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On Random and Systematic Errors of a Star Tracker

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ABSTRACT

The error of the attitude of many modern star trackers is determined by systematic errors. In this paper we consider the effect of random and different systematic errors of sensors on the attitude accuracy of star trackers. The results of modeling of principal systematic errors, first of all, due to underestimation of the dark current inhomogeneity, are discussed. It is shown that only this systematic error can decrease the attitude accuracy of a star tracker by a factor of 3–4. Methods of taking into account the main systematic errors and of the decreasing their effect on the star trackers attitude accuracy are proposed.

INTRODUCTION

Recently, more attention is paid to the study of systematic errors and calibrations of stars trackers[1].

Attitude errors of a star tracker can be divided into two classes: the first class – random errors (they can't be avoided by any signal processing), while the second – systematic errors that could be significantly reduced by calibrations and by careful image processing.

Modern star trackers can reach higher attitude accuracy without changing the design and electronics if systematic errors are compensated by the signal processing. The actual-to-calculated random error ratio is about 10 – 30 for modern star sensors.

Accurate taking into account the errors by means of the star tracker image processing will allow us to reach the desired accuracy with less weight and cost of the equipment.

Note that this approach to the design of star trackers requires accurate calibrations of the device, both ground- and space-based. Also it is necessary to measure and use some additional parameters while the image processing (for example, measuring the temperature of the photo sensor, or color-indices of stars in the catalog may be needed, etc.).

The designers of micro- and nano-satellites impose requirements on star trackers such as miniaturization with higher update frequency and attitude accuracy. The fulfillment of these requirements results in the diameter reducing of the star tracker lens and this leads to a decrease in the signal from the star.

Attitude accuracy depends mainly on the ratio of the signal from the star to the sum of random and systematic noises. Since there is no way to increase the signal, to keep or even raise the signal-to-noise ratio it is necessary to reduce the systematic part of the noise.

In some cases, in order to increase the attitude accuracy, image processing of stars with the low (5-10) signal-to noise-ratio is required.

It is possible to improve the attitude accuracy by changing the star tracker design. However, in this paper we do not consider the problems of optimal design of star trackers, but concentrate on the image processing.

MAIN SOURCES OF ERRORS

The measured coordinates of a star directly depend on the image brightness distribution on the sensor's surface. Hence, inexact reduction of this distribution leads to an additional error in the position of observed stars and consequently in the star tracker attitude.

The problem of correct reduction of the image obtained with a sensor is particularly important for high-precision instruments. This implies taking into account a number of possible errors sources. Below the most important of them are discussed.

Random errors

Random errors are related to the sensor intrinsic noise of different nature and with the shot noise of the flux from the star. They are fundamentally unavoidable, but their effect can be significantly reduced:

1. The effect of shot noise decreases with the increase of exposure time if it is possible for the particular experiment.

2. The effect of random dark signal component can be reduced by cooling the sensor (thereby reducing the average dark current).

3. Readout noise of the sensor can be reduced by selecting the appropriate mode of operation of the device, for example, reading out with a lower frequency.

Systematic errors

Systematic errors are due to both properties of the particular sensor and the technique of the star image processing. We distinguish the following sources of errors:

1. *Image pixelization*. It leads to the fact that while calculating the star's image center (photocenter) position, weighted sums of signals in pixels are used instead of the convolution of the model point spread function (PSF) with the obtained image. This error can be eliminated for a certain PSF by calculating the correction to the measured coordinates. This correction is a function of the measured coordinates.

2. *The limitation of the analyzed area while coordinates measuring*. This leads to the fact that the PSF wings are not taken into account in the analysis when measuring the coordinates using bounded area of the image. As a result, the measured coordinates are systematically shifted to the center of the analyzed area. The typical value of the error might be 0.1 – 0.01 pixel when analyzing the area of 3 – 5 pixels. To eliminate this error it is possible to calculate the correction using the measured position of a photocenter inside the central pixel of the area.

3. *The inhomogeneous sensitivity of the sensor elements* (i.e. Pixel Response Non-Uniformity). Typically, the value of such inhomogeneity of the sensitivity is a few percent, which is very substantial for high-precision instruments. It is taken into account by measuring the flat field during the instrument calibration and by calculation of the relative sensitivity ratio for each pixel. Then, during the sensor operation, these coefficients are used to correct the measured signal in each pixel.

4. *Inhomogeneous sensitivity within a sensor element*. This effect is caused by the pixel structure of the sensors with front illumination through the electrodes. It is significant in the case when the star image size is about one pixel. The error can be taken into account by means of calibration, which determines a correction to the photocenter position when it shifts within a pixel. Similarly, we can take into account the dependence of the sensitivity of the sensor element on the incident angle.

5. *The inhomogeneity of dark current in various sensor elements*. Error caused by this factor may be substantially reduced, either by cooling the device to a temperature at which dark current can be neglected, or by calculating the accumulated dark signal based on the calibration data of dark current of sensor elements.

The applied model of the dark current does not include some effects that contain so-called telegraph noise, which is a spontaneous change in the dark current of sensor elements[2]. This can increase dispersion of the pixel dark signal by a factor of tens. In order to take into account such systematic errors special methods of processing need to be developed or the stars images containing such pixels should be excluded.

6. *The effect of "hot" pixels* – elements with abnormally high dark current. "Hot" pixels are due to defects of the silicon structure formed mainly through removal of atoms from the lattice by high-energy cosmic-ray and the solar wind particles. These elements generally have a different from the other temperature dependence of the dark current. Their distribution over the photosensitive area of the sensor varies with time, primarily due to the addition of such new elements. To take them into account usually a list with the coordinates and the dark current is made that is applied during the process of photometric reduction. The easiest way to eliminate this effect in the analysis of sensor data is the exclusion from the consideration of stars which images contain "hot" pixels.

7. *The displacing effect of electron traps*. This effect results in a systematic shift of the photocenter, which is the greater, the smaller the signal from the star is. The effect results also in the generation of high dark charge in the area behind a bright star. The effect is directly related to the procedure of the charge transfer and occurs only in the CCD.

8. *The aberrations of the optical sensor system that does not effect on PSF form, but effect on the position of the image on the focal plane (field curvature, distortion, declination and displacement of the sensor etc.)*. These aberrations entail a shift of the star image and are eliminated in the image processing by application of correction polynomials depending on the coordinates in the focal plane. The parameters of polynomials are defined during the geometric calibrations.

9. *The aberrations of the optical sensor system that lead to PSF form dependence on the image position on the focal plane (coma, astigmatism)*. The errors caused by these effects are also eliminated by means of geometric calibrations and by applying appropriate correction polynomials.

10. *Chromatic aberration*. They lead to a shift of the measured coordinates due to the change of the effective wavelength of the radiation of the stars with different spectra. Chromatic aberration results in the fact that beams of different wavelengths are focused at different points, so PSF looks differently at different wavelengths. As a result the photocenter position of the star depends on its spectral class.

In order to take into account the influence of the spectrum of each star it is necessary to store the additional value characterizing spectrum in the navigation catalogue. This value can be a color index of stars, such as B-V.

11. *The differences in the spectral sensitivity of individual sensor elements* (i.e. Spectral Response Non-Uniformity). The heterogeneity of the pixels sensitivity appears differently in different wavelengths. To take it into account, calibration of the sensor sensitivity in several wavelengths is required. While image processing, these monochromatic sensitivity maps are combined with weights depending on the color index of the star.

13. *Inhomogeneity of the signal gains*. In CMOS device, signal amplifiers are present in each element of the sensor; ADCs are present usually in each column. CCDs may have few output amplifiers (typically 1 to 4) and the same quantity of ADC. To take into account the spread of signal transformation it is necessary to make a list or map of their gains.

14. *Bias non-uniformity*. To eliminate this error the map of bias needs to be stored and taken into account while image processing.

Some of the systematic errors cannot be distinguished as they produce the same effect, e.g. the shift of measured center as a function of the position. In this case a general correction for all these factors is applied.

Further, we assume that these systematic errors are independent of each other and the total dispersion is the sum of the dispersions of the individual errors.

CONSIDERATION OF PARTICULAR CASES

Pixelization and bounded area errors

The error related to the bounding of the analyzed area for the coordinate measurements and the error due to the pixelization can be considered together. To estimate the value of this error and its dependence on the position, following calculation was carried out.

In general case, the formula for determining the coordinates of a photo center can be written as a convolution of the PSF with the corresponding coordinate:

$$\bar{x}_c = \frac{\iint_{-\infty}^{+\infty} x f(x-x_c, y-y_c)dxdy}{\iint_{-\infty}^{+\infty} f(x-x_c, y-y_c)dxdy}, \quad \bar{y}_c = \frac{\iint_{-\infty}^{+\infty} y f(x-x_c, y-y_c)dxdy}{\iint_{-\infty}^{+\infty} f(x-x_c, y-y_c)dxdy}, \qquad (1)$$

where $f(x-x_c, y-y_c)$ is the distribution function of the star's signal at the focal plane, x, y – frame coordinates, x_c, y_c – the coordinates of the center of the star image (photocenter) in the case of an "ideal" device, free from systematic errors described above. If the function $f(x,y)$ is symmetric with respect to the arguments, the coordinates of the "ideal" image center and calculated by the formula (1) are coincide.

Actually, the photocenter coordinates are calculated on the basis of counts per pixel in the small frame area. In most cases, the formula of the sum of the weighted average of the coordinates with weights being proportional to the signal at each pixel is used:

$$\tilde{x}_c = \frac{\sum_{i,j} x_j F_{i,j}}{\sum_{i,j} F_{i,j}}, \quad \tilde{y}_c = \frac{\sum_{i,j} y_j F_{i,j}}{\sum_{i,j} F_{i,j}}, \qquad (2)$$

where i, j are the numbers of row and column of the pixel in the frame, $F_{i,j}$ – the signal at that pixel, x_j, y_i – coordinates of the pixel center, the summation is performed over all the pixels of the area. Weights can be chosen to be proportional to a function of the signal in the pixel, such as a squared signal.

Taking a specific model of distribution$f(x,y)$, the measured signal in the pixel can be calculated by means of the corresponding integral of the function:

$$F_{i,j} = \int_{y_i-\frac{\Delta}{2}}^{y_i+\frac{\Delta}{2}} \int_{x_j-\frac{\Delta}{2}}^{x_j+\frac{\Delta}{2}} f(x-x_c, y-y_c)dx\,dy. \qquad (3)$$

Shift of x and y, due to the discretization and bounding of the analyzed area can be calculated using the following formulae:

$$\delta x = \tilde{x}_c - \bar{x}_c, \quad \delta y = \tilde{y}_c - \bar{y}_c. \qquad (4)$$

As PSF a Gaussian with different values of the parameter α, which describe the width of the image, was taken:

$$f(x,y) = \frac{S}{0.62\pi\alpha^2} e^{-\frac{x^2+y^2}{0.62\alpha^2}}, \qquad (5)$$

where S is the total signal from the star, α – the radius of the circle with 80% of the signal from the star.

Figure 1 shows the dependence of standard deviation of the photo centers positions calculated by (2) for areas of the coordinates determination of three sizes: 3×3, 5×5, 7×7 pixels. Both axes have logarithmic scale. We can see that there is a minimum of the error for image sizes of about 2 pixels.

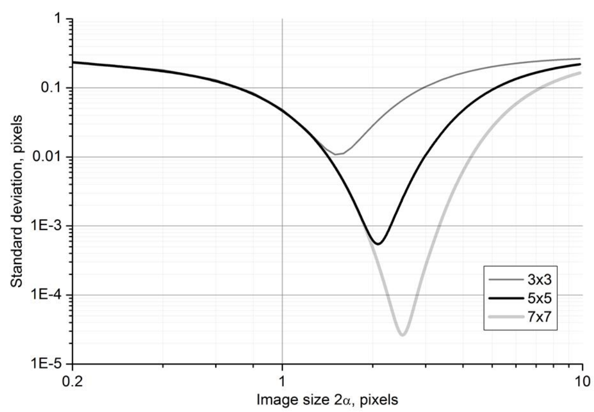

Figure 1. Error of star image position (by formula(2)) depending on image size

Figures 2 and 3 show the coordinates shift functions determined by (4) for areas of 5x5 pixels depending on the actual position of the star image in the central pixel. Functions are built for several sizes of the star image; they are listed in the legend and are in pixels.

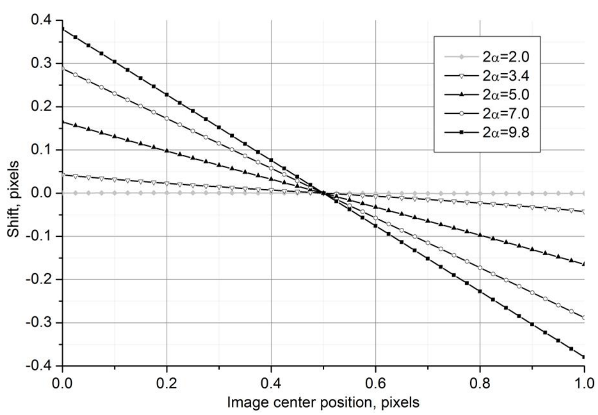

Figure 2. Coordinates shift for large sizes of stars images

Note that, by measuring the PSF and its variation across the field of view, the corrections to the measured coordinates can be calculated in the same way in case of aberration errors.

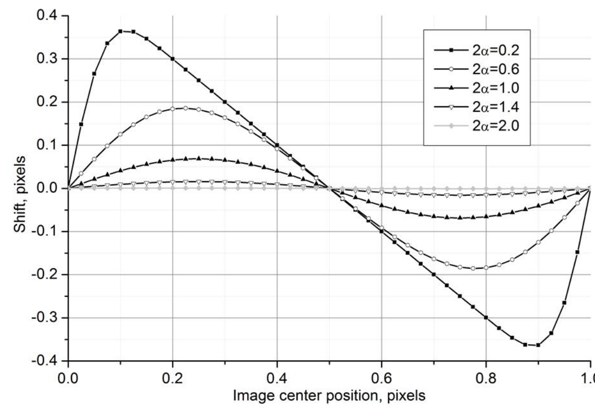

Figure 3. Coordinates shift for small sizes of stars images

Formula (2) can be applied with square-law weighting coefficients:

$$\tilde{x}_c = \frac{\sum_{i,j} x_j F_{i,j}^2}{\sum_{i,j} F_{i,j}^2}, \quad \tilde{y}_c = \frac{\sum_{i,j} y_j F_{i,j}^2}{\sum_{i,j} F_{i,j}^2}. \qquad (6)$$

Simulation shows that (6) is more robust and gives less outliers at small values of the signal-to-noise ratio, but the (2) is more exact at large values. According to the (6), dependence of the coordinate determination error on the image size is shown in Figure 4.

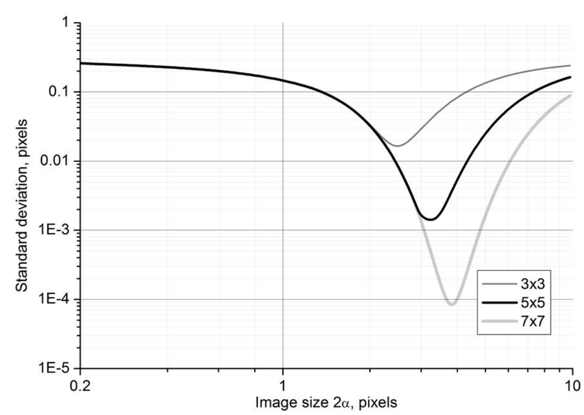

Figure 4. Error of star image position (by formula (6)) depending on image size

Random error

Assume that all systematic errors in the device are zero or taken into account while processing. Then, the coordinate error is inversely proportional to the signal-to-noise ratio. In formulae (2) and (6) for the coordinates determination, the factor is close to 1.

$$\sigma_{x,y} \cong \frac{\Delta}{SNR}, \quad (7)$$

where Δ is the pixel size, SNR – signal-to-noise ratio.

The signal-to-noise ratio is determined by the following expression:

$$SNR \cong \frac{S_{n\times n}}{\sqrt{S_{n\times n} + n^2\left(\sigma_{bg}^2 + \sigma_{th}^2 + \sigma_{rd}^2\right)}}, \quad (8)$$

where $S_{n\times n}$ is a signal from the star contained within a square $n\times n$ pixels, σ_{bg}^2 – dispersion of the sky background noise, σ_{th}^2 – dispersion of the dark signal, σ_{rd}^2 – dispersion of the readings noise, all values are expressed in electrons.

According to (7) and (8) to reduce the coordinates errors it is necessary to reduce the size of the area where the coordinates are determined, to reduce the level of dark current and dispersion of the readings noise, and to choose a brighter stars.

Dispersion of the readings noise is a feature of the device and determined by operation mode, mainly by the frequency. Therefore, the possibilities for controlling the reading noise are limited.

Inhomogeneity of dark current

To determine the effect of inhomogeneity of dark current of the sensor elements, the algorithm of determining the coordinates of photocenters for model images with inhomogeneous dark current was tested. The series of images, where the accumulated dark signal of the element was drawn at random from a Gaussian distribution with an average dark level of 250 electrons per cell, was generated. The standard deviation to the average level ratio was varied from 0 to 0.25 for the various series of images. In the procedure of determining the coordinates, the dark signal was found in the part of the frame which was not occupied with a star image (along the perimeter of the square containing the image of the star). For each series of a few thousand of frames standard deviation of photo center position was calculated.

Figure 5 shows the dependence of error of the photocenters positions on the value of dark signal non-uniformity (DSNU). Here DSNU is the dark current standard deviation to mean value ratio. The curves are shown for the three signal levels that, in the absence of dark current inhomogeneity, have given the signal-to-noise ratio – *SNR*. The results for two variants of the procedure of determining the coordinates photo centers are shown: Pow=1 by (2) and Pow=2 by (6).

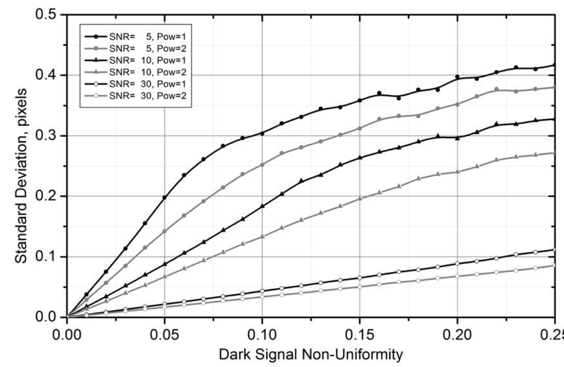

Figure 5. Effect of dark current inhomogeneity on the position error

At high signal levels, the coordinate determination error caused by the dark current inhomogeneity is directly proportional to the value of DSNU (two lower curves in Figure 5). At low signal levels (*SNR* = 3) and high DSNU values, given dependence differ from linear. This is due to the fact that at the large dark current inhomogeneity, the probability of the star image detecting decreases. While testing, for the star image detection, signal over the noise exceeding criterion of $SNR_{det}>3$ was applied. Figure 6 shows the probability of the star image detection with *SNR*=5 and *SNR*=10 depending on dark current non-uniformity.

Initial *SNR*=3 is reduced to 0.9 and the initial *SNR*=10 – to 3.0 with increasing of DSNU to 0.25.

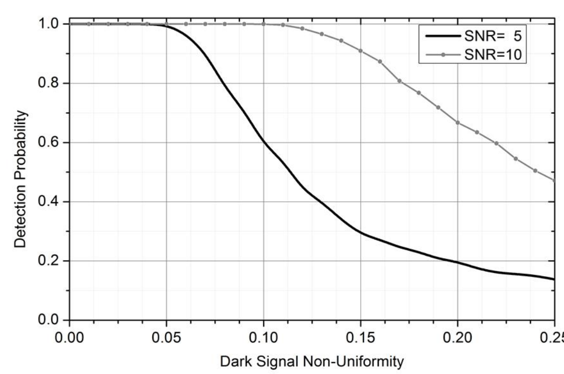

Figure 6. Effect of dark current inhomogeneity on the star image detection

The photocenter position error for stars with *SNR*=10, specified by the Poisson noise only according to the (1), should be 0.1 pixel. At the same time in case of DSNU equal to 0.25 error in determining the photocenter position is about 0.3 pixels (see Figure 5). Thus the position error is 3 times greater than in case of a homogeneous dark current.

Cooling of a sensor

The possible solution of precision degeneracy problem caused by inhomogeneity of the mean dark signal within sensor is the cooling of the sensor down to temperatures that implies neglecting the dark current during analysis. Thus, accordingly to the properties of E2V CCD 47-20 BI AIMO[3] (which we use in our design of highly precise star tracker), it's cooling down to -20°C is enough to suppress mean dark signal in each element to 1 electron per second. However, low temperature does not affect "hot" elements of the sensor that have anomalously high level of a dark signal generation.

At the same time, the possibility of cooling imposes additional requirements on the design of a star tracker and its space platform. First of all the problem of (thermoelectric) cooler mounting and powering have to be solved. Then, the system of heat dumping is needed. These requirements are especially difficult to satisfy while one manufactures small sized micro-trackers for micro- and nano-satellites.

Next, if CCD is used in star tracker design as a sensor, the problem of charge transfer inefficiency becomes important. Image readout from CCD is realized through parallel transfer of collected charge between sensor's elements. However, charge transfer efficiency is highly depended on the CCD temperature and decreases with cooling. The mechanism of such behavior is based on existing of electronic traps – the defects in sensor's lattice. Such traps are able to "capture" input electrons and "emit" them after short time. Various traps characterized by different values of charge capacity and typical times of capture and emission processes[4].

As a result, traps add some blur to final CCD image as it shown in Figure 7. In this example the dark signal of "hot" element passed a trap while being read. The trap takes part of the charge from "hot" pixel and slowly emits electrons afterwards.

The blurred image obviously leads to the decrease of star tracker accuracy. The strength of the effect depends on the relations between typical times of charge capture, emission and transfer. And the problem is that, for low temperatures, transfer time becomes less than time of emission which increases the strength of the blur.

The mentioned defects in CCD structure (as traps mentioned above) tend to recover itself with time. And rapidity of this process is higher for high temperatures (so called "burn effect").

Figure 7. Part of CCD dark frame

Finally, we conclude that accurate choice on the sensor's temperature regime is quite important for a final star tracker precision.

Dark signal pre-calculation

Another valuable method for suppressing the dark signal influence on the final tracker precision is the pre-calculation of collected dark charge in each element of the sensor within the appropriate model. For the typical case one can assume that relative temperature distribution over sensor's surface remains constant and relations between temperature and dark current are similar for all elements.

There are two basic stages in the pre-calculation procedure. First stage is realized during the calibration of a dark signal. The map of values of dark generation over sensor can be obtained from the analysis of "dark frames" – a set of successive frames exposed for different times and read out from the sensor screened from any light pollution. These values are normalized to the signal from sensor's elements that are always dark since they are additionally covered by manufacturer with oblique material ("covered" elements).

This map of relative dark currents, stored in tracker's permanent memory, is assumed to be independent on sensor's temperature.

Then, while tracker operating, the amount of collected dark charge is calculated for each pixel multiplying corresponding value from the stored map by actually measured signal from appropriate "covered" elements. The resulting value then must be subtracted from this pixel's value.

CALIBRATIONS

During ground-based preparation of a star tracker, a number of hardware (sensor, optics scheme etc.) calibrations are needed. Their results are stored as tables of parameters of correction functions in the tracker's permanent memory. At the same time the space-based calibrations are also may be useful during flight. The main reason for the latter is accidental occurring of new defects in the sensor's structure due to interactions with cosmic rays (mostly with high-energy protons of solar wind). This leads to temporal variability of dark generation levels and relative sensitivity of elements of the sensor. And their changes are more significant if the tracker operates within radiation belt of the Earth. A part of such defects, however, as was mentioned above, are able to be restored within time.

But, in general, periodical space-based recalibration of dark signal and sensitivity is needed for high precision trackers.

DISCUSSION AND CONCLUSION

In this paper we shortly describe main sources of systematic errors that suppress star tracker's final precision. A number of other systematic errors may be also mentioned but were not considered above since their estimation is very hard, they can be investigated during additional ground and space-based calibration. Note here, that ability of taking into account tracker's systematic errors influence the final design of the device and its data processing system including processor (CPU) efficiency, volume of memory and complexity of algorithms of data reduction and analysis.

Also, a space-based recalibrations require some tracker's constructive features as shutter-like device for dark signal calibrations and source of uniform light pollution for calibration of sensor's sensitivity.

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