Small Satellite Capability Analysis: Communications Subsystem

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ABSTRACT
With improved technology, small satellites have opened space to customers such as universities conducting research, companies testing products, and countries unable to support a traditional large space program. As more small satellites operate in space, a theoretical concept of their capabilities is useful not only for satellite designers, but also current spacecraft operators and space situational awareness observers. This study examines the maximum communication capability of a S-band geostationary small satellite as part of a larger Small Satellite Capability Analysis study. Power production using body-mounted solar cells is maximized for cubic satellites with characteristic lengths from ten centimeters to one meter. The maximum data rate is calculated using link analysis and QPSK coding procedures. Multiple ground station sizes and allocated volumes for the transmitter are examined, and ideal conditions are used throughout the study. While the maximum data rate increases with satellite size and ground station antenna size, this procedure gives a “physics-only” solution that is far beyond the capabilities of current satellite components. Technological limits are applied to the solution to find the realistic maximum communication capability. A few general trends are apparent in the results. The input power required by the transmitter determines the minimum size satellite that can support a given transmitter. If only a small portion of the satellite volume is allocated for the transmitter, the transmitter volume also dictates the minimum size satellite that can support a given transmitter. The supported transmitter determines the output power and maximum bandwidth, which determine the resulting data rate. These limitations are the areas of technological development necessary to expand the future communication capability of small satellites. A trendline of allocated payload volumes based on satellite size is also developed to indicate the capability of the analyzed small satellite to accomplish other tasks. This is combined with the communication performance of satellites (with small transmitter-allocated internal volumes) to create a 3-D solution space for the overall capability of small satellites.

EXPLANATION OF STUDY
As technological advances allow components to get smaller and smaller, the capabilities of small satellites continue to grow. These satellites can be designed, built, and launched in the span of a few years for a fraction of the cost of a traditional satellite, expanding the potential satellite market. This design methodology is so new that both new and old industry members are unfamiliar with the potential and limits of these satellites. This Small Satellite Capabilities Analysis study addresses this question.

By necessity, an analytical study of this size must be split into several parts. This portion of the study focuses on the communications subsystem, as well as the power subsystem’s coupling with the communications subsystem. The communications subsystem is chosen because it is essential to satellite missions. With few exceptions, the only way to communicate with satellites once in orbit is through electromagnetic waves. Even within the constraints of the communication subsystem, the analysis must be restricted for this individual study since there are too many variables to address. Therefore this analysis will focus on the single-channel S-band downlink capabilities of a geostationary satellite. An S-band frequency (2.25 GHz) was chosen because these frequencies are popular for small commercial satellites trying to attain high data rates. 

Frequencies between 2.20 GHz and 2.29 GHz are reserved in the United States by the Federal Communications Commission (FCC) for “space operations, earth exploration satellites, and space research”.

Similarly, the Space Operational Service reserves the same frequencies in Europe. Unlike most current communications satellites, single channel transmission is used to analyze the capability of a generic satellite. Multiple channels would constrain the study to a specific mission. A geostationary orbit (GEO) was chosen since the trade space for low-Earth orbits has already been studied (many small satellites have already transmitted data in low-Earth orbits). Lower
altitude satellites are able to transmit at much higher data rates due to the decreased range, but have limited access time to the ground station. In addition, GEO is especially popular for communications satellites since a single GEO satellite can cover around a third of the Earth’s surface.\textsuperscript{4,5,6} The areas above a GEO satellite’s highest visible latitude (±81.3°) are sparsely populated.\textsuperscript{7} A cube-shaped satellite with body-mounted solar cells was also chosen to maintain consistency. Data from other satellites was adjusted to fit this profile by changing the surface area to maintain the same volume. The length of a side is considered the characteristic length of the satellite. This study will analyze cubic satellites with characteristic lengths between 0.10 and 1.00 meters.

At this point it is beneficial to address the approach taken in this study. This is a parametric analysis intended to establish the communication capabilities of a small satellite in GEO under ideal conditions. The conditions of this study are set to maximize the potential of the satellite to transmit large amounts of data within the constraints of commercially available technology. This is not a design exercise for a specific satellite, which would instead consider the worst-case mission scenarios and proceed accordingly. An actual satellite will transmit less data than these results due to the assumptions made in this analysis. In addition, regulations for effective/equivalent isotropic radiated power (EIRP), bandwidth, and orbit location will not be addressed since the goal is to study the capabilities of a satellite.\textsuperscript{5} What this analysis does provide is a technical upper limit for current commercial small satellite performance, which is useful both in the initial design and analysis of satellite missions.

To achieve this goal, the following assumptions were made:

1. Eccentricity and inclination are assumed to be 0 and 0° respectively. In reality this can only be approached, but the orbit control effort can be ignored since it is relatively small.\textsuperscript{5} Additionally, orbital perturbations of the GEO satellite will not be considered.\textsuperscript{4}

2. The satellite has no redundant hardware. This is typical of many small satellites, which operate under an acceptable risk philosophy.\textsuperscript{1} This is in contrast to the traditional, large, expensive GEO satellite, which contains multiple redundancies to reduce risk.\textsuperscript{4}

3. No concentrators are used on the solar cells and panels. These are large, cumbersome, and beyond the capabilities of many small satellite programs.\textsuperscript{8}

4. The satellite is assumed to be in a maximum transmission mode and therefore sends the maximum possible power to the communications system.

5. The satellite is not in an eclipse or penumbra. GEO has two 45-day eclipse periods each year, which last from a few minutes to 72 minutes in length per day. These are negligible in this general maximum capability study, which is for an arbitrary time period.\textsuperscript{1,5,9} Additionally, it is assumed there are no eclipses of the sun by the moon, which is much rarer.\textsuperscript{4}

6. A connectionless transmission is used, meaning there is no retransmission or acknowledgement of transmission.\textsuperscript{4}

7. The same antenna is used to transmit and receive signals, increasing the available area for solar cells to produce power. The deduction in gain is accounted for in the analysis by the antenna gain. There is no interference between the uplink and downlink signals because only the downlink signal is being transmitted.

8. Only one right-hand circularly polarized signal is transmitted. This polarization decreases some of the attenuation effects in the atmosphere. While techniques such as cross-polarization and multiplexing exist, these will not be analyzed here for generality.\textsuperscript{5,9} In addition, this study focuses on small commercial satellites, which generally do not have the volume or the resources for such a system.

9. No additional amplifiers are used due to size and volume constraints. For satellites with larger characteristic lengths, this assumption may be unrealistic depending on the rest of the satellite’s design.

10. The signal is a digital transmission. Digital signals tend to have higher efficiencies and more data processing alternatives.\textsuperscript{5} The energy is also more evenly spread through the signal because of digital data’s random nature, although scrambling can also help with this.\textsuperscript{5} The data is assumed to be initially digital so quantizing noise is ignored.\textsuperscript{5}

**Small Satellite Background**

Small satellites have been around since Sputnik over half a century ago.\textsuperscript{10} However, over the years...
industry has shifted to bigger, more expensive satellites. Recently small satellites have experienced a resurgence in popularity due to advancements in technology, especially in miniaturization of electronics. In fact, some small technological components of today are more capable than their larger, more traditional counterparts from a few years ago. The design priorities for these satellites are also very different than most traditional satellites. These satellites often use commercial off-the-shelf (COTS) parts, smaller design teams, and accept more design risks. This approach combined with launch opportunities as a secondary payload drastically reduces the cost of such a satellite. As a consequence, the satellite industry is now expanding to countries that cannot afford a full space program of their own, companies seeking to test their products in space, and universities conducting research and training of future spacecraft designers. Satellites with the shorter characteristic lengths examined in this study have been used to prove technologies such as microinstruments, signal trackers, tethers, and even an AIS (automatic identification system). They have also hosted cameras and environment monitoring instruments for “small scale” science. Current research is investigating formation flying of small satellites, which could lead to the creation of large constellations of small satellites. Further technology development, especially in miniaturization and low-power equipment, will further increase these capabilities.

While the industry has divided small satellites into different categories, the definitions between these categories are ambiguous and not well defined. The most commonly accepted definition is based on the mass of the satellite. Picosatellites are between 0.1 kilograms (kg) and 1 kg, nanosatellites are between 1 kg and 10 kg, microsatellites are between 10 kg and 100 kg, and minisatellites are between 100 kg and 500 kg. The minimum satellite considered in this study is the 1 U CubeSat, which is a 10 cm cube just over 1 kg in mass.

**COMMUNICATION LINK ANALYSIS**

Link analysis (often called a link budget), examines the transmission of electromagnetic waves between transmitting and receiving antennas. It accounts for the power gains and losses encountered along the propagation path, as well as the minimum signal power needed at the receiving antenna to properly interpret the signal. In essence, it demonstrates whether the communication link between the two antennas can achieve the desired performance. Many of the terms are expressed in decibels. This is a remnant of the days of severely limited computing power, but is still used today since it is more intuitive than the linear form due to the magnitude of some of the terms.

**SIGNAL POWER**

The signal power is measured at the reference point between the signal power and the noise sections of the communications link. The reference point is generally taken at the receiving antenna output terminals. Sometimes the receiver terminals are used, but as long as the signal and noise powers are defined at the same point, the link analysis result is the same. The signal power includes the transmitted power and all of the power gains and losses before the reference point, while all the power effects after the reference point are represented as noise. The signal power must exceed the noise power to be distinguishable. This quantity is sometimes represented as energy per bit, which is the signal power divided by the signal bandwidth. Traditionally, signal power is represented as the sum in decibels of the transmitted power, antenna gain, and various losses that occur before the link analysis reference point (discussed later).

**Transmitted Power**

This study uses a cube-shaped satellite with body-mounted solar cells. Such a satellite has two sources of power: solar cells which produce power from solar radiation and a battery that stores and deposits energy. Normally these sources would be sized to support the power needs of the satellite, which would minimize the mass and volume needed to obtain a certain level of performance. However, in this analysis the produced power is maximized to achieve maximum communication capability, represented by the data rate. The amount of power produced by the solar array depends on the characteristic length of the satellite, which determines the surface area. The satellite is covered by solar cells (except for the antenna) to maximize solar cell power output. Since small satellites have much smaller thermal gradients, and therefore much simpler thermal subsystems, no surface area is allocated for thermal dispersion.

The power produced by the solar cells can be approximated in two ways. The continuous method is given by Eq. (1), where A is the available area, $P_{\text{sun}}$ is the sun’s irradiance at the Earth and the total efficiency ($\eta$) is composed of the pointing efficiency, cell efficiency, and array efficiency.

$$P_{\text{sa}} = A \eta P_{\text{sun}}$$
In addition to the cell packing factor (for mounting, etc.), the array efficiency also contains a component due to the array temperature. The efficiency falls 0.5% for each degree Celsius higher than 28°C. The temperature is taken as 58°C (an average GEO temperature of 53°C plus an additional 5°C since the solar cells are body-mounted). Beginning of life values are used since the study wishes to find the maximum possible performance. Since the sun will only illuminate half of the satellite at a time, only half of the total power will be used. This averages the produced power between when the antenna side is facing the sun and when it is not.

The discrete method accounts for the fact that there are minimum solar cell sizes and string lengths. Each side of the satellite must contain whole numbers of these (since small satellites use COTS parts), which means there is leftover surface area until another whole unit can be added. First the number of cells in a string must be determined. The number of cells is the ratio of the voltage supply to the cell voltage, rounded up to the nearest integer. This is then multiplied by the area of the cell to get the area of the string. Now the number of strings on a side of the satellite is calculated. This is the ratio of the area of the side to the area of the string, multiplied by the cell packing factor and rounded down to the nearest integer. Note one of the sides must leave room for the antenna. The power generated on a side is given in Eq. (2), where $S$ is the number of strings, $C$ is the number of cells on a string, $P_{\text{cell}}$ is the power generated per cell, and $\eta$ is the pointing efficiency and the temperature component of the array efficiency.

$$P_{\text{side}} = S(C)P_{\text{cell}}\eta$$  

(2)

The power generated per solar cell is the product of the cell voltage and the cell current. All six sides are summed to get the total power. However, only half of the power is used because only half of the satellite is illuminated at a time.

Both methods use the same pointing efficiency to account for the fact that the sunlight is not orthogonal to the solar cell. In Appendix A, the optimum angle between the sun line and solar cell normal for overall power production is calculated as 54.73561°. For angles less than 30°, the cosine of the angle is used as the pointing efficiency. For angles greater than 30° the performance begins to decrease. Between 50° and 80°, the modified pointing efficiency is given by Eq. (3). Above 80°, negligible power is produced.

$$PE = -0.369(\cos\phi)^3 + 0.637(\cos\phi)^2 + 0.750 \cos \phi - 0.015$$  

(3)

The effect of minimum cell size is shown in Fig. 1. The discrete line draws away from the continuous line until the next whole string is added, at which point it returns to the continuous line because the solar cell-covered area is now the same.

![Figure 1: Area Comparison Between Methods](image-url)

Currently, the most popular solar cell for small satellites is the triple-junction gallium arsenide (GaAs) solar cell. Even though this solar cell is more expensive than some of its counterparts, it has a higher efficiency because it uses red light, infrared light, and infrared photons to produce power. Each layer of the cell is a semiconductor designed for different wavelengths. It is also more resistant to radiation and temperature cycles. However, not all of the power produced will be transmitted. Power is lost to other functions necessary during the communication mode as well as to inefficiencies. Therefore, to maintain ideal but realistic results the minimum values for these effects are used. Specific values used in this study are listed in Appendix B. The remaining power is used to power the transmitter.

**Why Not Use the Battery in this Analysis?**

A battery does not produce energy; it stores energy for use at a later time. Except for the initial discharge of the battery, which is ignored in this study due to the negligible contribution over time, the solar cells are the only means of producing energy for the satellite. Adding power from the battery temporarily boosts the transmitted power and therefore the data rate. However, while the battery is charging, the power used to charge the battery reduces the amount of data transferred.
during that time. As proven later, the data transferred is linearly related to the power transmitted. Therefore, the increase in transmitted data during battery discharge is the same as the decrease of data during battery charging, so the battery does not affect the overall transmission capability of the satellite. Duty cycling, such as transmitting only while the battery is discharging, does not help because the battery cannot recharge enough in the charging time to transmit more than if only solar power is used. In other words, the communication capability for GEO satellites depends on the energy generated by the satellite, not the method of energy transfer. Other orbits have limited transmission windows and could possibly benefit from battery power, but these are outside this study’s scope.

The proof for the exclusion of the battery from the analysis is shown below. Start with Eq. (4) (from the link analysis), where \(DR\) stands for data rate, \(POW\) stands for total power transmitted at that particular point in time (due to solar cells and battery), and \(Q\) stands for the rest of the link budget parameters (described later in this paper).

\[
DR = POW(t)Q
\]  

(4)

\(Q\) is constant since ideal values are assumed for this study. Therefore the total data achieved over time \(T\) is shown in Eq. (5), where \(P\) is the power generated by the solar cells, \(V\) is the voltage of the battery, \(I\) is the current of the battery, the subscript \(c\) stands for battery charging mode, and the subscript \(d\) stands for battery discharging mode.

\[
\int_0^T DR(t)dt = \int_0^T (P_c(t) + V_c(t)I_c(t) + P_d(t) + V_d(t)I_d(t))Q dt
\]

(5)

The charging parameters are zero when the battery is discharging, and the discharging parameters are zero when the battery is charging. The integral can be rewritten as Eq. (6), where \(T_c\) is the time the battery takes to fully charge, and \(T_d\) is the time the battery takes to fully discharge. This can be further separated into individual integrals, as shown in Eq. (7).

\[
\int_0^T DR(t)dt = \int_0^{T_c} (P_c(t) + V_c(t)I_c(t))Q dt + \int_{T_c}^{T_{c+d}} (P_d(t) + V_d(t)I_d(t))Q dt
\]

(6)

\[
\int_0^T DR(t)dt = \int_0^{T_c} P_c(t)Q dt + \int_{T_c}^{T_{c+d}} P_d(t)Q dt + \int_0^{T_c} V_c(t)I_c(t)Q dt + \int_{T_c}^{T_{c+d}} V_d(t)I_d(t)Q dt
\]

(7)

Note that the third integral on the right side of Eq. (7) represents the power needed to charge the battery during one battery cycle and the fourth integral represents the power discharged by the battery during one battery cycle. These are also the only integrals that use battery performance. The power needed to charge the battery is slightly larger than the power discharged by the battery, however since the battery power discharged is idealized in this study these two integrals cancel each other out.\(^8\)

Since the solar power produced by the solar cells is assumed constant whether the battery is charging or discharging, the expression for data collected is now Eq. (8), where \(P\) is the constant power produced by the solar cells.

\[
\int_0^T DR(t)dt = \int_0^T PQ dt
\]

(8)

This shows the power \((POW)\) used in the original expression from the link analysis is only dependent on the solar cells. While it is good for the battery to be included for power smoothing, other operational modes, etc., it does not affect the overall maximum communication performance for a GEO satellite.

### Antenna Gain

The design of an antenna can increase the power of the signal it is transmitting or receiving by concentrating the signal’s energy in a particular direction.\(^5\) Gain is the increase in signal strength measured with respect to a hypothetical isotropic antenna.\(^6,11\) Normally, the mass and power of the transmitting antenna is coupled. Larger antennas generally produce more gain, which means they require less power but more mass.\(^7\) However, to constrain the problem within the scope of this study and have consistency across all satellite sizes, only patch antennas are examined. Patch antennas are common for extremely small satellites because they are generally light-weight and non-deployable.\(^14,15\) Patch antennas can also be circularly polarized, which helps decrease signal power loss in the atmosphere.\(^5,16\) Since the standard size of a patch antenna is a square with sides equal to half the wavelength of the signal, the antenna mass and power coupling is not a factor in this analysis.\(^14\) The signal will be maintained along the boresight of the antenna, which is the direction of maximum gain.\(^5\)

After a survey of the available patch antennas, the Microstrip Patch Antenna by Antenna Development Corporation was chosen because it had the highest gain (8 dB) for dual frequency-transmits and receives and smallest beamwidth \((70^\circ))\(^16\).

Beamwidth is the angle over which the maximum gain occurs, generally defined as the conical angle...
for which the gain drops by half (known as the 3 dB bandwidth). Note that the angle of view of the Earth from GEO is 17.5°, so the more directed the beam is, the less energy misses the ground station, and even the Earth. The rest of the signal is broadcast into space, wasting signal power. The Microstrip Patch Antenna also is small enough to fit on all the considered satellites, is circularly polarized, and had a maximum bandwidth (20 MHz) and output power (10 W) that outperforms the transmitter limitations (discussed later).

The receiver antenna is assumed to be a circular parabolic reflector, which is the most common type of ground station antenna because it has good gain, good beamwidth, and is built more easily than some of its counterparts. The gain is given by Eq. (9), where A is the area of the antenna, λ is the wavelength of the signal, D is the diameter of the antenna, and η is the efficiency of the antenna.2,4,11

\[ G = \eta \frac{4\pi A}{\lambda^2} - \eta \left( \frac{rD}{\lambda} \right)^2 \]  

(9)

Antenna efficiency is the product of many factors, such as surface irregularities and differences in antenna properties. The overall efficiency is taken as 70%, although it can often be as low as 55%.4 This efficiency multiplied by the physical aperture area gives the effective aperture area.4,5 Note from Eq. (9) that the larger the diameter of the antenna, the larger its gain. The beamwidth for a parabolic dish with circular aperture of frequency f (in GHz) and diameter D (in meters) is given in Eq. (10).

\[ \theta = \frac{21^\circ}{(f \text{GHz})^2D} \]  

(10)

Four different cases will be examined to represent the variety of ground stations available. Ground station antennas tend to be between 0.6 m and 30 m.4 Ground station antennas with diameters of 15 meters, 3 meters, 1 meter, and 0.5 meters are examined. One might be tempted to just build a very large ground station antenna with large gain. However, there tends to be an upper limit for practical ground antenna sizes. Not only do mechanical and financial challenges make larger antennas prohibitive, but these large antennas also tend to have beamwidths so small (see Eq. (10)) that it is difficult to maintain a communication link with the satellite.2,10

**Line Loss**

Present in both the satellite and ground station, line loss accounts for inefficiencies in transmitting the signals through wiring, equipment, etc. between the transmitter and the antenna. Other line losses, such as power lines to the transmitter, are accounted for elsewhere. While this study’s focus is optimal conditions, it must be tempered with reality, so the minimum line loss of 1 dB is used for the transmitting equipment. A similar loss is observed in the receiving equipment, but this is included in the system temperature calculation for noise.

**Free Space Loss**

Free space loss is the conventional term for the combination of two different power effects. The first is the inverse square law of radiation, which states the signal’s power flux density decreases proportionally to the inverse of the distance from the source squared. An electromagnetic signal is actually transmitted in all directions. This means that the power is spread across the area of a growing sphere as it moves through space, reducing the received power at the ground station. (Gain in different directions is accounted for in the antenna gain term.) The power flux density is given in Eq. (11), where R is the range between the two antennas and EIRP is the combination of the transmitted power, transmitting antenna gain, and the line loss.

\[ PFD = \frac{EIRP}{4\pi R^2} \]  

(11)

Since the elevation is assumed to be 90°, the range is minimized as the altitude of the satellite. It is assumed that the ground station is at sea level to generalize the results. The second part is the spreading loss, which is the wavelength’s effect given by Eq. (12) with λ as the signal wavelength.

\[ Spreading \ Loss = \frac{\lambda^2}{4\pi} \]  

(12)

Multiplying the two terms (and then dividing by EIRP to make the power ratio form) gives the total “free space loss” in Eq. (13).

\[ L_s = \left( \frac{\lambda}{4\pi R} \right)^2 \]  

(13)

**Propagation Path Loss**

The Earth’s atmosphere’s effect, particularly oxygen absorption, is small for frequencies from 1 GHz to 4 GHz. Propagation effects, which include refraction and attenuation, are taken as -0.04 dB at 90° elevation.
**Radome Loss**

Antennas generally use radomes to protect the antenna hardware from both Earth and space environments. Both the transmitting and receiving antenna are assumed to have radomes, which contribute a 1 dB loss each.\(^5\)

**Factors Not Included in Signal Power**

The pointing error represents the error from misaligned antennas. The pointing loss is dependent on the absolute pointing error and the half-power beamwidths of the antennas.\(^4\) It is generally small for well-designed systems (antennas with large beamwidths, ground station antennas with tracking systems, etc.), so for this study it is assumed that the pointing error is zero.\(^5\) Note that this is also sometimes called the “depointing error”.\(^4\)

Polarization loss is caused by differing antenna polarizations.\(^2,4\) Antennas and electromagnetic signals have different polarizations depending on the motion of their electric fields. The axial ratio defines the polarization, but there are some special cases.\(^4\) Linear polarizations have stationary electric fields (vertical or horizontal) and circularly polarized polarizations have rotating electric fields (right or left).\(^1,4\) Losses between opposite polarizations are so large that completing the communication link is practically impossible, while the loss between linear and circularly polarized antennas is only 3 dB.\(^4,9\) A slight loss also happens when a non-linear polarized signal has a pointing error, making the signal appear slightly differently polarized to the receiving antenna than in reality.\(^1\) However, like the pointing error this is ignored to examine optimum behavior.

Depolarization losses have the same results as polarization losses. However, instead of the losses being caused by mismatched signals and/or hardware, depolarization occurs when the signal’s polarization is changed by a medium along the transmission path.\(^5\) Main causes of depolarization are rain, high altitude, ice particles, and multipath.\(^5,17\) Depolarization losses also depend on the type of modulation used.\(^18\)

Multipath is caused by reflections or abnormal tropospheric conditions that distort the signal.\(^3,11\) A ground station antenna at 90° elevation with no nearby obstructions will have negligible multipath effects.\(^4\) If rain were included, it would contain several different effects, including absorption of the signal power, depolarization due to scattering, and increased system temperature (all negligible at this frequency).\(^1,2,17\) Scintillation is mostly brief drop-outs and loss of synchronization, and is quite small if elevation is greater than 10°.\(^6,17\) Ionospheric attenuation, along with other ionospheric effects, is small at S-band frequencies, and is therefore set at zero.\(^1,2,5\) Dispersion occurs when a medium the signal travels through changes the frequency phase across the signal’s bandwidth.\(^17\) Atmospheric dispersion can determine the largest usable bandwidth for a signal.\(^17\)

**BAND NOISE POWER**

The noise is the sum of the power effects after the reference point. It represents all unwanted power contributions that must be overcome to detect the signal.\(^4\) These include radiation sources, receiver components, and other sources of interference.\(^4\) It is the sum in decibels of Boltzmann’s constant, the system temperature, and the signal bandwidth (which is solved for in this study).\(^2\) This is also represented by noise power spectral density (No), which is the band noise power divided by bandwidth (this corresponds to the energy per bit representation).\(^5\) Noise power spectral density is usually assumed white Gaussian overall, but constant over relatively small bandwidths.\(^4,9\) Therefore the noise power spectral density is assumed constant for the available bandwidth, and the total amount of noise depends on the amount of bandwidth.\(^4,6,11\)

There are two main categories of noise: antenna noise (created between the transmitting and receiving antenna as well as noise from the receiving antenna) and receiver noise (from the physical receiver as well as the connections and other equipment between the receiving antenna and the receiver).\(^5\) A lot of this is thermal noise caused by the motions of electrons in devices and black body emissions.\(^5,6,17\) Other sources are represented by the temperature of a resistor that produces the same noise power as the source under consideration.\(^4\) This source’s “temperature” represents the strength of the noise from a noise source.\(^11\) The total noise is represented by the system temperature.\(^1,4,5,6,11\)

**System Temperature**

The system temperature is the combination of the component temperatures referenced to the link analysis reference point, usually the receiver antenna outputs.\(^6,7\) The noise temperatures of different sources are added after being scaled by the gain of the components.\(^6\) To scale, the component temperature is divided by the gain of each device between it and the reference point.\(^5,6\) Therefore the
effect of each component temperature depends not only on the temperature magnitude, but also on the devices between the component and the reference point. This means the first few devices in the receiver chain tend to comprise most of the receiver temperature.\(^5\) Note that since the reference point is the receiving antenna output terminals, it does not need to be scaled. The system temperature is the sum of the antenna temperature (which includes noise effects in addition to the thermal temperature) and the receiver temperature.\(^4\) This results in a system temperature of 351.815 K, or 25.463 dB, calculated in the following sections.

**Temperature Representations**

There are two popular ways to represent noise sources. The first way uses the effective (or equivalent) noise temperature, which is the temperature of a passive resistor that creates the same noise power density as the noise source.\(^5,9\) This is the component temperature. The second way is to use a noise figure, which scales the reference temperature, typically 290 K, to produce the actual noise effect.\(^4,6,7,9\) The noise figure, sometimes called the noise factor, is the ratio of the signal power/noise power ratio at the input of the component to the signal power/noise power ratio at the output of the component.\(^5\) Note that if the component has a gain, it is included in the ratio at the output.\(^5,7\) The noise figure is then used to calculate the effective temperature. The effective temperature can be defined in terms of the noise figure using Eq. (14).\(^5,11\)

\[
T_e = T_{\text{ref}} (NF - 1) \quad (14)
\]

Along the same lines, the noise figure can be written in terms of the temperatures by rearranging Eq. (14) to create Eq. (15).\(^4,5,7,9,17\)

\[
NF = 1 + \frac{T_e}{T_{\text{ref}}} \quad (15)
\]

**Antenna Temperature**

Like the overall system temperature, antenna temperature also has many sources. These sources fall into two categories: component noise, which is from the antenna structure itself, and sky noise, which is from a source observed by the antenna.\(^4,5\) The component efficiency losses are often around 1-2 dB, but are generally represented in the receiver antenna efficiency and therefore ignored in the system temperature.\(^5\) However, the antenna’s physical temperature, taken as the reference temperature, must still be included in the antenna temperature.\(^4\)

Natural sources of sky noise include galactic noise, atmospheric gas, hydrometers (rain, clouds, etc.), lightning, cosmic background radiation, the sun, the moon, celestial radio sources, and reflected radiation.\(^5,17\) Human sources include other communication links, machinery, and electronic devices such as radar.\(^5,17\) Even things like power transmission lines and ignition sources in an internal combustion engine can add noise to the system.\(^17\) Fortunately for this study, most terrestrial systems operate at or below 2 GHz.\(^17\) While ideal conditions are used in this study, statistical methods are generally used in specific satellite design due to the unpredictable nature of these sources.\(^7\)

Terrestrial sky noise develops when the signal passes through an absorbing medium along the signal path (i.e., propagation path loss). According to Planck’s law, any body above 0 K emits electromagnetic radiation.\(^9\) Therefore in addition to attenuating the signal (accounted for in the link analysis), the medium also emits radiation.\(^5,17\) This energy is incoherent and broadband.\(^17,18\) For terrestrial sources, the component temperature is 0 K for no attenuation and approaches the medium’s thermal temperature as attenuation increases.\(^18\) The main terrestrial natural source at this frequency is atmospheric gas since “clear sky” conditions are assumed (no rain, clouds, etc.).\(^4,5,17\) “Clear sky” is a good approximation because weather is negligible for a large percentage of time and locations at this frequency.\(^4\) Gaseous attenuation depends on elevation, altitude, and water vapor content, but is generally small in S-band frequencies.\(^4\) This is one of the reasons S-band is commonly used for satellite communications.\(^7\) Atmospheric gas noise temperature is taken as 1.75 K.\(^4,5\)

For extraterrestrial noise above 2 GHz, the main sources are the sun, moon, galactic noise, cosmic background, and a few radio stars.\(^2,5\) For GEO orbits, the sun is ignored because it only aligns with the satellite near the equinoxes for a short time each day, whereas this study is for a general period of time.\(^4,5\) Similarly, the satellite is rarely aligned with the moon or the Milky Way, and the main radio stars that contribute to antenna noise are not visible from GEO-pointing ground stations.\(^7\) Galactic and cosmic background noises are the only inputs used, calculated as 2.175 K and 2.89 K respectively.\(^5\) Note that while these are used in this system temperature calculation, galactic and cosmic noise are often considered negligible above 2 GHz.\(^9,11\)
A ground station antenna will absorb some of the Earth’s temperature in its sidelobes. The effect decreases as the elevation increases. This study’s ground station antenna has a 90° elevation. For general antennas, this sidelobe absorption can be 10 K or more. However, it is important to note that a few K won’t make a big difference to the overall performance because it is converted into decibels. Therefore an idealized Earth reflection absorption of 0 K is assumed. For this study, it is assumed that there are no man-made sources of noise to operate under ideal circumstances. This is the same as assuming the ground station antenna is very isolated. Therefore, multipath is also negligible.

There are other atmospheric noise effects that are negligible at S-band frequencies and are therefore not considered here. The two main sources of ionospheric attenuation (background ionization and scintillation) are negligible at this frequency. Defocusing (the spreading of the wave due to refraction) is negligible for elevations of 90°. Also ignored are: rain attenuation, cloud and fog attenuation, rain and ice depolarization, and amplitude/phase variations.

**Receiver Temperature**

The receiver temperature can be just as complicated as the antenna temperature since it consists of all the component temperatures after the reference point. However, these temperatures are scaled by the gains of the components between the examined component and the reference point. A common practice is to place a preamplifier at the receiving antenna output terminals, which means the other receiver component temperatures are divided by the preamplifier’s gain. Many systems use a low-noise amplifier (LNA), which has a low noise temperature and high gain. This study uses a LS-2200 Series S-Band LNA with a temperature of 48.487 K at the ambient reference temperature. The component temperatures after this LNA are multiplied by 5.0119x10^-6 and are therefore negligible. This also reduces receiver line loss by physically shortening the lines between the antenna and the amplifier, so the pre-LNA line loss is also negligible. Note that the receiver temperature contains some components excluded from the link budget due to the separation point between the signal power and noise in the communication link, including demodulation and receiver line power losses. However, these are also negligible due to the LNA.

**Bandwidth**

Bandwidth is the width of the frequency spectrum of the signal. While there is a single carrier frequency, the modulation process produces other frequencies. The spectrum depends on baseband signal, modulation type, and power transmitted from the transmitter antenna. A larger bandwidth means more data can be transmitted, but the noise will be higher. To reduce the noise, the bandwidth is filtered to isolate the main lobe of the signal transmission. This must be done carefully because too little bandwidth will not be able to convey all the transmitter data, while too much bandwidth will add noise that can overpower the signal. To optimize performance, theoretical Nyquist filtering is assumed (the filter is ideally rectangular, with no “roll off”). While there is no ideal Nyquist filter, it can be approached by complex filter methods and is therefore used as an idealizing assumption. This reduces interference with other sources, although it can introduce intersymbol interference. This overall procedure maximizes the data rate while eliminating the noise from unusable bandwidth. This main lobe is the same size as the bandwidth calculated with the link analysis.

Note that during specific satellite design projects, bandwidth is traded with the satellite power system, which is not designed solely to maximize communication capability. The relationship between the bandwidth and transmitted power is shown in Fig. 2 for a 0.5 m ground station antenna. The amount of bandwidth is linearly proportional to the transmitted power because the other link analysis parameters are idealized as constants. Since bandwidth is proportional to the data rate (discussed later), the data rate is also proportional to the transmitter power (used in Eq. (4) for the battery proof).

**EB/NO DESIGN**

Eb/No is the ratio of signal power to noise power for the communication link. In decibels, this is the difference between the signal power and the noise power, or the energy per bit and noise spectral density. Eb/No is the same no matter where along the communication link it is calculated. The design Eb/No is also equal to the sum of the required Eb/No, the implementation loss, and the link margin in decibels. The required Eb/No is the signal power to noise power ratio needed to detect the signal. If the link is properly designed, the implementation loss and link margin both have acceptable values.
Implementation Loss and Link Margin

Implementation loss represents the loss due to unexpected continuous effects, generally due to subsystem processing and hardware.\(^5,9\) Link margin is used to represent a bit of “extra” power included in the link to account for unexpected transient power losses.\(^2\) Examples are high solar activity, bad weather, and interference.\(^3\) The two terms are sometimes combined into an overall link margin.\(^1,6\)

While this study is intended to examine ideal conditions, an assumption of zero implementation loss or link margin would not represent a continuous link because the signal would fade in and out. Therefore, the minimum conventional acceptable implementation loss of 1 dB and link margin of 3 dB are used in this study.\(^1,11\)

Figure 3 shows the relationship between link margin and transmitted power. Note that this is with a set data rate of 5 kbps (and a 0.5 m ground station antenna), whereas this study will maximize data rate with a set link margin. The link margin is linearly related to transmitted power for a set data rate. However, since link margin is expressed in decibels and the transmitted power is expressed in linear Watts, the relationship appears as shown in Fig. 3.

MODULATION

The goal of this study is to maximize data rate, an indicator of communication capability. However, the link analysis solves for the available bandwidth instead. Data rate is dependent on the type of modulation as well as bandwidth. While there is a theoretical maximum, given by the Shannon-Hartley Theorem, it has not been approached closely except with incredibly complex coding and processing, so it is not used.\(^2,4\) Fortunately, on-board processing capabilities have progressed enough that the modulation is not a limiting factor for most common coding schemes.\(^1\) Baseband signals containing the digital data modulate carrier signals at the transmission frequency.\(^4\) Since noise produces bit errors in the baseband signal, the bit error rate (BER) is commonly used to indicate the quality of a digital transmitted signal.\(^2,4,5\) As the name suggests, BER is the ratio of error bits to the total received bits.\(^4\) Most BER limits are defined by specific mission parameters, but BER = 10\(^{-5}\) is used since it is a common guideline.\(^2\) BER is also important in link analysis because required Eb/No is the power ratio required to achieve a given BER with a certain modulation.\(^2\)

QPSK

Phase shift keying (PSK) is generally used for satellite transmission because it performs well and uses all of the signal power.\(^5,11\) For this study, quadrature phase-shift keying (QPSK) is used because it is a PSK with good spectral efficiency (defined later) and a constant envelope modulation (which allows the amplifier to operate at saturation).\(^1,4\) As discussed in further detail in Appendix C, QPSK uses four phases.\(^6\) Initially, this halves the data rate, but by offsetting the symbols (essentially double layering the data), the data rate is doubled back to its original value while still halving the required bandwidth.\(^5\) Higher order PSKs reduce the bandwidth more, but they have a higher required Eb/No.\(^4,5\) This is because additional power is needed to differentiate between the symbols since the phase states are no longer orthogonal.\(^4,5\) Some COTS space rated technologies have their own proprietary coding, but these often require a matching ground station.\(^29\) Small satellites do not tend to use these because they want to maximize the number of potential ground stations (AMSAT, universities, etc.).
**Error Correction**

Another way to reduce the required Eb/No is to add error-correction coding that provides a way to differentiate between weak/noisy symbols. Error-correction coding reduces required Eb/No and BER, but requires more bandwidth since extra data is added.\(^4\) When done properly, this results in an overall increase in the data rate, even with the extra bits.\(^4\) The reduction in required Eb/No is often called “decoding gain”.\(^4\) The mass and power of the satellite are also generally reduced since less power is required and current processor capabilities can achieve this type of coding relatively easily.\(^2,6,10,11\)

One of the most popular methods is forward error correction (FEC) coding, which adds extra redundant bits used by the receiver to check for errors. With these extra bits, single event upsets can be corrected, however double errors can cancel each other and pass undetected.\(^4,6\) On of the more popular FEC methods is BCH block codes, where each code word is formed from a calculated polynomial.\(^4\)

The code rate represents the ratio of original information bits to total number of bits (original data and FEC bits).\(^1,4\) Many different possible code rates exist, but this study uses a code rate of 0.5 since many commercial transmitters have this capacity built in and it achieves good performance.\(^7\) Spectral efficiency is the product of the code rate and the number of bits per symbol.\(^1,4\) It is the ratio of actual data rate (not counting FEC bits) to bandwidth, which indicates how effectively the bandwidth is used.\(^1\) For QPSK with 0.5 code rate, the spectral efficiency is 1.\(^4\)

The original theoretical required Eb/No for QPSK is 9.6 dB.\(^3\) Coherent demodulation is used because even though differential demodulation simplifies the demodulator, it degrades performance.\(^3\) Viterbi decoding is used to improve performance by reducing required Eb/No to 4.5 dB.\(^6\) A 1 dB implementation margin still needs to be included.\(^5\)

**LIMITATIONS**

While the results from the previous analysis are theoretically sound (shown in Figure 4), they are not realistic with today’s technology. Now that the physical constraints have been analyzed, the hardware constraints must be examined. Note that since these constraints are based on engineering limits for the hardware, it will be possible for the constraints to be modified as new technology is developed.

Even though communications and power subsystems are the main focus of this study, certain other subsystems are necessary for successful satellite operation. For this study, the transmitter can occupy 100%, 50%, or 20% of the satellite volume for each trend respectively (hereafter referred to as COM volume). This is to analyze different missions with different priorities for communication capability. For example, there are potential applications where a transmitter is the main instrument on a satellite. However, the limited volume of a small satellite can sometimes be an advantage instead of a disadvantage. For example, CubeSat thermal control is generally achieved by just connecting the elements to the structure for dissipation within predicted thermal “limits”.\(^10\)

Due to engineering constraints, the communication capability depends on the type of transmitter the satellite can support. The minimum size satellite that can support a transmitter is determined by the transmitter’s longest dimension, volume, mass, and whether the satellite can produce enough power to run the transmitter. The maximum data rate is determined by the output power of the transmitter, which is input into the link analysis. The bandwidth limit of the transmitter can also become a factor for some of the more bandwidth-limited transmitters. Antennas have this limit as well, but the selected antenna outperforms the transmitters.\(^16\)

**COMMUNICATIONS RESULTS**

These previous principles have now been applied to the small satellite analysis problem. As shown in Fig. 5, the power produced by the satellite increases with the characteristic length of the satellite. Note the small steps indicating discrete cell sizes. This behavior decreases in effect as the size of the satellite increases (and the side area becomes much
larger than the cell area). The power output from the transmitter is shown in Fig. 6 for the 0.5 m ground station antenna, 20% COM volume case. Note that this behavior originates from the transmitter limitations (discussed later) such as the power produced by the satellite. There is also leftover power that is not used by the transmitter, shown in Fig. 7 for this example. This figure is included to show the capability of the satellite to conduct operations at the same time as transmitting, although the primary measure of this is available payload volume (shown later).

The rest of the link analysis parameters besides transmitted power are fixed for this study, therefore the resulting bandwidth is linearly dependent on the transmitted power. If the transmission can be completed with a 3 dB link margin and a positive bandwidth (which is linearly proportional to the data rate), then the transmission is a success. However, while the minimum link margin was chosen to optimize the bandwidth produced, link margin is one of the more arbitrary choices in satellite design. Therefore, the results for 3 dB and 6 dB link margin are shown in Fig. 8, since 6 dB is also a common link margin choice. The successes and resulting bandwidths for both link margins are shown in Fig. 8 for the example case of 0.5 m ground station antenna and 20% COM volume. (Note Fig. 8 uses the transmitter trendlines developed in Appendix D.) The higher link margin slightly changes the trend, but overall it remains the same with a smaller data rate. This was also shown in Fig. 4 for the physics-only results. The rest of the study will only use a link margin of 3 dB.

The effective maximum data rate is then calculated from the produced bandwidth. The results are shown in Fig. 9 for a ground station antenna of 0.5 m. Also note the vast performance reduction in Fig. 9, which shows performance with current technology, compared to Fig. 4, which is based purely on idealized physics.
Ground stations of different sizes are analyzed since the receiver antenna size affects the gain of the antenna and therefore the possible data rate. As the ground station size increases, so does the maximum data rate. These results are shown in Fig. 10-12. Note while similar overall trends are produced for each ground station size, there are some differences in the transmitter trendlines (detailed in Appendix D). Note the output powers range from 1 to 5 W.

Figure 9: Communication Capability for 0.5 m Ground Station Antenna

Figure 10: Communication Capability for 1 m Ground Station Antenna

Figure 11: Communication Capability for 3 m Ground Station Antenna

Figure 12: Communication Capability for 15 m Ground Station Antenna

The main limitations for the communication performance are shown in Figure 13. The minimum size satellite that can support a transmitter is mainly determined by its produced power. Since the transmitters have set input powers, the relevant result of the solar power production is whether enough power is produced to run the transmitter. If the satellite is able to produce enough power, it is large enough to support the transmitter in the idealistic 100% and 50% COM volume scenarios.

Figure 13: Limitations for Communication Capability
The only time the transmitter volume is a limiting factor is for the 20% COM volume case, where the transmitter must share most of the satellite volume with other hardware. Also note the COM volume only changes the maximum data rate for satellites with very small characteristic lengths.

The maximum communication performance is mainly determined by the output power of the transmitter, which is input into the link analysis. This is within the maximum bandwidth limit except for the 15 m ground station antenna case. In this case, the ground station antenna adds so much gain that the bandwidth limit becomes a factor for some transmitters. The Comtech AeroAstro transmitter, which has the largest output power, is not used for this case because of its bandwidth limit. In addition, the bandwidth and volume constraints mean that the TXS384 transmitter is sometimes replaced with the Clyde Space CubeSat transmitter, which produces the same output power but has a larger bandwidth capability. The increase in gain still raises the maximum data rate compared to the smaller ground station cases. However, such a large ground station is rare for many small satellite operators, such as universities, to have regular access to.

A few trends are apparent from Fig. 5. For a satellite with small characteristic length and with other subsystems (and therefore less volume for the transmitter), establishing a communication link is not possible. While communication is possible for some small characteristic lengths, it is limited. Also remember that this is an idealized analysis, which does not account for pointing errors, weather, component life degradation, etc. Therefore it is risky to build such satellites for GEO operations since realistically communication is unlikely. Between the characteristic lengths of 0.25 m and 0.35 m, the communication capability expands rapidly. Communication is more reliable in this region, and small increases in size lead to great gains in communication capability. Finally, the communication capability reaches a maximum around a characteristic length of 0.35 m for this range of satellite sizes. After this characteristic length, the transmitter survey did not reveal any higher performing transmitters that fit within the small satellite profile. However, amplifiers can be added to the system to boost communication capacity further (an element outside the scope of this study). Perhaps more importantly, more of the satellite resources can be diverted to other tasks as the satellite size increases while still maintaining the same communication performance.

**AVAILABLE PAYLOAD**

Now that the power and communications performance have been developed, it remains to be seen how much room is left for a payload. While the 100% and 50% COM volume cases don’t have much spare volume, the 20% COM volume case still has potential payload volume. Available payload is a study parameter because it represents the capability of the satellite to do something besides communicate. Since the study is for general missions instead of a specific goal, it is of interest to examine if the maximum communication performance limits the satellite’s capability to do other operations. Available payload is a very generic way to look at this, since it is assumed that other operations would be conducted when the satellite is not in maximum communication mode.

Note that the majority of the available data is for the mass of the available payload. For small satellites, volume is a bigger constraint than mass because it is complicated by minimum component sizes. Unfortunately, not enough information was found to calculate the volume trendline directly. However, since the volume is so constrained, the mass density can be assumed fairly constant and used to transform mass values to volume values. The payload mass is divided by the total mass to get a payload percentage. By assuming constant density, the volume of the payload can be found as the same percentage of the total satellite volume.

The full payload mass calculation is described in Appendix E. Data from previous satellite missions was collected and then a best-fit trendline was used to approximate expressions between the variables. The resulting equations are shown in Eq. (16-18), where cl stands for characteristic length, m stands for mass, P stands for payload percentage, $V_{sat}$ standing for satellite volume, and APV stands for available payload volume. Results are shown in Fig. 14 and Appendix E.

\[ m = 242.27(c l^{2.382}) \text{ kg} \]  \hspace{1cm} (16)

\[ P = 40.229(m^{-0.135}) \% \] \hspace{1cm} (17)

\[ APV = P(V_{sat}) = 40.229(m^{-0.135})(c l^{3}) \] \hspace{1cm} (18)

For the 20% COM volume case, this can be combined with the previous data rate results to get a “space” of successful communication performance and payload volume. The boundary of the solution space is shown in Fig. 15-18 for each ground station antenna size. Every point below the maximum boundary is a successful solution.
Figure 14: Payload Volume

Figure 15: 3-D Solution Space Boundary for 0.5 m Ground Station Antenna

Figure 16: 3-D Solution Space Boundary for 1 m Ground Station Antenna

Figure 17: 3-D Solution Space Boundary for 3 m Ground Station Antenna

Figure 18: 3-D Solution Space Boundary for 15 m Ground Station Antenna

FUTURE WORK

Unfortunately, the scope of the study means that there are many unexamined aspects. For example, orbits other than GEO orbits not only have different link analysis values, but also have limited transmission windows for ground stations. This means that battery power can be used to boost transmission during these time periods since the satellite will not be transmitting continuously. The ability to add amplifiers to satellites with larger characteristic lengths could also greatly affect communications capability. More characteristic lengths, available communication volume percentages, and ground station antenna sizes can also be examined. Finally, this study is one part of the overall Small Satellites Capability Analysis, so coupling with other subsystems (analyzed in different parts of the study) has not been examined.

CONCLUSIONS

As this study shows, the limits for communication capability for small satellites are hardware-based instead of physics-based. The produced power limits the data rate for satellites with shorter characteristic lengths, while transmitter volume also becomes a factor for small transmitter-allocated internal volume cases. Communication is not reliable from
GEO for most of these satellites, while it quickly becomes much more feasible for satellites of around 0.3 m characteristic length. The larger small satellites are limited by transmitter output power, and in the cases of large ground stations, by maximum bandwidth. Conditions such as link margin, solar cell instrumentation, etc. can shift the trendline as a whole but the relative behavior remains the same. Not only does this study create a guideline for small satellite communication capability, but these limitations also identify key technological limitations to be overcome to expand small satellite communication performance.

ACKNOWLEDGMENTS

I’d like to thank Brian Engberg and Jeffrey Ganley from the Air Force Research Laboratory and Johnathan Jones from Applied Technology Associates for their support throughout this project.

REFERENCES


APPENDICES

A: Derivation of Optimum Sun Angle

N is the vector normal to the satellite surface. S is the sun line vector. These two vectors' geometry is similar and is described in Fig. A. This results in Eq. (A1) for the N and S vectors. The dot product of these two vectors gives the cosine of the sun angle, shown in Eq. (A2).

\[ \vec{N} = \begin{bmatrix} \sin \beta \cos \alpha \\ \sin \beta \sin \alpha \end{bmatrix}, \quad \vec{S} = \begin{bmatrix} \sin \psi \cos \theta \\ \sin \psi \sin \theta \end{bmatrix} \]  \hspace{1cm} (A1)

\[ \vec{N} \cdot \vec{S} = \cos \phi = \sin \beta \cos \alpha \sin \psi \cos \theta + \sin \beta \sin \alpha \sin \psi \sin \theta + \cos \beta \cos \psi \]  \hspace{1cm} (A2)

Since there are six N vectors (one for each side of the cube), the six dot products would normally be summed. The definitions of the N vector angles for each N vector are given in Table A. However, if the dot product is negative the side is blocked from the sun due to the geometry of the satellite. Therefore only the positive dot products are summed to get the Eq. (A3).

\[ P = \sin \psi \cos \theta + \sin \psi \sin \theta + \cos \psi \]  \hspace{1cm} (A3)

Table A: Vector Angles

<table>
<thead>
<tr>
<th>Angle</th>
<th>t</th>
<th>f</th>
<th>k</th>
<th>-t</th>
<th>-f</th>
<th>-k</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0°</td>
<td>90°</td>
<td>N/A</td>
<td>180°</td>
<td>270°</td>
<td>N/A</td>
</tr>
<tr>
<td>β</td>
<td>90°</td>
<td>90°</td>
<td>0°</td>
<td>90°</td>
<td>90°</td>
<td>180°</td>
</tr>
</tbody>
</table>

To maximize this function, the partials are taken with respect to each angle and set equal to zero. Angle combinations that satisfy both Eq. (A4) and Eq. (A5) are possible solutions.

\[ \tan \psi = \cos \theta + \sin \theta \]  \hspace{1cm} (A4)

\[ \sin \psi (\cos \theta - \sin \theta) = 0 \]  \hspace{1cm} (A5)

There are two sets of angles that satisfy these equations: \( \psi = 180°, \theta = 135° \) and \( \psi = 54.73561°, \theta = 45° \). These two sets are plugged back into Eq. (A3) to get P = -1 and 1.732051 respectively. Therefore the second angle set corresponds to the maximum power generated. These angles are plugged back into Eq. (A2) to find the sun angle. The optimum sun angle turns out to be 54.73561° for all of the illuminated sides.

B: Power Generation Coding Information

These are values used in this particular study. The sun irradiance is assumed to be 1366 W/m² at the Earth.¹ The packing density is assumed to be the common value of 90%.⁶ Out of the produced power, 1% is sent to the power equipment, 1.6% to the command and data handling equipment, and 3.6% to the attitude determination and control equipment.¹,⁶ The rest is sent to the transmitter. The lines between the solar cells and the transmitter have an efficiency of 80%.⁶ The voltage supply is assumed to be 5 V, a common voltage for small satellites.¹³ The solar cell is assumed to be the Spectrolab Ultra Triple Junction Solar Cell. It has a 28.3% cell efficiency, 2.35 V cell voltage, and generates 163 A/m².¹¹ Each cell has a 26.62 cm² area, with a length of 3.95 cm and a width of 6.89 cm.²¹,²²

Extra References:


C: QPSK

For quaternary coding (used in QPSK), each symbol contains two bits, which results in four possible symbols.\(^5\) It is a multi-level coding system, where binary bit streams are combined to reduce bandwidth.\(^5\) The digital modulator, part of the communication signal processing chain, modulates a digital bit stream onto a sinusoidal carrier for transmission.\(^5\) A QPSK modulator separates the data bit stream into two bit streams, which are then treated as separate BPSK (binary phase shift keying) streams.\(^5\) Odd bits go to the in-phase channel and even bits to the quadrature channel.\(^5\) The bit duration (time per bit) is doubled in these streams since every other original data bit is included, so the bit rate is halved.\(^5\) The quadrature bit stream is mixed with a 90° phase-led version of the carrier signal while the in-phase bit stream is mixed with the original carrier signal.\(^5\) The two bit streams are then recombined to be transmitted.\(^5\) The demodulator reverses this process.\(^5\) The bandwidth required is halved because the modulated bit duration was doubled, but the overall data rate is the same as BPSK.\(^5\) Coherent detection is assumed in the demodulator, meaning the only noise accepted is the in-phase noise.\(^5\) However, since there are two streams of data, each with orthogonal phases, the BER stays the same.\(^5\)

D: Transmitter Trendline

The transmitter trendline is formed by surveying the available technology and then forming a trendline from the optimal results. The data from the transmitters is in Table D. Different COM volumes were analyzed to see how sensitive the analysis was to this parameter to volume divisions within the satellite. Three cases (100%, 50%, and 20%) were chosen arbitrarily to represent a range of COM volumes. Still, it is important to examine multiple cases, especially for specific designs where maximizing communication performance must be balanced by other payloads, tasks, etc. Different ground station antenna sizes (15 m, 3 m, 1 m, and 0.5 m) were also examined. The trendline and the used transmitters are shown in Table D. Note that the data is for all cases unless stated otherwise.

The initial transmitter trendline is calculated with the link analysis. Then the largest dimension, volume, and mass constraints from the transmitter are added one at a time. The final trendline incorporates all of these. The main time these constraints matter is the volume constraint for the 20% case. This is because the main limitation to supporting the transmitter is power generation: a satellite big enough to generate enough operational power is big enough to (ideally) hold the transmitter. An example of the graph created by the transmitter trendline code is shown in Fig. D. Note this does not include eliminated transmitters.

Table D: Transmitter Trendline

<table>
<thead>
<tr>
<th>Transmitter</th>
<th>Min. Characteristic Length (cm)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>TXS384 S-Band Transmitter, Clyde Space &amp; Innovative Solutions in Space B.V.</td>
<td>0 for 50%, 100% cases; not used in 20% case</td>
<td>(28)</td>
</tr>
<tr>
<td>CubeSat S-Band TX, Clyde Space</td>
<td>13.5, only used for 20% cases &amp; 15 m cases</td>
<td>(29)</td>
</tr>
<tr>
<td>TX-2400 S Band Transmitter, SpaceQuest</td>
<td>27.0</td>
<td>(30)</td>
</tr>
<tr>
<td>T170A, L-3 Communications</td>
<td>30.2</td>
<td>(31)</td>
</tr>
<tr>
<td>S-Band Downlink Transmitter (4 W), Surrey Satellite Technology, Ltd.</td>
<td>33.0</td>
<td>(29,32,33)</td>
</tr>
<tr>
<td>S-Band Transponder, Comtech Aero/Astro</td>
<td>34.4, not used in 15 m cases</td>
<td>(34)</td>
</tr>
</tbody>
</table>

![Figure D: Transmitter Trendline for 0.5 m Ground Antenna and 20% Communications %](http://www.clyde-space.com/cubesat_shop/transceivers_rx_tx/transceivers/178_s-band-transmr)

Extra references:

E: Calculation of Available Payload Volume

As described previously, the trendline for available payload volume was calculated by forming a best-fit trendline to a survey of past and current small satellites. Two trendlines are combined: one relating the mass of the satellite to its characteristic length and one relating the payload mass to the total mass of the satellite (to calculate the payload percentage). This data is listed in Tables E1 and E2. Note that the available payload volume trendline only applies for the 20% COM volume case and other similar cases. The resulting trendlines are shown Fig. E1-E3, Eq. (16-18), and Fig. 14. Note that since the satellite is designated as a cube, its volume is the characteristic length cubed.

Table E1: Mass v. Size of Satellite

<table>
<thead>
<tr>
<th>Name</th>
<th>Source</th>
<th>Mass (kg)</th>
<th>Characteristic Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CubeSat 1U</td>
<td>(12)</td>
<td>1.333</td>
<td>0.1</td>
</tr>
<tr>
<td>CubeSat 2U</td>
<td>(12)</td>
<td>2.667</td>
<td>0.12599</td>
</tr>
<tr>
<td>CubeSat 3U</td>
<td>(12)</td>
<td>4</td>
<td>0.14422</td>
</tr>
<tr>
<td>AMSAT small micro</td>
<td>(6)</td>
<td>10</td>
<td>0.23</td>
</tr>
<tr>
<td>AMSAT large micro</td>
<td>(6)</td>
<td>125</td>
<td>0.56462</td>
</tr>
</tbody>
</table>

Extra references:

Table E2: Payload Percentage of Satellite

<table>
<thead>
<tr>
<th>Name</th>
<th>Source</th>
<th>Payload %</th>
<th>Total Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INP-XS</td>
<td>(23)</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>GOMX-1U</td>
<td>(24)</td>
<td>27.5</td>
<td>1</td>
</tr>
<tr>
<td>COTS CubeSat</td>
<td>(10)</td>
<td>23.077</td>
<td>1.3</td>
</tr>
<tr>
<td>INP-S</td>
<td>(23)</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>GOMX-2U</td>
<td>(24)</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>INP</td>
<td>(23)</td>
<td>50</td>
<td>3</td>
</tr>
<tr>
<td>GOMX-3U</td>
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<td>50</td>
<td>3</td>
</tr>
<tr>
<td>CanX-2</td>
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<tr>
<td>NTS</td>
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<td>GNB</td>
<td>(27)</td>
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<tr>
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<td>60</td>
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<td>39.75</td>
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<tr>
<td>INDEX</td>
<td>(10)</td>
<td>15.278</td>
<td>72</td>
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<tr>
<td>GALILEO</td>
<td>(11)</td>
<td>11.652</td>
<td>904.78</td>
</tr>
</tbody>
</table>

Figure E1: Satellite Mass

Figure E2: Payload Percentage

Figure E3: Satellite Volume


