods

The objective of Bayesian modeling is to update our knowledge regarding the selected process by evaluating the posterior distribution and marginal posterior distributions. To calculate these distributions, however, requires the knowledge of the normalizing constant expressed in $[2.3]$. For all but the simplest models, the determination of this normalizing constant is intractable and another method must be used. Markov Chain Monte Carlo methods provide a means to simulate realizations from a target (the posterior) distribution, thereby circumventing the need to analytically solve for the normalizing constant. Markov Chain Monte Carlo has been adopted by Bayesian statisticians starting with Gelfand and Smith (1990) because it allows sampling from otherwise intractable mathematical expressions for the posterior distribution. More detailed introductions to MCMC are provided in Gelman et al. (2004) and Robert (2007).

In particular, a combination of Gibb’s sampling (Casella and George, 1992) and a Metropolis-Hastings sampling (Metropolis et al., 1953; Hastings, 1970; Chib and Greenberg, 1995) will provide a means to numerically solve for the posterior distributions of interest.
All of the code required to develop these models will be written in the R programming language (R Development Core Team, 2010).

References


Fig. 2.1: Schematic of bed load transport
Abstract

Fluvial sediment transport studies have long underscored the difficulty in reliably estimating transport model parameters, collecting accurate observations, and making predictions due to measurement error, natural variability, and conceptual model uncertainty. Thus, there is a need to identify modeling frameworks that accommodate these realities while incorporating functional relationships, providing probability-based predictions, and accommodating for conceptual model discrimination. Bayesian statistical approaches have been widely used in a number of disciplines to accomplish just this, yet, applications in sediment transport are few. In this paper we propose and demonstrate a Bayesian statistical approach to a simple sediment transport problem as a means to overcome some of these challenges. This approach provides a means to rigorously estimate model parameter distributions, such as critical shear, given observations of sediment transport; provides probabilistically-based predictions that are robust and easily interpretable; facilitates conceptual model discrimination; and incorporates expert judgment into model inference and predictions. We demonstrate a simple uni-size sediment transport model and test it against simulated observations for which the ‘true’ model parameters are known. Experimental flume observations were also used to assess the proposed model’s robustness. Results indicate that such a modeling...
approach is valid and presents an opportunity for more complex models to be built in the Bayesian framework.

3.1 Introduction

Among the challenges communicated in the literature regarding sediment transport, dealing with uncertainty is a recurring theme. The scope of this uncertainty encompasses many aspects, including our ability to make accurate measurements (Bunte and Abt, 2001), the effect of natural variation or stochasticity on the process (Singh et al., 2009), as well as our ability to distill the true underlying relationship that governs the complex phenomenon into a conceptual model (Gomez and Church, 1989). Because of these uncertainties, there is a need to identify a framework in which sediment transport models can accommodate notions of uncertainty, variability, and error.

In recent years, Bayesian statistical models have been employed to address these challenges in a wide-variety of disciplines. Researchers in ecology, hydrology, and atmospheric and environmental science have increasingly used the Bayesian framework to model complex phenomena—for example, distributed rainfall-runoff models, species invasion dynamics, and uncertainty estimation in climate models (Hooten and Wikle, 2007; Vrugt et al., 2008; Cressie et al., 2009; Smith et al., 2009; Renard et al., 2010; Wikle and Hooten, 2010). Despite the mature models described in the hydrology and other literature, Bayesian methods have not been widely applied to sediment transport problems. This modeling framework was first mentioned by Griffiths (1982) in the context of geomorphology and has since been employed by only a few, including Kanso et al. (2005), Fox and Papanicolaou (2008), and Wu and Chen (2009). Wu and Chen (2009) also recognized the limited use of Bayesian methods for sediment transport problems and supposed that that
this may be due to a lack of demonstration to the sediment transport community.

3.1.1 Determinism and Statistics

For proper context, it is helpful to draw a distinction between purely deterministic and traditional and Bayesian statistical methods. Purely deterministic methods (in the sense of LaPlacian determinism (de LaPlace, 1825) assume that any functional or physically-based relationship used in an analysis is completely representative of the system and that any associated parameter values are fixed and known. Regarding our ability to discern this true underlying process, George Box stated that “all models are wrong, but some are useful.” Modern statistical methods (traditional and Bayesian alike) recognize this and provide means to make predictions and inference using noisy observations and imprecise conceptual models.

Bayesian models, like traditional methods such as maximum likelihood and regression techniques, are robust statistical methods that provide a means for parameter inference and prediction. When employed in very simple scenarios, such as the current sediment transport model, both approaches are likely to yield convergent solutions (see Chapter 4 of this dissertation). As deterministic models become more involved—the number of latent parameters increase and/or nested functions are employed—Bayesian models constructed hierarchically provide elegant and robust solutions, as evidenced by the number of applications in the hydrology literature in recent years. In this paper we provide a simple example of how the Bayesian framework can be employed in sediment transport and discuss several of the motivating factors for why they should be employed. Our thesis is that the simplicity of the proposed model will provide a demonstrated foundation upon which future sediment transport models that incorporate increased complexity may be built.
3.1.2 Challenges in Sediment Transport

Sediment transport is faced with many of the same challenges faced by the hydrology community, which has enjoyed wide use of Bayesian methods. Below, we outline a few of these challenges as they relate to the field of sediment transport that can be addressed through Bayesian methods.

Like other disciplines, sediment transport modeling involves the estimation of model parameters. For instance, many transport models require the specification of the Critical Shields number—a quantity that cannot be directly measured (Wilcock, 1988). Deterministic approaches to modeling transport entails the point-estimation of model parameters. Mueller et al. (2005), however, noted that uncertainties in specifying a single critical shear can lead to large errors since the process is nonlinear. This estimate can be made using statistical methods, but it is not uncommon to adjust it by eye (Wilcock, 1988; Wilcock and Crowe, 2003; Mueller et al., 2005; Gaeuman et al., 2009; Erwin et al., 2011) due to some nonlinear regression techniques giving too much weight to observations far from critical [see Wilcock (1988) for a discussion on this]. Additionally, Buffington and Montgomery (1997) compiled data from eight decades of incipient motion studies and created lists of reported critical shear values. They noted that the early researchers in the field (Shields, 1936; Einstein, 1950; Grass, 1970; Gessler, 1971; Paintal, 1971) acknowledged that the threshold at which sediments move is “inherently a statistical problem” and asserted that a frequency distribution of critical shear values for any given grain size is more appropriate than any single value (the physical basis for which is described in Buffington et al. (1992) and Johnston et al., 1998). Lavelle and Mofjeld (1987) questioned the idea of whether or not a single threshold exists at all and stated that stochastic aspects of transport lead us to reason that the critical shear concept will not be sharply defined and that it is most appropriately described
using statistical distributions. Critical shear will not be sharply defined in applied settings because of constantly changing hydraulic conditions, including changing bed topography, turbulence, varying supplies of bed material from upstream processes, and lastly irreducible noise brought about by stochasticity (Grass, 1970; Kirchner et al., 1990; Knighton, 1998; Gomez and Phillips, 1999; McLean et al., 1999; Bunte and Abt, 2005; Hicks and Gomez, 2005; Diplas et al., 2008).

Making observations of transport events is also problematic. In addition to the expense and difficulty in making sediment transport observations, measurements of sediment transport are prone to substantial uncertainty (Gaeuman et al., 2009; Wilcock, 2001). Variation is inevitably caused by sampling error as well as natural stochasticity. Variability, distinct from uncertainty, is an intrinsically important characteristic of natural systems and should be considered when modeling these systems (Clement and Piegay, 2005).

Recent work in hydrology underscores the difficulty of characterizing structural uncertainty (Renard et al., 2010), uncertainty due to model lack of fit and the same holds for sediment transport. Numerous bed load sediment transport relations have been suggested [see, for example, Gomez and Church (1989) or Garcia (2008)], thus there is a need to select one relationship over another, and therefore some degree of uncertainty and judgment are associated with this selection.

Lastly, sediment transport is known to be variable even under steady-state conditions (Knighton, 1998; Hicks and Gomez, 2005; Turowski, 2010), and predictions that are based on a purely deterministic framework do not necessarily account for this variability. In this case, predictive distributions with quantifiable probabilities are more appropriate than a single line fitted through a spread of observations.
3.2 Purpose and Objectives

The goal of this research is to incorporate and address many of the elements introduced in the previous section and develop them into a fluvial sediment transport model as a demonstration of the Bayesian statistical method applied to sediment transport problems, which can then be used as a foundation on which to build more complex models. In particular, our objectives are to:

1. Demonstrate the development and implementation of a Bayesian sediment transport model that, when given transport observations, makes it possible to:
   (i) estimate a credible interval (the Bayesian analog to confidence intervals, which is interpreted as the probability the realized parameter value is found in the specified interval) for the critical shear parameter, $\tau_c$; (ii) estimate a credible interval for the variance parameter, $\sigma^2$, a measure of model misspecification, measurement error, and random variation; (iii) provide sediment transport predictions delineated in credible intervals; (iv) compare different process models for fit via quantitative metrics; and (v) be easily extended or generalized to accommodate more complex descriptions.

2. Perform simulation studies to evaluate the proposed framework. Specifically:
   (i) simulate synthetic data according to established transport relationships with multiplicative noise according to $\sigma^2$, (ii) validate the model—verify that the model can recover the parameters that were specified when the synthetic data were generated, and (iii) explore the effect of various specifications for prior information on model inference.

3. Evaluate the model using observed transport data from flume studies with uni-size sediment. (i) estimate model parameters, $\tau_c$ and $\sigma^2$, (ii) evaluate
different process models, and (iii) provide a sediment rating curve in terms of credible intervals.

3.3 Formulation of Bayesian Model

Though Bayesian methods have appeared the broader literature, we take a pedagogical approach here with a specific focus on sediment transport. To clearly explain and illustrate how such a model is developed and implemented we have divided this section into the following subtopics: (1) governing sediment transport relations, which describe the mathematical formulations we used to model sediment transport from a physical and deterministic reference; (2) the specification of the Bayesian model, with considerations to basic Bayesian modeling concepts as well as likelihood and prior distribution selection; (3) data simulation, which discloses the process for creating synthetic data from the governing sediment transport relations and known parameters; (4) computational methods which provide a means to integrate the multi-dimensional relationships that naturally result from a Bayesian model; and lastly (5) a means for model evaluation whereby the predictive capabilities of the sediment transport model can be assessed. These subtopics will be described in what follows.

3.3.1 Governing Sediment Transport Relations

We emphasize at this point that the present research is not about modeling the fine-resolution physics of sediment entrainment—rather, it is about larger-scale bulk transport. While there are several options for the physical description of sediment transport, this paper uses an excess shear model due to its wide recognition in the literature.

The basic principle of an excess shear model is that the amount of sediment
transport is related, non-linearly, to the difference between the force induced on the
grains due to the overlying movement of water and the amount of force required to
move sediment of an arbitrary size (e.g., Wilcock et al., 2009). Mathematically, this
relationship is:

\[ q^* = a(\tau^* - \tau_c^*)^b, \]  

(3.1)

where \( q^* \) is the Einstein transport parameter, \( \tau^* \) is the Shields number, and \( \tau_c^* \) is the
critical Shields number. The parameters \( a \) and \( b \) are empirical coefficients and can
take on a range of values reflecting the numerous different proposed transport
relations. While the individual components of (4.1) are entirely non-dimensional,
these parameters can be expressed in dimensional terms:

\[ q^* = \frac{q_s}{\sqrt{(s - 1)gD^3}}, \]  

(3.2)

where \( q_s \) is unit sediment discharge in m\(^2\)/s, \( s \) is specific gravity of the sediments, \( g \)
is acceleration due to gravity in m/s\(^2\), \( D \) is particle diameter in m. The Shields
number, \( \tau^* \), (as described above) is represented by:

\[ \tau^* = \frac{\tau}{(s - 1)\rho gD}, \]  

(3.3)

where \( \tau \) is the grain shear stress in Pa, and \( \rho \) is the fluid density in kg/m\(^3\). The
critical Shields number quantifies how much shear is required to move an arbitrary
particle and is defined as:

\[ \tau_c^* = \frac{\tau_c}{(s - 1)\rho gD}. \]  

(3.4)
where \( \tau_c \) is the critical shear stress in Pa. One method to estimate shear stress experienced by the grains, \( \tau \), is presented in Wilcock (2001):

\[
\tau = 0.052 \rho (gSD_{65})^{0.25} u^{1.5}, \tag{3.5}
\]

where \( \rho \) and \( g \) are in S.I. units as noted above, \( u \) is the mean flow (depth-averaged) velocity in m/s, \( S \) is the surface slope, and \( D_{65} \) is the 65th percentile grain size in m. Since the model proposed here is for a bed of uni-size grains, \( D_{65} \) simply reduces to \( D \). With the components of (4.1) defined, a fully-dimensional relationship can be derived in which \( q_s \) can be isolated:

\[
q_s = a \sqrt{(s-1)gD^3} \left( \frac{0.052(gSD)^{0.25} u^{1.5}}{(s-1)gD} - \frac{\tau_c}{(s-1)\rho gD} \right)^{b}. \tag{3.6}
\]

There exist several variations on the excess shear transport model presented in (4.1) that were explored in this research. While incorporating any number of them would be straight-forward, we limit the number of models discussed in this paper to two for pedagogical reasons. These process models are included below as (3.7) and (3.8) from Meyer-Peter and Müller (1948) and Wong and Parker (2006) respectively:

\[
q^* = 8(\tau^* - \tau_{c}^*)^{3/2}, \tag{3.7}
\]

\[
q^* = 4.93(\tau^* - \tau_{c}^*)^{1.6}. \tag{3.8}
\]
3.3.2 Specification of Bayesian Transport Model

Bayesian Model Fundamentals

Bayesian modeling is derived from Bayes’ Theorem, which, in a statistical context can be written as:

\[ [\theta | z] = \frac{[z | \theta] [\theta]}{\int [z | \theta] d\theta}, \quad (3.9) \]

where the square brackets are a conventional Bayesian notation denoting a probability distribution, and the denominator is an unknown normalizing constant. The model in (3.9) is conventionally expressed as a proportionality, \( [\theta | z] \propto [z | \theta][\theta] \), since the computational methods employed in this paper are able to resolve the underlying posterior distribution without knowing the normalizing constant.

Were we to observe, or collect data, on the process \( z \), the model in (3.9) tells us that it would be controlled by some underlying parameter \( \theta \). The posterior distribution, \( [\theta | z] \), describes the distribution of values that \( \theta \) can assume, given the observations of \( z \). It is the posterior distribution that allows us to make inference on the model parameters and thereby establish credible intervals of values for \( \theta \). The right-hand side of (3.9) is comprised of two parts: the likelihood and the prior. The likelihood, \( [z | \theta] \) (i.e., the data model) describes the distribution of the observations, given \( \theta \). The likelihood should describe the structure of the process, \( z \), given the fact that it is controlled by a parameter \( \theta \). Lastly, the prior distribution, \( [\theta] \), encompasses what is known about \( \theta \) before considering the new observations—it can be the summary of all previous research in the literature on \( \theta \), or it can simply limit possible ranges of values for \( \theta \).

This basic example can be extended to include a vector or parameters, \( \theta \), such that:

\[ [\theta | z] \propto [z | \theta][\theta], \quad (3.10) \]
provided that either an appropriate multivariate prior, or individual prior distributions are specified for each element in $\theta$.

**Bayesian Transport Model**

A Bayesian model for sediment transport can be written as:

$$
[\tau_c, \sigma^2 | \log(q_{s,o})] \propto \left( \prod_{i=1}^{n} [\log(q_{s,o,i}) | \tau_c, \sigma^2] \right) [\tau_c][\sigma^2], \quad (3.11)
$$

where, $q_{s,o} = (q_{s,o,1}, \ldots, q_{s,o,n})'$. The model specified in (6.8) makes inference on multiple parameters, $\tau_c$ and $\sigma^2$, given a set of observations, $q_{s,o}$, of sediment transport. The product of the $i$ likelihoods represents the joint distribution of sediment transport observations. Also, the construction of the model above assumes independence between $\tau_c$ and $\sigma^2$, thereby allowing us to take the product of the two distributions as their joint prior density (i.e., $[\tau_c, \sigma^2] = [\tau_c][\sigma^2]$). If the parameters cannot be assumed independent, the priors may be specified by: $[\tau_c, \sigma^2] = [\tau_c|\sigma^2][\sigma^2]$ or some other conditional decomposition where the final prior takes the form of a Jeffreys [Jeffreys 1946] or some other default prior.

While it is theoretically possible to analytically determine the form of the posterior distributions of Bayesian models, this determination is impractical in all but the most simple model formulations [see Griffiths (1982) for geomorphic examples of analytically-tractable models]. In the case of this model, an analytical derivation of the joint posterior distribution was not attempted due to inherent nonlinearities and the posterior distribution was determined using the sampling methods explained in the Computational Methods section of this paper.
Likelihood

Irrespective of whether one chooses to derive the posterior analytically or not, the likelihood and prior distributions must be specified a priori. The likelihood in this model is specified as a function of the governing equations, including the parameter $\tau_c$, and additive noise (controlled by $\sigma^2$) in log-space. Specifically, the likelihood is denoted as:

$$
\log(q_{s,o,i})|\tau_c, \sigma^2 \sim N(\log(q_{s,i}), \sigma^2),
$$

where each observation, $\log(q_{s,o,i})$, has a mean value defined by the $\log()$ of (4.2) and variance $\sigma^2$. The selection of a normal likelihood is ideal from a computational standpoint because it allows the use of an efficient Gibbs sampler explained in a later section. The $\log()$ in the likelihood is a modeling assumption in which the error structure is assumed to be additive in log-space and is not related to the log-likelihood commonly encountered in maximum likelihood techniques.

The model in (6.9) implies that the sediment transport process generally follows the governing relationship specified in (4.2), but with some variability. In this case, the variance combines natural variation of the process, measurement error, and model misspecification into one term, but makes it possible to separate out the variability in sediment transport due to $\tau_c$ being a random variable as opposed to a fixed value. Further separation of variance can be achieved through hierarchical models (Cressie et al., 2009).

Priors

The prior distributions for $\tau_c$ and $\sigma^2$ must accommodate the physical realities of the parameters they represent. For instance, $\sigma^2$ must have positive real support
(negative values of variance are not reasonable), thus an inverse gamma distribution would be an appropriate selection as a prior (i.e., \( \sigma^2 \sim I.G.(r, q) \)), where the inverse gamma density is:

\[
P(\sigma^2| r, q) = \frac{1}{r^q \Gamma(q)} (\sigma^2)^{-(q+1)} e^{-\frac{1}{r\sigma^2}},
\]

where \( r \) and \( q \) are hyperpriors related to the mean and variance of an inverse gamma distribution such that \( r > 2, q > 2 \), and:

\[
E[\sigma^2] = \frac{1}{r(q-1)},
\]

\[
Var[\sigma^2] = \frac{1}{r^2(q-1)^2(q-2)}.
\]

Given a prior mean \( (\mu_{\sigma^2}) \) and variance \( (\sigma_{\sigma^2}^2) \) for the prior distribution of \( \sigma^2 \) in (6.8), the corresponding values for \( r \) and \( q \) can be determined by:

\[
r = \frac{\sigma_{\sigma^2}^2}{\mu_{\sigma^2}(\mu_{\sigma^2}^2 + \sigma_{\sigma^2}^2)},
\]

and

\[
q = \frac{1}{\mu_{\sigma^2} r} + 1.
\]

The prior distribution for \( \tau_c \), like that of \( \sigma^2 \), also has physical constraints that need to be captured in the specification. First, \( \tau_c \) must be positive; and second, if observations of sediment transport are made, then \( \tau_c \) (a measure of the critical shear stress averaged over the instantaneous Reynolds stresses) is necessarily less than the minimum shear at which transport was observed. Intuitively, the lowest flow at which sediments are moving obviously induces grain shear greater than critical, otherwise the grains would not move. This is an advantage of the Bayesian
approach over nonlinear regression. Constrained optimization of the least square functions for traditional approaches, though straightforward in principle, is often difficult in practice. The Bayesian approach seamlessly incorporates any constraint within the context of the prior. Given these constraints, a truncated normal distribution was selected for the prior on $\tau_c$ (i.e., $\tau_c \sim T.N.(\mu_{\tau_c}, \sigma_{\tau_c}) \tilde{b} \tilde{a}$). The density of the truncated normal is:

$$P(\tau_c|\mu_{\tau_c}, \sigma_{\tau_c}, \tilde{a}, \tilde{b}) = \frac{1}{\sigma_{\tau_c}} \phi \left( \frac{\tau_c - \mu_{\tau_c}}{\sigma_{\tau_c}} \right) \frac{\Phi \left( \frac{\tilde{b} - \mu_{\tau_c}}{\sigma_{\tau_c}} \right) - \Phi \left( \frac{\tilde{a} - \mu_{\tau_c}}{\sigma_{\tau_c}} \right)}{\Phi \left( \frac{\tilde{b} - \mu_{\tau_c}}{\sigma_{\tau_c}} \right) - \Phi \left( \frac{\tilde{a} - \mu_{\tau_c}}{\sigma_{\tau_c}} \right)},$$

(3.18)

where $\mu_{\tau_c}$ and $\sigma_{\tau_c}$ are location and shape parameters, $\tilde{a}$ is the lower-bound, $\tilde{b}$ is the upper-bound, and $\phi()$ and $\Phi()$ are respectively the probability density function and cumulative density function of the standard normal distribution.

### 3.4 Computational Methods

As mentioned previously, an analytic solution for the posterior shown in (6.8) is impractical due to the fact that the truncated normal prior for $\tau_c$ will not result in a conjugate (analytically tractable) joint posterior distribution. Therefore a computational method must be employed that makes it possible to obtain samples from the posterior distribution. In this research, Markov chain Monte Carlo (MCMC) was used to obtain these samples and approximate the posterior distribution of interest.

#### 3.4.1 Markov Chain Monte Carlo

The purpose of MCMC for model fitting is to construct a Markov chain that has a stationary and ergodic distribution that coincides with the posterior distribution. After a number of iterations, the Markov chain will converge to the target
distribution. Once a chain has converged to the target distribution, subsequent realizations from the chain coincide with the target distribution, and thus samples can be collected to approximate posterior quantities. Two different approaches to sampling from the posterior are used: Metropolis-Hastings and Gibbs sampling. Because a complete description of these methods is beyond the scope of this paper, only the practical considerations relating to these methods are presented. Casella and George (1992) and Chib and Greenberg (1995) provide helpful starting points for more-detailed descriptions of these algorithms. With reference to the latter condition of convergence, methods for assessing convergence are discussed in Gelman et al. (2004) and Robert (2007).

3.4.2 Sampling Algorithm for Transport Model

In order to fit the model presented in (6.8), an MCMC algorithm using a Gibbs sampler (Casella and George, 1992) for the parameter $\sigma^2$ and an M-H sampler (Metropolis et al., 1953; Hastings, 1970; Chib and Greenberg, 1995) for the parameter $\tau_c$ was implemented.

Because the inverse gamma prior for $\sigma^2$ is conjugate with the normal likelihood, the full conditional distribution for $\sigma^2$ can be determined as follows:

$$\begin{align*}
[\sigma^2 | \cdot] & \propto \prod_{i=1}^{n} [q_{s,o,i}|\tau_c, \sigma^2][\sigma^2] \\
& \propto \prod_{i=1}^{n} N(q_{s,o,i}, \sigma^2) I.G.(r, q) \\
& \propto \prod_{i=1}^{n} \left( \frac{1}{\sqrt{2\pi} \sigma^2} \exp \left\{ -\frac{1}{2} \frac{(q_{s,o,i} - q_{s,i})^2}{\sigma^2} \right\} \right) \\
& \times \left( \frac{\sigma^2}{\Gamma(q)} \right)^{(r+1)/2} \exp \left\{ -\frac{1}{\tau_c} \right\} \\
& \times \frac{\Gamma((n/2) + q + 1)}{\Gamma(q) \Gamma((n/2) + q + 1)} \\
& \times \exp \left\{ -\frac{1}{\sigma^2} \left( \frac{\sum_{i=1}^{n} (q_{s,o,i} - q_{s,i})^2}{2} + \frac{1}{\tau_c} \right) \right\},
\end{align*}$$

(3.19)

with the bottom-most proportionality in (3.19) having the form of an inverse
gamma kernel (compare to (3.13)), therefore the full-conditional distribution for $\sigma^2$ in the MCMC algorithm is $[\sigma^2|\cdot] = I.G.(\bar{r}, \bar{q})$. The updated parameters are then:

$$\bar{r} = \left( \frac{\sum_{i=1}^{n}(q_{s,o,i} - q_{s,i})^2}{2} + \frac{1}{r} \right)^{-1},$$ (3.20)

and

$$\bar{q} = \frac{n}{2} + q,$$ (3.21)

where $r$ and $q$ are the hyperpriors from (3.16) and (3.17).

Because of the inability to analytically determine the full conditional for $\tau_c$, the M-H algorithm was adopted for the critical shear step in the MCMC sampling procedure. This algorithm requires a set of observations, $q_{s,o}$, the parameters for our prior distributions: $\mu_{\sigma^2}$ and $\sigma_{\sigma^2}^2$ (and their corresponding $r$ and $q$ parameterizations from (3.16) and (3.17)) for the variance parameter; and $\mu_{\tau_c}$, $\sigma_{\tau_c}^2$, $\bar{a}$, and $\bar{b}$ for the critical shear parameter. The MCMC algorithm is provided in Appendix A.

### 3.5 Model Evaluation

The goals of model evaluation were to determine the ability of the Bayesian transport model to: (1) make correct inference on the parameters $\tau_c$ and $\sigma^2$ (parameter identifiability), (2) discriminate between competing process models, and (3) provide reasonable transport predictions. To test this, several strategies were employed. The first was to use synthetic observations whose true underlying parameter values were known. This makes it possible to compare the inferred parameters to the true, known parameters of the synthetic observations. Next, we used transport observations for uni-size sediment in a laboratory flume. Unlike the synthetic observations, the underlying parameter values and process model are unknown. Using the laboratory observations, predictive distributions were
calculated and compared to the observed transport events. To help discriminate between competing models, we calculated the Deviance Information Criterion (DIC), which is a measure of model fit to parsimony. Lastly, we evaluated the model’s sensitivity to different prior specifications.

### 3.5.1 Sensitivity to Prior Specification

The model described in (6.8) requires the specification of prior distributions for $\tau_c$ and $\sigma^2$, which are respectively truncated normal and inverse gamma. The truncated normal distribution is very flexible and can take on numerous different shapes, resulting from the specification of different prior parameter values. The truncated normal distribution is specified as: $\tau_c \sim T.N.(\mu_{\tau_c}, \sigma_{\tau_c})$. The hyperpriors, $\mu_{\tau_c}, \sigma_{\tau_c}, \tilde{b},$ and $\tilde{a}$ control the shape of the probability density function. The goal of a sensitivity study is to try different parameterizations of $\tau_c$ and assess how they influence posterior inference.

The same argument applies for the prior of $\sigma^2$, though, the inverse gamma distribution is not nearly as flexible in terms of the types of shapes it can assume. The prior for $\sigma^2$ is specified as: $\sigma^2 \sim I.G.(r, q)$ though, instead of specifying $r$ and $q$ directly, we will specify hyperpriors $\mu_{\sigma^2}$ and $\sigma_{\sigma^2}$ for the mean and variance. These hyperpriors are then used to calculate $r$ and $q$ through (3.16) and (3.17).

### 3.5.2 Posterior Prediction

Checking the posterior predictive distribution (PPD) makes it possible to evaluate how well model predictions fit the observations. Formally, the posterior predictive distribution is defined as the posterior distribution integrated over the parameters (Gelman et al., 2004). In our multi-parameter sediment transport model, the posterior predictive distribution, $[\log(\tilde{q}_{s,o})|\log(q_{s,o})]$, where $\tilde{q}_{s,o}$ is a
vector of unobserved events, is found by:

\[
\int \int \left[ \log(\tilde{q}_{s,o}) \log(q_{s,o}), \tau_c, \sigma^2 \right] \mid \tau_c, \sigma^2 \mid \log(q_{s,o}) \right] d\tau_c d\sigma^2. \tag{3.22}
\]

The PPD can be used to either make predictions of sediment transport for future observables, or to evaluate how well the model fits the actual process observations. The calculation of (3.22) is done through composition sampling (Tanner, 1996). Using the PPD, we can make predictions under the same conditions as our observations and compare the predictions to the observations.

### 3.5.3 Model Discrimination

Due to the large number of prospective transport models, there exists uncertainty as to which model will result in the best fit. The Deviance Information Criterion was described in Spiegelhalter et al. (1998) and provides a metric by which model fits and parsimony can be compared. The ‘best’ model is one in which DIC is minimized. The DIC is convenient since it can be easily calculated using MCMC samples and does not require any intractable analytic solutions.

### 3.6 Experimental Setup

To achieve the goals specified above, four distinct data scenarios were simulated. These synthetic observations were generated using (4.2), where realizations for the ‘true’ parameters, \( \tau_c \) and \( \sigma^2 \), were assigned. Table 3.1 shows all the parameter values used to simulate observations. Numerous deterministic approaches for estimating \( \tau_c \) have been published in the literature, with the first being Shields (1936) followed by approximations to Shields’ data in Brownlie (1981) in which the
following relation was proposed for the critical Shields Number:

\[ \tau_c^* = 0.22 R_{ep}^{-0.6} + 0.06 \cdot \exp\{-17.77 R_{ep}^{-0.6}\}, \quad (3.23) \]

where \( R_{ep} \) is the particle Reynolds Number:

\[ R_{ep} = \frac{\sqrt{gRDD}}{\nu}, \quad (3.24) \]

where \( R \) is the submerged specific gravity \((R = s - 1)\) and \( \nu \) is the kinematic viscosity of water. The relation in (3.23) provides a method to specify a plausible \( \tau_c \) using the previously-designated parameter values proposed in Table 3.1 and is only used for the purpose of data simulation and not for parameter inference.

Prescribing prior parameters for \( \sigma^2 \) is more direct. Values of \( \sigma^2 \) ranging from 0 to approximately 1 provide realistic relationships between shear and sediment discharge. Both low- and high-variance datasets were simulated, with values: \( \{\sigma^2 : 0.05, 1.10\} \).

In addition to synthetic observations, real flume observations from Smart (1984) were used in our Bayesian transport model. The observations reported in Smart (1984) are of uni-size sediment transport in planar beds and steep slopes, and therefore, are appropriate for this simple uni-size Bayesian sediment transport model.

3.7 Results and Discussion

3.7.1 Simulation and Sensitivity Studies

Four distinct data scenarios were simulated for model testing purposes, each consisting of a unique combination of low- and high-variance with many and few
observations. Specifically, data scenario 1 consisted of 21 observations with a variance of 0.05 (many low-variance observations); data scenario 2 had six observations with a variance of 0.05 (few low-variance observations); data scenario 3 consisted of 21 observations with a variance of 1.10 (many high-variance observations); lastly, data scenario 4 had 6 observations with a variance of 1.10 (few high-variance observations). These data scenarios shown in Figure 3.1.

In addition to four data scenarios, four distinct prior specifications were used in model fits. Histograms of these prior densities are illustrated in Figure 3.2. These priors were designed to represent a variety of prior scenarios: (a) a diffuse prior (known informally as a non-informative prior) that has a lower-bound at zero and an upper-bound above the minimum shear at which transport was observed to account for accidental sampling at low flows (prior 1), (b) a diffuse prior on compact support and whose mean is known to be inaccurate (prior 2), (c) an informative prior with an inaccurate mean and low variance (prior 3), and (d) an informative prior whose density increases with proximity to the lowest measured shear (prior 4). Table 3.2 shows the specified prior means and 95% credible intervals for each $\tau_c$ prior scenario. Because each data scenario has a true variance of either 0.05 or 1.10, priors for $\sigma^2$ were set at the known value for each data scenario with a small variance ($\mu_{\sigma^2} = \{0.05, 1.10\}$, $\sigma^2_{\sigma^2} = 0.15$) to isolate the effects of prior specification on critical shear. For data scenarios 1 and 2, the prior for $\sigma^2$ has a mean of 0.05. For data scenarios 3 and 4 the prior mean is 1.10. Thus, both the high- and low-variance data scenarios will have accounted for the true variance so that the effects of different critical shear priors can be evaluated.

The simulated observations were also used in a more robust test of parameter identifiability. A diffuse prior (prior 1) was assumed for critical shear and a set of three vague priors (a prior variance of 1000) for $\sigma^2$ were tested (see Table 3.3). Prior
1A has an expected value of 0.001 (an underestimate by a factor of 50 for data scenarios 1 and 2 and an underestimate by a factor of 1100 for data scenarios 3 and 4). Prior 1B has an expected value of 0.275 (overestimates the true variance of data scenarios 1 and 2 by a factor of 5.5 and underestimates the true variance of data scenarios 3 and 4 by a factor of 4). Prior 1C has an expected value of 5 (100 times larger than the true variance for data scenarios 1 and 2 and 4.5 times larger than the true variance for data scenarios 3 and 4). Table 3.3 presents the expected values, variances, and 95% credible intervals for these priors.

Inference on Model Parameters

The main purpose of the simulation studies was to determine the robustness of posterior inference on $\tau_c$ given different data and prior scenarios. Table 3.4 contains the simulation results, including the expected value of the posterior estimate of critical shear and critical Shields Number along with their 95% credible intervals grouped by scenario. Model misspecification is not a factor in these simulation results since the Meyer-Peter Müller (MP-M) relation (Meyer-Peter and Müller, 1948) was used for both data simulation and parameter estimation.

In each model fit, expert opinion must inform the Bayesian sediment transport model through the prior, and there are many different ways of parameterizing the prior distribution for $\tau_c$. The diffuse prior (prior 1) shown in Figure 3.2(a) is the least informative of all the priors; it simply states that critical shear must be greater than zero and is necessarily less than some shear at which sediment transport has been observed. All values within this range are equally probable under the diffuse prior. Prior 2, the diffuse prior on compact support shown in Figure 3.2(b), gives nearly-uniform probability to critical shear values over a restricted range of the support in prior 1. The lower-bound of prior 2 is no longer zero, but was set to 1.55
Pa. The upper-bound was also changed from 10 Pa to 4.45 Pa. Given the fact that the true critical shear of the simulated results is 6.72 Pa, the specification of prior 2 is known to exclude the true critical shear value and will prohibit the M-H sampler from exploring the portion of the parameter space outside of this compact support.

Prior 3, shown in Figure 3.2(c), was specified to be inaccurate in its mean, but not overly-limiting in its lower- and upper-bound. So, unlike prior 2, the specification of prior 3 permits any value between zero and 10 Pa to be possible, but not with equal probability; the most probable values for $\tau_c$ are centered at 3.00 Pa with a small variance. Prior 4, seen in Figure 3.2(d), favors critical shear values proximal to the upper-bound of 10 Pa. Prior 4 is useful for situations in which the first observation of sediment transport is supposed to be near critical.

Using these prior scenarios, we observe similarities and disparities in the posterior distributions across data scenarios. First, accurate inference was never obtained using the diffuse prior on compact support (prior 2). As was mentioned above, the specification of prior 2 restricted the lower- and upper-bounds thereby excluding the true critical shear value from the set of possible parameter values. What one observes in Table 3.4 is that no amount of data can overcome this restriction—data scenario 1 to data scenario 4 all result in unacceptable inference. Unless there are compelling (e.g., physically realistic) reasons to do so, priors using compact support should be avoided. The other priors, however, have varying performance across data scenarios (see Table 3.4).

Prior 1 can serve as a default for situations where no specific information regarding critical shear is known. The practical effects of prior 1 simply constrain possible values for critical shear between a lower-bound of zero and an upper-bound of 10 Pa with equal probability. The results in Table 3.4 show that using a diffuse prior is a very safe modeling strategy to employ in any of the data scenarios. Even
for the most limiting data set (data scenario 4), where observations are few and variance is high, a diffuse prior results in acceptable inference on $\tau_c$. The posterior distributions show that where there are more observations, the inferred critical shear values approach the true value used to generate the data.

The prior specification with the widest range of results was prior 3. This prior is informative in that a region of the interval $[0, 10]$ was given higher density than others (see Figure 3.2(c)). Prior 3, however, provides misleading information for critical shear. While the true value of $\tau_c$ is 6.72 Pa, the 95% credible interval for prior 3 is $[2.02, 3.98]$, as shown in Table 3.2. Unlike prior 2 which had compact support, the support of prior 3 maintains the lower-bound of zero and the upper-bound of 10 Pa. While this prior is misleading, it does not prohibit the M-H sampler from exploring the entire interval $[0, 10]$. What we see is that, in situations where there are sufficient data—data scenario 1, for example—the misleading prior is overwhelmed by the observations and the posterior still correctly infers the true value of the critical shear parameter. With fewer observations, and increasing variance, however, prior 3 asserts increased influence over the posterior distribution, resulting in unacceptable inference in the remaining data scenarios. Thus, inaccurate priors do not pose insurmountable problems when there are sufficient observations, while situations with few observations will result in unacceptable inference under prior 3.

The final prior (prior 4, Figure 3.2(d)) shows uniformly acceptable posterior inference for all data scenarios. Like priors 1, 3, and 4, it has support $[0, 10]$ but gives higher probability to critical shear values close to the upper-bound. In data collection scenarios where low transport rates are collected, as suggested in Wilcock (2001) for reference shear, prior 4 is particularly useful since expert opinion can inform the model that critical shear is somewhere near the lowest transport
observation.

More informative priors, like that of prior 4, can bolster inference in scenarios with only a few low transport rate observations. Table 3.5 shows the posterior inference on critical shear using three progressively more informative priors, presented in Figure 3.3. The diffuse prior is similar to prior 1 in that it gives nearly-equal probability to any critical shear value between the lower- and upper-bound. The vague prior, similar to prior 4, provides some information to the model regarding where the true value for critical shear is expected. The informative prior gives preference to values close to the true value. What one sees in Table 3.5 is that, using the same observations (data scenario 4) posterior inference is bolstered by prior information. The posterior expectation for critical shear approaches the true value as the information carried in the prior increases. Further, the range of the 95% credible interval also decreases. In sum, well-specified priors provide more precise estimates of critical shear.

The preceding analysis is helpful to isolate the sensitivity of posterior inference on critical shear since the variance parameter was assumed known. In all practical exercises, however (such as the flume experiment whose results follow in subsequent paragraphs), this is not reasonable. To this end, we fit three models to data scenarios 1 and 3 using the diffuse prior (prior 1) for critical shear and three vague priors for the variance parameter, each with an inaccurate mean. This analysis makes it possible to determine how sensitive posterior inference on model parameters is affected by inaccurate priors for $\sigma^2$ and a diffuse prior for $\tau_c$. Table 3.6 shows the results of this analysis.

The true variance parameter for data scenario 1 is 0.05, and 1.1 for data scenario 3. The priors on $\sigma^2$ however, have mean values that range from being 1100 times smaller than the true variance to 100 times larger than the true variance. On one
extreme, the model fit to data scenario 3 using prior 1A, which assumed a prior mean of 0.001, had a posterior mean of 0.91 and a 95% credible interval that contained the true realized value of 1.1. Prior 1B provided nearly identical inference despite a prior mean of 0.275 compared to 0.001. Prior 1C was an overestimate of $\sigma^2$ resulting in a posterior mean closer to the true value of 1.1 but with a relatively wider credible interval. For data scenario 1, whose true variance was 0.05, prior 1A, an underestimate by a factor of 50, performed well, inferring a mean value of 0.06 and a tight credible interval. Prior 1B resulted in posterior inference in which the lower-bound of the credible interval contained 0.05, and prior 1C resulted in unacceptable inference since the 95% credible interval did not contain the 0.05. The results in Table 3.6 indicate that it is far better to have a vague prior for $\sigma^2$ that underestimates the true parameter value (even by 1100 times) than it is to overestimate. It should be noted that in all these model fits, regardless of the prior on $\sigma^2$, posterior inference on critical shear always contained the true value for $\tau_c$, only with varying degrees of precision as shown in the associated credible intervals.

**Model Discrimination**

Because there exist numerous bed load relations in the literature, there is a tyranny of choice when it comes time to model sediment transport. Indeed, *Gomez and Church* (1989) noted that there are more bed load relations than there are good datasets with which to test them. A general criterion for a desirable model is one that is able to provide good fit to observations while minimizing the number of parameters used to obtain that fit. The Deviance Information Criterion provides a metric that quantitatively measures model fit and parsimony allowing competing models to be compared. A simulation test was performed to verify the utility of the DIC in model discrimination for sediment transport. Synthetic data were generated
using the MP-M bed load relation and two different model fits were performed; one
used as a process model the MP-M equation and the other used the updated
parameterization of the MP-M relation proposed in Wong and Parker (2006)
introduced earlier as (3.7) and (3.8). These results are presented in Table 3.7. In
comparing models, the magnitude of the DIC is irrelevant; it is used as a relative
measure between models. Table 3.7 shows us that the MP-M correctly infers the
true realized value of $\tau_c$ while the W-P model underestimates the realized value.
This pattern is repeated in the flume observations for reasons discussed later. The
MP-M model also has a smaller DIC value than the W-P model—a result we would
expect because the MP-M relation was used in the data simulation step of this test.
Lastly, the MP-M model has a credible interval that is narrower than that for the
W-P model, so based on these results the MP-M model can be identified as the
‘better’ model. This demonstrates that the DIC can provide a guideline by which
competing scientific theories may be compared.

3.7.2 Flume Observations

The simplicity of the model presented in (4.2) requires that any flume
observations used must meet several criteria. First, the flume observations must be
for uni-size (or near uni-size) sediments; second, because the proposed model does
not account for bed forms, the transport must occur over a planar bed; and third,
since the phenomenon of interest is bed load transport of gravel, a uni-size bed
comprised of grains larger than approximately 8 mm were sought. The experiments
published in Smart (1984) meet these criteria and were used to evaluate the
suitability of the proposed model. Specifically, we used the 10.5 mm bed load
results with our Bayesian transport model—this experiment consists of 26 sediment
transport observations in steep flumes for a nearly-uniform distribution of grains.
Unlike the simulation studies, the true parameter values of critical shear and variance are unknown for the Smart (1984) data. Given the results from the simulation studies, however, a general modeling strategy was developed. The simulation studies showed that the use of a diffuse prior (prior 1 in Figure 3.2(a)) is a defensible default prior. Accordingly, the prior for critical shear was specified as: $\tau_c \sim T.N.(10, 5000, 0, 20)$. The prior for the variance parameter was specified using judgment as to its mean and variance. A scatterplot of the transport observations demonstrated a low variance, similar to the low variance scenarios 1 and 2 in the simulation studies. Using these simulation studies as a reference, a prior for the variance was specified to have a mean value of 0.1, and a variance of 1000—the $(r, q)$ hyperpriors were then solved for using Equations (3.16) and (3.17). These priors are graphed as Figure 3.4.

Two different models were fit each using a different process model included as (3.7) to (3.8). Tables 3.8 and 3.9 summarize the posterior inference on both critical shear and variance for each model fit. Also presented are the DIC values computed for each model fit. Lastly, Figure 3.5 shows the posterior predictive distribution for each model fit. The plots on the left (Figures 3.5(a) and (c)) show the 68%, 90%, and 95%, credible intervals of the predictive distribution, represented in darkening shades of grey. The plots on the right (Figures 3.5(b) and (d)) show a partitioning of variability in predictions due to: (1) the fact that critical shear is being modeled as a random variable instead of a fixed value, and (2), the variance parameter $\sigma^2$ which is a multiplicative (additive in log-space) error term representing measurement error, natural variability of transport, and model misspecification. The combined region in Figures 3.5(b), and (d) is the same 95% credible interval shown in Figures 3.5(a) and (c).
Inference on $\tau_c$ and $\sigma^2$

We see in Table 3.8 that the inferred critical shear parameter is not independent of the bed load relation used in the model. The MP-M model infers a critical shear of 0.080 Pa, which is reasonable given the steep slopes of the flume studies \cite{Cao1986}. The W-P model, however, infers an entirely distinct posterior distribution for critical shear—the 95% credible interval for the MP-M models does not overlap with that of the W-P model. This is due to the fact that the MP-M and W-P models have differing values for the $a$ and $b$ coefficients in an excess shear formulation. At the most basic level, given values of $a$, $b$, and $\tau^*$ it is expected that solutions for $\tau^*_c$ in (4.1) would yield distinct results. Traditionally, the MP-M and W-P equations assume that the critical shields number is fixed and known at 0.047. Given the difficulty in specifying the critical shields number in practice, the present research advocates the measurement of $q^*$ and $\tau^*$ and the estimation of $\tau^*_c$ based on the measurements and prior knowledge. We see that the existence of a single critical shear value that is portable from one transport relation to another is therefore unreasonable because different process models may result in distinct posterior parameter spaces.

The inferred parameter values for $\sigma^2$, as shown in Table 3.9, however, are in agreement between the MP-M and W-P models at 0.14. Further, the posterior credible intervals for $\sigma^2$ under the MP-M and W-P models are consistent.

Model Discrimination

The DIC values, as reported in Tables 3.8 and 3.9, though somewhat inconclusive, seem to indicate that the MP-M model provides the better fit for the number of parameters estimated since it has the smallest value, 25.85 compared to 26.17. The models fitted to the flume observations do not exhibit as large a
difference in their DIC values as was the case for the simulated observations in Table 3.7, possibly inferring only a slight superiority of the MP-M model. Both of the process models used in this research are broadly defined as excess shear models which have the form \( q^* = a(\tau^* - \tau_c^*)^b \) for \( \tau_c^* < \tau^* \), and 0 otherwise. The Bayesian formulation presented in this paper models the parameters \( a, b, \) and \( \tau^* \) as fixed, and \( \tau_c^* \) as a random variable and variance \( \sigma^2 \) according to (6.8). The MP-M and W-P models fit into the definition of the excess shear models since the parameters \( a \) and \( b \) are specified directly.

**Posterior Prediction**

A major difference between the predictive results of a purely deterministic approach to sediment transport modeling and those from a Bayesian model is that, in the latter, the predictions are functions of random variables, as shown in (3.22), and are therefore random variables themselves. This characteristic means that predictions in a Bayesian transport model can be represented in terms of probabilities. The plots contained in Figure 3.5 show the PPD’s for the model runs and provide prediction ranges that can be assigned a probability of occurrence. What this shows is that, like purely deterministic approaches, the Bayesian framework can employ deterministic equations to make predictions, but unlike deterministic approaches, it does so using probability distributions for the parameters and predictions. The results in Figure 3.5 are very informative since the parameter and measurement uncertainty have been fully propagated through the analysis resulting in a range of credible predictions. The PPD represents what can be reasonably predicted given the parameter, measurement, and structural uncertainty related to the sediment transport phenomenon.

Figures 3.5(a) through (d) show subtle changes in shape over the range of
observed transport events due to the their respective process models. The MP-M model consists of two distinct slopes—higher slope at lower shears that then flattens out at higher shears. The W-P model, however, is generally flatter throughout due to the fact that $\tau_c$ is much lower than for the MP-M equation. The W-P relation tends to under-predict relative to the observed transport rates whereas the MP-M relation tends to slightly over-predict. This is seen in plots (a) and (c) of Figure 3.5 since the observed transport rates are located generally on the low-side of the PPD. Alternately, the observations in the W-P relation are located generally higher than the PPD. These discrepancies derive from the deterministic models and do not represent an inadequacy of the inferential framework (Box and Tiao, 1992).

Figures 3.5(b) and (d) show how much variation in the posterior predictions can be partitioned to the underlying distribution of $\tau_c$ (as shown in the light band running down the center of the PPD), as well as what can be attributed to the underlying variability, measurement error, and model misspecification shown by the darker area. The Bayesian transport model in (6.8) is admittedly simple in its construction, and more complicated models may account for a different partitioning of variability to different terms. The implications of this partitioning suggest that some of the variability is reducible (measurement error and model misspecification) while another portion is not (natural variability).

Given the model results using the flume data, the MP-M relation was identified as the most reasonable relation for these observations. This assessment is based in part on the MP-M model having a slightly lower DIC value, but more so because it identifies a tighter credible interval than the W-P relation, and further because this credible interval is centered on values that are consistent with our understanding of critical shear in steep flumes.
3.8 Implications and Conclusions

3.8.1 General

Having tested our model framework using both simulated and laboratory observations, we now conclude by broadly discussing sediment transport models, characteristics of the Bayesian framework, and the connections between the two in order to identify where a Bayesian approach to sediment transport modeling addresses specific needs of the discipline.

Early work recognized that a threshold for transport is better described by a statistical distribution than a fixed parameter. In many applied sediment transport problems, there is considerable difficulty in specifying justifiable values for critical shear, especially when one accounts for the multiplicity of bed arrangements and varying hydraulic conditions that affect incipient motion. To further complicate this, results from this research show that inferred values of critical shear are not independent of the process model used, therefore a given critical shear value (or distribution of values) that provide the best fit for one relation may result in a very poor fit for a different bed load relation. Additionally, Wilcock (2001) notes that when estimating sediment transport rates, strictly formula-based approaches are notoriously inaccurate and sampling campaigns are expensive. One current approach to sediment transport modeling is described in Wilcock (2001), who proposes the use of a deterministic transport relation that is calibrated through easily-obtained transport samples (i.e., low-transport observations). In this approach, the reference shear parameter, a surrogate for critical shear, is adjusted until a best fit to the observations is obtained. This method employs the use of observations, expert judgment, and a deterministic relationship to provide predictions. Once this model is calibrated, predictions are described by a line fitted
to the observations, but deviations from these predictions due to parameter, measurement, and structural uncertainty are not quantified.

A Bayesian sediment transport model is very easily adapted to the approach described above since expert opinion still gets to inform the model as to the values of critical shear through the prior, and transport observations are used to update the expert opinion in a formal way that incorporates variability and error. The reference shear approach requires the specification of a single value for $\tau_r$, whereas a Bayesian approach models critical shear as a random variable arising from a probability distribution. Describing critical shear as a random variable is more consistent with the realities of sediment transport, as discussed by Shields (1936), Einstein (1950), and later Buffington and Montgomery (1997). Further, a Bayesian approach to sediment transport modeling is very efficient since it provides a single framework that robustly: (1) estimates model parameters, (2) makes probability-based predictions, (3) incorporates expert knowledge, and (4) provides a means for model discrimination. The results of our model make it possible to explicitly account for predictive uncertainty in sediment transport formulae, given observations, thereby allowing prediction uncertainty to be incorporated in subsequent analyses, such as sediment budgets and river restoration modeling, for example.

Because of its ability to account for uncertainty, this framework would be particularly useful in geomorphological work related to river restoration. Stewardson and Rutherford (2008) observed that there exists a tendency for river managers to assume that the physical components of river restoration, such as sediment transport, are well-understood with minimal errors. They then demonstrate that there is “unreasonable confidence” in deterministic approaches to sediment transport and conclude that “rarely is any thought given to the best approach to modelling, including the calculation of input parameters to minimise uncertainties
in restoration decisions.” Additionally, Wheaton et al. (2008) noted that the river management community has largely brushed uncertainties aside. Because of this, the interactions between scientists and river managers are often tenuous (Wilcock and Crowe, 2003). To ameliorate this, Wilcock and Crowe (2003) concluded that uncertainty in model predictions must be clearly communicated so that management decisions can be better informed. Research in this area can either “ignore the uncertainty and hope that it is not debilitating for the project at hand, or accept the uncertainty and use it as a feature of the research” (Graf, 2008). A Bayesian approach to sediment transport modeling, such as that offered here, robustly addresses the challenges expressed above.

Further, the interpretation of the predictive distributions is intuitive. Whereas traditional statistical methods report confidence intervals that represent the percent of time the constructed interval will contain the true, fixed parameter value, Bayesian credible intervals are statements of probability, for example, the probability that the parameter of interest is contained in this interval is $(1 - \alpha)\%$. This simple interpretation of model results facilitates the “clear communication of uncertainty” recommended by Wilcock and Crowe (2003).

Lastly, because Bayesian models provide posterior predictive distributions, intuitive assessments of predictive uncertainty are avoided. Of the numerous studies published on decision-making under uncertainty, Tversky and Kahneman (1973) discusses how intuitive assessments of confidence are biased by representativeness and input consistency. A conclusion of their research is that individuals are prone to experience a high degree of confidence in highly-fallible judgments—stated otherwise, intuitive assessments of the credibility of predictions are usually highly-overconfident. The posterior predictive distribution developed in (3.22), however, avoids the shortcomings of intuitive assessments of uncertainty, and
provides a robust measure of the uncertainty in model predictions.

3.8.2 Model Extensions for Future Implementations

The simple model disclosed in this research demonstrates how a Bayesian sediment transport model may be developed. The present model is simple for pedagogical purposes and certainly does not represent the full complexity that is possible. Some specific extensions that are being pursued by the current authors are multi-fraction bed load transport which incorporate description of how different grain sizes are transported; inclusion of turbulence structures and hiding effects; mixture models and model averaging for model inference or to accommodate unknown error structures ([Xiao et al., 2011]); development of hierarchical implementations that account for different error sources; evaluation of implications in constructing sediment mass balances; and incorporating uncertainty into model forcing observations (such as streamflow).

The established hydrology literature demonstrates potential uses for the Bayesian framework in sediment transport as well. [Renard et al., 2010] provides a comprehensive example of how Bayesian approaches can be leveraged to inform model selection, error structure, and how uncertainty can inform a model—such an approach applied to sediment transport would be a valuable contribution. Pre-MCMC hydrology literature illustrate the potential use of decision theory in hydrology and how it can inform decision based on customizable loss functions that account for the capital and maintenance costs of over-design and the monetary consequences of inadequate design (Davis et al., 1972; Duckstein and Szidarovszky, 1977) thereby providing an optimal decision from a decision-theoretic perspective.

Given the broad challenges faced in the sediment transport community, it is our belief that Bayesian models can provide a tool for innovation.
References


Table 3.1: Values for fixed parameters.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>s</th>
<th>S</th>
<th>T, °C</th>
<th>ρ, kg/m³</th>
<th>ν, 10⁶ m²/s</th>
<th>g, m/s²</th>
<th>D, mm</th>
<th>u, m/s</th>
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<td>3/2</td>
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<td>1.519</td>
<td>9.81</td>
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Table 3.2: Priors for $\tau_c$. Critical Shields Number in parentheses.

<table>
<thead>
<tr>
<th>Prior</th>
<th>$E[\tau_c]$</th>
<th>95% C.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00 (0.039)</td>
<td>0.26–9.75 (0.002–0.075)</td>
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<tr>
<td>2</td>
<td>3.00 (0.023)</td>
<td>1.62–4.38 (0.013–0.034)</td>
</tr>
<tr>
<td>3</td>
<td>3.00 (0.023)</td>
<td>2.51–3.49 (0.019–0.027)</td>
</tr>
<tr>
<td>4</td>
<td>8.17 (0.063)</td>
<td>0.26–9.75 (0.002–0.075)</td>
</tr>
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</table>

Table 3.3: Vague priors for $\sigma^2$.

<table>
<thead>
<tr>
<th>Prior</th>
<th>$E[\sigma^2]$</th>
<th>Var[\sigma^2]</th>
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<td>1A</td>
<td>0.001</td>
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<tr>
<td>1B</td>
<td>0.275</td>
<td>1000</td>
<td>0.049–1.132</td>
</tr>
<tr>
<td>1C</td>
<td>5.0</td>
<td>1000</td>
<td>0.910–20.40</td>
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</tbody>
</table>

Table 3.4: Posterior inference results for $\tau_c$. Critical Shields Number in parentheses. For each data scenario the true value of $\tau_c$ is 6.72 Pa ($\tau^*_c = 0.052$).

<table>
<thead>
<tr>
<th>Data Scenario</th>
<th>Prior</th>
<th>$E[\tau_c]$</th>
<th>Posterior</th>
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<tr>
<td>1</td>
<td>1</td>
<td>6.75 (0.052)</td>
<td>6.70–6.78 (0.052–0.052)</td>
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<td></td>
<td>2</td>
<td>4.18 (0.032)</td>
<td>3.45–4.45 (0.027–0.034)</td>
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<tr>
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<td>3.32 (0.026)</td>
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<td>4</td>
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<td>6.61–6.71 (0.051–0.052)</td>
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<tr>
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<td>3.56–4.44 (0.027–0.034)</td>
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<td>3.01–6.25 (0.023–0.048)</td>
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<td>6.39–6.80 (0.049–0.053)</td>
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<td>6.04–6.82 (0.047–0.053)</td>
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<td>4.02 (0.031)</td>
<td>2.62–4.44 (0.020–0.034)</td>
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<tr>
<td></td>
<td>3</td>
<td>3.55 (0.027)</td>
<td>2.50–4.63 (0.019–0.036)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6.57 (0.051)</td>
<td>6.05–6.82 (0.047–0.053)</td>
</tr>
</tbody>
</table>
Table 3.5: Comparison of priors for data scenario 4. Critical Shields Number in parentheses. True value of \( \tau_c \) is 6.72 Pa (\( \tau_c^* = 0.052 \)).

<table>
<thead>
<tr>
<th>Prior</th>
<th>( E[\tau_c] )</th>
<th>95% C.I.</th>
<th>C.I. Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffuse</td>
<td>6.54 (0.051)</td>
<td>6.04–6.82 (0.047–0.053)</td>
<td>0.78 (0.006)</td>
</tr>
<tr>
<td>Vague</td>
<td>6.57 (0.051)</td>
<td>6.05–6.82 (0.047–0.053)</td>
<td>0.77 (0.006)</td>
</tr>
<tr>
<td>Informative</td>
<td>6.62 (0.051)</td>
<td>6.27–6.84 (0.048–0.053)</td>
<td>0.57 (0.005)</td>
</tr>
</tbody>
</table>

Table 3.6: Posterior inference on model parameters using priors 1A–1C for \( \sigma^2 \) and prior 1 (diffuse) for \( \tau_c \). Expected value of posterior distribution—values in parentheses denote the 95% credible interval. True critical shear value is 6.72 Pa; true variance for data scenario 1 is 0.05 and 1.1 for data scenario 3.

<table>
<thead>
<tr>
<th>Prior</th>
<th>Parameter</th>
<th>Data Scenario 1</th>
<th>Data Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>( \tau_c )</td>
<td>6.75 (6.71–6.78)</td>
<td>6.64 (6.40–6.80)</td>
</tr>
<tr>
<td></td>
<td>( \sigma^2 )</td>
<td>0.06 (0.03–0.10)</td>
<td>0.91 (0.50–1.62)</td>
</tr>
<tr>
<td>1B</td>
<td>( \tau_c )</td>
<td>6.75 (6.70–6.79)</td>
<td>6.64 (6.40–6.79)</td>
</tr>
<tr>
<td></td>
<td>( \sigma^2 )</td>
<td>0.08 (0.05–0.15)</td>
<td>0.92 (0.50–1.65)</td>
</tr>
<tr>
<td>1C</td>
<td>( \tau_c )</td>
<td>6.73 (6.60–6.83)</td>
<td>6.63 (6.33–6.81)</td>
</tr>
<tr>
<td></td>
<td>( \sigma^2 )</td>
<td>0.52 (0.91–20.5)</td>
<td>1.36 (0.76–2.42)</td>
</tr>
</tbody>
</table>

Table 3.7: Deviance Information Criterion results for competing simulation models. Critical Shields Number in parentheses. True value of \( \tau_c \) is 6.72 Pa (\( \tau_c^* = 0.052 \)).

<table>
<thead>
<tr>
<th>Model</th>
<th>DIC</th>
<th>( E[\tau_c] )</th>
<th>95% C.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP-M</td>
<td>46.95</td>
<td>6.70 (0.052)</td>
<td>6.63–6.76 (0.051–0.052)</td>
</tr>
<tr>
<td>W-P</td>
<td>72.16</td>
<td>6.25 (0.048)</td>
<td>5.71–6.57 (0.044–0.051)</td>
</tr>
</tbody>
</table>

Table 3.8: Comparison of process model performance on the Smart (1984) 10.5 mm flume observations—\( \tau_c \). Critical Shields Number in parentheses.

<table>
<thead>
<tr>
<th>Model</th>
<th>DIC</th>
<th>( E[\tau_c] )</th>
<th>95% C.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyer-Peter Müller</td>
<td>25.85</td>
<td>13.87 (0.080)</td>
<td>12.52–14.94 (0.072–0.086)</td>
</tr>
<tr>
<td>Wong-Parker</td>
<td>26.17</td>
<td>8.00 (0.046)</td>
<td>5.10–10.66 (0.029–0.062)</td>
</tr>
</tbody>
</table>
Table 3.9: Comparison of process model performance on the Smart (1984) 10.5 mm flume observations—$\sigma^2$.

<table>
<thead>
<tr>
<th>Model</th>
<th>DIC</th>
<th>$E[\sigma^2]$</th>
<th>95% C.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyer-Peter Müller</td>
<td>25.85</td>
<td>0.14</td>
<td>0.08–0.23</td>
</tr>
<tr>
<td>Wong-Parker</td>
<td>26.17</td>
<td>0.14</td>
<td>0.08–0.24</td>
</tr>
</tbody>
</table>

Fig. 3.1: Plot of simulated observation scenarios. Each data scenario is a distinct combination of high/low variance and many/few observations. (a) Data scenario 1: $N = 21$, $\sigma^2 = 0.05$, (b) data scenario 2: $N = 6$, $\sigma^2 = 0.05$, (c) data scenario 3: $N = 21$, $\sigma^2 = 1.10$, (d) data scenario 4: $N = 6$, $\sigma^2 = 1.10$. 
Fig. 3.2: Four prior scenarios. Vertical line (red) denotes true $\tau_c$ in simulated data scenarios. Parameterization follows: $T.N.(\mu_{\tau_c}, \sigma_{\tau_c}, \tilde{a}, \tilde{b})$. (a) Prior scenario 1, (b) prior scenario 2, (c) prior scenario 3, and (d) prior scenario 4.
Fig. 3.3: Progressively informative priors for data scenario 4.

Fig. 3.4: Prior distributions for Smart (1984) observations. Vertical line (red) denotes the minimum shear that produced transport.
Fig. 3.5: Posterior predictive intervals for Smart (1984) observations. Plots (a) and (b) use the MP-M bed load relation. Plots (c) and (d) use the W-P relation.
CHAPTER 4
TRADITIONAL AND BAYESIAN STATISTICAL MODELS IN FLUVIAL SEDIMENT TRANSPORT

Abstract

The characterization of sediment transport is an important problem that has been actively studied for some time. Numerous approaches have been demonstrated in the literature including mechanistic models, probabilistic arguments, machine learning algorithms, and empirical formulations. Most implementations of sediment transport relations are deterministic in nature and require the specification of model parameters. These parameters are traditionally assumed fixed (i.e., a single value) and subsequent predictions are not necessarily representative due to uncertainty as they are fixed (i.e., a line) as well. In this paper we present a Bayesian statistical sediment transport model and compare its ability to infer critical shear values from observations to nonlinear regression. This approach provides several advantages, namely: (1) parameters are not constrained to be normally-distributed as is required in many traditional approaches; (2) estimates of parameter variability are easily obtained and interpreted from distributions that arise naturally from the estimation and prediction process; (3) predictive distributions, or probability densities of predictions, are easily obtained through Bayesian methods and provide a robust way to sediment transport probabilistically centered on a deterministic formulation.


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4.1 Introduction

Accurate prediction in fluvial sediment transport has been an active area of research for many years now due to its implications in engineering, ecology, and river management. Its characterization, however, is impeded by nonlinearities, uncertainty in measurements, and process variability.

Generally, sediment transport relations can be defined in two ways. First, they can be formulated through physics and mechanics of particle movements or they can be defined empirically using observations and predictor variables. These relations, however they may be derived, can then be used either deterministically or stochastically. A deterministic approach assumes fixed values for model parameters resulting in a single prediction of transport for any given set of parameter values—this is the most common method in practice (Lopez and Garcia 2001). The stochastic approach assigns probability densities for model parameters. These distributions can be representative of the uncertainty of the underlying fixed parameter assumption. This results in an ensemble of predictions realized from combinations of the individual parameter distributions. Both approaches require values to be specified for parameters either as point values in the former or distributions in the latter. This specification is accomplished through expert judgment, selection through calibration data, or some combination of the two.

Most contemporary bed load equations are the result of laboratory experiments, for example Gilbert (1914), Meyer-Peter and Müller (1948), Guy and Simons (1966), and contain parts that are deterministic and empirical. In application, it is assumed that these governing relationships adequately explain the variation in the observations so they are often applied in a deterministic framework using fixed parameters. For example, the original formulation of the Meyer-Peter Müller equation uses 0.047 as the value for the critical Shields parameter. Laboratory
studies indicate that threshold is well-defined for precisely-known bed configurations (e.g., angle of repose) under steady conditions (Dey 1999), however, it has long been recognized that sediment transport is a variable and uncertain process with parameters that are not fixed (Kirchner et al. 1990) owing to different bed-states and other unknowable conditions. This phenomenon prompted research into stochastic and probabilistic formulations of transport. For example, H. A. Einstein proposed a probabilistic framework in which entrainment and travel distance were derived analytically through the manipulation of probability densities (Einstein 1950). Hamamori (1962) analytically derived a probability distribution for bed load that accounts for bed forms and, more recently, Turowski (2010) followed up by deriving a distribution of bed load transport rates based on probabilistic assumptions of waiting times using either the exponential or Poisson distribution.

Other statistical approaches include multiple linear regression (Sinnakaudan et al. 2006) and nonlinear regression. Asselman (2000) noted that many approaches to fitting sediment rating curves involves a power law, for example $Y = \theta_1 Q^{\theta_2}$ where the response $Y$ is a nonlinear function of discharge $Q$ and parameters $\theta_1$ and $\theta_2$, whose values generally hold no direct physical meaning and whose estimation usually involve the least squares criterion. Machine learning algorithms for sediment transport models have also been developed (e.g., Dogan et al. 2009). These approaches differ in that the underlying function is assumed unknown and is estimated using suspected predictor variables. Despite demonstrating good fits to observed values, statistical approaches to sediment transport have been criticized as ‘black-box’ methods (Asselman 2000) because model parameters are often physically uninterpretable with no explanatory power.

In this paper we discuss a Bayesian statistical approach—a relatively new method in fluvial sediment transport. Bayesian methods represent a class of
statistical methods that, for sediment transport, address several shortcomings of previous approaches. Historically, nonlinear models have been calibrated through nonlinear regression using weighted and non-weighted algorithms. These approaches assume model parameters—or some transformation—are normally-distributed and seldom does one see the possibility to use other distributions. Further, these models assume the underlying parameters are fixed (Lopez and Garcia 2001). Bayesian methods relax the normally-distributed assumption on the parameters and provide a formal mechanism to incorporate prior knowledge into the model. Bayesian methods assume that parameters are random variables arising from a probability density rather than a single fixed value. Griffiths (1982) first proposed using Bayesian inference models in geomorphology—including sediment transport—but only discussed fully conjugate models where solutions could be derived analytically through arguments of calculus; more complex models were unattainable. The paper of Gelfand and Smith (1990), however, demonstrated the applicability of Markov chain Monte Carlo (MCMC) methods to overcome this limitation.

To date, there are few instances in which Bayesian statistical models have been used in sediment transport. Kanso et al. (2005) employed Bayesian statistics in parameter estimation for sediment erosion in Parisian sewers, Fox and Papanicolaou (2008) used Bayesian methods for watershed erosion processes, and Wu and Chen (2009) demonstrated how Bayesian methods can be utilized in sediment entrainment problems. Wu and Chen (2009) note the sparse use of Bayesian applications in hydraulic engineering and conjecture that this is largely due to a lack of demonstration as well as its foreign resemblance to traditional deterministic approaches familiar to engineers. Chapter 3 of this dissertation provides a pedagogical example of how Bayesian methods may be employed in sediment transport problems and Chapter 5 demonstrates the implications of Bayesian
models on sediment budgets for the Snake River, WY.

In what follows, we compare a traditional curve fitting approach using frequentist nonlinear least squares (NLS) estimation with a Bayesian sediment transport model. The comparison includes the inferential and predictive capabilities of each method as well as a discussion on the potential benefits of modeling sediment transport in the Bayesian framework.

4.2 Methods

4.2.1 Model Formulation

Using $R$ (R Development Core Team 2010), three statistical formulations were tested—one nonlinear and two Bayesian regressions. All incorporate a deterministic model of sediment transport.

An excess shear model was selected as the deterministic function of sediment transport for the simulated and flume observations. This model is:

$$q^* = a(\tau^* - \tau_c^*)^b,$$

(4.1)

where $q^*$ is the normalized dimensionless unit sediment discharge (Einstein transport parameter), $\tau^*$ is the Shields number, $\tau_c^*$ is the critical Shields number, and $a$ and $b$ are empirical parameters, with values of $a = 8$ and $b = 1.5$ corresponding to the well-known Meyer-Peter and Müller (1948) bed load equation. This relationship can be brought into dimensional space, and if we substitute the skin friction relation provided in Wilcock (2001), this equation becomes:

$$q_s = 8\sqrt{(s-1)gD^3} \left( \frac{0.052(gSD)^{0.25}u^{1.5}}{(s-1)gD} - \frac{\tau_c}{(s-1)\rho gD} \right)^{1.5},$$

(4.2)
where $q_s$ is unit sediment discharge in $\text{m}^2/\text{s}$, $s$ is specific gravity of the sediments, $g$ is acceleration due to gravity in $\text{m}/\text{s}^2$, $D$ is particle diameter in $\text{m}$, $S$ is channel slope in $\text{m}/\text{m}$, $\tau_c$ is critical shear stress in Pa, and $u$ is depth-averaged velocity in $\text{m}/\text{s}$.

The methods described in the following sections use (4.2) as the governing relationship. It is assumed that all parameters except critical shear are adequately measurable. Thus, using simulated and laboratory observations each method infers parameter value(s) for $\tau_c$ that make the observed transport events most likely.

In nonlinear least squares regression parameters are iteratively selected according to the least squares minimization criterion, $\min \left( \sum_{i=1}^{n} \left( \log(q_{s,o,i}) - \log(\hat{q}_{s,i}) \right)^2 \right)$ where $q_{s,o,i}$ is an observed value and $\hat{q}_{s,i}$ is a predicted value. The log transformation is used to stabilize the variance required for NLS. The use of NLS requires that parameters and residuals be normally distributed. When these conditions are met, valid predictions and parameter estimates can be inferred. Interested readers are referred to Bates and Watts (1988) for detailed information on NLS.

The Bayesian approach used here is described in Chapter 3. The basic formulation of the Bayesian model is:

$$\begin{align*}
\tau_c, \sigma^2 | \log(q_{s,o}) & \propto \left( \prod_{i=1}^{n} \left[ \log(q_{s,o,i}) | \tau_c, \sigma^2 \right] \right) \left[ \tau_c \right] \left[ \sigma^2 \right], \quad (4.3)
\end{align*}$$

where $q_{s,o}$ is a vector of $n$ observed transport events, and $\sigma^2$ is a variance term representative of measurement error, model misspecification, and natural variability. Equation (6.8) shows that the joint probability distribution (posterior) of $\tau_c$ and $\sigma^2$ given the log of observed sediment transport events $q_{s,o}$ is proportional to the product of the observation likelihoods and priors for $\tau_c$ and $\sigma^2$. Formula (6.8) is expressed as a proportionality instead of an equality—a simplification afforded by MCMC. The likelihood—the distribution from which observations arise—is assumed
to be lognormally distributed; that is, \( \log(q_{s,o,i}) | \tau_c, \sigma^2 \sim N(\log(q_{s,i}), \sigma^2) \), where \( q_{s,i} \) is governed by (4.2) and the symbol ‘\( \sim \)’ reads ‘is distributed as.’ This specification implies that sediment transport generally behaves according to the \( \log \) of (4.2), but with variance \( \sigma^2 \) due to natural variability, measurement error, and model misspecification.

One feature of Bayesian methods is that they incorporate prior knowledge on parameters via prior distributions. Priors for \( \sigma^2 \) and \( \tau_c \) allow modelers to assume that parameters arise from any valid probability distribution. Here, \( \sigma^2 \) is assumed to follow an inverse gamma (I.G.) distribution, \( \sigma^2 \sim I.G.(r,q) \), where \( r \) and \( q \) are shape and scale parameters. The inverse gamma distribution is appropriate because it is only defined for positive real numbers, like variances. Critical shear is assumed to follow a truncated normal distribution, \( \tau_c \sim T.N.\left(\mu_{\tau_c}, \sigma_{\tau_c}, a, b\right) \), where \( \mu_{\tau_c} \) is the prior mean, \( \sigma_{\tau_c} \) is the prior standard deviation, \( a \) is the lower-bound, and \( b \) is the upper-bound. The truncated normal is selected because it constrains the parameter space of \( \tau_c \) (negative and excessively high critical shear values make no physical sense) while retaining the ability to be informative or uninformative in model assumptions.

The Bayesian statistical model in (6.8) cannot be solved analytically. Markov Chain Monte Carlo methods, however, provide a means to draw samples from the posterior distribution in such models. In particular, we used a Gibbs sampler and a Metropolis-Hastings update to achieve this. Readers are referred to Chapter 3 for further information regarding these computational methods.

4.2.2 Method Testing

To test these methods we used simulated observations as well as observations from laboratory flume experiments. Four different data scenarios were simulated, as
shown in Figures 1(a) through (d). Two different prior distributions for $\tau_c$ were specified for the Bayesian model for each data scenario. The first, a diffuse prior, shown in Figure 2(a), assumes all values on the interval $[0, 10]$ are equally probable. The second prior shown in Figure 2(b) gives preference to values centered on 6.72 Pa with a small variance. A vague prior having a mean of 0.07 and a variance of 150 was also specified for the variance parameter $\sigma^2$ in the simulation study.

These methods were also tested using laboratory flume data of a uni-size sediment transport experiment by Smart (1984), included as Figure 1(e). As before, two prior distributions for $\tau_c$ and one for $\sigma^2$ were specified for the flume observations. The prior distributions for critical shear mimic the shapes in Figure 2, but with different parameter values. The diffuse prior for critical shear is $T.N.(10, 5000, 0, 20)$, and the informative prior is $T.N.(14, 0.75, 0, 20)$. A vague prior for $\sigma^2$ having a mean of 0.10 and a variance of 100 was also specified for the flume data.

4.3 Results and Discussion

Table 4.1 presents the inferred values and confidence/credible intervals for critical shear resulting from each method. The critical shear value that was realized for the simulated data scenarios is 6.72 Pa. Figure 3 shows the resulting predictions using the NLS and BM P2 methods on the flume data. The shaded regions in Figure 3 represent the 68%, 90%, and 95% credible intervals for the Bayesian predictive distribution. The dash-dot line running down the center of the posterior predictive distribution (PPD) represents the predictions from NLS. The inset figures labeled (a) through (c) are the probability densities of the PPD at the three cross-sections denoted by the short vertical lines with corresponding labels (a) through (c). The dashed vertical lines in the inset plots represent the 0.025 and 0.975 quantiles—the
solid vertical line denotes the observed transport rate at that shear cross-section.

In Table 4.1 we see point estimates (posterior mean for Bayesian estimates) and 95% inference regions for each method and scenario. For the simulated data, NLS exhibits identically distributed (i.i.d.) residuals—that is, $\epsilon \overset{i.i.d.}{\sim} N(0, \sigma^2)$, thereby validating parameter inference and prediction.

Table 4.1 also shows that for low-variance data scenarios (1 and 2) all approaches provide nearly identical inference. Differences in estimates appear in data scenarios 3 and 4, being comprised of noisier (more realistic) observations. The Bayesian model with the informative prior (BM P2) gives more accurate and precise inference in these last two data scenarios. The Bayesian model with an uninformative prior also infers a credible interval just as precise or better than the confidence interval for NLS in data scenarios 3 and 4.

Regarding the effect of prior knowledge on posterior inference, critics of Bayesian methods suggest that prior distributions reduce the objectivity of the analysis because we assign distributions to the model parameters. Box and Tiao (1992), however, observe that traditional approaches (such as NLS) are implemented “as if it were believed a priori that the probability distribution of the data was exactly Normal, and that each observation had exactly the same variance, and was distributed exactly independently of every other observation.” This, in effect, is the prior knowledge about the parameters implicit in traditional analysis techniques. The Bayesian approach exposes all the components, even prior assumptions, of the inferential system to appropriate scrutiny. Box and Tiao (1992) further state that “if non-Normality was suspected, for example, it might be sensible to postulate that the sample came from a wider class of parent distributions of which the Normal was a member. The consequential analysis could be difficult via sampling theory but is readily accomplished in a Bayesian framework.”
For the flume data, Table 4.1 shows that the Bayesian methods result in more precise estimates of the realized value of critical shear than NLS. The NLS and BM P2 predictions in Figure 3 show the most obvious difference between the two methods: NLS predicts as a line through the points and BM P2 predicts as a distribution of transport events for a given shear value. The distributions shown in Figure 3(a) through (c) can be used to obtain sediment discharge quantiles thereby associating an exceedence probability of some transport rate at a given shear. Often we are more concerned with the occurrence of extreme events rather than the mean behavior of a system and the Bayesian approach provides a readily-interpretable predictive distribution for a given shear value. Further, this distribution arises naturally from the model and is not assumed to follow any particular distribution. Prediction intervals for nonlinear regression can be achieved through a linear approximation of the nonlinear regression after which ordinary least squares (OLS) prediction intervals are derived from the approximation (Bates and Watts 1988). The prediction intervals of the approximated model are constrained by normality. The net effect is that for instances where parameter normality is justified and the linear approximation to the nonlinear model is adequate, the two models give comparable results. When these assumptions are not justified, however, the nonlinear regression approach is not guaranteed to be reliable while the Bayesian posterior will adapt to the underlying distribution.

The utility of Bayesian methods has also been considered in the broader hydrology literature for its implications in decision theory (Davis et al. 1972; Duckstein and Szidarovszky 1977), a branch of statistics that addresses how to make optimal decisions considering risk and cost. These studies demonstrate how Bayesian methods can be used to balance the costs of over- and under-design with risk of failure and its associated costs. Similar questions could be posed in the field
of sediment transport (e.g., Lopez and Garcia 2001). For instance, in river restoration many system reaches are located downstream of dams, and are cut off from natural sediment supply. River managers may augment the sediments (e.g., Trinity River, CA; Colorado River, AZ) to maintain habitat and avoid degradation but this involves estimating sediment mobilization. The approach demonstrated in this paper could be used in these situations in conjunction with loss and risk functions to arrive at scientifically optimal decisions. There is considerable difficulty, however, in developing relevant loss functions for these scenarios due to the complexities of river behavior—this must be developed further before these concepts can be implemented.

4.4 Conclusions

The purpose of this technical note is to highlight some of the potential benefits Bayesian statistical approaches can offer fluvial sediment transport analyses. While we do not suppose it is the only way forward, there are several key advantages to this approach that make it attractive. First, regarding model parameters as random variables has strong support in the literature and is more consistent with observation than fixed values. Second, Bayesian approaches permit parameters to be easily modeled from any valid distribution and the normality constraints for residuals and parameters in traditional methods are not an issue. Third, the proposed model provides robust predictions of transport as distributions, which are not constrained by normality, making it possible to easily determine the mean and extreme behaviors of the process. Fourth, Bayesian parameter inference regions for simulated and flume observations were either consistent with or more precise than regions inferred by NLS. Lastly, the Bayesian formulation explicitly communicates all model components to appropriate criticism, such as choices for priors, whereas the prior
assumptions for traditional approaches are implicit in the use of the method.

References


Table 4.1: Inferred critical shear. BM-D = Bayesian model, diffuse prior; BM-I = Bayesian model, informative prior; NLS = Nonlinear least squares. “C.I.” denotes the credible interval for BM-D and BM-I and confidence interval for NLS.

<table>
<thead>
<tr>
<th></th>
<th>Data Scenario 1</th>
<th>Data Scenario 2</th>
<th>Data Scenario 3</th>
<th>Data Scenario 4</th>
<th>Flume Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_c$</td>
<td>95% C.I.</td>
<td>$\tau_c$</td>
<td>95% C.I.</td>
<td>$\tau_c$</td>
</tr>
</tbody>
</table>
Fig. 4.1: Data scenarios. (a) data scenario 1, low variance, many observations; (b) data scenario 2, low variance, few observations; (c) data scenario 3, high variance, many observations; (d) data scenario 4, high variance, few observations; (e) Smart (1984) 10 mm observations.
Fig. 4.2: Prior distributions on $\tau_c$: (a) diffuse prior (BM-D), (b) informative prior (BM-I). Vertical line denotes the realized value of $\tau_c$ at 6.72 Pa in simulations. Truncated normal distribution is parameterized as $T.N.(\mu, \sigma, \tilde{a}, \tilde{b})$. 
Fig. 4.3: Predictions for Smart (1984) observations. Shaded regions in the main plot represent the 68%, 90%, and 95% credible intervals on the predictive distribution. Black line (dash-dot linetype) is the NLS prediction. Inset plots (a) through (c) correspond to cross-sections of the predictive distribution at the observations denoted by black triangles and labels '(a)', '(b)', or '(c)'. Inset plots show the predictive distribution as a density function, the 0.025 and 0.975 quantiles as black dashed lines, and the observed transport value as a solid vertical line.
CHAPTER 5
ACCOUNTING FOR UNCERTAINTY IN CUMULATIVE SEDIMENT TRANSPORT USING BAYESIAN STATISTICS

Abstract

That sediment transport estimates have large uncertainty is widely acknowledged. When these estimates are used as the basis for a subsequent analysis, such as cumulative sediment loads or budgets, treatment of uncertainty requires careful consideration. The propagation of uncertainty is a problem that has been studied in many other scientific disciplines. In recent years, Bayesian statistical methods have been successfully used to this end in hydrology, ecology, climate science, and other disciplines where uncertainty plays a major role—their applications in sediment transport, however, have been few. Previous work demonstrated how deterministic sediment transport equations can be brought into a probabilistic framework using Bayesian methods. In this paper, we extend this basic model and apply it to sediment transport observations collected on the Snake River in Wyoming, USA. These data were used previously to develop a 50-year sediment budget below Jackson Lake dam. We revisit this example to demonstrate how viewing sediment transport probabilistically can help better characterize the propagation of uncertainty in the calculation of cumulative sediment transport. We present the development of probabilistic sediment rating curves that rely on


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deterministic sediment transport equations and then show how these can be used to compute the distribution of sediment input and output for each year from 1958 to 2007. The Bayesian approach described provides a robust way to quantify uncertainty and then propagate it through to subsequent analyses. Results show that transport uncertainty is quantified naturally in the Bayesian approach, making it unnecessary for modelers to assume some specified error rate (e.g., ± 5%) when developing estimates of cumulative transport. Further, we demonstrate that a Bayesian approach better constrains uncertainty and allows sediment deficit and surplus to be examined in terms of quantified risk.

5.1 Introduction

Estimates of sediment transport rate are widely known to have large uncertainty (Gomez and Church, 1989; Wilcock, 2001). Uncertainty poses a particular challenge for cases in which the cumulative transport is of interest. These include the delivery of sediment to reservoirs and other receiving waters, the supply of sediment to a river reach of concern, and the balance of input and output such that the net storage of sediment in a reach can be determined. Because these estimates involve propagating uncertainty over time, typically as a function of water discharge, defining a model that describes the uncertainty of the transport estimate is necessary.

The topic of uncertainty is widely treated in many scientific disciplines, and recent advances in statistical and computational methods have created a new set of tools at the disposal of researchers. One such method that is gaining prominence in diverse scientific fields is that of Bayesian statistical models. These tools provide a formal and theoretically solid framework in which deterministic process functions can be incorporated into a probabilistic framework thereby facilitating the
quantification of parameter, structural, and predictive uncertainty. This approach has been used extensively in other disciplines, but has not been widely used in sediment transport applications. Griffiths (1982) was an early proponent of this approach and demonstrated geomorphically relevant examples, but these examples could be solved analytically. More complex models were unattainable until Geman and Geman (1984) and Gelfand and Smith (1990) demonstrated the use of Markov Chain Monte Carlo (MCMC) methods in Bayesian statistical analysis.

Chapter 3 outlined the development of a simple Bayesian statistical model for sediment transport and demonstrated some of the benefits of this approach, including the ability to incorporate deterministic functions into a probability framework and to use prior knowledge and make predictions as probability distributions. In this paper we extend the basic Bayesian sediment transport model developed in Chapter 3 and apply it to sediment transport observations made on a large gravel-bed river. Using the model predictions, we outline the development of a probabilistically based sediment mass balance using this modeling framework, and we evaluate the implications of viewing sediment transport probabilistically on the calculation of long-term sediment budgets.

5.1.1 Sediment budgets

One of the most challenging calculations in sediment transport is to determine the mass balance, or sediment budget, of a reach as the difference between input and output over a defined period of time. The cumulative transport at upstream and downstream sections, as well as any significant tributaries, must be determined; and an effective means is required to combine estimates of variance (due to errors or natural variability) at each section into a credible variance estimate for the net change in sediment storage. An alternative approach is to directly measure changes
in sediment storage through field resurveys of channel morphology as well as the use of repeat aerial photography comparisons (e.g., Ashmore and Church 1998, Eaton 2001, Gaeuman 2003). These methods, however, are not feasible for systems for which an historical survey, aerial or otherwise, is unavailable.

Thus, an alternate approach to constructing a sediment budget is to use the existing streamflow record and sediment rating curves to quantify annual sediment yield. This mass balance approach has been employed on the lower Colorado River (Schmidt 1999, Topping et al. 2000, Hazel et al. 2006), the Sacramento River (Singer and Dunne 2004), the Toutle River (Major 2004), the upper Green River below Flaming Gorge dam (Andrews 1986, Grams and Schmidt 2005), the Fraser River (McLean et al. 1999a), and the Ebro River (Vericat and Batalla 2006). While the reasoning behind constructing a sediment mass balance using historic streamflow records and sediment rating curves is sound, significant uncertainty is associated with any sediment rating curve. A common challenge concerns application of sediment transport relations beyond the period of available observations.

Sediment rating curves that do not account for the variability of bedload will at best provide a quantification of the mean behavior of the system, and often the extremes are of the most concern. What is not quantified in traditional approaches to sediment budgets is the variability one can expect to see in the sediment influx and outflux, as well as the magnitude of the difference between the influx and outflux required in order to be considered significant. In the absence of this information, a sediment mass balance may be indeterminate (e.g., Grams and Schmidt 2005). While the approach to sediment budgets offered in this paper does not overcome the problems associated with non stationary sediment rating curves, it does address the issue of bedload variability for a given flow condition.

Although the uncertainties associated with estimates of sediment transport
developed from sediment rating curves have been widely acknowledged no universally accepted strategy for rigorously quantifying uncertainty in long-term estimates of sediment transport has been identified. Previous efforts to grapple with uncertainty in cumulative estimates of sediment transport include assuming that direct measurements of sediment transport are accurate to within some specified percentage of the total transport rate (Topping et al., 2000; Major, 2004; Grams and Schmidt, 2005), calculation of confidence intervals on the rating relation (Vericat and Batalla, 2006), or implementation of an error propagation analysis (Dunne et al., 1998). McLean et al. (1999a) used a Monte Carlo simulation to assess the precision of the Fraser River sediment sampling program. The Monte Carlo analysis was used to compute the coefficient of variation for replicate samples collected at a given vertical in an attempt to quantify the variability in transport rates resulting from both actual fluctuations in transport and sampling errors. In contrast, Singer and Dunne (2004) emphasized the role of streamflow variability in interannual estimates of bed material load. Toward this end, they coupled a stochastic streamflow model with calibrated sediment transport formulae to quantify variability in decadal estimates of bed material flux.

In what follows, we discuss the advantages of a Bayesian modeling framework over alternative approaches for quantifying uncertainty in sediment transport problems.

5.1.2 Bayesian models

Bayesian statistical models have gained increased prominence over the last 25 years owing in large part to advances in computing power and the development of sophisticated numerical methods. This modeling framework possesses several desirable properties that make it well suited for describing complex phenomena,
such as sediment transport.

The first of these properties relates to the treatment of latent model parameters—those values which we cannot measure directly but must infer from observation. In deterministic approaches, model parameters (such as critical shear in sediment transport) are treated as fixed but unknown, and any variation around this fixed value is a metric of the uncertainty of the true single fixed value. It has long been acknowledged, however, that the threshold at which sediments move is not a fixed value but is, in fact, a probability distribution (e.g., Grass, 1970; Gessler, 1971; Paintal, 1971; Kirchner et al., 1990). Treating critical shear as a fixed value is therefore inappropriate and is largely for computational convenience. A Bayesian approach makes it possible to model (i.e., estimate or infer values of) latent parameters as random variables arising from probability distributions. The ability to model parameters, such as critical shear, as a random variable reconciles the long-established concepts of threshold with an appropriate modeling framework.

Because the underlying model parameters in a Bayesian approach are random variables, functions of these parameters (including predictions) are also random variables. The significance of this is that model predictions can be defined probabilistically. Deterministic approaches to sediment transport using fixed parameter values generally involve fitting a line through a distribution of observations. What is not quantified in the deterministic approach, however, is the extent to which deviations from this fitted line can be expected—it is well documented that sediment transport is variable even at steady-state conditions (Knighton, 1998; Hicks and Gomez, 2005; Turowski, 2010). One approach, then, is to make a set of Monte Carlo simulations using probability distributions for the model parameters thereby resulting in an ensemble of model predictions. This approach assumes that (i) the parameters are fixed and their variance is a reflection
of uncertainty in the point estimate; and (ii) that all of the observed variability in sediment transport is directly because of parameter uncertainty. This approach will yield useful results only if the specified parameter distributions are correct.

Returning to a Bayesian approach, distributions for the latent parameters (prior distributions) are assigned just as in the forward stochastic approach and these distributions are ‘updated’ by incorporating observations—sometimes called data assimilation (e.g., Wikle and Berliner (2007)—of sediment transport events into the Bayesian statistical model. These updated distributions, called posterior distributions, are estimates of the underlying parameters in light of the observations and prior distributions. Posterior distributions are weighted combinations of prior knowledge and information from observation—the more observations you have, the less weight is placed on prior distributions. These inferred parameter values can then be used to calculate a posterior predictive distribution (PPD) as opposed to a single predictive line. Thus, the Bayesian model accomplishes both parameter estimation and stochastic prediction in one theoretical framework.

Bayesian statistical models, while based on probability, do not solely rely upon probability distributions or empirical relations without regard to the relevant physics of the process. In recent years, statistical approaches to bedload transport employing machine learning algorithms have been demonstrated (Bhattacharya et al., 2007; Dogan et al., 2009; Sasal et al., 2009) wherein the nonlinear functions that govern transport are derived using sets of covariates and parameters trained to the observed bedload discharges. While machine learning algorithms seek to estimate an unknown function from covariates, the Bayesian model described in this paper incorporates deterministic equations. Machine learning techniques often result in good model fits, though the resulting parameters do not necessarily carry the same interpretability as do models that are based on equations derived from the
physics of the phenomenon—this is especially true when the covariates are orthogonally transformed, such as in principal components analysis. As will be shown in what follows, incorporating deterministic or physics-based reasoning into Bayesian models is straightforward. This allows experts to select the deterministic models that are most appropriate and still retain the ability to model the phenomenon probabilistically.

Last, a Bayesian sediment transport model makes it possible to partition out variability—that is, to quantify the various sources of variability in the transport process. For instance, because model parameters are treated as random variables arising from probability distributions, we can expect some variance in predictions based on this alone. Further, sediment transport is spatially and temporally variable due to constantly changing hydraulic conditions, including changing bed topography, turbulence, varying supplies of bed material from upstream processes, and lastly irreducible noise brought about by stochasticity (Grass, 1970; Kirchner et al., 1990; Knighton, 1998; Gomez and Phillips, 1999; McLean et al., 1999b; Bunte and Abt, 2005; Hicks and Gomez, 2005; Diplas et al., 2008). An additional contributor of variance in model predictions is the conceptual model itself that we use to describe the process. Invariably, the models we use to describe physical systems are simplifications of reality, and many ways to describe the same process exist (Gomez and Church, 1989). Thus, the selection of a particular deterministic relationship will have an effect on the variability of predictions. Finally, data collection error will also contribute to observed variability in fluvial transport (McLean et al., 1999b; Diplas et al., 2008). The model presented in what follows distinguishes between variability owing to the model parameters being random variables, and variability because of stochasticity, measurement error, and model misspecification.
5.2 Study area

The Bayesian model described above is used here to quantify uncertainty associated with sediment rating curves developed from bedload transport data collected on the Snake River in northwestern Wyoming. The bedload transport data were collected for the purpose of developing a sediment budget for a 16-km stretch of the Snake River within Grand Teton National Park (GTNP) (Erwin et al., 2011). The study reach begins ∼9 km downstream from Jackson Lake Dam (JLD) at the confluence with Pacific Creek, shown in Fig. 5.1 and extends to Deadman’s Bar.

The upper Snake River is a wandering, gravel-bed river. Through the study area, the Snake River and its tributaries flow through outwash produced during the Pinedale glaciation. Much of the Snake River is flanked by Pleistocene outwash terraces, which intermittently confine the valley (Love et al., 2003). Deadman’s Bar is located in one of these confined areas, which makes it an ideal sampling location because the reach planform has changed very little over the last 50 years (Nelson, 2007). This eliminates any complications caused by transitions to multiple threads in the channel, or changes in sinuosity or braiding index.

Jackson Lake originally formed during the last glaciation when it was impounded by a recessional moraine. In 1906, JLD was constructed at the outlet of Jackson Lake to capitalize on this natural water storage location. The JLD increased the level of Jackson Lake by 11.9 m, creating 109 m$^3$ of storage in Jackson Lake reservoir. Importantly, the dam did not change the sediment supply to the upstream end of the study area because Jackson Lake existed prior to construction of JLD and the supply here has effectively been zero for thousands of years. Thus, the primary source of coarse sediment to the Snake River in GTNP is tributaries.

The upper Snake River and its tributaries drain the Teton and Absaroka Mountains and the Yellowstone Plateau. Although stream flow of the Snake River
immediately downstream from JLD is entirely determined by dam releases, flow farther downstream results from the combined effects of dam releases and natural inflow from tributaries. The annual flood in the watershed is driven by spring snowmelt and typically occurs in May or June. However, the average annual peak flow is now significantly less than the unregulated peak flow due to dam operations (Erwin et al., 2011).

The field data set we used was collected from tethered rafts on three large gravel-bed rivers (see Fig. 5.2). The challenges in collecting the transport samples were exceptional, not least to collect sufficient samples to represent a highly variable transport field, and the resulting transport data demonstrate considerable scatter not uncommon in field data sets. Although other gravel rivers can be found with transport data sets that are more abundant and empirically define a tighter sediment rating curve, the abundance and scatter in the data is not a primary consideration in the basic purpose of this paper, which is to demonstrate application of a Bayesian approach in developing a more robust characterization of uncertainty and a more credible estimate of cumulative transport. In fact, the Snake River and its tributaries provide an appropriate illustration of how the Bayesian approach can be used to more effectively determine cumulative transport rates with field sediment transport data.

Developing a long-term sediment budget for the Snake River using the transport data of Erwin et al. (2011) faces two important challenges that merit attention. First, transport rates at the budget outlet (Snake River) were measured in 2007 whereas transport rates at the budget inlets (Buffalo Fork and Pacific Creek) were measured in 2006. Second, the cumulative sediment transport and sediment budgets are developed by applying transport observations from two years to a fifty-year flow record. This first issue requires the assumption that transport rates at the sites
were similar in both years. The second issue requires the more severe assumption that the relation between flow and transport remain stationary over entire period of record. There is no specific information to inform these assumptions, although several factors suggest that the assumption is worth making. First, the broad patterns of sediment supply to the reach have remained unchanged throughout the Holocene (Jackson Lake Dam altered flows to the reach, but not sediment supply). Second, no significant changes in the reach planform have been observed since 1945 (Nelson 2007). Third, sediment supply to the reach is abundant and tracer experiments indicate that common floods are capable of fully mobilizing the bed material. Together, these conditions suggest a transport (rather than supply) limited reach that has been in place for centuries, such that an assumption of stationary sediment rating curves is plausible.

The issue of rating curve stationarity is relevant to the accuracy of the long-term sediment budget determined from short-term data. This is a common and fundamental challenge that no method, statistical or otherwise, can resolve. Our goal in this paper is not to present and interpret a sediment budget for the reach (this has already been done by Erwin et al. 2011), but to demonstrate an approach that allows uncertainty in the budget calculation to be more effectively evaluated and applied to calculations of cumulative load. Any error arising from the assumption of stationarity does not impair this effort.

5.3 Methods

In this section we describe the methods used to develop the probabilistic sediment rating curves and sediment budget. Sediment input from tributaries and output from the mainstem were quantified across a range of discharges allowing the construction of predictive sediment distributions for each location. These
distributions were then used to construct long-term estimates of sediment influx and outflux using historical streamflow records.

5.3.1 Sediment transport observations

Three sampling sites were established for measuring bedload transport through the study reach. In 2006, bedload transport was measured on Buffalo Fork and Pacific Creek for the purpose of determining sediment inputs to the study reach. In 2007, bedload transport was measured on the Snake River at Deadman’s Bar in an effort to quantify sediment outputs.

The sampling methods are described in detail in [Erwin et al., 2011] but are summarized here. At each sampling site, transport rates were measured using a raft-based sampling platform and a Toutle River 2 (TR-2) bedload sampler ([Childers, 1999] [Wallick et al., 2010]). Samples were collected using a modified version of the equal width increment (EWI) method, where each complete measurement consisted of one pass across the channel where 10–12 samples were taken at equally spaced intervals across the active bed. The sampler remained on the bed for 30–240 seconds at each vertical, and the time interval remained constant for each sample. All samples were sieved and weighed in $1/2 - \phi$ size classes. Channel conditions remained consistent during the 2-year sampling period so as to justify the assumption that there was no pronounced shift in the sediment rating curve from one year to the next.

5.3.2 Bayesian sediment transport model

Fundamentals

The basic premise of Bayesian models is discussed below. If we were to assume
some generic process $z$ as a function of some parameter $\theta$, a Bayesian model of this process would be

$$[\theta|z] = \frac{[z|\theta][\theta]}{\int[z|\theta][\theta]d\theta}$$  \hfill (5.1)

where the square brackets ‘[]’ denote a probability distribution, and the vertical bar ‘|’ denotes a ‘given’ such that $[\theta|z]$ is interpreted as the distribution of the model parameter $\theta$ given observations of the process $z$. The model in Eq. (5.1) is conventionally expressed as a proportionality, $[\theta|z] \propto [z|\theta][\theta]$, because the denominator in Eq. (5.1) is a fixed but unknown normalizing constant and MCMC does not require this normalizing constant to be known.

In words, the Bayesian model above allows us to make inference on the parameter $\theta$ by using the intrinsic information contained about it in observations of the process $z$. Given observations of $z$, it is possible to back calculate values of $\theta$, but algebraic answers only return a single value. Because the Bayesian model treats parameters as distributions, the Bayesian answer is analogous to the back calculated parameter values for $\theta$, except that it is a distribution instead of a single value. This back calculated distribution is the posterior distribution, $[\theta|z]$, which reads as ‘the distribution of $\theta$ given observations of $z$’. The right-hand side of Eq. (5.1) consists of the likelihood and the prior. The prior distribution, $[\theta]$, is supplied to the model by the user. It is a statement of what is known about $\theta$ before the new observations were collected. This could be a summary of the literature on the parameter, an educated guess, or it can serve to constrain the parameter space to a physically plausible range. The likelihood, $[z|\theta]$, sometimes called the data model, represents the distribution of observations given the parameter $\theta$. The likelihood describes the structure of the process being modeled and recognizes the fact that observations of $z$ are dependent on the parameter $\theta$. 
Readers are referred to Chapters 3 and 4 for more information on Bayesian models, and specifically Bayesian sediment transport.

**Sediment transport governing equations**

As was mentioned previously, the Bayesian framework integrates deterministic functions into a probabilistic framework. It was established in Erwin et al. (2011) that the Parker (1979) and Parker (1990) models represented the observed Snake River transport data well. Further, Wilcock (2001) advocated their use because they are well suited to predicting sediment transport over different ranges of grain shear stresses. These equations are used in the present paper to provide a representative comparison to the work done in Erwin et al. (2011) (see the cited publications for further justification). These relationships are

\[
W^* = \begin{cases} 
11.2 \left(1 - 0.846 \frac{\tau'}{\tau_r}\right)^{4.5}, & \text{for } \frac{\tau'}{\tau_r} > 1 \text{ (Parker, 1979)} \\
0.0025 \left(\frac{\tau'}{\tau_r}\right)^{14.2}, & \text{for } \frac{\tau'}{\tau_r} < 1 \text{ (Parker, 1990)}
\end{cases}
\]  

(5.2)

where \(W^*\) is the dimensionless transport rate, \(\tau_r\) is the reference shear (a surrogate for critical shear), and \(\tau'\) is the skin friction. The value for \(W^*\) is defined as

\[
W^* = \frac{g q_s (s - 1)}{\left(\frac{\tau'}{\rho}\right)^{1.5}}
\]  

(5.3)

where \(g\) is gravity, \(q_s\) is the unit bedload transport rate, \(s\) is the specific gravity (2.65), and \(\rho\) is the density of the water. The relationship used to calculate skin
friction was derived in Erwin et al. (2011) and is provided here as a reference

\[ \tau' = 17k^{1.5} (SD_{65})^{0.25} Q^{1.5m} \]  \hspace{1cm} (5.4)

where \( S \) is the slope, \( D_{65} \) is the 65th percentile grain size, \( Q \) is discharge, and \( k \) and \( m \) are empirical coefficients determined in Nelson (2007). Lastly, the conversion to dimensionless shear stress from shear stress follows:

\[ \tau_r^* = \frac{\tau_r}{(s-1) \rho g D_{50}} \]  \hspace{1cm} (5.5)

Using the equations specified in Eq. (5.2) through Eq. (5.4), we can solve for the dimensional transport rate, \( Q_s \), resulting in

\[ Q_s = \begin{cases} 
11.2 \left( 1 - 0.846 \frac{\tau_r}{\rho} \right)^{4.5} \left( \frac{\tau'}{\rho} \right)^{1.5} \frac{W}{g(s-1)}, & \text{for } \frac{\tau'}{\tau_r} > 1 \text{ (Parker, 1979)} \\
0.0025 \left( \frac{\tau'}{\tau_r} \right)^{14.2} \left( \frac{\tau'}{\rho} \right)^{1.5} \frac{W}{g(s-1)}, & \text{for } \frac{\tau'}{\tau_r} < 1 \text{ (Parker, 1990)} 
\end{cases} \]  \hspace{1cm} (5.6)

where \( W \) is the channel width.

**Bayesian model formulation**

The basic model in Eq. (5.1) was adapted for sediment transport, resulting in the following:

\[ [\tau_r, \sigma^2 | \log(Q_{s,o})] \propto \prod_{i=1}^{n} [\log(Q_{s,o,i}) | \tau_r, \sigma^2] [\tau_r][\sigma^2] \]  \hspace{1cm} (5.7)

where \( Q_{s,o} \) is the vector of total cross section sediment discharge observations,
\( Q_{s,o} = (Q_{s,o,1}, \ldots, Q_{s,o,n})' \), assumed to be generally governed by Eq. (5.6). The model specified in (6.8) makes inference on both \( \tau_r \) and \( \sigma^2 \), respectively, the reference shear stress and the variance.

The likelihood is the distribution from which the observations arise, and this distribution must be specified directly. Section 3.2.2 outlined the deterministic models used to describe the dynamics of sediment transport, and we refer to those equations as we construct the likelihood. In words, the following likelihood specification means that we believe sediment transport generally behaves according to the models developed in Parker (1979) and Parker (1990) while providing a term for the noise associated with this process, in measurements as well as in natural variability. To this end, we specified a normally distributed likelihood whose mean is defined by Eq. (5.6) with variance \( \sigma^2 \), as shown below:

\[
\log(Q_{s,o,i}|\tau_r, \sigma^2) \sim N(\log(Q_{s,i}), \sigma^2), \text{ for each observation, } i, \tag{5.8}
\]

where ‘\( \sim \)’ is a statistical notation meaning ‘is distributed as’. The likelihood specified above integrates a deterministic relationship that describes sediment transport into a probabilistic framework.

Because the model specified in Eq. (6.8) makes inference on two parameters, two prior distributions must be specified. In selecting prior distributions for parameters, care must be taken to ensure that the support of the distribution matches that of the parameter. For instance, we know that \( \sigma^2 \) must be greater than zero but less than infinity; and so an inverse gamma distribution (\( \sigma^2 \sim I.G.(r,q) \), where \( r \) and \( q \) are ‘hyperpriors’) is an appropriate selection. For the reference shear parameter, we know that it, too, must be greater than zero but less than any transport event with \( W^* < 0.002 \) (by construction). Thus, a lower- and upper-bound can be placed on \( \tau_r \).
To this end, a truncated normal distribution was specified as the prior for reference shear \((\tau_r \sim \text{T.N.}(\mu_{\tau_r}, \sigma_{\tau_r}, \tilde{a}, \tilde{b}))\), respectively representing a mean, standard deviation, lower-bound, and upper-bound for \(\tau_r\). One advantage of the truncated normal distribution is that it is very flexible and can take on a variety of shapes depending on the specified hyperprior values \((\mu_{\tau_r}, \sigma_{\tau_r}, \tilde{a}, \tilde{b})\), as shown in Fig. 5.3.

**Parameter inference and prediction**

One of the goals of formal statistical methods (both classical and Bayesian) is to make inference on latent or unobservable parameters. In order to do this, we must make observations of the process of interest after which we may then infer parameter values in context of the new observations. The posterior distribution in Eq. (6.8) is an updated joint distribution of the reference shear and variance parameters after considering the prior distributions (as specified by the expert) and the information available in the new observations. This distribution can be used to identify a credible interval or a range of values between which there is a \((1 - \alpha)\%\) probability of containing the realized parameter value. The term “credible interval” is a term in Bayesian statistics that is analogous to a confidence interval from classical statistics. The reason for this distinction is rooted in the interpretation of what probability truly represents. Readers are referred to any textbook on Bayesian statistics for a more detailed explanation (e.g., Gelman et al., 2004; Robert, 2007). This credible interval is computed by finding the \(\alpha/2\) and \(1 - \alpha/2\) quantiles for the \((1 - \alpha)\%\) credible interval. The distribution of these parameters is not constrained by requirements of normality (see Chapter 4). Most classical statistical methods assume that model parameters (or transformations thereof) are normally distributed. These assumptions are relaxed in a Bayesian approach and only prior distributions on the parameters are specified. As a result, these specifications of the
prior distributions are based more on scientific merit than computational convenience (Box and Tiao 1992). The posterior distribution for these parameters arises naturally from the prior distributions and likelihood and represents the updated knowledge on the parameters. These updated parameter distributions can then be used for prediction.

Prediction in the Bayesian framework is nicely integrated into the theoretical framework. The posterior predictive distribution represents the probability distribution of future, in this case, sediment transport events given the previously observed events. Mathematically, this is expressed as

$$\log(Q_{s,o}) | \log(Q_{s,o}) = \int \int \log(\tilde{Q}_{s,o}) | \log(Q_{s,o}), \tau_r, \sigma^2 | \tau_r, \sigma^2 | \log(Q_{s,o}) d\tau_r d\sigma^2 \quad (5.9)$$

The solution to Eq. (5.9) defines a distribution of sediment transport rates for a given flow condition. In essence, it is the probabilistic analog of a fitted line through observations. This distribution recognizes the natural variability in sediment transport noted in the literature (e.g., Hamamori 1962; Knighton 1998; Hicks and Gomez 2005).

**Computational methods**

Because a solution to the posterior distribution in Eq. (6.8) is analytically intractable, the posterior must be sampled using MCMC with a Gibbs sampler and Metropolis-Hastings update to simulate observations from the posterior distribution. Readers are referred to Metropolis et al. (1953); Hastings (1970); Casella and George (1992); Chib and Greenberg (1995) for further information on these methods. The integration required for the posterior predictive distribution in Eq. (5.9) is performed by composition sampling of the MCMC posterior distributions.
(Tanner, 1996). More detailed information is available in Chapter 3. The R programming environment (R Development Core Team, 2010) was used to implement this model.

5.3.3 Sediment budget calculation

The approach to developing a probabilistic sediment budget is very similar to traditional approaches except that, in the probabilistic approach, we assume that a distribution of sediment transport rates—instead of a single prediction—may occur for any given flow condition (as defined by the posterior predictive distribution).

The first step is to get the time-series of streamflow (daily mean discharge) at the location where a sediment rating curve has been developed. Assuming a period of record of \( I \) years, take day \( j \) of year \( i \) and determine the streamflow. Using this discharge, get the corresponding sample of sediment transport rates from the Bayesian rating curve. Because the posterior predictive distribution consists of a set of samples (as opposed to some analytic equation) these \( M \) samples (\( M \) should be some large number) are stored as the \( j^{th} \) column in a matrix—each column corresponding to the sediment flux for day \( j \) in year \( i \). This is repeated for each day in year \( i \). Because annual sediment yield is simply the sum of all the daily sediment fluxes, the distribution of annual sediment yield is the sum across all \( J \) columns of the matrix—this results in a single column of \( M \) summations comprising the distribution of annual sediment yield (it should be noted that Nelson (2007) showed that there was no progressive channel change over the period from 1945–2002, thus we assumed that lateral inputs are trivial relative to the tributary inputs.) This process is repeated for all \( I \) years.

For the Snake River sediment budget presented here, annual sediment yield was calculated for Pacific Creek and Buffalo Fork to determine sediment inputs to the
study area and for the Snake River at Deadman’s Bar, to determine sediment output. Because JLD completely disrupts sediment supply from the mainstem to the study area, inputs are calculated as the sum of tributary inputs. The sediment budget was calculated for 1958 to the present, the period influenced by modern rules for dam releases. Because these influxes and outfluxes are distributions, their quantiles can be calculated that determine some interval for annual sediment yield at a \((1 - \alpha)\)% credibility.

### 5.4 Results

Sediment transport observations and the Bayesian transport model specified in Eq. (6.8) were used to estimate the parameter distributions for \(\tau_r\) and \(\sigma^2\) at three river sampling sites: Buffalo Fork, Pacific Creek, and the Snake River at Deadman’s Bar (see Fig. 5.1). Table 5.1 shows the prior distributions used for each site and Fig. 5.4 shows the prior and posterior distributions for \(\tau_r\) and \(\sigma^2\) at Deadman’s Bar. The priors and posterior results for Buffalo Fork and Pacific Creek mimic the general shapes shown in Fig. 5.4 but with shifted locations. The prior for reference shear is diffuse and is functionally uniform over the specified interval. The prior for the variance parameter has a large variance as well to make it vague, though the inverse gamma distribution’s shape is less flexible than the truncated normal. These priors were selected so that very little information is assumed, thereby allowing the observations to provide the most information about the inferred parameter values. Table 5.2 presents the inferred values and credible intervals for reference shear and variance from the Bayesian model; values and intervals for reference shear from Erwin et al. (2011); and channel characteristics for each sampling location. Using these posterior parameter distributions for reference shear and variance, the posterior predictive distribution was calculated for each cross-section. These
predictive distributions are presented in Figs. 5.5(A) through 5.5(C) and are plotted as areas defining the 68%, 90%, and 95% credible intervals (moving from the inner to the outer region) for each sampling location. A 95% credible region is interpreted as the region in which future observables have a 95% probability of occurring.

Fig. 5.6 is a schematic diagram that compares the assumptions associated with a forward stochastic and a Bayesian model for sediment transport. The top half of Fig. 5.6 (dashed boxes) corresponds to the forward stochastic model and the bottom half to the Bayesian approach. The ensemble predictions for Deadman’s Bar shown in Fig. 5.6(G) uses a uniform distribution of reference stresses on the expert-defined uncertainty envelopes from Erwin et al. (2011). The ensemble predictions were constructed using 7000 samples (to match the number of MCMC samples from the Bayesian model) of reference shear. These predictions assume τ_r to be the only random variable (no log-additive noise). Fig. 5.6(H) is the PPD for Deadman’s Bar. Fig. 5.6(H) assumes τ_r and σ^2 are random variables and that the log() of sediment transport predictions have constant variance defined by σ^2 (see Figs. 5.6(B) and 5.6(F)). While the Bayesian model described in this paper only assumes two parameters, Fig. 5.6 illustrates that this is extensible to an arbitrary number of parameters (see Figs. 5.6(C) and 5.6(D)).

Figures 5.7 and 5.8 show the credible intervals for the sediment mass balance. The sediment inputs are represented by the red regions in Figs. 5.7 and 5.8. The outflux is represented by the black regions in the same figures. Inset plot (A) of Figures 5.7 and 5.8 represents the (1 − α)% credible region for annual sediment yield. For any given year there is a distribution of sediment influx and outflux, and inset plot (A) shows the width of the (1 − α)% credible interval for these distributions. Sediment influx and outflux, however, is not uniformly distributed over this interval. Fig. 5.9 shows actual predictive distributions of influx and
outflux for the years 1960 and 1984 at $\alpha = 0.05$ credible level along with the $\alpha/2$ and $1 - \alpha/2$ quantiles shown as vertical lines. These lines are used to establish the credible interval width in inset plot (A) of Figs. 5.7 and 5.8. Figure 5.9(A) shows that the 1960 influx and outflux credible intervals overlap for $\alpha = 0.05$, while Fig. 5.9(B) shows a distinct gap between the 1984 influx and outflux values for $\alpha = 0.05$. These elements help define what constitutes a ‘significant’ difference in influx and outflux and are also seen in Fig. 5.7(A) for 1960 and 1984. Inset plot (B) of Figs. 5.7 and 5.8 shows the cumulative sediment mass balance at the chosen credible level starting in 1950. These distributions are found by summing the preceding annual sediment yields up to the selected year for both influx and outflux. Inset plot (C) shows the posterior distribution of cumulative sediment yield at the end of the period of record, 2007. This distribution (with both traditional quantiles and highest posterior density (HPD) interval [Box and Tiao 1992] denoted by the vertical lines) shows the probability density function for the difference between the cumulative influx and the cumulative outflux for the study reach.

5.5 Discussion

5.5.1 Bayesian sediment transport model

The model proposed in this research is an extension of the Bayesian formulation presented in Chapter 3. Here we demonstrate that by modifying the deterministic equation specified as the mean value of the likelihood function, Eqs. (6.9) and (5.2), the simple Bayesian formulation presented in Chapter 3 can be used to model sediment transport in a large gravel-bed river. Using non informative priors for the model parameters, we obtained estimates for $\tau_r$ and $\sigma^2$, as shown in Table 5.2. From a numerical methods standpoint, the MCMC algorithm converged quickly on the
parameter space and exhibited the necessary properties—e.g., mixing, acceptance rate; (Gelman et al., 2004)—characteristic of stable and valid computations. The posterior predictive distributions shown in Fig. 5.5 are further evidence of model validity; the predictive distributions appear to be reasonable given the observed transport rates. By all accounts, the Bayesian method infers reasonable parameter values and produces predictions that are sensible given the observations.

Regarding parameter inference, we can compare the inferred values from the Bayesian model to the values fitted manually in Erwin et al. (2011). One characteristic of these parameter estimates is that while the point estimates are generally the same for reference shear between the two approaches, the credible interval of the Bayesian estimates is much tighter than the manually fitted uncertainty envelopes used in Erwin et al. (2011). This discrepancy is largely owing to the fact that in the manually fitted calibration all of the observed variation is assumed to be solely attributable to variation in \( \tau_r \) (see Fig. 5.6). The Bayesian model, however, partitions between parameter variability (as expressed by the credible interval for \( \tau_r \)) and variability because of random noise, measurement error, and model misspecification (expressed by the variance parameter \( \sigma^2 \)).

While the uncertainty envelopes used in Erwin et al. (2011) is a more simplistic approach to quantifying uncertainty in transport predictions, they only define an upper and lower limit for reference shear and do not of themselves define a distribution. What happens between these bounds is not defined. It could be flat (uniformly distributed), skewed (e.g., lognormally or gamma distributed), or symmetric (e.g., normally distributed). To produce stochastic predictions using these uncertainty envelopes requires defining these distributions, and the resulting ensemble of predictions is directly dependent on these assumptions. Figure 5.6 outlines how one might use such uncertainty envelopes in a Monte Carlo analysis by
assuming a uniform distribution between the two limits. A uniform distribution for \( \tau_r \) was sampled many times between these bounds, and the ensemble of predictions is shown in Fig. 5.6(G). Adopting a Bayesian approach to this problem, however, still allows the modeler to define, to their best knowledge, the shape and location of the parameter distributions (as in Fig. 5.6(C)) but the resulting predictions are based on the posterior distribution (Fig. 5.6(D)) and not on these prior distributions. Figure 5.4 shows the difference between the prior distributions for both parameters and the updated posterior distributions. By collecting observations and incorporating them into the statistical framework, the prior knowledge (or ignorance) was updated to yield parameter estimates that produce the most realistic results. The results in Figs. 5.6(G) and (H) show that the more simplistic approach produces predictions with decreasing variance as discharge increases. The PPD in (H) has constant variance (in log space) at all discharges due to the specification of the likelihood. This difference is related to the fact that the forward stochastic approach in this comparison does not assume any variance structure of the observations—it assumes all variance is explained by the governing equation and variation in reference shear. The Bayesian approach incorporates a variance structure. In simple models, where only one or two parameters are involved, an argument could be made that a modeler could iteratively calibrate the parameter distributions in a forward stochastic model to produce a reasonable ensemble of predictions; but this becomes infeasible with many parameters or highly nonlinear systems such as sediment transport. To be clear, a forward stochastic approach is entirely capable of making the same predictions of a Bayesian model. Certainly, if the forward stochastic model had two parameters instead of one and the distributions of these parameters matched those of the posterior distribution of the Bayesian model then the results would be indistinguishable. In order for this to
occur, however, the distributions for the parameters would need to be identical. The benefit of the Bayesian approach is that the stochastic prediction (formally, the posterior predictive distribution) and the parameter estimation (the posterior distribution) are developed in a single theoretical framework. It is conceivable that one could implement a type of non-linear regression/estimation for the model parameters for use in the forward stochastic model, but the results of non-linear regression require parameter normality, something not always guaranteed. Further, the estimate from a non-linear regression method is a fixed value and assumes normal (or t-distribution) errors. The Bayesian approach allows you to inform the model via priors, have them updated formally into the posterior distribution, and then make predictive distributions. Further, the MCMC algorithm is much more efficient in identifying the joint parameter space than a manual, iterative approach, especially because the estimates are distributions and not a point value. This difficulty only increases with dimensionality.

A further consideration that warrants the present estimation-prediction framework for sediment transport is that the threshold at which sediments begin to move is highly dependent on evolving bed configurations that are generally unknowable at the scale of large gravel-bed rivers such as the Snake (Kirchner et al., 1990; Eaton and Church, 2010). In a laboratory setting, the critical shear parameter is often well defined by its size and known bed characteristics (e.g., angle of repose, Dey, 1999). In the absence of this information, however, defining a grain shear required to produce movement is complicated by the unknown and evolving bed configurations. Thus the inferred reference (or critical) shear on large river systems averages over these unknowns and behaves less as a physically-meaningful parameter for the individual grains and more like a calibration parameter for the bed as an ensemble. As such, the inferred values are unlikely to be portable from
one river reach to another.

### 5.5.2 Implications for sediment budgets

As described in section 3.3, one source of uncertainty in sediment budgets is the uncertainty in sediment transport rating curves. The probabilistic rating curves described in the preceding sections form the basis on which the probabilistic sediment budget is constructed and allows us to robustly propagate uncertainty at different levels. The first level of uncertainty relates to prior knowledge of parameter values, and this is considered during the specification of the prior distributions. The next level pertains to the resulting uncertainty in the posterior distributions; this uncertainty is propagated through to the construction of the sediment rating curves. The final step in this paper is where uncertainty in the rating curves is propagated into yearly and cumulative sediment budget calculations.

Grams and Schmidt (2005) presented an example of developing a sediment budget for the Green River below Flaming Gorge dam and discussed the various uncertainties associated with this analysis. One of the conclusions of their paper is that “full consideration of the uncertainty in the sediment budget indicates that the budget is better described as indeterminate, which is not equivalent to an equilibrium condition” (Grams and Schmidt 2005). This conclusion was based on expert-defined error magnitudes that were applied to the measured annual loads (e.g., 5% and 10%) and underscores the reality that a simple description of system uncertainty may obscure the underlying behavior of a river system. A Bayesian approach to sediment budgets differs in that the uncertainty associated with sediment yields is estimated and not assumed. The estimation of sediment yield variability arises naturally from the specification of priors for the relevant parameters, as well as the observations that are collected to calibrate the model. At
the very basic level, an expert assessment of uncertainty with respect to the
dparameters that govern the process can be stated to initiate the analysis (even if
very approximately). After that, the uncertainties related to prediction and
sediment budget calculations are not assumed because they are explicitly quantified
in the posterior distributions. Another consideration when doing probabilistic
sediment budgets is that the shape of the distributions will also have an effect on
uncertainty. If error rates are assumed to be uniform (e.g., any value ±5% is equally
probable) or heavy-tailed (i.e., more probability mass is located away from the
center of the distribution) subsequent calculations will be less certain than if a
normal distribution were assumed. When one assumes an error structure rather
than estimating it, these distributions must be specified by the modeler and may
influence the outcome. The approach promoted in this paper removes this burden
from the modeler and allows the prior distributions and the observations to
determine the form and bounds of these distributions as it naturally performs the
tasks of parameter estimation and stochastic prediction.

Erwin et al. (2011) noted that the magnitude of their uncertainty in sediment
flux calculations did not allow them to determine whether or not the reach is in
sediment deficit or surplus. They noted, however, that the difference between influx
and outflux was sufficiently large in 11 of the 50 years that a deficit could be
inferred. A surplus could be demonstrated in 1 year. These are the results of
manually defining an upper and lower reference shear that results in a region that
contains 90% of the sediment transport observations. The results shown in Figs. 5.7
and 5.8 show that, at 95% credibility, the system is in deficit for 15 of the years and
in surplus for two years. At 90% credibility (figure not shown), the number of years
in sediment deficit increases to 20 and two years in surplus. The 68% credible
interval of sediment yield indicates that 37 years are in deficit and two are in
surplus. Constructing sediment budgets probabilistically in a Bayesian framework provides a robust way to propagate uncertainty and answer questions about how different influx and outflux need to be in order to be considered a real signal and not just noise. The ability to distinguish between variability attributable to noise and variability as a characteristic system behavior addresses the uncertainty challenge posed by Grams and Schmidt (2005).

Questions regarding the sediment budget can be asked and answered at a given tolerance for risk while explicitly accounting for variability and uncertainty. At the limit of taking the credible interval to 0%, the predictive distributions in Figs. 5.7 and 5.8 reduce to the purely deterministic budget calculations. At this level, every year is either in deficit or surplus. If, for a given year, the influx is 10,000 m$^3$ and the outflux is 10,005 m$^3$, we intuitively know that this is unlikely to be a significant difference. But at what difference does significance begin and at what significance level? We observe that the uncertainty in sediment rating curves is often the focus of analysis, yet this uncertainty is not always propagated through to estimates of annual yield (e.g., McLean et al., 1999a; Major, 2004; Vericat and Batalla, 2006). Without a robust method to quantify first the uncertainty of the sediment rating curve and its distribution and second how this uncertainty affects estimates of sediment yield, point estimates of sediment yield will always be distinct and left to intuition to determine what constitutes a material difference versus an immaterial difference. Fig. 5.9 shows the predictive distributions for sediment influx and outflux in the study reach for 1960 and 1984. This figure illustrates how calculating the quantiles of the predictive sediment yield distributions facilitates the distinction between significant and non significant differences. In 1960, the 95% credible interval on the influx and outflux overlap—thus one can conclude that at 95% credibility no significant difference is present. The values for 1984, however, illustrate that
because the credible regions do not overlap the distance between the influx 0.975 quantile and the outflux 0.025 quantile is the sediment deficit that can be justifiably reported at the 95% credible level. These quantiles form the boundaries shown in Fig. 5.7(A) and the regions shown in Figs. 5.7 and 5.8 are not flat.

Over the study time period (1958–2007), we accumulated the sediment yields to develop a cumulative sediment yield distributions shown as (B) in Figs. 5.7 and 5.8. This cumulative distribution sums the preceding years’ annual yield distributions (e.g., Fig. 5.9) to produce a new distribution. These annual loads are then accumulated from 1958 to 2007. The total difference in sediment influx and outflux, shown as (C) in the same figures, demonstrates that every computation on the sediment budget data is done as a distribution. Thus, we can say with 95% credibility that by 2007 the study reach was in sediment deficit by at least 600,000 m$^3$ and at most 1,400,000 m$^3$. These intervals can be reduced by relaxing the credibility requirements (compare the quantiles in Fig. 5.8(C) to Fig. 5.7(C)). The analysis provided in this paper confirms the conclusion of Erwin et al. (2011) that the study reach on the Snake River is generally in deficit (on a yearly basis) but goes further to provide a robust estimate of the accumulated sediment deficit over the period of record.

5.6 Conclusions

The main purposes of this paper were to demonstrate the applicability of the Bayesian sediment transport model developed in Chapter 3 to a large gravel-bed river, provide increased visibility of the Bayesian approach because of its ability to model complex systems and to accommodate uncertainty in predictions and parameters, and evaluate the suitability of the Bayesian approach as the basis for constructing probabilistic sediment budgets.
The model results indicate that the Bayesian approach to sediment transport is readily extensible to large gravel-bed rivers and provides a robust method to incorporate expert knowledge, estimate model parameters, and define sediment transport predictions in terms of probabilities. This approach naturally incorporates deterministic models, thereby providing a physically-based approach to model systems that are generally predictable by governing relationships but have elements of randomness associated with them. The model described here is able to partition out variability attributable to parameter variability as well as stochasticity and measurement error. More simplistic approaches, such as the forward stochastic model described in Fig. 5.6, implicitly assume that all variability is because of parameter uncertainty and would be difficult to calibrate manually in a nonlinear system where an error structure was incorporated and or more model parameters were introduced. The ability of MCMC to search parameter spaces of complex and nonlinear systems provides an attractive option for quantifying parametric uncertainty of such systems.

The posterior predictive distributions produced from the Bayesian analysis form the foundation of the probabilistic sediment budgets described in this paper. Because the uncertainty associated with sediment transport for a given flow is robustly quantified and propagated into the PPD, the experts modeling the system need not assume error rates of annual sediment loads for a given location. This further removes the necessity for modelers to choose the type of distribution for the uncertainty (e.g., normal, uniform, etc.). This is particularly attractive because uniformly distributed error rates (e.g., ±5%) may be unnecessarily conservative, potentially resulting in indeterminate sediment budgets. The Bayesian sediment budget shown in Figs. 5.7 and 5.8 indicate that either deficit or surplus can be reliably inferred for 17 years at the 95% credible interval. Erwin et al. (2011)
estimated this number to be 13 at 90% confidence. Because the sediment yields are probability distributions, a credibility interval of sediment flux can be determined for any given credibility level or tolerance for risk. Credible intervals for influx, efflux, sediment transport, sediment accumulation/evacuation are easily obtained at any risk profile by adjusting the quantiles appropriately. Further, the approach described in this paper makes it easy to calculate the cumulative sediment yield as a distribution in which uncertainties have been fully propagated from the initial model assumptions.

While the approach that we promote here has advantages over deterministic and some simple uncertainty estimation schemes, the Bayesian model itself is quite modest. This general approach offers significant opportunities for more complex models to be constructed for fluvial sediment transport that estimate more parameters, incorporate competing deterministic functions, use different variance structures, and evaluate the suitability of different submodels such as those used for the partitioning of grain shear from total shear. One weakness of the current model is that it assumes stationary (time-independent) sediment rating curves for each location—while certainly not ideal, this assumption was driven by necessity because no other records from which the sediment rating curves could be estimated for previous years were available. This is not a reality unique to the data of Erwin et al. (2011) but is reflective of many applied geomorphic studies at large. Provided, however, that adequate data to develop time-dependent sediment rating curves are available, this could be integrated into the sediment budget framework presented above. Future work could also focus on the uncertainty of other model unknowns, such as grain size distributions and variability of estimated water discharges, though this uncertainty may be extremely small relative to the others. The current paper discusses the use of Bayesian statistics for bedload transport, and we believe that
this approach will also prove useful in other areas of the field such as suspended sediment transport, multi-fraction bed load, determination of the optimal deterministic model for a dataset, and general prediction and inference related to sediment transport phenomena.

References


Dogan, E., Tripathi, S., Lyn, D.A., Govindaraju, R.S., 2009. From flumes to rivers: can sediment transport in natural alluvial channels be predicted from observations at the laboratory scale? Water Resources Research 45, W08433.


Table 5.1: Prior distributions for $\tau_r$ and $\sigma^2$. The truncated normal distributions are specified as $T.N.(\mu_{\tau_r}, \sigma_{\tau_r}, \tilde{a}, \tilde{b})$. The inverse gamma distribution is specified as $I.G.(\mu_{\sigma^2}, \sigma_{\sigma^2})$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Snake River</th>
<th>Buffalo Fork</th>
<th>Pacific Creek</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\tau_r]$ $\sim$</td>
<td>$T.N.(7, 1000, 0, 15)$</td>
<td>$T.N.(7, 1000, 0, 15)$</td>
<td>$T.N.(10, 1000, 0, 20)$</td>
</tr>
<tr>
<td>$[\sigma^2]$ $\sim$</td>
<td>$I.G.(0.5, 50)$</td>
<td>$I.G.(0.5, 50)$</td>
<td>$I.G.(0.5, 50)$</td>
</tr>
</tbody>
</table>

Table 5.2: Inferred and fixed parameters for bed load transport function. Top section displays results from the posterior distributions. Middle section shows selected parameters from [Erwin et al. (2011)](Erwin+et+al.+2011). Bottom section shows channel properties for each site. Point estimate is the expected value of the posterior distribution and values in parentheses comprise the 95% posterior credible interval for the Bayesian estimates. Values without credible intervals are assumed fixed and known.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Snake River</th>
<th>Buffalo Fork</th>
<th>Pacific Creek</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_r$, Pa</td>
<td>9.38 (9.05–9.66)</td>
<td>9.67 (9.14–10.15)</td>
<td>14.30 (13.97–14.64)</td>
</tr>
<tr>
<td>$\tau_r^*$</td>
<td>0.017 (0.016–0.018)</td>
<td>0.033 (0.031–0.035)</td>
<td>0.042 (0.041–0.043)</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>1.90 (1.35–2.73)</td>
<td>1.25 (0.81–1.92)</td>
<td>0.62 (0.36–1.08)</td>
</tr>
<tr>
<td>$\tau_r$, Pa $^\dagger$</td>
<td>9.80 (8.30–11.30)</td>
<td>9.50 (8.50–12.50)</td>
<td>14.20 (13.20–14.20)</td>
</tr>
<tr>
<td>$\tau_r^*$ $^\dagger$</td>
<td>0.018 (0.015–0.021)</td>
<td>0.032 (0.029–0.043)</td>
<td>0.042 (0.039–0.045)</td>
</tr>
<tr>
<td>Water surface slope, $S$</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.0035</td>
</tr>
<tr>
<td>$D_{65}$, mm</td>
<td>50</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>$D_{50}$, mm</td>
<td>34</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>Active channel width, m</td>
<td>70</td>
<td>45</td>
<td>43</td>
</tr>
<tr>
<td>Num. observations</td>
<td>62</td>
<td>39</td>
<td>24</td>
</tr>
</tbody>
</table>

$^\dagger$Values as reported in [Erwin et al. (2011)](Erwin+et+al.+2011)
Fig. 5.1: The Snake River in Grand Teton National Park. The study area extends from the confluence of the Snake River with Pacific Creek downstream to Deadman’s Bar. Locations of bedload sampling sites are indicated by stars. Modified from Erwin et al. (2011).
Fig. 5.2: Sediment transport sampling on the Snake River at Deadman’s Bar.
Fig. 5.3: Potential prior distributions for reference shear. Prior (A) specifies a mean value for reference shear centered on 10 Pa, with some variance; prior (B) assumes that every value between zero and the upper-bound is equally likely; prior (C) gives higher probability to values near the upper limit of the prior distribution. Priors are parameterized according to $T.N.(\mu_{\tau_r}, \sigma_{\tau_r}, \tilde{a}, \tilde{b})$.

Fig. 5.4: Prior (dash-dot) and posterior (solid) distribution densities for reference shear and variance at Deadman’s Bar on the Snake River. (A) Reference shear; (B) variance.
Fig. 5.5: Probabilistic sediment rating curves. (A) Snake River at Deadman’s Bar, (B) Buffalo Fork, and (C) Pacific Creek. The shading (lightest to darkest) represents the 68%, 90%, and 95% credible intervals.
forward stochastic approach

Bayesian approach

Specify parameter distributions, \( \theta \)

Update priors using \( Q_{s,o} | \theta, \sigma^2 \)

Composition sampling

Monte Carlo simulations

Ensemble predictions

\[
Q_{s,o} = f(\theta)
\]

where \( \theta = \{\theta_1, \theta_2, \ldots, \theta_m\} \)

\[
\log(Q_{s,o}) \sim N(\log(f(\theta)), \sigma^2)
\]

\( \log(Q_{s,o}) = \log(f(\theta)) + \epsilon \)

\( \epsilon \sim N(0, \sigma^2) \)

\[
\hat{Q}_s = f(\theta)
\]

\[
\log(\hat{Q}_s) = \log(f(\theta)) + \epsilon
\]

\( \epsilon \sim N(0, \sigma^2) \)

Fig. 5.6: Flowchart comparing workflows of a forward stochastic model and a Bayesian model for sediment transport. Forward stochastic model workflow goes from (A) to (C) to (E) to (G). The Bayesian model goes from (B) to (C) to (D) to (F) to (H). The ensemble predictions in (G) assume a uniform distribution of \( \tau_r \) with upper and lower limits defined by Erwin et al. (2011) shown in Table 5.2 at Deadman’s Bar. (H) PPD for Deadman’s Bar. The density curves shown on the right of (G) and (H) are the distributions of sediment transport at the water discharges denoted by the vertical line (red) on the rating curves. The thin horizontal lines correspond to the 68%, 90%, and 95% prediction intervals.
Fig. 5.7: Sediment budget using 95% credible interval. (A) Annual sediment yield. Shaded regions represent the 95% credible interval on the posterior predictive distribution of total annual sediment flux. A 'significant' difference between the influx and outflux occurs where the regions do not overlap. (B) Cumulative sediment yield starting in 1950. (C) Net change in sediment storage over period of record.
Fig. 5.8: Sediment budget using 68% credible interval. (A) Annual sediment yield. Shaded regions represent the 68% credible interval on the posterior predictive distribution of total annual sediment flux. A ‘significant’ difference between the influx and outflux occurs where the regions do not overlap. (B) Cumulative sediment yield starting in 1950. (C) Net change in sediment storage over period of record.
Fig. 5.9: Posterior predictive distributions of annual transport for 1960 and 1984. Dashed lines (red) denote sediment influx, solid lines (black) denote outflux. Vertical lines correspond to the 0.025 and 0.975 quantiles for each distribution.
Abstract

Sediment transport in rivers naturally involves the movement and redistribution of a range of sediment sizes rather than a single size fraction. Sediment modeling approaches may elect to describe the process using a single fraction representing the bulk transport, or multiple fractions may be described using appropriate transport theory. Previous work in developing and applying Bayesian sediment transport models have considered only a single fraction or bulk transport represented by a single characteristic grain size, therefore developing a multi-fraction Bayesian sediment transport model is a logical next step in providing more representative transport models. The research in this chapter describes the development and testing of such a model—one that extends previous concepts from one fractional dimension into \( k \) fractional dimensions. The model here uses the Wilcock-Crowe equation as its deterministic base and the testing dataset comprises the series of flume observations that were used to originally develop the Wilcock-Crowe equation. The Bayesian multi-fraction model is able to infer the posterior distributions for reference stresses for each of the size fractions modeled and provide a robust estimate of parameter variability. Predictive distributions of both bulk and fractional transport are also derived. Further, the inferred posterior distributions are used to evaluate the hiding function and Shields variation curve developed originally with the Wilcock-Crowe equations, and it is shown that the hiding function resulting from the Bayesian analysis is consistent with the original specification, despite subtle changes in inferred reference stresses of the test
sediments. The Bayesian model results are consistent with the original manually-calibrated equations and offer an advancement in that they offer estimates of parameter and predictive uncertainty to multi-fraction transport problems.

6.1 Introduction

It is intuitively understood that sediment transported in natural river systems contains a range of grain sizes. For simplicity, many transport formulas are based on equations that do not distinguish between different size classes. These approaches, when applied to sediment transport of natural rivers, aggregate the different size fractions into a single class—simply, the sediment transported—and use a characteristic grain size (e.g., $D_{65}$ or $D_{50}$) to represent the actual range of sizes in the sediment. Such simplifications can be useful but they do not capture the sub-processes that may influence overall transport. In particular, it has been demonstrated that the transport rate can be heavily influenced by the fines content of sediment mixtures (Einstein, 1950; Egiazaroff, 1965; Parker et al., 1982; Wilcock and Crowe, 2003). Previous work on Bayesian sediment transport models adopted an aggregate or lumped approach to modeling the transport of single size fractions (Chapter 3) and bulk transport of multiple size fractions (Chapter 5). These papers demonstrated that Bayesian statistics can be employed to provide a robust estimate of the predictive and parametric uncertainty for single fraction and bulk sediment transport. The purpose of this paper is to make the incremental yet necessary transition from a single fraction Bayesian transport model to one that accounts for multi-fraction transport of an arbitrary number of particle sizes. This increase in dimensionality requires extension of previously-developed code to account for and estimate uncertainty in the sediment transport phenomenon. To this end, the Wilcock-Crowe equations (Wilcock and Crowe, 2003) were selected to be the
deterministic “center” of the model developed in this paper and the laboratory flume data that were used in their development [Wilcock et al. 2001] were used as a test data set. The Wilcock-Crowe equations are based on surface grain size, rather than bulk grain size that includes the subsurface. Hence, the transport rates are scaled to the size population immediately available for transport. This method avoids the ambiguity associated with grain sorting processes that cause a discrepancy between sediments under transport and those comprising the bulk bed mixture. Further, these equations have been used in numerous applications in the literature (for example, Gaeuman et al. 2009) thereby making it a good candidate for use in a Bayesian framework.

6.2 Background

The previous chapters of this dissertation provided context for the motivation in developing Bayesian sediment transport models and demonstrated their utility in parameter estimation, incorporating prior knowledge, quantifying uncertainty, and in making probability-based predictions. For example, Chapter 5 of this dissertation modeled sediment transport in the Snake River using a bulk transport approach despite the obvious simplification that the Snake does not transport a single grain size. The unit volumetric discharge of sediment was modeled without regard to the different size classes actually moving on the bed and this analysis was used to develop a useful and robust sediment budget of the river. In systems where it is important to characterize the individual rates of movement for different size classes, this uni-size or bulk transport approach is not sufficient. For example, the Trinity River in northern California is a system that is actively being managed and ‘restored’ after having been impacted by pervasive hydraulic mining and the construction of a large dam that diverted a large fraction of the annual runoff from
the river basin. One effect of this history was an increase in fine sediments into the previously-gravel bedded Trinity River. Among the challenges to the Trinity River Restoration Program (TRRP), evacuation of fine sediments from the channel is a primary target, but as fine grain particles move, so do larger grains, thus quantifying the relative proportion of fine-grained to coarse-grained sediment transport becomes a relevant problem in this context. In this setting, a bulk fraction model is inadequate and a multi-fraction approach is necessary.

It is established in the literature that relative differences in grain sizes in mixed sediments results in preferential movement of specific size classes (e.g., Einstein, 1950; Egiazaroff, 1965; Parker et al., 1982; Jackson and Beschta, 1984; Ikeda and Iseyo, 1988). Wilcock and Crowe (2003) developed and introduced an advancement in modeling these dynamics through experimentation and the derivation of new transport equations. After making observations of multi-fraction sediment experiments in a flume, the observations were used to calibrate the transport equations and the inferred parameters were explored to develop new relationships that describe the relative size effects in mixed sediments. The research presented in this chapter builds on the work of Wilcock and Crowe (2003) and others by incorporating these equations into a Bayesian model. Further, present research relies upon observations for ‘learning’ about the parameters much in the same way Wilcock and Crowe (2003) learned about the parameters through a manual calibration.

The procedure promoted in this research serves to provide a tool to robustly estimate model parameters (given some observations of the process) for better prediction and quantification of parametric and predictive uncertainty. This method is offered as an alternative to a manual expert calibration that relies on an individual adjusting model parameters until the model predictions are consistent with the observations. It is not the intention of this research to suggest that expert
input has no place in modeling and that the purely expert-calibration is invalid. In fact, the Bayesian estimation and predictive architecture allows for incorporation of expert opinion and knowledge in ways that traditional statistical techniques do not. However, even under the assumption that Bayesian parameter estimates are no worse than the manually-calibrated parameter values, the Bayesian method presents a more robust way of quantifying both parametric and predictive uncertainty compared to manual calibration. Further, the inferred parameter values can then be used in a similar fashion as was done in Wilcock and Crowe (2003) to explore the inferred parameters and data to evaluate the extent to which the functional relationships derived in Wilcock and Crowe (2003) are supported or challenged by the Bayesian estimates.

6.3 Objectives

The scope of this last chapter includes three general objectives: 1) develop a multi-fraction sediment transport model by extending from one fraction to k fractions and improving upon the previous Bayesian models by including a more suitable prior and a dimensionally scalable MCMC algorithm; 2) test the newly-developed model using laboratory flume data and validate the Bayesian model results against previous work and other statistical methods to verify that the model output is reasonable; and 3) discuss any differences between the Bayesian model output and the validation data analysis, explore what is gained by adopting a Bayesian approach over previous approaches, and draw conclusions regarding the suitability of the Bayesian multi-fraction transport model.

6.4 Model Formulation

The Bayesian model proposed in this research (this and previous chapters) can
be broadly described as comprising two parts. First is the deterministic model that is assumed to generally describe the mean behavior of the phenomenon. Second is the Bayesian model into which the deterministic model is nested such that the parametric and predictive uncertainty of the deterministic equations can be estimated.

In referring to models, it is helpful to make brief mention of the usage of the term ‘model’. In this paper, the term is used to describe both the sediment transport equations it is believed the process follows (i.e., the sediment transport model as defined by the Wilcock-Crowe equations) as well as the statistical model (e.g., a linear or nonlinear regression model), which in this case is a Bayesian statistical model. The Wilcock-Crowe equations and the Bayesian statistical formulation that are combined in this paper constitute the overall sediment transport model that is being promoted.

The following sections describe both the deterministic, or sediment transport-based theory as well as the statistical formulation that form the basis for the approach offered in this research. Readers are referred to Wilcock et al. (2001) and Wilcock and Crowe (2003) for further information on the Wilcock-Crowe equations and chapters 3 through 5 for further background details on Bayesian sediment transport models.

### 6.4.1 Sediment Transport Governing Equations

Because the model in this chapter uses both the data from Wilcock et al. (2001) and then the transport equations from Wilcock and Crowe (2003), a brief summary of these papers is warranted.

Wilcock et al. (2001) performed a series of 38 sediment transport experimental runs in a flume 0.6 m wide and 7.9 m long. Previous work performed included 10
additional runs for another sediment mixture, thereby totaling 48 flume runs with five sediments in which flow, transport rate, and bed surface grain size were measured over a range of different water discharges. The sediments that comprised these 48 runs varied in bulk sand content (particle sizes smaller than 2 mm), from 6% to 34%. The five sediment mixtures of which there were 9 or 10 experiments in each—J06 (6.2%), J14(14.9%), J21(20.6%), J27(27.0%), and BOMC, or Bed of Many Colors (34.3%)—are referred to frequently in the sections that follow. The flume experiments themselves were brought to a steady-state condition in a water and sediment recirculating flume prior to measurement of discharge and fractional sediment transport. Wilcock and Crowe (2003) utilized these data to develop a surface-based sediment transport equation that accounts for the sorting processes that result in grain hiding in mixed sediments. The transport equations, which are discussed below, were calibrated by eye—that is, the model parameters (reference stress of each size fraction) were adjusted until the model fit the observations as judged by expert opinion and experience. These calibrated reference stresses were then used in subsequent analysis that explored concepts such as the hiding function and the variation of Shields parameter as a function of sand content.

As demonstrated in previous chapters the Bayesian framework integrates deterministic functions into a probabilistic framework. Wilcock and Crowe (2003) proposed a set of equations that accounts for the transport of different size fractions on the surface of a river bed and their interactions as a function of the proportion of fines contained in the bed surface. These equations are used as the deterministic function that provides the mean value of the likelihood, as described in previous chapters. As is common to other sediment transport equations, sediment transport
can be described in non-dimensional terms:

\[
W_j^* = \begin{cases} 
0.002 \phi^{7.5}, & \text{for } \phi < 1.35, \\
14 \left(1 - \frac{0.894}{\phi^{0.5}}\right)^{4.5}, & \text{for } \phi \geq 1.35 
\end{cases} 
\]  

(6.1)

where \(W_j^*\) is the dimensionless transport rate of the \(j^{th}\) size fraction, \(\phi = \frac{\tau'}{\tau_{r,j}}\), \(\tau_{r,j}\) is the reference stress of the \(j^{th}\) size fraction, and \(\tau'\) is the skin friction. This equation is similar in form to the equations proposed by \textit{Parker} (1979, 1990) used in Chapter 5 and can be adapted for use in a bulk transport setting by dropping the \(j\) subscript. The value for \(W_j^*\) is defined as:

\[
W_j^* = \frac{gq_{s,j}(s - 1)}{F_j u^*_3}, 
\]  

(6.2)

where \(g\) is the acceleration due to gravity, \(q_{s,j}\) is the unit bed load transport rate of the \(j^{th}\) size fraction, \(s\) is the specific gravity (2.65), \(F_j\) is the proportion of size \(j\) on the bed surface, \(u^*_3 = \left(\frac{\tau'}{\rho}\right)^{0.5}\), and \(\rho\) is the density of the water.

One of the key features of the multi-fraction model is the hiding function. This concept goes as far back as \textit{Einstein} (1950) and describes the effect of relative grain size on transport rate. Generally, fine grains in a mixed sediment will be shielded or hidden from the boundary shear stress thereby reducing mobility of the finer size fractions. Coarser fractions are generally more mobile because of the smoothing effect fine grains have on the bed surface. The interaction between the \(j^{th}\) size fraction reference shear stress and that of the geometric mean size reference shear stress.
stress was estimated in [Wilcock and Crowe (2003)] and is defined as:

\[
\frac{\tau_{r,j}}{\tau_{r,sm}} = \left( \frac{D_j}{D_{sm}} \right)^b,
\]

(6.3)

where \(\tau_{r,sm}\) is the geometric mean particle size of the surface, \(D_{sm}\) is the geometric surface mean diameter, and the exponent \(b\) in (6.3) is:

\[
b = \frac{0.67}{1 + \exp\{1.5 - \frac{D_j}{D_{sm}}\}}.
\]

(6.4)

The remaining part of the model is prediction of \(\tau_{r,sm}\), which [Wilcock and Crowe (2003)] found depended on the fraction of sand in the bed surface. It was determined that the Shields parameter in the sediment experiments varied with sand content by:

\[
\tau_{r,sm} = 0.021 + 0.015\exp\{-20F_s\},
\]

(6.5)

where the conversion to dimensionless shear stress from shear stress follows:

\[
\tau_{r,j}^* = \frac{\tau_{r,j}}{(s - 1)\rho gD_{sm}}.
\]

(6.6)

### 6.4.2 Multi-fraction Bayesian Transport Model

Using the [Wilcock et al. (2001)] data, two Bayesian models were developed. The first model evaluates the bulk sediment transport, that is, it does not partition transport out into different size fractions and therefore infers a reference stress that is representative of the mixture as a whole (bulk transport). This model is identical to that proposed in chapters 3 and 5 with the exception of the specification of the prior distribution for the variance parameter and the deterministic function for the likelihood mean. The bulk transport model is described as:
where $\tau_{r,bulk}$ is the reference shear stress of the bulk transport, $\sigma^2_{bulk}$ is the variance of the bulk transport, and $Q_{s,o,i,bulk}$ is the vector of $n$ observations (for $i = 1 \ldots n$) of sediment transport summed across all size fractions for a given flow condition (shear stress). This first model evaluates the total bed load transport of all size fractions for a given flow condition in aggregate.

The second model takes into account transport of different size fractions and is the multi-fraction sediment transport model that distinguishes transport between one size fraction to another. This model is very similar to (6.7) except that the joint likelihood is the product over $m$ size fractions and $n$ observations of each size fraction. Mathematically, this is represented as:

$$[\tau_r, \sigma^2 | \log(Q_{s,o})] \propto \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \log(Q_{s,o,i,j}) \bigg| \tau_{r,j}, \sigma^2_{j} \right),$$

(6.8)

where $Q_{s,o,i,j}$ is an $n \times m$ matrix of sediment transport observations for each size fraction ($i = 1 \ldots n$ observations and $j = 1 \ldots m$ size fractions).

The likelihood, which describes the distribution from which the data are derived, has for a mean value the deterministic function of sediment transport using the Wilcock-Crowe equations:

$$\log(Q_{s,o,i,j}) | \tau_{c,j}, \sigma^2_{j} \sim N(\log(Q_{s,i,j}), \sigma^2_{j}),$$

(6.9)

where $Q_{s,i,j}$ is defined by 6.1 As demonstrated in chapters 3 and 5, the mean value of the likelihood can be any function. In the absence of a pre-defined model, the
mean value of the likelihood could be some statistical function of forcing variables with unknown coefficients. In this case, the number of model parameters to estimate increases and priors would need to be specified for each of the new parameters.

The parameters to be estimated in both the bulk and multi-fraction models are the reference stress(es) and standard deviation(s). The prior reference stress is distributed as:

\[ \tau_{r,j} \sim TN(\mu_j, \psi_j, a_j, b_j), \]  

(6.10)

where \( \mu_j, \psi_j, a_j, \) and \( b_j \) are hyperpriors of the truncated normal distribution on \( \tau_{r,j} \).

For the bulk transport model, the same specification applies by simply dropping the \( j \) subscripts. The prior standard deviation is distributed as:

\[ \log(\sigma_j) \sim N(\eta_j, \gamma_j^2), \]  

(6.11)

where \( \eta_j \) and \( \gamma_j^2 \) are hyperpriors of the normal distribution on \( \log(\sigma_j) \). As before, dropping the \( j \) subscript specifies the prior for the bulk model.

Chapters 3 through 5 all assumed that the prior for the variance parameter was an inverse gamma distribution. The present model deviates from this specification—the main reason being that the previous research observed that the inverse gamma distribution, while conjugate with the likelihood thereby resulting in cleaner MCMC steps, was always very near zero and the inverse gamma distribution’s shape may have imposed some constraints on the inferred posterior distributions. The prior specification for \( \sigma \) in this research was explored to address this shortcoming.

**MCMC Algorithm**

The models described in the previous chapters employed a two parameter
MCMC algorithm consisting of a Gibbs sampler (Casella and George, 1992) for the variance parameter (because the prior for $\sigma^2$ was conjugate with the likelihood) and a Metropolis update for the reference/critical stress. The Metropolis update (Chib and Greenberg, 1995) was manually tuned by trial and error. The model described in this chapter has many more dimensions that make manually tuning the MCMC algorithm prohibitive. In previous chapters a single tuning parameter for the critical/reference stress was all that was required to make the Markov chain converge on the target distribution. The work in this paper uses five different sediment experiments, each consisting of 13 different size fractions. Further, the modification of the prior on $\sigma$ precludes the use of a Gibbs sampler because (6.11) is not conjugate with the likelihood, therefore two Metropolis updates are required for each MCMC iteration. In short, a manual tuning in the present work would require the specification of 130 tuning parameters. To avoid this, an adaptive MCMC algorithm was employed. Adaptive MCMC algorithms essentially tune themselves such that the jumping variance (the magnitude of the random walk through the parameter space) scales based on a pre-defined optimal acceptance ratio. There are numerous adaptive MCMC algorithms in the literature of varying complexity (e.g., Haario et al., 2001; Roberts and Rosenthal 2009; Vrugt et al., 2009). For this model, a parsimonious adaptive MCMC algorithm was selected. Atchadé and Rosenthal (2005) outlines several adaptive algorithms and provides the framework for the algorithm used in for this model. Algorithm 4.1 of Atchadé and Rosenthal (2005) was implemented for this model. In short, this algorithm evaluates the acceptance probability and adjusts the adaptation parameter to fit a predefined optimal acceptance rate of 0.234 for this two-parameter model. Where before it was required to directly specify a tuning variance, $\sigma^2_{\text{tune}}$, a seed tuning variance is specified that is generally appropriate and is selected by trial and error, and this initial tuning
variance is adapted (modified) at each MCMC sample such that the long-term acceptance probability is optimal. The adapted tuning variance is defined as:

$$\sigma_{n+1} = \sigma_n + \gamma_n (\alpha(X_n, Y_{n+1}) - \bar{\tau}),$$

(6.12)

where $\gamma_n = \sigma_{n-1}/n$ and effectively scales the adaptation such that with each iteration the tuning variance adapts less and less, $\alpha(x, y) = \min\left(1, \frac{\pi(y)}{\pi(x)}\right)$, $\pi(x)$ is the likelihood of parameter value, $X_n$ is the Markov chain for the parameter of interest, $Y_{n+1}$ is a new proposal $Y_{n+1} \sim N(X_n, \sigma_n^2)$, and $\bar{\tau} \equiv 0.234$. A random variable, $U \sim Uniform(0, 1)$ is sampled, and the newly proposed value is accepted when $U \leq \alpha(X_n, Y_{n+1})$ and rejected otherwise in which case $X_{n+1} = X_n$.

### 6.5 Results

The purpose of this paper is to develop and test a Bayesian multi-fraction model centered on the Wilcock-Crowe equations using the laboratory flume data used to originally develop the Wilcock-Crowe equations. Using these data provides a convenient way to evaluate the inferred reference stresses from the Bayesian model as well as compare the resulting relationships of reference stress to sand content and the hiding function proposed by Wilcock and Crowe (2003).

The results that follow are in three sections. The first briefly describes the dataset and provides visualizations and analysis that validates that the data were properly read into the Bayesian model code. It also serves as an illustration to the reader of the nature of the data. The remaining two sections describe, respectively, the results of the bulk and fractional transport models.

#### 6.5.1 Experimental Data

The Wilcock-Kenworthy-Crowe (WKC) data used in this research was
downloaded from the original paper’s FTP site hosted by the American Geophysical Union (AGU), at ftp://agu.org/apend/wr/2001WR000683/. These text files were formatted so that they could be read into the R computing platform \textit{R Development Core Team} (2012). Numerous data checks were performed to verify that the data had been read in correctly (due to the complex and multi-dimensional nature of the data). These checks included mathematical as well as visual checks. Several plots of the data were created for comparison against published figures for verification. Figures 6.1 and 6.2 are two such figures that correspond to published figures from the literature. Figure 6.1 shows the particle size distributions for each sediment mixture, which is labeled in the legend. This figure shows that the BOMC sediment is generally finer than all the others with J06 being the coarsest. The shape of this figure compared to that of Figure 1 of \textit{Wilcock and Crowe} (2003) shows that the data are the same between the two analyses. Figure 6.2 shows the bulk transport rates for the 9 to 10 experiments for each sediment mixture. This figure is comparable to Figure 5(c) of \textit{Wilcock et al.} (2001). The comparison of the two plots confirms that the data used in this experiment are consistent with those used in the derivation of the deterministic equations that will be used in the likelihood of the Bayesian model. We also observe in this figure the general pattern of transport in which the finer sediment mixtures are transported at lower shear stresses. Fractional transport data were verified using several different plot comparisons that are not included in this manuscript for mainly editorial reasons.

6.5.2 Bulk Transport

The bulk transport model fitted to the observations is similar to the analysis performed in Chapter 5 in which a truly multi-fraction sediment mixture is modeled using a single fraction model. The total transport rate, integrated across all grain
sizes, is the observation and prediction in such an analysis. The reference shear stress and variance inferred from this model represent the integrated or bulk behavior of the sediment mixture as an ensemble. Each different sediment mixture was analyzed in the model described by (6.7). The inferred reference stresses and variances are summarized in Table 6.1.

Figure 6.3 shows the typical prior parameterizations for the reference stress and standard deviation parameters in the likelihood of both the bulk and fractional transport models. These priors are considered to be generally vague and were used in all of the models that were fitted. As before, these priors can be changed to carry more specific information for consideration by the Bayesian model.

Figure 6.4 displays the diagnostic plots that resulted from the bulk transport model run for the J06 sediment. The left-most column shows the MCMC trace plots which shows the random walk through the parameter space for all of the 10,000 MCMC samples. In these columns we see good mixing—that is, the random walk through the parameter space looks random and there are no areas of the parameter space where the chain gets stuck for some period of time. Further, convergence is quick and there appears to be no autocorrelation among the data points—another indicator of valid MCMC results. The middle column shows the posterior histograms after discarding the first 2,000 samples to allow the Markov chain to converge or “burn-in.” The MCMC chain burn-in refers to the number of samples that are discarded to account for the bias imposed by the starting value of the MCMC chain. The chain is burned-in once it has found the target distribution—subsequent samples constitute samples from the target distribution and can be used to estimate the target probability density. The right-most column shows the tuning variance adaptation trace plot. This is the parameter that auto-adjusts at each iteration. There are initially (in the first 15 or so MCMC
samples) large swings in the trace plot followed quickly by convergence into a stable tuning variance value. This stable value is what would have normally been specified by manually tuning through trial and error for each parameter and sediment. Trace plots for additional sediment runs are available and exhibit similar characteristics, but are not included here.

Figure 6.5 shows the similarity collapse of the bulk transport data. If a single transport function applies to all mixtures, then dividing each stress by the reference stress should collapse all transport observations about a common trend. The similarity collapse relies on the inferred reference stresses for each bulk sediment mixture, which are included in Table 6.1 along with the inferred variances and number of observations for each sediment. If the observed stresses for a given transport rate are not divided by the mean reference stress for each sediment, then the points would plot similarly to Figure 6.2. The inverse of the mean inferred reference stresses provides the normalizing constant to make all the transport observations collapse into a single series of points. The similarity collapse for the bulk transport model results demonstrates that when each sediment mixture and transport observation is scaled by the inferred reference stress, then all of the points collapse into a single relationship. The line in Figure 6.5 (which is the calibrated Wilcock-Crowe equation (6.1) for the bulk transport data) fits the data very well for \( W^* \) values less than 0.01. Higher values, however, seem to be over predicted for bulk transport starting around \( W^* \) values greater than 0.03 indicating a model deficiency in the case of bulk sediment transport.

Individual posterior predictive distributions (PPDs) were calculated for each sediment mixture, and these PPDs are included as Figure 6.6. As a validation/check on the results of the Bayesian parameter estimates, a non-linear least squares regression was fitted to the observed data, and the resulting rating curves are
represented by the red lines of the same figure. This overlay demonstrates that parameter inference between the Bayesian and NLS approaches agree, so this can be taken as a type of validation that the Bayesian model was correctly implemented in the code. In this instance, the residuals of the NLS analysis were satisfactory so as to justify reliance on the NLS estimates. Further, as shown in Figure 6.4, the posterior distribution for reference stress looks normally-distributed, thereby satisfying the normality requirements of NLS. Figure 6.6 also illustrates one of the primary differences and motivations for modeling sediment transport as a Bayesian—predictive uncertainty is robustly defined in the Bayesian approach. In order to do the same for nonlinear regression a linear approximation of the nonlinear regression must be performed after which ordinary least squares (OLS) prediction intervals are derived from the approximation (Bates and Watts, 1988).

Individually, the transport rating curves may also suggest systematic over prediction evidenced by the individual rating curves for J21 and J27, though it is less convincing than the evidence provided by the similarity collapse in Figure 6.5.

6.5.3 Fractional Transport

Developing the model code to accommodate multiple size fractions involved both a great deal of data management to ensure that the proper subsets of the input data matrix were read correctly, but also, the model output required non-trivial amounts of checking to ensure that the model results were reliable. As for the bulk transport model, the fractional model requires trace and diagnostic plots to be analyzed. For the sake of brevity, only the trace plots for J14 are presented formally here. These trace plots are considered representative of the overall process.

Figures 6.7 through 6.12 are the diagnostic plots for the fractional model fitted to the J14 sediment mixture. Figure 6.7 shows the trace plot of the Markov chains
for each size fraction. As is evidenced by this plot, the 10,000 MCMC samples were performed for each size fraction where there was at least one observation for that size class. Here, we observe good mixing with an absence of the chain preferentially searching a small subset of the parameter space that seems unreasonable. This provides good validation of the numerical method and instills confidence regarding the validity of the posterior distribution.

Figure 6.8 shows the evolution of the tuning parameter that results from the adaptive MCMC portion of the code. There are initially significant deviations from the seed value for the tuning variance, but then a relatively quick and distinct convergence of the parameter is observed. For all cases, the number of burn-in samples was set to 2,000. By the 2,000th iteration, the tuning variances were stable and adapting less and less with each iteration. We observe in the tuning parameter trace plot for the 45.3 mm size class, however, that the chain seems to be trending upwards. This is likely due to the fact that are few observations in the larger size classes, therefore the algorithm has more uncertainty with these Markov chains. While we would like to see a nice flat convergence for the 45.3 mm size class (as is generally the case in the other size fractions), the tuning parameter of itself is not the inferential goal but simply the variance parameter of the random walk through the parameter space. Because the trace plots for the 45.3 mm size class in Figure 6.7 possesses all the desirable properties of a convergent Markov chain, the trending in the tuning variance need not be considered an issue that is materially affecting the posterior distribution.

Figure 6.9 shows the posterior distributions for J14, by each size fraction, along with the prior distribution plotted as a solid (red) line for reference. Compared to the prior information supplied, which was in essence a flat or uniform distribution over the interval \((0, 20]\), The information provided by the observations, even where
there were very few observations (as is the case for the coarser grain sizes, such as 45.3 mm, see Table 6.3 for a complete disclosure of the number of observations for each size fraction, by sediment mixture) there is still significant learning about the reference stress. One weakness of the current model, however, is that flume runs that did not result in any movement of a particular grain size (large ones for example) are not accounted for in the current model—that is, only those instances where sediment moved are considered a data point even though there is important information contained in those experiments where nothing moved. The current model does not use any of the information contained in the zero-transport experimental runs. This is possible, however, in the current code by specifying a lower limit in the prior distribution based on the experimental runs where there was no transport—those discharges, and the associated shear stress, are necessarily less than the reference stress, otherwise there would have been movement. Incorporating this information into the model in a more formal way (i.e., developing a model that accounts for zero transport events) would be a valuable improvement in future models.

Similarly, the model diagnostic plots for \( \sigma_j \), figures 6.10 through 6.12, also support the conclusions of valid posterior convergence of the AMCMC algorithm for reasons that applied to \( \tau_{r,j} \).

The original derivation of the Wilcock-Crowe equations was based on the “manually” calibrated reference stress values that were fitted by eye in Wilcock and Crowe (2003) because it was observed that optimized solutions for the parameters did not provide as good a fit visually and seemed to be overly-influenced by outliers. The Bayesian model produces random variable estimates of reference shear—the point values (means) are shown in Table 6.2 and the graphical representation of the posteriors is shown in Figure 6.13. From these densities, the inferred data can be
visualized in a number of ways. Figure 6.14 shows the original Wilcock and Crowe (2003) point estimates for reference stress as well as error bars for each point value plotted along side the posterior mean values referenced in Table 6.2. Because there is a lot of overlap in the data in Figure 6.14, each sediment was plotted on its own, with credible interval regions being displayed by the shaded regions of Figure 6.15 and the red points, lines, and error bars representing the original estimates from Wilcock and Crowe (2003). This figure provides a clear comparison of the inferred reference stresses to those specified originally in Wilcock and Crowe (2003). There is a gap in the J06 plot of Figure 6.15, so there is a gap in the credible interval in between the smallest particle size (plotted at 0.707 mm) and the rest of the grain sizes. The point estimate, with error bars for the respective credible intervals was plotted and jittered (offset) to the side so they two values could be easily compared.

Figures 6.16 through 6.18 illustrate the overlap of the individual sediments’ credible regions at varying credibility levels. The sediments are most certainly distinct at the lower end of the grain sizes, but become increasingly convoluted with increasing grain size. What is also apparent from these figures is that mixtures with less sand are generally flatter, signifying an equal mobility regime where all particles begin to move at the same shear stress. Sandier mixtures exhibit a preferential movement, and sandier mixtures, in general, are more easily moved than their coarser counterparts.

There are some structural differences when one looks closely at each individual data series, however. Sediment mixture J27 shows a much more marked dip for the 4.0 mm size class (see also Figure 6.13 for better detail on this). The reference stresses from Wilcock and Crowe (2003) do show a slight decrease for this size fraction as well, but it is not as emphasized as the results from this research. On the whole, the original results shown in Figure 6.14 are more monotonic than the results
of the current analysis. One underlying goal of Wilcock and Crowe (2003) was to further research the effect of sand content on transport and it was assumed (and demonstrated) that increased sand contents increased transport rates. With an overarching theory of increased transport with sand content, one would believe the relationship to be positive monotonic, so the bumps and deviations in the inferred reference stresses may be experimental noise. Nonetheless, the trends shown in figures 6.14 through 6.18 are what the data bear. Monotonic curves could conceivably be drawn through the credible intervals shown in figures 6.16 through 6.18, though perhaps less for the 68% confidence region. This overall positive monotonic relationship that one would suppose from the prevailing theory on sand content and transport, while not directly inferred from the analysis, is still likely given the variability of the parameters.

Figures 6.19 through 6.23 show the fractional rating curves for the five different sediments. The results in these figures indicate that, when there is data, the model is able to infer a reference stress distribution that produces predictions consistent with the observations. While the rating curves generally seem to be appropriately drawn, there is one suspect rating curve, and that is shown in Figure 6.21 for the 32.0 mm size fraction. This size fraction had only two observations basically on top of each other (the two points are somewhat difficult to discern in the plot due to their relative proximity), so the posterior distribution is surprisingly tight compared to the 45.3 mm posterior of the same figure, which only has one observation. The effect of having two points right on top of each other is that the variance is probably not representative of the true process. This shortcoming is not unique to a Bayesian approach, but is a general limitation and constraint that accompany these data. To compensate for this, a more specific prior for the variance parameter could be specified that is more in line expectations.
The inferred reference stresses in Table 6.2 can be used to construct a plot of the Wilcock-Crowe hiding function, which plots the ratio of the reference stress for a given size fraction to the reference stress of the surface geometric mean particle size against the ratio of the grain diameter of a given size fraction to the ratio of the geometric mean surface grain diameter. Thus, Figure 6.24 shows how the reference stress for a given size fraction can collapse into a relationship when scaled by the geometric mean particle size reference stress and diameter. This relationship was a central finding of Wilcock and Crowe (2003) (see Figure 4 of the referenced publication) and resulted from the manually calibrated reference stresses. In this research, the reference stresses were inferred from the Bayesian multi-fraction model and yield a similar trend.

Another central result from Wilcock and Crowe (2003) (see Figure 5 of their paper) was the observation that the reference shear stress of a sediment mixture varies with sand content. It was observed that the reference stress of the geometric mean particle size of the mixture—the grain size used to scale the hiding function—decreased with increasing sand content and a model was fitted to describe this change in reference stress and is included below as (6.13).

\[
\tau_{rm}^* = \theta_1 + \theta_2 \exp[-\theta_3 F_s] \tag{6.13}
\]

with the published result of:

\[
\tau_{rm}^* = 0.021 + 0.015 \exp[-20 F_s] \tag{6.14}
\]

Figure 5 of Wilcock and Crowe (2003) was updated using the inferred parameter values for reference stress and is included as Figure 6.25. The original formulation for describing the change in the reference stress of the mean particle size class is
plotted as a dashed line in Figure 6.25. Because there is significant variability in the right-most point (corresponding to the inferred mean reference stress for BOMC) that is not an issue in Figure 5 of Wilcock and Crowe (2003), a new relationship was fitted using NLS to these data points for a new trend. The results of this regression are included as (6.15).

\[ \tau_{rm}^* = 0.024 + 0.016 \exp[-28F_s] \] (6.15)

While a new equation can be fitted to the plotted points, this does not change the fact that there the right-most point exerts leverage and requires the entire flat portion of (6.15) and Figure 6.25 to be elevated. To explore this further Figure 6.25 was updated to reflect the fact that instead of point values, the Bayesian method infers distributions. Thus, the posterior distribution of the geometric mean surface grain size reference stress was used to construct a credible interval, plotted in Figure 6.26. Here we see that the variation in the reference stress alone could account for significant spread in the resulting equations (6.13) through (6.15).

Lastly, Figure 6.27 shows the fractional similarity collapse that was constructed using the inferred reference stresses for each grain size class and sediment mixture. This figure is comparable to Figure 6 of Wilcock and Crowe (2003) and demonstrates that the estimated parameters from the Bayesian model are able to give valid results when compared to the manually-estimated parameter values used to construct the original similarity collapse of Figure 6 in Wilcock and Crowe (2003).
6.6 Discussion

6.6.1 General

The general conclusion that can be drawn from the results section is that the Bayesian model possesses reasonable inferential and predictive power for multi-fraction sediment transport. The first indication of this success is in the positive indications from the numerical method diagnostic plots, the trace plots and posterior histograms. A second and more convincing indication of model validity is the general correlation of the manually/expert-fitted reference stress curves with those inferred from the model.

Several figures communicate this feature of the analysis, including figures 6.14 and 6.15. The latter figure, in particular, is interesting. There is no instance in which the error bars and credible interval infer a distinct region. The error bars set by Wilcock and Crowe (2003) are generally conservative relative to the 95% credible interval. For example, J06, J14, and J27 have numerous points that assume wider error bars than the credible interval. Small diameters of J06 have almost twice the interval width. One possible explanation for this is that the Wilcock and Crowe (2003) ranges were manually determined and represent values (no formal confidence interval associated with it) that provide a reasonable fit. Thus, all of the variability in the predictions was assumed to come from variation in the reference stress parameter. In the Bayesian model, there are two components that contribute to overall variation—reference stress and the variance parameter. Thus, the reference stress credible intervals of the Bayesian model represent true parameter variability, and the variance parameter represents “other” variability (model misspecification, measurement error, etc.) whereas the reference stress error bars defined in Wilcock and Crowe (2003) had to account for total variability in just one parameter.
The largest discrepancies between the original error bars and the Bayesian credible interval, however, are for the very large diameters. Especially for the 45.3 mm size class of J14, J21, and J27. The original error bars are much smaller than the Bayesian estimate. This likely is because there are so few transport observations of the larger size classes, so calibrating to a single point is problematic.

There are a couple instances where the original estimates are consistently adjacent to the credible interval boundary. The original reference stresses for J27 are consistently on the 95th percentile boundary, and even seem to follow the jogs up and down on that same percentile almost all the way to the top. The credible intervals seem to track very well on this sediment as well.

The overall difference between the manually-fitted reference stresses and those inferred by the Bayesian model, is that the manual values are much smoother and are closer to monotonicity than the Bayesian estimates. As discussed briefly before, one would expect a positive monotonic relationship between sand and transport, so while the manual fitting is likely influenced by this overarching thought, the Bayesian estimates are independent of the monotonic assumption. What can be said, however, is that the credible regions, as drawn in figures 6.13 through 6.18 could in most instances (J14 seems like the weak link) achieve a positive monotonic trend.

What is also interesting to note is that the fractional posterior densities in Figure 6.13 show numerous posteriors with hints of bi-modality in the reference stress parameter. There are also some other posteriors that resemble a heavy-tailed distribution, such as a Cauchy much more than they do a Gaussian distribution (see, for example, the posterior for the 45.3 mm size fraction of J21). As was discussed in Chapter 4, traditional methods of parameter inference (e.g., NLS or weighted least squares) would not be easily modified to accommodate this type of
result, whereas this is a feature of the inferred data that can easily be resolved because of the Bayesian methodology through a simple evaluation of the posterior distribution. Further, the original error bars in Figure 3 of Wilcock and Crowe (2003) do not specify how the values are distributed between the limits, whether it be normally-distributed, uniformly, or some other distribution. This information is, however, provided by the Bayesian approach.

The success of the adaptive component of the MCMC algorithm is quite a significant model development over previous codes. Because previous models only had a few sediments and only one size fraction, manually tuning these models for numerical stability, despite being an iterative trial and error process, was a manageable modeling cost. In order to run numerous sediments each with 13 size fractions using this established tuning practice would have made fitting the model impractical and would have defeated the entire purpose of the model development. Further, previous models only required the specification of one tuning variance, for the reference stress, because the prior distribution for the sediment transport variance parameter (6.11) was an inverse gamma distribution, which is conjugate with the likelihood (6.9) and therefore can be sampled via the Gibbs sampler for which a tuning variance is not required. Because the prior for $\sigma$ was changed to a non-conjugate prior distribution, the MCMC algorithm then requires two tuning variances. To fit all the fractional models in this experiment requires the tuning of 130 parameters—doing so via the adaptive MCMC algorithm is much more practical than doing so manually by trial and error.

6.6.2 Hiding Function and Similarity Collapse

Using the calibrated model parameters from their analysis, Wilcock and Crowe (2003) explored the relationships between the reference stresses and grain size and
sand content. They developed a hiding function, (6.3) and (6.4), similar in spirit to those previously developed by Einstein (1950) and Egiazaroff (1965) to describe the shear stress needed to entrain different size fractions of a given mixture. Figure 6.24 shows the original hiding function as developed by Wilcock and Crowe (2003) and shows the plotted points using the Bayesian estimates for $\tau_{r,sm}$. The hiding function fitted to the inferred reference stresses does not suggest any deficiencies of the original function. By comparing Figure 3 of Wilcock and Crowe (2003) to Figure 6.24 in the present analysis, the only real noticeable feature is that the Bayesian fit seems to have more uncertainty than the original fit as judged by the spread in the data points in the figure.

The ability to predict $\tau_{r,sm}$ was identified by Wilcock and Crowe (2003) as one of the key requirements for a surface-based transport method because the hiding function and similarity collapse are dependent upon it. To this end Wilcock and Crowe (2003) developed a curve, included in this paper as (6.14) and in figures 6.25 and 6.26, plotted sand content versus $\tau_{r,sm}$ and fitted a nonlinear relationship that included an asymptote for large $X$. The notion that $\tau_{r,sm}$ would decrease with increasing sand content was a departure from previous thought, and was clearly demonstrated in Wilcock and Crowe (2003). However, the ultimate relationship of reference stress and sand content that was fitted in Wilcock and Crowe (2003) assumed fixed values for the reference stresses. Further, the decay rate and asymptote is highly sensitive to small perturbations of the data, especially changes in $\tau_{r,sm}$ for J27 and BOMC (the farthest two points on the right in Figure 6.25). As established in figures 6.13 through 6.18 the reference stresses are distributions and are not fixed. The analysis that developed (6.14) assumed that the reference stress for the geometric mean surface particle size was a fixed point. Using the posterior mean values for the mean surface stresses, the present analysis resulted
in a different relationship, with the reference stress for BOMC being higher than what was originally specified in [Wilcock and Crowe (2003)] for the same sediment, which lead to (6.14). There is extreme sensitivity to the reference stress for BOMC in (6.14) because it is a leverage point—slight shifts in its value result in large changes to the trend. The reference stress itself is a random variable, and if we account for this, the regions defined in (6.26) seem to support the validity of the original equation, especially when one considers that for each of the 9 to 10 flume experiments for a given sediment, the mean surface diameter, while well-defined, also had some variability in it.

To some extent, the validity of (6.14) is somewhat of a moot point under a Bayesian approach because this method of parameter estimation can infer that value directly—the hiding function and similarity collapse still depend on the mean surface size reference stress, but this parameter is estimated directly from the model results. But what is also worth mentioning, is that the original Shields variation curve proposed by [Wilcock and Crowe (2003)], (6.14) and the dashed line in figures 6.25 and 6.26 is wholly contained in the credible regions shown in Figure 6.26—the new relationship proposed in (6.15) exceeds the credible interval for a surface sand content of 20% in Figure 6.26. Thus, while prediction of $\tau_{r,sm}$ using the proposed equations is not necessarily needed for a Bayesian implementation of this sediment transport model, the original equation is supported by the Bayesian model results.

The final component of the multi-fraction model is the similarity collapse, shown as Figure 6.27. Each fractional transport observation is plotted using each size fraction’s posterior mean value divided into the observed shear stress, giving the x-axis ratio $\tau/\tau_{r,j}$. The results shown in Figure 6.27 indicate that the model parameters as estimated by the Bayesian algorithm provide a well-defined similarity collapse, similar to that originally produced in Figure 6 of [Wilcock and Crowe]
Any of the sensitivities that were observed in the hiding and Shields parameter variation plots seem to not be an issue in the similarity collapse indicating that the general similarity phenomenon is, perhaps, a robust characteristic of sediment transport rather than an experiment-specific result. In the linear portion of the curve (from x-values of 0.3 to 2) the data points are tightly and symmetrically clustered about the deterministic prediction, and the over-prediction that was observed in the bulk similarity collapse in Figure 6.5 does not appear to be an issue in the multi-fraction collapse. The original similarity collapse of [Wilcock and Crowe (2003)] does not have this same symmetry in the linear region of the plot. However, the collapse shown in Figure 6.27 does seem to suggest a small bias (under-prediction) based on the data—something that the original collapse in [Wilcock and Crowe (2003)] does not seem to have.

### 6.6.3 Motivation and Implementation on New Datasets

The primary motivation for this work is quite simple: to quantify uncertainty. The process of sediment transport is nonlinear and is governed by factors that are more random variables than fixed values. Deterministic predictions of sediment transport, while correct “on average”, do not explicitly account for the full range of possibilities for a given set of conditions. The results of this paper demonstrate that the Bayesian approach provides valid estimates of model parameters for multi-fraction sediment transport with their associated uncertainty.

[Wilcock and Crowe (2003)] made several advances in establishing new relationships that describe sediment transport dynamics, and did so on a deterministic platform. It is widely accepted in the field of sediment transport that randomness and variability are key components of the process, so to create a model that makes it possible to leverage advances in multi-fraction transport prediction
(such as the Wilcock-Crowe equations) with the ability to robustly infer the parameter sets and predictions as random variables (the Bayesian approach) provides an answer to problem of accounting for and managing uncertainty in sediment transport processes.

Kirchner et al. (1990) observed that for a given bed configuration a multiplicity of transport rates was possible. This phenomenon, while understood and widely accepted, has generally found no general solution. Sediment transport predictions may be calibrated to observations, but the calibration results in a series of point predictions (a line) for a continuum of shear stresses. This problem of fitting a relationship is generally challenging enough in practice that doing formal uncertainty studies is largely left out. Without a formal method of quantifying the uncertainty, the only practical option is to do a manual perturbation analysis to see what range of parameter values provide. Doing this type of analysis for 65 different sediment runs (13 sediment fractions for 5 different sediments) is time-consuming and perhaps prone to typographical errors. Further, as discussed in a previous section, doing this for one parameter (reference stress) is likely feasible, but trying to manually include an additional parameter (e.g., variance) and trying to manually partition out parametric and other variability quickly becomes impractical.

The onerous task of uncertainty estimation is eased by applying a Bayesian model. The magnitude of this task multiplies with each new parameter that is added to the mix. Figure 6.15 illustrated that there is a discrepancy between manual ‘trial-and-error’ uncertainty estimates and the quantiles associated with the Bayesian posterior. In this comparison of one uncertainty estimate to another, it is not the author’s intention to say that the Bayesian uncertainty bound is the absolute truth in comparison to the manual estimate. But what can be said is that the MCMC algorithm’s ability to draw thousands of samples from the region of the
parameter space with the highest likelihood is most certainly indefatigable when compared to an individual’s tolerance for mundane trial-and-error tasks. By drawing thousands of samples, the shape of the parameter’s density becomes apparent and the ability to propagate that through to subsequent analyses is relatively simple. Furthermore, during the sampling of these values the random walk component ensures that the relevant areas of the parameter space are considered as suitable candidate values, in a way searching all the corners and regions for likely parameter values.

In the present paper, the Wilcock-Crowe equation is essentially being calibrated to observations with the primary calibration parameter being reference stress. There are numerous ways of calculating the boundary stress of a river or flume, but what is largely unknown is how the total boundary stress is partitioned out into skin friction. This partitioning is largely due to random and constantly changing bed configurations. Thus, observed transport events used to calibrate the shear stress required for transport avoids the many issues associated with trying to partition total boundary stress into skin friction (the stress that is acting on the surface grains) and form drag (the shear stress acting on the dunes, bars, and banks of the river, which does not contribute to grain movement). The approach offered in this paper provides a way to robustly calibrate to the estimate the reference stress that effectively accounts for the drag partition problem.

In addition to estimation of parametric uncertainty, being able to quantify the risk of predictions is extremely valuable. There are conceivably many situations in which the preferential transport of one size fraction over another is of interest. This may be to either remove a certain fraction (e.g., removal of fines for spawning habitat improvement), retain a certain fraction (e.g., contaminated sediment mobility), or simply assess mobility of a given size class or classes (e.g., exogenous
material of different material properties, such as specific gravity). The bulk transport approach as adopted in previous chapters does not permit these types of questions to be answered. The model developed in this paper, however, would allow an estimation of risk on a size class basis. Where traditional methods provide a deterministic estimate of mobility, the approach described in this chapter could show a risk-based mobility assessment and makes it possible to achieve an estimate of the likelihood of mobilizing grain size 'x' if we mobilize grain size 'y'—traditional methods cannot do this.

Taking the method developed in this paper and applying it to additional sets of data is an important next step that needs to occur in order to test its applicability and utility. One of the main advantages of applying this method is that, once the code is setup, incorporating new datasets is generally simple—the hardest part is usually making sure that all the data are formatted and entered correctly. For future use with a new dataset, the fractional transport data (of whatever number of fractions) could be evaluated using (6.8). In this capacity, the general form of the sediment transport curve would be the Wilcock-Crowe equation, which would essentially be calibrated for the new dataset. Because the calibration (which is really the Bayesian parameter estimation procedure) directly specifies the reference stresses for all of the particle size classes, certain functions of the Wilcock-Crowe equations, as specified in Wilcock and Crowe (2003), would no longer be necessary. One of these would be the variation of Shields stress as a function of sand content. Not needing (6) of Wilcock and Crowe (2003) simplifies the overall approach and likely has the overall effect of reducing uncertainty because, as was shown previously, the variation of Shields stress with sand content was highly sensitive to the inferred reference stress of BOMC. The results of an additional dataset would allow for comparison of variation with sand content, and perhaps each subsequent
dataset could be included to develop a series of relationships that quantify the variation of Shields stress with sand content.

### 6.7 Conclusions

This paper describes the motivation for and development of a multi-fraction Bayesian sediment transport model. The model was developed using the Wilcock-Crowe equations and utilized sediment transport data from mixed sediment flume experiments disclosed in [Wilcock et al.](2001). The Bayesian model was fitted to these transport observations and the resulting posterior distributions were analyzed and compared to the results from [Wilcock and Crowe](2003), which is the publication where the Wilcock-Crowe equations were developed and explained. The bulk model posterior distributions were also compared to NLS estimates of model parameters as a verification that the Bayesian code was working correctly and the derived relationships originally produced as the Wilcock-Crowe equations were evaluated using the Bayesian posterior distribution results. It was found that the Bayesian model generally provides an accurate and useful tool to robustly estimate model parameters, though the detailed portions of the Wilcock-Crowe equations (i.e., the hiding function and Shields parameter variation curve) were observed to be extremely sensitive to small deviations in inferred reference stress for the sandier mixtures.

From a modeling perspective, this new model increased the dimensionality of the model from 2 parameters (bulk reference stress and variance) to 2 x k size fractions dimensions. Also, the prior distribution for the variance parameter was changed from an inverse gamma distribution used in prior work with single fraction/aggregated transport to a normal distribution to overcome some constraints identified in previous chapters. This research utilized 5 mixed sediments,
of 13 fractions each, resulting in an estimation of 130 parameters. This increase in dimensionality is significant as it presents a more complete description of sediment transport processes, but brings along with it an increased demand on the MCMC algorithm. While multiple Markov chains could be spawned (for example, one for each size fraction), each Markov chain must be run sequentially thereby making parallel computations for a single fraction off limits. Previous research manually tuned the MCMC algorithm by specifying via trial and error the tuning variances for the reference stress parameter. Because previous research did not consider numerous sediments and size fractions, manually tuning was an acceptable task. In the multi-fraction model, manually specifying 130 parameters would be onerous so an adaptive algorithm was developed to mitigate the increase in number of dimensions. The AMCMC algorithm showed good convergence on both the model parameters as well as the numerical method tuning parameter to the degree that model diagnostic plots confirm convergence on the target posterior distributions.

Predictive distributions were developed for both the bulk and fractional transport rating curves. The fractional transport rating curves showed greater variance for larger particle sizes, owing to the fact that there are fewer observations of large particle movement than small particle movement. Future work needs to provide a better way of managing the zero-transport events so that the information from the experiments in which there was no measurable transport are incorporated into the parameter estimation scheme.

The model developed in this chapter is suitable for use in multi-fraction sediment transport, and the next steps involve applying the methodology on transport events in gravel bedded rivers where the transport of multiple fractions is being measured. Further applications of this concept involve the development of a two-fraction (sand and gravel) model, application to suspended sediment transport,
and development of model code that is distributable for wider use by practitioners.

References


Ikeda, H., and F. Iseya (1988), Experimental Study of Heterogeneous Sediment Transport, Environmental Research Center papers, Environmental Research Center, University of Tsukuba, Tsukuba.


Table 6.1: Data summary for sediment mixtures. Inferred bulk transport posterior means for $\tau_r$, in Pa and $\sigma$, in m$^3$/m/s, number of observational runs for each sediment, and the proportion sand in both the surface and total mixture.

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<th>J21</th>
<th>J27</th>
<th>BOMC</th>
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Table 6.2: Inferred fractional transport posterior means for $\tau_{r,j}$, in Pa.

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Table 6.3: Number of observations of fractional transport.

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Table 6.4: Inferred fractional transport posterior means for $\sigma_j$, in $m^3/m/s$.

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Fig. 6.1: Particle size distribution. Compare to Fig. 1 of Wilcock and Crowe (2003).
Fig. 6.2: Bulk transport. Compare to Fig. 5a of WKC.
Fig. 6.3: Prior distributions for parameters in transport models.
Fig. 6.4: Trace plots for J06 sediment. Results are representative of all bulk sediments. Red line represents the prior distribution as specified in Figure 6.3.
Fig. 6.5: Bulk similarity collapse.
Fig. 6.6: Bulk rating curves. Red line shows the NLS prediction. Color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.
Fig. 6.7: Fractional trace plot of $\tau_{r,j}$ for J14.
Fig. 6.8: Fractional trace plot of $\sigma_{\text{tune}, \tau, j}$ for J14.
Fig. 6.9: Fractional posterior distributions of $\tau_{r,j}$ for J14. Red line represents the prior distribution as specified in Figure 6.3.
Fig. 6.10: Fractional trace plot of $\sigma_j$ for J14.
Fig. 6.11: Fractional trace plot of $\sigma_{\text{tune},i,j}$ for J14.
Fig. 6.12: Fractional posterior distributions of $\sigma_j$ for J14. Red line represents the prior distribution as specified in Figure 6.3.
Fig. 6.13: Inferred reference stress densities.
Fig. 6.14: Comparison of original reference stress curves from Wilcock and Crowe (2003) to inferred curves from Bayesian model. Error bars are those from Wilcock and Crowe (2003).
Fig. 6.15: Individual sediment comparisons of original reference stress curves from Wilcock and Crowe (2003) to inferred curves from Bayesian Model. Error bars are those from Wilcock and Crowe (2003). Credible intervals are color coded from darkest to lightest for 95%, 90%, and 68%. The 0.707 mm value was jittered so as to not overplot for J06.
Fig. 6.16: Inferred reference stresses at 68% credible interval.
Fig. 6.17: Inferred reference stresses at 90% credible interval.
Fig. 6.18: Inferred reference stresses at 95% credible interval.
Fig. 6.19: Posterior predictive distribution for J06, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.
Fig. 6.20: Posterior predictive distribution for J14, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.
Fig. 6.21: Posterior predictive distribution for J21, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.
Fig. 6.22: Posterior predictive distribution for J27, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.
Fig. 6.23: Posterior predictive distribution for BOMC, color coded darkest to lightest for the 95%, 90%, and 68% credible intervals.
Fig. 6.24: Hiding function using inferred mean surface size reference stresses.

Fig. 6.25: Shields stresses plotted against $\tau_{r,sm}$.
Fig. 6.26: Shields stresses plotted against $\tau_{rsm}$ with credible intervals, color coded darkest to lightest for 95%, 90%, and 68%.
Fig. 6.27: Fractional similarity collapse.
CHAPTER 7

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The research presented in this dissertation describes a method for addressing the challenge of characterizing uncertainty in sediment transport problems. Uncertainty in sediment transport has been widely recognized in the literature as affecting our ability to apply theoretical or semi-empirical models developed in a laboratory to actual sediment transport problems. Predictive, model selection, and parameter uncertainties are the main constituents to the overall problem affecting engineering and applied geomorphological studies related to sediment transport. In particular, the literature indicates that the river management community, of which sediment transport is a large part, has generally disregarded uncertainty and its implications for river projects (Wheaton et al. 2008). Indeed, Graf (2008) stated that uncertainty can either be ignored and hoped to not be debilitating to a project, or the reality of uncertainty can be accepted and incorporated as a feature of the research. The goal of this dissertation was to pursue the latter.

Chapter 3 outlined a basic Bayesian sediment transport model which was applied to synthetic and laboratory flume data. A fundamental difference between Bayesian statistical models and traditional approaches is that in a Bayesian approach the model parameters, such as critical or reference shear, are believed to be random variables instead of fixed parameters. The notion of critical shear, one of the main model parameters in sediment transport, being a random variable is consistent with conceptual descriptions of the phenomenon and is well-established and justified in the literature therefore the Bayesian approach is a more appropriate representation than traditional fixed descriptors. Using a general excess shear model
in conjunction with Bayesian theory, the model in this chapter was applied to uniformly-sized sediments. By analyzing synthetically-generated data it was demonstrated that the Bayesian approach was able to correctly estimate the true parameter values used to simulate the synthetic transport observations. A study was made that evaluated the effects measurement and model noise with the number of observations as well as the choice of priors. It was demonstrated that vague priors, such as those that are effectively uniform over some interval are effective default priors for these models. This chapter demonstrated that unrepresentative prior information, even when strongly specified, is overcome by the model provided that there are sufficient observations to refute prior knowledge. In addition to representing model parameters as random variables, the posterior predictive distribution is an extremely helpful concept for the field of sediment transport. Because the overall process is very nonlinear and intrinsically variable even under steady state conditions, making deterministic predictions is unrepresentative of the distribution of sediment transport rates that occur; therefore a predictive distribution that provides a probability density of transport rates for a given flow condition is conceptually more desirable than purely deterministic predictions and has practical advantages that make it extremely useful. For example, using the approach prescribed in this dissertation it is possible to robustly estimate the probability of sediment mobilization for a given flow condition—something that was not readily apparent previously. Further, the Bayesian approach makes these types of interpretations and predictions intuitive as it merely requires the evaluation of the posterior or posterior predictive distributions. This chapter also demonstrated how Bayesian information theory can measure the suitabilities of a competing deterministic equations through the use of the deviance information criterion.

The goal of chapter 4 was to compare and contrast Bayesian parameter
estimation with traditional methods, nonlinear least squares in particular. Wu and Chen (2009) noted that Bayesian methods have not yet gained wide acceptance in the fields of hydraulics and sediment transport, and conjectured that this is in part due to lack of exposure and familiarity as well as its foreign resemblance to approaches that are already familiar with engineers. Thus, chapter 4 draws the comparisons and underlines the distinctions that make a Bayesian approach worth considering. This chapter shows that the model parameters can be easily assumed to originate from any valid probability function, something that is not easily done using traditional methods. It should be noted that while NLS is widely used for many applications, it assumes that model parameters are exactly normal. The NLS approach also does not provide a robust method to characterize predictive uncertainty, whereas the Bayesian model can be used to produce a posterior predictive distribution.

Chapter 5 analyzes a sediment budget developed by Erwin et al. (2011) on the Snake River in Wyoming. Sediment budgets are often based on historic flow records and sediment transport-streamflow rating curves. This chapter demonstrates how the Bayesian approach addresses a fundamental challenge in applied geomorphology, that of propagating uncertainty through an analysis. The research in this chapter develops sediment rating curves at three different locations on the Snake as well as her tributaries to aide in development of a sediment mass balance. Naturally, sediment transported in the Snake does not consist of only one sediment size fraction, but the model approach was to make predictions of the bulk transport by mass and volume without partitioning into different size fractions. This simplification was part of the original research in Erwin et al. (2011) and is a oft-invoked simplification in applied geomorphology. Using the bulk transport rating curves, the inflow predictive distributions are added together and compared to the
outflow predictive distributions to develop a cumulative sediment yield over the streamflow period of record. Where the original research had to rely on subjective assessments of uncertainty envelopes, the uncertainty estimates of the Bayesian approach were shown to be more robust as they not only characterize the credibility limits, but also the shape of the distribution between the limits. Annual sediment yield distributions were accumulated to show that the Snake river is in sediment deficit and calculated a probabilistic estimate of the magnitude of this deficit over the period of record. This chapter demonstrated that more sophisticated deterministic models are trivially incorporated into the Bayesian statistical framework and have practical benefits that merit wider use.

The last topic of the dissertation, discussed in chapter 6, was the development and testing of a multi-fraction sediment transport model using data originally used for the Wilcock-Crowe equations. Because sediment transported in natural systems consists of a distribution of sediments due to the mixed nature of the bed, whose distribution can have significant effects on transport, it is sometimes necessary to implement a more sophisticated or complete model for a particular analysis. Extending the model from a uni-size or bulk sediment fraction to multiple fraction involved significant improvements to previous model code, including the amelioration of the MCMC algorithm to make it practically scalable to predict on multiple size fractions. To this end, an adaptive MCMC algorithm was introduced thereby making the entire approach more scalable and robust. The new code demonstrated that parameter inference and probabilistic predictions can be extended to $k$ fractions. Using the inferred parameters, the original formulations of the Wilcock-Crowe equations were evaluated—in some instances confirmed and in others challenged. The uncertainty bounds of the inferred reference shear values were compared to the original uncertainty estimates and were found to reduce the
amount of parameter uncertainty to lower levels than previously assumed.

The models developed in this dissertation served to introduce and familiarize the discipline to a relevant modeling tool that has not yet been widely used in the field of sediment transport. The main significance of this research is that the models outlined here provide a basic template for and example of how to more robustly characterize the uncertainty associated with sediment transport problems. As discussed in the introductory and literature review chapters, river restoration projects are increasingly trying to manage sediments in such a way that the natural functioning of the rivers are restored along with the associated ecological functions. In this industry, the characterization of uncertainty is recognized to be a critical component, but there exist very few frameworks in which this can be quantified and evaluated. From an environmental perspective, these methods can be used to predict the mobility risk associated with contaminated sediments within a system to help inform management strategies. The methods discussed here also demonstrated that having better-informed estimates of uncertainty, such as was the case with the Snake river sediment budget, we can have a clearer picture of what is occurring in the system after having accounted for the uncertainty. As shown in the last chapter, the ability to evaluate established theory by using the model output (both the posterior distributions and the derived predictive distributions) permit us to understand more fully the applicability and robustness of the deterministic equations that are often applied in practice. Applying and extending the models developed in this dissertation can reasonably improve the standard of practice of sediment management and the design of engineering structures to withstand a better-constrained design event than would be done without.

While the implementation of these models will likely add valuable information to sediment-related work, these models are just the beginning in terms of
sophistication and completeness. There are numerous extensions and improvements that can be made to increase applicability and use. At the very least, the current model codes developed here could be coded into a software distribution for use by practitioners. One of the biggest impediments to wide-spread adoption of this approach is the generally lack of specialized statistical knowledge required to implement. By producing a code package that can read in a set of data and perform the needed analysis a major stumbling block to adoption can be mitigated. While providing the code publicly makes it available for wide-use, there still exists a need for the end users to understand the concepts and caveats of the models. In part, the papers developed for this dissertation serve that purpose, though with more formal code releases these considerations will require more disclosure.

The models themselves can and should be improved by extending to new relations and structural improvements. Modifying the way the current code treats observations of no transport would be a major improvement. For example, in the last chapter it was observed that in an experiment of 10 independent observations only one resulted in transport. In the current model, this translates into only one data point. Just because no movement occurred in an experiment does not mean that it contained no information. Adapting the current models to account for the zero transport observations would be a valuable improvement in model parameter estimation and prediction.

Most of the effort of this dissertation was spent simply trying to prove the concept that the Bayesian method to sediment transport will address much of the predictive and inferential uncertainty that is currently a challenge. Only in the last chapter was the method used to evaluate existing relationships and propose changes. Using a Bayesian approach to evaluate and work through structural uncertainty would also be an important improvement worthy of additional research.
In the wider field of hydrology, Bayesian methods have been successfully used to develop new understandings and relationships as a result of increased visibility to practitioners and maturation of the models. The general field of sedimentation engineering and applied geomorphology has not fully committed to this avenue of research most certainly for a number of reasons, but primarily due to lack of visibility, unfamiliarity among practitioners, and inaccessibility to the specialized code required to carry out the analysis. The products of this dissertation, it is hoped, can serve to facilitate this area of research and provide the basis for future work.

References


APPENDICES
1. Select a starting value for critical shear, $\tau_c(0)$.

2. For iterations, $k = 1, 2, \ldots N$:

   (a) Calculate $q_s|\tau_c(k - 1)$ using (4.2).

   (b) Calculate updated parameters, $\tilde{r}$ and $\tilde{q}$ using (3.20) and (3.21).

   (c) Obtain $k^{th}$ sample for $\sigma^2 \sim I.G.(\tilde{r}, \tilde{q})$.

   (d) Sample $\tau_{cs} \sim T.N.(\tau_c(k - 1), \sigma^2_{\text{tune}})^{\tilde{b}}$.

   (e) Calculate $q_{ss}|\tau_{cs}$ using (4.2).

   (f) Calculate the ratio, $\rho$, via:

   $$\rho = \frac{[q_{ss}|q_0, \sigma^2(k)]|[\tau_{cs}|\mu_{\tau_c}, \sigma^2_{\tau_c}]}{[q_s|q_0, \sigma^2(k)]|[\tau_c(k - 1)|\mu_{\tau_c}, \sigma^2_{\tau_c}]}.$$ 

   (g) Accept/reject proposed value, $\tau_{cs}$:

   $$\tau_c(k) = \begin{cases} \tau_{cs}, & \text{with probability } \rho \\ \tau_c(k - 1), & \text{with probability } 1 - \rho. \end{cases}$$

This algorithm is repeated $N$ times until an adequate number of samples are obtained from the target distribution. The total number of samples from the target distribution is equal to the number of samples, $N$, less the number of iterations required for burn-in, $N_{\text{burn-in}}$ (the number of samples required for convergence on
target density). These samples from the target distribution can then be used to make inference on the parameters of interest: $\tau_c$ and $\sigma^2$. 
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Logan, UT 84322-5210

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**Education**

**Ph.D. Civil and Environmental Engineering**
Utah State University, College of Engineering  
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Advisor: David K. Stevens, P.E., Ph.D.
Dissertation: Applications of Bayesian Statistics in Fluvial Bed Load Transport.

**M.S. Statistics**
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GPA 3.9/4.0
Advisor: Mevin B. Hooten, Ph.D

**B.S. Civil and Environmental Engineering**
University of Utah, College of Engineering  
GPA 3.5/4.0
Advisor: Lawrence K. Reaveley, P.E., Ph.D.
Senior Project: Design of parking structure and UTA TRAX realignment on the University of Utah campus.

**Professional Experience**

**Chief Engineer, Water Resources**
Freeport-McMoRan Copper & Gold  
Nov 2011 – Present
Currently working as Chief Engineer of water resources for the corporate technical services department. Duties consist of overseeing and managing engineering work and studies at company operations worldwide and providing technical expertise to help guide strategic planning, operational improvement, and engineering activities for special projects. Managing work in advancing and applying a 2-dimensional coupled (hydraulics and sediment transport) model of a large river system; designing passive stormwater treatment systems to achieve high water quality standards; working in developing a more robust bathymetric data collection method for all of our sites; developing a project whose goal is to restore an impacted river through geomorphic restoration principles; development and improvement of stochastically-based water use models; guiding work to improve estimation of surface evaporation processes on tailings storage facility beaches; managing a project that develops a regional surface/subsurface hydrology model; project management of consultants and programs, managing $30 M in design and construction projects over the next 5 years, managing approximately $1 M in consulting activities for water related projects at mine sites mainly in Indonesia and Africa.

**Self Employed**
Civil Engineer, I.T. Consultant, Patent Drafter  
May 2004 – Aug 2009
Worked with engineering consulting firms setting up sophisticated engineering software packages (Bentley Microstation and Digital Interplot Server) to bring them into compliance with state project delivery guidelines. Provided general I.T. support for engineering offices. Worked closely with an intellectual property attorney to draft figures and drawings required for patent prosecution, both international and domestic.
H. W. Lochner, Inc., Salt Lake City, UT  
Jan 2001 – Jun 2005

Civil Engineer  
Worked as a design civil engineer for a transportation engineering firm. Performed hydrologic and hydraulic design, urban hydrology, stormwater conveyance, attenuation and retention; gravity-flow irrigation design in pipes and ditches; and modeling with PondPack, StormCAD and FlowMaster. Designed according to federal manuals including TR-55, AASHTO Model Drainage Manual and various HEC publications. Developed excellent drafting skills in Microstation and AutoCAD and gained experience with ESRI’s ArcMap software and integration of CAD drawings into ArcMap. Acquired experience in utility design, location, and conflict resolution as well as roadway geometric design, maintenance of traffic, and signing and striping. Used web development software to design public and project stakeholder websites in public outreach efforts. Developed proficiency in the development of html, asp and cfml web pages. Served as technical writer for various engineering reports, such as traffic justification and environmental analysis.

Consolidation Coal Company, Richlands, VA  
May 2000 – Aug 2000

Engineer’s Aide  
Underground surveying, general underground labor – belt maintenance, belt-line setup, longwall setup. Lots of shoveling.

Research Experience  
Utah State University – Utah Water Research Laboratory, Logan, UT  
Ph.D. Student  
Aug 2005 – May 2013

Developed a series of Bayesian statistical sediment transport models that merge deterministic equations into a probabilistic framework, thereby permitting the quantification of parametric, predictive, and model selection uncertainty in a strongly nonlinear process. Bayesian statistical methods have widespread use in hydrology but have not been applied to sediment transport problems until recently. The models that were developed were applied to synthetic, flume, and data collected on river systems and showed key inferential and predictive advantages or traditional approaches that have been utilized in fluvial geomorphology approaches to date.

Master’s Student  
Aug 2009 – May 2011

Developed an approach to fluvial sediment transport modeling using a Bayesian statistical framework. The resulting model explicitly accounts for uncertainty in measurements, parameters, and model misspecification errors while allowing deterministic functions to describe the process. The Bayesian transport model provides risk-based predictions and a means to compare competing scientific theories while incorporating expert judgment into model inference and prediction.

Graduate Research Assistant  
Aug 2005 – Nov 2011

Co-developer and lead programmer of data-visualization software (IIMS Time-series Analyst) for the U.S. Bureau of Reclamation. Developed desktop and web-based versions that display and analyze large time-series datasets stored in enterprise-class database systems. Met regularly with project partners to guide development direction and priorities of cooperative agreement. Coordinated programming efforts with program partners and sub-contractors. Responsible for the hiring, supervision, and direction of one programmer. Provided software training meetings to stakeholders and client.
Professional Affiliations
Golden Key International Honors Society
Chi Epsilon Honors Engineering Society
American Society of Civil Engineers
American Statistical Association
American Geophysical Union
Society for Mining Metallurgy and Engineering (Chapter president, University of Utah 2000-2001)

Honors and Awards
U.S. Department of Education GAANN Fellowship recipient ($75,000)
Utah Water Research Laboratory Graduate Research Assistantship ($80,000)
Distinguished Browning Scholar
Browning Scholar
College of Engineering Dean’s List
Finalist—AWRA-Utah Student Paper Competition, 2011
Best Oral Presentation—Intermountain Graduate Research Symposium 2011
Best Paper—AWRA-Utah Student Paper Competition, 2011

Teaching Experience
Instructor – Engineering Statistics in R
Co-developed and co-taught (with Ph.D. advisor) a weekly seminar on engineering statistics in the R software environment. Attendees consisted of graduate students and faculty from several departments. Researchers would present their computational and statistical challenges. Appropriate solutions in R were subsequently coded and presented.

Instructor – CEE 6670: Environmental Process Laboratory
Laboratory and classroom sessions taught in conjunction with a graduate-level course in physical/chemical processes in Environmental Engineering. Prepared and delivered lectures for class of nine graduate students; solely responsible for grading of lab reports, writing and grading exams, and laboratory setup. Course focused not only on technical aspects of physical and chemical processes in engineering, but also on developing technical writing skills.

Student review summary (out of 175 questions):
Excellent (104/175), Very Good (58/175), Good (11/175), Fair (2/175), Poor (0/175), Very Poor (0/175). Average rating of effectiveness in teaching the subject matter was 5.4/6.0 where 6 denotes “Excellent” and 5 denotes “Very Good.”

Guest Lecturer – CEE 6660: Statistics for Environmental Engineers
Prepared and gave lectures relating to statistical concepts and computational techniques in the R programming language, helped students with homework problems, directed computing laboratory sessions, prepared and supervised environmental tracer laboratory experiments.

Teaching Assistant – CEE 6630: Process Dynamics
Directed students in computational exercises in Matlab and R.

Teaching Assistant – CEE 6660: Statistics for Environmental Engineers
Directed students in computational exercises in the R programming language.
GAANN Fellowship Instruction  
Fall 2005 – Fall 2006

Three semester-long preparatory seminars on the topics: Effective Engineering Instruction, Successful Faculty Strategies, and Grant and Proposal Writing. Courses taught by tenured faculty and provided opportunities for classroom skills practice and development of professional habits and strategies suitable for academic and research activities at a university.

**Publications**

**Schmelter, M.L.,** P.R. Wilcock, M.B. Hooten, and D.K. Stevens (in preparation), Multi-fraction Bayesian bed load sediment transport model

**Schmelter, M.L.** (2013), Applications of Bayesian statistics in fluvial bed load transport, Ph.D. Dissertation, Utah State University, Department of Civil and Environmental Engineering.


**Schmelter, M.L.** (2011), Accounting for Uncertainty in Sediment Transport Using Bayesian Statistical Models, AWRA-Utah Section Student Conference and Paper Competition.


**Presentations**


Schmelter M. L. (2010), Bayesian Sediment Transport Modeling. Presented at Special Seminar for Fluvial Geomorphology Laboratory, Department of Watershed Science, College of Natural Resources, Utah State University, Logan, UT, 3 Sept.

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Spoken Languages
English – Native.
French – Fluent, written and oral.
German – Novice.

Computer Languages
Microsoft Visual Studio: VB.NET (advanced), ASP.NET, AJAX, C# (novice).
Database: Oracle, MySQL, SQL Server.
Statistics: R (expert), SAS.
Mathematics: Maple, Matlab, Mathcad.
**Software Proficiency**

*Engineering*: AutoCAD, Microstation, In-Roads, Descartes, Digital Interplot, StormCAD, PondPack, FlowMaster, CulvertMaster, HEC-HMS, HEC-RAS, ESRI ArcMap 10, ArcHydro, TauDEM, GoldSim

*General*: Microsoft Office, LaTeX, TeXniccenter.


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