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Modeling the Extended Low and Sudden High Periods of Activity in Infectious Diseases

Introduction

Since 2019, COVID-19 has caused over 6 million deaths worldwide [1]. This, and other infectious diseases, have deeply impacted the lives of individuals all across the world both health wise and economically.

Understanding how a disease might reemerge after a long period of low activity has profound consequences in epidemiology. Predicting activity will help with the planning for deadly pandemics/epidemics like COVID-19 and Influenza.

Background

If susceptible and infected individuals are homogeneously mixed, we can assume that the risk of a susceptible person catching the pathogen from an infected person is higher when individuals are in close physical proximity, which can be interpreted as if the population is at a high density. Conversely, at low population densities, the risk of catching the pathogen from an infected person may be relatively lower.

This means that the disease has a positive density-dependent relationship between the per capita growth rate of infected individuals and the population density. This relationship is reminiscent of the Allee effect in ecology.

The Model

To model the positive density dependent relationship, we used a a Stochastic Differential Equation (SDE) with the deterministic part coming from an Allee Effect Ordinary Differential Equation. This equation has been shown to model COVID-19 well [2].

We denote *I* as the number of active infected individuals in the human population, and consider the deterministic equation for its evolution in time:

$$\frac{dI}{dt} = h + rI^{1+q}\left(1 - \frac{I}{K}\right) - \mu I$$

The parameters in this equation include a positive immigration rate of infected individuals, h > 0, the recovery rate, μ > 0, and the maximum possible number of individuals that can catch the disease before a health infrastructure emergency is declared, K > 0. Table 1 shows the parameter estimates selected in our simulations and the parameter estimated that modeled COVID-19 well with a logistic equation [2].

The Stochastic Model is the deterministic equation shown below, but with an added term, σI, that is associated with environmental noise in the system:

$$\frac{dI}{dt} = h + rI^{1+q}\left(1 - \frac{I}{K}\right) - \mu I + \sigma I\,dW$$

Adding environmental noise to the Allee Effect Ordinary Differential Equation produced graphs similar to those in Figures 2 and 3.

Results

Using the Stochastic Model, we made simulated runs that mirrored the pattern of COVID-19 data seen in Utah (see Figure 1).

Table 1- Comparison of parameter estimates.

Parameter Estimates from Model	Parameter Estimates from Paper [2]
h = 0.02 (arbitrarily chosen)	h = N/A
0.07 > r > 0.08	0.099 > r > 0.195
0.009 > q > 0.015	0.6 > q > 1.2
μ = 0.045 [4]	N/A
0.175 > σ > 0.225	N/A

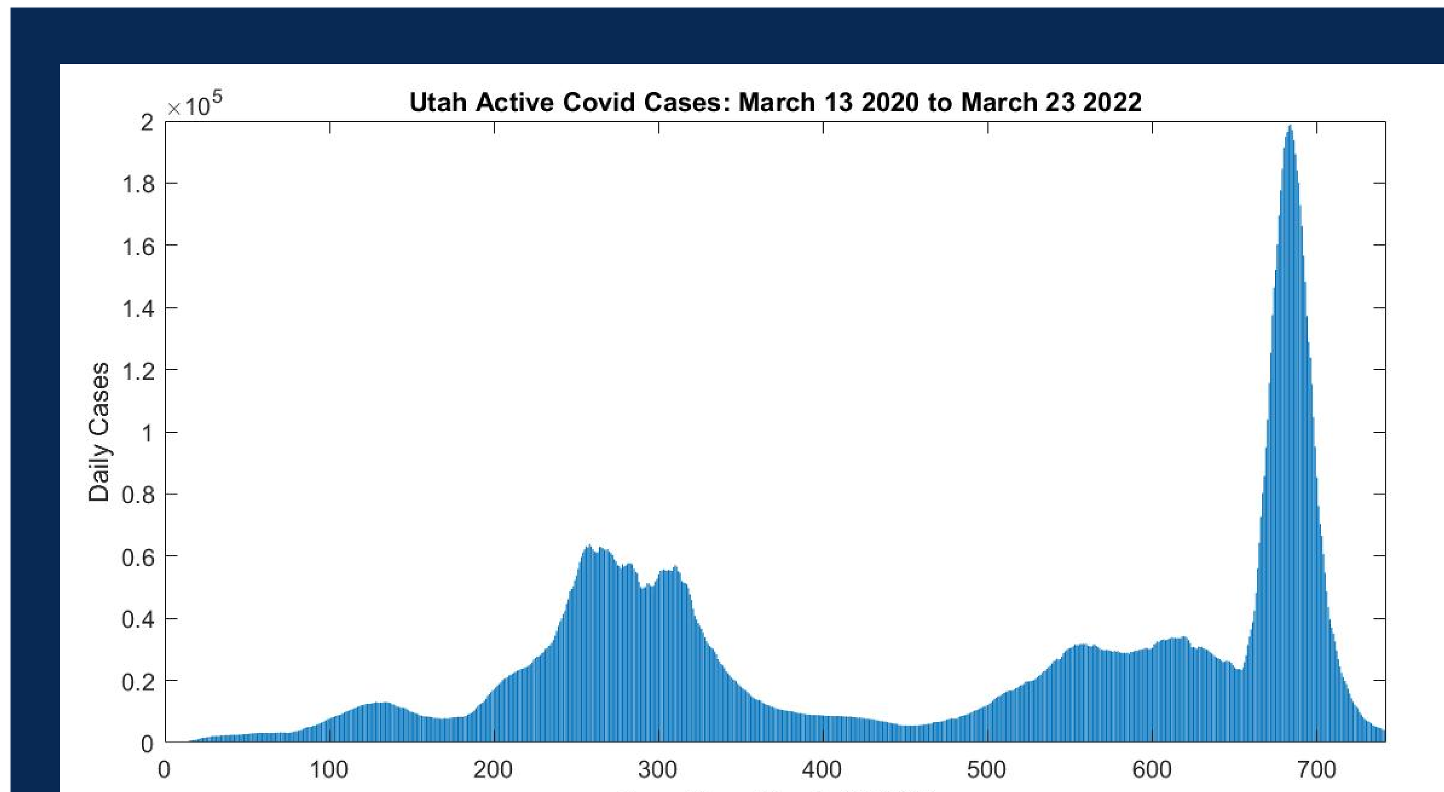

Figure 1- *Active Infected Individuals with COVID-19 in Utah (taken from 'Utah Department of Health' website)*

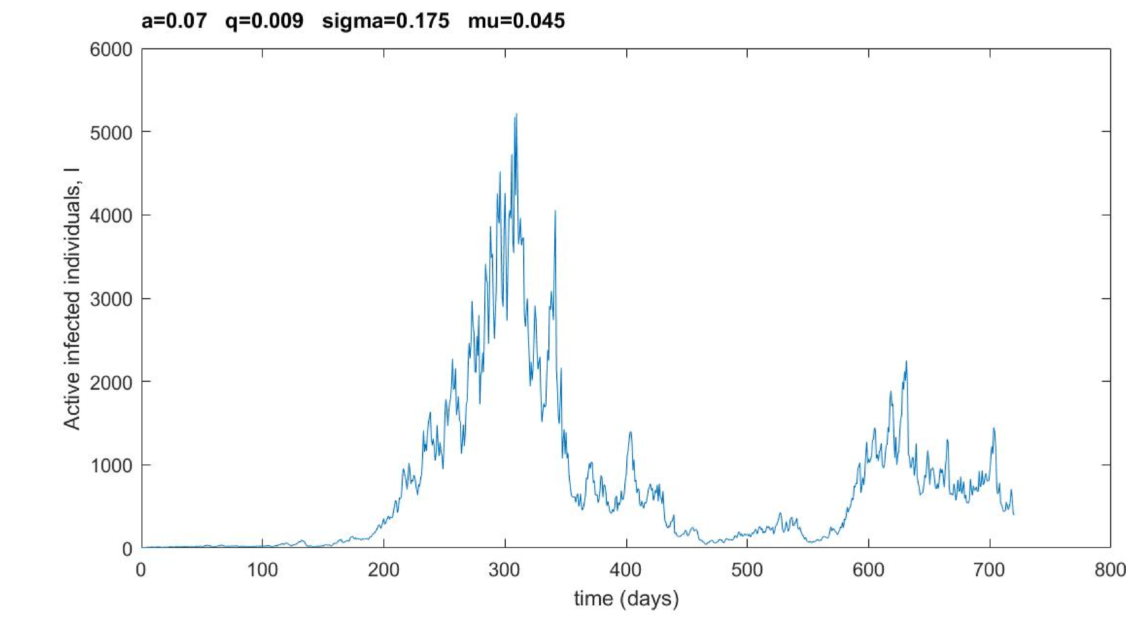

Figure 2- Sample run from Model. r = 0.07, q = 0.009, σ = 0.175

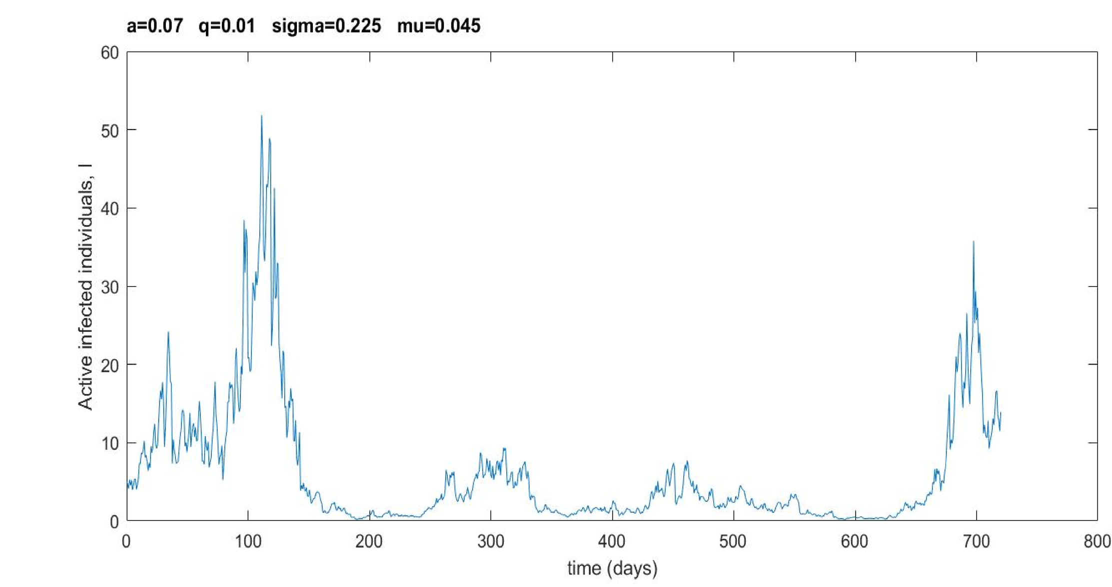

Figure 3- Sample run from Model. r = 0.07, q = 0.01, σ = 0.225,

Conclusions

The model with added environmental noise generates sample runs that have similar patterns of "lows" and "highs" to the real-world COVID-19 data from the state of Utah. In addition, many parameters give a biological interpretation that can be applied to the data.

Furthermore, the model reveals that **stochasticity** is responsible for the ups and downs in the paths, a feature absent from the typical deterministic model.

Future Work

For the current model, we assumed that r, or the growth rate, was a constant. There is evidence to suggest that the growth rate often depends on density with an inverse relationship. Making the growth rate a function on density would likely improve the current model.

Another term, d, has been used in both the Allee Effect equation and to model COVID-19 with a logistic equation [2]. While parameter d doesn't have a biological interpretation, it is a scaling parameter that sometimes helps the data fit better.

$$(1 - \frac{I}{K})^d$$

References
1. https://www.worldometers.info/coronavirus/coronavirus-death-toll/
2. E. Pelinovsky, A. Kurkin, O. Kurkina, M. Kokoulina, and A. Epifanova (2020) *Logistic equation and COVID-19*, Chaos, Solitons and Fractals, 140, 110241.5
3. L.F. Gordillo and P.E. Greenwood (2022) Allee effects plus noise induce population dynamics resembling binary Markov highs and lows
4. Paul, S., Lorin, E. Estimation of COVID-19 recovery and decease periods in Canada using delay model. *Sci Rep* **11**, 23763 (2021). https://doi.org/10.1038/s41598-021-02982-w