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Gravitational Gauge Theory and the Existence of Time

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Abstract. General relativity may be formulated as a gauge theory more than one way using the quotient manifold approach. We contrast the structures that arise in four gravitational gauge theories, three of which give satisfactory gauge theories of general relativity. Of particular interest is the quotient of the conformal group of a flat space by its Weyl subgroup, which always has natural symplectic and metric structures in addition to the requisite manifold. This quotient space admits canonically conjugate, orthogonal, metric submanifolds distinct from the original space if and only if the original flat space has signature $n, -n$ or 0 . In the Euclidean cases, the resultant configuration space must be Lorentzian. This gives a 1-1 mapping between Euclidean and Lorentzian submanifolds, with induced Euclidean gravity or general relativity, respectively.

1. General relativity as a gauge theory

Direct unification of gravity with the standard model requires general relativity to be expressed as a gauge theory. Moreover, it is reasonable to expect that string or some other TOE will reduce to a gauge theory of general relativity in an appropriate limit. We therefore consider various ways to accomplish such a gauge theory [1]-[8].

While we expect the correct gauge theory limit to be supersymmetric, we restrict our attention here to the bosonic sector. The obvious choice is to gauge the Poincaré group, but it seems more natural to choose a simple group. In addition, there are metric, symplectic and complex structures required by quantum theory that a unified approach might predict. Surprisingly, one of the gauge theories we present below gives rise to these structures at a group-theoretic level.

As additional motivation, we recall the result of Ehlers, Pirani and Schild [9]. Their goal was to find the most general geometry consistent with the observable motions of particles and light. To accomplish this, they showed that the free-fall paths of particles determine a projective connection, while the free-fall paths of light determine a conformal connection. Combining these results by demanding that in the limit of high velocity, the particle geodesics coincide with the lightlike trajectories, they found that spacetime should have a Weyl connection. At this fundamental level, the local symmetry should therefore include dilatations as well as the Lorentz group. This strongly suggests that we gauge the Weyl or conformal group instead of the Poincaré group. If the dilatational gauge field, the Weyl vector, is pure gauge then we are free to make the usual definition of the second (currently, a set number of oscillations of hyperfine Cesium 133 transitions).

We examine four gauge theories of gravity, three of which lead to general relativity. Before describing these, we briefly review the quotient manifold method of gravitational gauging [3]-[5].

Let \mathcal{G} be a Lie group, and \mathcal{H} a Lie subgroup of \mathcal{G} . Then the quotient \mathcal{G}/\mathcal{H} is a manifold, \mathcal{M} , and the Maurer-Cartan equations for the group give a connection on \mathcal{G} . We modify the connection so that



the Maurer-Cartan equations now define the curvature 2-forms,

$$\mathbf{R}^A = d\omega^A - \frac{1}{2}c_{BC}^A \omega^B \wedge \omega^C$$

Once we have made the choice of \mathcal{G} and \mathcal{H} , this construction determines

- the physical arena, \mathcal{M}
- the local symmetry group, \mathcal{H}
- the relevant tensor fields, \mathbf{R}^A
- any structures inherited from \mathcal{G}

While other structures may be imposed, we consider only those which arise directly from properties of the gauge group. To complete a gravity theory, we construct an \mathcal{H} -invariant action from the available tensors. While there is some arbitrariness in this step, the goal of reproducing general relativity is restrictive. Each connection form is then varied independently to find the field equations.

We illustrate the method with the Poincaré group.

The Maurer-Cartan equations for the Poincaré group are

$$\begin{aligned} d\tilde{\omega}_b^a &= \tilde{\omega}_b^c \wedge \tilde{\omega}_c^a \\ d\tilde{\mathbf{e}}^a &= \tilde{\mathbf{e}}^c \wedge \tilde{\omega}_c^a \end{aligned}$$

Now we take the quotient of the Poincaré group by its Lorentz subgroup. The result is a principal fiber bundle with n -dim base manifold and local Lorentz symmetry. We generalize the solder form $\tilde{\mathbf{e}}^a \rightarrow \mathbf{e}^a$ and the spin connection $\tilde{\omega}_b^a \rightarrow \omega_b^a$, leading to the addition of curvature and torsion 2-forms to the Maurer-Cartan equations,

$$\begin{aligned} \mathbf{R}_b^a &= d\omega_b^a - \omega_b^c \wedge \omega_c^a \\ \mathbf{T}^a &= d\mathbf{e}^a - \mathbf{e}^c \wedge \omega_c^a \end{aligned}$$

The generalization of the connection is restricted only by integrability and the condition that curvature and torsion be horizontal, i.e., $\mathbf{R}_b^a = \frac{1}{2}R_{bcd}^a \mathbf{e}^c \wedge \mathbf{e}^d$ and $\mathbf{T}^a = \frac{1}{2}T_{bc}^a \mathbf{e}^b \wedge \mathbf{e}^c$. The integrability condition, found by taking the exterior derivative of the curvature expressions, gives the Bianchi identities. Finally, the most general action linear in the curvature and torsion is

$$S = \int (\alpha \mathbf{R}^{ab} + \beta \mathbf{e}^a \wedge \mathbf{e}^b) e_{abc\dots d} \wedge \mathbf{e}^c \wedge \dots \wedge \mathbf{e}^d$$

Variation of the solder form and spin connection leads to vanishing torsion, and the vacuum Einstein equation with cosmological constant.

2. Four gauge theories of gravity

We repeat similar constructions for three additional quotients, giving the following four gauge theories of gravity (see also [6, 7]):

Group, \mathcal{G}	Subgroup, \mathcal{H}	dim \mathcal{M}	Name
Poincaré	Lorentz	n	Poincaré
Inhomogeneous Weyl	Weyl	n	Weyl geometry
Conformal	Inhomogeneous Weyl	n	Weyl conformal gravity
Conformal	Weyl	$2n$	Biconformal gravity

These are the only Lorentz covariant quotients with \mathcal{G} no larger than conformal. There are substantial differences between these gravity theories. We compare, in turn, the following features:

- (i) Number and form of the curvatures
- (ii) Action and the relationship to general relativity
- (iii) Induced structures
 - (a) Killing metric
 - (b) Symplectic form
 - (c) Spacetime signature: the existence of time

2.1. Number and form of the curvatures

The number of curvatures equals the number of generators of the original group, \mathcal{G} , and the form of each curvature depends on the dimension of the quotient manifold, \mathcal{M} . In all cases we take the torsion (the curvatures associated with translations) to vanish.

As we have seen, the Poincaré gauging leads to Riemann curvature of the usual form, with the possible addition of torsion.

Weyl geometry[10]-[13] increases the number of curvatures by one – the dilatational curvature, given by the curl of the Weyl vector. The new gauge field also enters the relationship between the solder form and the spin connection, so with vanishing torsion we have

$$\begin{aligned}\mathbf{R}_b^a &= \mathbf{d}\omega_b^a - \omega_b^c \wedge \omega_c^a \\ 0 &= \mathbf{d}\mathbf{e}^a - \mathbf{e}^c \wedge \omega_c^a - \mathbf{e}^a \wedge \boldsymbol{\omega} \quad (\mathbf{T}^a = 0) \\ \Omega &= \mathbf{d}\boldsymbol{\omega}\end{aligned}$$

where $\boldsymbol{\omega}$ is the Weyl vector. The form of \mathbf{R}_b^a in terms of the spin connection, ω_b^a , is the same as for Poincaré although the spin connection now has some dependence on the Weyl vector; the dilatational curvature is also expanded in terms of the solder form, $\Omega = \frac{1}{2}\Omega_{ab}\mathbf{e}^a \wedge \mathbf{e}^b$. This is the minimal extension of the usual Riemannian picture that is consistent with the findings of Ehlers, Pirani and Schild[9].

For both of the conformal gaugings, there are additional generators for the special conformal transformations and hence corresponding additional curvatures. There are also additional contributions to the Lorentz and dilatational curvatures because of the new special conformal gauge fields, \mathbf{f}_a . The curvatures are now:

$$\begin{aligned}\Omega_b^a &= \mathbf{d}\omega_b^a - \omega_b^c \wedge \omega_c^a - \mathbf{f}_b \wedge \mathbf{e}^a + \eta_{bc}\eta^{ad}\mathbf{f}_d \wedge \mathbf{e}^c \\ 0 &= \mathbf{d}\mathbf{e}^a - \mathbf{e}^c \wedge \omega_c^a - \mathbf{e}^a \wedge \boldsymbol{\omega} \quad (\mathbf{T}^a = 0) \\ \mathbf{S}_a &= \mathbf{d}\mathbf{f}_a - \omega_a^c \wedge \mathbf{f}_c - \boldsymbol{\omega} \wedge \mathbf{f}_a \\ \Omega &= \mathbf{d}\boldsymbol{\omega} - \mathbf{e}^a \wedge \mathbf{f}_a\end{aligned}$$

For Weyl conformal gravity ([11], and more recently, [14] and references therein) each of the curvatures Ω_b^a , \mathbf{S}_a and Ω is expanded quadratically in the solder form as before, e.g., $\Omega_b^a = \frac{1}{2}\Omega_{bcd}^a \mathbf{e}^c \wedge \mathbf{e}^d$. However, since the biconformal gauging[7],[15]-[17] leaves both \mathbf{e}^a and \mathbf{f}_a spanning the base manifold, the curvatures of the biconformal geometry have additional terms,

$$\Omega^A = \frac{1}{2}\Omega_{ab}^A \mathbf{e}^a \wedge \mathbf{e}^b + \Omega_b^{Aa} \mathbf{f}_a \wedge \mathbf{e}^b + \frac{1}{2}\Omega^{Aab} \mathbf{f}_a \wedge \mathbf{f}_b$$

Here, Ω^A refers to any of the curvatures, Ω_b^a , \mathbf{S}_a , Ω . The additional properties of the biconformal gauging compensate somewhat for this added complexity. Most importantly, while it is well-known that Weyl conformal gravity does not reproduce general relativity, the biconformal gauging does.

2.2. Action functionals and the relationship to general relativity

Each of these gravitational gauge theories has a different relationship to general relativity, largely because of necessary differences in the allowed action functionals.

As we have seen, the most general action linear in the curvature in Poincaré gravity is the Einstein-Hilbert action together with a cosmological constant. Palatini variation then implies vanishing torsion and the usual vacuum Einstein equation results.

For Weyl geometry, we may write an action linear in the curvature if we introduce a scalar field. This gives the n -dim generalization of the action used by Dirac in discussing the Large Numbers Hypothesis[18]-[20],

$$S = \int \left(\kappa^2 g^{\alpha\beta} R_{\alpha\beta} - \frac{\beta}{2} \kappa^{2\left(\frac{n-4}{n-2}\right)} g^{\alpha\mu} g^{\beta\nu} \Omega_{\alpha\beta} \Omega_{\mu\nu} - \frac{\alpha}{2} g^{\alpha\beta} D_\alpha \kappa D_\beta \kappa - \lambda \kappa^{\frac{2n}{n-2}} \right) \sqrt{-g} d^n x$$

Dirac varied this action in second-order formalism, assuming a Weyl connection. However, varying the metric and connection independently, this action leads to a pure-gauge Weyl vector and hence describes a Riemannian geometry. This may also be seen by rescaling the metric by an appropriate power of the scalar field. A nontrivial Weyl geometry is obtained only with an action quadratic in the Lorentz curvature. The Dirac/Weyl action above is completely satisfactory if our goal is to write a locally scale covariant gauge theory of general relativity. The triviality of the Weyl vector is exactly the condition we need in order to allow a global definition of a time standard. With this definition, we recover general relativity.

Weyl conformal gravity, by contrast with the other theories discussed here, does not readily reproduce general relativity. Actions linear in the curvature become inconsistent, and we must write quadratic terms. Regardless of the initial form of these quadratic terms, the gauge field of special conformal transformations may be eliminated from the problem. When this is done, the action always reduces to a linear combination of the square of the Weyl curvature and the square of the dilatational curvature. In 4-dim we have

$$S = \int \left(\alpha \Omega^a_b \wedge \Omega^b_a + \beta \Omega \wedge^* \Omega \right)$$

and even this expression does not generalize to higher dimensions. We must either go to k^{th} order in the curvature in $2k$ -dimensions, or we must use a Yang-Mills type of action,

$$S_{YM} = \int \left(\alpha \Omega^a_b \wedge^* \Omega^b_a + \beta \Omega \wedge^* \Omega \right)$$

In none of these cases does the theory reproduce general relativity. The Yang-Mills action gives a field equation involving the divergence of the Weyl curvature tensor, but also has an energy-momentum type source built quadratically from the curvature. The energy-momentum term arises from the variation of the metric in the Hodge dual and volume form. It can be shown that by placing a certain constraint on the curvature the resulting solutions describe conformally Ricci flat spacetimes, but the curvature constraint restricts solutions to a subset of the possible Petrov types. As we shall show below, these difficulties come with no added benefit: the additional gauge fields are auxiliary and no new structures are introduced.

Biconformal gauging was first introduced by Ivanov and Niederle[7] when they realized the drawbacks of Weyl conformal gravity. There have been numerous subsequent developments, including the introduction of an action linear in the curvature[16],

$$S = \int \left(\alpha \Omega^a_b + \beta \delta^a_b \Omega + \gamma \mathbf{e}^a \wedge \mathbf{f}_b \right) \wedge \varepsilon^{be\dots f}_{ac\dots d} \mathbf{e}^c \wedge \dots \wedge \mathbf{e}^a \wedge \mathbf{f}_e \wedge \dots \wedge \mathbf{f}_f$$

This action is valid in any dimension. It works because of the symplectic structure discussed below, which allows the volume element to be dimensionless. Naturally, the field equations arising from this action are complicated both by the number of equations and by the number of fields in the $2n$ -dimensional base space. Nonetheless, it has been shown that the field equations generically describe general relativity on the co-tangent bundle of an n -dim Riemannian spacetime with pure-gauge Weyl vector[21]. As with the linear action for the Weyl geometry, choosing a global time standard reduces the theory to general relativity.

2.3. Additional structures

We now examine structures which arise from the original choice of gauge group, \mathcal{G} .

2.3.1. Killing metric The Killing metric of a semi-simple Lie group is a non-degenerate symmetric form constructible from the generators, G_A , as

$$K_{AB} = \lambda \text{tr}(G_A G_B)$$

where λ is any convenient constant. Since all of the groups considered here are subgroups of the conformal group, we can find the consequences for each gauging from the conformal Killing metric,

$$K_{AB} = \begin{pmatrix} \delta_b^a \delta_d^c - \eta^{ac} \eta_{bd} & 0 & 0 & 0 \\ 0 & 0 & \delta_b^a & 0 \\ 0 & \delta_a^b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where the upper left corner gives the Killing metric of the Lorentz group, the off-diagonal Kronecker deltas span the translations and special conformal transformations, and the 1 in the lower right arises from the dilatations. The question we ask is whether the restriction of this metric to the base manifold induces a non-degenerate metric there.

For the Poincaré, Weyl, and Weyl conformal gaugings, the answer is no. The base manifold corresponds to the zero in the second row and second column of K_{AB} , and since that part of the Killing metric is off-diagonal, there is no induced metric in any of these gauge theories. Naturally, we may introduce the original metric by hand after gauging.

In contrast to this, the restriction of K_{AB} to the biconformal base manifold is the non-degenerate form,

$$K_{\Sigma\Delta} = \begin{pmatrix} 0 & \delta_b^a \\ \delta_a^b & 0 \end{pmatrix}$$

giving biconformal geometry a natural, conformally invariant metric.

2.3.2. Symplectic form A symplectic form is defined as a closed, nondegenerate 2-form. Such a form can exist only on even dimensional manifolds. If present it gives a way to divide fields into canonically conjugate pairs, which for convenience we call configuration/momentum pairs.

There is no reason to expect a symplectic form in any of these geometries, and certainly not in the Poincaré, Weyl or conformal Weyl cases because these theories exist in odd dimensions. Nor is there any evidence for a symplectic form in the structure equations. However, in the biconformal gauging the Maurer-Cartan equation for the Weyl vector is

$$d\omega = e^a \wedge f_a$$

The left side shows that this 2-form is closed, since it is exact, while the right side is necessarily nondegenerate. The presence of this symplectic form survives into the curved spaces: generic solutions to the field equations have symplectic structure. This reinforces the interpretation of these solutions as co-tangent bundles, and has been used to express Hamiltonian particle dynamics[22] and quantum mechanics[23] in biconformal spaces.

2.3.3. Spacetime signature: the existence of time As we have seen, the extension to Weyl geometry offers only local dilatations as compensation for moving beyond Poincaré gauge theory, and neither the Poincaré nor the Weyl group is semisimple. Though many interpretations of Weyl geometry have been proposed, none has successfully provided new insights or become generally accepted.

The Weyl conformal gauging has also failed to introduce interesting new structures, and more importantly, fails to reproduce general relativity in any straightforward way. Even if we accept a modified theory of gravity, the Weyl conformal gauging requires a different action in every dimension.

The situation is considerably different for the biconformal gauging. As we have shown, the same linear-curvature action reproduces general relativity in any dimension, there is a natural symplectic form, and the conformal Killing metric has a nondegenerate restriction to the space. These elements

combine to give the interpretation of a biconformal geometry as an expression of general relativity on the spacetime co-tangent bundle.

As might be expected, biconformal spaces are closely related to twistor spaces, \mathbb{C}^n . When the torsion vanishes, biconformal geometries have a complex structure, and may be identified with twistor spaces. However, because they arise from gauge theory, biconformal geometries have additional structure. The additional structure has an interesting consequence for our understanding of time.

Clearly, twistor space has numerous submanifolds of arbitrary signature. In particular, it has both Euclidean and Lorentzian submanifolds. However, there is no preferred map between these.

The case is different in biconformal spaces, where, using the metric and symplectic form, it becomes possible to ask for orthogonal, canonically conjugate, metric submanifolds. It turns out that these exist only in special circumstances, and the solution provides a unique map between Euclidean and Lorentzian submanifolds[24]. In the next section, we explore this construction in some detail.

3. The signature theorem

Since biconformal spaces have a symplectic form and a metric, we may ask for the existence of well-defined configuration and momentum subspaces[24]. Consider the conformal group of a compactified, n -dim space with metric η_{ab} of signature $s = p - q$, where $p + q = n$. The quotient of the conformal group by its Weyl subgroup is then a flat biconformal space of dimension $2n$.

We demand the following properties:

- (i) The configuration space, \mathcal{S} , and the momentum space, \mathcal{P} are spanned by canonically conjugate basis forms.
- (ii) The spaces \mathcal{S} and \mathcal{P} are submanifolds.
- (iii) The momentum space \mathcal{P} is conformally flat.
- (iv) The configuration space \mathcal{S} has non-degenerate induced metric.
- (v) The spaces \mathcal{S} and \mathcal{P} are orthogonal.

Conditions 1, 2 and 3 insure that the biconformal space is a phase space[25]. The condition 4 requires the configuration space metric to arise as the restriction of the Killing metric to \mathcal{S} , while condition 5 makes this induced metric unambiguous.

It is known that the solutions to the torsion-free biconformal field equations describe co-tangent bundles. Therefore, condition 3 is a consequence of the field equations. Here, we consider only flat biconformal space and impose condition 3 by hand. Since the results of this depend only on the signatures of the submanifolds, the results continue to hold for arbitrary biconformal spaces subject to the field equations.

It is straightforward to find the general form of a basis for orthogonal, canonically conjugate, subspaces. The result, up to changes of basis within the subspaces separately, is

$$\begin{aligned}\psi^a &= \frac{1}{2\alpha} \mathbf{e}^a - \alpha h^{ab} \mathbf{f}_b \\ \chi_a &= \frac{1}{2\alpha} h_{ab} \mathbf{e}^b + \alpha \mathbf{f}_a\end{aligned}$$

where $h^{ab} h_{bc} = \delta_c^a$ and h_{ab} is symmetric. With the Killing metric holding for the original $(\mathbf{e}^a, \mathbf{f}_a)$ basis, we find that h_{ab} becomes the configuration space metric. It follows from the non-degeneracy of the Killing metric that the momentum space has metric $-h^{ab}$.

We next impose conformal flatness of the subspace spanned by χ_a . It follows that h_{ab} is necessarily of one of two forms:

$$\begin{aligned}1. h^{ab} &= \frac{1}{nh} \eta^{ab} \\ 2. h^{ab} &= \frac{(n-2)}{hu^2} \left(-2u^a u^b + u^2 \eta^{ab} \right)\end{aligned}$$

where $u^2 = \eta_{ab}u^a u^b$ and $h = \eta_{ab}h^{ab}$. The first solution is the expected projection from the biconformal space back to the original space. The second solution will be recognized as a signature-changing transformation of the original metric.

Before considering the conformally flat solutions in more detail, we impose the final condition. Requiring the subspaces \mathcal{S} and \mathcal{P} to be submanifolds places integrability conditions on the basis forms. Each set of basis forms must be involute,

$$\begin{aligned} \mathbf{d}\psi^a &\sim \psi^a \\ \mathbf{d}\chi_a &\sim \chi_a \end{aligned}$$

However, for general h_{ab} , the structure equations for ψ^a and χ_a contain terms $(\mathbf{D}h^{ab})\chi_b$ and $\mathbf{D}h_{ab}\psi^b$, respectively. In order for these equations to be in involution, the covariant derivative $(\mathbf{D}h^{ab})\chi_b$ must have vanishing $\psi^b\psi^c$ term, and $\mathbf{D}h_{ab}\psi^b$ must have vanishing $\chi^b\chi^c$ term. The vanishing of these quadratic terms, along with the signature-changing form of h_{ab} required by conformal flatness has the *unique* solution

$$h_{ab} = \lambda \left(y_a y_b - \frac{1}{2} y^2 \eta_{ab} \right)$$

where y_a comprise half of the biconformal coordinates. A check shows that with this solution the basis forms are indeed involute.

Finally, we must consider whether this solution for h_{ab} actually does produce a consistent metric on the configuration space. Since y_a is a coordinate on a subspace with metric η_{ab} , y_a will take on both timelike and spacelike values unless η_{ab} is Euclidean. In general, the resulting signature, s' , of h_{ab} will be

$$\begin{aligned} s' &= s - 2 \quad y_a \text{ spacelike} \\ s' &= -s - 2 \quad y_a \text{ timelike} \end{aligned}$$

These are consistent only if $s = 0$, or if the original space is Euclidean, $s = \pm n$. We therefore have a consistent solution for h_{ab} in only two cases. The signature of h_{ab} is $n - 2$ if the original space is Euclidean, and -2 if the original space has vanishing signature.

If the original dimension is $n = 4$, both cases lead to Lorentzian spacetimes; for other dimensions, a Euclidean initial space always leads to a Lorentzian spacetime. This establishes a 1-1 correspondence between initial Euclidean spaces and Lorentzian spacetimes. Since η_{ab} also provides a solution, we can project from the biconformal space to either of these configuration spaces. In particular, we note that a gravitational solution on the biconformal space will induce Euclidean gravity on the one hand and solutions to general relativity on the other. Again, we stress that our restriction to flat biconformal spaces must generalize smoothly to curved spaces since the results depend only on signature.

As a final observation, notice that there is a particular subset of phase space coordinates, y_a , which determine the direction of the light cones in the configuration space. This will undoubtedly have consequences for the existence of tachyons – for example, if y_a is a momentumlike direction, then the momentum necessarily points inside the lightcone by definition.

4. Summary

\mathcal{G}	\mathcal{H} = Local symmetries	General relativity	Scaling	Physical fields (tensors)	Structures on $\mathcal{M} = \mathcal{G}/\mathcal{H}$ arising from \mathcal{G}
Poincaré	Lorentz	Yes	No	Solder form Curvature	• Spacetime
Weyl	Lorentz Dilatations	Yes	Yes	Solder form Curvature	• Spacetime
Conformal	Lorentz Dilatations Spec. conf.	No (Weyl conformal gravity)	Yes	Solder form Weyl Curvature	• Spacetime
Conformal	Lorentz Dilatations	Yes	Yes	Solder form Curvature Symplectic form Co-torsion Dilatation	• ST cotangent bundle • Symplectic structure • Induced Killing metric • Complex structure • Time (derived Lorentz sym.)

References

- [1] Utiyama R 1956 *Phys. Rev.* **101** p 1597
- [2] Kibble T W B 1961 *J. Math. Phys.* **2** p 212
- [3] Kobayashi S and Nomizu K 1963 *Foundations of Differential Geometry* (New York: Wiley)
- [4] Ne'eman Y and Regge T. 1978 *Phys. Lett. B* **74** p 54
- [5] Ne'eman Y and Regge T 1978 *Riv. Nuovo Cim.* **1N5** p 1
- [6] Ivanov E A and Niederle J 1982 *Phys. Rev. D* **25** p 976
- [7] Ivanov E A and Niederle J 1982 *Phys. Rev. D* **25** p 988
- [8] Isham C 1999 *Modern Differential Geometry for Physicists* (Hackensack: World Scientific)
- [9] Ehlers J, Pirani A E and Schild A 1972 *General Relativity* ed O'Raifeartaigh L (Oxford: Oxford University)
- [10] Weyl H 1918 *Sitzung. d. Preuss. Akad. d. Wissensch.* p 465; 1923 *The Principle of Relativity, Chapter XI* (Dover)
- [11] Weyl H 1952 *Space-Time-Matter* (New York: Dover)
- [12] Weyl H 1921 *Nature* 106; 1918 *Math. Zeitschr.* 2 p 384; 1918 *Ann. d. Physik*, 54 p 117 ; 1919 *Ann. d. Physik*, 59 p 101; 1920 *Phys. Zeitschr.* 21 p 649; 1921 *Ann. d. Phys.* 65 p 541; 1921 *Phys. Zeitschr.* 22 p 473; 1929 *Zeit. f. Physik*, 56 p 330
- [13] Adler R, Bazin M, Schiffer M 1965 *Introduction to General Relativity*, (New York: McGraw Hill) p 401
- [14] Bouchami I J and Paranjape M B 2008 *Physical Review D* **78** 044022
- [15] Wheeler J T 1998 *J. Math. Phys.* **39** p 299
- [16] Wehner A and Wheeler J T 1999 *Nucl. Phys. B* **557** p 380
- [17] Anderson L B and Wheeler J T 2004 *Nucl. Phys. B* **686** p 285
- [18] Dirac P A M 1951 *Proc. Roy. Soc.* **A209** p 291
- [19] Dirac P A M 1952 *Proc. Roy. Soc.* **A212** p 330
- [20] Dirac P A M 1973 *Proc. Roy. Soc.* **A333** p 403
- [21] Wheeler J T *in preparation*
- [22] Wheeler J T 2007 *Canadian Journal of Physics*, **85** (4) p 307
- [23] Anderson L B and Wheeler J T 2006 *Int. J. Geom. Meth. Mod. Phys.* **3** p 315
- [24] Spencer J A and Wheeler J T 2008 arXiv:0811.0112v1 [gr-qc]
- [25] Abraham R H and Marsden J E 1978 *Foundations of Mechanics* (New York: Addison-Wesley)