SIMPLIFIED APPROACH TO THE PROBLEM OF STABILITY
OF SOIL SLOPES UNDER HORIZONTAL EARTHQUAKE
AND PORE PRESSURE

by

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LIST OF SYMBOLS

1. Angle of internal friction \( \phi \)
2. Acceleration due to gravity \( g \)
3. Arc length \( L \)
4. Buoyant unit weight of soil \( \gamma_b \)
5. Construction pore pressure factor \( C \)
6. Chord length \( L \)
7. Earthquake acceleration coefficient \( \alpha \)
8. Equivalent unit weight \( \gamma_E \)
9. Friction angle modified due to earthquake (horizontal) \( \phi_{me} \)
10. Friction angle modified due to pore pressure \( \phi_{mp} \)
11. Force due to cohesion \( C \)
12. Factor of safety \( F \)
13. Horizontal earthquake force \( F_E \)
14. Initial angle of internal friction without earthquake or pore pressure \( \phi_i \)
15. Moment arm ratio \( k/d \)
16. Saturated unit weight of soil \( \gamma_s \)
17. Specific gravity of soil \( G \)
18. Slope of embankment \( i \)
19. Sudden draw-down factor \( \beta \)
20. Seepage factor \( \gamma \)
21. Stability number \( c/F \gamma_H \)
22. Unit cohesion \( c \)
23. Unit weight of soil at any moisture content \( \gamma_t \)
24. Unit weight of soil \( \gamma \)
25. Unit weight of water \( \gamma_{w} \)
26. Void ratio \( e \)
27. Weight of sliding mass \( W \)
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CHAPTER I
INTRODUCTION

A problem that has confronted engineers for a great number of years is that of determining the stability of soil slopes. Many factors introduce complications into stability analysis. Most embankments contain heterogeneous soils, often of several types. This and other complications usually necessitate the use of a simplified cross section. At the same time, it is often necessary to adopt simplified average soil characteristics which represent actual characteristics as best as possible. Those steps which bear on the choice of the simplified section and the simplified soil characteristics are always important in stability analysis work. These steps are independent, however, of the actual analysis. From this point on, simplified conditions only are considered, since the main object herein is to explain methods of analysis once the simplifications are made. In this analytical approach the following simplified conditions and assumptions are carried out:

1. An average or typical cross section is used. It is assumed that no shearing stresses act on the plane of the section and, therefore, that a two-dimensional case exists. The entire mass is assumed to be composed of one type of uniform soil.

2. It is assumed that the shearing strength of each individual soil occurring in the cross section may be represented by an expression in the form of Coulomb's empirical law:

   \[ S = c_e + \bar{\sigma} \tan \phi_e \]

   where \( c_e \) and \( \phi_e \) are the effective cohesion and the effective friction angle, respectively, that apply for each soil under
the existing conditions. The normal pressure \( \bar{p} \) is the
intergranular pressure on the failure plane.

3. The intergranular pressure used in the shearing strength
expression is obtained by deducting the neutral pressure from
the total pressure existing at that point, provided that the
earthquake is not considered. To get the effective inter-
granular pressure with the horizontal earthquake being considered
it is necessary to subtract the normal component of the earth-
quake pressure from the above intergranular pressure. The
neutral pressure or the pore pressure at any point is given by
the flow-net in case of seepage, by the full hydrostatic pressure
in case of sudden draw-down and by

\[
\frac{U_a \Delta V}{V_{ao} + hV_w - \Delta V}
\]

case of consolidation during construction (10). In the above
expression

\( U_a \) = Initial pore pressure (usually considered as
atmospheric pressure—absolute pressure)

\( V_{ao} \) = Initial volume of free air in the soil mass
(per cent of total initial volume)

\( \Delta V \) = \( V_{ao} - V_{ac} \) = Volume change from consolidation
(per cent of total initial volume)

\( V_w \) = Volume of water in the soil mass (per cent of
total initial volume)

\( h \) = Capacity of water to dissolve air from Henry's
Law (approximately 0.02)

All stability analyses are based on the concept that an embankment fails
unless the resultant resistance to shear on every surface traversing the
embankment is greater than the resultant shearing force exerted on that
surface by the mass above, along with the earthquake force when this is to be considered. The surface which is most liable to fail is called the critical surface.

**Proposed research**

The $\phi$-circle method developed by the late D. W. Taylor provides a relation between the stability number ($c/F\gamma H$) and slope angle $\phi$ for different values of $\phi$ (angle of internal friction). But, Taylor in his mathematical analysis did not consider the possibility of failure of slopes from other considerations such as

1. Earthquake (horizontal)
2. Pore pressure due to construction.

So the major investigation of this dissertation is to find some relationships such that one can utilize Taylor's stability charts even when one is to consider horizontal earthquake or pore pressure or both. The writer here analyzes the following cases:

1. Earthquake (horizontal)
2. Pore pressure
   a. Due to sudden draw-down
   b. Due to seepage
   c. Due to construction.

This study is limited to a simple slope with ideal conditions.
CHAPTER II
REVIEW OF LITERATURE

Various methods of stability computations have been developed. All assume homogeneous soil, constant angle of slope, level top surface and shearing strength as expressed by Coulomb's equation. The most common methods may be summarized as follows:

1. The Culmann method assumes rupture will occur on a plane. It is of interest only as a classical solution, since actual failure surfaces are invariably curved.

2. The Resal-Frontard method assumes the soil mass to act as a slope of infinite extent, and by use of conjugate stress relationships results in an equation for the rupture surface. This method has been criticized because the results indicate that the mass above the rupture surface is not in static equilibrium at the point of failure with all shearing strength being utilized.

3. Brahtz method of stability analysis: In the development of the science of soil mechanics, as in the development of most of the physical sciences, the gap between theory and application has at some points been wide and difficult to bridge. One school of thought has developed methods of analysis which, though easily applied, fail to take into account some of the important physical properties of the soil mass. A second school of thought has deduced methods, which although they take into account many of the physical properties of the materials, involve intricate mathematical computation, and
are, therefore, somewhat abstruse and difficult of application. 
Brahtz' method of stability analysis belongs to the second 
group.

4. The spiral method assumes the rupture to be a logarithmic 
spiral.

5. The circular arc method was first proposed by K. E. Petterson, 
based on his study of the failure of a quay wall in Goeteborg 
in 1915 or 1916. The justification that circular arcs are 
close approximations of actual rupture surfaces comes from field 
investigations of a large number of actual slides, especially 
in railroad cuts by the Swedish Geotechnical Commission. This 
method has been widely accepted as satisfactory, and several 
methods of procedure based on the circular arc have been 
proposed, among which are the following:

a. Method of slices (7): This method was advanced by the 
Swedish Geotechnical Commission and developed in quite some 
detail by Professor W. Fellenius. The main objection to 
this or any other graphical method such as that given by 
A. W. Bishop (11) and later developed by M. Arnold (15) 
rests in the fact that the most dangerous circle of an 
infinite number of possible circles must be found, and, 
thus, the graphical procedure must be repeated for a large 
number of circles.

b. $\phi$-circle method (1): This method was proposed some years 
ago by Professors Glennon Gilboy and Arthur Casagrande, its 
initial use being in the development of a completely 
graphical solution of the slope problem. In the mathemat-
ical solutions, the late D. W. Taylor set up a relation
between the dimensionless number \((c/F \gamma H)\) and slope angle \(i\) for different values of \(\phi\) (angle of internal friction).

In that above dimensionless number, \(F\) is the factor of safety with respect to cohesion, \(c\) is the actual unit cohesion for the soil in question, and \(\gamma\) is the effective unit weight of soil. This basic dimensionless expression is called the "stability number." This method is a simple convenient bridge between theory and application. But, Taylor in his mathematical analysis did not consider all possibilities such as earthquake, pore pressure, etc. The purpose of this analysis is to develop a simple, clear and objective solution for the horizontal earthquake case and various types of pore pressure cases.
CHAPTER III
DIMENSIONAL AND MATHEMATICAL ANALYSIS

Dimensional analysis

Stability of a soil slope is dependent on

\[ H = \text{height} = L \]
\[ i = \text{slope angle} \equiv (\text{dimensionless}) \]
\[ c = \text{soil cohesion} \equiv FL^{-2} \]
\[ \phi = \text{friction angle of soil} \equiv (\text{dimensionless}) \]
\[ \gamma = \text{total unit weight of soil} \equiv FL^{-3} \]

\( \pi \)-term relationship to determine the critical height \( H \) at which a landslide will occur is given by:

\[ H = f_1(c, \gamma, i, \phi) \]

i.e.,

\[ H = C_\alpha \left( c_1 \gamma c_2 i c_3 \phi c_4 \right) \]

where

\( c_1, c_2, c_3, c_4 \) are constant indices.

So,

\[ L = (FL^{-2})^1 (FL^{-3})^2 \]

Then,

\[ L : 1 = -2c_1 - 3c_2 \]
\[ F : 0 = c_1 + c_2 \]

So,

\[ H = C_\alpha \left( c_1 \gamma^{-1} i c_3 \phi c_4 \right) \]

Or,

\[ \frac{c_\phi}{F_H} = f(i, \phi) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1) \]

If one introduces \( F \), the factor of safety with respect to cohesion, then above equation (1) becomes:

\[ \frac{c_\phi}{F_H} = f(i, \phi) = \text{function of } i \text{ and } \phi. \]
This dimensionless quantity, $\frac{c}{F_H}$, is called the stability number. It depends on slope angle $i$ and friction angle $\phi$, and one will write it as $\frac{c}{F_H}$. Two different factors of safety had been proposed for use in stability problems. Fellenius and most other investigators used the ratio of actual shearing strength to critical shearing strength, which was in agreement with the usual concept of factor of safety and was called the true factor of safety. The other factor had been used by Jaky and Rendulic, and might be described as the ratio between actual cohesion and critical cohesion.

Of the two items which comprise the shearing strength, namely cohesion and friction, the ratio between actual and critical cohesion has bearing on the cohesion only and will be called herein the factor of safety with respect to cohesion. In setting up mathematical solutions, the use of this factor proves to be simpler, thus it is used in the derivations given herein. However, the true factor of safety may easily be adapted in the results of these solutions. The friction angle, $\phi$, as used here, is the developed obliquity. If a true factor of safety, $F_T$, is desired to apply to both cohesion and friction, then the developed obliquity must be $\phi/F_T$. Expressed mathematically the above may be summarized as follows:

$$\frac{c}{F_H} = f(i, \phi)$$

Mathematical analysis

The mathematical solution was given by D. W. Taylor (1), first for the case of circles passing through the toe of the slope, and followed by the case of circles passing below the toe of the slope. In this analysis the forces considered were:

1. The weight of the mass, $W$, acting vertically downward through the center of gravity of the mass.
Figure 1. Sketches showing elements of the $\Phi$-circle method for circle passing through the toe of slope.

2. The resultant cohesion, $C$:
   Its magnitude is $c_1 \overline{L}$, where $c_1$ is the unit cohesion required for equilibrium and $\overline{L}$ is the length of the chord $AB$. Its line of action is parallel to the chord $AB$ and its moment arm, $a$, is described by

   \[ a = \frac{R}{L} \]

   where $\overline{L}$ is the length of arc $AB$. Thus, the line of action of $C$ may be found, and its position is independent of the magnitude of the cohesion.

3. The resultant force, $P$, is transmitted from grain to grain of the soil across the arc $AB$. The force polygon is as shown in the sketch.

   From Figure 1 (a)

   \[ C = c_1 \overline{L} = \frac{c_1 \overline{L}}{F} = \frac{2c_1}{F} R \sin \beta \]

   where, $F = \text{factor of safety}$.

   \[ W = \gamma R^2 \beta - \gamma R^2 \sin \beta \cos \beta + \frac{H^2}{2} (\cot \phi - \cot i) \]

   \[ R = \frac{H}{2} \csc \phi \csc \beta \]

   \[ (1) \]

   \[ (2) \]

   \[ (3) \]
which reduced and combined with (3) gives

\[ Wd = \frac{H^2}{12} \left[ 1 - 2 \cot^2 i + 3 \cot i \cot \alpha - 3 \cot i \cot \beta + 3 \cot \alpha \cot \beta \right] \]  

(4)

From Figure 1 (c)

\[ \alpha' = d \cosec \gamma \]  

(5)

\[ \alpha' = a \cosec (\alpha - u) = R \cosec \beta \cosec (\alpha - u) \]  

(6)

\[ \alpha' = R \sin \phi \cosec (u - v) \]  

(7)

From Figure 1 (b)

\[ \frac{W}{C} = \cos \frac{(\alpha - v)}{\sin v} = \cos \alpha \cot v + \sin \alpha \]  

(8)

Substituting (3) in (2) and the result in (4)

\[ H^2 = \frac{1/2 \cosec^2 \alpha (R \cosec \alpha \cot \beta - \cot \beta) + \cot \alpha - \cot i}{1/3 (1 - \cot^2 i) + \cot i (\cot \alpha \cot \beta) + \cot \alpha \cot \beta} \]  

(9)

From (5), (6) and (3)

\[ \cot \gamma = \frac{H^2}{2d} \cosec \alpha \cosec \gamma \cosec^2 \beta - \tan \alpha \]  

(10)

From (5), (7) and (3)

\[ \sin (u - v) = \frac{H}{2d} \sin u \cosec \alpha \cosec \beta \sin \phi \]  

(11)

Placing (3) in (1) and (2), then setting (8) equal to the ratio of (2) and (1)

\[ \frac{c}{\gamma} = \frac{1/2 \cosec^2 \alpha (R \cosec \alpha \cot \beta - \cot \beta) + \cot \alpha - \cot i}{2 \cot \alpha \cot \beta + 2} \]  

(12)

The solution for the case where the rupture surface passes below the toe of the slope is almost the same as for the above case. The only important difference is that another variable enters which is designated by \( n \), and is shown in Figure 2.
Figure 2. Elements of $\phi$-circle method for circle passing below the toe of the slope.

Here, $n = \frac{1}{2} \left( \cot \alpha - \cot \beta - \cot i + \sin \phi \csc \alpha \csc \beta \right)$ \hspace{1cm} (13)

$\frac{c}{F \overline{H}} = \frac{1}{2} \csc \alpha \left( \frac{\beta \csc \beta - \cot \beta}{2} - \cot \alpha - \cot i - 2n \right)$ \hspace{1cm} (14)

$D = \frac{1}{2} \left( \csc \alpha \csc \beta - \cot \alpha \cot \beta + 1 \right)$ \hspace{1cm} \hspace{1cm} (15) \hspace{1cm} (Depth factor)

Figure 7 gives the chart for stability number vs slope angle for different angles of internal friction. Figure 8 gives the chart showing effect of depth limitation, DH on stability number $\frac{c}{F \overline{H}}$ for $\phi = 0$. This DH is defined as the maximum depth of a point through which the slip circle passes. Now the question is: can these charts be used when horizontal earthquake or pore pressure is introduced in the problem?

The writer has analyzed the above cases in the following way:

**Case I: Earthquake (horizontal)**

Figure 3.
If one analyzes the effect of horizontal earthquake force on the stability of soil slopes, it can be shown that it increases the overturning force and decreases the resisting force. If now, one considers the effective unit weight of the sliding mass as $\gamma_E$, then the effective weight $W_E$ ($\gamma v$) will give the same overturning moment as by the horizontal earthquake force, $F_E$ ($\alpha W = \alpha \gamma v$) and the weight, $W$.

In other words, from Figure 16, by taking moment about the center of the circle

$$W_E d = Wd + F_E t$$

Or $\gamma_E Vd = \gamma Vd + \alpha \gamma Vl$

Or $\gamma_E = \gamma + \alpha \gamma l/d = \gamma (1 + \alpha l/d)$ .... (1)

Where $V$ is the volume of the sliding mass, $\gamma$ is the given unit weight of the soil, $l$ is the moment arm of the force $F_E$ and $d$ is the moment arm of $W$, the weight of sliding mass.

It is this $\gamma_E$ which will be used in the stability number. Now, if one analyzes a small elementary strip on the sliding mass with weight, $W_j$ and horizontal earthquake force of $\alpha W_j$, then it can be seen that the resisting force other than cohesion is $(W_j \cos \theta_j - \alpha W_j \sin \theta_j)$ tan $\phi_i$, where tan $\theta_j$ is the slope of the sliding circle at the $j$th element, and $\phi_i$ is the given angle of internal friction.

So, the total resisting force is

$$\sum (W_j \cos \theta_j - \alpha W_j \sin \theta_j) \tan \phi_i$$

i.e., $\gamma \sum [(y_2 - y_1)_j \cos \theta_j - \alpha (y_2 - y_1)_j \sin \theta_j] \Delta x \tan \phi_i$ .... (2)

The total effect due to earthquake force being taken care of by equivalent unit weight, $\gamma_E$ and the modified friction angle, $\phi_{mE}$, the resisting force is given by $\gamma_E \sum [y_2 - y_1)_j \cos \theta_j] \Delta x \tan \phi_{mE}$ .... (3)

Now equating (2) with (3),
\[ \gamma \sum [(y_2 - y_1)_j \cos \theta_j - \alpha (y_2 - y_1)_j \sin \theta_j] \tan \phi_i = \gamma^E \sum [(y_2 - y_1)_j \cos \theta_j] \tan \phi_{mE} \]

Or, \[ m = \frac{\tan \phi_{mE}}{\tan \phi_i} = \frac{\gamma^E}{\gamma} \left[ 1 - \frac{\alpha \sum (y_2 - y_1)_j \sin \theta_j}{\sum (y_2 - y_1)_j \cos \theta_j} \right] \]

(4)

Where \( m \) is the ratio of the tangent of the modified friction angle to the tangent of the initial or given friction angle. So from (1) and (4),

\[ m = \frac{1}{(1 + \alpha l/d)} \left[ 1 - \frac{\alpha \sum (y_2 - y_1)_j \sin \theta_j}{\sum (y_2 - y_1)_j \cos \theta_j} \right] \]

(5)

So, for different values of \( \alpha \), different values of \( m \) will be obtained for a given \( \phi_i \) and a given slope. Now, from this \( m \), \( \phi_{mE} \) can be obtained. And, it is with this value of \( \phi_{mE} \) one will look into Taylor's chart when horizontal earthquake is to be considered. The slope with the new slip circle (obtained with modified \( \phi \)) with equivalent unit weight \( \gamma^E \) will give the same resisting moment and overturning moment as given with the original slip circle (obtained with initial \( \phi \)) with the weight \( W \) and the earthquake force, \( F_E \).

For a wide range of \( \alpha \), the value of \( \phi_{mE} \) lies between \( \phi_i \) and 0 i.e., \( \phi_i > \phi_{mE} > 0 \). It can be seen that the failure circle given by \( i \) and \( \phi_i \) and that by \( i \) and 0 are very close to each other, or in other words, the discrepancy involved is very small. So, \( L_i \approx L_m \), then \( c_i \) (cohesive force) \( \approx \) \( c_m \) (cohesive force) for small values of \( c \) (unit cohesion), The total resisting moment for the initial case with earthquake is given by equation (2) and that given for the modified case by equation (3). From equations (1), (2) and (3) one can get the equation (5), and from there one gets the values of \( \phi_{mE} \) (modified friction angle due to earthquake). By successive approximation one will also get the same result. For example, let \( i = 30^\circ \) and \( \phi_i = 20^\circ \) and for which \( l/d = 1.77 \) from Figure 9. From equation (5), the solution of
which is shown in Appendix under heading, \( i = 30^\circ, \phi_i = 20^\circ \)

\[
m = \frac{1 - 0.48\alpha}{1 + 1.77\alpha} = \frac{\tan \phi_m}{\tan \phi_i}
\]

when \( \alpha = 0.10 \), then \( m = 0.81 \), and then \( \phi_m = 16.5^\circ \).

Now, using \( \phi = 16.5^\circ \), \( \phi/d = 1.82 \) (for Figure 9.)

For \( \alpha = 0.10 \), \( m = 0.80 \).

Then \( \tan \phi_m = 0.80 \times \tan 20^\circ = 16.5^\circ \) (approximate).

The equation (5) can be expressed in the form of integration as follows:

\[
m = \frac{1}{(1 + \alpha l/d)} \left[ 1 - \alpha \int \frac{(y_2 - y_1) \sin \theta}{(y_2 - y_1) \cos \theta} dx \right]
\]

Or, \( m = \frac{1}{(1 + \alpha l/d)} \left[ 1 - \alpha \frac{\text{Numerator}}{\text{Denominator}} \right] \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5) \)

Equation of the circle is

\[
(x - h)^2 + (y - k)^2 = R^2 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)
\]

Equation of the line OA is

\[
y = x \tan i \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7)
\]

Equation of the line AB is

\[
y = (2R \sin \beta \sin \alpha) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (8)
\]

Where, \( h = R \sin (\beta - \alpha) \)

\([K = R \cos (\beta - \alpha) \]

From equation (6)

\[
\frac{dy}{dx} = \frac{x - h}{y - k} = \tan \theta
\]

So,

\[
\sin \theta = \frac{x - h}{R} ; \quad \cos \theta = \frac{y - k}{R} \cdot \frac{\sqrt{R^2 - (x - h)^2}}{R}
\]

So, in equation (5)

\[
\text{Numerator} = \int_0^{2R \sin \beta \sin \alpha} (y_2 - y_1) \sin \theta \, dx
\]
Numerator = \int_0^{2R \sin \beta_o \sin \alpha_o / \tan i} \left[ x \tan i - (K - \sqrt{R^2 - (x-h)^2}) \right] \frac{x-h}{R} \, dx

+ \int_0^{2R \sin \beta_o \cos \alpha_o - (K - \sqrt{R^2 - (x-h)^2})} \frac{x-h}{R} \, dx

2R \sin \beta_o \sin \alpha_o / \tan i

Numerator = \frac{\tan i}{R} \left[ \frac{x^3}{3} - h \frac{x^2}{2} \right] \left| \begin{array}{c}
2R \sin \beta_o \sin \alpha_o \tan i \\
0 \\
- \frac{K}{R} \frac{(x-h)^2}{2} \\
0 \end{array} \right| 

+ \frac{1}{R} \left[ -\frac{1}{3} \left[ R^2 - (x-h)^2 \right]^{3/2} \right] \left| \begin{array}{c}
2R \sin \beta_o \sin \alpha_o / \tan i \\
0 \\
2R \sin \beta_o \sin \alpha_o \tan i \\
0 \end{array} \right|

Numerator = (\tan i) R^2 \left[ \frac{3}{3} \sin^3 \frac{\beta_o \sin^3 \alpha_o}{\tan^3 i} - \sin (\beta_o - \alpha_o) \right] 

- \cos \left( \frac{\beta_o - \alpha_o}{2} \right) R^2 \left| \begin{array}{c}
2 \sin \beta_o \sin \alpha_o \tan i \\
0 \\
- \sin (\beta_o - \alpha_o) \end{array} \right|^2 - \sin^2 (\beta_o - \alpha_o)\]

- \frac{1}{3} R^2 \left[ \cos^2 (\beta_o - \alpha_o) + 4 \sin \beta_o \sin \alpha_o \sin (\beta_o - \alpha_o) \right]
\[ - \frac{4 \sin^2 \beta_0 \sin^2 \alpha_o}{\tan^2} \left( \frac{3}{2} \right) - \cos^3 \left( \beta_o - \alpha_o \right) - \cos \left( \frac{\beta_o + \alpha_o}{2} \right) R^2 \left[ \sin^2 \beta_o + \alpha_o \right] \]
\[ - \left[ \frac{2 \sin \beta_o \sin \alpha_o}{\tan i} - \sin \left( \beta_o - \alpha_o \right) \right] \]
\[ - \frac{\sin^2 \beta_o \cos \alpha_o \sin \left( \beta_o - \alpha_o \right)^{3/2}}{\tan^2 i} \]
\[ \sin^2 \beta_o \cos \alpha_o \sin \left( \beta_o - \alpha_o \right) \]
\[ \frac{3}{2} \left( \cos^2 \left( \beta_o - \alpha_o \right) - 4 \sin^2 \beta_o \cos^2 \alpha_o + 4 \right) \]
\[ \sin \beta_o \cos \alpha_o \sin \left( \beta_o - \alpha_o \right) \]
\[ \sin \left( \beta_o - \alpha_o \right) \]
\[ \tan i \]
\[ \text{Denominator} = \int_0^{2R \sin \beta_0 \cos \alpha_o \tan i} \left[ \begin{array}{c} \tan i - \left( K - \sqrt{R^2 - (x-h)^2} \right) \frac{\sqrt{R^2 - (x-h)^2}}{R} \end{array} \right] \]
\[ + \int_0^{2R \sin \beta_0 \cos \alpha_o \tan i} \left( 2R \sin \beta_0 \tan i \right) \]
\[ \sqrt{R^2 - (x-h)^2} \]
\[ \frac{dx}{\tan i} \]
\[ \text{Denominator} = \int_0^{2R \sin \beta_0 \cos \alpha_o \tan i} \frac{\tan i}{R} \left( h - K \right) \]
\[ \sqrt{R^2 - (x-h)^2} \]
\[ + \frac{1}{R} \int_0^{2R \sin \beta_0 \cos \alpha_o \tan i} \left( R^2 - (x-h)^2 \right) \]
\[ \frac{dx}{\tan i} \]
\[ \text{Denominator} = \frac{\tan i}{R} \left[ \frac{1}{2} \left( R^2 - (x-h)^2 \right) \right] \]
\[ + \frac{2R \sin \beta_0 \cos \alpha_o \tan i}{2R \sin \beta_0 \cos \alpha_o \tan i} \]
\[ \frac{2R \sin \beta_0 \sin \alpha_o \tan i}{R} \]
\[ \frac{dx}{\tan i} \]
\[ \frac{1}{2} \left( (x-h) \sqrt{R^2 - (x-h)^2} + R^2 \sin^{-1} \left( \frac{x-h}{R} \right) \right) \]
\[ \frac{dx}{\tan i} \]
\[ \text{Denominator} = \tan i \left[ \frac{1}{2} \left( R^2 - (x-h)^2 \right) \right] \]
\[ + \frac{2R \sin \beta_0 \sin \alpha_o \tan i}{R} \]
\[ \frac{dx}{\tan i} \]
\[ \frac{1}{2} \left( (x-h) \sqrt{R^2 - (x-h)^2} + R^2 \sin^{-1} \left( \frac{x-h}{R} \right) \right) \]
\[
\frac{1}{R^2} \left[ \frac{R^2}{x} - \frac{(x-h)^3}{3} \right] + \frac{2R \sin \beta_0 \sin \alpha_0}{\tan \iota} \\
+ \frac{2R \sin \beta_0 \sin \alpha_0 - R \cos(\beta_0 - \alpha_0)}{\frac{1}{2} [(x-h)^2 - \sqrt{R^2 - (x-h)^2 + R^2 \sin^{-1} (\frac{x-h}{R})]}} + \frac{2R \sin \beta_0 \cos \alpha_0}{\tan \iota} \\
+ \frac{1}{R^2} \left[ \frac{R^2}{x} - \frac{(x-h)^3}{3} \right] + \frac{2R \sin \beta_0 \cos \alpha_0}{\tan \iota} \\
\text{Denominator} = \frac{\tan \iota}{3} R^2 \left[ \left( \cos^2(\beta_0 - \alpha_0) + 4 \frac{\sin \beta_0 \sin^2 \alpha_0}{\tan^2 \iota} \right) \right]^{3/2} \\
- \frac{\cos^3(\beta_0 - \alpha_0)}{\sin(\beta_0 - \alpha_0) \tan \iota - \cos(\beta_0 - \alpha_0)} \\
+ \frac{2}{R^2} \left[ \frac{2 \sin \beta_0 \sin \alpha_0}{\tan \iota} - \sin(\beta_0 - \alpha_0) \right] \\
\left( \cos^2(\beta_0 - \alpha_0) - 4 \frac{\sin \beta_0 \sin^2 \alpha_0}{\tan^2 \iota} \right) \right]^{1/2} \\
+ \frac{4}{\tan \iota} \sin \beta_0 \sin \alpha_0 \sin(\beta_0 - \alpha_0) \\
+ \sin^{-1} \frac{2 \sin \beta_0 \sin \alpha_0}{\tan \iota} - \sin(\beta_0 - \alpha_0) \\
+ \sin(\beta_0 - \alpha_0) \cos(\beta_0 - \alpha_0) + (\beta_0 - \alpha_0)] \\
+ R^2 \left[ \frac{2 \sin \beta_0 \sin \alpha_0}{\tan \iota} - \frac{1}{3} \left( \frac{2 \sin \beta_0 \sin \alpha_0}{\tan \iota} - \sin(\beta_0 - \alpha_0) \right) \right]^{3} - \frac{1}{3} \sin^3(\beta_0 - \alpha_0)] \\
- \frac{\cos(\beta_0 + \alpha_0)}{2} R^2 \left[ \sin(\beta_0 + \alpha_0) \right]
\]
\[
\begin{align*}
\cos^2 (\beta_0 - \alpha_0) &= 4 \sin^2 \beta_0 \cos^2 \alpha_0 + 4 \sin \beta_0 \\
\cos \alpha_0 \sin (\beta_0 - \alpha_0) = 1/2 + \sin^{-1} \left( \frac{\sin \beta_0 \sin \alpha_0}{\tan i} \right) \\
- \sin (\beta_0 - \alpha_0) &= \left( \frac{\cos^2 (\beta_0 - \alpha_0)}{\tan i} \right) \\
-4 \sin^2 \beta_0 \sin^2 \alpha_0 &= 4 \sin \beta_0 \sin \alpha_0 \\
\sin (\beta_0 - \alpha_0) &= \left( \frac{2 \sin \beta_0 \sin \alpha_0}{\tan i} \right) \\
- \sin (\beta_0 - \alpha_0) &= \left( \frac{2 \sin \beta_0 \sin \alpha_0}{\tan i} \right)^3 \\
+ R^2 [2 \sin \beta_0 \cos \alpha_0 - \frac{1}{3} \sin^3 (\beta_0 + \alpha_0)] \\
-2 \frac{\sin \beta_0 \sin \alpha_0}{\tan i} &= \frac{1}{3} \left( \frac{2 \sin \beta_0 \sin \alpha_0}{\tan i} \right)^3 \\
- \sin (\beta_0 - \alpha_0) &= \left( \frac{2 \sin \beta_0 \sin \alpha_0}{\tan i} \right)^3 \\
\end{align*}
\]

Let \( A = \sin \beta_0 \frac{\sin \alpha_0}{\tan i} \)

\( B = \sin (\beta_0 - \alpha_0) \);

\( C = \cos (\beta_0 - \alpha_0) \);

\( D = \sin (\beta_0 + \alpha_0) \);

\( E = \cos (\beta_0 + \alpha_0) \);

\( F = \sin \beta_0 \cos \alpha_0 \);

\( G = \sin \beta_0 \sin \alpha_0 \);

Then,

\[
\begin{align*}
\text{Numerator} &= \left[ \frac{8}{3} \frac{G^3}{A^3} - 2 \frac{G}{A} \cdot B \cdot A^2 \right] - \frac{C}{2} \left( (2A-B)^2 - B^2 \right) \\
&\quad + \frac{1}{3} \left[ c^2 + 4AB - 4A^2 \right]^{3/2} - \frac{1}{3} \left[ c^2 + 4AB - 4A^2 \right]^{3/2} \\
&\quad - \frac{1}{2} \left[ d^2 - (2A-B)^2 \right]^{3/2} - \frac{1}{3} \left[ (c^2 - 4A^2 - 4FB)^2 - (c^2 - 4A^2)^2 \right]^{3/2} \\
&\quad - \frac{2}{3} \left[ d^2 - (2A-B)^2 \right]^{3/2} - \frac{2}{3} \left[ (c^2 - 4A^2 + 4AB)^2 + 4FB \right]^{3/2} \\
\end{align*}
\]
And, thus, the solution of equation (5) is obtained.

Case II: Pore pressure

(a) Pore pressure due to sudden draw-down:

Figure 4. Draw-down case.
The force $W$ of Figure 1 (b) represents the effective weight of the sliding mass. For the submerged or buoyant slope, Figure 4 (a) shows the forces acting according to the $\phi$-circle solution, wherein $W_b$, the effective weight for this case, equals the product of the area of the sliding mass and the buoyant unit weight. The force polygon is shown in full lines in Figure 4 (b). The weight, $W_o$, which is equal to the weight of a mass of water of the same total volume as the sliding mass, must be present in the submerged case, but it has no effect since it is just balanced by forces $E_1$ and $E_2$, the resultant water pressures across the rupture arc and the slope, respectively. The lines of action of these three forces $W_o$, $E_1$ and $E_2$ are shown in Figure 4 (c). It may be noted that the moments of $W_o$ and $E_2$ about 0 must just balance each other. It is correct to speak of $W_o$ as an overturning force, but in this instance its overturning effect is just counterbalanced by the resisting effect of $E_2$.

The submerged case may be transformed into the sudden draw-down case by the sudden removal of the force $E_2$. Since the moment of $E_2$ just balances that of $W_o$, removal of $E_2$ introduces an additional overturning tendency equal to the moment of $W_o$. The weight $W_o$ at the instant of sudden draw-down is carried by a temporary excess of pressure in the water, and intergranular stresses can replace this hydrostatic excess only as fast as the necessary strains in the mass can develop. So, with the assumption that $W_o$ is not carried by the soil skeleton, and, moreover, since $E_1$ being normal to the slip circle passes through the center of it, no friction can be developed to help in resisting the shearing stresses it induces. Thus, the overturning forces acting are $W_o$ which is resisted by cohesion and friction together. The force diagram for $W_b$ alone is
shown in Figure 4 (a) while that for $W_o$ is shown in Figure 4 (d). Thus, the cohesion required to overcome the combined overturning effects of $W_b$ and $W_o$ is the sum of $C_b$ and $C_o$ of Figures 4 (a) and 4 (d). 

$$\text{(1)}$$

Figure 5. Force diagram for sudden draw-down case.

The writer adopts the following graphical method:

In Figure 5 (b) $AC = W_b$ (buoyant weight). This $W_b$ is balanced by $C_1A$ and $C_1A$ (cohesion required for $W_b$). $CB = W_o$ (weight of a mass of water of the same volume as the sliding mass).

$$AC + CB = W_b + W_o = W_s$$

From B, BK is drawn parallel to $0'O$. From C, CT is drawn parallel to $C_1A$ to meet BK at T. Then CT is the cohesion required to give resisting moment to overcome the overturning moment due to $W_o$. Then $C_2C_1$ is added to $C_1A$. So, $CT = C_2C_1 + C_1A = C_o + C_b$. This CT is the total cohesion to overcome the effect of total weight, $W_s = W_b + W_o$. Then $BC_2$ gives rise to $\phi_{mP}$, the new friction angle obtained from pore pressure due to sudden draw-down. From point $0'$, $0'S$ is drawn parallel to $BC_2$. Then a small circle is drawn such that $0'S$ is a tangent to this circle. This small circle gives rise to $\phi_{mP}$ (modified friction angle under pore pressure due to sudden draw-down). In Figure 5 (b),

$$\frac{CB}{AB} = \frac{W_o}{W_s} = \beta (\text{sudden draw-down factor}), \text{ as defined by the writer.}$$

So, for different values of $\beta$, one will get different values of $\phi_{mP}/ \phi_1$. 

and it is this value of $\phi_{mP}$ one uses in Taylor's chart. Modified slip circle corresponding to $\phi_{mP}$ can be used in the analysis. And, the unit weight to be used in stability number is the saturated unit weight, $\gamma_s$.

(b) Pore pressure due to seepage: Its effect is similar to that of sudden draw-down. Here, in place of $\beta$, one can use seepage factor as defined by the writer.

$$\delta = \frac{\gamma_w a_1}{\gamma_t (A-a) + \gamma_s a} = \text{seepage factor}$$

Figure 6. Effect of seepage.

Where

$\gamma_w$ = Unit weight of water
$\gamma_t$ = Unit weight of soil at any moisture content
$\gamma_s$ = Saturated unit weight of soil
$a_1$ = The area between the pore pressure curve and slip circle obtained by $\phi$-circle method
$a$ = The area between the phreatic line and the slip circle
$A$ = Total area of sliding mass.

In a similar manner to that of sudden draw-down, a chart of $\delta$ vs $\phi_{mP}/\phi_i$ can be obtained for the case of pore-pressure due to seepage. It is this value of $\phi_{mP}$ that one will use in Taylor's chart. And, the unit weight to be used in stability number is

$$\gamma_E = \frac{\gamma_t (A-a) + \gamma_s a}{A}$$

as suggested by the writer.
(c) Pore pressure due to construction: Its procedure is also similar to that of sudden-draw-down. Only instead of $\beta$, one uses $\xi$ which is expressed as a ratio of the pore pressure force to the total weight on the rupture plane. If the height of the soil above any point on the rupture plane is $h$, and the height due to pore water pressure is $h_1$ above the same point, then as defined by the writer

$$\xi = \frac{\gamma_w h_1}{\gamma_t h}$$

Where

- $\gamma_w =$ Unit weight of water
- $\gamma_t =$ Unit weight of soil at any moisture content
- $\xi =$ Construction pore pressure factor.

If the effect of pore pressure due to construction is 10 per cent of the full hydrostatic pressure then

$$\xi = 0.10 \frac{\gamma_w}{\gamma_t}.$$  

So then, like those of Cases II (a) and II (b) a chart of $\xi$ vs $\phi_{mp}/\phi_i$ can be obtained. This chart will give the friction angle modified due to construction pore pressure. And the unit weight of soil in this case is $\gamma_t$. 
Taylor's stability chart gives the relation between stability number, \( \frac{c}{F} \gamma H \) and slope angle, \( i \) for different values of \( \phi \). From this relationship one can find factor of safety, \( F \), for a given soil and a given slope with a given height or one can find the height of the embankment where the factor of safety is specified. The writer has tried to develop certain relationships with the help of which one can still use the Taylor's chart even when one is to consider either horizontal earthquake or pore pressure effect in the problem of slope stability.

Case I: Effect of horizontal earthquake

The writer has analyzed the effect of horizontal earthquake in 40 different cases and as a typical example only one is shown in Figure 16, for \( i = 30^\circ \), \( \phi_i = 10^\circ \), and \( H = 50' \).

In all these examples two different heights of embankment, namely 50' and 75', were chosen. The results obtained were the same for the two heights as shown in the charts of \( l/d \) vs \( i \), with a given \( \phi_i \) (initial angle of friction without the effect of earthquake). This shows the validity of the relationship of the dimensionless quantities.

The procedure that the writer has adopted in this analysis is as follows and the graphical construction is as shown in Figure 16.

First, the earth embankment is drawn to a scale. Then, corresponding to the given slope, \( i \), and the given initial friction angle, \( \phi_i \), the critical slip circle is drawn with the help of Table I (given by
Taylor). This circle has its center at 0 which is determined by the values of $\alpha_o$ and $\beta_o$ (as given in Table I), and it passes through the toe, A, and meets the embankment at B. And so, AB is the rupture arc. At any point L on the slip circle, angle $\angle OLK$ is drawn such that $\angle OLK = \phi_i$ (given initial friction angle when there is no earthquake).

Then the $\phi$-circle is drawn with its center at 0, such that it touches LIK at K. After the critical slip circle and the friction circle are drawn, the next step is to find the center of gravity of the combined sliding mass; and this is done in the following way:

Referring to Figure 16, from 0, OS is drawn perpendicular to AB, the chord of length L.

$\overline{OS} = \text{Radius, } R$

$\overline{OB} = \text{Radius, } R$

Now, one is to find the center of gravity of the circular segment.

$x^2 + y^2 = R^2$

$x = r\cos \psi$

$y = r\sin \psi$

$R \sin \theta_1 = D$

or, $\theta_1 = \sin^{-1} \frac{D}{R}$

$\overline{y}$ (the distance of the c-g from the center 0 along OS) = $\frac{\int y \, dy \, dx}{\int dy \, dx}$

$= \frac{\int_{\phi_1}^{\pi/2} \left[ \frac{D \cos \psi}{\sin \psi} \right] \sin \psi \, d\psi}{\int_{\phi_1}^{\pi/2} \left[ \int_0^R \frac{r'}{D \cosec \psi} \right] \, d\psi}$

$= \frac{2}{3} D \left( \frac{R}{D} \right)^3 \frac{\cos \theta_1 - \cot \theta_1}{(\pi/2 - \theta_1 - \cot \theta_1)}$

This $\overline{y}$ is then marked at $\theta_2$ along OS. Then the next step is to find the C. G. of the triangular portion by graphical method and then marked at
Let the distance \( a_1a_2 = d \). The combined C. G. lies at \( a_0 \) on \( a_1a_2 \) at a distance of \( a = \frac{A_1d}{A_1 + A_2} \) from \( a_2 \), where 

\[
A_1 = \text{Area of the triangular portion} \\
A_2 = \text{Area of the circular sector}
\]

Thus, the weight, \( W \), and the horizontal earthquake force, \( F_E (\propto W) \) act at \( a_0 \). The moment arm of \( W \) is \( d \) and that of \( F_E \) is \( l \). After knowing \( l \) and \( d \) chart for \( l/d \) vs \( i \) is prepared. Then 

\[
\gamma_E = \sqrt{(1 + \alpha l/d)}
\]

is determined. Now, according to the analysis of Case I in Chapter III,

\[
m = \frac{\tan \phi_m E}{\tan \phi_1} = \frac{1 - \alpha \text{Numerator}}{\frac{1}{\text{Denominator}} + \alpha l/d}
\]

is calculated.

For example, given \( i = 30^\circ \) and \( \phi_1 = 15^\circ \), Figure 9 gives \( l/d = 1.808 \). Then to calculate \([1 - \alpha \text{Numerator} \text{Denominator}]\) one has to take the values of \( \alpha_0 \) and \( \beta_0 \) corresponding to the given \( i \) and \( \phi_1 \). In this case, \( \alpha_0 = 27^\circ \), \( \beta_0 = 39^\circ \) from Table I. So,

\[
A = \frac{\sin \beta_0 \sin \alpha_0}{\tan i} = \frac{\sin 39^\circ \sin 27^\circ}{\tan 30} = 0.494
\]

\[
B = \sin (\beta_0 - \alpha_0) = \sin 12^\circ = 0.2079
\]

\[
C = \cos (\beta_0 - \alpha_0) = \cos 12^\circ = 0.9781
\]

\[
D = \sin (\beta_0 + \alpha_0) = \sin 66^\circ = 0.9135 \quad E = \cos (\beta_0 + \alpha_0) = 0.4067
\]

\[
F = \sin \beta_0 \cos \alpha_0 = \sin 39^\circ \cos 27^\circ = 0.561
\]

\[
G = \sin \beta_0 \sin \alpha_0 = \sin 39^\circ \sin 27^\circ = 0.285
\]

\[
\frac{8}{3} GA^2 - 2ABG = 0.126
\]

\[
2CA^2 - 2ABC = 0.275
\]

\[
C^2 + 4AB - 4A^2 = 0.389
\]

\[
2A - B = 0.780
\]

\[
c^2 - 4F^2 + 4FB = 0.165
\]
BG - AC = -0.424

\[
\sin^{-1} (2A-B) = 51.3^\circ = \frac{\pi}{180} \times 51.3 = 0.895 \text{ radian}
\]

BC = 0.2079 \times 0.9781 = 0.203

\[
\sin^{-1} (B) = 12^\circ = \frac{\pi}{180} \times 12 = 0.210 \text{ radian}
\]

\[
\sin^{-1} (D) = 66^\circ = \frac{\pi}{180} \times 66 = 1.150 \text{ radian}
\]

\[
\frac{\text{Numerator}}{R^2} = 0.0934; \text{ So, Numerator} = R^2 (0.0934)
\]

\[
\frac{\text{Denominator}}{R} = 0.200; \text{ So, Denominator} = R^2 (0.200)
\]

Then,

\[
1 - \frac{\text{Numerator}}{\text{Denominator}} \alpha = 1 - \frac{R^2(0.0934)}{R^2(0.200)} \alpha = 1 - 0.467 \alpha
\]

So,

\[
\alpha = \frac{1 - 0.467 \alpha}{1 + 1.808 \alpha}
\]

Hence, for different values of \( \alpha \), different values of \( \alpha \) will be obtained. These are shown in Figures 10 to 14.

**Case II: Effect of pore pressure**

For the effect of pore pressure in general, the writer has analyzed four different cases, namely

1. With \( i = 30^\circ \)

   \( \phi = 15^\circ \)

   \( H = 50' \)

2. With \( i = 45^\circ \)

   \( \phi = 20^\circ \)

   \( H = 50' \)

3. With \( i = 30^\circ \)

   \( \phi = 20^\circ \)

   \( H = 75' \)

4. With \( i = 30^\circ \)

   \( \phi = 20^\circ \)

   \( H = 50' \)
Figure 17 is shown as a typical one. In all these cases, the main idea is to find a relationship between the pore pressure factor and $\phi_{mp}/\phi_i$ (ratio of modified friction angle to initial friction angle). All these four cases have given same result which shows it can be extended dimensionally to other values.

(a) **Pore pressure due to sudden draw-down.** The writer has adopted the following procedure in establishing a relationship between sudden draw-down factor, $\beta (\gamma_w/\gamma_s)$ and $\phi_{mp}/\phi_i$ (ratio of friction angle modified under pore pressure due to sudden draw-down to the initial friction angle). As in Figure 16, the C. G. of the sliding mass is determined and $0'$ is located in this case. Procedure for the graphical construction is followed as given by the writer in Case II (a) of Chapter III. For different values of $\beta$, different values of $\phi_{mp}/\phi_i$ are obtained. When these points are plotted (as shown in Figure 15), they are found to fit a straight line such that

$$\frac{\phi_{mp}}{\phi_i} + \beta = 1$$

or

$$\frac{\phi_{mp}}{\phi_i} = 1 - \beta = 1 - \frac{\gamma_w}{\gamma_s} = \frac{\gamma_0}{\gamma_s}$$

or

$$\frac{\phi_{mp}}{\phi_i} = \frac{G-1}{G+e}$$

Where

$G =$ The sp. gr. of soil,

$e =$ The void ratio of soil.

(b) **Pore pressure due to seepage.** Similar procedure to that of sudden draw-down is adopted in this case to establish the relationship between the seepage factor, $\delta$ (as defined in Case II (b) of Chapter III) and $\phi_{mp}/\phi_i$ (ratio of the friction angle modified under pore pressure due to seepage to the initial friction angle without seepage). Equivalent unit weight, $\gamma_E$ is determined from the formula given by the writer in Case II (b) of Chapter III.
(c) **Pore pressure due to construction.** This procedure is also similar to that of sudden draw-down. A relationship is established between construction pore pressure coefficient, \( \xi \) (as defined by the writer in Case II (c) of Chapter III) and \( \phi_{mp}/\phi_i \) (ratio of friction angle modified under construction pore pressure to the initial friction angle without pore pressure).
CHAPTER V

RESULTS OF THE ANALYSIS

The results of the stability analysis by $\phi$-circle method considering only the external force $W$, are tabulated in Table I. These data given by Taylor (1) are verified with a digital 1620 computer. Results of the writer's analysis considering earthquake are tabulated in Table 2, and the results are plotted in Figure 10 to Figure 14 as $\mathbf{m}$ vs $\mathbf{\alpha}$ for different $\phi_i$'s and different constant slope angle $i$. Also, the results of $(\mathbf{j}/\mathbf{d})$ vs $i$ are plotted for different $\phi_i$'s in Figure 9. Results of the analysis done by the writer for the case of pore pressure are tabulated in Table 3, and the results are plotted in Figure 15 as $\mathbf{\phi_m}/\phi_i$ vs $\beta$ (or $\delta$ or $\xi$).
### Table 1. Data on critical circles by the $\phi$ -circle method (1).

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
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<td>$i$</td>
<td>$\phi$</td>
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| 75° | 0° | 41.8° | 25.9° | | | 0.219 | 0.219 | The final column is after $\phi$-circle rotation on the assumption that intensity of $\rho$-force is equal to zero at A and B and varying sinusoidally between $-\frac{1}{2}$ and $\frac{1}{2}$.
| 10° | 0° | 47.5° | 23.5° | | | 0.173 | 0.173 | |
| 15° | 0° | 50 | 23° | | | 0.152 | 0.152 | |
| 20° | 0° | 53 | 22° | | | 0.134 | 0.134 | |
| 25° | 0° | 56 | 22° | | | 0.117 | 0.117 | |
| 60° | 0° | 35.3° | 35.4° | | | 0.191 | 0.191 | |
| 10° | 0° | 38.5° | 34.5° | | | 0.162 | 0.162 | |
| 15° | 0° | 41 | 33° | | | 0.138 | 0.138 | |
| 20° | 0° | 44 | 31.5° | | | 0.116 | 0.116 | |
| 25° | 0° | 46.5° | 30.2° | | | 0.097 | 0.097 | |
| 45° | 0° | (28.2°) | (44.7°) | | | 1.062 (0.170) (0.170) | |
| 10° | 0° | 34 | 42.1° | | | 1.026 | 1.026 | |
| 15° | 0° | 36.1 | 37.2° | | | 1.001 | 1.001 | |
| 20° | 0° | 38 | 34.5° | | | 0.975 | 0.975 | |
| 25° | 0° | 40 | 31° | | | 0.941 | 0.941 | |
| 30° | 0° | (20°) | (53.4°) | | | 1.301 | 1.301 | |
| 10° | 0° | 25 | 44° | 0.29 | | 1.332 | 1.332 | |
| 15° | 0° | 27 | 39° | | | 1.092 | 1.092 | |
| 20° | 0° | 28 | 31° | | | 1.038 | 1.038 | |
| 25° | 0° | 29 | 25° | | | 1.003 | 1.003 | |
| 15° | 0° | (10.6°) | (60.7°) | | | (2.117) (0.145) (0.145) | |
| 10° | 0° | (12.5°) | (47°) | | | (1.549) (0.072) (0.068) | |
| 11 | 0° | (47.5°) | 0.55 | | | 1.697 | 1.697 | |
| 14 | 0° | (34°) | | 0.04 | | 1.222 | 1.222 | |

**All Values**

Figures in parentheses are values for most dangerous circle through the toe when a more dangerous circle exists which passes below the toe.
(A) Typical cross section and failure arc in Zone A. Critical circle passes through toe and stability number represented in chart by full lines.

(B) Typical cross section showing cases considered in Zone B.

Case 1: The most dangerous of the circles passing through the toe, represented by full lines in chart. Where full lines do not appear, this case is not appreciably different from Case 2.

Case 2: Critical circle passing below the toe, represented by long dashed lines in chart, where long dashed lines do not appear, the critical circle passes through the toe.

Case 3: Surface of ledge or a strong stratum at the elevation of the toe (D = 1); represented by short dashed lines in chart.

Figure 7. Chart of stability numbers.
Figure 8. Chart of stability numbers for the case of zero friction angle and limited depth.
Table 2. It is furnished by the writer from graphical and mathematical analysis for the horizontal earthquake force with the help of data from Table 1. (Reference to Figure 16 and Appendix.)

| \( \phi_i \) | \( l/d \) | \( \alpha \) | \( m \) | Remarks |
|——|——|——|——|——|
| 75° | 5° | 0.946 | 0.0 | 1.000 | The values of \( l/d \) are computed by taking average of the values on two dams of height 50' and 75'. Both the dams give almost the same results. This shows that the results can be extended for any dimension. |
| | | | 0.1 | 0.840 |
| | | | 0.2 | 0.700 |
| | | | 0.3 | 0.586 |
| | | | 0.4 | 0.490 |
| | | | 0.5 | 0.403 |
| | | | 0.6 | 0.326 |
| | | | 0.7 | 0.258 |
| | | | 0.8 | 0.200 |
| 10° | 0.838 | | 0.0 | 1.000 |
| | | | 0.1 | 0.830 |
| | | | 0.2 | 0.690 |
| | | | 0.3 | 0.575 |
| | | | 0.4 | 0.467 |
| | | | 0.5 | 0.375 |
| | | | 0.6 | 0.290 |
| | | | 0.7 | 0.215 |
| | | | 0.8 | 0.148 |
| 15° | 0.755 | | 0.0 | 1.000 |
| | | | 0.1 | 0.840 |
| | | | 0.2 | 0.700 |
| | | | 0.3 | 0.578 |
| | | | 0.4 | 0.470 |
| | | | 0.5 | 0.375 |
| | | | 0.6 | 0.288 |
| | | | 0.7 | 0.210 |
| | | | 0.8 | 0.140 |
| 20° | 0.670 | | 0.0 | 1.000 |
| | | | 0.1 | 0.835 |
| | | | 0.2 | 0.690 |
| | | | 0.3 | 0.560 |

\[ m = \frac{1-0.9\alpha}{1+0.84\alpha} \]

\[ m = \frac{1-0.9\alpha}{1+0.755\alpha} \]

\[ m = \frac{1-1.08\alpha}{1+0.67\alpha} \]
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\( 25^\circ \) 0.590

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\( 60^\circ \) 5° 1.130

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\( 15^\circ \) 0.934

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\( 20^\circ \) 0.838

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<td>$\Phi_1$</td>
<td>$l/d$</td>
<td>$\alpha$</td>
<td>$m$</td>
<td>Remarks</td>
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</table>

25° 1.720

| 0.0 | 1.000 |
| 0.1 | 0.810 |
| 0.2 | 0.670 |
| 0.3 | 0.560 |
| 0.4 | 0.470 |
| 0.5 | 0.400 |
| 0.6 | 0.340 |
| 0.7 | 0.300 |
| 0.8 | 0.255 |

$m = \frac{1 - 0.492\alpha}{1 + 1.72\alpha}$

15° 5° 4.708

| 0.0 | 1.000 |
| 0.1 | 0.670 |
| 0.2 | 0.495 |
| 0.3 | 0.390 |
| 0.4 | 0.320 |
| 0.5 | 0.270 |
| 0.6 | 0.230 |
| 0.7 | 0.200 |
| 0.8 | 0.176 |

$m = \frac{1 - 0.20\alpha}{1 + 4.708\alpha}$

10° 3.850

| 0.0 | 1.000 |
| 0.1 | 0.670 |
| 0.2 | 0.492 |
| 0.3 | 0.374 |
| 0.4 | 0.294 |
| 0.5 | 0.230 |
| 0.6 | 0.186 |
| 0.7 | 0.150 |
| 0.8 | 0.120 |

$m = \frac{1 - 0.64\alpha}{1 + 3.85\alpha}$
Figure 9. Moment arm ratio \( l/d \) vs. slope angle \( i \) and initial friction angle \( \phi_i \).
Figure 10. Friction angle ratio $m$ vs. horizontal earthquake acceleration coefficient $\alpha$ for a slope of 15°.
Figure 11. Friction angle ratio $m$ vs. horizontal earthquake acceleration coefficient $\alpha$ for a slope of 30°.
Figure 12. Friction angle ratio $m$ vs. horizontal earthquake acceleration coefficient $\alpha$ for a slope of $45^\circ$. 
Figure 13. Friction angle ratio $m$ vs. horizontal earthquake acceleration coefficient $\alpha$ for a slope of 60°.
Figure 14. Friction angle ratio $m$ vs. horizontal earthquake acceleration coefficient $\alpha$ for a slope of $75^\circ$. 
Table 3. Results of the analysis considering pore pressure are tabulated below.

<table>
<thead>
<tr>
<th>$\beta$ or $\delta$</th>
<th>$\phi_{mp}/\phi_i$</th>
<th>$\phi_{i}=15^\circ$</th>
<th>$\phi_{i}=20^\circ$</th>
<th>$\phi_{i}=25^\circ$</th>
<th>$\phi_{i}=30^\circ$</th>
</tr>
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<td>1.00</td>
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<td>0.94</td>
<td>0.91</td>
<td>0.90</td>
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<td>0.72</td>
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<td>0.60</td>
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<tr>
<td>0.4</td>
<td>0.60</td>
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<tr>
<td>0.5</td>
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<td>0.50</td>
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<td>0.6</td>
<td>0.42</td>
<td>0.40</td>
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<tr>
<td>0.7</td>
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<td>0.30</td>
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<tr>
<td>1.0</td>
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</table>

The above data are plotted on graph paper and they seem to fit a straight line passing through (1,0) and (0,1).

So, the equation of the line is

$$\beta + \frac{\phi_{mp}}{\phi_i} = 1$$  \hspace{1cm} (1)

or

$$\delta + \frac{\phi_{mp}}{\phi_i} = 1$$  \hspace{1cm} (2)

or

$$\epsilon + \frac{\phi_{mp}}{\phi_i} = 1$$  \hspace{1cm} (3)

From (1)

$$\frac{\phi_{mp}}{\phi_i} = 1 - \beta = 1 - \frac{\gamma_w}{\gamma_s} = \frac{\gamma_b}{\gamma_s} = \frac{G-1}{G+e}$$

so, in case of sudden draw-down

$$\frac{\phi_{mp}}{\phi_i} = \frac{G-1}{G+e}$$
Figure 15. Ratio of developed friction angle (due to pore pressure) and initial friction angle vs.
sudden draw-down or construction pore pressure coefficient.
CHAPTER VI
APPLICATION TO DESIGN

In the preceding chapters possible design procedures are outlined. As a means of further explaining and clarifying and at the same time verifying this method with the existing but time-consuming slip-circle slice method, the following design problem is taken.

The problem in Figure 18 is a typical section of a dam with the following soil properties
\[ \phi_i = 10^\circ; \quad \gamma_t = 120 \text{ p.c.f.}; \quad \gamma_s = 134.8 \text{ p.c.f.}; \quad c = 1000 \text{ p.s.f.} \]

Design number 1

The stability analysis of the upstream slope is to be done for the following cases, first by slip-circle slice method and then by \( \phi \)-circle method.

Case I with earthquake (\( \alpha = 0.1 \))

Case II with pore pressure due to sudden draw-down

Case III with pore pressure due to construction (\( \xi = 0.20 \))

Design number 2

The stability analysis of the downstream slope is to be done for the case, namely, pore pressure due to seepage. For this one needs to draw the flow net and at least to draw the phreatic line and the equipotential lines. These in turn will give the areas \( A, a, a_1 \) in case of \( \phi \)-circle method. Then, one can compute
\[ \delta = \frac{\gamma_w a_1}{\gamma_t (A-a) + \gamma_s a} \]
Solution, Design Number 1

Case I. By slip-circle slice method: (Figure 18)

\[
F = \frac{\phi_i \left[ \gamma_c (\sum N - \sum E_t) \right]}{\gamma_t (\sum T + \sum E_t)}
\]

\[
= \frac{1,000 \times 317 + 0.1763 \times [120 \times (10,160 - 236)]}{120 \times (2360 + 1,016)}
\]

\[
= 1.30
\]

Case II. By \( \phi \)-circle method:

From Figures 10 and 11 (by interpolation) corresponding to \( \alpha \) (slope angle) = 16° and \( \phi_i = 10° \), for \( \gamma = 0.10 \tan \phi_m / \tan \phi_i = 0.68 \).

So, \( \phi_m = 6.90 \). Let the true factor of safety be \( F = 1.2 \), then \( \phi_m \)-actual = \( \frac{6.90}{1.02} = 5.8° \). Corresponding to this \( \phi_m \)-actual and \( \alpha = 16° \), \( c / F \gamma H = 0.05 \) (from Figure 7) . . . . . . . . . (1)

Now, \( \gamma_E \) (for \( \alpha = 0.10 \)) = \( \gamma_t (1 + \alpha \ell / d) \) and corresponding to \( \phi_i = 10° \) and \( \alpha = 16° \), \( \ell / d = 3.85 \). So, \( \gamma_E = 120 (1 + 0.10 \times 3.85) \) = 166 p.c.f. So, from (1) \( F = \frac{1000}{166 \times 100 \times 0.05} = 1.2 \). So the true factor of safety is 1.2.

Case II. By slice method: (Figure 18)

\[
F = \frac{\phi_i \left[ \gamma_c (\sum N - \sum E_t) \right]}{\gamma_t (\sum T + \sum E_t)}
\]

\[
= \frac{1,000 \times 317 + 0.1763 \times [134.8 \times 10,160 - 62.5 \times 11,880]}{134.8 \times 2,360}
\]

\[
= 1.34
\]

Case II. By \( \phi \)-circle method:

From the Figure 15, for \( \beta = \frac{\gamma_m}{\gamma_s} = \frac{62.5}{134.8} = 0.465 \), \( \phi_m / \phi_i = 0.535 \). So, \( \phi_m = 0.535 \times 10° = 5.35° \). Let \( F \) (true factor of safety) be 1.25. Then \( \phi_m \)-actual = \( \frac{5.35}{1.25} = 4.27° \). Corresponding to this \( \phi_m \)-actual and \( \alpha = 16° \), \( \frac{c}{F \gamma H} = 0.06 \) (from Figure 7), or

\[
F = \frac{10000}{0.05 \times 134.8 \times 100} = 1.25 \]. So, true factor of safety = 1.25.
Case III. By slice method:

If \( \varepsilon = 0.2 \), then \( \frac{f_w h_1}{f_t h} = 0.2 \), or \( \frac{h_1}{h} = \frac{0.2 \times 120}{62.5} = 0.385 \).

So, pore pressure due to construction is 38.5 per cent of full hydrostatic pressure. Now, \( F \) (factor of safety)

\[
= \frac{317,000 + 0.1763 [120 \times 10,160 - 62.5 \times 0.385 \times 11,880]}{120 \times 2,360}
\]

\[= 1.70 \]

Case III. By \( \phi \)-circle method:

From Figure 15, corresponding to \( \xi = 0.2 \), \( \phi_{\text{mp}} = 0.8 \).

So, \( \phi_{\text{mp}} = 8^\circ \). Let \( F = 1.55 \), then \( \phi_{\text{actual}} = \frac{8}{1.55} = 5.2^\circ \). Now, for

\( i = 16^\circ \) and \( \phi_{\text{actual}} = 5.2^\circ \), \( c/F \gamma_H = 0.054 \). Or, \( F = \frac{1000}{0.054 \times 120 \times 100} \)

\[= 1.55 \]. So, true factor of safety = 1.55.

Solution, design number 2

Pore pressure due to seepage by slip-circle slice method: (Figure 19)

\[
F, S, = \frac{1,000 \times 264 + 0.1763 [134.8 \times 2950 + 120 \times 3850 - 62.5 \times 2360]}{134.8 \times 1,004 + 120 \times 1,181}
\]

\[= 1.40 \]

Pore pressure due to seepage by \( \phi \)-circle method: (Figure 20)

\[
\delta = \frac{f_w a_l}{f_t (A-a) + f_a} = \frac{62.5 \times 3,250}{120 (12,400 - 5,300) + 134.8 \times 5,300}
\]

\[= 0.13, \text{ corresponding to } \delta = 0.13, \phi_{\text{mp}} = 0.87. \text{ So, } \phi_{\text{mp}} = 0.87\]

\[x 10 = 8.7^\circ. \text{ Assuming } F = 1.25, \phi_{\text{actual}} = \frac{8.7}{1.25} = 7^\circ. \text{ So, corresponding to } i = 22^\circ, \phi_{\text{actual}} = 70^\circ, c/F \gamma_H = 0.063 \ldots \] (1)

Now, \( \gamma_E = \frac{f_t (A-a) + f_a}{A} = \frac{120 (12,400 = 5,300) + 134.8 \times 5,300}{12,400} \)

\[= 126 \text{ tbs/cf}. \text{ From (1), } F = \frac{1000}{0.063 \times 125 \times 100} = 1.25. \text{ Hence, true factor of safety is 1.25.} \]
CHAPTER VII
SUMMARY AND CONCLUSION

The $\phi$-circle method employed to get the relation between the stability number, $c/FH$ and slope angle, $i$, for the different $\phi$'s, makes the problem of stability analysis of earth slopes simpler compared to many other methods so far used in the design.

When the slope angle and the friction angle are known, $c/FH$ may be obtained directly from Figure 7, while for zero friction angle, if there is a limitation in the depth to which a rupture surface may extend, Figure 8 will furnish the value.

Now, these two figures, namely Figure 7 and Figure 8, are based only on the external load $W$, the weight of the sliding mass. If now the problem is complicated by introducing the following cases, namely (1) earthquake, (2) pore pressure, (a) due to sudden draw-down, (b) due to seepage, (c) due to construction, then the problem is to get the modified friction angles in those cases.

In case of earthquake Figures 10 to 14 as supplied by the writer from his mathematical and graphical analyses of 40 different cases give the relation between $m$ (the ratio of the tangent of the modified friction angle due to earthquake to the tangent of the friction angle) and $\chi$ (the coefficient of earthquake acceleration) for a given $i$ (slope angle) and a given $\phi_i$ (initial friction angle). It is this modified friction angle which will be used to obtain the stability number in Figure 7. And, also in this case, the unit weight of soil in the stability number is the equivalent unit weight, $\gamma_E$, and, $\gamma_E = \gamma (1 + \chi l/d)$ where $l/d$ is the
moment arm ratio. Figure 9 supplied by the writer gives the relation between \((l/d)\) and slope angle \((i)\) for different \(\phi_i\)'s.

In the case of pore pressure due to sudden drawdown, \(\beta (\frac{\gamma_w}{\gamma_s})\) (the sudden draw-down factor) is calculated and the corresponding \(\phi_{mp}/\phi_i\) (ratio of friction angle modified due to pore pressure to the initial friction angle) is obtained from Figure 15. The modified friction angle \((\phi_{mp})\) is used to obtain the stability number in Figure 7. The unit weight in this case is the saturated unit weight of the soil. In this connection one thing can be observed. Figure 15 represents a straight line whose equation may be written as

\[
\frac{\phi_{mp}}{\phi_i} + \beta = 1, \text{ or } \frac{\phi_{mp}}{\phi_i} = 1 - \beta = 1 - \frac{\gamma_w}{\gamma_s}
\]

or

\[
\frac{\phi_{mp}}{\phi_i} = \frac{\gamma_b}{\gamma_s}
\]

or

\[
\frac{\phi_{mp}}{\phi_i} = \frac{G-1}{G\gamma_e} \text{ (in case of sudden draw-down)}
\]

where \(G\) is the specific gravity of the soil.

In case of pore pressure due to seepage, seepage factor \(\delta\)

\[
\left(\frac{\gamma_w}{A-a} + \gamma_s\right) \text{ i.e. } \frac{\gamma_w}{\gamma_t (A-a) + \gamma_s a} \text{ approximately}
\]

is calculated after drawing the flow net, mainly the phreatic line and the equipotential lines, and the critical circle by \(\phi\)-circle method with the help of Table I. Then, corresponding to this, \(\delta \cdot \phi_{mp}/\phi_i\) is obtained from Figure 15. And, this \(\phi_{mp}\) is used in Figure 7 for the determination of the stability number. The equivalent unit weight \((\gamma_E)\) to be used in the stability number, is given by

\[
\gamma_E = \frac{\gamma_t (A-a) + \gamma_s a}{A}
\]

In case of pore pressure due to construction, \(\varepsilon = \frac{\gamma_w h_1}{\gamma_t h} = \text{ construction pore pressure coefficient, where} \)
\( \gamma_w \) = unit weight of water
\( \gamma_t \) = unit weight of soil at any moisture content
\( h \) = full hydrostatic pressure head
\( h_1 \) = pore pressure head expressed as a per cent of full hydrostatic pressure head.

Knowing \( \epsilon \), \( \Phi_{\text{mp}} / \Phi_i \) can be obtained from Figure 15 and then with this \( \Phi_{\text{mp}} \), stability number may be obtained from Figure 7. The unit weight used in this stability number is \( \gamma_t \), the unit weight of soil mass at field moisture content. The dependability of the results of such solutions will depend entirely on the conditions. In a few ideal cases the results may be very dependable. More often there will be questionable items, such as the possibility of surface cracking at the top of the slope. In this case the results can be accepted only as rough indications. If good judgment is used in estimating and attempting to evaluate the unknown factors whatever the conditions may be, the results should be of value in arriving at logical conclusions.

For slight variations of soil properties within the sliding mass, average values for cohesion, unit weight and friction angle may be used.

Although stability problems involve more questionable items than most engineering problems, it is seldom feasible to make use of large factors of safety. Conditions are often such that about the largest factor of safety that may be chosen is 1.5. This method being a simple and handy one is advantageous to the designer for designing an earth embankment whether it is an earth dam, railway cut, highway embankment, canal embankment or any such earth slope.
Graphical analysis for the earthquake case done as in Figure 16, gives the following informations.

1. a. For \( i = 15^\circ \)
   \[ \phi_1 = 5^\circ \]
   \[ H = 50' \]
   Measured: \( R = 180 \text{ ft.} \)
   \[ D = 123 \text{ ft.} \]
   \[ \theta_1 = 90^\circ - \beta_o = 42.5^\circ \]
   \[ y = \frac{2}{3} x D x \frac{(\frac{R}{D})^3}{(\frac{R}{D})^2 (\frac{H}{2} - \theta_1) - \cot \theta_1} \]
   \[ = 146 \text{ ft.} \] (This is fixed as \( a_1 \))

And, \( a_2 \) is fixed by the intersection of the medians of the triangle ABC.

\[ a_1 a_2 = 34'; \]

\[ A_1 = 1,915 \text{ sq. ft.} \]
\[ A_2 = 11,200 \text{ sq. ft.} \]

Total 13,115 sq. ft.

\[ x = \frac{1,915 \times 34'}{13,115} = 5 \text{ ft.} \]

\[ \theta = 2 \beta_o = 95^\circ = 1.65 \text{ radius} \]

\[ L = R \theta = 180 \times 1.65 = 297.5 \text{ ft.} \]

\[ a = \frac{R}{L} = \frac{180 \times 297.5}{264} = 203.5 \text{ ft} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>138.5</td>
<td>30'</td>
<td>4.616</td>
</tr>
</tbody>
</table>

b. For \( i = 15^\circ \)
   \[ \phi_1 = 5^\circ \]
   \[ H = 75' \]
   \[ R = 272 \text{ ft.} \]
   \[ D = 184 \text{ ft.} \]
   \[ y = 220 \text{ ft.} \]
\[ a_1 a_2 = 52.5 \text{ ft.} \]
\[ A_1 = 4510 \text{ sq. ft.,} \quad A_2 = 25,750 \text{ sq. ft.} \]
\[ \bar{X} = \frac{4510 \times 52.5}{4510 + 25,750} = 7.8 \text{ ft.} \]

\[ \theta = 2 \beta_0 = 95^\circ = 1.65 \text{ radians} \]
\[ \hat{L} = R\theta = 272 \times 1.65 = 450 \text{ ft.} \]
\[ a = \frac{\hat{L}}{L} = \frac{272 \times 450}{401.5} = 306 \text{ ft.} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>209'</td>
<td>43.5'</td>
<td>4.80</td>
</tr>
</tbody>
</table>

2. a. For \( i = 15^\circ \)

\[ \phi_i = 10^\circ \quad \text{H} = 50' \]

\[ R = 187 \text{ ft.} \]
\[ D = 155.5 \text{ ft.} \]
\[ \bar{y} = 169 \text{ ft.} \]

\[ a_1 a_2 = 33.25 \text{ ft.} \]
\[ A_1 = 312 \text{ sq. ft.;} \quad A_2 = 4,820 \text{ sq. ft.} \]
\[ \bar{X} = \frac{312 \times 33.25}{312 + 4,820} = 2.02 \text{ ft.} \]

\[ \theta = 2 \beta_0 = 2 \times 34 = 65^\circ = 1.18 \text{ radians} \]
\[ \hat{L} = R\theta = 187 \times 1.18 = 221 \text{ ft.} \]
\[ a = \frac{\hat{L}}{L} = \frac{187 \times 221}{209.5} = 197 \text{ ft.} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.25'</td>
<td>42'</td>
<td>3.870</td>
</tr>
</tbody>
</table>

b. For \( i = 15^\circ \)

\[ \phi_i = 10^\circ \quad \text{H} = 75' \]

\[ H = 75' \quad R = 276 \text{ ft.} \]
\[ D = 229.5 \text{ ft.} \]
\[ \overline{y} = 245 \text{ ft.} \]
\[ a_1 a_2 = 41.6 \text{ ft.} \]
\[ A_1 = 700 \text{ sq. ft.}; \quad A_2 = 10,300 \text{ sq. ft.} \]
\[ X = \frac{700 \times 416}{700 + 10,300} = 2.65 \text{ ft.} \]
\[ \Theta = 2 \beta_0 = 68^\circ = 1.18 \text{ radians}; \quad \overline{L} = R \Theta = 326 \text{ ft.} \]
\[ a = \frac{\overline{R}}{L} = \frac{276 \times 326}{308} = 294 \text{ ft.} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>235'</td>
<td>61.5'</td>
<td>3.83</td>
</tr>
</tbody>
</table>

3. a. For \( \theta = 30^\circ \)
\[ \phi_1 = 5^\circ \]
\[ H = 50' \]
\[ R = 86.2 \text{ ft.} \]
\[ D = 57.8 \text{ ft.} \]
\[ \overline{y} = 68.7 \text{ ft.} \]
\[ a_1 a_2 = 19 \text{ ft.} \]
\[ A_1 = 768 \text{ sq. ft.}; \quad A_2 = 2,450 \text{ sq. ft.} \]
\[ X = \frac{768 \times 19}{768 + 2,450} = 4.55 \text{ ft.} \]
\[ \Theta = 2 \beta_0 = 96^\circ = 1.67 \text{ radians} \]
\[ \overline{L} = R \Theta = 86.2 \times 1.67 = 144 \text{ ft.} \]
\[ a = \frac{\overline{R}}{L} = \frac{86.2 \times 144}{128.2} = 96.7 \text{ ft.} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>59'</td>
<td>28'</td>
<td>210</td>
</tr>
</tbody>
</table>

4. a. For \( \theta = 30 \)
\[ \phi_1 = 10^\circ \]
\[ H = 50' \]
\[ \alpha_0 \quad \beta_0 \]
\[ 25^\circ \quad 48^\circ \]
\[ R = 87' \]
\[ D = 62.5'' \]
\[ \bar{y} = 67 \text{ ft.} \]
\[ a_1 a_2 = 15.25 \text{ ft.} \]
\[ A_1 = 578 \text{ sq. ft.;} \quad A_2 = 2,028 \text{ sq. ft.} \]
\[ \bar{x} = \frac{578 \times 15.25}{578 + 2,028} = 3.38 \text{ ft.} \]
\[ \theta = 2 \beta_o = 88^\circ = 1.53 \text{ radians} \]
\[ L = R \theta = 87 \times 1.53 = 133.5 \text{ radians} \]
\[ a = \frac{R \theta}{L} = \frac{87 \times 133.5}{120.5} = 96' \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.8'</td>
<td>30'</td>
<td>1.96</td>
</tr>
</tbody>
</table>

b. For \( i = 30^\circ \)
\[ \phi_i = 10^\circ \]
\[ H = 75' \]
\[ R = 127.5' \]
\[ D = 92.5' \]
\[ \bar{y} = 107 \text{ ft.} \]
\[ a_1 a_2 = 27.5 \text{ ft.} \]
\[ A_1 = 1,140 \text{ sq. ft.;} \quad A_2 = 4,320 \text{ sq. ft.} \]
\[ \bar{x} = 5.8 \text{ ft.} \]
\[ \theta = 2 \beta_o = 88^\circ = 1.53 \text{ radians} \]
\[ L = R \theta = 127.5 \times 1.53 = 195 \text{ ft.} \]
\[ a = \frac{R \theta}{L} = \frac{127.5 \times 195}{176.5} = 141 \text{ ft.} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>92'</td>
<td>48'</td>
<td>1.92</td>
</tr>
</tbody>
</table>

5. a. For \( i = 30^\circ \)
\[ \phi_i = 15^\circ \]
\[ H = 50' \]
\[ R = 88.5 \text{ ft.} \]
\[ D = 69.5 \text{ ft.} \]
\[ \overline{y} = 77.5 \text{ ft.} \]
\[ a_{1a_2} = 18.36 \text{ ft.} \]
\[ A_1 = 327 \text{ sq. ft.} \]
\[ \overline{X} = 3.16 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 78^\circ = 1.36 \text{ radians} \]
\[ \widehat{L} = R \theta = 88.5 \times 1.36 = 120 \text{ sq. ft.} \]
\[ a = \frac{R \theta}{\widehat{L}} = \frac{88.5 \times 120}{110.5} = 96 \text{ ft.} \]

\[
\begin{array}{ccc}
    l & d & l/d \\
    66' & 36.8' & 1.795 \\
\end{array}
\]

b. For \( i = 30^\circ \)
\[ \phi_1 = 15^\circ \]
\[ \phi_0 = 27^\circ \]
\[ \theta_0 = 39^\circ \]

\[ \overline{y} = 113 \text{ ft.} \]
\[ a_{1a_2} = 23 \text{ ft.} \]
\[ A_1 = 780 \text{ sq. ft.} \]
\[ \overline{X} = 4.15 \text{ ft.} \]
\[ \theta = 78^\circ = 1.36 \text{ radians} \]
\[ \widehat{L} = R \theta = 133 \times 1.36 = 181 \text{ ft.} \]
\[ a = \frac{R \theta}{\widehat{L}} = \frac{133 \times 181}{187} = 144 \text{ ft.} \]

\[
\begin{array}{ccc}
    l & d & l/d \\
    96.5' & 53' & 1.820 \\
\end{array}
\]
6. a. For $i = 30^\circ$

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28°</td>
<td>31°</td>
</tr>
</tbody>
</table>

$y = 94.5$ ft.
$\bar{y} = 94.5$ ft.

$D = 89.4$ ft.
$\bar{D} = 89.4$ ft.

$\phi_1 = 20^\circ$
$H = 50'$
$R = 94.5$ ft.

\[ a_1 a_2 = 16.5 \text{ ft.} \]

$A_1 = 238 \text{ sq. ft.}; \quad A_2 = 1,108 \text{ sq. ft.}$

$\theta = 2 \beta_0 = 62^\circ = 108 \text{ radians}$

$\bar{L} = R \theta = 104 \times 1.08 = 112$ ft.

\[ a = \frac{R \bar{L}}{L} = \frac{104 \times 112}{107} = 109 \text{ ft.} \]

<table>
<thead>
<tr>
<th>$l$</th>
<th>$d$</th>
<th>$L/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>81'</td>
<td>46'</td>
<td>1.760</td>
</tr>
</tbody>
</table>

b. For $i = 30^\circ$

$y = 139$ ft.
$\bar{y} = 139$ ft.

$D = 136.8$ ft.
$\bar{D} = 136.8$ ft.

$\phi_1 = 20^\circ$
$H = 75'$
$R = 159$ ft.

$\bar{y} = 139$ ft.
$\bar{y} = 139$ ft.

\[ a_1 a_2 = 23.35 \text{ ft.} \]

$A_1 = 593 \text{ sq. ft.}; \quad A_2 = 2,600 \text{ sq. ft.}$

$\bar{A} = 4.33$ ft.
$\bar{A} = 4.33$ ft.

$\theta = 2 \beta_0 = 62^\circ = 108 \text{ radians}$

$\bar{L} = R \theta = 159 \times 1.08 = 171$ ft.

\[ a = \frac{R \bar{L}}{L} = \frac{159 \times 171}{163} = 167 \text{ ft.} \]

<table>
<thead>
<tr>
<th>$l$</th>
<th>$d$</th>
<th>$L/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120'</td>
<td>67.5'</td>
<td>1.780</td>
</tr>
</tbody>
</table>
7. a. For \( i = 30° \)

\[ \phi_i = 25° \]

\[ H = 50' \]

\[ R = 122.5 \text{ ft.} \]

\[ D = 110.8 \text{ ft.} \]

\[ y = 111 \text{ ft.} \]

\[ a_1a_2 = 15 \text{ ft.} \]

\[ A_1 = 106.2 \text{ sq. ft.}; \quad A_2 = 785 \text{ sq. ft.} \]

\[ X = 1.79 \text{ ft.} \]

\[ \theta = 2 \beta_o = 50° = 0.87 \text{ radian} \]

\[ L = R \theta = 106.2 \text{ ft.} \]

\[ a = \frac{H}{L} = \frac{122.5 \times 106.2}{109.8} = 125 \text{ ft.} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.5'</td>
<td>55.3'</td>
<td>1.72</td>
</tr>
</tbody>
</table>

8. a. For \( i = 45° \)

\[ \phi_i = 5° \]

\[ H = 50' \]

\[ R = 72.8 \text{ ft.} \]

\[ D = 54.2 \text{ ft.} \]

\[ y = 61 \text{ ft.} \]

\[ a_1a_2 = 14.35 \text{ ft.} \]

\[ A_1 = 820 \text{ sq. ft.}; \quad A_2 = 1,208 \text{ sq. ft.} \]

\[ X = 5.32 \text{ ft.} \]

\[ \theta = 2 \beta_o = 84.2° = 1.46 \text{ radians} \]

\[ D = R \theta = 72.8 \times 1.46 = 106 \text{ ft.} \]

\[ a = \frac{H}{L} = \frac{72.8 \times 106}{97.5} = 79.2 \text{ ft.} \]
9. a. For \( i = 45^\circ \)
\[ \phi_1 = 10^\circ \]

\[
\begin{array}{c|c|c}
\hline
l & d & l/d \\
47' & 3.13' & 1.505 \\
\hline
\end{array}
\]

\[ H = 50' \]
\[ R = 70.5 \text{ ft.} \]
\[ D = 54.5 \text{ ft.} \]

\[ \overline{y} = 61.4 \text{ ft.} \]
\[ a_{1a_2} = 14.2 \text{ ft.} \]
\[ A_1 = 562 \text{ sq. ft.} \]
\[ \overline{x} = 4.9 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 79.4^\circ = 1.38 \text{ radians} \]
\[ \widehat{L} = R\theta = 70.5 \times 1.38 = 97.5 \text{ ft.} \]
\[ a = \frac{R\theta}{L} = \frac{70.5 \times 97.5}{90} = 76.4 \text{ ft.} \]

\[
\begin{array}{c|c|c}
\hline
l & d & l/d \\
46' & 34.5' & 1.330 \\
\hline
\end{array}
\]

b. For \( i = 45^\circ \)
\[ \phi_1 = 10^\circ \]

\[
\begin{array}{c|c|c}
\hline
\alpha_0 & \beta_0 \\
34^\circ & 39.7^\circ \\
\hline
\end{array}
\]

\[ H = 75' \]
\[ R = 106' \]
\[ D = 81.7' \]

\[ \overline{y} = 90 \text{ ft.} \]
\[ a_{1a_2} = 19.25 \text{ ft.} \]
\[ A_1 = 1,475 \text{ sq. ft.} \]
\[ \overline{x} = 7.43 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 79.4^\circ = 1.38 \text{ radians} \]
\[ \widehat{L} = R\theta = 143.5 \text{ ft.} \]
\[ a = \frac{R \sin \phi_i}{L} = \frac{106 \times 143.5}{134.75} = 113 \text{ ft.} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>67.3'</td>
<td>50.5'</td>
<td>1.330</td>
</tr>
</tbody>
</table>

10. a. For \( i = 45^\circ \)
\[ \phi_i = 15^\circ \]
\[ H = 50' \]
\[ R = 70 \text{ ft.} \]
\[ D = 56.2 \text{ ft.} \]
\[ y = 59.3 \text{ ft.} \]
\[ a_1 a_2 = 11.5 \text{ ft.} \]
\[ A_1 = 495 \text{ sq. ft.} \]
\[ \bar{x} = 4.24 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 74.4^\circ = 1.295 \text{ radians} \]
\[ \tau = R \theta = 70 \times 1.295 = 90.6 \text{ ft.} \]
\[ a = \frac{R \sin \phi_i}{L} = \frac{70 \times 90.6}{84.7} = 75 \text{ ft.} \]

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>44'</td>
<td>36.5'</td>
<td>1.200</td>
</tr>
</tbody>
</table>

b. For \( i = 45^\circ \)
\[ \phi_i = 15^\circ \]
\[ H = 75' \]
\[ R = 104.8 \text{ ft.} \]
\[ D = 83.8 \text{ ft.} \]
\[ y = 94 \text{ ft.} \]
\[ a_1 a_2 = 21 \text{ ft.} \]
\[ A_1 = 1100 \text{ sq. ft.} \]
\[ \bar{x} = 7.55 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 74.4^\circ = 1.29 \text{ radians} \]
\[ L = R \theta = 104.8 \times 1.29 = 135 \text{ ft.} \]
\[ a = \frac{R \theta}{L} = \frac{104.8 \times 135}{126.5} = 112 \text{ ft.} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>68.2'</td>
<td>56'</td>
<td>1.22</td>
</tr>
</tbody>
</table>

11. a. For \( i = 45^\circ \)
\[ \phi_1 = 20^\circ \]
\[ H = 50' \]
\[ R = 71 \text{ ft.} \]
\[ D = 59 \text{ ft.} \]
\[ y = 63.5 \text{ ft.} \]
\[ a_{1a_2} = 12.38 \text{ ft.} \]
\[ A_1 = 350 \text{ sq. ft.} \]
\[ X = \frac{350 \times 12.38}{1068} = 4.05 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 69^\circ = 1.20 \text{ radians} \]
\[ L = R \theta = 85.2 \text{ ft.} \]
\[ a = \frac{R \theta}{L} = \frac{71 \times 85.2}{80.5} = 75.1 \text{ ft.} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>46'</td>
<td>40.5'</td>
<td>1.135</td>
</tr>
</tbody>
</table>

b. For \( i = 45^\circ \)
\[ \phi_1 = 20^\circ \]
\[ H = 75' \]
\[ R = 108 \text{ ft.} \]
\[ D = 89.5 \text{ ft.} \]
\[ y = 93 \text{ ft.} \]
\[ a_{1a_2} = 16.8 \text{ ft.} \]
\[ A_1 = 832 \text{ sq. ft.} \]
\[ A_2 = 1,620 \text{ sq. ft.} \]
\[ X = 5.7 \text{ ft.} \]
\[ \theta = 2 \beta_o = 69^\circ = 120 \text{ radians} \]
\[ L = R \theta = 129.6 \text{ ft.} \]
\[ a = \frac{R}{L} = \frac{108 \times 129.6}{121.5} = 115 \text{ ft.} \]

<table>
<thead>
<tr>
<th></th>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>68'</td>
<td>59.6'</td>
<td>1.140</td>
</tr>
</tbody>
</table>

12. a. For \( i = 45^\circ \)
\[ \phi_1 = 25^\circ \]
\[ H = 50' \]
\[ R = 75.6 \text{ ft.} \]
\[ D = 65 \text{ ft.} \]
\[ \bar{y} = 66.5 \text{ ft.} \]
\[ a_1 a_2 = 11.1 \text{ ft.} \]
\[ A_1 = 225 \text{ sq. ft.;} \quad A_2 = 531 \text{ sq. ft.} \]
\[ \bar{x} = 3.3 \text{ ft.} \]
\[ \theta = 2 \beta_o = 62^\circ = 1.08 \text{ radians} \]
\[ L = R \theta = 81.5 \text{ ft.} \]
\[ a = \frac{R}{L} = \frac{75.6 \times 81.5}{78} = 79 \text{ ft.} \]

<table>
<thead>
<tr>
<th></th>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>48'</td>
<td>44.4'</td>
<td>1.080</td>
</tr>
</tbody>
</table>

13. a. For \( i = 60^\circ \)
\[ \phi_1 = 5^\circ \]
\[ H = 50' \]
\[ R = 70 \text{ ft.} \]
\[ D = 58 \text{ ft.} \]
\[ \bar{y} = 60.3 \text{ ft.} \]
\[ a_1 a_2 = 10.5 \text{ ft.} \]
\[ A_1 = 826 \text{ sq. ft.;} \quad A_2 = 630 \text{ sq. ft.} \]
\( \bar{x} = 5.98 \text{ ft.} \)
\( \theta = 2 \beta_o = 69^\circ = 1.20 \text{ radians} \)
\( \widehat{L} = R \theta = 70 \times 1.20 = 84 \text{ ft.} \)
\( a = \frac{RL}{L} = \frac{70 \times 84}{79.5} = 74 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.2'</td>
<td>36.8'</td>
<td>1.12</td>
</tr>
</tbody>
</table>

b. For \( i = 60^\circ \)
\( \phi_1 = 5^\circ \)
\( H = 75' \)
\( R = 106 \text{ ft.} \)
\( D = 87.5 \text{ ft.} \)
\( \bar{y} = 93.7 \text{ ft.} \)
\( a_1a_2 = 18 \text{ ft.} \)
\( A_1 = 1,890 \text{ sq. ft.}; \quad A_2 = 1,436 \text{ sq. ft.} \)
\( \bar{x} = 10.2 \text{ ft.} \)
\( \theta = 2 \beta_o = 69^\circ = 1.2 \text{ radians} \)
\( \widehat{L} = R \theta = 106 \times 1.2 = 127.2 \text{ ft.} \)
\( a = \frac{RL}{L} = \frac{106 \times 127.2}{120} = 112.2 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.5'</td>
<td>55.8'</td>
<td>1.14</td>
</tr>
</tbody>
</table>

14. a. For \( i = 60^\circ \)
\( \phi_1 = 10^\circ \)
\( H = 50' \)
\( R = 70.2 \text{ ft.} \)
\( D = 59.2 \text{ ft.} \)
\( \bar{y} = 62.5 \text{ ft.} \)
\( a_1a_2 = 10.8 \text{ ft.} \)
A_1 = 760 sq. ft.; A_2 = 624 sq. ft.
\bar{x} = \frac{760 \times 10.8}{1,384} = 5.92 \text{ ft.}
\theta = 2 \beta_o = 66^\circ = 1.145 \text{ radians}
\hat{L} = R \theta = 80.5 \text{ ft.}
a = \frac{R \hat{L}}{L} = \frac{70.2 \times 80.5}{76} = 74.5 \text{ ft.}

\begin{tabular}{|c|c|c|}
\hline
l & d & l/d \\
\hline
41.2' & 39.8' & 1.035 \\
\hline
\end{tabular}

b. For i = 60^\circ
\phi_1 = 10^\circ
H = 75'
R = 105 \text{ ft.}
D = 88 \text{ ft.}
\bar{y} = 92 \text{ ft.}
a_{1\alpha_2} = 16 \text{ ft.}
A_1 = 1,710 \text{ sq. ft.; } A_2 = 1,385 \text{ sq. ft.}
\bar{x} = 8.85 \text{ ft.}
\theta = 2 \beta_o = 66^\circ = 1.145 \text{ radians}
\hat{L} = R \theta = 105 \times 1.145 = 120 \text{ ft.}
a = \frac{R \hat{L}}{L} = \frac{105 \times 120}{114} = 110.5 \text{ ft.}

\begin{tabular}{|c|c|c|}
\hline
l & d & l/d \\
\hline
60.8' & 59.5' & 1.020 \\
\hline
\end{tabular}

15. a. For i = 60^\circ
\phi_1 = 15^\circ
H = 50'
R = 70 \text{ ft.}
D = 60 \text{ ft.}
\( \bar{y} = 64.8 \text{ ft.} \)
\( a_1 a_2 = 11.8 \text{ ft.} \)
\( A_1 = 630 \text{ sq. ft.} ; \quad A_2 = 523 \text{ sq. ft.} \)
\( \bar{x} = 6.5 \text{ ft.} \)
\( \theta = 2 \beta = 63^\circ = 1.1 \text{ radians} \)
\( \hat{L} = R \theta = 70 \times 1.1 = 77 \text{ ft.} \)
\( a = \frac{R \theta}{L} = \frac{70 \times 77}{72.4} = 74.5 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.5'</td>
<td>43.5'</td>
<td>0.930</td>
</tr>
</tbody>
</table>

b. For \( i = 60^\circ \)
\( \phi_i = 15^\circ \)
\( H = 75' \)
\( R = 104.5 \text{ ft.} \)
\( D = 89.5 \text{ ft.} \)
\( \bar{y} = 94.5 \text{ ft.} \)
\( a_1 a_2 = 16.25 \text{ ft.} \)
\( A_1 = 1,410 \text{ sq. ft.} ; \quad A_2 = 1,208 \text{ sq. ft.} \)
\( \bar{x} = 8.7 \text{ ft.} \)
\( \theta = 2 \beta = 63^\circ = 1.095 \text{ radians} \)
\( \hat{L} = R \theta = 114.5 \)
\( a = \frac{R \theta}{L} = \frac{104.5 \times 114.5}{109} = 110 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>61'</td>
<td>64'</td>
<td>0.938</td>
</tr>
</tbody>
</table>

16. a. For \( i = 60^\circ \)
\( \phi_i = 20^\circ \)
\( H = 50' \)
\( R = 68.2 \text{ ft.} \)
\( D = 59 \text{ ft.} \)
b. For $i = 60^\circ$

\[ \phi_1 = 20^\circ \]

\[ H = 75' \]

\[ R = 102 \text{ ft.} \]

\[ D = 88 \text{ ft.} \]

\[ \bar{y} = 90.5 \text{ ft.} \]
\[ a_1 a_2 = 14.5 \text{ ft.} \]

\[ A_1 = 1,110 \text{ sq. ft.;} \quad A_2 = 1,050 \text{ sq. ft.} \]

\[ \bar{x} = 7.45 \text{ ft.} \]

\[ \theta = 2 \beta_0 = 60.4^\circ = 1.05 \text{ radians} \]

\[ L = R \theta = 107 \text{ ft.} \]

\[ a = \frac{R \theta}{L} = \frac{102 \times 107}{104} = 105 \text{ ft.} \]

\[
\begin{array}{ccc}
\hline
l & d & l/d \\
\hline
55' & 65.8' & 0.834 \\
\hline
\end{array}
\]
\( \bar{y} = 58.5 \text{ ft.} \)
\( \bar{a_1}a_2 = 9.62 \text{ ft.} \)
\( A_1 = 330 \text{ sq. ft.}; \quad A_2 = 370 \text{ sq. ft.} \)
\( \bar{x} = 4.54 \text{ ft.} \)
\( \theta = 2 \beta_o = 60^\circ = 1.046 \text{ radians} \)
\( \bar{L} = R \theta = 65.5 \times 1.046 = 68.2 \text{ ft.} \)
\( a = \frac{R \bar{L}}{L} = \frac{65.5 \times 68.2}{68} = 68.5 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.8'</td>
<td>45.5'</td>
<td>0.720</td>
</tr>
</tbody>
</table>

b. For \( i = 60^\circ \)
\( \phi_1 = 25^\circ \)

\( H = 75' \)
\( R = 98.2 \text{ ft.} \)
\( D = 85 \text{ ft.} \)
\( \bar{y} = 90.3 \text{ ft.} \)
\( \bar{a_1}a_2 = 16.25 \text{ ft.} \)
\( A_1 = 758 \text{ sq. ft.}; \quad A_2 = 872 \text{ sq. ft.} \)
\( \theta = 2 \beta_o = 60^\circ = 1.046 \text{ radians} \)
\( \bar{L} = R \theta = 102.5 \text{ ft.} \)
\( a = \frac{R \bar{L}}{L} = \frac{98.2 \times 102.5}{98.2} = 102.5 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.8'</td>
<td>69'</td>
<td>0.735</td>
</tr>
</tbody>
</table>

18. a. For \( i = 75^\circ \)
\( \phi_1 = 5^\circ \)

\( H = 50' \)
\( R = 83.5 \text{ ft.} \)
\( D = 75.8 \text{ ft.} \)
\( \bar{Y} = 115 \text{ ft.} \)
\( a_1a_2 = 14.25 \text{ ft.} \)
\( A_1 = 2,215 \text{ sq. ft.} \); \( A_2 = 860 \text{ sq. ft.} \)
\( \bar{X} = 10.25 \text{ ft.} \)
\( \theta = 2 \beta_o = 50^\circ = 0.87 \text{ radian} \)
\( \bar{L} = R \theta = 110 \text{ ft.} \)
\( a = \frac{R \bar{L}}{L} = \frac{126.5 \times 110}{106.5} = 131 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>72.5'</td>
<td>77'</td>
<td>0.946</td>
</tr>
</tbody>
</table>

19. a. For \( i = 75^\circ \)
\( \phi_i = 10^\circ \)

\( \alpha_o \) | \( \beta_o \) |
| 47.5° | 23.5° |

\( H = 50' \)
\( R = 85 \text{ ft.} \)
\( D = 78 \text{ ft.} \)
\( a_1a_2 = 11 \text{ ft.} \)
\( A_1 = 865.8 \text{ sq. ft.} \); \( A_2 = 344.2 \text{ sq. ft.} \)
\( \bar{X} = 7.88 \text{ ft.} \)
\( \theta = 2 \beta_o = 47^\circ = 0.82 \text{ radian} \)
\( \bar{L} = R \theta = 69.5 \text{ ft.} \)
\( a = \frac{R \bar{L}}{L} = \frac{85 \times 69.5}{67.5} = 87.5 \text{ ft.} \)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( d )</th>
<th>( l/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>46.6'</td>
<td>56'</td>
<td>0.832</td>
</tr>
</tbody>
</table>

b. For \( i = 75^\circ \)
\( \phi_i = 10^\circ \)

\( \alpha_o \) | \( \beta_o \) |
| 47.5° | 23.5° |

\( H = 75' \)
\( R = 128.8 \text{ ft.} \)
\( D = 118.4 \text{ ft.} \)
\(a_{1a_2} = 14.7 \text{ ft.}\)

\(A_1 = 1,965 \text{ sq. ft.}\)

\(\bar{x} = 10.7 \text{ ft.}\)

\(\theta = 2 \theta_0 = 47^\circ = 0.82 \text{ radian}\)

\(L = R \theta = 105.5 \text{ ft.}\)

\(a = \frac{R \bar{r}}{L} = \frac{128.8 \times 105.5}{101.8} = 133.5 \text{ ft.}\)

<table>
<thead>
<tr>
<th>(l)</th>
<th>(d)</th>
<th>(l/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>71'</td>
<td>84'</td>
<td>0.845</td>
</tr>
</tbody>
</table>

20. a. For \(i = 75^\circ\)

\(\phi_1 = 15^\circ\)

\(H = 50'\)

\(R = 84 \text{ ft.}\)

\(D = 77.4 \text{ ft.}\)

\(\bar{y} = 78.5 \text{ ft.}\)

\(a_{1a_2} = 9.6 \text{ ft.}\)

\(A_1 = 765 \text{ sq. ft.}\)

\(\theta = 46^\circ = 0.8 \text{ radian}\)

\(L = R \theta = 67.2 \text{ ft.}\)

\(a = \frac{R \bar{r}}{L} = \frac{84 \times 67.2}{65.6} = 86.2 \text{ ft.}\)

\(\bar{x} = 6.85 \text{ ft.}\)

<table>
<thead>
<tr>
<th>(l)</th>
<th>(d)</th>
<th>(l/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>44'</td>
<td>58'</td>
<td>0.756</td>
</tr>
</tbody>
</table>

b. For \(i = 75^\circ\)

\(\phi_1 = 15^\circ\)

\(H = 75'\)

\(R = 123 \text{ ft.}\)

\(D = 113.4 \text{ ft.}\)
$\bar{y} = 114$ ft.
$a_1 a_2 = 13.75$ ft.
$A_1 = 1700$ sq. ft.; $A_2 = 675$ sq. ft.
$\bar{x} = 9.85$ ft.
$\theta = 2 \beta_0 = 46^\circ = 0.8$ radian
$\bar{L} = R \theta = 98.4$ ft.
$a = \frac{\bar{R}}{\bar{L}} = \frac{123 \times 98.4}{97} = 125$ ft.

<table>
<thead>
<tr>
<th>$l$</th>
<th>$d$</th>
<th>$l/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.8'</td>
<td>84.6'</td>
<td>0.755</td>
</tr>
</tbody>
</table>

21. a. For $\phi = 75^\circ$
$\phi_1 = 20^\circ$

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$53^\circ$</td>
<td>$22^\circ$</td>
</tr>
</tbody>
</table>

H = 50'
R = 82.7 ft.
D = 76.7 ft.

$\bar{y} = 77.5$ ft.
$a_1 a_2 = 9.30$ ft.
$A_1 = 660$ sq. ft.; $A_2 = 268$ sq. ft.
$\bar{x} = 6.62$ ft.
$\theta = 2 \beta_0 = 44^\circ = 0.765$ radian
$\bar{L} = R \theta = 63.4$ ft.
$a = \frac{\bar{R}}{\bar{L}} = \frac{82.7 \times 63.4}{62.4} = 84$ ft.

<table>
<thead>
<tr>
<th>$l$</th>
<th>$d$</th>
<th>$l/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.3'</td>
<td>61'</td>
<td>0.665</td>
</tr>
</tbody>
</table>

b. For $\phi = 75^\circ$
$\phi_1 = 20^\circ$

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$53^\circ$</td>
<td>$22^\circ$</td>
</tr>
</tbody>
</table>
\[ H = 75' \]
\[ R = 126.5 \text{ ft.} \]
\[ D = 117.5 \text{ ft.} \]
\[ \bar{y} = 118 \text{ ft.} \]
\[ a_1 = \frac{13.1 \text{ ft.}}{2} \]
\[ A_1 = 1,460 \text{ sq. ft.}; \quad A_2 = 598 \text{ sq. ft.} \]
\[ \bar{x} = 9.25 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 44^\circ = 0.765 \text{ radian} \]
\[ L = R \theta = 96.5 \text{ ft.} \]
\[ a = \frac{R \bar{L}}{L} = \frac{126.5 \times 96.5}{93.8} = 130 \text{ ft.} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>62'</td>
<td>92'</td>
<td>0.675</td>
</tr>
</tbody>
</table>

22. a. For \( \theta = 75^\circ \)
\[ \phi_1 = 25^\circ \]
\[ \begin{array}{cc}
\phi_1 & \phi_2 \\
25^\circ & 22^\circ \\
\end{array} \\
\frac{\phi_1}{\phi_2} = \frac{25^\circ}{22^\circ} \]

\[ H = 50' \]
\[ R = 81.3 \text{ ft.} \]
\[ D = 75.5 \text{ ft.} \]
\[ \bar{y} = 77.5 \text{ ft.} \]
\[ a_1 = 9.75 \text{ ft.} \]
\[ A_1 = 554 \text{ sq. ft.}; \quad A_2 = 242 \text{ sq. ft.} \]
\[ \bar{x} = 6.8 \text{ ft.} \]
\[ \theta = 2 \beta_0 = 44^\circ = 0.765 \text{ radian} \]
\[ L = R \theta = 62 \text{ ft.} \]
\[ a = \frac{R \bar{L}}{L} = \frac{81.3 \times 62}{61} = 82.5 \text{ ft.} \]

<table>
<thead>
<tr>
<th>l</th>
<th>d</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.5'</td>
<td>61.5'</td>
<td>0.61</td>
</tr>
</tbody>
</table>
22. b. For $i = 75^\circ$

\[ \phi_1 = 25^\circ \]

\[ R = 121 \text{ ft.} \]

\[ D = 112.2 \text{ ft.} \]

\[ y = 115 \text{ ft.} \]

\[ a_1a_2 = 14.5 \text{ ft.} \]

\[ A_1 = 1,195 \text{ sq. ft.} \]

\[ A_2 = 540 \text{ sq. ft.} \]

\[ X = 10 \text{ ft.} \]

\[ \theta = 2 \beta_o = 44^\circ = 0.765 \text{ radian} \]

\[ L = R \theta = 92.5 \text{ ft.} \]

\[ a = \frac{HE}{L} = \frac{121 \times 92.5}{91} = 123 \text{ ft.} \]

<table>
<thead>
<tr>
<th>$l$</th>
<th>$d$</th>
<th>$l/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.3'</td>
<td>92.5'</td>
<td>0.59</td>
</tr>
</tbody>
</table>
STABILITY ANALYSIS DUE TO EARTHQUAKE (HORIZONTAL)

INFORMATION:

\[ l = 30^\circ \quad \beta \]
\[ \alpha = 10^\circ \quad 29^\circ 45' \]
\[ H = 80' \]

TO FIND THE C.G. OF THE CIRCULAR PORTION:

FROM THE DRAWING - \( R = 87' \)

\[ D = 62.5' \quad (05) \]
\[ \sin \beta = 0.72 \]
\[ \tan \beta = 0.39 \]
\[ \tan \alpha = 0.625 \quad \text{(approx)} \]

THE C.G. (C) OF THE TRIANGULAR PORTION IS GIVEN BY THE INTERSECTION OF THE MEDIAN.

SO THEN \( C \) IS MEASURED AS 1525 FT.

THE AREA OF THE TRIANGLE \( (A) = 5.78 \text{ FT}^2 \)

THE AREA OF THE CIRCULAR PORTION \( (A) = 2.028 \text{ FT}^2 \)

THE COMBINED C.G. IS GIVEN BY

\[ C = (A_1 + A_2) / (A_1 + A_2) \]

TO CALCULATE \( C \), THE LENGTH (\( d \))

\[ d = 2 \quad 68' \quad 113 \text{ RADIAN} \]

\[ c = 2 \quad 83' \quad 133 \text{ FT} \]

\[ C = 2 \quad 87' \quad 196' \]

\[ \frac{L}{d} = \frac{2}{2} = 1 \quad 30' \quad 1.96 \]
STABILITY ANALYSIS DUE TO PORE PRESSURE

INFORMATION:

\[ \theta = 45^\circ \]
\[ \phi = 30^\circ \]
\[ \beta = 75^\circ \]
\[ \delta = 45^\circ \]
\[ \gamma = 30^\circ \]
\[ \lambda = 15^\circ \]

\[ H = 50' \]
\[ R = 75' \]
\[ D = 50' \]
\[ V = 45' \]
\[ X = 45' \]
\[ Y = 45' \]

\[ \begin{array}{ccccccccccc}
0 & 0.00 & 0.03 & 0.05 & 0.07 & 0.08 & 0.10 \\
0 & 0.09 & 0.12 & 0.15 & 0.18 & 0.20 & 0.22 \\
\end{array} \]

\[ \begin{array}{ccccccccccc}
0 & 0.00 & 0.03 & 0.05 & 0.07 & 0.08 & 0.10 \\
0 & 0.09 & 0.12 & 0.15 & 0.18 & 0.20 & 0.22 \\
\end{array} \]

\[ \beta \quad \text{Sudden Drawdown Factor} \]
\[ \delta \quad \text{Seepage Factor} \]
\[ \epsilon \quad \text{Construction Pore Pressure Coefficient} \]

**Figure 1a**

**Figure 1b**

**In Case of Drawdown**

\[ C = \frac{\gamma' \cdot H}{\gamma} \]

**In Case of Pore Pressure**

\[ C = \frac{\gamma' \cdot H}{\gamma} \]

\[ \gamma' \text{ = Saturated unit weight of soil.} \]

\[ \gamma \text{ = Unit weight of Water.} \]

\[ \delta \text{ = Total area within slip circle.} \]

\[ \phi \text{ = Total area below the phreatic line.} \]

\[ \phi \text{ = Total area between the slip circle and pore pressure line.} \]

**In Case of Pore Pressure Due to Construction**

\[ C = \frac{\gamma' \cdot H}{\gamma} \]

\[ \gamma' \text{ = Pore fluid pressure in excess of soil.} \]

\[ \gamma \text{ = Total unit weight of soil.} \]

\[ \epsilon \text{ = Pore fluid pressure in excess of soil expressed as a percent of total unit weight of soil.} \]
STABILITY ANALYSIS OF U/S SLOPE WITH

- EARTHQUAKE
- PORE PRESSURE DUE TO SUDDEN DRAWDOWN
- PORE PRESSURE DUE TO CONSTRUCTION

BY SLIP CIRCLE METHOD.

(TAKING e = 0.6)
(PORE PRESSURE = 0.6 TIMES THE WEIGHT ON THE SLIDE ACTING IN THE DIRECTION OF THE NORMAL)

INFORMATION

\( \beta = 30^\circ \)
\( \gamma = 1240 \text{ kN/m}^2 \)
\( h = 20 \text{ m} \)
\( e = 0.6 \)
\( c = 4000 \text{ kN/m}^2 \)
\( \tan \beta = 0.1733 \)

\[ \text{E.P. DUE TO SUDDEN DRAWDOWN} = (100 \times 30 \times 0.6) + 8560 \text{ ft}^2 \]

\[ \text{E.P. DUE TO CONSTRUCTION} = (100 \times 30 \times 0.6) + 8560 \text{ ft}^2 \]

FIGURE 18

SCALE
STABILITY ANALYSIS OF D/S SLOPE WITH PORE PRESSURE DUE TO SEEPAGE
BY SLIP CIRCLE METHOD

HOR. DISTORTION SCALE: $\sqrt{r^2 + \frac{1}{2}}$

Fig. 10
STABILITY ANALYSIS OF D/S SLOPE WITH PORE PRESSURE DUE TO SEEPAGE

BY Ø-CIRCLE METHOD.

CHECKING FOR SEEPAGE:

\[
\delta = \frac{K_{12} - 0}{K_{12}(A - a) + K_{0}} = 0.13
\]

\[
T_{1} = \frac{K_{12}(A - a) + K_{0}}{A}
\]

\[
T_{0} = 120 \text{ lb/ft}^2
\]

\[
T_{a} = 14.8 \text{ lb/ft}^2
\]

\[
T_{w} = 62.0 \text{ lb/ft}^2
\]

AREA: A = 42 × (15 × 125) = 12,400 sq ft

AREA: 0 = 15 × (15 × 25) = 5,625 sq ft

AREA: 02 = 17 × (15 × 25) = 3,650 sq ft

FIGURE 20

SCALE
VERIFICATION OF TABLE 1 (After D. W. Taylor)

BY 1620 COMPUTER PROGRAM

DATA ON CRITICAL CIRCLES BY PHI-CIRCLE METHOD

1 READ, AI, FI, X, Y
   RAD=3.1415927/180.
   AT=RAD*AI
   FI=RAD*FI
   X=RAD*X
   Y=RAD*Y
   COTX=(X)/SIN(X)
   COTY=COS(Y)/SIN(Y)
   COTI=COS(ALT)/SIN(ALT)
   SINFI=SIN(FI)
   SECX=1/cos(X)
   CSCX=1/sin(X)
   CSCY=1/sin(Y)
   AN=5*(COTX-COTY-COTI+SENF*CSCT*CSCT)
   IF(AN)3,4,4
3   BN=AN
   AN=O
   GO TO 5
4   BN=AN
5   UNUM=5*CSCT*CSCT*(Y*CSCT*CSCT-COTI)*COTX-COTI-X.*AN
   UDEN=(1.-2.*COTI*COTI)/3. +COTI*(COTX-COTI)+COTX*COTY
   UDEN=UNUM+2.*AN*(AN-SINF)*CSCT*CSCT
   COTU=UNUM*Y*SECX*CSCT*CSCT*CSCT/UDEN-1./COTX
   U=ATAN(l./COTU)
   SINU=SIN(U)
   SINF=UNUM*SINF*CSCT*CSCT*CSCT/SINF/UDEN
   COSB=SQR(l.-SINB*SINB)
   B=ATAN(SINB/COSB)
   V=U-B
   COTV=COS(V)/SIN(V)
   CFWH=UNUM/(2.*COTX*COTV+Z)
   D=5*(CSCT*CSCT-COTX*COTY+1.)
   PUNCH, BN, D, CFWH
   GO TO 1
END
<table>
<thead>
<tr>
<th>DATA CARDS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
</tr>
<tr>
<td>i</td>
<td>0</td>
</tr>
<tr>
<td>90.0</td>
<td>5</td>
</tr>
<tr>
<td>90.0</td>
<td>10</td>
</tr>
<tr>
<td>90.0</td>
<td>15</td>
</tr>
<tr>
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