Design and Optimization of a Disaggregated Constellation for Space Situational Awareness

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Contributions of this paper

1. Propose a sequential optimization method to solve a computationally intractable problem

2. Provide an analytical method to evaluate mission performance

3. Apply unique optimization approach in combining multiple objective functions to create a single value proposition

Conclusion: Novel economic niche for CubeSats with performance comparable to national assets
The Orbital Debris Problem

- 21,000+ objects currently in JSPOC catalog [USAF 2012]

Objects under 10 cm are not tracked due to lack of consistent observation
- Est. 500,000 objects > 1 cm in diameter [Liou 2010]
- Est. 200,000,000 objects > 1 mm in diameter [NASA ODPO 2012]
The Orbital Debris Problem

- Orbital debris in the 1-10cm regime poses a significant threat to space assets [Christiansen 2003]
  - STS-7 window impact first confirmed damage, 1983 [Liou 2010]
  - STS-118 radiator impact largest confirmed damage, 2007 [Lear 2008]

Only 7% of Earth-orbiting space objects are operational assets.

93% of the objects in Earth orbit serve no purpose other than endangering operational assets [McKnight et al. 2013]

Orbital debris detection is of utmost importance to continued commercial and federal uses of space. [Kessler et al. 2010]
Benefits of a Disaggregated CubeSat Constellation

CubeSats scale well and are much cheaper

If the effectiveness of space-based CubeSat SSA can be proven, then the possibilities for such a constellation must be strongly considered

Inherent benefits to disaggregated networks [3]
- Survivability
- Robustness
- Resilience

Space Fence – ground-based radar arrays
$100s M USD [1]

SBSS – planned constellation for on-orbit SSA
$1B USD [2]

[1]: Lockheed Martin press release 2014
[2]: Boeing and AF Space Command fact sheet 2015
[3]: Boeing and AF Space Command fact sheet 2015
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Passive Optical Sensor Design Goals

Design Goals
- Detect many objects
- Detect objects accurately
- Detect dim objects

Decision Criteria
- FOV
- IFOV
- Limiting magnitude

Sizing Parameters
- D: Lens Diameter
- l: Focal Length
- p: Pixel Size

\[ FOV_h = n_{p,h} IFOV \]
\[ FOV_v = n_{p,v} IFOV \]
\[ IFOV = 2 \tan^{-1} \left( \frac{p}{2ND} \right) \]

\[ m_v = -2.5 \log_{10} \left[ \frac{\text{SNR}_{\text{alg}} \left[ \sqrt{m_i \omega N D (q_{\text{sky}} + q_{\text{dark}})} \right]^{\frac{1}{2}}}{\Phi_0 \tau_{\text{atm}} \tau_{\text{opt}} \left( \frac{\pi D^2}{4} \right) \text{QE} \sqrt{p}} \right] \]

[Coder 2015]
Sizing Parameter Tradeoff

= favorable point

- Limiting Magnitude
- IFOV

- FOV
- IFOV

I: Focal Length (meters)

p: Pixel Size (micrometers)
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Where can we see an object?

Line of Sight – Can the observer see the object?

\[
o \cdot \hat{v} + \sqrt{||o||^2 - ||R_e||^2} \geq 0
\]

Illumination – Is the object illuminated?

\[
r \cdot \hat{s} > 0
\]

\[
||\hat{s} \times r|| \leq R_e
\]

Solar Phase Angle – Is the object reflecting light to the observer?

\[
\psi = \arccos \left( \frac{v \cdot s}{||v|| \cdot ||s||} \right)
\]

Coordinate system and constraints developed in [Worthy et al. 2013]
How far away can we see an object?

We can fix payload and environmental parameters such that SNR is the only free variable:

- $q_{\text{min}}$ – minimum photon flux through an optic necessary to detect an object
- $q_{\text{opt}}$ – photon flux through an optic
- $b$ – space object parameters: area, albedo, and reflectivity

The following derivation can then be carried out based on equations from [Worthy 2013, Coder et al 2015]:

\[
q_{\text{min}} = \frac{\text{SNR}^2_{\text{min}}}{2t_{\text{int}}} \left( t_{\text{int}} + \sqrt{t_{\text{int}}^2 + 4 \frac{t_{\text{int}}^2}{\text{SNR}^2_{\text{min}}}} c \right)
\]

where $c = m \left( 1 + \frac{1}{z} \right) \left[ (q_{\text{sky}} + q_{\text{dark}})t + \frac{\sigma^2}{n^2} \right]$

\[
q_{\text{opt}} = \Phi_0 \tau_{\text{opt}} \left( \frac{\pi D^2}{4} \right) QE
\]

\[
b = A \alpha a
\]

\[
m_{SO} = m_{\text{sun}} - 2.5 \log \left( \frac{b}{\rho^2} \right)
\]

\[
m_{SO} = -2.5 \log_{10} \left( \frac{q_{\text{min}}}{q_{\text{opt}}} \right)
\]

\[
m_{\text{req}} = m_{\text{sun}} + 2.5 \log_{10} \left( \frac{q_{\text{min}}}{q_{\text{opt}}} \right)
\]

\[
m_{\text{req}} = 2.5 \log_{10} \left( \frac{b}{\rho^2} \right)
\]
How far away can we see an object?

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ</td>
<td>( \frac{\pi}{3} )</td>
<td>radians</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>0.1</td>
<td>seconds</td>
</tr>
<tr>
<td>( \Phi_0 )</td>
<td>( 5.636 \times 10^{10} )</td>
<td>photons/s/m²</td>
</tr>
<tr>
<td>( D )</td>
<td>0.075</td>
<td>meters</td>
</tr>
<tr>
<td>( N )</td>
<td>0.8</td>
<td>N/A</td>
</tr>
<tr>
<td>( f )</td>
<td>N*D</td>
<td>meters</td>
</tr>
<tr>
<td>( QE )</td>
<td>0.6</td>
<td>N/A</td>
</tr>
<tr>
<td>( z )</td>
<td>1024<em>1280</em>1</td>
<td>pixels</td>
</tr>
<tr>
<td>( m )</td>
<td>5</td>
<td>pixels</td>
</tr>
<tr>
<td>( q_{p,\text{dark}} )</td>
<td>0.5</td>
<td>photons/s/pixel</td>
</tr>
<tr>
<td>( q_{p,\text{sky}} )</td>
<td>0.5</td>
<td>photons/s/pixel</td>
</tr>
<tr>
<td>( \tau_{\text{opt}} )</td>
<td>0.9</td>
<td>N/A</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>9</td>
<td>photons/s/pixel</td>
</tr>
<tr>
<td>( n )</td>
<td>1</td>
<td>N/A</td>
</tr>
<tr>
<td>( \text{SNR}_{\text{min}} )</td>
<td>4</td>
<td>N/A</td>
</tr>
</tbody>
</table>

1 cm diameter object:

\[
A = \pi r^2 = \pi (0.5 \text{ cm})^2
\]

\[
\rho \leq \rho_{\text{max}} = \sqrt{\frac{b}{10^{-4} m_{\text{req}}}}
\]

An SNR value of 4 is used as a conservative estimate, all other values are baselined from the RECONSO system and a nominal space object defined by [Mulrooney et al. 2008]
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Performance Simulation

- Produce set of $N$ observer satellites
  - $P$ planes are integer multiple of $N$ satellites
  - Share semi-major axis, $a$, and inclination, $i$
  - Walker Delta configuration
- Propagate 15,106 objects in the Space Object Catalog\(^1\) and $N$ observers (Worthy, et. All 2013)
  - Fixed step two body propagator
  - 24 hour simulation period
- Perform photometric calculations
  - 10 cm object size
  - Albedo of 0.175
  - Evaluate daily unique detections

1: www.space-track.org: accessed 3/30/2015
Constellation Cost Estimate

- **Parametric Cost Estimating Relationships**
  - Small Satellite Cost Model (Maher, et. al. 2002)
  - Microcosm Sizing Models (Larson, et. al. 1999)

- **Bottoms Up Cost Estimates**
  - COTS Equipment\(^1\)
  - Secondary Payload Launch\(^2\)

\[
\text{Cost} = C_{Dev} + C_{Prod} + C_{Launch} + C_{Ops}
\]

\[
C_{Prod} = \frac{TFU \cdot N^{1 + \frac{\log(\text{learning})}{\log(2)}}}{(1 + R)^T}
\]

\[
C_{Ops} = \frac{0.08 \cdot C_{Prod} + Rent \cdot t_{Ops} + 2 \cdot M}{(1 + R)^T}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{Dev})</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>(C_{TFU})</td>
<td>$200,000</td>
</tr>
<tr>
<td>Learning</td>
<td>0.90</td>
</tr>
<tr>
<td>(C_{Launch})</td>
<td>$545,000</td>
</tr>
<tr>
<td>(C_{GS,rent})</td>
<td>$3,000/month</td>
</tr>
<tr>
<td>(M)</td>
<td>$110,000</td>
</tr>
<tr>
<td>Rate</td>
<td>6%</td>
</tr>
<tr>
<td>Total Lifetime</td>
<td>3 years</td>
</tr>
</tbody>
</table>

1: Clyde-Space, Tyvak, ISIS
2: www.spaceflightservices.com
• Traditional multi-objective optimization problems lack prioritization
• Mission performance and lifecycle cost are related to form single objective function
• Prevailing market price of mission performance is used to determine revenue (Blake, et. al. 2014)

\[
\text{Combined Objective Function}
\]

\[
\text{with: } NPV = \frac{Revenue}{(1 + R)^{T_1}} - \frac{Cost}{(1 + R)^{T_2}}
\]

\[
\min_{N,a,i} f(x) = -NPV
\]

\[
\text{subject to: } a, i \in \mathbb{R}
\]

\[
N \in \mathbb{N} \leq 36
\]

\[
\begin{bmatrix}
1 \\
6578 \\
0
\end{bmatrix} \leq \begin{bmatrix}
N \\
a \\
i
\end{bmatrix} \leq \begin{bmatrix}
36 \\
7678 \\
100
\end{bmatrix}
\]
Semi-Major Axis

- Semi-Major Axis (km):
  - 6600
  - 6400
  - 6800
  - 7000
  - 7200
  - 7400
  - 7600

- Economic Return ($M):
  - 4
  - 6
  - 8
  - 10

- Unique Detections, % of SOC:
  - 20
  - 25
  - 30
Orbital Inclination

- Economic Return ($M)
  - 4
  - 6
  - 8
  - 10

- Unique Detections, % of SOC
  - 20
  - 25
  - 30
Number of Satellites

Economic Return ($M)

Unique Detections, % of SOC

Number of Satellites

Economic Return
SOC Coverage
Optimization Performance

- Non-differentiable function requires heuristic algorithm
- Discontinuous and nonlinear
- Genetic algorithm employed to handle these problems
  - Random nature avoids local minima (Goldberg, 1989)
  - Heritage in Constellation design (Ely, et. al. 1999)

<table>
<thead>
<tr>
<th>Function</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Size</td>
<td>20</td>
</tr>
<tr>
<td>Generations</td>
<td>15</td>
</tr>
<tr>
<td>Elitism Count</td>
<td>2</td>
</tr>
<tr>
<td>Selection Method</td>
<td>Roulette</td>
</tr>
<tr>
<td>Crossover Fraction</td>
<td>75%</td>
</tr>
<tr>
<td>Mutation Fraction</td>
<td>1%</td>
</tr>
<tr>
<td>Constraint Evaluation</td>
<td>Exterior Penalty</td>
</tr>
</tbody>
</table>
Nominal case provides optimal configuration of:
4 Satellites, 7200 km, at 77° @ $12.1M
Series of optimization algorithms performed to explore uncertainty with respect to driving parameters
Conclusion

• Space Situational Awareness remains a critical task for the continued effective utilization of space

• Sequential optimization enables manageable design of a computationally difficult problem

• Analytical method for mission performance provides formal definition for mission design

• A unified optimization approach presents a feasible business case for commercial operation
Thank You!
Whole bunch of math

End goal: Define detectable range, $\rho$, as a function of SNR

1. Use Eq. (3) to change Eq. (1) into a function of photon flux $q$

2. Photon flux is a function of range and lens parameters

3. Sub Eq. (4) into Eq. (1)

4. Define $m_{\text{req}}$ and sub into Eq. (1), giving Eq. (9)

5. End result: Eq. (10)

Equations and relations from [Worthy et al. 2013, Coder et al. 2015, Mulrooney et al. 2008]