A Method of Estimating Fishing Pressure and Harvest as Used on Logan River, Utah

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A METHOD OF ESTIMATING FISHING PRESSURE AND HARVEST
AS USED ON LOGAN RIVER, UTAH

by

Albert Frank Regenthal

A thesis submitted in partial fulfillment
of the requirements for the degree
of
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in
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* Utah State Agricultural College, Utah Fish and Game Commission, United States Fish and Wildlife Service and Wildlife Management Institute, cooperating.
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INTRODUCTION

Background of study

"The time is fast approaching when conservationists and sportsmen alike will no longer be satisfied with the haphazard methods which have all too frequently prevailed in the past," stated Davis (1938). These words might well apply to creel census. The ordinary methods of creel census, although widely used in fisheries management, give neither reliable estimates of fishing pressure nor of total harvest.

Completely accurate statements of fishing pressure and harvest can only result from an enumeration of all anglers and their catch. On most bodies of water it has not been economically feasible for management agencies to effect a complete census of fishermen and harvest. Practically, absolute accuracy is not necessary for management. The development of creel census methods resulting in estimates of known accuracy, yet requiring only a practical outlay of time and money, would be a step toward more effective management.

Thoreson (1948) estimated the number of anglers using the Logan River by taking sample counts of the angler's cars. He found that the automobiles were more readily seen than the fishermen themselves. Estimates of the average number of fishermen per car, average success, and average fisherman day were based on samples of the creel. Total fishing pressure and total harvest were then estimated as follows:

Total fishermen = Total fishermen's cars x average number of anglers per car, and

Total harvest = Total fishermen x average success per unit of time x units of time per angler.
Fechacek (1949) elaborated upon the methods initiated by Thoreson and depended upon a large sample count to give reliability to his estimates. It was found that limits of accuracy could be set on the estimate of average fishermen per car. Seasonal and daily variations in fishing pressure were demonstrated.

Objectives and scope of the study

The primary purpose of this study is to aid in the development of an economically practical method, based on car counts, that will yield estimates of fishing pressure and harvest with known accuracy.

The study is divided into two phases, each of which contributes a share of the data necessary for the calculation of the desired estimates.

The first phase is the automobile estimate. It is utilized principally in the determination of the amount and distribution of fishing pressure. The challenge of counting anglers without actually seeing them makes this phase of interest.

The second phase is the creel census. Data collected in this phase are employed in the calculation of total harvest of fish. In addition to its use in this study, the creel census serves as a valuable public relations tool and with the car count estimate, serves as a check on the efficiency of the stocking program.

This study was conducted during the regular 1950 fishing season, June 14 to October 5, inclusive on the Logan River drainage, Utah.

Description of study area

The study area includes the Logan River drainage from the headwaters of Beaver Creek at the Idaho state line to the first dam east of Logan City. The river flows in a southwesterly direction through the Wasatch Mountains, and with its tributaries, constitutes nearly 70
linear miles of fishable stream (Figure 1). The river is paralleled by road throughout the study area, and for 25 miles by improved highway. Almost all parking areas along the river are visible from the road.

The Logan River is a swiftly flowing stream, with few pools, that drops from an altitude of 8,500 feet at the state line to 4,600 feet as it enters Cache Valley. The average gradient is approximately 70 feet per mile and the average depth 0.74 feet (Brown, 1935). The bottom consists primarily of boulders and coarse rubble. Water temperatures, from April to October, range from 37 to 56 degrees fahrenheit (Fleener, 1950), and average 52 degrees fahrenheit during June and July (Brown, 1935).

Species of fish present

There are seven species of fish present in the Logan River above the mouth of the canyon. Five species figure prominently in the creel. These are: the rainbow trout (Salmo gairdneri irideus), brown trout (Salmo trutta fario), cutthroat trout (Salmo clarki), brook trout (Salvelinus fontinalis), and the mountain whitefish (Prosopium williamsoni). The Utah sculpin (Cottus bairdi semiscaber), and the locally rare smallfin redside shiner (Richardsonius balteatus hydrophlox) do not appear in the creel. The taxonomy is by Miller (1949). In addition, a hybrid (rainbow x cutthroat) is present in about the central third of the river.¹/

¹/ Identification verified by Dr. R. R. Miller, Associated Curator of Fishes, Univ. of Mich., Ann Arbor, in letter to Dr. William F. Sigler, Professor of Wildlife Management, Utah State Agricultural College, Logan, Utah, March 27, 1949.
Figure 1. Divisions of Logan River used in fisheries investigations, Cache National Forest, Utah, 1950 (Scale 1" = 2.73 miles)
REVIEW OF LITERATURE

Creel census has been widely used as a tool in fisheries management. Phenicie and Bishop (1950) describe creel census as "...the constant check which will indicate efficient management." Many researchers have contributed their efforts to aid in the development of methods to economically obtain, and critically analyze, creel census data.

Methods of estimating vital statistics of fish populations were advanced by Ricker (1948). Mottley (1946) suggested a method of adapting creel census to the estimation of biological populations. Tarzwell and Miller (1942) discussed weekly cycles and seasonal trends in fishing pressure, and techniques of sampling. The use of statistically sound experimental designs for creel census studies was advocated by Mottley (1942).

Distribution curves are encountered in many studies concerning populations. The application and use of the Poisson distribution curve in the analysis of fishery data received the attention of Garwood (1938) and Ricker (1937).
METHODS OF PROCEDURE

Automobile estimate

A satisfactory technique for counting fishermen without actually seeing them would be of interest to researchers and management. On many lakes and streams the automobiles of anglers must be parked where they can easily be seen and counted, while vegetation or topography hide the fishermen from view. In this study, as in those of Thoreson and Peacock, car count samples form the basis for the estimates of fishing pressure and harvest.

There are two assumptions upon which the validity of the automobile estimate technique is predicated:

1. There is a random movement of fishermen along the stream and in and out of the study area.

2. The automobiles of fishermen can be seen and recognized.

There are three variables in fishing pressure that influence the number of automobiles along the stream at any given time. Fishing pressure varies: first, according to the time of season; second, according to the day of the week; third, according to the time of day.

An experimental design that would evaluate these variables was devised.

The fishing season was divided into ten intervals (Table 10). Each interval was of two weeks duration with the exception of opening day and the last two days of the season. Opening day was regarded as a separate interval because of the unusual fishing pressure normally exerted on that day.
Within each seasonal interval a division was made between weekdays and holidays. Saturdays and Sundays, as well as other legal and local holidays, were placed in the latter classification. No further breakdown was believed necessary since past studies on the river indicated little significant difference in fishing pressure among different days within each division.

Each legal 16-hour fishing day was divided into four equal periods. This division was arbitrarily selected because past studies (Thoreson, 1948; Pechacek, 1949) indicated that the average fisherman day approximated four hours. The periods were designated as follows:

Period A - 5:00 a.m. to 9:00 a.m.
Period B - 9:00 a.m. to 1:00 p.m.
Period C - 1:00 p.m. to 5:00 p.m.
Period D - 5:00 p.m. to 9:00 p.m.

The 114 days of the 1950 fishing season multiplied by the four daily periods gave a total of 456 periods in which automobiles could be counted. Since it was impractical to count anglers' cars during each period of every fishing day, the feasible method was to take a sample number of counts. The sampling technique used in 1948 and 1949 was such that no limits of accuracy could be set on the total estimates of anglers' cars. Counts were made in over 50 percent of the periods to increase the reliability of the estimates. This year the sample size was reduced. A total number of 135 sample counts, or approximately 30 percent of the counts possible, were distributed throughout the season so that they approximately paralleled the seasonal, weekly, and daily trends in fishing pressure found by Pechacek in 1949 (Table 8). The distribution of samples made on the basis of the percentage of total fishing pressure...
exerted, in (a) each seasonal interval, (b) each weekly division within the intervals, and (c) each daily period within the weekly divisions. An example of the sampling distribution for one representative interval is given in table 1.

Table 1. Allocation of sample counts July 15 - July 28. (Seasonal interval 4), Logan River Study, 1950. Based on 135 total counts.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Number</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of total seasonal pressure, 1949</td>
<td>13.6</td>
<td></td>
</tr>
<tr>
<td>Number of counts, 1950</td>
<td>18*</td>
<td></td>
</tr>
<tr>
<td>Weekly division</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of total pressure in interval, 1949</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of counts, 1950</td>
<td>9*</td>
<td>9*</td>
</tr>
<tr>
<td>Daily period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of pressure in weekly division, 1950</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of counts, 1950</td>
<td>2 2 2 2 3</td>
<td>2 2 2 3*</td>
</tr>
</tbody>
</table>

* Rounded to nearest whole number.

After the number of counts for the weekly divisions within each interval had been determined, the specific weekdays or holidays to be sampled were picked at random. Wherever possible, since it did not affect the statistical design, two counts were made on the same day as a matter of economy. The sample counts calculated for the daily periods in each weekly division were scheduled so that the four-hour time interval in each period was randomly sampled.

As a check on the validity of the assumption that there is a random movement of fishermen along the stream and in and out of the study area,
40 additional car counts were scheduled. The same method of distributing these check counts was used as was employed in scheduling the regular car counts. Each check count followed a regular count by a time interval varying from 15 minutes to an hour.

**Creel census**

Three items essential to the estimation of fishing pressure and harvest were obtained from the creel census in this study. These were: fishermen per car, rate of fishing success, and length of fisherman day. Total number of anglers' cars, the fourth necessary item, was estimated from the car count samples. Estimates of harvest and fishing pressure were calculated:

- **Fishing pressure** = Anglers' cars x anglers per car
- **Harvest** = Anglers x rate of fishing success x fisherman day.

The creel form used in 1950 was designed to collect data for an analysis of the stocking program and the creel in addition to the items needed in the study. One form was used to represent each angler contacted. Fishing party (Figure 2, No. 3), was used synonymously with the number of anglers in one car.

The study area was divided into 25 sections to aid in the collection and analysis of data (Table 2, Figure 1). The sections, selected arbitrarily, varied in length from 0.4 to 3.6 miles and were bounded by prominent landmarks, Right Fork and Temple Fork, sections 12 and 18, were not included in either the creel census or car counts.

The creel census was conducted in conjunction with the car counts. The same seasonal intervals, weekly divisions and daily periods were utilized in recording and analyzing creel data. A total of 519 creel samples were distributed throughout the fishing season so that they approximately
Figure 2. Creel census form used on Logan River study area, 1950.
paralleled the trends in fishing pressure.

Anglers were contacted purely at random. No regard was given to time of day, weather, success, location on stream, or sex of angler.
Table 2. Divisions of the Logan River drainage employed in collection of creel census and car count data, 1950.

<table>
<thead>
<tr>
<th>Section number</th>
<th>Description or name</th>
<th>Miles from 1st dam to 2nd dam in miles</th>
<th>Road length in miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>First impoundment (state)</td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>3</td>
<td>Power plant race to 2nd dam</td>
<td>.9</td>
<td>1.9</td>
</tr>
<tr>
<td>4</td>
<td>Second impoundment</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>5</td>
<td>Second impoundment to 3rd dam</td>
<td>2.8</td>
<td>1.5</td>
</tr>
<tr>
<td>6</td>
<td>Third impoundment</td>
<td>4.1</td>
<td>.8</td>
</tr>
<tr>
<td>8</td>
<td>Malibu to Beardean</td>
<td>5.4</td>
<td>1.0</td>
</tr>
<tr>
<td>9</td>
<td>Beardean to Card R. S.</td>
<td>6.4</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>Card R. S. to Preston Valley</td>
<td>7.6</td>
<td>1.0</td>
</tr>
<tr>
<td>11</td>
<td>Preston Valley to Juniper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Right Hand Fork</td>
<td>8.6</td>
<td>1.0</td>
</tr>
<tr>
<td>13</td>
<td>Juniper Lodge Bridge to China Row</td>
<td>9.6</td>
<td>1.35</td>
</tr>
<tr>
<td>14</td>
<td>China Row to Jardine</td>
<td>10.4</td>
<td>2.3</td>
</tr>
<tr>
<td>15</td>
<td>Jardine Juniper foot trail</td>
<td>12.7</td>
<td>1.4</td>
</tr>
<tr>
<td>16</td>
<td>Lower dugway bridge to upper dugway bridge</td>
<td>14.1</td>
<td>.8</td>
</tr>
<tr>
<td>17</td>
<td>Upper dugway bridge to Temple Fork road bridge</td>
<td>14.9</td>
<td>.7</td>
</tr>
<tr>
<td>18</td>
<td>Temple Fork</td>
<td>15.6</td>
<td>2.6</td>
</tr>
<tr>
<td>19</td>
<td>Temple Fork Road Bridge to Ricks Spring bridge</td>
<td>15.6</td>
<td>.6</td>
</tr>
<tr>
<td>20</td>
<td>Ricks Spring to Highway cattle guard</td>
<td>16.2</td>
<td>1.6</td>
</tr>
<tr>
<td>21</td>
<td>Highway cattle guard to Tony Grove S.C. bridge</td>
<td>17.7</td>
<td>1.8</td>
</tr>
<tr>
<td>22</td>
<td>Tony Grove S.C. bridge to Red Banks bridge</td>
<td>19.5</td>
<td>1.5</td>
</tr>
<tr>
<td>23</td>
<td>Red Banks bridge to Franklin Basin Road Jot.</td>
<td>21.0</td>
<td>1.7</td>
</tr>
<tr>
<td>24</td>
<td>Franklin Basin Road Jot. to Beaver Creek Road</td>
<td>22.7</td>
<td>5.6</td>
</tr>
<tr>
<td>25</td>
<td>Beaver Creek Road to Idaho line</td>
<td>26.3</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Entire stream or any combination of divisions</td>
<td></td>
<td>29.7</td>
</tr>
</tbody>
</table>
ANALYSIS

Estimate and variance of anglers' cars

The three variables in fishing pressure that influence the number of anglers' cars along the stream at any given time are; (a) time of the season, (b) day of the week, and (c) time of the day.

For analysis, car count data for holidays and weekdays were separated and individually treated. Such a procedure eliminated from the analysis the variation between these days of the week. Table 3 gives a summary of car count data for holidays. Car count data for weekdays is summarized in Table 4.

The problem, and the attempted solution, of estimating the total anglers' cars and evaluating the effect of the variations in fishing pressure at different times in the season and at different times of day is outlined briefly below:

The Model:

\[ Y_{ijk} = \mu X_0 + s_i X_1 + p_j Z_j + \epsilon_{ijk} \]

\[ Y_{ijk} = \text{No. of cars observed in the } k\text{th observation of the} \]
\[ i\text{th interval of the season and the } j\text{th period of the day.} \]

\[ X_0 = 1 \text{ for every observation} \]
\[ X_1 = 1 \text{ for } i\text{th interval of season, } 0 \text{ for all others.} \]
\[ Z_j = 1 \text{ for } j\text{th period of day, } 0 \text{ for all others.} \]
\[ \epsilon_{ijk} = \text{a random variable with mean } = 0, \text{ variance } = \delta^2, \]
\[ \text{normally and independently distributed.} \]

\[ i = 1, 2, \ldots, 10; j = 1, 2, 3, 4; k = 1, 2, \ldots, h_{ij} \]

Constants to be estimated:
\[ \lambda = \text{a fixed effect} \]
\[ s_i = \text{effect of the } i\text{th interval of the season} \]
\[ p_j = \text{effect of the } j\text{th period of the day} \]

Table 3. Car counts and cars counted on holidays, by weekly divisions and seasonal intervals, Logan River, 1950

<table>
<thead>
<tr>
<th>Interval</th>
<th>Number of counts</th>
<th>Number of cars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 4. Car counts and cars counted on weekdays, by weekly divisions and seasonal intervals, Logan River, 1950

<table>
<thead>
<tr>
<th>Interval</th>
<th>Number of counts</th>
<th>Number of cars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
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<tr>
<td>5</td>
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<td>7</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

The normal equations were formed in the usual way, the set was solved by absorbing the \( s_i \), writing the \( p_j \) equations in terms of
$(\lambda, \sigma_1)$ and letting $p_4 = 0$. This left a $3 \times 3$ matrix which was inverted by the abbreviated Doolittle method and the least squares estimates of $\lambda$, $\sigma_1$, and $p_4$ rather easily found assuming in addition that $SS1 = 0$.

The estimate of total fishermen cars on the river throughout the entire season equaled

$$4 \sum (\bar{w}_i) (\lambda, \sigma_1)$$

Where

$w_i =$ number of days in the $i$th interval and the factor $4 =$ number of periods per day.

A detailed description of the solution is given in the appendix (Demonstration 1).

The total number of anglers' cars on the Logan River in 1950 was estimated to be $11,774 \pm 1,283$ cars ($P \pm .95$). On weekdays the estimated total cars was $5,419 \pm 1,139$. The estimate of total cars on the river on holidays was $6,355 \pm 637$ cars. A summary of the estimated cars by weekly division and seasonal interval is given in Table 5.

The estimate of the mean number of cars per period $(\bar{\lambda})$, a fixed effect, was 49.05 cars per holiday period and 17.73 cars per weekday period.

The estimate of the mean number of anglers' cars per period was changed in each seasonal interval by the effect of the interval upon that estimate. On holidays effects of seasonal interval ranged from $87.45$ cars in interval number 1 to $-32.53$ cars in interval number 9 (Table 6). On weekdays the effect ranged between $6.76$ cars in the 6th interval to $-12.44$ cars in the 9th interval.

The effect of the daily period also changed the estimate of mean number of cars per period. On holidays the estimate of mean cars per period was increased by $7.35$ cars in period C and decreased by $12.32$
Table 5. Estimated anglers' automobiles by seasonal
interval and weekly division, Logan River,
1950

<table>
<thead>
<tr>
<th>Seasonal interval</th>
<th>Holidays</th>
<th>Weekdays</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>548</td>
<td>*</td>
<td>548</td>
</tr>
<tr>
<td>2</td>
<td>724</td>
<td>766</td>
<td>1,490</td>
</tr>
<tr>
<td>3</td>
<td>1,019</td>
<td>922</td>
<td>1,941</td>
</tr>
<tr>
<td>4</td>
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<td>1,954</td>
</tr>
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<td>5</td>
<td>855</td>
<td>799</td>
<td>1,654</td>
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<tr>
<td>6</td>
<td>551</td>
<td>980</td>
<td>1,561</td>
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<td>7</td>
<td>704</td>
<td>528</td>
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<td>358</td>
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<td>734</td>
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<tr>
<td>9</td>
<td>266</td>
<td>211</td>
<td>477</td>
</tr>
<tr>
<td>10</td>
<td>205</td>
<td>*</td>
<td>205</td>
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</tbody>
</table>

Total 6,555 5,419 11,774
* No weekdays in interval.

Table 6. Effect of the time of season on the mean
number of anglers' cars per period by holi-
days and weekdays, Logan River, 1950

<table>
<thead>
<tr>
<th>Seasonal interval</th>
<th>Holidays (cars)</th>
<th>Weekdays (cars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$87.45</td>
<td>...</td>
</tr>
<tr>
<td>2</td>
<td>$11.09</td>
<td>$1.62</td>
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<td>$6.71</td>
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<td>5</td>
<td>$4.25</td>
<td>$2.24</td>
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<td>6</td>
<td>$12.86</td>
<td>$6.76</td>
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<td>7</td>
<td>$13.36</td>
<td>$5.07</td>
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<td>$12.44</td>
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<tr>
<td>10</td>
<td>$23.97</td>
<td>...</td>
</tr>
</tbody>
</table>
cars in period A. On weekdays the range of effect was from a $\pm 6.72$
cars in period D to $-10.55$ cars in period A (Table 7).

Table 7. Effect of the time of day on the mean number
of anglers' cars per period, by holidays
and weekdays, Logan River, 1950

<table>
<thead>
<tr>
<th>Period</th>
<th>Holidays</th>
<th>Weekdays</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-12.52</td>
<td>-10.55</td>
</tr>
<tr>
<td>B</td>
<td>1.35</td>
<td>0.72</td>
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<tr>
<td>C</td>
<td>7.35</td>
<td>5.15</td>
</tr>
<tr>
<td>D</td>
<td>6.35</td>
<td>6.72</td>
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</tbody>
</table>

Estimate and variance of fishermen per car

The number of fishermen per car taken from each of the 519 creel
samples were tabulated as follows:

<table>
<thead>
<tr>
<th>Anglers per car</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>180</td>
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<tr>
<td>2</td>
<td>198</td>
</tr>
<tr>
<td>3</td>
<td>87</td>
</tr>
<tr>
<td>4</td>
<td>44</td>
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<td>4</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

The frequency distribution of the number of anglers-per-car-minus-
one was plotted and resembled a Poisson distribution (Figure 5). One
angler was subtracted from each car in order to form the zero anglers-
per-car group necessary for calculation. Following the procedure out-
lined in Chapter 16 of Snedecor (1946), the expected distribution was
calculated and the Chi-square test applied. Chi-square was 4.40079
with four degrees of freedom. The deviation of the sample from the ex-
pected distribution was not significant.

The mean of the sample distribution was 1.0597. The variance of
the mean was 0.00204. The standard deviation of the mean (standard error)
was calculated as 0.046. Replacing the one angler subtracted, the mean
Figure 3. Frequency distribution of anglers-per-car-minus-one, on the Logan River Study Area, 1950."
number of anglers per car, at the 95 percent confidence interval, fell between 1.9862 and 2.1332, or 2.0597 ± 3.57 percent.

**Estimate of total fishermen**

The mean number of anglers per car multiplied by the estimated total number of cars gives the estimated total number of anglers that used the stream during the fishing season. Thus, 2.0597 anglers per car multiplied by 11,774 cars gave an estimated total of 24,261 fishermen that used the study area in 1950.

**Variance of estimate of total fishermen**

The estimate of total fishermen was the product of the total number of cars and the average number of anglers per car. The variance of that estimate, a combination of the variances of each of the components, can be expressed:

\[
\frac{V_T}{T^2} = \frac{V_X}{X^2} + \frac{V_Y}{Y^2}
\]

Where

\begin{align*}
T &= \text{total estimated anglers,} \\
X &= \text{total estimated cars, and} \\
Y &= \text{mean anglers per car}
\end{align*}

From the preceding analysis of the car count and creel census

\begin{align*}
T &= 24,261 \\
X &= 11,774 \\
Y &= 2.0597 \\
V_T &= 425,872.304 \\
V_Y &= 0.00204
\end{align*}

When values were substituted and the equation solved the variance of the total estimated fishermen was 2,088,970. The standard deviation was 1,445. Fiducial limits at approximately the 95 percent confidence
interval were

\[ T = 24,251 \times t (1.045) \]

\[ T = 24,251 \times 2.659, \text{ or} \]

\[ T = 24,251 \times 11.71 \text{ percent.} \]

**Correction of estimate of total fishermen**

Car counts were scheduled so that they should have randomly sampled the four-hour time interval in each daily period. The analysis of the car count assumed that the periods had been randomly sampled. Actually, periods A and D were not randomly sampled in this study.

The legal length of the fishing day was 16 hours. The hours of daylight, however, decreased from 15.06 hours on the first day of the season to 11.45 hours on the closing day. The car counts scheduled early in period A and late in period D could not be made until the earliest and latest possible moments, respectively, that it was light enough to see the cars to be counted. It was observed that anglers rarely fished in those portions of the legal fishing day from 5 A.M. until 20 minutes before sunrise and from 20 minutes after sunset to 9 P.M. Car counts scheduled during those time intervals should have shown few, if any, anglers' cars but the counts actually taken to represent the scheduled counts resulted in the recording of a sizeable number of cars. While this error in no way invalidates the design of the experiment it did result in a larger estimate of anglers' cars in this instance.

To correct for the error the average length of the actual fishing day from 20 minutes before sunrise to 20 minutes after sunset was calculated for each seasonal interval. The average length of the actual fishing day was converted to percentage of legal fishing day and the estimate of anglers' cars for each seasonal interval reduced proportionately (Table 11). The corrected estimate of total anglers' cars was
When the 10,850 oars were multiplied by the mean number of anglers per car (2.0597) a new estimate of 22,548 fishermen resulted.

This estimate, however, was based on an assumed 4-hour fisherman day. The actual fisherman day, calculated from the creel forms of those anglers who had completed fishing (Figure 2, No. 12), averaged 3.35 hours.

The total number of anglers estimated on the basis of an assumed fisherman day can be corrected to give an estimate based on the actual fisherman day by use of the formula:

\[
\text{Anglers} = \frac{\text{Assumed fisherman day} \times \text{Anglers}}{\text{Actual fisherman day}}
\]

On the Logan River in 1950, the corrected estimate of total anglers was 26,684.

**Estimate of total harvest**

Total harvest may be expressed as the product of the total number of anglers times the average catch per angler per unit of time, times the units of time per angler.

From items number 9 and 11 on the creel forms recorded in 1950, it was determined that each fisherman caught an average 0.6732 fish per hour of fishing effort. The average fisherman day was 3.35 hours and the total number of fishermen was estimated as 26,684. Thus, 26,684 anglers times 3.35 hours, times 0.6732 fish per hour gave an estimated harvest of 60,625 fish.

**Check counts of anglers' cars**

The total number of cars counted during the 40 check counts was 1885 cars. Cars counted during the 40 corresponding scheduled car counts numbered 1,794. The difference in total number of cars counted
was 4.8 percent. Since the check counts followed the car counts by time
intervals ranging from 15 minutes to 1 hour, the small difference in
total cars counted lends confidence to the assumption that there was a
random movement of fishermen along the stream and in and out of the
study area.

**Distribution of fishing pressure**

The estimated number of anglers' automobiles along the stream in
any given seasonal interval, weekly division, or daily period can be
used as an index of fishing pressure exerted in that time period.

The seasonal distribution of fishing pressure on the Logan River in
1950 followed the same general trend as that observed in 1949 (Figure 4).
The distribution of car counts in this study, then, closely approximated
the seasonal distribution in fishing pressure.

An estimated 53.96 percent of total fishing pressure was exerted on
the river on holidays. Weekdays contributed 46.02 percent. It should
be noted, however, that there were only 37 holidays as compared with 78
weekdays. Fishing pressure on any one holiday then was slightly more
than twice that of any weekday. A further breakdown of weekly fishing
pressure is given in Table 10.

Over the entire fishing season, more anglers fished in period D
than any other period. It was followed, in order of decreasing pres-
sure, by periods C, B, and A. There was some variation in daily fishing
pressure in different weekly divisions (Table 8).

Table 8. Distribution of daily fishing pressure by weekly division
and for the entire season, Logan River, 1950.

<table>
<thead>
<tr>
<th>Period</th>
<th>Percent of total fishing pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Holidays</td>
</tr>
<tr>
<td>A</td>
<td>18.14</td>
</tr>
<tr>
<td>B</td>
<td>24.16</td>
</tr>
<tr>
<td>C</td>
<td>29.14</td>
</tr>
<tr>
<td>D</td>
<td>28.55</td>
</tr>
</tbody>
</table>
Figure 4. Distribution of fishing pressure by seasonal intervals, based on car count estimates, Logan River, 1949 and 1950.
SUMMARY AND CONCLUSIONS

1. Because reliable estimates of total fishing pressure and total harvest are of vital importance to management agencies, a study was conducted in 1950 on the Logan River drainage to aid in the development of a practical method of obtaining those estimates with known accuracy.

2. Samples of anglers' cars, which on the Logan River were more practically counted than anglers themselves, formed the basis for the estimate of total anglers' cars. The sample counts were distributed so that they approximately paralleled the seasonal, weekly, and daily variations in fishing pressure. A creel census was conducted in conjunction with the car counts to obtain the estimates of anglers per car, average angler success, and average fisherman day necessary to estimate the total fishermen and total harvest.

3. Statistical analysis of the car count and creel data showed that 11,774 \( \pm \) 1,283 cars transported an average of 2.0597 \( \pm \) 0.0735 anglers per car, or a total of 24,251 \( \pm \) 2,839 anglers to the study area.

4. Because darkness encroached upon the early morning and late evening hours of the legal fishing day, fishermen did not utilize the full 16 hours. Corrections for the partial utilization of the legal fishing day, an error not inherent in the experimental design, changed the estimate of total cars to 10,880.

5. The mean length of the fisherman day was 3.55 hours. The assumed fisherman day for analysis was four hours. Corrections for the difference in length of fisherman day resulted in a revised estimate of 26,884 anglers.
6. The average angler was successful in catching 0.6782 fish per hour of fishing effort. The 26,684 anglers on the Logan River in 1950 caught an estimated total of 60,625 fish.

7. In this study the number of holidays per seasonal interval varied, as did the number of weekdays. Analysis would be considerably simplified if the number of holidays in each interval were equal and the number of weekdays per interval also equal.

8. Further study is needed to set fiducial limits to the estimates of average angler success and total harvest, and to simplify the analysis.
APPENDIX
Table 9. Distribution of seasonal, weekly and daily fishing pressure,
Logan River, 1949.

<table>
<thead>
<tr>
<th>Seasonal interval</th>
<th>Percent of total pressure</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Holidays</td>
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<td>June 12 - June 24</td>
<td>ABC</td>
</tr>
<tr>
<td></td>
<td>Weekdays</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>3</td>
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</tr>
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<td>June 25 - July 8</td>
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<tr>
<td>4</td>
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<td>ABC</td>
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<td>5</td>
<td>Holidays</td>
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<td>ABC</td>
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<td>Weekdays</td>
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</table>

Table 9. Distribution of seasonal, weekly and daily fishing pressure, Logan River, 1949. (Continued)

<table>
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<th>Seasonal interval</th>
<th>Percent of total pressure</th>
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<th></th>
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</thead>
<tbody>
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<td></td>
<td>Seasonal</td>
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<td>August 6 -</td>
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<td>B</td>
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<td>C</td>
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<td></td>
<td></td>
<td>D</td>
<td>42.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Weekdays</td>
<td>100</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* No weekdays in interval.

Estimation of the number of cars on the Logan River Study Area, (Holidays) 1950.

Model:

\[ Y_{ijk} = \lambda x_0 + s_1 x_1 + P_j z_j + \epsilon_{ijk} \]

Where, \( \lambda \) = a fixed effect,

\( s_1 \) = effect of the ith interval of the season, and

\( P_j \) = effect of the jth time of day, are unknown constants;

And \( \epsilon_{ijk} \) is a random variable with mean = 0, variance = \( \sigma^2 \) and uncorrelated.

\[ Y_{ijk} \] = number of cars observed in the kth observation of the ith interval of the season and the jth period of the day.

\( X_0 \) = 1 for every observation

\( X_1 \) = 1 for ith interval of season, 0 for all others

\( Z_j \) = 1 for jth period of day, 0 for all others

\( i = 1, 2, 3, \ldots, I \)

\( j = 1, 2, 3, \ldots, M \)

\( k = 1, 2, 3, \ldots, n_{ij} \)

Find those values for the parameters which will minimise the \( \text{Se}^2_{ijk} \) as follows:

\[ \text{Se}^2_{ijk} = \sum (Y_{ijk} - \lambda x_0 - s_1 x_1 - P_j z_j)^2 \]

Set:

\[ \frac{\partial \text{Se}^2_{ijk}}{\partial \lambda} = -2 X_0 [Y_{ijk} - \lambda x_0 - s_1 x_1 - P_j z_j] = 0 \]

\[ \frac{\partial \text{Se}^2_{ijk}}{\partial s_1} = -2 X_1 [Y_{ijk} - \lambda x_0 - s_1 x_1 - P_j z_j] = 0 \]

\[ \frac{\partial \text{Se}^2_{ijk}}{\partial P_j} = -2 Z_j [Y_{ijk} - \lambda x_0 - s_1 x_1 - P_j z_j] = 0 \]

This gives a set of \( (I \times I \times M) \) linear equations in \( (I \times I \times M) \) unknowns.
The equations are not independent, however, since the sum of the \( s_i \) equations and the sum of the \( p_j \) equations equal the \( \mu \) equation. Hence, two additional equations are necessary to render the set solvable. A wide variety of supplementary equations are possible which will give a unique solution.

Write the estimating or normal equations as follows:

Information matrix

<table>
<thead>
<tr>
<th>Equation ( \mu )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( \ldots )</th>
<th>( s_i )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( \ldots )</th>
<th>( p_j )</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( sx^2 )</td>
<td>( sx_0x_2 )</td>
<td>( sx_0x_2 )</td>
<td>( \ldots )</td>
<td>( sx_0x_1 )</td>
<td>( sx_0z_1 )</td>
<td>( sx_0z_2 )</td>
<td>( \ldots )</td>
<td>( sx_0z_j )</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>( sx_0x_2 )</td>
<td>( sx_1^2 )</td>
<td>( 0 )</td>
<td>( \ldots )</td>
<td>( 0 )</td>
<td>( sx_1z_1 )</td>
<td>( sx_1z_2 )</td>
<td>( \ldots )</td>
<td>( sx_1z_j )</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( sx_0x_2 )</td>
<td>( 0 )</td>
<td>( sx_1^2 )</td>
<td>( 0 )</td>
<td>( sx_2z_1 )</td>
<td>( sx_2z_2 )</td>
<td>( \ldots )</td>
<td>( sx_2z_j )</td>
<td>( sy_{1jk}x_2 )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( s_i )</td>
<td>( sx_0x_2 )</td>
<td>( 0 )</td>
<td>( \ldots )</td>
<td>( sx_1^2 )</td>
<td>( sx_1z_1 )</td>
<td>( sx_1z_2 )</td>
<td>( \ldots )</td>
<td>( sx_1z_j )</td>
<td>( sy_{1jk}x_i )</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>( sx_0z_1 )</td>
<td>( sx_1z_1 )</td>
<td>( sx_2z_1 )</td>
<td>( \ldots )</td>
<td>( sx_1z_1 )</td>
<td>( sz_1^2 )</td>
<td>( 0 )</td>
<td>( \ldots )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>( sx_0z_2 )</td>
<td>( sx_1z_2 )</td>
<td>( sx_2z_2 )</td>
<td>( \ldots )</td>
<td>( sx_1z_2 )</td>
<td>( sz_2^2 )</td>
<td>( 0 )</td>
<td>( \ldots )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( p_j )</td>
<td>( sx_0z_j )</td>
<td>(sx_1z_j )</td>
<td>( sx_2z_j )</td>
<td>( \ldots )</td>
<td>( sx_1z_j )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( sz_j^2 )</td>
<td>( \ldots )</td>
</tr>
</tbody>
</table>

Example (Based on Table 3):

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>Season interval</th>
<th>Period of day No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( B )</td>
<td>( C )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>( 2 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( 3 )</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>( 4 )</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>( 5 )</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>( 6 )</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>( 7 )</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>( 8 )</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>( 9 )</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>( 10 )</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>( A )</td>
<td>16</td>
<td>11</td>
</tr>
<tr>
<td>( B )</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>( C )</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>( D )</td>
<td>21</td>
<td>1</td>
</tr>
</tbody>
</table>
The equations can be solved in many ways. With the objectives in mind and considering the labor involved in finding a solution it seems desirable to absorb the $s_1$ equations by writing the $p_j$ equations in terms of ($A/s_1$) and solving the reduced set by inverting the resulting matrix.

\[
\begin{align*}
A/s_1 &= 1/4 \left[ 546 - 1p - 1p_2 - 1p_3 - 1p_4 \right] = \\
A/s_2 &= 1/5 \left[ 319 - 0 - 2p_2 - 2p_3 - 1p_4 \right] = \\
A/s_3 &= 1/14 \left[ 717 - 3p_1 - 4p_2 - 3p_3 - 4p_4 \right] = \\
A/s_4 &= 1/11 \left[ 606 - 3p_1 - 3p_2 - 2p_3 - 5p_4 \right] = \\
A/s_5 &= 1/10 \left[ 538 - 2p_1 - 3p_2 - 2p_3 - 3p_4 \right] = \\
A/s_6 &= 1/7 \left[ 246 - 2p_1 - 2p_2 - 1p_3 - 2p_4 \right] = \\
A/s_7 &= 1/8 \left[ 281 - 2p_1 - 2p_2 - 2p_3 - 2p_4 \right] = \\
A/s_8 &= 1/4 \left[ 84 - 1p_1 - 1p_2 - 1p_3 - 1p_4 \right] = \\
A/s_9 &= 1/7 \left[ 128 - 1p_1 - 2p_2 - 2p_3 - 2p_4 \right] = \\
A/s_{10} &= 1/4 \left[ 108 - 1p_1 - 0 - 1p_3 - 2p_4 \right] = 
\end{align*}
\]

Now the four $p_j$ equations can be written in terms of ($A/s_1$) as follows:

\[
\begin{align*}
p_1 &= 1(A/s_1) \neq 0(A/s_2) \neq 3(A/s_3) \neq 3(A/s_4) \neq 2(A/s_5) \\
&= 2(A/s_6) \neq 2(A/s_7) \neq 1(A/s_8) \neq 1(A/s_9) \neq 1(A/s_{10}) \neq 16_{pl} = 57 \\
p_2 &= 1( ) \neq 2( ) \neq 4( ) \neq 3( ) \neq 3( ) \neq 57 \\
&= 2( ) \neq 2( ) \neq 1( ) \neq 2( ) \neq 0( ) \neq 20_{p2} = 95 \\
p_3 &= 1( ) \neq 2( ) \neq 3( ) \neq 2( ) \neq 2( ) \neq 1( ) \neq 17_{p3} = 95 \\
&= 1( ) \neq 2( ) \neq 1( ) \neq 2( ) \neq 1( ) \neq 21_{p4} = 110 \\
p_4 &= 1( ) \neq 1( ) \neq 4( ) \neq 3( ) \neq 3( ) \neq 21_{p4} = 110 \\
&= 2( ) \neq 2( ) \neq 1( ) \neq 2( ) \neq 2( ) \neq 21_{p4} = 110 
\end{align*}
\]

Substituting the values for ($A/s_1$) in the above equations and collecting terms yields the four equations as follows:

\[\]
Similarly for equations P2, P3, and P4.

These computations can be completed more quickly by using the following table:

<table>
<thead>
<tr>
<th></th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
<th>P₄</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>s₁</td>
<td>1(1/4)</td>
<td>1(1/4)</td>
<td>1(1/4)</td>
<td>1(1/4)</td>
<td>546</td>
</tr>
<tr>
<td>s₂</td>
<td>0(0/5)</td>
<td>2(2/5)</td>
<td>2(2/5)</td>
<td>1(1/5)</td>
<td>319</td>
</tr>
<tr>
<td>s₃</td>
<td>3(3/14)</td>
<td>4(4/14)</td>
<td>3(3/14)</td>
<td>4(4/14)</td>
<td>717</td>
</tr>
<tr>
<td>s₄</td>
<td>3(3/11)</td>
<td>3(3/11)</td>
<td>2(2/11)</td>
<td>3(3/11)</td>
<td>606</td>
</tr>
<tr>
<td>s₅</td>
<td>2(2/10)</td>
<td>3(3/10)</td>
<td>2(2/10)</td>
<td>3(3/10)</td>
<td>538</td>
</tr>
<tr>
<td>s₆</td>
<td>2(2/7)</td>
<td>2(2/7)</td>
<td>1(1/7)</td>
<td>2(2/7)</td>
<td>246</td>
</tr>
<tr>
<td>s₇</td>
<td>2(2/8)</td>
<td>2(2/8)</td>
<td>2(2/8)</td>
<td>2(2/8)</td>
<td>281</td>
</tr>
<tr>
<td>s₈</td>
<td>1(1/4)</td>
<td>1(1/4)</td>
<td>1(1/4)</td>
<td>1(1/4)</td>
<td>84</td>
</tr>
<tr>
<td>s₉</td>
<td>1(1/7)</td>
<td>2(2/7)</td>
<td>2(2/7)</td>
<td>2(2/7)</td>
<td>128</td>
</tr>
<tr>
<td>s₁₀</td>
<td>1(1/4)</td>
<td>0(0/4)</td>
<td>1(1/4)</td>
<td>2(2/4)</td>
<td>108</td>
</tr>
</tbody>
</table>

N = 16  20  17  21
Y₁ = 571  957  938  1107
The new set of $p_j$ equations are then obtained as follows:

$\begin{align*}
\begin{bmatrix}
16 - 1(1/4) - 0(0/5) - 3(3/14) & \ldots & - 1(1/4)
\end{bmatrix} P_1 \\
- \begin{bmatrix}
1(1/4) / 0(2/5) / 3(4/14) / 3(3/11) & \ldots & / 1(0/4)
\end{bmatrix} P_2 \\
- \begin{bmatrix}
1(1/4) / 0(2/5) / 3(3/14) / 5(2/11) & \ldots & / 1(1/4)
\end{bmatrix} P_3 \\
- \begin{bmatrix}
1(1/4) / 0(1/5) / 3(4/14) / 3(3/11) & \ldots & / 1(2/4)
\end{bmatrix} P_4 \\
= \begin{bmatrix}
571 - (1/4) 546 - (0.5) 319 - 3(14) 717 \ldots - 1/4
\end{bmatrix}
\end{align*}$

$\begin{align*}
\begin{bmatrix}
1(1/4) / 2(0/5) / 4(3/14) / 3(3/11) & \ldots & / 0(1/4)
\end{bmatrix} P_1 \\
- \begin{bmatrix}
20 - 1(1/4) - 2(2/5) - 4(4/14) & \ldots & - 0(0/4)
\end{bmatrix} P_2 \\
- \begin{bmatrix}
1(1/4) / 2(2/5) / 4(3/14) / 3(2/11) & \ldots & / 0(1/4)
\end{bmatrix} P_3 \\
- \begin{bmatrix}
1(1/4) / 2(1/5) / 4(4/14) / 3(3/11) & \ldots & / 0(2/4)
\end{bmatrix} P_4 \\
= \begin{bmatrix}
957 - 1/4(546) - (2/5) 319 - 4(14) 717 \ldots - 0(4/4)
\end{bmatrix}
\end{align*}$

$\begin{align*}
\begin{bmatrix}
1(1/4) / 2(0/5) / 3(3/14) / 2(3/11) & \ldots & / 2(1/4)
\end{bmatrix} P_1 \\
- \begin{bmatrix}
1(1/4) / 2(2/5) / 3(4/14) / 2(3/11) & \ldots & / 1(0/4)
\end{bmatrix} P_2 \\
- \begin{bmatrix}
17 - 1(1/4) - 2(2/5) - 3(3/14) - 2(2/11) & \ldots & - 1(1/4)
\end{bmatrix} P_3 \\
- \begin{bmatrix}
1(1/4) / 2(1/5) / 3(4/14) / 2(3/11) & \ldots & / 1(2/4)
\end{bmatrix} P_4 \\
= \begin{bmatrix}
938 - (1/4) 546 - 2(5) 319 - 3(14) 717 \ldots - 1(4/4)
\end{bmatrix}
\end{align*}$

$\begin{align*}
\begin{bmatrix}
1(1/4) / 1(0/5) / 4(3/14) / 3(3/11) & \ldots & / 2(1/4)
\end{bmatrix} P_1 \\
- \begin{bmatrix}
1(1/4) / 1(2/5) / 4(4/14) / 3(3/11) & \ldots & / 2(0/4)
\end{bmatrix} P_2 \\
- \begin{bmatrix}
1(1/4) / 1(2/5) / 4(3/14) / 3(2/11) & \ldots & / 2(1/4)
\end{bmatrix} P_3 \\
- \begin{bmatrix}
21 - 1(1/4) - 1(1/5) - 4(4/14) - 5(3/11) & \ldots & - 2(2/4)
\end{bmatrix} P_4 \\
= \begin{bmatrix}
1107 - (1/4) 546 - (1/5) 319 - (4/14) 717 \ldots - 2(4/4)
\end{bmatrix}
\end{align*}$

The reduced equations:

$\begin{align*}
\begin{array}{c|c|c|c|c|c}
\hline
\text{P}_1 & \text{P}_2 & \text{P}_3 & \text{P}_4 & \text{Sum} \\
\hline
12.174678 & -4.132465 & -3.409740 & -4.632465 & \phantom{-}\text{-198.857} \\
-4.132467 & 14.196107 & -4.659740 & -5.403893 & \phantom{-}36.737 \\
-3.409740 & -4.659738 & 12.829221 & -4.759738 & 112.511 \\
-4.632467 & -5.403893 & -4.759740 & 14.796107 & 123.068 \\
\hline
\end{array}
\end{align*}$
The sum of both right and left sides of the four equations are zero.

Supplement the set with the auxiliary equation:

\[ P_4 = 0. \]

And strike out the \( p_4 \) equation and the \( p_4 \) column, leaving the following equations for solution by inverting the matrix:

\[
\begin{align*}
P_1 - P_4 &\neq P_2 - P_4 &\neq P_3 - P_4 = \text{Sum} \\
\text{P1} & 12.174676 & -4.152465 & -3.409740 = 4.6325 \\
\text{P2} & -4.152465 & 14.196107 & -4.659740 = 5.4039 \\
\text{P3} & -3.409740 & -4.659738 & 12.829221 = 4.7898 \\
\end{align*}
\]

Obtain the inverse matrix by the abbreviated Doolittle method as described by Dwyer with results as follows:

Inverse \( C_{ij} \) matrix

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & 0.11158970 & 0.04793386 & 0.04706776 = -198.837 \\
2 & 0.0479336 & 0.10056652 & 0.04925647 = -56.757 \\
3 & 0.04706775 & 0.04925647 & 0.10835058 = 112.511 \\
\end{array}
\]

The following regression coefficients are computed by the method described by Snedecor (1946):

\[
\begin{align*}
P_1 - P_4 &= -18.6534 \\
P_2 - P_4 &= -7.6824 \\
P_3 - P_4 &= 1.0219 \\
P_4 &= 0 \\
\end{align*}
\]

Assume: \( P_1 \neq P_2 \neq P_3 \neq P_4 \neq 0 \), instead of \( P_4 = 0 \)

Then:

\[
\begin{align*}
P_1^1 &= P_1 - \frac{P_1 \neq P_2 \neq P_3 \neq P_4}{4} = -18.6534 - (-6.3285) = -12.3249 \\
P_2^1 &= P_2 - \frac{P_1 \neq P_2 \neq P_3 \neq P_4}{4} = -7.6824 - (-6.3285) = -1.3539 \\
P_3^1 &= P_3 - \frac{P_1 \neq P_2 \neq P_3 \neq P_4}{4} = 1.0219 - (-6.3285) = 7.3504 \\
\end{align*}
\]
\[ P_4^1 = P_4 - P_1 / P_2 / P_3 / P_4 = 0 - (-6.3285) = 6.3285 \]

Substitute these values in the \( (\mu / s_1) \) equations and solve for \( (\mu / s_1) \), with results as follow:

\[
\begin{align*}
\mu / s_1 &= 136.50 \\
\mu / s_2 &= 60.14 \\
\mu / s_3 &= 50.86 \\
\mu / s_4 &= 55.76 \\
\mu / s_5 &= 53.30 \\
\mu / s_6 &= 36.19 \\
\mu / s_7 &= 35.12 \\
\mu / s_8 &= 21.00 \\
\mu / s_9 &= 16.52 \\
\mu / s_{10} &= 25.08 \\
\end{align*}
\]

Finally impose the condition:

\[ S_{s_1} = 0 \]

and solve for \( \mu \) and \( s_1 \).

\[ 10 / S_{s_1} = S (\mu / s_1) \]

Since \[ S_{s_1} = 0, \]

\[ = 1/10 S (\mu / s_1) \]

\[ = 1/10 (490.47) = 49.05 \]

Substitute this value for \( \mu \) and solve for each \( s_1 \), with results as follow:

\[
\begin{align*}
s_1 &= 87.45 \\
s_2 &= 11.09 \\
s_3 &= 1.81 \\
s_4 &= 6.71 \\
s_5 &= 4.25 \\
s_6 &= -12.86 \\
s_7 &= -13.98 \\
s_8 &= -28.05 \\
s_9 &= -32.53 \\
s_{10} &= -23.97 \\
\end{align*}
\]

To obtain the number of cars for each seasonal interval

\[ Y_{ijk} = \mu / s_1 / (P_j) \]

There were a different number of holidays to be sampled in each seasonal interval

Let \[ w_i = \text{Number of holidays in the } i\text{th interval, and} \]
\[ a = \text{Number of periods per day} = 4 \]

Then

\[ Y_{ijk} = \text{aw}_i (\lambda \mu / s_1). \]

And where:

\[
\begin{align*}
  i = 1, w = 1 & \quad Y = (4) (1) (49.05 \neq 87.45) = 548 \\
  i = 2, w = 3 & \quad Y = (4) (3) (49.05 \neq 11.09) = 724 \\
  i = 3, w = 5 & \quad Y = (4) (5) (49.05 \neq 1.81) = 1,019 \\
  i = 4, w = 5 & \quad Y = (4) (5) (49.05 \neq 6.71) = 1,117 \\
  i = 5, w = 4 & \quad Y = (4) (4) (49.05 \neq 4.25) = 855 \\
  i = 6, w = 4 & \quad Y = (4) (4) (49.05 - 12.86) = 581 \\
  i = 7, w = 5 & \quad Y = (4) (5) (49.05 - 13.93) = 704 \\
  i = 8, w = 4 & \quad Y = (4) (4) (49.05 - 28.05) = 338 \\
  i = 9, w = 4 & \quad Y = (4) (4) (49.05 - 32.53) = 286 \\
  i = 10, w = 2 & \quad Y = (4) (4) (49.05 - 25.97) = 205
\end{align*}
\]

The total number of cars estimated for holidays on the Logan River in 1950 was 6,355.

**Analysis of variance**

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>Ssq.</th>
<th>Msq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitting constants ( R(\lambda, s_i, p_j) )</td>
<td>13</td>
<td>( R(\lambda, s \ p) )</td>
<td></td>
</tr>
<tr>
<td>Remainder</td>
<td>60</td>
<td>difference</td>
<td>s^2</td>
</tr>
<tr>
<td>Total</td>
<td>73</td>
<td>( Y^2 ijk )</td>
<td></td>
</tr>
</tbody>
</table>

\[ R(\lambda, s, p) = T_g / s_{igj} / p_{ijg} \]

Where

\[
\begin{align*}
  T_g & = \text{total cars counted on holidays} = 3,573 \\
  G_i & = \text{total cars counted in ith interval of season} \\
  G_j & = \text{total cars counted in the jth period of day} \\
  R(\lambda, s, p) & = 223,473.5191 \\
  Y^2 ijk & = 248,723
\end{align*}
\]

Thus:
The coefficient of variation then becomes:
\[
C.V. = \frac{s \times 100}{\bar{X}}
\]
\[
C.V. = \frac{20,34517}{49.05} \times 100 \approx 41.48\%
\]

**Variance of estimate of total cars, holidays**

The quantity it is desired to estimate is the population parameter \( \mu \) which can be expressed
\[
m = \frac{\sum_{i=1}^{4} \sum_{j=1}^{m} \sum_{s_{1}} \sum_{s_{2}} \sum_{s_{3}} \sum_{s_{4}} P_{ij}}{1/4 (P_{1} \neq P_{2} \neq P_{3} \neq P_{4})}
\]

Where \( \mathbf{W} = \mathbf{S w} \)

The normal equations are:
\[
N_{i} \hat{\mu} - \sum_{j} S N_{i} s_{1} j \neq \sum_{j} S N_{j} p_{j} \neq \bar{Y}_{i}...
\]
\[
N_{i} \hat{\mu} - \sum_{j} s_{1} s_{i} j \neq \sum_{j} S m_{i j} p_{j} \neq \bar{Y}_{i}...
\]
\[
N_{i} \hat{\mu} - \sum_{j} S N_{i} s_{1} j \neq \sum_{j} N_{j} p_{j} \neq \bar{Y}_{i}...
\]

Substitute \( m \) for \( \mu \) in these equations.

Then
\[
\mu = m - \frac{1}{4} \mathbf{W}^{-1} - 1/4 (P_{1} \neq P_{2} \neq P_{3} \neq P_{4})
\]

Using the appropriate relations to give a matrix with full rank, remove \( s_{10} \) and \( p_{4} \). This will give
\[
\begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1p} \\
S_{12} & S_{22} & \cdots & S_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
S_{1p} & & \cdots & S_{pp}
\end{bmatrix}
\]
Where \( p = 1/9 \times 3 = 13 \).

Example:

With information matrix

| \( \lambda \) | Season interval | Period of day | Total 
|---|---|---|---|
| \( \lambda \) | 1 | 2 | \( \ldots \) | 9 | 10 | A | B | C | D | Total 
| A | 74 | 4 | 5 | \( \ldots \) | 7 | 4 | 16 | \( \ldots \) | 17 | 21 | 3,578 
| B | 4 | 4 | 0 | \( \ldots \) | 0 | 0 | 1 | \( \ldots \) | 1 | 1 | 546 
| C | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) 
| D | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) 

The matrix becomes

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( P_1 )</th>
<th>( \ldots )</th>
<th>( P_5 )</th>
</tr>
</thead>
</table>
| A | 74 | 2.0000 | -1.0000 | \( \ldots \) | -1.0000 | -2.5 | \( \ldots \) | -1.6 | = 1 
| B | 4 | 3.8919 | -0.3243 | \( \ldots \) | -0.4324 | 0 | \( \ldots \) | 0 | = 0 
| C | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) 

Find the first element (\( c_{11} \)) of the inverse of this matrix which satisfies the equations:

\[
\begin{align*}
  c_{11} s_{11} &\neq c_{12} s_{12} \neq \ldots \neq c_{1p} s_{1p} = 1 \\
  c_{11} s_{12} &\neq c_{12} s_{22} \neq \ldots \neq c_{1p} s_{2p} = 0 \\
  \vdots &\neq \vdots \\
  c_{11} s_{1p} &\neq c_{12} s_{2p} \neq \ldots \neq c_{1p} s_{pp} = 0
\end{align*}
\]

The equations are most easily solved by iteration, and \( c_{11} \) found to be 0.011. The variance of \( \hat{m} \) will then be \( c_{11} s^2 \).

An estimate of the population parameter (\( \hat{m} \)) may be written

\[
\hat{m} = \frac{S}{\sum \frac{w_i}{w}} \neq 1/4 (P_1 \neq P_2 \neq P_3 \neq P_4), \text{ or}
\]

\[
\hat{m} = \text{Total estimated cars} \neq \frac{4 \times S w_1}{w}
\]
Substituting values

\[ \hat{m} = 42.939 \]

Fiducial limits of the population parameter \((m)\), for each period, with 60 degrees of freedom and at the 95 percent confidence interval can be expressed

\[ m = \hat{m} \pm t_{0.05} \left(\frac{s^2}{m}\right) \]

Where

\[ s^2 \hat{m} = c_{11} (s^2) \]

Substituting values

\[ m = 42.939 \pm 2.011(421.4241) \]

\[ m = 42.939 \pm 4.306 \]

For the entire season of 148 periods, then

\[ m = 6.355 \pm 637, \text{ or} \]

\[ m = 6.355 \pm 10.02\% \]

Estimation of the number of cars on the Logan River Study Area, (Weekdays), 1950.

Following the procedure outlined above for holidays, with information matrix

| \( \mu \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 1 | 12 | 9 | 9 | 14 | 7 | 5 | 4 | 1 | 11 | 11 | 16 | 23 | | | | | | | | | | | | | | | |
| 2 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 3 | 5 | | | | | | | | | | | | | | | |
| 3 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 5 | | | | | | | | | | | | | | | |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 4 | 3 | 4 | | | | | | | | | | | | | | | |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | | | | | | | | | | | | | | | |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | | | | | | | | | | | | | | | |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | | | | | | | | | | | | | | | |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | | | | | | | | | | | | | | | |
| A | 11 | 2 | 1 | 2 | 3 | 1 | 1 | 1 | 0 | 11 | 0 | 0 | 0 | | | | | | | | | | | | | | |
| B | 11 | 2 | 1 | 2 | 4 | 1 | 1 | 0 | 0 | 11 | 0 | 0 | 0 | | | | | | | | | | | | | | |
| C | 16 | 3 | 3 | 3 | 3 | 2 | 1 | 2 | 0 | 0 | 0 | 16 | 0 | | | | | | | | | | | | | | |
| D | 23 | 5 | 4 | 3 | 4 | 3 | 2 | 1 | 1 | 0 | 0 | 0 | 23 | | | | | | | | | | | | | | |

with total Saqs = 42,855,
total weekdays = 77,
and with
\[ w_1 = 0 \quad w_6 = 10 \]
\[ w_2 = 10 \quad w_7 = 9 \]
\[ w_3 = 9 \quad w_8 = 10 \]
\[ w_4 = 9 \quad w_9 = 10 \]
\[ w_5 = 10 \quad w_{10} = 0. \]

Compute the estimate of total cars on the Logan River on weekdays as 5,419 cars. By season interval compute the estimates as follow:

\[ s_2 = 766 \quad s_6 = 980 \]
\[ s_3 = 922 \quad s_7 = 528 \]
\[ s_4 = 837 \quad s_8 = 376 \]
\[ s_5 = 799 \quad s_9 = 211 \]

Analysis of variance

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>Ssq.</th>
<th>Msq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitting constants R (( \mu, s_1, p_j ))</td>
<td>11</td>
<td>33,786.9495</td>
<td></td>
</tr>
<tr>
<td>Remainder</td>
<td>50</td>
<td>9,068.0505</td>
<td>171.0952</td>
</tr>
<tr>
<td>Total</td>
<td>61</td>
<td>42,855.0000</td>
<td></td>
</tr>
</tbody>
</table>

The coefficient of variations becomes:

\[ C.V. = \frac{s}{\mu} \times 100 \]

\[ C.V. = \frac{13.0805}{17.78} \times 100 = 73.77\% \]

Fiducial limits

With the cill value equal to 0.020, degrees of freedom equal 49, and at the 95 percent confidence interval, find the fiducial limits of the estimate of total cars on weekdays as

\[ 5,419 \pm 1,139 \text{ cars, or} \]

\[ 5,419 \pm 21.02\%. \]
Variance of estimate of total anglers' cars, holidays and weekdays combined, Logan River, 1950

The total estimated number of anglers' automobiles, holidays and weekdays combined was 17,774. The variance of that estimate can be expressed

\[ V(T) = (an_h)^2 \sigma_{sh}^2 + (n_w)^2 \sigma_{sw}^2 \]

Where

\[ n_h = \text{Total holidays} \]
\[ n_w = \text{Total weekdays} \]
\[ x_h = \text{Average cars per period, holidays} \]
\[ x_w = \text{Average cars per period, weekdays} \]
\[ x = \text{Total cars on river, entire season} \]
\[ \sigma_{x_h}^2 = c_{11h} \sigma_h^2 \]
\[ \sigma_{x_w}^2 = c_{11w} \sigma_w^2 \]

Then \[ V(X) = (4 \times 37)^2 (0.011) (421.4247) + (4 \times 77) (0.020) (171.0952) = 428,155.494 \]

and fiducial limits of the estimated total cars at the 95 percent confidence interval (approximately) can be computed.

Total cars = 11,774 ± 1,285 \[ \sqrt{428,155.494} \]

= 11,774 ± 1,285 or

= 11,774 ± 10.9%
Table 10. Distribution of fishing pressure by seasonal interval and weekly division, Logan River, 1950

<table>
<thead>
<tr>
<th>Interval</th>
<th>Date</th>
<th>Percent of total fishing pressure</th>
<th>Seasonal</th>
<th>Holiday</th>
<th>Weekly</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>June 17</td>
<td>4.65</td>
<td>4.65</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>June 18 - June 30</td>
<td>12.66</td>
<td>6.15</td>
<td>6.51</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>July 1 - July 14</td>
<td>16.49</td>
<td>8.65</td>
<td>7.83</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>July 15 - July 28</td>
<td>16.80</td>
<td>9.49</td>
<td>7.11</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>July 29 - Aug. 11</td>
<td>14.05</td>
<td>7.26</td>
<td>6.79</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Aug. 12 - Aug. 25</td>
<td>15.26</td>
<td>4.93</td>
<td>8.32</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Aug. 26 - Sept. 8</td>
<td>10.46</td>
<td>5.98</td>
<td>4.48</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Sept. 9 - Sept. 22</td>
<td>6.06</td>
<td>2.87</td>
<td>3.19</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Sept. 23 - Oct. 6</td>
<td>4.05</td>
<td>2.26</td>
<td>1.79</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Oct. 7 - Oct. 8</td>
<td>1.72</td>
<td>1.72</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td>100.00</td>
<td>53.96</td>
<td>46.02</td>
<td></td>
</tr>
</tbody>
</table>

* No weekday in interval.
Table 11.  Estimate and corrected estimate of total anglers' cars by seasonal interval and weekly division, Logan River, 1950.

<table>
<thead>
<tr>
<th></th>
<th>Season interval</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Estimated cars, holidays</td>
<td>548</td>
<td>724</td>
<td>1,019</td>
<td>1,117</td>
<td>855</td>
<td>581</td>
<td>704</td>
<td>338</td>
<td>266</td>
<td>203</td>
</tr>
<tr>
<td>Estimated cars, weekdays</td>
<td>...</td>
<td>766</td>
<td>922</td>
<td>837</td>
<td>799</td>
<td>960</td>
<td>528</td>
<td>376</td>
<td>211</td>
<td>...</td>
</tr>
<tr>
<td>Estimated cars, totals</td>
<td>546</td>
<td>1,488</td>
<td>1,939</td>
<td>1,952</td>
<td>1,652</td>
<td>1,859</td>
<td>1,250</td>
<td>712</td>
<td>475</td>
<td>201</td>
</tr>
<tr>
<td>Average length of actual</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fishing day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of legal 16 hour</td>
<td>15.75</td>
<td>15.75</td>
<td>15.63</td>
<td>15.32</td>
<td>14.88</td>
<td>14.35</td>
<td>15.73</td>
<td>15.13</td>
<td>12.50</td>
<td>12.12</td>
</tr>
<tr>
<td>fishing day</td>
<td>98.3</td>
<td>98.3</td>
<td>97.7</td>
<td>95.7</td>
<td>93.0</td>
<td>89.7</td>
<td>85.8</td>
<td>82.1</td>
<td>78.1</td>
<td>75.7</td>
</tr>
<tr>
<td>Corrected estimated total</td>
<td>537</td>
<td>709</td>
<td>994</td>
<td>1,067</td>
<td>793</td>
<td>519</td>
<td>802</td>
<td>276</td>
<td>206</td>
<td>152</td>
</tr>
<tr>
<td>cars, holidays</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Corrected estimated total</td>
<td>537</td>
<td>1,463</td>
<td>1,896</td>
<td>1,858</td>
<td>1,536</td>
<td>1,598</td>
<td>1,056</td>
<td>585</td>
<td>371</td>
<td>152</td>
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<tr>
<td>cars, weekdays</td>
<td>...</td>
<td>754</td>
<td>901</td>
<td>791</td>
<td>743</td>
<td>879</td>
<td>453</td>
<td>309</td>
<td>165</td>
<td>...</td>
</tr>
<tr>
<td>Corrected estimated total</td>
<td>537</td>
<td>1,463</td>
<td>1,896</td>
<td>1,858</td>
<td>1,536</td>
<td>1,598</td>
<td>1,056</td>
<td>585</td>
<td>371</td>
<td>152</td>
</tr>
<tr>
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<td>Title and Details</td>
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<tr>
<td>Dwyer, Paul S.</td>
<td>The solution of simultaneous equations. Psychometrika 6(2):101-129. (original not seen)</td>
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<td>The statistical analysis of creel census data. Amer. Fish. Soc. Trans., 76:290-300.</td>
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