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Energy Extraction from Black Holes

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This work was made possible through a UNSG consortium Fellowship, as well as support through the USU physics department and my advisor, Dr. M. J. Rodriguez

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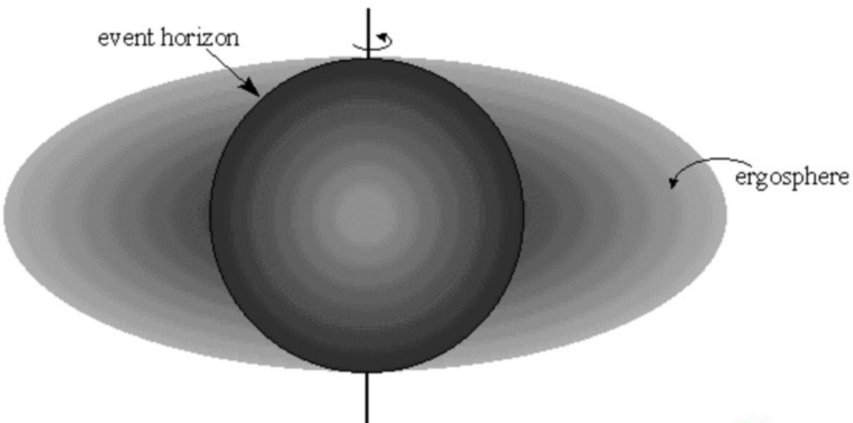
4D Black Holes

$$ds^2 = -\left(1 - \frac{r_s r}{\Sigma}\right) c^2 dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r_s r a^2}{\Sigma}\sin^2\theta\right)\sin^2\theta\, d\phi^2 - \frac{2 r_s r a \sin^2\theta}{\Sigma} c\,dt\, d\phi$$

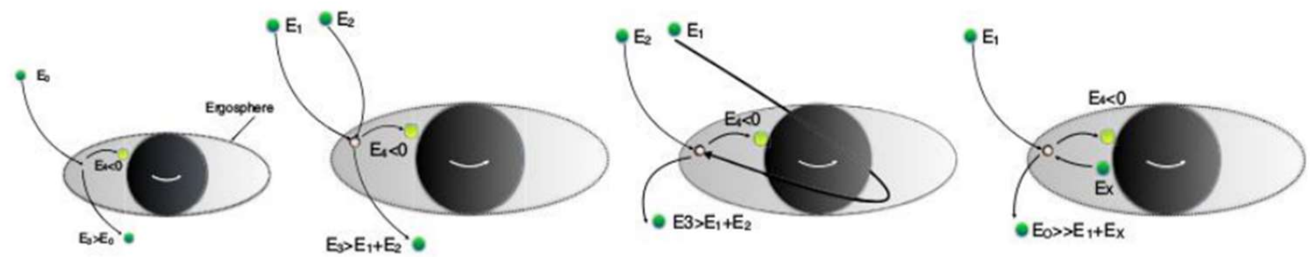


Timelike objects within the ergosphere rotate relative to asymptotic infinity.

Closed, null surface with Killing vectors: $\{\partial_t, \partial_\phi\}$

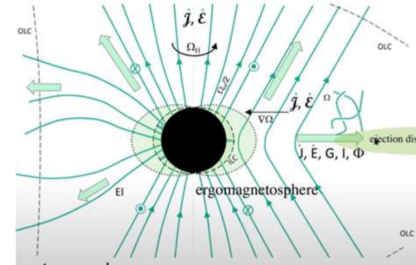
Asymptotic (Thermodynamic) Charges: $\{4\pi r_s, r_s a\}$

<u>Penrose Process:</u>



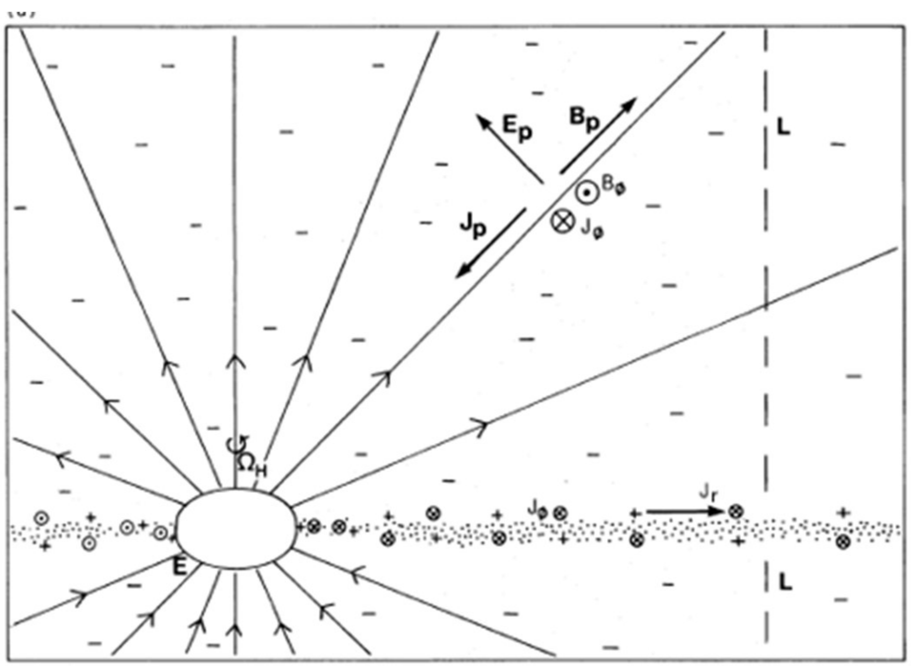
Intuitively, there is a “rotational induction” within the ergosphere; this allows energy to be extracted from the J= Ma (thermodynamic) charge when the scattering orbit has some oppositely induced states.

4D Magnetospherics



- In fact, considering the ideal Magnetohydrodynamic limit:

Fluid Field: $$S = S_{BH} + S_{matter} \approx S_{BH} + S_{EM} = \int \sqrt{-g}\left(R + \frac{1}{4}F^2 - A \cdot J\right)$$



Gravitationally Saddled:

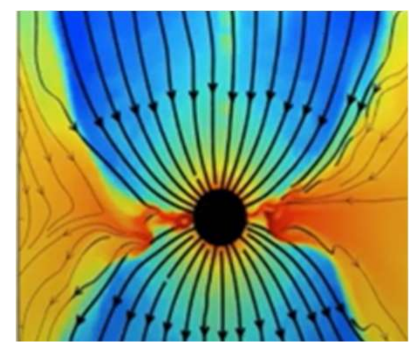
$$\delta S \approx \delta S_{EM}$$

This action gives vacuum (source free) E&M in curved spacetime:

$$F_{\mu\nu} J^{\nu} = 0$$

Using the spacetime symmetries, these four equations can be reduced to one, the Stream Equation:

$$\partial_\theta\left(\frac{I^2}{8\pi^2} - \frac{A\sin^2\theta}{2\Sigma^2}(\Omega_F - \Omega_H)^2(\partial_\theta \psi)\right) = 0$$



Perturbation Method

- Under Cosmic Censorship, here there is only one thermodynamic ratio:

$$\Delta(r) = r^2 - r_s r + a^2 \quad \Rightarrow \frac{r_+}{M} = \frac{1}{2}\left(1 + \sqrt{1-\alpha^2}\right) \qquad \alpha = \frac{J}{M^2}$$

- Then, model symmetries (equatorial) can be used to initialize perturbative solution seeds.

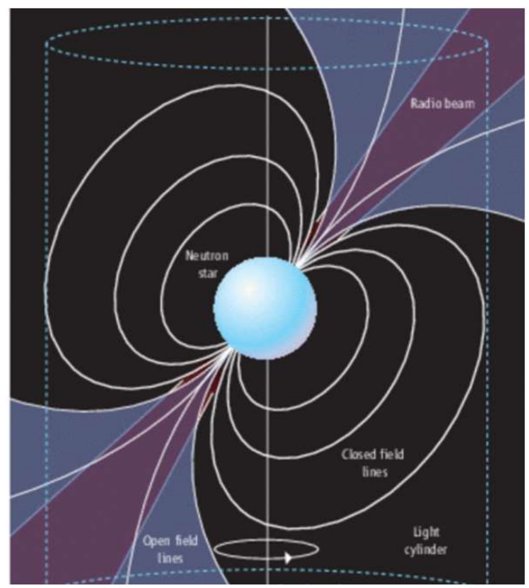
Today, only consider monopole solutions:

$$\Psi_\phi = \Psi_\phi^{(0)} + \alpha^2 \Psi_\phi^{(1)} + O(\alpha^4) \qquad \frac{\alpha r}{r_0} \ll 1$$

$$M\omega_\phi = \alpha\omega^{(1)} + O(\alpha^3)$$

$$MI = \alpha I^{(1)} + O(\alpha^3) \qquad 0 < \theta < \frac{\pi}{2}$$

$^{(i)}$ Indicates the perturbative solution order; e.g., $^{(0)}$ indicates the family of support fields for the α=0-solution.



Then, to control stability of the stability of the light surfaces, the solution can be matched to a di-pole far-field perturbation.

$$F = \frac{I}{2\pi(-g^T)^{1/2}}\epsilon_P + d\psi \wedge \eta \qquad \eta = d\varphi - \Omega_F(\psi)dt$$

$$\chi_F = \partial_t + \Omega_F(\psi)\partial_\phi$$

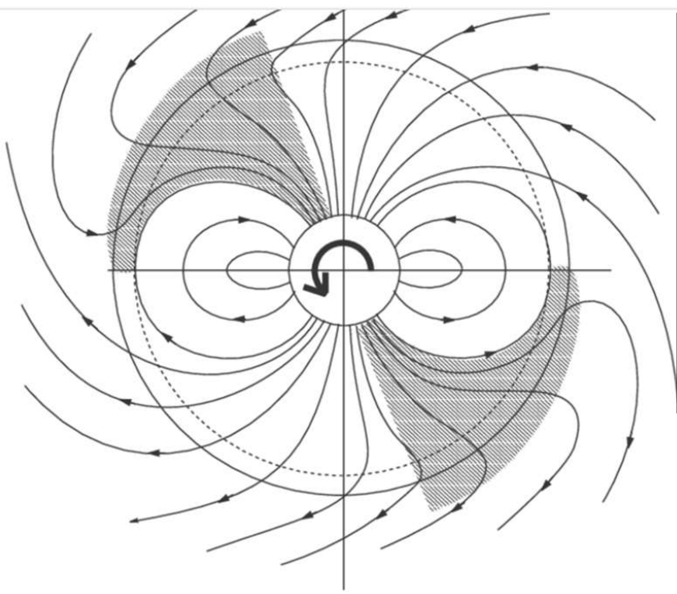
Physics (on-Sheet)

- Co-rotation vector, orthogonal to *η*, determines a relative frequency measure:

$$\chi \cdot F = -(\chi \cdot \eta)d\psi = (\Omega_F - \Omega_H)d\psi$$

And also the F^2-invariant:

$$F^2 = \frac{2B_T^2}{-g^T}\left(1 - \frac{|\chi_F|^2|d\psi|^2}{2B_T^2}\right)$$

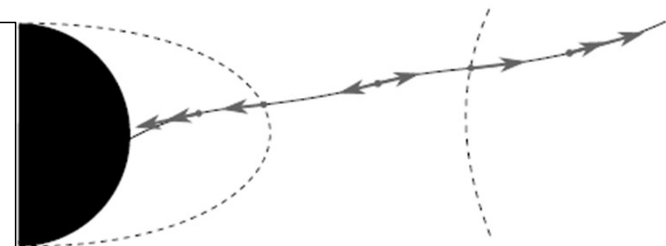


<u>On-Sheet Scattering</u>

χ_F is **always** a timelike field sheet Killing Vector.

$$u \cdot F = 0 \qquad u^2 = -1$$

- Thus, it measures field sheet conservation laws: $\chi \cdot p = p_t + \Omega_F p_\phi$
  - 2-D Dynamics: only need to fix one toroidal invariant.



Energy Extraction

$$\frac{d\mathcal{L}}{dt} = 2\pi \int_0^\pi (\Omega_H - \Omega_F)(\psi_{,\theta})^2 \sqrt{\frac{g_{\varphi\varphi}}{g_{\theta\theta}}}d\theta$$

$$\frac{d\mathcal{E}}{dt} = 2\pi \int_0^\pi \Omega_F(\Omega_H - \Omega_F)(\psi_{,\theta})^2 \sqrt{\frac{g_{\varphi\varphi}}{g_{\theta\theta}}}d\theta$$

Kerr: $$ds^2 = -\left(1-\frac{r_s r}{\Sigma}\right)c^2 dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r_s r a^2}{\Sigma}\sin^2\theta\right)\sin^2\theta\, d\phi^2 - \frac{2 r_s r a \sin^2\theta}{\Sigma} c\, dt\, d\phi$$

5D, Meyers-Perry Black Hole (Single Spinning)

- Consider the d=5 Meyers-Perry metric with a single spin turned off (b=0):

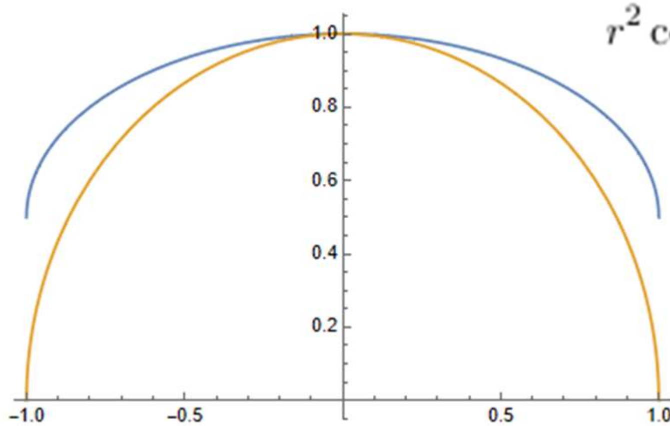
$$ds^2 = -\left(1 - \frac{m}{\Sigma(r,\theta)}\right)dt^2 + \sin\theta^2\left(a^2 + r^2 + \frac{a^2 m \sin\theta^2}{\Sigma(r,\theta)}\right)d\phi^2 + \frac{-2am\sin\theta}{\Sigma(r,\theta)}dt d\phi + r^2\cos\theta^2 d\psi^2 + \frac{r^2\Sigma(r,\theta)}{\Delta(r)}dr^2 + \Sigma(r,\theta)d\theta^2$$

$$\Delta(r) = r^2\left(r^2 + a^2 - m\right) \qquad \Sigma = r^2 + a^2\cos^2\theta$$

Perturbation Scale:

$$r_0 = \sqrt{m} \qquad \frac{r_+}{r_0} = \sqrt{1-\alpha^2} \quad \Rightarrow \quad \alpha = \frac{a}{\sqrt{m}}$$

Notice the Kerr horizon has mass-support near extremality (blue), while the b=0 MP (yellow) does not: presumably, this makes the fitting more dynamic



Notice the extra angle's metric function can be directly included into the "Cosine-complex" that dominates near the symmetric zeros of $g_{t\phi}$ component, especially at large radius:

$$r^2\cos^2\theta d\psi^2 + \frac{r^2\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 :$$

$$= \Sigma\left(\left(\frac{r d\psi}{a}\right)^2 + \left(\frac{dr}{\frac{\sqrt{\Delta}}{r}}\right)^2 + d\theta^2\right) - (ar d\psi)^2$$

Thus, in the b=0 case, the extra coordinate is an r-strong (cylindrical) support space (U(1)-internal) that smoothly envelopes to 0 near the anti-symmetric zeros of $g_{t\phi}$

5D Flat

Kerr:

$$\Psi_\phi^{(0)} = 1 - \cos\theta$$
$$\Psi_\phi^{(1)} = f(r)\frac{\sin^2\theta}{2}$$

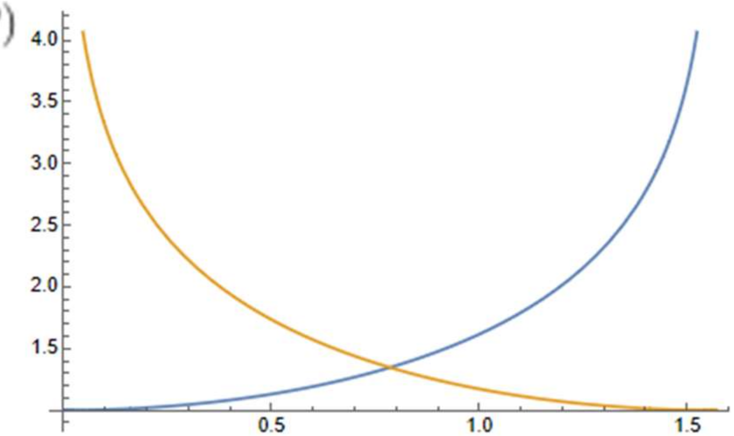
- Considering constant frequency, $\omega^{(0)} = c_3$, zero current $I^{(0)}=0$, non-rotating, $\alpha = 0$, solutions, the stream equation can be represented:

$$(j_\mu F^{\mu,\theta})r^4 = \mathcal{L}_\psi \Psi_\psi H[r,\theta] + \mathcal{L}_\phi \Psi_\phi G[r,\theta] + K[r,\theta]\left(\omega_\phi \delta_\phi[\Psi_\phi] + \omega_\psi \delta_\psi[\Psi_\psi]\right) = 0$$

$$\mathcal{L}_\psi \Psi_\psi \equiv r\,\partial_r\left[\left(\frac{r^2-m}{r}\right)\partial_r \Psi_\psi\right] + \cot\theta\,\partial_\theta\left[\frac{1}{\cot\theta}\partial_\theta \Psi_\psi\right] \qquad \mathcal{L}_\phi \Psi_\phi \equiv r\,\partial_r\left[\left(\frac{r^2-m}{r}\right)\partial_r \Psi_\phi\right] + \tan\theta\,\partial_\theta\left[\frac{1}{\tan\theta}\partial_\theta \Psi_\phi\right]$$

Looking for radially-independent solutions yields: $\Psi_\phi^{(0)} = 1 - c_1 \log(\cos\theta)$

$$\Psi_\psi^{(0)} = 1 - c_2 \log(\sin\theta)$$



<u>5D, Split Field Perturbation:</u>

$$\Psi_\phi^{MP} = \Psi_\phi^{(0)} + \alpha^2 \Psi_\phi^{(1)} + O(\alpha^4)$$
$$r_0 \omega_\phi^{MP} = \alpha \omega^{(1)} + O(\alpha^3)$$
$$r_0 I^{MP} = \alpha I^{(1)} + O(\alpha^3)$$

Comparing the $\phi^{(0)}$-order solutions leads us to try the same $\phi^{(1)}$-order perturbation:

Keep the φ-flux field $^{(0)}$-order:

$$r_0 \omega_\psi^{MP} = O(\alpha^3)\ , \qquad \Psi_\psi^{MP} = \Psi_\psi^{(0)} + O(\alpha^4)$$

$$\Psi_\phi^{(1)} = f(r)\frac{\sin^2\theta}{2}$$

Solutions

$$\Psi_\phi^{(0)} = 1 - c_1 \log(\cos\theta) \qquad \Psi_\phi^{(1)} = f(r)\frac{\sin^2\theta}{2}$$
$$\Psi_\psi^{(0)} = 1 - c_2 \log(\sin\theta)$$

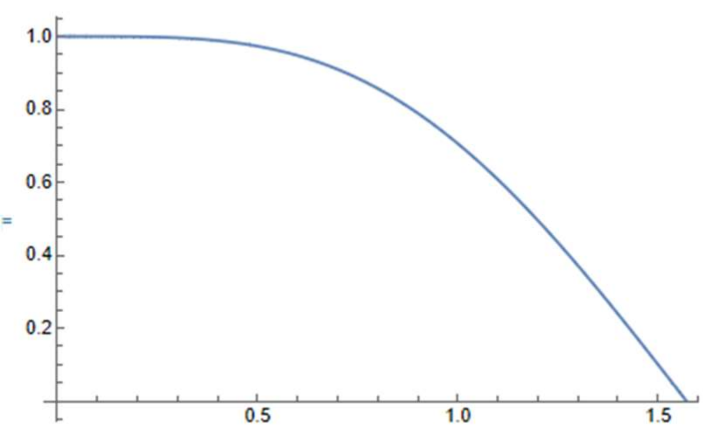
- It can be shown that

$$j_\mu F^{\mu,t} = O(a^3)\ , \qquad j_\mu F^{\mu,\phi} = \frac{c_1 a}{m}\frac{\sec\theta\,\partial_r I}{r^3}\ , \qquad j_\mu F^{\mu,\psi} = -\frac{a c_2 \csc^2(\theta)\partial_r I}{m r^3}\ , \qquad j_\mu F^{\mu,r} = O(a^3)$$

and that

$$I = \sqrt{\lambda}\sqrt{1-\sin(\theta)^4} \qquad\qquad c_2 = 0$$

induces



$$f''(r) - \frac{(m+r^2)\,f'(r)}{r^3 - mr} + \frac{4f(r)}{m-r^2} - \frac{4(c_1^2\,(c_3^2 r^6 - 2c_3 m^2 r^2 + m^3) + \lambda r^4)}{c_1 m r^2\,(m-r^2)^2} = 0$$

Frobenius type (Singular-fixed point) ODE: we can always generate an exact OPE. These global solutions are usually considered too rigid, but recent results suggest they may indeed be adaptable.

Generally, we may extend a dipole field inward and match to our split field; recent results may also indicate novel extensions.

Conclusion and Ongoing work

- Perturbative techniques can work in 5D: the non-spinning solution tower has been established.
- Further, constant-frequency power dissipation solutions seem to work;
  - Indeed, there are a number of potentially novel features in the emergent dynamics and topological thermalizations within the matching region
- Results forthcoming

Thank you

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