Time-Inconsistent Preferences and the Welfare Effects of Financing Unfunded Social Security With Consumption Taxation

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Time-Inconsistent Preferences and the Welfare Effects of Financing Unfunded Social Security with Consumption Taxation

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

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in

Economics

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Time-Inconsistent Preferences and the Welfare Effects of Financing Unfunded Social Security with Consumption Taxation*

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Abstract

A sizable body of evidence suggests that individuals make retirement preparation plans for the future, but then they persistently fail to follow through and prepare adequately to fund their retirement. In parallel, observational and experimental evidence suggests that people discount the future hyperbolically, and a hyperbolic discount function leads to inadequate preparation for retirement in modeling applications. In this paper, I construct a life-cycle model of consumption, saving, and intensive labor supply in which the representative individual possesses a hyperbolic discount function. The model exhibits time-inconsistent dynamic optimization as the individual persistently formulates, breaks, and then re-formulates consumption, saving, and labor supply plans for the future, thus leading to lower levels of accumulated savings for retirement compared to what was planned originally. Because social security programs are commonly justified on grounds of helping to provide resources in retirement, I study the question of whether or not a revenue-neutral social security program that is financed via consumption taxation might improve well-being compared to the status quo of a payroll-tax financed program (like what is currently operated in most advanced economies). I find and report that the gains to well-being can be sizable for a large set of the parameter space that is studied, although notable cases exist in which welfare losses can be incurred.

Keywords: social security, hyperbolic discounting, naiveté, insufficient saving for retirement, time-inconsistent dynamic optimization

JEL Classification: C61, D15, D91, H55

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1 Introduction

The social security program in the United States (and in most other advanced economies) operates as a pay-as-you-go (PAYGO) or unfunded program that is currently financed with payroll taxation. In this paper, I re-visit the question of whether or not individuals would be better off if social security was financed alternatively with consumption taxation in place of payroll taxation. Specifically, I examine this question with a behavioral model in which individuals have time-inconsistent preferences as a result of possessing a hyperbolic discount function.

An existing literature examines the welfare effects of a consumption-tax financed social security program. Gahramanov and Tang (2013) find that a consumption tax-financed program results in increased welfare in an overlapping-generations general-equilibrium model with individuals who have exponential discount functions. One of the key outcomes in their model is that individuals face little incentive to alter their labor supply decision in a consumption-tax financed program compared to one that is financed with payroll taxation.

Heijdra and Ligthart (2002) study this same question in an overlapping-generations general-equilibrium model and find that a consumption-tax financed program is generally welfare-improving across much (but not all) of the studied parameter space. In addition, they report that the welfare gains accrue to younger (and unborn) generations, while the welfare of older generations is reduced in the model. A consumption-tax financed program hurts those who are seasoned workers or retired, and the welfare losses are most pronounced for those generations who are either retired or close to retiring. Related to this point, they report that a payroll-tax financed program is more likely to receive political support given that a consumption-tax financed program generates welfare gains to younger (and unborn) generations only.

Additional studies by Auerbach and Kotlikoff (1987), Fuster et al. (2008), Lehmus (2011), and others, report similar findings. Auerbach and Kotlikoff (1987) were the first to study this question in an overlapping-generations model, as opposed to a representative-agent infinite-horizon model. Fuster et al. (2008) study an overlapping-generations environment and model individuals who do and who do not care about their future posterity. They find that fiscal policy (including social security) that is financed via consumption taxation generates welfare gains if individuals do care about their future posterity, and they also report that consumption taxation triggers welfare losses if individuals care only about their own life cycle. Lehmus (2011) reports welfare gains from a consumption-tax financed program, compared to a payroll-tax financed program with parameters set to the social security program in Finland.

The phenomenon of hyperbolic discounting of future rewards has been documented in the fields of experimental psychology and behavioral economics as a robust behavioral trait that people appear to possess, as opposed to exponential discounting (Kirby and Maraković 1995; Myerson and Green 1995; Frederick et al. 2002; Rachlin 2006). Following the early findings of Strotz (1955), the use of
a hyperbolic discount function leads to preferences that are time-inconsistent, thus representing the idea that people can make plans and then fail to follow through with their plans. In modeling applications, hyperbolic discounting is often used synonymously to describe present-biased, impulsive behavior (O’Donoghue and Rabin 1999). In economic applications hyperbolic discounting has been applied as a superior modeling explanation (compared to exponential discounting) to many human behaviors like excessive borrowing early in life and insufficient retirement savings (Loewenstein and Prelec 1992; Laibson 1997; Angeletos et al. 2001; Tanaka and Murooka 2012).

Previous work concludes that a consumption-tax financed social security program typically results in higher welfare compared to a payroll-tax financed program. However, prior work has examined this question exclusively in modeling environments with exponential discounting. In contrast, I construct a life-cycle model of consumption, saving, and intensive labor supply in which the modeled individual possesses a hyperbolic discount function. Behavior exhibits time-inconsistent dynamic optimization as the individual repeatedly formulates, breaks, and then re-formulates consumption, saving, and labor supply plans for the future, thus leading to lower levels of accumulated savings for retirement compared to what was planned originally. I find that replacing a payroll-tax financed social security program with a revenue-neutral consumption-tax financed program can lead to sizable life-cycle welfare gains for such individuals over a large set of the parameter space that is studied, although notable cases exist in which welfare losses can be incurred.

2 Theoretical Model of Time-Inconsistent Decision Making

2.1 Environment

Age is a continuous variable that is indexed by $t$. The representative individual starts work at age $t = 0$, retires exogenously at age $t = T$, and passes away with certainty at age $t = T$. The individual has the possibility to earn a gross wage-income flow $w(t) = wq(t)$ for $t \in [0, T]$ where $w$ is the market wage and $q(t)$ is a hump-shaped longitudinal age-efficiency profile with the following form

$$q(t) = q_0 + q_1 t + q_2 t^2, \text{ for } t \in [0, T],$$

which is a quadratic polynomial. The profile is parameterized with

$$q_0 = 1,$$
$$q_1 = 0.0315,$$
$$q_2 = -0.00062,$$

in order to be consistent with the salient properties of the life-cycle income profile from Gourinchas and Parker (2002). These parameter values indicate that the quadratic wage-income profile will
peak at age 50 (which is \( t = 25 \) in our model since \( t = 0 \) corresponds to age 25) and that the ratio of peak wage-income to initial wage-income is 1.4, as in the Gourinchas and Parker (2002) sample. The private savings asset, \( k(t) \), grows at a constant rate of interest, \( r \). The individual receives social security benefits, \( b \) for \( t \in [T, \bar{T}] \), which can be financed with a tax on labor supply (payroll tax) at rate \( \theta_w \) for \( t \in [0, T] \) or with a tax on consumption at rate \( \theta_c \) for \( t \in [0, T] \). Social security benefits are financed on a pay-as-you-go basis. The representative individual has perfect foresight over the entire life cycle, passes away with no assets, and constructs a consumption program, \( c(t) \) for \( t \in [0, \bar{T}] \), and intensive labor supply program, \( l(t) \) for \( t \in [0, \bar{T}] \), that maximizes discounted lifetime utility, recognizing that \( w(t)l(t) \) is labor-income earnings which are endogenous to the model. As such, both labor taxation and consumption taxation are distortionary in the model. Regarding the discounting of utility in decision problems, \( F(x) = (1 + \beta x)^{-1} \) is the standard hyperbolic discount function for a discounting delay of length \( x \), following the experimental-observational findings of Mazur (1987).

### 2.2 Time-Inconsistent Dynamic Optimization during Working Phase

From the perspective of any planning instant, \( t_0 \in [0, T] \), the representative individual solves

\[
\max_{\{c(t), l(t)\}} \left\{ \int_{t_0}^{T} F(t - t_0) \left[ \ln c(t) + \gamma \ln(1 - l(t)) \right] \, dt + \int_{T}^{\bar{T}} F(t - t_0) \ln c(t) \, dt \right\},
\]

subject to

\[
\frac{dk(t)}{dt} = rk(t) + (1 - \theta_w)w(t)l(t) - (1 + \theta_c)c(t), \quad \text{for } t \in [t_0, T],
\]

\[
\frac{dk(t)}{dt} = rk(t) + b - (1 + \theta_c)c(t), \quad \text{for } t \in [T, \bar{T}],
\]

\[
k(t_0) \text{ given},
\]

\[
k(\bar{T}) = 0,
\]

where \( \gamma \) is the weight on leisure utility in the instantaneous utility function, relative to a weight of 1 for consumption utility.

The dynamic optimization problem expressed by equations (5)–(9) can be solved via the Maximum Principle for two-stage optimal control problems (due to a switch in the objective functional and in the laws of motion) with two control variables and with a fixed-endpoint condition.\(^1\) The planned consumption path (that the individual intends to follow) and the planned labor supply

\(^1\)See Tomiyama (1985) for the technical details of two-stage optimal control problems.
path (that the individual intends to follow) are given as

\[
\hat{c}(t; t_0) = \frac{k(t_0)e^{-r_{t_0}} + \int_{t_0}^{T} (1 - \theta_w)w(v)e^{-rv} \, dv + \int_{T}^{T} be^{-rv} \, dv}{\int_{t_0}^{T} F(v - t_0) \, dv + \gamma \int_{t_0}^{T} F(v - t_0) \, dv} \frac{F(t - t_0)e^{rt}}{1 + \theta_c}, \quad \text{for } t \in [t_0, T],
\]

(10)

\[
\hat{l}(t; t_0) = 1 - \frac{k(t_0)e^{-r_{t_0}} + \int_{t_0}^{T} (1 - \theta_w)w(v)e^{-rv} \, dv + \int_{T}^{T} be^{-rv} \, dv}{\int_{t_0}^{T} F(v - t_0) \, dv + \gamma \int_{t_0}^{T} F(v - t_0) \, dv} \frac{\gamma F(t - t_0)e^{rt}}{(1 - \theta_w)w(t)}, \quad \text{for } t \in [t_0, T],
\]

(11)

from the perspective of any planning instant \( t_0 \in [0, T] \) during the working phase where \( v \) is a dummy variable of integration. Note that the optimal controls are contingent on the current state, \( k(t_0) \), which is dependent on the entire past history of behavior on the interval \([0, t_0]\). The derivation of these optimal controls is outlined in the Appendix.

A key characteristic of the model is that the representative individual is allowed to re-visit their plans and re-optimize at every instant on account that the individual represented here is assumed to be naive (irrational), meaning that they do not anticipate the fact that they will engage in time-inconsistent decision making. More specifically, the individual naively believes that their future selves will follow through with the optimal controls denoted by (10)–(11), being unaware of their future propensity to deviate from the plans given the hyperbolic discount function. As such, the individual’s actual consumption and actual labor supply will coincide with their planned consumption and planned labor supply only at the instant when the plans are formulated, \( t_0 \). The actual consumption and actual labor supply amounts at the planning instant \( t = t_0 \) can therefore be found by replacing all \( t \) in (10) and (11) with \( t_0 \). This yields equations (12)–(13),

\[
c(t_0; t_0) = \frac{k(t_0)e^{-r_{t_0}} + \int_{t_0}^{T} (1 - \theta_w)w(v)e^{-rv} \, dv + \int_{T}^{T} be^{-rv} \, dv}{\int_{t_0}^{T} F(v - t_0) \, dv + \gamma \int_{t_0}^{T} F(v - t_0) \, dv} e^{r_{t_0}} \frac{1}{1 + \theta_c}, \quad (12)
\]

\[
l(t_0; t_0) = 1 - \frac{k(t_0)e^{-r_{t_0}} + \int_{t_0}^{T} (1 - \theta_w)w(v)e^{-rv} \, dv + \int_{T}^{T} be^{-rv} \, dv}{\int_{t_0}^{T} F(v - t_0) \, dv + \gamma \int_{t_0}^{T} F(v - t_0) \, dv} \frac{\gamma e^{r_{t_0}}}{(1 - \theta_w)w(t_0)}, \quad (13)
\]

noting that \( F(0) = 1 \), given that there is no discounting over a discounting delay of length 0.

Since the model is constructed in continuous time, every point in time is itself a planning instant. Therefore, all \( t_0 \) can be replaced with \( t \) in (12) and (13) to acquire the actual consumption path
and the actual labor supply path, both as functions of actual savings, $k(t)$. These actual time paths during the working phase are expressed as

$$c(t) = \left( \frac{k(t)e^{-rt} + \int_t^T (1 - \theta_w)w(v)e^{-rv} \, dv + \int_t^T be^{-rv} \, dv}{\int_t^T F(v-t) \, dv + \gamma \int_t^T F(v-t) \, dv} \right) \frac{e^{rt}}{1 + \theta_c}, \quad \text{for} \ t \in [0, T], \quad (14)$$

$$l(t) = 1 - \left( \frac{k(t)e^{-rt} + \int_t^T (1 - \theta_w)w(v)e^{-rv} \, dv + \int_t^T be^{-rv} \, dv}{\int_t^T F(v-t) \, dv + \gamma \int_t^T F(v-t) \, dv} \right) \frac{\gamma e^{rt}}{(1 - \theta_w)w(t)}, \quad \text{for} \ t \in [0, T], \quad (15)$$

where $k(t)$ is the actual savings asset balance at any date $t$ in the working phase of the life cycle. It important to recognize that (14) and (15) are the envelopes of a continuum of initial planned consumption values and labor supply values, respectively. These expressions are implicit functions of the actual savings asset at time $t$, which in turn is a function of the time path of actual consumption and actual labor supply behavior on the past interval $[0, t]$ via

$$\frac{dk(t)}{dt} = rk(t) + (1 - \theta_w)w(t)l(t) - (1 + \theta_c)c(t). \quad (16)$$

Therefore, to completely identify the actual consumption path and actual labor supply path of the representative individual at each and every age $t \in [0, T]$, the system of equations needs to be solved: (14), (15), and (16) coupled with the initial condition $k(0) = 0$. Solving this system of equations characterizes the actual consumption, saving, and labor supply behavior of the representative individual. In practice, discretized-differential approximation is used to calculate the time path of $k(t)$. This is accomplished by transforming equation (16) into differential form such that exact or true changes in the savings asset path, $\Delta k(t)$, will approximately follow

$$dk(t) = [rk(t) + (1 - \theta_w)w(t)l(t) - (1 + \theta_c)c(t)]dt \quad (17)$$

given a discrete change in time of $dt = \Delta t$ in advancing from one period of the life cycle to the next. This therefore implies that the savings asset path will follow

$$k(t + dt) = k(t) + \Delta k(t) \approx k(t) + dk(t) = k(t) + [rk(t) + (1 - \theta_w)w(t)l(t) - (1 + \theta_c)c(t)]dt, \quad \text{for} \ t \in [0, T], \quad (18)$$
given the condition $k(0) = 0$. Note that the approximation becomes more precise as $dt = \Delta t \to 0$. 

6
2.3 Time-Inconsistent Dynamic Optimization during Retirement Phase

From the perspective of any planning instant, \( t_0 \in [T, \bar{T}] \), the representative individual solves

\[
\max_{\{c(t)\}} \left\{ \int_{t_0}^{T} F(t - t_0) \ln c(t) \, dt \right\}, \tag{19}
\]

subject to

\[
\frac{dk(t)}{dt} = rk(t) + b - (1 + \theta_c)c(t), \quad \text{for } t \in [T, \bar{T}], \tag{20}
\]

\[k(t_0) \text{ given},\tag{21}\]

\[k(\bar{T}) = 0, \tag{22}\]

where \( l(t) = 0 \) for all \( t \in [T, \bar{T}] \) by assumption of retirement.

Application of the Maximum Principle for optimal control problems with a fixed-endpoint condition yields the planned consumption path that the individual intends to follow,

\[
\hat{c}(t; t_0) = \left( \frac{k(t_0)e^{-rt_0} + \int_{t_0}^{T} be^{-rv} \, dv}{\int_{t_0}^{T} F(v - t_0) \, dv} \right) \frac{F(t - t_0)e^{rt}}{1 + \theta_c}, \quad \text{for } t \in [t_0, T], \tag{23}
\]

from the perspective of any planning instant \( t_0 \in [T, \bar{T}] \) during the retirement phase. Note again that the optimal control is contingent on the current state, \( k(t_0) \), which is dependent on the entire past history of behavior on the interval \([0, t_0]\). The derivation of the optimal control for the retirement phase is also outlined in the Appendix.

As outlined above, the representative individual is allowed to re-visit their plans and re-optimize at every instant. Therefore, the individual’s actual consumption during retirement coincides with their planned consumption only at the instant when a plan is formulated, \( t_0 \). The actual consumption amount at the planning instant \( t = t_0 \) is therefore found by replacing all \( t \) in (23) with \( t_0 \). This yields equation (24),

\[
c(t_0; t_0) = \left( \frac{k(t_0)e^{-rt_0} + \int_{t_0}^{T} be^{-rv} \, dv}{\int_{t_0}^{T} F(v - t_0) \, dv} \right) e^{rt_0}, \tag{24}
\]

remembering again that \( F(0) = 1 \). And since every point in time is itself a planning instant, all \( t_0 \) can be replaced with \( t \) in (24) to identify the actual consumption path as a function of actual
savings, \( k(t) \). The actual time path during the retirement phase is expressed as

\[
c(t) = \left( \frac{k(t)e^{-rt} + \int_{t}^{\bar{T}} be^{-rv} \, dv}{\int_{t}^{T} F(v - t) \, dv} \right) \frac{e^{rt}}{1 + \theta_c}, \quad \text{for } t \in [T, \bar{T}],
\]

where \( k(t) \) is the actual savings asset balance at any date \( t \) in the retirement phase of the life cycle. It is important to recognize that (25) is the envelope of a continuum of initial planned consumption values, and it is an implicit function of the actual savings asset at time \( t \), which in turn is a function of the entire history of actual consumption (and actual labor supply while working) on the interval \([0, t]\) via

\[
\frac{dk(t)}{dt} = rk(t) + b - (1 + \theta_c)c(t).
\]

Similar to the procedure outlined above, to identify the actual consumption path of the representative individual at each age \( t \in [T, \bar{T}] \), the system of equations needs to be solved: (25) and (26) coupled with the initial condition \( k(T) \) given, which is determined by the history of all prior behavior. Solving this system of equations characterizes the actual consumption and saving behavior of the representative individual during the retirement phase. Discretized-differential approximation is employed again to calculate the time path of \( k(t) \). Similar to what was outlined above, this is accomplished by transforming equation (26) into differential form such that exact or true changes in the savings asset path, \( \Delta k(t) \), will approximately follow

\[
dk(t) = [rk(t) + b - (1 + \theta_c)c(t)] dt
\]

given a discrete change in time of \( dt = \Delta t \) in advancing from one period of the life cycle to the next. This therefore implies that the savings asset path will follow

\[
k(t + dt) = k(t) + \Delta k(t) \\
\approx k(t) + dk(t) \\
= k(t) + [rk(t) + b - (1 + \theta_c)c(t)] dt, \quad \text{for } t \in [T, \bar{T}],
\]

where the condition \( k(T) \) is determined by all prior behavior. Note again that the approximation becomes more precise as \( dt = \Delta t \to 0 \).
3 Numerical Simulation

3.1 Parameterization

In this section quantitative simulation results from the theoretical model with time-inconsistent dynamic optimization are reported and discussed. Unless stated otherwise, the following parameter values are selected and used in numerical exercises. The dates of retirement and death are $T = 40$ and $\bar{T} = 55$ respectively. These dates are selected to represent an individual who starts work at age 25, retires at age 65, and passes away at age 80. The parameter values of the quadratic age-efficiency profile, expressed as equations (2)–(4), are used to match the characteristics of the life-cycle income profile reported in Gourinchas and Parker (2002). Given that labor supply, and therefore, labor earnings are endogenous, the weight on leisure utility is selected as $\gamma = 1.185$, following McGrattan and Prescott (2017, 2018). The life cycle is simulated at a grid step-size of $dt = 0.1$, corresponding to a step-size of one-tenth of a year. In all quantitative experiments in which the social security program in the model is financed via labor-earnings taxation (payroll taxation), the tax rate is set to $\theta_w = 0.106$, reflecting the combined employer-employee OASI payroll tax rate in the United States. As a result, the pension benefit received by the individual in the model replaces approximately 28 percent of average labor earnings over the course of the simulated life cycle, which is a little lower than (but still in the approximate neighborhood of) the replacement rate for the average household in the United States. Sensitivity analysis is performed on the following key parameters: $r$, $\beta$, and $\gamma$.

3.2 Financing of Pay-As-You-Go Social Security

Social security benefits received during retirement take the following form if the program is financed via taxation on labor-earnings (payroll taxation),

$$ b_w = \frac{\int_0^T \theta_w w(t) l(t) \, dt}{T - \bar{T}}. $$

(29)

The numerator of (29) is total payroll taxes collected across all workers, where $l(t)$ for $t \in [0, T]$ is given by (15), while the denominator is the total number of retirees. Alternatively, if the social security program is financed via consumption taxation, then social security benefits received during retirement take the form,

$$ b_c = \frac{\int_0^T \theta_c c(t) \, dt + \int_{T}^{\bar{T}} \theta_c c(t) \, dt}{T - \bar{T}}. $$

(30)
The numerator of (30) is total consumption taxes collected across all workers and across all retirees, where $c(t)$ for $t \in [0, T]$ is given by (14) and $c(t)$ for $t \in [T, \bar{T}]$ is given by (25). Lastly, the consumption tax rate, $\theta_c$, is calibrated to always satisfy the following condition in all quantitative experiments,

$$
\theta_c = \arg \min \left\{ \left( \int_0^T \theta_w w(t) l(t) \, dt - \left[ \int_0^T \theta_w c(t) \, dt + \int_T^T \theta_w c(t) \, dt \right] \right)^2 \right\},
$$

subject to $\theta_w = 0.106$, thus preserving revenue neutrality in all quantitative comparisons across alternatively financed social security programs.

### 3.3 Numerical Results and Implications

The welfare effects of replacing the current payroll-tax financed social security program with another, equally sized, consumption-tax financed program are examined. In all quantitative experiments, time paths for $w(t)$, $w(t)l(t)$, $k(t)$, $l(t)$, and $c(t)$ are simulated. The time path for $k(t)$ is approximated using discretized-differential approximation, whereas the derived solutions of $l(t)$ for $t \in [0, T]$, of $c(t)$ for $t \in [0, T]$, and of $c(t)$ for $t \in [T, \bar{T}]$ are employed. Because social security tax revenues are collected from the endogenous decisions of labor supply and consumption, social security benefits are therefore determined endogenously in the model on account that labor supply decisions and consumption decisions are formulated taking social security benefits as given. To model the endogenous determination of social security benefits, a guess and feedback approach is used to identify a given level of social security benefits that satisfies behavioral rules and model parameters.

To help describe and explain the behavior of a hyperbolic discounter, Figures 1, 2, and 3 depict the planned and actual consumption paths, labor supply paths, and savings asset paths over the life cycle of a representative time-inconsistent individual. As depicted with the blue dashed line in each of these figures, the individual’s initial plan is to reduce consumption for the next several years in order to experience significantly increased consumption after retirement. The individual simultaneously plans, when young, to increase their labor supply for the next several years in order to allow for decreased labor supply later during the working years as they get closer to retirement, in addition to enjoying higher levels of consumption later in life as a result of having worked more intensively when younger. Such consumption and labor supply plans lead to the initial planned savings asset balance path depicted by the blue dashed line in Figure 3.

However, the black solid lines in Figures 1, 2, and 3, depict the actual choices of the individual, noting that the green solid line in Figure 1 depicts actual labor income earnings as a result of the actual labor supply choices of the individual. Early on, the individual consumes more and works less...
than what they had originally planned, thus leading to lower savings than what they had planned to accumulate for retirement. With deviations from planned behavior, the individual continually re-optimizes and makes new labor supply plans and consumption plans. A select number of these plans are also depicted in these figures, with the plan from the perspective of \( t_0 = 10 \) shown in dashed gray, the perspective of \( t_0 = 20 \) shown in dashed yellow, and the perspective of \( t_0 = 30 \) shown in dashed red. This is because the individual is present-biased and therefore prioritizes current consumption and current leisure, at the expense of future consumption and future leisure, thus preferring to consume more and work less early in life even though they know that it would benefit them later in life to have worked and have saved more.

For each unique ordered triplet over the values for \( r, \beta, \) and \( \gamma \) within the examined parameter space, the individual’s actual labor supply path and consumption path are used to calculate the total tax revenue generated from a payroll-tax financed social security program with a payroll tax rate of \( \theta_w = 0.106 \). The consumption tax rate, \( \theta_c \), is then calibrated according to equation (31) so that the actual labor supply and consumption choices of an otherwise identical individual generate an equivalent amount of tax revenue to finance social security benefits via a consumption-tax financed program that is counterfactual. This exercise is performed for each unique ordered triplet value for \( r, \beta, \) and \( \gamma \) in order to preserve revenue neutrality, thus ensuring a fair comparison in all relative welfare calculations that result from participation in a payroll-tax financed social security program versus a consumption-tax financed program.

To measure welfare changes that could result, in theory, from participating in a consumption-tax financed social security program relative to a payroll-tax financed program, lifetime well-being or welfare is measured from the perspective of a benevolent social planner who is paternalistic. Welfare is defined as the experienced lifetime utility of an individual, and consumption utility flows and leisure utility flows are discounted exponentially from the perspective of a social planner (zero social discounting is also examined for purposes of sensitivity analysis).\(^2\) Consistent with this specification of welfare, a consumption-equivalent compensating variation, \( \Psi \), is used to measure the effects on welfare that might result from participating in a consumption-tax financed social security program, relative to a program that is financed with payroll taxation. The compensating variation, \( \Psi \), measured as the percentage change in lifetime consumption, solves the following equation,

\[
\int_0^T e^{-\rho t} \left[ \ln((1 + \Psi)c(t; \theta_w, 0)) + \gamma \ln(1 - l(t; \theta_w, 0)) \right] dt + \int_T^\infty e^{-\rho t} \ln((1 + \Psi)c(t; \theta_w, 0)) dt \\
= \int_0^T e^{-\rho t} \left[ \ln c(t; 0, \theta_c) + \gamma \ln(1 - l(t; 0, \theta_c)) \right] dt + \int_T^\infty e^{-\rho t} \ln c(t; 0, \theta_c) dt. \tag{32}
\]

This measure calculates the percentage amount of consumption by which a benevolent social planner

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\(^2\)See Samuelson (1975), Harsanyi (1977), and Akerlof (2002) regarding welfare if individuals are not rational.
would need to add or subtract from the lifetime consumption of an individual who participates in a payroll-tax financed social security program, in order to equalize their lifetime well-being to that of an otherwise identical individual who participates in a consumption-tax financed program that is counterfactual. Instances in which the compensating variation is negative in value reflect the idea that an individual would experience damage to lifetime well-being as a result of being able to hypothetically participate in a consumption-tax financed social security program, and instances in which the compensating variation is positive in value capture the idea that welfare would be improved via hypothetical participation in a consumption-tax financed program.

As an illustrative example, the behaviors of otherwise identical individuals who participate in each of the two alternative tax-financed social security programs are depicted in Figures 4 and 5 for the ordered triplet \( r = 0.05, \beta = 0.08, \) and \( \gamma = 1.185 \). Figure 4 depicts that while labor supply, and therefore labor-income earnings, are nearly identical across payroll-tax financed and consumption-tax financed programs, the individual consumes less throughout their life cycle under a payroll-tax financed program compared to participating in a consumption-tax financed program. Figure 5 depicts the savings asset balance over the life cycle across each of the two alternatively financed programs. The savings asset balance is slightly higher at the beginning of the life cycle under a payroll-tax financed program, but participation in the consumption-tax financed program leads to choices that result in a higher total savings asset balance at retirement. At these parameter values (and also assuming that the hypothetical social planner discounts at the rate \( \rho = r = 0.05 \)), the compensating variation is \( \Psi = 1.72\% \) which indicates that an individual would enjoy 1.72 percent higher lifetime consumption if they were able to participate in a consumption-tax financed social security program, relative to a payroll-tax financed program.

The main quantitative findings of this paper are reported in Tables 1, 2, and 3, noting that the top panel of each table differs from the bottom panel with respect to \( \rho = r \) compared to \( \rho = 0 \). Measurements of welfare with \( \rho = r \) imply that the social planner prefers a smooth age-consumption profile as the ideal consumption allocation over the life cycle. In contrast, measurements of welfare with \( \rho = 0 \) imply that the social planner does not discriminate across the different age-dated selves of an individual.\textsuperscript{3} Table 1 reports the calibrated revenue-neutral consumption tax rate and the corresponding compensating variation over a sizable range of values for \( r \) and \( \beta \), given a weight on leisure utility of \( \gamma = 1.185 \), following McGrattan and Prescott (2017, 2018). Parameterizations in which the modeled individual violates the assumptions of the model regarding feasible labor supply behavior are disregarded and denoted with the symbol, †.

What can be seen from Table 1 is that higher values of the interest rate and lower values of the hyperbolic discount function parameter both result in higher welfare gains for the individual in a consumption-tax financed program, compared to a payroll-tax financed program. It makes sense

\textsuperscript{3}A social discount rate of \( \rho = 0 \) is consistent with the criterion of Ramsey (1928) in which non-zero social discounting based on pure time preference is classified as unethical.
that a higher value of the interest rate and/or a lower value of the discount function parameter would provide the individual with incentives to save more, thus improving their savings asset balance in order to consume later in life. For example, looking at Table 1 corresponding to the column of \( \beta = 0.10 \), an individual makes time-inconsistent labor supply and consumption choices with the same degree of time inconsistency, while the interest rate is varied. At lower values of the interest rate, say at \( r = 0.03 \) or \( r = 0.04 \), the amounts that they have saved do not grow as quickly, leaving less savings available to enable consumption later in life that could be taxed, which leads to higher calibrated values of the revenue-neutral consumption tax rate. For example, this can be seen for the triplet of \( r = 0.04 \) (with \( \beta = 0.10 \) and \( \gamma = 1.185 \)) in which the calibrated revenue-neutral consumption tax rate is \( \theta_c = 0.1278 \) with a corresponding compensating variation of \( \Psi = -1.23\% \).

Higher interest rates result in the opposite, increasing total savings to the point that the calibrated revenue-neutral consumption tax rate is much lower than the payroll tax rate of \( \theta_w = 0.106 \). For example, this can be seen for the triplet of \( r = 0.07 \) (with \( \beta = 0.10 \) and \( \gamma = 1.185 \)) in which the calibrated revenue-neutral consumption tax rate is only \( \theta_c = 0.0723 \) with a corresponding compensating variation of \( \Psi = 4.17\% \).

Quantitative findings follow the same pattern when examining the parameter space in other dimensions. At lower values of the interest rate, the degree by which an individual hyperbolically discounts the future is critical to determining if a consumption-tax financed social security program is welfare-improving or not. For example, at the triplet \( \beta = 0.07 \) (with \( r = 0.04 \) and \( \gamma = 1.185 \)) the calibrated revenue-neutral consumption tax rate is \( \theta_c = 0.1051 \) with a corresponding compensating variation of \( \Psi = 0.82\% \), whereas at the triplet \( \beta = 0.10 \) (with \( r = 0.04 \) and \( \gamma = 1.185 \)) the calibrated revenue-neutral consumption tax rate is \( \theta_c = 0.1278 \) with a corresponding compensating variation of \( \Psi = -1.23\% \).

Comprehensive results of sensitivity analysis are reported in Tables 1, 2, and 3. Tables 2 and 3 report sensitivity analysis with respect to the utility weight on leisure, \( \gamma \). Specifically, Table 2 reports findings for a value of \( \gamma = 2 \) (reflecting a higher preference for leisure compared to what is reported in Table 1), and Table 3 reports findings for a value of \( \gamma = 0.5 \) (reflecting a lower preference for leisure compared to what is reported in Table 1). Lastly, as can be seen in the bottom panels of Tables 1, 2, and 3, the role of zero social discounting on the measured welfare effects is minimal as a result of participation in a consumption-tax financed social security program relative to a payroll-tax financed program.

For illustrative purposes, Figures 6, 7, and 8 representatively depict a common feature as to when participation in a consumption-tax financed social security program is welfare-improving and when it is not. Higher degrees of time inconsistency (i.e., higher values of \( \beta \)), that lead to higher levels of accumulated debt over the life cycle accompanied with lower levels of spending in retirement, correspond generally to cases where participation in a consumption-tax financed social security program is NOT welfare-improving. The blue lines in Figures 6, 7, and 8 correspond to a
lower degree of time inconsistency ($\beta = 0.09$ given $r = 0.05$ and $\gamma = 1.185$) generating a welfare improvement from participation in a consumption-tax financed program ($\Psi = 0.94\%$), as compared to the red lines that correspond to a higher degree of time inconsistency ($\beta = 0.12$ given $r = 0.05$ and $\gamma = 1.185$) generating a welfare loss from participation ($\Psi = -1.36\%$).

4 Concluding Remarks

Previous work reports that a consumption-tax financed social security program provides welfare gains compared to a payroll-tax financed program, if individuals are modeled to possess an exponential discount function. I construct a partial-equilibrium model with individuals who have time-inconsistent preferences as a result of possessing a hyperbolic discount function, and I find that a consumption-tax financed social security program can lead to welfare gains, although this finding is sensitive to select parameters of the model. In future work, it would be interesting to examine how these findings might change as a result of adding retirement choice and/or lifetime uncertainty to my modeling framework. Also, it would be interesting to examine how the findings adjust if my model was placed into a general-equilibrium setting. A general-equilibrium setting would capture additional welfare effects via market price adjustments as a result of reforming to a consumption-tax financed program. Specifically, it is very possible that reform of a social security program that is financed via alternative distortionary tax instruments can lead to changes in equilibrium objects, such as aggregate labor supply, aggregate physical capital, and aggregate income.

Appendix

Derivations for Working Phase

Solving the time-inconsistent dynamic optimization problem specified by equations (5)–(9), involves using the Maximum Principle for two-stage optimal control problems with a fixed-endpoint condition (Tomiyama 1985). From the perspective of any planning instant $t_0 \in [0, T]$ during the working phase, the Hamiltonians and optimality conditions are the following,

\[
H_1(t) = F(t - t_0) \ln c(t) + F(t - t_0)\gamma \ln(1 - l(t)) \\
+ \lambda_1(t) \left[ r k(t) + (1 - \theta_w) w(t) l(t) - (1 + \theta_c) c(t) \right], \quad \text{for } t \in [t_0, T], \quad (A1)
\]

\[
H_2(t) = F(t - t_0) \ln c(t) + \lambda_2(t) \left[ r k(t) + b - (1 + \theta_c) c(t) \right], \quad \text{for } t \in [T, \bar{T}], \quad (A2)
\]
\[
\frac{\partial H_1(t)}{\partial c(t)} = \frac{F(t-t_0)}{c(t)} - (1+\theta_c)\lambda_1(t) = 0, \quad \text{for} \ t \in [t_0, T], \quad (A3)
\]

\[
\frac{\partial H_1(t)}{\partial l(t)} = -\frac{F(t-t_0)\gamma}{(1-l(t))} + \lambda_1(t)(1-\theta_w)w(t) = 0, \quad \text{for} \ t \in [t_0, T], \quad (A4)
\]

\[
\frac{\partial H_1(t)}{\partial k(t)} = r\lambda_1(t) \overset{\text{set}}{=} -\frac{d\lambda_1(t)}{dt}, \quad \text{for} \ t \in [t_0, T], \quad (A5)
\]

\[
\frac{\partial H_1(t)}{\partial \lambda_1(t)} = r(k(t) + (1-\theta_w)w(t)l(t) - (1+\theta_c)c(t) \overset{\text{set}}{=} \frac{dk(t)}{dt}, \quad \text{for} \ t \in [t_0, T], \quad (A6)
\]

\[
\frac{\partial H_2(t)}{\partial c(t)} = \frac{F(t-t_0)}{c(t)} - (1+\theta_c)\lambda_2(t) = 0, \quad \text{for} \ t \in [T, \bar{T}], \quad (A7)
\]

\[
\frac{\partial H_2(t)}{\partial k(t)} = r\lambda_2(t) \overset{\text{set}}{=} -\frac{d\lambda_2(t)}{dt}, \quad \text{for} \ t \in [T, \bar{T}], \quad (A8)
\]

\[
\frac{\partial H_2(t)}{\partial \lambda_2(t)} = r(k(t) + b - (1+\theta_c)c(t) \overset{\text{set}}{=} \frac{dk(t)}{dt}, \quad \text{for} \ t \in [T, \bar{T}], \quad (A9)
\]

\[
\lambda_1(T) = \lambda_2(T). \quad (A10)
\]

Solving (A5) and (A8) yields,

\[
\lambda_1(t) = a_1e^{-rt}, \quad \text{for} \ t \in [t_0, T], \quad (A11)
\]

\[
\lambda_2(t) = a_2e^{-rt}, \quad \text{for} \ t \in [T, \bar{T}]. \quad (A12)
\]

Using the switchpoint condition, (A10), yields

\[
a_1e^{-rT} = a_2e^{-rT}, \quad (A13)
\]

meaning that

\[
a_1 = a_2 \rightarrow a, \quad \text{for} \ t \in [t_0, \bar{T}]. \quad (A14)
\]

Thus, (A11) and (A12) can be combined

\[
\lambda(t) = ae^{-rt}, \quad \text{for} \ t \in [t_0, \bar{T}], \quad (A15)
\]

which means that (A3) and (A7) can also be combined

\[
\frac{F(t-t_0)}{c(t)} - (1+\theta_c)\lambda(t) = 0, \quad \text{for} \ t \in [t_0, \bar{T}]. \quad (A16)
\]
With substitution and rearrangement

\[ c(t) = \frac{F(t - t_0)}{(1 + \theta_c)ae^{-rt}}, \quad \text{for } t \in [t_0, \bar{T}]. \quad (A17) \]

For convenience, (A17) can be rewritten further, given \( \frac{1}{a} \equiv z, \)

\[ c(t) = z \left( \frac{F(t - t_0)e^{rt}}{1 + \theta_c} \right), \quad \text{for } t \in [t_0, \bar{T}]. \quad (A18) \]

Rewriting (A4) with substitution yields

\[ l(t) = 1 - z \left( \frac{\gamma F(t - t_0)e^{rt}}{(1 - \theta_w) w(t)} \right), \quad \text{for } t \in [t_0, T]. \quad (A19) \]

Solving the state equation (A6) yields the general solution,

\[ k(t) = e^{rt} \left\{ q_1 + \int_{t_0}^{t} [(1 - \theta_w) w(v)l(v) - (1 + \theta_c) c(v)] e^{-rv} dv \right\}, \quad \text{for } t \in [t_0, T], \quad (A20) \]

where \( v \) is a dummy variable of integration. Using (8) the initial condition, \( k(t_0) \) given, identifies the constant of integration,

\[ q_1 = k(t_0) e^{-r t_0} - \int_{t_0}^{t} [(1 - \theta_w) w(v) l(v) - (1 + \theta_c) c(v)] e^{-rv} dv, \quad (A21) \]

which yields the particular or definite solution,

\[ k(t) = e^{rt} \left\{ k(t_0) e^{-r t_0} + \int_{t_0}^{t} [(1 - \theta_w) w(v) l(v) - (1 + \theta_c) c(v)] e^{-rv} dv \right\}, \quad \text{for } t \in [t_0, T]. \quad (A22) \]

Solving the state equation (A9) yields the general solution,

\[ k(t) = e^{rt} \left\{ q_2 + \int_{t_0}^{t} \left[ b - (1 + \theta_c) c(v) \right] e^{-rv} dv \right\}, \quad \text{for } t \in [T, \bar{T}]. \quad (A23) \]

Using (9) the fixed-endpoint condition, \( k(\bar{T}) = 0, \) identifies the constant of integration,

\[ q_2 = - \int_{T}^{\bar{T}} \left[ b - (1 + \theta_c) c(v) \right] e^{-rv} dv, \quad (A24) \]
which yields the particular or definite solution,

\[ k(t) = e^{rt} \int_T^t \left[ b - (1 + \theta_c) c(v) \right] e^{-rv} \, dv, \quad \text{for } t \in [T, \bar{T}]. \tag{A25} \]

Given the identity \( k(T) = k(T) \), evaluate both (A22) and (A25) at \( t = T \) and then equate,

\[ e^{rT} \left\{ k(t) e^{-rt_0} + \int_t^T \left[ (1 - \theta_w) w(v) l(v) - (1 + \theta_c) c(v) \right] e^{-rv} \, dv \right\} = e^{rT} \int_T^T \left[ b - (1 + \theta_c) c(v) \right] e^{-rv} \, dv. \tag{A26} \]

Making substitutions from (A18) and (A19) and rearranging yields

\[ k(t_0) e^{-rt_0} + \int_{t_0}^T (1 - \theta_w) w(v) \left[ 1 - z \left( \frac{\gamma F(v - t_0) e^{rv}}{1 - \theta_w} \right) \right] e^{-rv} \, dv + \int_T^T b e^{-rv} \, dv \]

\[ = \int_{t_0}^T (1 + \theta_c) \left[ z \left( \frac{F(v - t_0) e^{rv}}{1 + \theta_c} \right) \right] e^{-rv} \, dv. \tag{A27} \]

Solving for the unknown constant of integration \( z \) yields,

\[ z = \frac{k(t_0) e^{-rt_0} + \int_{t_0}^T (1 - \theta_w) w(v) e^{-rv} \, dv + \int_T^T b e^{-rv} \, dv}{\int_{t_0}^T F(v - t_0) \, dv + \gamma \int_{t_0}^T F(v - t_0) \, dv}. \tag{A28} \]

Inserting (A28) into (A18) and (A19) produces equations (10) and (11), the solutions to the time-inconsistent dynamic optimization problem during the working phase.

**Derivations for Retirement Phase**

Solving the time-inconsistent dynamic optimization problem specified by equations (19)–(22), involves using the Maximum Principle for one-stage optimal control problems with a fixed-endpoint condition. From the perspective of any planning instant \( t_0 \in [T, \bar{T}] \) during the retirement phase, the Hamiltonian and optimality conditions are the following,

\[ H_3(t) = F(t - t_0) \ln c(t) + \lambda_3(t) \left[ r k(t) + b - (1 + \theta_c) c(t) \right], \quad \text{for } t \in [t_0, \bar{T}], \tag{A29} \]

\[ \frac{\partial H_3(t)}{\partial c(t)} = \frac{F(t - t_0)}{c(t)} - (1 + \theta_c) \lambda_3(t) \overset{\text{set}}{=} 0, \quad \text{for } t \in [t_0, \bar{T}], \tag{A30} \]

\[ \frac{\partial H_3(t)}{\partial k(t)} = r \lambda_3(t) \overset{\text{set}}{=} - \frac{d \lambda_3(t)}{dt}, \quad \text{for } t \in [t_0, \bar{T}], \tag{A31} \]
\[ \frac{\partial H_3(t)}{\partial \lambda_3(t)} = rk(t) + b - (1 + \theta_c) c(t) \equiv \frac{dk(t)}{dt}, \quad \text{for } t \in [t_0, \bar{T}]. \tag{A32} \]

Solving (A31) yields,
\[ \lambda_3(t) = a_3 e^{-rt}, \quad \text{for } t \in [t_0, \bar{T}]. \tag{A33} \]

Substitute into (A30) and rearrange algebraically,
\[ c(t) = \frac{1}{a_3} \left( \frac{F(t - t_0)e^{rt}}{1 + \theta_c} \right), \quad \text{for } t \in [t_0, \bar{T}]. \tag{A34} \]

For convenience, (A34) can be rewritten further, given \( \frac{1}{a_3} \equiv m, \)
\[ c(t) = m \left( \frac{F(t - t_0)e^{rt}}{1 + \theta_c} \right), \quad \text{for } t \in [t_0, \bar{T}]. \tag{A35} \]

Solving the state equation (A32) yields the general solution,
\[ k(t) = e^{rt} \left\{ q_3 + \int_{t_0}^{t} [b - (1 + \theta_c) c(v)] e^{-rv} dv \right\}, \quad \text{for } t \in [t_0, \bar{T}]. \tag{A36} \]

Using (21) the initial condition, \( k(t_0) \) given, identifies the constant of integration,
\[ q_3 = k(t_0) e^{-rt_0} - \int_{t_0}^{t} [b - (1 + \theta_c) c(v)] e^{-rv} dv, \tag{A37} \]

which yields the particular or definite solution,
\[ k(t) = e^{rt} \left\{ k(t_0) e^{-rt_0} + \int_{t_0}^{t} [b - (1 + \theta_c) c(v)] e^{-rv} dv \right\}, \quad \text{for } t \in [t_0, \bar{T}]. \tag{A38} \]

Evaluate (A38) with equation (22), the fixed-endpoint condition \( k(\bar{T}) = 0 \), and simplify to yield
\[ k(t_0) e^{-rt_0} + \int_{t_0}^{T} [b - (1 + \theta_c) c(v)] e^{-rv} dv = 0. \tag{A39} \]

Making the substitution from (A35) and rearranging yields
\[ \int_{t_0}^{\bar{T}} (1 + \theta_c) \left[ m \left( \frac{F(v - t_0)e^{rv}}{1 + \theta_c} \right) \right] e^{-rv} dv = k(t_0) e^{-rt_0} + \int_{t_0}^{\bar{T}} be^{-rv} dv. \tag{A40} \]
Solving for the unknown constant of integration \( m \) yields,

\[
m = \frac{k(t_0) e^{-rt_0} + \int_{t_0}^{t} be^{-rv} \, dv}{\int_{t_0}^{t} F(v - t_0) \, dv}.
\]  

(A41)

Inserting (A41) into (A35) produces equation (23), the solution to the time-inconsistent dynamic optimization problem during the retirement phase.

References


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Utility Weight on Leisure: $\gamma = 1.185$  Social discount rate: $\rho = r$

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<td>0.1029</td>
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<td>0.1212</td>
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</tbody>
</table>

Utility Weight on Leisure: $\gamma = 1.185$  Social discount rate: $\rho = 0$

Note: The top number is the revenue-neutral consumption tax rate and the bottom number is the compensating variation (i.e., the percentage change in period consumption needed to equalize the lifetime utility of a hyperbolic discounter who participates in a payroll-tax financed social security program to that of an otherwise identical hyperbolic discounter who participates in social security that is consumption-tax financed with revenue neutrality). In comparisons, the payroll tax rate is set at 10.6 percent.

† Invalid quantitative findings due to predicted labor supply behavior that violates assumptions of the model regarding feasible labor supply behavior, namely that $l(t) \in [0, 1]$ for all $t \in [0, T]$. 

21
Table 2. Revenue-Neutral Consumption Tax Rate ($\theta_c$) and Compensating Variation ($\Psi$).

<table>
<thead>
<tr>
<th>Utility Weight on Leisure: $\gamma = 2$</th>
<th>Social discount rate: $\rho = r$</th>
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<tr>
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<td>†</td>
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Utility Weight on Leisure: $\gamma = 2$ | Social discount rate: $\rho = 0$

<table>
<thead>
<tr>
<th>$r$</th>
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<th>0.04</th>
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<td>†</td>
<td>†</td>
<td>†</td>
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<td>0.0762</td>
<td>4.94%</td>
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</tbody>
</table>

Note: The top number is the revenue-neutral consumption tax rate and the bottom number is the compensating variation (i.e., the percentage change in period consumption needed to equalize the lifetime utility of a hyperbolic discounter who participates in a payroll-tax financed social security program to that of an otherwise identical hyperbolic discounter who participates in social security that is consumption-tax financed with revenue neutrality). In comparisons, the payroll tax rate is set at 10.6 percent.

† Invalid quantitative findings due to predicted labor supply behavior that violates assumptions of the model regarding feasible labor supply behavior, namely that $l(t) \in [0, 1]$ for all $t \in [0, T]$. 
### Table 3. Revenue-Neutral Consumption Tax Rate ($\theta_c$) and Compensating Variation ($\Psi$).

<table>
<thead>
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<th>$r$</th>
<th>$\beta$</th>
<th>0.03</th>
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<th>0.07</th>
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<td>0.26%</td>
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<td>0.52%</td>
<td>0.64%</td>
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<td>0.95%</td>
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<td>0.117</td>
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<td>0.1179</td>
<td>0.1207</td>
<td>0.1233</td>
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<td>0.0903</td>
<td>0.0957</td>
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<td>0.1057</td>
<td>0.1104</td>
<td>0.1149</td>
<td>0.1192</td>
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<td>0.1272</td>
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</tr>
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<td>0.0766</td>
<td>0.0827</td>
<td>0.0888</td>
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<td>0.1061</td>
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**Utility Weight on Leisure: $\gamma = 0.5$**

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</table>

**Utility Weight on Leisure: $\gamma = 0.5$, Social discount rate: $\rho = r$**

Note: The top number is the revenue-neutral consumption tax rate and the bottom number is the compensating variation (i.e., the percentage change in period consumption needed to equalize the lifetime utility of a hyperbolic discounter who participates in a payroll-tax financed social security program to that of an otherwise identical hyperbolic discounter who participates in social security that is consumption-tax financed with revenue neutrality). In comparisons, the payroll tax rate is set at 10.6 percent.

† Invalid quantitative findings due to predicted labor supply behavior that violates assumptions of the model regarding feasible labor supply behavior, namely that $l(t) \in [0, 1]$ for all $t \in [0, T]$.  

---

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Figure 1.

Underlying parameter values: $r = 0.05$, $\theta_w = 0.106$, $\beta = 0.09$, and $\gamma = 1.185$.

Figure 2.

Underlying parameter values: $r = 0.05$, $\theta_w = 0.106$, $\beta = 0.09$, and $\gamma = 1.185$. 
Figure 3.

Savings Plans and Actual Savings Balance

Underlying parameter values: $r = 0.05$, $\theta_w = 0.106$, $\beta = 0.09$, and $\gamma = 1.185$.

Figure 4.

Earnings, $w(t)/l(t)$ and Consumption, $c(t)$

Underlying parameter values: $r = 0.05$, $\rho = r$, $\beta = 0.08$, and $\gamma = 1.185$ yielding $\Psi = 1.72\%$. 
Figure 5.

Savings Asset Balance, $k(t)$

Underlying parameter values: $r = 0.05$, $\rho = r$, $\beta = 0.08$, and $\gamma = 1.185$ yielding $\Psi = 1.72\%$.

Figure 6.

Earnings, $w(t)l(t)$ and Consumption, $c(t)$

Underlying parameter values: $r = 0.05$, $\rho = r$, and $\gamma = 1.185$. 
Figure 7.

Labor Supply, $l(t)$

- $\beta = 0.09, \Psi = 0.94\%$
- $\beta = 0.12, \Psi = -1.36\%$

Underlying parameter values: $r = 0.05, \rho = r$, and $\gamma = 1.185$.

Figure 8.

Savings Asset Balance, $k(t)$

- $\beta = 0.09, \Psi = 0.94\%$
- $\beta = 0.12, \Psi = -1.36\%$

Underlying parameter values: $r = 0.05, \rho = r$, and $\gamma = 1.185$. 