Spin Stability of Sounding Rocket Secondary Payloads Following High Velocity Ejections

Weston McClain Nelson

Utah State University

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SPIN STABILITY OF SOUNding ROCKET SECONDARY PAYLOADS FOLLOWING HIGH VELOCITY EJECTIONS

by

Weston M. Nelson

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Mechanical Engineering

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UTAH STATE UNIVERSITY
Logan, Utah
2013
Abstract

Spin Stability of Sounding Rocket Secondary Payloads Following High Velocity Ejections

by

Weston M. Nelson, Master of Science
Utah State University, 2013

Major Professor: Stephen A. Whitmore
Department: Mechanical and Aerospace Engineering

The Auroral Spatial Structures Probe (ASSP) mission is a sounding rocket mission studying solar energy input to space weather. ASSP requires the high velocity ejection (up to 50 m/s) of 6 secondary payloads, spin stabilized perpendicular to the ejection velocity. The proposed scientific instrumentation depends on a high degree of spin stability, requiring a maximum coning angle of less than 5º. It also requires that the spin axis be aligned within 25º of the local magnetic field lines. The maximum velocities of current ejection methods are typically less than 10m/s, and often produce coning angles in excess of 20º. Because of this they do not meet the ASSP mission requirements. To meet these requirements a new ejection method is being developed by NASA Wallops Flight Facility. Success of the technique in meeting coning angle and B-field alignment requirements is evaluated herein by modeling secondary payload dynamic behavior using a 6-DOF dynamic simulation employing state space integration written in MATLAB. Simulation results showed that secondary payload mass balancing is the most important factor in meeting stability requirements. Secondary mass payload properties will be measured using an inverted
torsion pendulum. If moment of inertia measurement errors can be reduced to 0.5%, it is possible to achieve mean coning and B-field alignment angles of 2.16° and 2.71°, respectively.
Public Abstract

Spin Stability of Sounding Rocket Secondary Payloads Following High Velocity Ejections

Weston M. Nelson

The Auroral Spatial Structures Probe (ASSP) mission is a sounding rocket mission studying the effects of solar energy on space weather. ASSP requires the high speed ejection (up to 50 m/s) of 6 secondary payloads to gather the required data. The scientific instruments on the secondary payloads require that the payloads are stable in flight with coning angles of less than 5°, where the coning angle is the amount the payload is allowed to “wobble” about its spin axis. The secondary payloads are also required to have their spin axes aligned with 25° of the local magnetic field lines. Current ejection methods do not meet the velocity requirement and are often lead to unstable flight. To meet the ASSP mission requirements, a new ejection method is being developed by NASA Wallops Flight Facility. This document describes how the flight stability of the secondary payloads was modeled using computer simulations. These simulations showed that to meet the stability requirements for ASSP the secondary payload mass properties must be accurately measured and the payloads balanced. If errors in mass property measurements can be reduced to 0.5%, it is possible to achieve mean coning and B-field alignment angles of 2.16° and 2.71°, respectively.
Acknowledgments

I would like to express my gratitude to Dr. Christine Hailey, Dr. Charles Swenson and Dr. Stephen Whitmore for providing excellent support to me throughout my research, and also to Dr. Chad Fish and the Space Dynamics Laboratory for giving me the opportunity to work on this project. Most importantly, I would like to thank my wife, Rebekah, for encouraging me in my hare-brained idea of leaving a good job with the Air Force and going back to school.

Weston M. Nelson
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Chapter 1

Introduction

Many sounding rocket missions eject secondary payloads for a variety of purposes: to create distributed sensor arrays, release targets for radar tracking, disperse chemicals to study atmospheric mixing, etc. The payload ejection method depends on the required ejection speed and directional accuracy, number of secondary payloads and degree of flight stability required. A short survey of some of the methods, proposed and employed, for deploying secondary payloads is outlined in the following paragraphs.

Conde [1] at the University of Alaska Fairbanks designed and executed a mission with the objective of tracking winds in the upper atmosphere. The mission used a sounding rocket to carry 16 secondary payloads, called “Ampules” to altitude where they were ejected using a combination of springs and small solid rocket motors. Once dispersed, each ampule released a vial of trimethyl-aluminum into the atmosphere. The released chemicals were then tracked using ground based sensors to map upper atmospheric winds. This ejection method yielded mixed results. The twelve ampules ejected using springs dispersed according to plan and yielded good results. However, the rocket deployed ampules failed to separate from the sounding rocket.

Lynch [2] at Dartmouth University used two different ejection methods to disperse two pairs of secondary payloads. Extensive pre-mission simulation predicted coning angles between 5° and 15°. The first pair was ejected forward and aft at 8 and 6 m/s respectively, along the local magnetic field line using a spring deployment. An initial spin rate of 4°/sec was supplied by the rotation of the launch vehicle with the spin axis in line with the ejection vector. The remaining pair of secondary payloads were ejected radially in the plane perpendicular to the local magnetic field lines. The spin axis of these secondary payloads was designed to be perpendicular to the ejection direction. To achieve this orientation, the
secondary payloads were mounted inside the main payload body, off-center from the spin axis. According to the original design, upon release of the secondary payloads, centripetal acceleration, caused by the spin of the main payload, was to accelerate the secondary payloads radially along a track, ejecting them at 2 m/s with a spin rate of 1.0 and 1.5 Hz respectively. Mechanical difficulties caused one of the radially ejected secondary payloads to release prematurely with a spin rate of 2.6 Hz and an ejection velocity of 6 m/s. The measured coning angles between 25 and 30 degrees were significantly higher than predicted. The increased coning angles were attributed to “tip-off” disturbances due to friction and other factors. Tip-off is defined as the introduction of angular rotation about any axis other than the desired spin axis.

Another option for releasing secondary payloads uses a deployment mechanism similar to the P-Pod [3], which was developed to deploy orbital cubesats by California Polytechnic State University. The P-Pod is a simple elongated box shaped container capable housing a 3U cubesat (30 cm X 10 cm X 10 cm) or a combination of smaller cubesats. The ejection sequence uses a simple release mechanism to open a spring-loaded door. The cubesat is then ejected from the P-Pod using a second compressed spring. Achieved on-orbit ejection velocities vary from 1.6 to 2.0 m/s depending on the mass of the cubesat and the stiffness of the ejection springs used. This method could be useful for some missions where wide dispersion of secondary payloads and spin stability are not critical.

The examples from the Conde [1] and Lynch [2] missions show that for low velocity ejections simple compressed spring ejection mechanisms work well. However, for ejections requiring high velocities or spin axes not aligned with the ejection vector, these ejection methods produce unacceptably large coning angles.

The Auroral Spatial Structures Probe (ASSP) mission, the subject program for this thesis, requires the ejection of 6 secondary payloads from the Main Payload at velocities several times higher than the missions described above (between 25 and 50 m/s). Like the radially ejected pair of secondary payloads from the Lynch mission [2], the ASSP secondary payloads will be spin stabilized around an axis perpendicular to the ejection velocity. How-
ever, the required ejection velocity of 50 m/s is too high and the maximum allowable coning angle of 5° is too low to be achieved using the technique developed on the Lynch mission. These unique requirements necessitate the development of a new low coning angle ejection technique, validated by simulation and test.

The major design duties for ASSP are split between the Space Dynamics Laboratory (SDL), North Logan, Utah, NASA Wallops Flight Facility (WFF), Virginia. SDL is charged with the mechanical and electrical design for the Secondary Payloads as well as the sensor design for both the Main and Secondary Payloads. WFF, an experienced provider of sounding rocket launch and support services, is in charge of launch vehicle configuration, launch operations, and the mechanical and electrical design of the Main Payload including the Secondary Payload ejection system. Mission execution will be a collaborative effort by both SDL and WFF. Secondary mission partner, Embry Riddle Aeronautical University, is assisting with the design of the Main Payload plasma density probes.
Chapter 2

Auroral Spatial Structures Probe (ASSP) Mission Summary

Before discussing the details of the ASSP Secondary Payload stability model, a brief summary of the mission science objectives, concept of operations and a basic description of the Main and Secondary Payload designs are presented to provide the reader with sufficient background information.

2.1 Science Objectives

Creating accurate models of space weather is a high national priority for both military and civilian applications. One important factor influencing space weather is the energy input to Earth's upper atmosphere from solar wind. This energy input at high latitudes drives magnetospheric features and events that propagate to all latitudes. These features and events vary both temporally and spatially across the Earth’s Polar Regions due to the changing solar wind-Magnetosphere interactions. One of the main mechanisms by which energy is deposited into the high latitude ionosphere is Joule heating. This phenomenon is a result of driven currents in the upper atmosphere dissipating energy in the form of heat. The energy deposited in the atmosphere by Joule heating, Q, is a function of the vectors for the local electric field (E-field), E, magnetic field (B-field), B, local current density, J, and the neutral wind, U [4] as shown in Eq. 2.1. Due to the lack of high fidelity in situ measurements, the spatial and temporal variation of the electric field on small scales is the least well understood of these factors.

\[
Q = J \cdot (E + U \times B) \tag{2.1}
\]

Large-scale E variations are easily measured with available ground- and space-based systems. Ground based radar measurements, by systems such as the Super Dual Auroral
Radar Network (SuperDARN) [5] can provide data on temporal scales as small as 1 minute and on spatial scales upward of 30 kilometers. Space based satellite observations refine the spatial resolution of measurements down to approximately 1000 meters, but are only capable of seeing temporal variation on scales as small as the satellite’s orbital period (approximately 90 minutes for low Earth orbit satellites). Weimer and Matsuo have created E-Field models using ground and space based measurements as inputs to theoretical models [6] covering spatial and temporal scales upward of 200 km and 160 minutes respectively. Other techniques such as Assimilative Mapping of Ionospheric Electrodynamics (AMIE) [7] use nonlinear least squares fit methods to match data from a variety of sources and models to create maps of Ionospheric electrodynamics. These observations, models and maps are used as inputs to simulations predicting space weather.

Condrescu pointed out that current models for Joule heating input to space weather use mean E-field patterns [8]. These mean patterns are established from data collected by ground and space based systems. However, this approach leads to variability and underestimation of Joule heating. Several studies have estimated that the error from using mean E-Field patterns ranges from 30% to 200% [9–11]. An example of the discrepancy between measured along-track E-field data from NASA’s Dynamics Explorer 2 satellite (DE 2) and the E-Field models used by Weimer and Matsuo [6] can be seen in Fig 2.1. The theoretical models, Weimer and Weimer+EOF, shown in the figure capture the large scale fluctuations in the E-field, but fail to account for smaller scale variation and spikes that are shown in the data from DE2.

If the spatial and temporal resolutions of the various measurement systems and models discussed are plotted on a logarithmic scale, it is apparent that a large region of data at small spatial and temporal scales is left unobserved by the ground and space based systems (see Fig 2.2). Ignoring the small scale field variations, illustrated in Fig 2.1, leads to the inadequacies of models discussed by Condrescu. The ASSP mission hopes to fill this gap in measurements by collecting observations of the variation in the E-field on small temporal and spatial scales within the region labeled “ASSP Time-Space Separated Observations” in
Fig. 2.1: Along-track E-field DE 2 satellite measurements vs theoretical models (Figure 10 in Matsuo) \[6\]

Fig 2.2. These observations will provide missing pieces of data that will allow the models such as those created by Weimer and Matsuo to more closely reflect reality. The improved E-field models can then be used as inputs to space weather models improving their accuracy.

### 2.2 ASSP Concept of Operations

ASSP consists of one Main Payload and six Secondary Payloads that, each equipped with sensors to measure the Earth’s electric and magnetic fields as well as plasma density. The payloads shown in Fig. 2.3, will be launched by NASA Wallops aboard a Terrier-Talos-Oriole-Nika sounding rocket out of Poker Flats, Alaska during a period of high Auroral electromagnetic activity. The launch azimuth will be directed toward the magnetic North Pole as shown in Fig 2.4, where the red line indicates the trajectory of the Main Payload. After burnout of the launch vehicle, the Secondary Payloads will be ejected from the Main Payload using gas springs, shown in Fig 2.5, on a timing schedule listed in the major mission events summary in Table 2.1. The velocity change applied by the gas springs will cause
spatial separation between Secondary Payloads to increase through the duration of the flight, as demonstrated by the green lines in Fig 2.4. Nominal range and altitude plots for the constellation trajectory are presented in Figs 2.6 and 2.7.

The constellation configuration was designed to allow ASSP to collect measurements on the short spatial and temporal scales required to fill the mission science objectives. To achieve the desired constellation configuration, the first and third pairs of Secondary Payloads will be ejected with a relative speed of 50 m/s and the second pair at half speed (25 m/s) oriented as shown in Fig. 2.8. The maximum stroke length for the gas spring is approximately 1 m, making an ejection velocity of 50 m/s an ambitious goal. The final achievable ejection velocity will only be known after gas spring testing is complete. The off-set angle of 6.75°, shown in the Fig. 2.8, helps compensate for Earth’s rotation.
beneath the constellation. Ejecting Secondary Payloads 1-4 along the same azimuth allows the Main Payload and these four Secondary Payloads to make five separate measurements along the same magnetic flux tube at different times. The local temporal variation of the E- and B-fields may be quantified using these time separated measurements of a given point along the trajectory. When measurements commence shortly after ejection the separation times will be small, but will increase to approximately 105 seconds by the end of the flight defining the coverage range on the temporal scale of Fig 2.2. Similarly, by comparing the measurements made by all six Secondary Payloads and the Main Payload at any given moment the spatial variation in the E- and B-fields may be quantified for that time. Ejecting
Table 2.1: Major mission events summary

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<thead>
<tr>
<th>Event</th>
<th>Time (s)</th>
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<th>Range (km)</th>
<th>Velocity (m/s)</th>
<th>Q (psf)</th>
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<td>Launch/Talos Ignition</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
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<td>Talos Burnout</td>
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<td>0.2</td>
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<td>Terrier Ignition</td>
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<td>Nose Cone Eject</td>
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<td>Nihka Ignition</td>
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<td>Sub Payload 1&amp;2 eject</td>
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<td>110 km down-leg</td>
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<td>109.9</td>
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<td>0.0</td>
<td>917.1</td>
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Fig. 2.6: Nominal constellation range/altitude vs. time

Fig. 2.7: Nominal constellation range vs. altitude
Fig. 2.8: Secondary Payload ejection directions and velocities [4]
Secondary Payloads 5-6 with cross-range velocity vectors allows the spatial field variations perpendicular to the launch azimuth to be estimated as well as along the launch azimuth. Initially, spatially separated measurements will be less than 100 m apart but separation will increase to approximately 60 km before atmospheric re-entry, defining the coverage range on the spatial scale of Fig 2.2. A summary of how all the major mission events fit together is displayed in Fig. 2.9.

2.3 Vehicle Design

2.3.1 Main Payload Description

Because this work focuses on Secondary Payload stability following ejection, the Main Payload description will be brief, followed by a detailed description of the Secondary Payload
Main Payload Sensor Description

The Main Payload sensor suite measures Earth's E and B-fields as well as plasma density. The E-field sensor consists of the four deployable rigid booms evenly spaced about the spin axis on the forward Main Payload section shown in Fig 2.10. Each boom is 2 m in length and uses the rocket body's spin and a spring loaded kick-off to provide the force necessary to deploy from the stowed configuration. The initial configuration is depicted in Fig. 2.3 to the final configuration seen in Fig. 2.10. Three evenly spaced booms approximately 0.5 m in length on the Main Payload aft section house 3 probes for measuring plasma density and a science grade magnetometer for B-field measurements.
Main Payload Attitude Determination and Control System

Attitude control is provided by the Magnetic NASROC Inertial Attitude Control System (MaNIACS). MaNIACS has been used on many previous WFF sounding rocket missions and has a well established flight heritage. It uses a combination of magnetic field and inertial sensors to create attitude solutions with better than 1° accuracy. Control authority is generated by a set of cold gas thrusters utilizing four tanks providing 800in³ of high pressure nitrogen (~3000 psi). MaNIACS stabilizes the Main Payload orientation throughout the flight and executes all reorientation maneuvers for the Main Payload prior to each Secondary Payload ejection as described in the Section 2.2. In addition to the data generated by the inertial measurement system, a GPS system provides position and velocity data via signals received through a single wrap-around GPS antenna.

Main Payload Telemetry System

The Main Payload telemetry system provides a single S-Band telemetry down-link at 4 Mbps (2.5Mbps science data) at a planned RF carrier frequency of 2279.5 MHz.

Secondary Payload Ejection System

The Secondary Payload ejection system consists of two sets of three gas springs, designed and built by NASA Wallops, located on the fore and aft ends of the Main Payload (see Fig. 2.3 and 2.5). The gas springs are constructed using two concentric hollow tubes, one nested inside the other. All three gas springs are connected to a shared plenum in an open configuration (see Fig. 2.11). The plenum and gas springs are filled with Nitrogen compressed to approximately 200 psi. Each Secondary Payload is mounted on a pair of specially designed forks shown in Fig. 2.12 located on top of the outer cylinder. A spring loaded retaining latch clamps the Secondary Payloads in the mounting forks before ejection. The ejection velocity of each gas spring is set by the gas pressure, the cross sectional area of the spring and the stroke length. Ejection occurs when a pyrotechnic device is fired releasing the spring loaded locking collar, located at the base of the gas spring, allowing it to rotate to the unlocked position. The pressurized gas in the combined spring-plenum
Fig. 2.11: Ejection system block diagram
volume then expands forcing the outer cylinder, and the attached Secondary Payload, to slide along the inner cylinder. The retaining latch holding the Secondary Payloads in the forks is then released as the outer cylinder accelerates. When the outer cylinder reaches the end of the stroke, it hits a stop made of crush material and the Secondary Payload separates from the rocket.

Following each ejection, gas from the high pressure tank, connected to the plenum via a regulator, brings the pressure back up to 200 psi. Given the accuracy of the regulator and temperature changes this value could vary by as much as 10 psi. The actual ejection occurs so quickly (~0.035 seconds) that there is no significant gas flow from the high pressure tank into the plenum during ejection. Allowing all three gas springs and the plenum to be interconnected creates a greater volume of gas available for expansion during each ejection and minimizes the pressure drop. Reducing the pressure differential during the gas spring expansion, compared to an isolated gas spring not attached to a plenum, allows for a more constant Secondary Payload acceleration profile and provides two main benefits. First,
it reduces the required gas spring pressure before ejection and second, the initial shock experienced by the Secondary Payload is minimized.

### 2.3.2 Secondary Payload Description

#### Secondary Payload Sensor Description

The Secondary Payloads are also equipped with a suite of SDL designed instruments for measuring plasma density as well as E and B-fields. The local E-field is sampled throughout the flight by measuring the potential difference between two isolated sensors immersed in the plasma. Each E-field sensor is attached to the end of a deployable flexible wire boom to maximize the distance between them (see Fig. 2.13). The sensors must be deployed many Debye lengths apart to be located in an independent plasma sheath region. Typically this distance is greater than a meter for ionospheric observations. Each sensor will independently achieve a potential relative to the surrounding plasma. The average electric field in space is

---

**Fig. 2.13: Secondary Payload sensor block diagram**

---
determined from the potential difference between the sensors and their separation distance [12–14].

The wire booms are initially coiled inside the E-field spool and are deployed shortly after Secondary Payload ejection. The use of flexible wire booms for space based measurements is an emerging technology that enables small, cubesat sized, secondary payloads to take measurements that previously required much larger instruments.

To keep the booms taut the Secondary Payloads will be given an initial angular velocity of 2 Hz with the spin axis perpendicular to the ejection velocity vector. As the wire booms deploy to their full length of 2 meters, for a total of 4 meters between probes, the moment of inertia (MOI) about the spin axis will increase, reducing the spin rate of the Secondary Payload to conserve angular momentum. In the fully deployed configuration (see Fig. 2.14), the spin rate will be ~0.5 Hz, providing enough centripetal acceleration on the E-field probes to keep the wire booms taut.

B-field measurements are provided by a 3-axis, science grade magnetometer installed on a nadir pointing rigid boom perpendicular to the wire E-field booms as shown in Fig. 2.13.

![Fig. 2.14: Secondary Payload with wire E-field booms deployed](image)

**Secondary Payload Attitude Determination and Control System**

Secondary Payload attitude will be reconstructed post-flight using data from an internal set of accelerometers, gyroscopes and the science magnetometer. A NASA Sounding Rocket Division (NSROC) supplied GPS will provide position and velocity data. The Sec-
Secondary Payloads do not have an active control system and will rely on mass balancing and minimizing tip-off from the ejection system to ensure stable flight.

If the Secondary Payloads experiences large coning angles, pendulum motion will be induced in the wire booms. This motion, if severe, could render the collected data difficult to interpret.

**Secondary Payload Telemetry System**

The Secondary Payloads have a 200 Kbps single S-Band telemetry down-links at unique carrier frequencies planned to be 2215.5, 2217.5, 2219.5, 2221.5, 2223.5 and 2225.5 MHz that will be utilized after ejection. Prior to ejection, telemetry data will be transmitted via the Main Payload telemetry section through an umbilical connection.
Chapter 3
Research Objectives

The main goal of the research here-in is to create an accurate model of Secondary Payload dynamic behavior immediately following ejection. This model can then be used to evaluate the effectiveness of the gas spring ejection technique in providing spin stabilized flight, meeting the mission requirement of less than $5^\circ$ coning angle and alignment to local B-field lines within $25^\circ$. Based on the results, problem areas can be identified and improvements made to the ejection technique.

To accomplish this goal, a 6 degree of freedom rigid body simulation was developed and validated. This simulation provides a framework where all the factors that potentially lead to unstable flight are simulated together and their effects on one another examined. A study was conducted to identify all potential disturbance torques and forces and determine which were significant for ASSP. Models to accurately describe each perturbing factor were then created and incorporated into the dynamic simulation.

Potential perturbing factors identified included mass imbalances in the Secondary Payloads, tip-off caused by the ejection mechanism itself, excitation of dynamic vibration modes by ejection loads (leading to time dependent variation in the mass properties of the Secondary Payloads) and instability of the Main Payload as an ejection platform.

Once the framework for conducting the analysis was in place, case studies were performed to answer the following questions.

1. Given the baseline design for ASSP, what are the expected 2 and 3-$\sigma$ levels for coning angle and B-field alignment?

2. Which factors are most significant in perturbing the Secondary Payload attitudes away from their “ideal” flight?
3. What are the feasible steps that may be taken to mitigate these disturbances?

4. What are the expected 2 and 3-σ levels for coning angle and B-field alignment after steps to mitigate the perturbations have been taken?

5. Is performance improved, and is the improvement worth the associated costs?
Chapter 4
ASSP Secondary Payload Dynamic Ejection Simulation
Description

4.1 Numerical Simulation Methodology

Secondary Payload motion was modeled using a formulation presented by Baruh [15] that employs a Newtonian formulation of the rigid body equations of motion in state space form. These equations are presented in Eqs. 4.1 to 4.13.

\[ \dot{v}_1 = -v_3 \omega_2 + v_2 \omega_3 + \frac{F_1}{m} \]  \hfill (4.1)
\[ \dot{v}_2 = -v_1 \omega_3 + v_3 \omega_1 + \frac{F_2}{m} \]  \hfill (4.2)
\[ \dot{v}_3 = -v_2 \omega_1 + v_1 \omega_2 + \frac{F_3}{m} \]  \hfill (4.3)

\[ \dot{A}_1 = (\cos \phi \cos \psi - \sin \phi \cos \theta \sin \psi)v_1 + (-\cos \phi \sin \psi - \sin \phi \cos \theta \cos \psi)v_2 + \sin \phi \sin \theta v_3 \]  \hfill (4.4)
\[ \dot{A}_2 = (\sin \phi \cos \psi - \cos \phi \cos \theta \sin \psi)v_1 + (-\sin \phi \sin \psi + \cos \phi \cos \theta \cos \psi)v_2 - \cos \phi \sin \theta v_3 \]  \hfill (4.5)
\[ \dot{A}_3 = \sin \theta \sin \psi v_1 + \sin \theta \cos \psi v_2 - \cos \theta v_3 \]  \hfill (4.6)

\[ \dot{\omega}_1 = \frac{(I_2 - I_3)}{I_1} \omega_2 \omega_3 + \frac{M_1}{I_1} \]  \hfill (4.7)
\[
\dot{\omega}_2 = \frac{(I_3 - I_1)}{I_2} \omega_1 \omega_3 + \frac{M_2}{I_2} \\
\] (4.8)

\[
\dot{\omega}_3 = \frac{(I_1 - I_2)}{I_3} \omega_1 \omega_2 + \frac{M_3}{I_3} \\
\] (4.9)

\[
\dot{e}_0 = \frac{-\omega_1 e_1 - \omega_2 e_2 - \omega_3 e_3}{2} \\
\] (4.10)

\[
\dot{e}_1 = \frac{\omega_1 e_0 + \omega_3 e_2 - \omega_2 e_3}{2} \\
\] (4.11)

\[
\dot{e}_2 = \frac{\omega_2 e_0 - \omega_3 e_1 + \omega_1 e_3}{2} \\
\] (4.12)

\[
\dot{e}_3 = \frac{\omega_3 e_0 - \omega_2 e_1 - \omega_1 e_2}{2} \\
\] (4.13)

In this formulation Eqs. 4.1 to 4.3 are the time derivatives of the body fixed translational velocities, \( \dot{v} \), Eqs. 4.4 to 4.6 are the time derivatives of the body’s position in the inertial frame, \( \dot{A} \), Eqs. 4.7 to 4.9 are the time derivatives of the body-fixed angular velocities, \( \dot{\omega} \), and Eqs. 4.10 to 4.13 are the time derivatives of the Euler Parameters, \( \dot{e} \), used to describe the orientation of the body-fixed frame relative to the inertial frame. This formulation aligns the body-fixed frame to the axes created by the principal moments of inertia, where the subscript 1 refers to the axis aligned with the largest principal moment of inertia and 3 refers to the smallest. This simplifies the rotational equations of motion by causing all terms containing products of inertia (POI) to go to zero. External forces and moments on the body are represented by the variables \( F \) and \( M \), respectively. The variables \( \phi, \theta \) and \( \psi \) are the rotation angles from a 3-1-3 Euler rotation sequence, \( m \) is the mass of the body and \( I_{1-3} \) are the principal moments of inertia. Collectively these equations make up the state vector derivatives.

Before describing specific simulation methodology, three coordinate systems must be
Fig. 4.1: 6 DOF simulation flow block diagram

defined. The first coordinate system is the Secondary Payload Geometry Coordinates. This is a right-handed body fixed coordinate system where the -z axis is aligned with the magnetometer boom and the +x axis is aligned with the #1 wire E-field boom (see Fig 2.13). It is convenient to define the initial conditions in this frame, but it should be remembered that the equations of motion are written in terms of the principal moments of inertia. This leads us to the next coordinate system, a body-fixed frame, aligned with the principal inertia axes of the body, referred to simply as the body frame in the remainder of this document. Finally, the Earth Centered Earth Fixed or ECEF coordinate system will be used to define the inertial frame. This simulation is only concerned with the first few seconds after ejection, so the Earth’s rotation may be neglected.

Before integration begins, an eigensolver is used to calculate the cosine matrix, $[c]$, needed to rotate the initial conditions from the Secondary Payload Geometry Coordinates to the body frame (see the flow chart of the simulation operation presented in Fig. 4.1). The rotated initial conditions are then fed into a fixed step 4th Order Runge Kutta integrator.
Table 4.1: Classical Orbital Elements for the translational degrees of freedom validation

<table>
<thead>
<tr>
<th>Semi-Major Axis</th>
<th>Eccentricity</th>
<th>Right Ascension</th>
<th>Inclination</th>
<th>Argument of Perigee</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000 km</td>
<td>0.2</td>
<td>60°</td>
<td>45°</td>
<td>30°</td>
</tr>
</tbody>
</table>

Table 4.2: Translational degrees of freedom validation; error summary

<table>
<thead>
<tr>
<th></th>
<th>Mean Position Error (m)</th>
<th>Mean Velocity Error (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>3.6361e-06</td>
<td>2.1059e-09</td>
</tr>
<tr>
<td>y</td>
<td>3.8267e-06</td>
<td>2.3979e-09</td>
</tr>
<tr>
<td>z</td>
<td>2.9860e-06</td>
<td>1.6288e-09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Position Error Standard Deviation (m)</th>
<th>Velocity Error Standard Deviation (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2.3899e-06</td>
<td>1.8911e-09</td>
</tr>
<tr>
<td>y</td>
<td>3.8764e-06</td>
<td>2.2746e-09</td>
</tr>
<tr>
<td>z</td>
<td>2.2428e-06</td>
<td>1.2560e-09</td>
</tr>
</tbody>
</table>

that calls the state vector derivatives. Time varying force and moment inputs can be fed into the integrator at each time step. Finally, the output values are rotated from the body frame to the the Secondary Payload Geometry frame for post processing.

4.2 Model Verification and Validation

Accurate implementation of the translational degrees of freedom and force inputs was validated by integrating an arbitrary satellite orbit for one complete period using a time step of 2 seconds and a simple point mass gravity model as the only force input. The same orbit was propagated again using a validated code based on Kepler’s Equations as presented by Shepperd [16]. Initial angular velocity for the orbiting body was set to zero, thereby locking the orientation of the body fixed coordinates with relation to the inertial coordinates. Locking the orientation of the coordinate systems allows the body fixed velocities from the simulation to be compared with the inertial velocities generated from the Keplarian model. The classical orbital elements for the test orbit are shown in Table 4.1. An overlay of orbit position and velocity are plotted in Figs. 4.2 and 4.3 and a summary of the absolute values of the errors between the Keplarian “truth” solution and the numerical integration are presented in Table 4.2. Visual inspection of the overlays reveals little if any difference in
Fig. 4.2: Orbit position overlay; translational degrees of freedom validation

Fig. 4.3: Orbit velocity overlay; translational degrees of freedom validation
the results of the two simulations. If calculated as percent errors the position and velocity errors are both on the order of $10^{-7}$%. However, even though the errors are very small, the question must be asked, “Are the errors due to the numerical accuracy of the solution or systemic errors in the programming?” To answer this question, a computational convergence study of position errors was performed using step sizes of $dt = 2, 4, 8, 16$ seconds. The Keplarian model was used as the truth solution and position errors were calculated using the relative error of the L2-norm. If there are no programming errors in the simulation, the computational convergence study should show 4th order accuracy, matching the order of accuracy of the integrator. The study results are plotted in Fig. 4.4. The slope of the linear fit in each plot equals the computational convergence rate or order of accuracy. The orders of accuracy for the position components range between 4.01 and 3.98, indicating that the only errors between the simulation output and the Keplarian model are the expected numerical integration errors due to step size.
A second test examines the accuracy of the relationship between the rotational and translational degrees of freedom. An initial angular velocity vector for the orbiting body of \( \omega = (0.5, 2, 1) \) Hz was supplied in this test. It should be remembered that the velocity derivatives, presented in Eqs. 4.1 to 4.3, are relative to the body fixed coordinates. Introducing non-zero initial angular velocities “unlocks” the body fixed coordinates from the inertial coordinates allowing them to rotate. If the body fixed velocity derivatives are integrated correctly, the inertial body positions should still match the test orbit regardless of the payload orientation. The integration step size must be reduced to many times smaller than the smallest angular rate period to reduce integration errors. Figure 4.5 shows an overlay of the Keplerian model and the numerical solution using a step size of 0.001 seconds over the first tenth of the orbit and a summary of the error results is displayed in Table 4.3. Again, a visual inspection of Fig. 4.5 and Table 4.3 shows the two simulations produce very similar results, but the introduction of angular rates has increased the error levels.
Table 4.3: Rotational degrees of freedom validation; error summary

<table>
<thead>
<tr>
<th></th>
<th>Mean Position Error (m)</th>
<th>Position Error Stand. Dev. (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.2551</td>
<td>x</td>
</tr>
<tr>
<td>y</td>
<td>5.2469</td>
<td>y</td>
</tr>
<tr>
<td>z</td>
<td>3.6521</td>
<td>z</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1905</td>
<td>4.6588</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.5156</td>
</tr>
</tbody>
</table>

by several orders of magnitude relative to the first test. To explain this increase in error, a computational convergence study was performed using step sizes of \( dt = 0.001, 0.002, 0.004, 0.008 \) and \( 0.016 \) seconds. The results of this study are summarized in Fig. 4.6. The orders of accuracy for the integration of the three position components range between 4.13 and 3.79, indicating that the only errors between the simulation output and the Keplerian model are the again the expected numerical integration errors due to step size.

A final test examines the accuracy of simulation response to moment inputs. This test is broken into two parts. The first part looks at the accuracy of moment integration about each axis independently. The second part checks for the appropriate response due to gyroscopic coupling when a moment is applied about a given axis when a body is already spinning about a different axis. The moment integration test applies a known moment profile about a single principal axis and compares the analytical integration of the moment to the numerical integration produced by the simulation. If the body starts from rest and the moment is applied about one axis, the angular rate derivatives reduce to the expressions shown in Eqs. 4.14 to 4.16,

\[
\alpha_x = \frac{M_x}{I_{xx}} \tag{4.14}
\]

\[
\alpha_y = \frac{M_y}{I_{yy}} \tag{4.15}
\]
where $\alpha$, $M$ and $I$ are the angular accelerations, moments and moments of inertia about the x, y and z axes. Applying the time varying moment profile shown in Eq. 4.17

$$M(t) = \sin (2\pi t)$$

(4.17)

to equations 4.14 to 4.16 and integrating over time yields the following analytical solution for the angular rate profile where $\omega$ is the angular rate and $I$ is the moment of inertia (MOI) of the axis about which the moment was applied.

$$\omega(t) = \frac{-\cos (\pi t) + 1}{\pi I}$$

(4.18)

A computational convergence study using step sizes of $dt = .001$, .002, .004 and .008
Fig. 4.7: Moment input computational convergence study showing 4th order accuracy was performed using the analytical solution in Eq 4.18 as a “truth” solution. The slope of the convergence plots for this study, shown in Fig. 4.7, demonstrate 4th order accuracy, verifying that moment inputs about a single axis are being appropriately accounted for.

The second half of the moment input test deals with the cross-coupling terms of the angular rate derivatives Eqs. 4.7 to 4.9. When a body has angular velocity components about any two perpendicular axes it will experience an angular acceleration about the third axis. The cross-coupling test, unlike the computational convergence tests, is a qualitative test verifying that cross-coupled reactions occur about the correct axis and have the appropriate sign. As a test case, the motion of an arbitrary test body, with moments of inertia of $I_{xx} = 0.1$, $I_{yy} = 0.1$ and $I_{zz} = 0.12$ and an initial angular velocity of $\omega_z = 4\pi$, was integrated for 2 seconds with a time step of 0.001 while experiencing a constant moment about the x-axis, $M_x = 1$. The resulting angular velocities were plotted in Fig. 4.8. The positive pitching moment causes a positive angular velocity $\omega_x$, and the cross-coupling term in Eq 4.8 then causes a positive angular velocity $\omega_y$. It should be noted that after about half
a second, \( \omega_y \) becomes large enough that the cross-coupling term in Eq 4.7 overpowers the moment term causing \( \omega_x \) to decrease. The same test was repeated with different initial velocity and moment combinations to verify the appropriate cross-couple reaction in Eq. 4.9.

The positive results from the tests described verify that the state space integration scheme presented by Baruh was implemented appropriately in this numerical simulation.

### 4.3 Disturbance Characterization

#### 4.3.1 Mass Property Characterization and Balancing

Based off of previous SDL experience, a design requirement was established that the ASSP Secondary Payloads be symmetric major axis spinners with an inertia ratio of the major to minor moments of inertia of at least 1.25. If standard notation for the inertia tensor, presented in Eq 4.19 is used, this means that given the ASSP geometry coordinate
system, \( I_{xx} = I_{yy} \), \( I_{zz}/I_{xx} = I_{zz}/I_{yy} \geq 1.25 \) and \( I_{xy} = I_{xz} = I_{yz} \approx 0 \).

\[
[I] = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{xy} & I_{yy} & I_{yz} \\
I_{xz} & I_{yz} & I_{zz}
\end{bmatrix}
\] (4.19)

Initial Secondary Payload mass property estimates were provided by developing a detailed CAD solid model of the Secondary Payload. The center of mass position of the solid model was adjusted using internal weights until it was within .005” of the geometric center of the Secondary Payload. The inertia tensor, relative to the ASSP Geometry Coordinate System, is presented in Eq 4.20.

\[
[I] = \begin{bmatrix}
.010601 & -.000013 & .000026 \\
-.000013 & .010601 & -.000055 \\
.000026 & -.000055 & .013255
\end{bmatrix} \left( \text{kg} \cdot \text{m}^2 \right)
\] (4.20)

Based off of this initial estimate of mass properties, \( I_{xx} \) and \( I_{yy} \) are identical, \( I_{zz}/I_{xx} = I_{zz}/I_{yy} = 1.25 \) and the products of inertia (POI) \( I_{xy}, I_{yz} \) and \( I_{xz} \) are more than two orders of magnitude smaller than the moments of inertia (MOI). The design requirements were achieved only by paying close attention to the mass properties of the Secondary Payload from the beginning of the design process.

A variety of methods exist for measuring the mass properties of an object, each with specific advantages and disadvantages. The center of mass of an object may be measured using the unbalanced moment, a three point, or mechanical repositioning method [17]. MOIs are measured using either inverted torsion pendulums or hanging wire pendulums and POI measurements may be conducted using either the spin balance or the moment of inertia method [17]. Factors influencing the selection of one method over another include cost, accuracy, ease of measurement, fixturing, availability of instrumentation, etc. The Space Dynamics Laboratory (SDL) Nanosat Operation Verification and Assessment test facility (NOVA lab) contains instruments for making three point center of mass measurements, and MOI measurements using an inverted torsion pendulum. The inverted torsion pendulum
may also be used to calculate the POIs using the moment of inertia method, described later in this section. Figure 4.9 shows the three point center of mass measurement table and Fig 4.10 depicts the inverted torsional pendulum MOI measurement table. Both instruments are designed for measuring the mass properties of cubesat sized spacecraft.

The center of mass measurement table is supported by three load cells. When the body to be measured is mounted to the table, weight measurements from the three sensors can be used to triangulate the position of the center of mass in the plane of the table. The body is then repositioned and a second set of measurements taken in a body plane perpendicular to the first. Using these two positions the body’s center of mass can be fully defined.

MOI measurements are conducted by mounting the body to be measured in a test fixture with known inertial properties and attached to the MOI table top. The MOI table top is mounted on a vertical shaft with known torsional stiffness, $k_\theta$, and very low damping. By disturbing the base from its resting position, the natural frequency, $\omega_n$, of the combined test setup and body may be calculated by measuring the oscillation period, $T$. 
The expression for the undamped natural frequency of a single degree of freedom system, shown in Eq. 4.22, may be expanded to represent the composite MOI of the body and the test setup in terms of the oscillation period as shown in Eq. 4.23

$$\omega_n = \frac{2\pi}{T}$$  \hspace{1cm} (4.21)

$$\omega_n = \sqrt{\frac{k_\theta}{I}}$$  \hspace{1cm} (4.22)

$$\frac{2\pi}{T} = \sqrt{\frac{k_\theta}{I_{\text{setup}} + I_{\text{body}}}}$$  \hspace{1cm} (4.23)

Solving for $I_{\text{body}}$ results in an expression (Eq. 4.24) for the MOI of the body axis aligned with the axis of rotation of the MOI table in terms of known parameters.

$$I_{\text{body}} = \frac{k_\theta T^2}{(2\pi)^2} - I_{\text{setup}}$$  \hspace{1cm} (4.24)
Using specially designed fixtures, shown in Figs. 4.11 to 4.13 the body is then reoriented so the body axis corresponding to the next MOI to be measured is aligned with the MOI table axis of rotation and the process is repeated.

POI for the body may be evaluated, using the MOI method, by taking 3 more MOI measurements about an axis rotated 45° between the x-y, x-z and y-z axes. The MOI resulting from this set of measurements will be referred to as $I_{Axy}$, $I_{Axz}$ and $I_{Ayz}$. The second set of measurements may be used in conjunction with the moments of inertia about the body axes to calculate the POI, $I_{xy}$, $I_{xz}$ and $I_{yz}$, using Eqs. 4.25-4.27 [18].

\begin{align*}
I_{xy} &= \frac{I_{xx} + I_{yy}}{2} - I_{Axy} \quad (4.25) \\
I_{xz} &= \frac{I_{xx} + I_{zz}}{2} - I_{Axz} \quad (4.26) \\
I_{yz} &= \frac{I_{yy} + I_{zz}}{2} - I_{Ayz} \quad (4.27)
\end{align*}

The vertical fixture, shown in Fig. 4.11, is used to measure $I_{zz}$. The horizontal fixture shown in Fig. 4.12 is used to measure $I_{xx}$, $I_{yy}$ and $I_{Axy}$ by rotating the Secondary Payload about the spin up bearings to align the payload with the desired measurement axis. Finally, $I_{Axz}$ and $I_{Ayz}$ can be measured using the 45° fixture displayed in Fig. 4.13.

Uncertainty levels of MOI and POI measurements are dependent on many factors including oscillation period measurement accuracy, test setup, repeatability and human error among other factors. Initial simulation runs assumed that measured MOI values would be within 1% of the actual value assuming a $3\sigma$ error range. The largest predicted POI values from the CAD model are more than two orders of magnitude smaller than the smallest MOI (see Eq. 4.20) so even a 1% variation in MOI measurements could cause a large variation in POI values. Using predicted MOI and POI values with Eqs. 4.25 to 4.27 it was found that a 1% error in MOI measurement could lead to errors in POI values greater than 1000%. As POI values get smaller the error value increases even more.
Fig. 4.11: Vertical MOI measurement fixture

Fig. 4.12: Horizontal MOI measurement fixture
Balancing procedures were developed to help minimize measurement errors and achieve the goal that the ASSP Secondary Payloads be major axis spinners with an inertia ratio of 1.25. Twenty pairs of threaded holes were incorporated into the design, as shown in Fig. 4.14, so that once inertia tensor measurements are completed weights can be added to compensate for offsets in the center of mass and minimize any mass imbalances. After weights are added, inertia tensor measurements will be repeated to verify the desired results were achieved and modify the balance weight configuration if required.

Because POI calculations are very sensitive to errors in MOI measurements, several safeguards have been incorporated into the balancing procedure to minimize errors. Some of these safeguards are summarized below.

- All measurements are conducted on an air bearing table to minimize the effect of external vibrations on the test setup
- Tare measurements are conducted after each fixture change
Fig. 4.14: Secondary Payload threaded balance holes and balance weights

- Period measurements are conducted using an optical sensor with 500 microsecond accuracy

- Period measurements are repeated 10 times for each payload orientation ($I_{xx}, I_{yy}, I_{zz}, I_{Axy}, I_{Axz},$ and $I_{Ayz}$)

- The mean of each period measurement set is used in MOI calculations

- Standard deviation of each period measurement set is used to approximate error range for each MOI calculation

Wiener of Space Electronics, a company specializing in mass property measurements, was able to achieve error levels as low as 0.35% using an inverted torsional pendulum [16]. By incorporating the safeguards described above into the balancing procedure, SDL hopes to achieve an error level of 0.5% in MOI measurements.
Fig. 4.15: Predicted gas spring velocity and acceleration profile

4.3.2 Ejection Mechanism Model

A numerical model of the gas spring expansion, created by NASA Wallops, provided an estimate for the expected acceleration profile experienced by the Secondary Payloads (see Fig. 4.15). This acceleration profile was critical for analyzing the expected vibrational modes experienced by the Secondary Payload as described in the next section, however, the exact acceleration profile does not affect any of the other perturbing factors. As was mentioned earlier, the 50 m/s target velocity may be lowered depending on the physical gas spring ground test results. If this is the case, it will not affect the validity of the stability analysis presented in this work.

Although the gas springs have guide rods and other mechanisms included in the design to minimize tip-off to the Secondary Payloads, it is likely that some minor unwanted angular acceleration will be induced during ejection. Data recorded by Secondary Payload accelerometers and gyros during ground ejection tests will be used to determine the actual
acceleration profile and estimate tip-off levels that may be experienced during on-flight ejections. However, until this data becomes available, a placeholder value of 60 rad/sec was used as a worst case tip-off angular rate.

4.3.3 Vibrational Modes

The Secondary Payload solid model was used to perform a structural and vibrational analysis, using the commercial software package FEMAP, to ensure the Secondary Payloads would be able to survive ejection loading. Additionally, there was concern that because the Science Boom consists of a tip-mass on a slender tube oriented perpendicular to the ejection direction, the high accelerations imparted by the gas springs could excite large vibrations. Significant vibration in the boom, or any internal component, would cause time dependent variation in the inertia tensor. This phenomenon could lead to significant instability in Secondary Payload attitude.

The numerical vibration analysis revealed that the first fundamental mode involves Science Boom vibration and has a frequency of 113.8 Hz. The question remained, would the relatively high frequency vibration of the Science Boom mass affect the dynamics of the overall body? The answer was found by reducing the problem to a single degree of freedom, enforced motion, vibration problem. The Secondary Payload was modeled as a body with mass \( m \), and the Science Boom was modeled as a spring with stiffness \( k \), and a viscous damper with a damping ratio \( \xi \), as shown in Fig. 4.16. The variable \( u \) is the motion of the Secondary Payload due to periodic “base motion,” \( w \), with an amplitude, \( A \), and a frequency, \( \Omega \). The base motion in this case is the Science Boom vibration and the forcing frequency is the first fundamental mode frequency. Boom stiffness may be approximated using the formula for bending stiffness in a uniform cross-section beam shown in Eq. 4.28,

\[
k = \frac{3EI}{L^3}
\]  

(4.28)

and the damping ratio, \( \xi \), is defined as
Fig. 4.16: Simplified Secondary Payload and Science Boom vibrational model

\[ \xi = \frac{c}{c_r} \]  

(4.29)

where \( c \) is the equivalent viscous damping of the boom and \( c_r \) is the critical viscous damping for the spring mass system. Structural components, such as beams, that have few joints and are made out of stiff materials, like composites, typically have low damping ratios. This analysis assumed a damping ratio of 0.02. The Secondary Payload mass, \( m \), and boom stiffness, \( k \), may be used to calculate the natural frequency, \( \omega_n \), of the spring-mass system, as shown in Eq. 4.30.

\[ \omega_n = \sqrt{\frac{k}{m}} \]  

(4.30)

The Science Boom physical properties are summarized in Table 4.4.

Craig and Kurduila state that “when \( \Omega \gg \omega_n \), the inertia of the mass keeps it from moving much, so that the relative motion consists primarily of the base moving relative to the mass” [19]. This seems to be the case here, where the forcing frequency is nearly an order of magnitude larger than the natural frequency of the spring mass system. This was
Table 4.4: Science Boom physical properties summary

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length ($L$)</td>
<td>.121 m</td>
</tr>
<tr>
<td>Outer Diameter</td>
<td>.013 m</td>
</tr>
<tr>
<td>Inner Diameter</td>
<td>.010 m</td>
</tr>
<tr>
<td>Area Moment of Inertia ($I$)</td>
<td>8.729e-10 m$^4$</td>
</tr>
<tr>
<td>$E$</td>
<td>22.4 GPa</td>
</tr>
<tr>
<td>$m$</td>
<td>3.4 kg</td>
</tr>
<tr>
<td>$k$</td>
<td>33.4 kN/m</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>15.8 Hz</td>
</tr>
</tbody>
</table>

verified by calculating the transmission ratio, $u/z$, using Eq. 4.31 [19], where $r$ is the ratio of the forcing frequency over the natural frequency.

$$TR = \frac{u}{z} = \sqrt{\frac{1 + (2\xi r)^2}{(1 - r^2)^2 + (2\xi r)^2}}$$

(4.31)

The transmission ratio for the case being considered is 2.03%. The total boom deflection during vibration is on the order of millimeters, so boom vibration was discounted as a significant perturbing factor, and was not included in the ejection dynamics simulation. The next lowest fundamental frequency involved several internal components and occurred at 315 Hz. By inspection of Eq. 4.31, it can be seen that if $r$ is increased $TR$ will be reduced even further, making internal vibration due to ejection even less significant to overall stability than the Science Boom vibration.

**4.3.4 Characterization of Rocket Body Instability**

Instability in the rocket body itself prior to ejection must be accounted for to accurately model the ejection process. Two primary sources of instability were identified. First, residual roll and transverse rates based off of attitude control performance will supply an angular velocity perturbation to the Secondary Payload. Second, any pressure imbalance between the fore and aft gas springs, when each pair of Secondary Payloads are ejected, will create a torque on the rocket body that will induce another perturbation to the initial angular velocity of the Secondary Payload.
Prior to ejection, the Main Payload attitude control system will de-spin the rocket body and align the z-axis of the Secondary Payload to be ejected with the local magnetic field line. Attitude control system performance simulations conducted by NASA Wallops Flight Dynamic Division show Main Payload alignment with the desired attitude will be within 0.8° roll and 0.5° in pointing. Main Payload angular rates will be below 0.5°/sec and 0.2°/sec in the roll and transverse directions. According to the flight control engineer at Wallops, these numbers represent the bounds of a range of possible values centered about zero with a uniform probability of occurrence. Due to some uncertainty in these values a worst case margin of 25% was assumed in the instability model.

Because the exact orientation of the spinning Secondary Payloads is unknown at the time of ejection, the contribution to the initial conditions of the roll alignment and rate are randomly distributed between the x and y axes of the ASSP Geometry Coordinate System. The contribution from the transverse alignment and rate is randomly distributed between all three axes.

As mentioned in Section 2.3.1, pressure in the gas springs may vary by as much as 10 psi. If it is assumed that both fore and aft gas springs fire simultaneously this will result in up to a 20 psi differential. For simulation purposes, a 25% uncertainty level was again assumed. An estimate of the angular rate induced in the Main Payload by a pressure imbalance during ejection may be obtained using the following equation where \( \omega_{\text{eject}} \) is the angular velocity induced by ejection pressure imbalance, \( dp \) is the pressure differential and is assumed to remain constant during ejection, \( la \) is the lever arm or the radial distance between the center of gravity of the Main Payload and the center of the gas spring, \( dt \) is the duration of the ejection, \( A_{\text{piston}} \) is the cross-sectional area of the gas spring and \( I_{\text{trans}} \) is the transverse MOI of the Main Payload.

\[
\omega_{\text{eject}} = \frac{A_{\text{piston}} \cdot dp \cdot la \cdot dt}{I_{\text{trans}}} \tag{4.32}
\]

As with the residual transverse rate previously discussed, this perturbation is randomly distributed between all three Secondary Payload axes.
Chapter 5

Simulation Results and Perturbation Minimization

5.1 Baseline Simulation

Because there are so many factors that influence the flight dynamics of the Secondary Payloads, a Monte Carlo approach was deemed appropriate to determine the perturbing factor limits bounding the ejection. Each Monte Carlo run of the simulation consisted of 1000 individual 5 second integrations using a time step of 0.001 sec. Perturbation levels for each integration were randomly selected from the error ranges described in Section 4.3. Mass properties from the CAD model were used as a baseline to which random errors were added.

An initial estimate of 1% errors in MOI measurements was used in the first simulation. The distribution of coning and B-field alignment angles are plotted in histogram form in Fig. 5.1. The distributions in both histograms are skewed to the left and appear to follow a log-normal distribution. The cumulative density function for a log-normal distribution may be calculated using Eq. 5.1

\[
CDF = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{\ln(x) - \mu}{\sqrt{2}\sigma} \right)
\] (5.1)

where \(\mu\) and \(\sigma\) are the mean and standard deviation of the variables natural logarithm. In this case the variables in question are coning angle and B-field alignment. The values for \(\mu\) and \(\sigma\) may be approximated using Eqs. 5.2 and 5.3

\[
\mu = \ln \left( \frac{m^2}{\sqrt{v + m^2}} \right)
\] (5.2)

\[
\sigma = \sqrt{\ln \left( \frac{v}{m^2} + 1 \right)}
\] (5.3)
where $m$ and $v$ are the mean and variance (standard deviation squared) of the log-normal distribution.

Using Eq. 5.1 the one sided 95 and 99.7 percentile maximum were calculated for both the coning and B-field alignment angles. These values are equivalent to the $2\sigma$ and $3\sigma$ maximum levels in Research Objective 4 outlined in Chapter 3. The mean, standard deviation, 95% and 99.7% maximum levels for both variables are shown in Table 5.1. Both the mean and 99.7% maximum coning angle for the baseline simulation exceed the $5^\circ$ limit set out by the science requirements, however the B-field alignment mean does stay within the desired $25^\circ$ limit, while the 99.7% maximum exceeds it by $22.55^\circ$.

Before any efforts were taken to reduce the maximum coning angle, additional studies were performed to determine which were the driving perturbation factors. The factors were divided into three groups. First, errors due to mass imbalance, second, angular rate perturbations including both ejection tip-off and rocket body instability and third, roll and
Table 5.2: Monte Carlo simulation results for perturbing factors in isolation

<table>
<thead>
<tr>
<th></th>
<th>Coning Angle Mean</th>
<th>Coning Angle Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Imbalance</td>
<td>10.93°</td>
<td>6.03°</td>
</tr>
<tr>
<td>Angular Rate Perturbation</td>
<td>2.82°</td>
<td>0.31°</td>
</tr>
<tr>
<td>Roll and Pointing Error</td>
<td>2.37°</td>
<td>0.00°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>B-Field Align Mean</th>
<th>B-Field Align Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Imbalance</td>
<td>12.87°</td>
<td>7.32°</td>
</tr>
<tr>
<td>Angular Rate Perturbation</td>
<td>3.83°</td>
<td>0.60°</td>
</tr>
<tr>
<td>Roll and Pointing Error</td>
<td>3.24°</td>
<td>0.58°</td>
</tr>
</tbody>
</table>

pointing errors. Monte Carlo simulations were run for each perturbing factor in isolation. The mean and standard deviation for each case are displayed in Table 5.2. It is obvious from the data in Table 5.2 that the most influential perturbing factor is by far mass imbalance.

5.2 Perturbation Minimization

Initial perturbations resulting from roll and pointing errors, tip-ff and ejection induced torques were shown to have minimal influence on Secondary Payload stability. Therefore, perturbation minimization efforts are focused on improving Secondary Payload mass properties.

A simulated mass property balance was carried out in the CAD environment using the solid model brass and tungsten weights as shown in Fig. 4.14. Using a set of standard weight sizes, the Secondary Payload was balanced accurately enough that $I_{xx}$ and $I_{yy}$ are within 0.7% of each other, $I_{zz}/I_{xx} = 1.24$ and $I_{zz}/I_{yy} = 1.24$ and the products of inertia (POI) $I_{xy}$, $I_{yz}$ and $I_{xz}$ are more than three orders of magnitude smaller than the moments of inertia (MOI). Given this information a new baseline inertia tensor was created (see Eq. 5.4).

$$[I] = \begin{bmatrix} .010789 & -.000001 & -.000003 \\ -.000001 & .010718 & .000004 \\ -.000003 & .000004 & .013393 \end{bmatrix} \left( \text{kg} \cdot \text{m}^2 \right)$$  (5.4)
This new baseline assumption is dependent upon the accuracy of the MOI measurements. Up to 1% error in MOI measurements was assumed in the baseline simulations. As was mentioned in Section 4.3.1, SDL hopes to achieve MOI measurement errors as low as 0.5% by implementing rigorous safeguards on the measurement procedure. Using the new balanced inertia tensor two additional cases were run, first with 1% MOI error as a maximum error bound, and second with 0.5% as an estimate for the minimum achievable error level.

Balancing the Secondary Payloads resulted in a marked improvement in the coning angle and B-field alignment distributions as displayed in Figs. 5.2 and 5.3. If a 1% MOI measurement error is assumed, balancing the payloads cuts the coning angle and B-field misalignment by more than half. If the MOI measurement error is reduced to 0.5%, both the coning angle and B-field misalignment are reduced by an additional 25% by balancing (see Figs. 5.1 and 5.3).
Fig. 5.3: Monte Carlo simulation results distributions after balancing with 0.5% MOI error estimate

Table 5.3: Monte Carlo simulation results after Secondary Payload balancing and a comparison to the baseline simulation

<table>
<thead>
<tr>
<th></th>
<th>Coning Angle</th>
<th>Coning Angle Reduction</th>
<th>B-Field Align</th>
<th>B-Field Align Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1% MOI Error</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>4.00º</td>
<td>62.5%</td>
<td>4.85º</td>
<td>62.0%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.94º</td>
<td>67.2%</td>
<td>2.54º</td>
<td>65.0%</td>
</tr>
<tr>
<td>95%</td>
<td>7.70º</td>
<td>64.8%</td>
<td>9.70º</td>
<td>63.5%</td>
</tr>
<tr>
<td>99.7%</td>
<td>12.70º</td>
<td>67.2%</td>
<td>16.65º</td>
<td>65.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Coning Angle</th>
<th>Coning Angle Reduction</th>
<th>B-Field Align</th>
<th>B-Field Align Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.5% MOI Error</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.16º</td>
<td>79.8%</td>
<td>2.71º</td>
<td>78.8%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.96º</td>
<td>83.8%</td>
<td>1.28º</td>
<td>82.4%</td>
</tr>
<tr>
<td>95%</td>
<td>4.00º</td>
<td>81.7%</td>
<td>5.15º</td>
<td>80.6%</td>
</tr>
<tr>
<td>99.7%</td>
<td>6.35º</td>
<td>83.6%</td>
<td>8.40º</td>
<td>82.3%</td>
</tr>
</tbody>
</table>
Chapter 6

Conclusion

As discussed earlier, secondary payloads are often ejected during sounding rocket missions for various purposes. Low speed ejections of these payloads ($<10$ m/s) have been successfully achieved, primarily using compressed springs. However, a need exists for an ejection system that can successfully achieve high separation velocities while allowing the ejected payload to maintain spin stability. A gas spring ejection system capable of achieving ejection velocities of 50 m/s has been developed by NASA’s Wallops Flight Facility for this purpose. A 6 degree of freedom, rigid body, dynamic simulation was created to assess the stability of spinning ASSP Secondary Payloads following ejection by the WFF designed gas spring system. Various tests were conducted to validate the simulation by using cases with known analytical solutions for comparison. These tests verified that the simulation is 4th order accurate.

The effects of several perturbing factors were considered while creating the simulation including: vibration of Secondary Payload components, tip-off from the gas springs, rocket body instability, and Secondary Payload mass imbalances. The lowest frequency vibration modes of the Secondary Payload were calculated and deemed too high to affect the overall dynamics of the body. Secondary Payload component vibration was, therefore, determined not to be a significant perturbing factor and was eliminated from the simulation. Gas spring tip-off will be characterized after physical testing of the system is complete. The results presented in this document use a placeholder value of $60^\circ$/s as an estimate for tip-off perturbation. Simulation runs, including the other perturbing factors in isolation, showed that the performance characteristics of today’s attitude control systems, such as MaNIACS, are accurate enough that attitude control errors are not a major perturbation to Secondary Payload stability. The most significant known perturbing factor to Secondary Payload
stability is by far the mass balance of the payload itself. Design features, incorporated into the Secondary Payload frame, allow mass imbalances to be minimized using external weights.

The limiting factor in the balancing process is the MOI measurement accuracy. MOI measurement procedures, including numerous safeguards were developed to minimize measurement errors. After Secondary Payload balancing, if a 1% error is assumed, the mean Secondary Payload coning angle will be within the 5° requirement, but the 95% and 99.7% maximum coning angles exceed the requirement by 2.7° and 7.7° respectively. If the error level is reduced to 0.5% the mean and 95% maximum coning angles meet the requirement and the 99.7% maximum coning angle exceeds the 5° requirement by only 1.35°. The 25° B-field alignment requirement is easily met in both cases for the mean, 95% and 99.7% maximum levels (see Table 5.3).

The results obtained from this simulation show that spin stabilized Secondary Payload flight using a gas spring ejection is possible. However, it should be noted that the assumptions involved in modeling many of the perturbing factors are based on data from previous missions and theoretical models. Improvement to the perturbation models will be made when physical test and flight data become available for comparison.

The last remaining question to be answered is, “Are the costs of improving the mass properties worth the effort?” It is estimated that it will take approximately two weeks to balance all six Secondary Payloads using SDL’s existing facilities. Resulting in a few thousand dollars cost in labor, fixturing, design time and other resources. Due to the sensitive nature of the wire E-field instrument design and the required sensitivity for the B-field measurements, the answer is yes. Large coning angles or misalignment from the local B-field vector could render collected data difficult to interpret.

Using the understanding of ASSP Secondary Payload ejection dynamics established here, the predicted coning and B-field alignment angles will serve as a starting point for studying the dynamic behavior of rapidly deployed wire booms. Once dynamic models for boom deployment are created, they can be combined with the results from this stability
study, as well as existing trajectory models, into an overall Secondary Payload dynamics model. The follow-on model will then be superimposed on existing models of the Earth’s electric and magnetic fields to provide ASSP with a theoretical baseline measurement set which can be compared to flight measurements.

The design approach and simulation methodology presented here will likely be useful to similar missions in the future. However, the methods laid out in this paper should not be taken as a panacea for correcting stability issues. Careful evaluation of the science requirements and the associated costs of payload balancing should be undertaken to determine acceptable levels of instability and the appropriate level of effort that should be put into balancing the payload.
References


2. Disbrow, M. E., “Postflight Mission Analysis Black Brant XII MK1 40.023 UE (Lynch/Dartmouth College/PFRR),” Black Brant XII Mk1 40.023 UE, Jan. 2010.


FAST and POLAR,” *Geophysical Research Letters*, Volume 25(12), 1998, pp. 2081-
2084.

14. Maynard, N. C., “Electric Field Measurements in Moderate to High Density Space
Plasmas with Passive Double Probes,” *American Geophysical Union Geophysical


16. Shepperd, S. W., “Universal Keplerian State Transition Matrix,” *Celestial Mechanics,

Conference of the Society of Allied Weight Engineers*, at Wichita, Kansas May 18-20,

18. Wiener, K., and Boynton, R., “Using the "Moment of Inertia Method to Determine
Product of Inertia,” *51st Annual Conference of the Society of Allied Weight Engineers*, at

pp. 81-113.
Appendix
MATLAB CODE

Main Driver Program

% Ejection Dynamics Simulation
% Weston Nelson - June 2013
% This rigid body dynamic simulation is customized for ASSP and
% predicts coning angles and B-field alignment. The derivation for
% the equations of motion is presented in "Analytical Dynamics" by
% Haim Baruh and are presented in state space derivative form. The
% variable integrated are defined below.
% Translation Variables in Body Fixed Frame
% X(1,n)                         % vx (m/s)
% X(2,n)                         % vy (m/s)
% X(3,n)                         % vz (m/s)
% Position Variables in Inertial Frame (ECI)
% X(4,1)                         % Ax coordinate (m)
% X(5,1)                         % Ay coordinate (m)
% X(6,1)                         % Az coordinate (m)
% Angular Velocity Variables in Body Fixed Frame
% X(7,1)                         % w1 (rad/s)
% X(8,1)                         % w2 (rad/s)
% X(9,1)                         % w3 (rad/s)
% Euler Parameters in Inertial Frame (ECI)
% X(10,1)                        % e0
% X(11,1)                        % e1
% X(12,1)                        % e2
% X(13,1)                        % e3

clear all; close all; clc

global Ip mass GMearth

% User Input

% Simulation Time settings
t_start = 0;                        % (sec)
t_final = 5;                        % (sec)
dt0 = .001;                         % time step (sec)
nsteps = ceil((t_final-t_start)/dt0)+1;    % number of simulation steps
monte_carlo_max = 100;

% Perterbation variables

% Randomization coefficient to get appropriate standard deviation
% rc_std = 1 for 1-sigma distribution
% rc_std = 1/n for n-sigma distribution
rc_std = 1/3; % 3-sigma distribution on error rates

uf = 1.25; % uncertainty factor add to estimates provided by Wallops

%Initial spin rate
wx0 = 0;
wy0 = 0;
wz0 = 4*pi;

% Pressure imbalance between opposing gas springs
dp = 20*uf; % psi

% Residual angular rates at time of ejection
w_roll = 0.5*uf; % deg/s
w_trans = 0.2*uf; % deg/s

% MANIACS alignment error
error_roll = 0.8*uf; % deg
error_point = 0.5*uf; % deg

% Tip Off (worst case)
w_tipoff = 60; % deg/s

% Total body mass
mass0 = 3.4; % (kg)
mass_error = 0.05; % (kg)

%MOI measurement accuracy
MOI_error = .01;
POI_error = 90.55;

% Earth's gravity
GMearth = 398600.4356e9; % m^3/s^2

% Initial Position Vector (ECI) (randomization not added)
r0 = [6378000; 0; 0]; % (m)

% Initial Velocity Vector (body)
% NOTE: MUST CHANGE RANDOMIZATION IF VY AND VZ ARE NOT NON-ZERO!!!!
vx0 = 50; % (m/s)
vy0 = 0;
vz0 = 0;
v_error = 0.01;

% initial Euler angles
phi0 = 0;
theta0 = 0;
psi0 = 0;

% NOTE: Inertia tensor may be edited on lines 122 to 127

%---------------------------------------------------------------------
% Preallocate memory for output vectors
max_cone = zeros(monte_carlo_max,1);
min_cone = zeros(monte_carlo_max,1);
max_B_angle = zeros(monte_carlo_max,1);

% Initialize data array
X       = zeros(13,nsteps);
Tsav    = zeros(1,nsteps);
phi     = zeros(1,nsteps);
theta   = zeros(1,nsteps);
psi     = zeros(1,nsteps);
w_body  = zeros(3,nsteps);
H       = zeros(3,nsteps);
coning  = zeros(1,nsteps);
B_angle = zeros(1,nsteps);

%%---------------------------------------------------------------------
for j = 1:monte_carlo_max

  % Base Moment of Inertia Tensor
  Ixx = 0.010789; % (kg*m^2)
  Iyy = 0.010718; % (kg*m^2)
  Izz = 0.013393; % (kg*m^2)
  Ixy = 0.00000267; % (kg*m^2)
  Iyz = 0.00000267; % (kg*m^2)
  Ixz = 0.00000267; % (kg*m^2)

  [Ixx,Iyy,Izz,Ixy,Iyz,Ixz]=inertia_variability(Ixx,Iyy,Izz,Ixy,Iyz,Ixz,MOI_error,POI_error,rc_std);

  mass = mass0*(1+mass_error*(rc_std*randn));

  % calculate perturbation to initial angular velocity
  [ w_pert ] = ang_rate_pert( dp,w_roll,w_trans,w_tipoff,rc_std);

  % Initial Angular Velocity Vector (ASSP geometry coordinates)
  w0 = [wx0;wy0;wz0]+w_pert; % (rad/s)

  % Initial Velocity Vector in inertia coordinates
  vx = vx0*(1+v_error*(rc_std*randn));
  vy = vx0*(v_error*(rc_std*randn));
  vz = vx0*(v_error*(rc_std*randn));

  % Inertia Tensor
  I_body = [Ixx Ixy Ixz; ...
            Ixy Iyy Iyz; ...
            Ixz Iyz Izz];

  % Calculate Principal Moments of Inertia
[c, I_principle] = eig(I_body);

Ip(1) = I_principle(1,1);
Ip(2) = I_principle(2,2);
Ip(3) = I_principle(3,3);

% Convert to w0 to principal coordinates/body coordinates
wp = c'*w0;

% Calculate initial Euler angles using Eq 7.5.3
theta0 = acos(c(3,3));
phi0 = asin(c(3,1)/sin(theta0));
psi0 = acos(c(2,3)/sin(theta0));

% initial Euler angles
[phi_pert, theta_pert, psi_pert] = pointing_error(...
    (error_roll, error_point);

phi(1) = phi0 + phi_pert;
theta(1) = theta0 + theta_pert;
psi(1) = psi0 + psi_pert;

%convert initial velocity from inertial to body coordinates
[ R ] = R_matrix_313( phi(1), theta(1), psi(1) );
v0 = R*[vx;vy;vz];

[e0,e1,e2,e3] = euler313_to_eparam(phi(1), theta(1), psi(1));

% Set initial conditions
% Translation Variables in Body Fixed Frame
X(1,1) = v0(1);                 % vx (m/s)
X(2,1) = v0(2);                 % vy (m/s)
X(3,1) = v0(3);                 % vz (m/s)

% Position Variables in Inertial Frame (ECI)
X(4,1) = r0(1);                 % Ax coordinate (m)
X(5,1) = r0(2);                 % Ay coordinate (m)
X(6,1) = r0(3);                 % Az coordinate (m)

% Angular Velocity Variables in Body Fixed Frame
X(7,1) = wp(1);                 % w1 (rad/s)
X(8,1) = wp(2);                 % w2 (rad/s)
X(9,1) = wp(3);                 % w3 (rad/s)

% Euler Parameters in Inertial Frame (ECI)
X(10,1) = e0;                   % e0
X(11,1) = e1;                   % e1
X(12,1) = e2;                   % e2
X(13,1) = e3;                   % e3

i = 1;
t = t_start;
dt = dt0;
Tsav(1) = t_start;

while t < t_final
    i = i+1;
    if (t+dt) > t_final
        dt = t_final - t;
    end

    x_old = X(:,i-1);
    x_new = rk45(@diffeq_6_DOF, x_old, t, dt);
    t = t + dt;

    X(:,i) = x_new;
    Tsav(i) = t;
    x_old = x_new;
    [phi(i), theta(i), psi(i)] = eparam_to_euler313...
        (X(10,i),X(11,i),X(12,i),X(13,i));

    % rotate angular velocity vector to body coordinates
    w_body(:,i) = c*X(7:9,i);

    % calculate the angular momentum vector
    H(:,i) = I_body*w_body(:,i);

    % calculate the coning angle
    temp = dot([0,0,1]',H(:,i)/norm(H(:,i)));
    if norm(temp) > 1
        temp = sign(temp);
    end
    coning(i) = acos(temp)*180/pi;    %coning angle (degrees)

    % calculate the B-field vector in body coordinates
    [ R ] = R_matrix_313( phi(i), theta(i), psi(i) );
    B_vec = R*[0;0;1];

    % find angle between B-field and science boom
    temp = dot([0;0;1],B_vec);
    B_angle(i) = acos(temp)*180/pi;     %B-field alignment(degrees)
end

% Record max/min coning angle from run
max_cone(j) = max(coning);
min_cone(j) = min(coning);

% Record max B-field alignemnt angle
max_B_angle(j) = max(B_angle);
end

mean_cone = mean(max_cone)
\[ \text{std}\_\text{cone} = \text{std}(\text{max}\_\text{cone}) \]

\[ \text{mean}\_\text{B}\_\text{angle} = \text{mean}(\text{max}\_\text{B}\_\text{angle}) \]
\[ \text{std}\_\text{B}\_\text{angle} = \text{std}(\text{max}\_\text{B}\_\text{angle}) \]

\begin{verbatim}
figure(1)
hist(max_cone,20)
xlabel('Coning Angle (deg)')
ylabel('Count')
title('Monte Carlo Coning Angle Distribution')

figure(2)
hist(max_B_angle,20)
xlabel('B-Field Alignment Angle (deg)')
ylabel('Count')
title('Monte Carlo B-Field Alignment Angle Distribution')
\end{verbatim}

**Inertia Variability Function:** adds randomized errors to the inertia tensor based on specified MOI and POI error levels

\begin{verbatim}
%This function outputs inertia tensor variables after adding on randomized
%variability within the levels of measurement accuracy

% Add random measurement error to MOI
Ixx = Ixx*(1+MOI_error*(rc_std*randn));
Iyy = Iyy*(1+MOI_error*(rc_std*randn));
Izz = Izz*(1+MOI_error*(rc_std*randn));

% Add random measurement error to POI
Ixy = Ixy*(1+POI_error*(rc_std*randn));
Iyz = Iyz*(1+POI_error*(rc_std*randn));
Ixz = Ixz*(1+POI_error*(rc_std*randn));
end
\end{verbatim}

**Angular Rate Perturbation Function:** adds randomized angular rate errors from MaNIACS, and gas spring ejection

\begin{verbatim}
function [ w_pert ] = ang_rate_pert( dp,w_roll,w_trans,w_tipoff,rc_std)
%This function calculates randomized angular rate perturbation inputs

% Worst Case perturbation inputs
% dp      = Pressure imbalance between opposing airsprings (psi)
% w_roll  = Residual roll rate at ejection (deg/s)
% w_trans = Residual transverse (pitch/yaw) rate at ejection (deg/s)
% w_tipoff= Angular velocity due to tipoff (deg/s)
\end{verbatim}
% This assumes a uniform distribution in realized main payload roll and % transverse rates (see note), and a 3-sigma normalized distribution on % the other perturbing factors.

% NOTE FROM ERNIE BOWDEN WITH REGARD TO w_roll and w_trans % They aren’t 1-sigma numbers, but they aren’t really normally % distributed at all. I’d take them as a uniform distribution centered % on 0. Also, the design will probably be refined, so they may change % some as we move forward, so some margin should be carried on the % rates. Will you be getting a coning angle vs payload body rates % function from your work?
% Ernie Bowden
% NSROC GNC Engineer
% Ernest.L.Bowden@nasa.gov
% Office: 757.824.2137
% Cell: 757.894.2136

% Main Payload Angular Velocity Rates at Ejection
w_roll = w_roll*pi/180;        % convert to rad/s
w_trans = w_trans*pi/180;      % convert to rad/s

% Generate a random roll rate within the ranges above
% r = a + (b-a).*rand; generates random number on interval [a,b]
w_roll = w_roll*(2*rand-1);
w_trans = w_trans*(2*rand-1);

% Angular rates induced by pressure imbalance
dp = dp*6894.75729;            % convert to Pa
dp = dp*rc_std*randn;
[w_eject] = ejection_imbalance( dp );

% Ejection Tip-off
w_tipoff = w_tipoff*pi/180;     % convert to rad/s

% Calculate factors to randomly distribute disturbances between axes
% randomly distribute w_roll between wx and wy
c1 = rand;
% randomly distribute w_trans between wx, wy and wz
c2 = rand;
c3 = rand;
c4 = c2*(2/3);
c5 = c3*(1-c2*(2/3));
c6 = (1-c2*(2/3)-c3*(1-c2*(2/3)));
% randomly distribute w_eject between wx, wy and wz
c7 = rc_std*rand;
c8 = rc_std*rand;
c9 = c7*(2/3);
c10 = c8*(1-c7*(2/3));
c11 = (1-c7*(2/3)-c8*(1-c7*(2/3)));
% randomly distribute w_tipoff between wx, wy and wz
c12 = rc_std*rand;
c13 = rc_std*rand;
c14 = c12*(2/3);
c15 = c13*(1-c12*(2/3));
c16 = (1-c12*(2/3)-c13*(1-c12*(2/3)));

w_pert = [c1*w_roll+c4*w_trans+c9*w_eject+c14*w_tipoff ;...
(1-c1)*w_roll+c5*w_trans+c10*w_eject+c15*w_tipoff;...
c6*w_trans+c11*w_eject+c16*w_tipoff ];

end

Pointing Error Perturbation Function: adds randomized pointing and roll position errors from MaNIACS

function [ phi_pert,theta_pert, psi_pert ] = pointing_error(error_roll,error_point)
%This function adds roll and pointing error into the initial orientation
% at the time of ejection

phi_pert = 0;

%convert errors to radians
error_roll = error_roll*pi/180;
error_point = error_point*pi/180;

%randomize level of pointing error
error_roll = error_roll*(2*rand-1);
error_point = error_point*(2*rand-1);

theta_pert = error_roll;  % accounts for roll error

%distribute pointing error between theta and psi
cl = rand;
theta_pert = theta_pert+cl*error_point;
psi_pert = (1-cl)*error_point;

end

313 Rotation Matrix Function: creates a 313 rotation matrix from euler angles (inertial to body)

function [ R ] = R_matrix_313( phi, theta, psi )
%creates a 313 rotation matrix from euler angles (inertial to body)

c_phi = cos(phi);
s_phi = sin(phi);

c_theta = cos(theta);
s_theta = sin(theta);
c_psi = cos(psi);
s_psi = sin(psi);

% Form rotation matrix
R = [c_phi*c_psi - s_phi*c_theta*s_psi, s_phi*c_psi + c_phi*c_theta*s_psi,
    -c_phi*s_psi - s_phi*c_theta*c_psi, -s_phi*s_psi + c_phi*c_psi*c_theta,
    s_phi*s_theta, -c_phi*s_theta, c_theta];

313 Euler angle to Euler parameter conversion function

function [e0,e1,e2,e3] = euler313_to_eparam(phi, theta, psi)
% 3-1-3 rotation sequence
% Ref. Eq 7.7.40 Analytical Dynamics

e0 = cos((phi+psi)/2)*cos(theta/2);
e1 = cos((phi-psi)/2)*sin(theta/2);
e2 = sin((phi-psi)/2)*sin(theta/2);
e3 = sin((phi+psi)/2)*cos(theta/2);

Euler parameter to 313 Euler angle conversion function

function [phi, theta, psi] = eparam_to_euler313(e0,e1,e2,e3)
% 3-1-3 rotation sequence
% Ref. Eq 7.7.42,44 Analytical Dynamics

phi = atan2(e3,e0) + atan2(e2,e1);
psi = atan2(e3,e0) - atan2(e2,e1);
theta = 2*asin(sqrt(e1^2+e2^2));

4th Order Runge Kutta Integrator: completes one integration step

function xnew = rk45(F,xold,t,dt)
y = xold;
x = t;
dydx = feval(F,y,x);

h=dt;
hh = h/2;
h6 = h/6;
xh = x + hh;

%keyboard
yt = y + hh*dydx;

dyt = feval(F,yt,xh);
yt = y + hh*dyt;

dym = feval(F,yt,xh);
yt = y + h*dym;
dym = dyt + dym;

dyt = feval(F,yt,x+h);
yout = y + h6*(dydx+dyt+2*dym);
xnew = yout;

State Space Derivative Function: calculates the state space derivatives needed for integration as formulated by Haim Baruh. Note: Because Secondary Payload ejections are exo-atmospheric no moment inputs were experienced by the body.

function [dx] = diffeq_6_DOF(X,t)

global Ip mass

% Reference: "Analytical Dynamics, Haim Baruh" Section 8.7
% Eqns 1-6 are translational equations of motion.
% Eqns 7-13 are rotational equations of motion using an Euler parameter formulation and a 3-1-3 rotation sequence.
% v1 dot, v2 dot, v3 dot - velocity derivative components in body fixed coordinates
% A1 dot,A2 dot,A3 dot - position derivatives in inertial coordinates
% w1 dot,w2 dot,w3 dot - angular rate derivatives about principles axis
% e0 dot, e1 dot, e2 dot, e3 dot - euler parameter derivatives about principles axis
%

dx=zeros(13,1);

% Unpack state vector
% Translation Variables in Body Fixed Frame
v1 = X(1);         % vx (m/s)
v2 = X(2);         % vy (m/s)
v3 = X(3);         % vz (m/s)

% Position Variables in Inertial Frame (ECI)
A1 = X(4);          % Ax coordinate (m)
A2 = X(5);          % Ay coordinate (m)
A3 = X(6);   % Az coordinate (m)
A = [A1;A2;A3];

% Angular Velocity Variables in Body Fixed Frame
w1 = X(7);   % w1 (rad/s)
w2 = X(8);   % w2 (rad/s)
w3 = X(9);   % w3 (rad/s)

% Euler Parameters in Inertial Frame (ECI)
e0 = X(10); % e0
e1 = X(11); % e1
e2 = X(12); % e2
e3 = X(13); % e3

[phi, theta, psi] = eparam_to_euler313(e0,e1,e2,e3);

[R] = R_matrix_313( phi, theta, psi );

%Calculate force and moment vectors in ECI
[F] = ASSP_force( A );
[M] = ASSP_moment( t );
M = [0;0;0];

%Rotate to body coordinates
F = R*F;

% Derivatives for translational equations of motion
% Eq 8.7.5 Analytical Dynamics - Haim Baruh
dx(1) = -v3*w2 + v2*w3 + F(1)/mass;   % v1 dot
dx(2) = -v1*w3 + v3*w1 + F(2)/mass;   % v2 dot
dx(3) = -v2*w1 + v1*w2 + F(3)/mass;   % v3 dot
dx(4:6) = R*[v1;v2;v3];

% Derivatives for rotational equations of motion
% Eq 8.7.2 Analytical Dynamics - Haim Baruh
dx(7) = (Ip(2)-Ip(3))*w2*w3/Ip(1) + M(1)/Ip(1);  % w1 dot
dx(8) = (Ip(3)-Ip(1))*w1*w3/Ip(2) + M(2)/Ip(2);  % w2 dot
dx(9) = (Ip(1)-Ip(2))*w1*w2/Ip(3) + M(3)/Ip(3);  % w3 dot

% Euler parameter derivatives
dx(10) = 0.5*(-w1*e1 - w2*e2 - w3*e3);   % e0 dot
dx(11) = 0.5*(w1*e0 + w3*e2 - w2*e3);   % e1 dot
dx(12) = 0.5*(w2*e0 - w3*e1 + w1*e3);   % e2 dot
dx(13) = 0.5*(w3*e0 + w2*e1 - w1*e2);   % e3 dot