

## Orbit Determination from Two Line Element Sets of ISS-Deployed CubeSats

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### ABSTRACT

Deploying nanosatellites from the International Space Station (ISS) has become prevalent since the addition of the NanoRacks CubeSat Deployer in early 2014. Since then, 61 CubeSats have been deployed from the ISS, with the majority coming from the Planet Labs Flock 1 and Flock 1B constellations. CubeSats often rely on two-line elements (TLEs) made publicly available by the Joint Space Operations Center (JSpOC) for orbit determination and conjunction assessments, so the accuracy of JSpOC TLEs for ISS-deployed CubeSats is important to examine. In this work, the accuracy of TLEs of Flock 1B satellites are analyzed by comparison to orbits as derived from two-way ranging. Ten Flock 1B satellites are examined for the month of September 2014, using 634 TLEs from start date to end date across the flock. Prior TLE assessments for CubeSats in LEO have estimated error to be within 1 km. We found that error for ISS-deployed CubeSats is substantially higher than prior estimates. Using only forward propagation with the most recent TLE, as is the case for operational TLE use, median error in position is found to be 4.52 km with a first quartile of 2.01 km and a third quartile of 10.6 km. The 1- $\sigma$  in-track propagation error after one day ranges from 10-30 km among the 10 satellites, and the two-day 1- $\sigma$  error ranges from 20-70 km. To improve TLE accuracy for on-orbit operations, a batch least squares estimation technique is used to estimate some or all elements of the current TLE based on prior TLEs. It is shown that this method can improve the propagation of a TLE significantly, particularly in cases of sparse updates, with up to 95% error reduction. This can potentially enable operations in cases where they would otherwise be lost due to inaccurate orbital knowledge.

### INTRODUCTION

For many small satellite operators, the primary method of orbit determination is through publicly available two-line element sets (TLEs) published by the Joint Space Operations Center (JSpOC)<sup>†</sup>. A two-line element consists of a satellite identifier, an epoch, six orbital

elements, and a B\* term related to the ballistic coefficient. TLEs are designed for use with the Standard General Perturbations 4 (SGP4) model, which is also made available for public use<sup>1,2</sup>.

CubeSat operators commonly use SGP4 for both ground station tracking and on-orbit propagation with TLE updates provided by JSpOC several times a day. For satellite operators who rely on JSpOC TLEs as the only source of orbital information, successful operations depend on the accuracy of those TLEs.

JSpOC TLEs are also commonly used for conjunction assessments, which are crucial in mitigating potential space debris or loss of space assets. The Center for Space Standards & Innovation (CSSI) has provided SOCRATES – Satellite Orbital Conjunction Reports Assessing Threatening Encounters in Space – as a tool to predict possible satellite conjunctions based on JSpOC TLEs<sup>3</sup>. Estimating TLE covariance is a challenge for obtaining realistic conjunction assessments<sup>4</sup>. Improved understanding of TLE accuracy can help satellite operators avoid unwanted close approaches.

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<sup>†</sup> Disclaimer from space-track.org: Two-line element (TLE) set is the mean Keplerian orbital element at a given point in time for each space object reported. A TLE is generated using the simplified general perturbations theory and is reasonably accurate for long periods of time. A TLE available to the public should not be used for conjunction assessment prediction. Satellite operators are directed to contact the Joint Space Operations Center at 805-605-3533 for access to appropriate data and analysis to support operational satellites. This site may be inaccessible for short periods of time for routine maintenance and updates. The U.S. government reserves the right to limit both access duration and data amounts for any user. U.S. government does not warrant the accuracy or completeness of this website or that the website will be uninterrupted, error free, that defects will be corrected, or that the website or server will be free of viruses, or other technical problems.

The accuracy of TLEs has been analyzed for several types of satellites with orbits in low and medium Earth orbits<sup>5-7</sup>. It is useful to examine propagation error in the reference frame of the satellite (known Hill's frame or local vertical local horizontal). In Hill's frame, the error is split into the in-track, cross-track, and radial components. The in-track axis is parallel to the local horizontal in the direction of the orbital velocity vector, the radial axis is formed by the local vertical in parallel with the position vector, and the cross-track axis forms a right-handed set. It has been noted in prior studies that the predominant error is the in-track component<sup>5-7</sup>.

A thorough analysis by T. S. Kelso compared precision ephemerides from the GPS constellation to published TLEs<sup>5</sup>. It was found that a bias can develop in propagation that is consistent across TLEs, so that the in-track error is consistently positive or negative rather than zero mean. If there is a consistent bias, it can be estimated and eliminated. However, Kelso also noted that this bias varies between satellites, so each satellite must be examined individually for this trend.

Several studies have focused specifically on the accuracy of JSpOC TLEs for small satellites in low Earth orbit (LEO), typically by comparison with on-board GPS data. The TLEs of a small satellite (5" x 5" x 10") in a low orbit below 350 km were compared against a GPS arc of 213.5 minutes<sup>6</sup>. Using the JSpOC TLE closest in time, position errors of less than one kilometer were seen. Several small satellite missions above 600 km have also been examined, again comparing published TLEs with on-orbit GPS data, and the in-track error was found to be correlated with the B\* term of the TLE<sup>7</sup>. This finding suggests that it may improve the TLEs to use an average of the B\* term, or the best estimate from prior analysis. TLEs were again observed to be accurate to within a kilometer for several days near the epoch.

However, to date there has not been a published study of JSpOC TLE accuracy for small satellites deployed from the International Space Station (ISS). Through NanoRacks, 61 CubeSats have utilized this method of deployment, and it appears that TLE accuracy for this orbit is not consistent with numbers cited in previous studies. At this relatively low altitude of 420 km or less, atmospheric drag is a significant perturbation that is challenging to characterize. If published TLEs are not updated every few hours, they rapidly become inaccurate as seen in our analysis, particularly in the in-track direction. Propagation errors seen in this analysis of ISS-deployed CubeSats occasionally grow to hundreds or even thousands of kilometers, which is severe enough to prevent satellite operations absent another method of orbit determination.

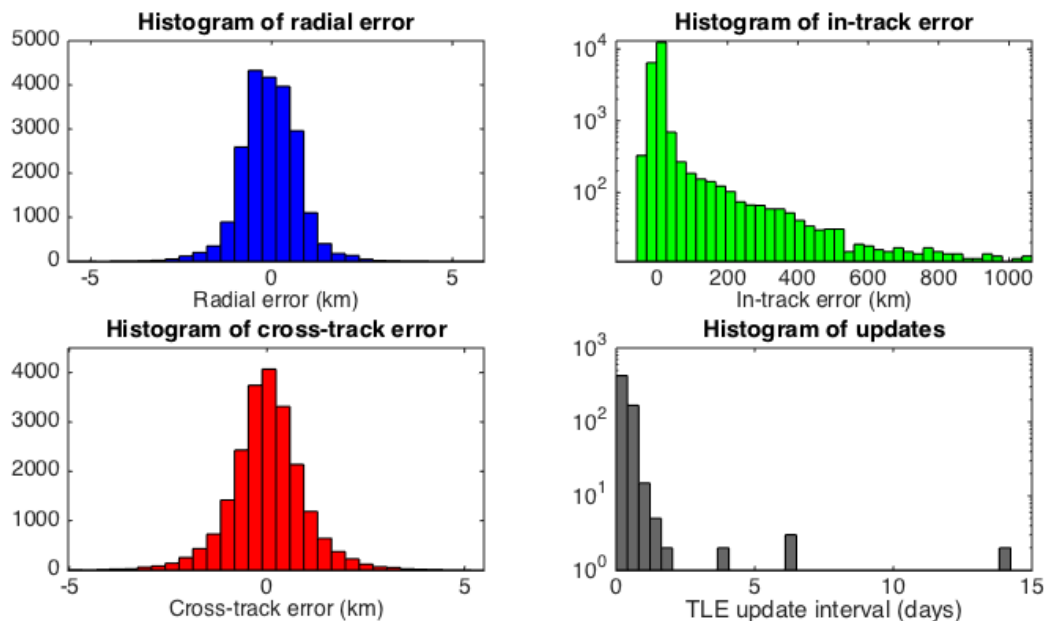
To assess TLE accuracy from an ISS-deployed orbit, orbital data was provided to the author by Planet Labs. Planet Labs aims to produce a constellation of small satellites that can image the Earth once a day, and they are the largest consumer of NanoRacks deployments from the ISS. In February 2014, they deployed 28 satellites of their Flock 1 from the ISS. Planet Labs released another set of satellites, Flock 1B, from the ISS starting in late August 2014. Planet Labs internally noted inconsistency in published TLEs, which became the motivation for this study. They utilize a radio for two-way ranging and generate an independent set of orbital elements from this data, which provides a comparison for assessing JSpOC TLE accuracy. Planet Labs recently made their orbital ephemerides available for public use, including comparison to most recent JSpOC TLEs (they can be accessed at <http://ephemerides.planet-labs.com/>).

There are three primary objectives of this work. The first objective is to provide statistics regarding JSpOC TLE accuracy for the benefit of satellite operators. The second objective is to assess a self-consistency metric<sup>3</sup> that provides a variance estimate of TLE error. The third aim is to propose an estimation technique to improve the accuracy of a current TLEs based on prior, recent TLEs. The aim of these tools is to enable small satellite operators to better understand and utilize TLEs in practice.

## TLE ACCURACY

Published TLEs do not come with an associated accuracy metric. In cases where satellite operators rely on TLEs for orbit determination, this becomes problematic. It is difficult to plan for science operations or calculate a link budget without a good sense of orbital accuracy. Statistical estimates of TLE error for CubeSats in LEO have been limited in scope, only covering several TLE updates for single satellites<sup>6,7</sup>.

To characterize TLE accuracy, additional orbital data from Planet Labs Flock 1B was provided. The data consists of satellite position derived from two-way radio ranging measurements, which Planet Labs estimates to be accurate to around 1 km. The accuracy of these measurements is not as good as on-board GPS. However, given the large magnitude of errors seen from JSpOC TLEs this uncertainty in the "truth set" is acceptable in most cases, but certainly limits the precision of error estimates. The orbital data used here was also utilized by Planet Labs for satellite operations, and is treated as a "truth set" for JSpOC TLE comparison. TLEs are compared against ranging data for 10 satellites for the entire month of September 2014. This period covers 634 TLEs across all satellites.



**Figure 1: Histogram of TLE forward propagated errors and update intervals for 10 Flock 1B CubeSats in September 2014.**

Since the aim of this work is to aid satellite operations, TLEs are only propagated forward in time. This is intended to mimic the operational approach of always using the most recently published TLE. The Planet Labs state vector is acquired at 20-minute intervals, and each is compared to the most recent available TLE. Resulting error is split up into radial, in-track, and cross-track errors, which are shown in Figure 1. Also included in Figure 1 are the update intervals between TLEs (note that the histogram of in-track error and update times are plotted on a log scale for clarity). To maintain accuracy, TLEs should be updated several times a day for CubeSats in LEO, but major gaps are seen between updates for Flock 1B. The longest gap between updates is just over two weeks, which results in the heavy tail for in-track errors in Figure 1.

The errors are certainly correlated in time and are not well fit by Gaussian distributions. They can be described simply by the median and quartile ranges to give a sense of the error bounds. The error statistics are summarized in

Table 1. This table indicates that across all satellites, position error was under 2.01 km for 25% of the time, under 4.52 km for 50% of the time, and above 10.6 km for 25% of the time. Overall, the errors seen in Flock 1B are substantially worse than previous estimates of less than 1 km in position error. This presents a significant operational challenge for satellites that rely on TLEs for orbit determination.

**Table 1: Forward-propagated TLE error statistics for entire set of Flock 1B satellites.**

Error	Q1 (km)	Median (km)	Q3 (km)
Radial component	-0.54	-0.07	0.45
In-track component	-3.45	0.32	6.26
Cross-track component	-0.50	0.00	0.51
Total error	2.01	4.52	10.60

### SELF-CONSISTENCY ANALYSIS

In this section, a self-consistency analysis technique is discussed for the estimation of TLE variance as a function of propagation interval. The method is presented, followed by application to the Flock 1B satellites in September 2014. The  $1\text{-}\sigma$  in-track propagation error ranges from 10-30 km after one day and 20-70 km after two days. The self-consistency analysis is found to approximate the error well, to within 10%.

#### Method

A self-consistency measure has been proposed<sup>5</sup> in which each TLE is propagated to the epoch of TLEs that precede and follow it. A TLE at time  $t_i$  is propagated to the time of a second TLE at  $t_j$ . If each TLE is considered to be “truth” at its epoch, the variance of a propagated TLE can be estimated as a function of the propagation interval. This approach

underestimates the error of the TLE at its epoch, which by default is zero, but remains useful in examining the effect of propagation.

Comparison between chronological JSpOC TLEs can also be used so that the operator knows the general accuracy of the TLEs. The propagation interval is a major factor in reducing accuracy, but there are also significant variations in TLE accuracy at the epoch. Observing the TLEs for Flock 1B satellites shows that there are periodically large spikes in error, which may persist several days or more until settling back down to nominal accuracy. A possible hypothesis is that with the number of CubeSats in the ISS orbit, occasional “cross-tagging” or observation mismatch occurs. This would skew the accuracy of the TLE until more reliable measurements replace erroneous ones. Regardless of the cause, it is useful for the operator to recognize when TLEs may be unreliable. Chronological self-consistency can indicate when this occurs.

### Results

Propagation errors for the Flock 1B TLEs are, in general, substantially worse than in prior literature<sup>6,7</sup>. The 1- $\sigma$  in-track propagation error after one and two days is shown in Table 2. JSpOC TLE updates for ISS-deployed CubeSats are very frequent compared to higher orbits (e.g. several times a day in LEO vs. several times a week in GEO), which can be explained by the large propagation errors that rapidly accumulate. The median update time for Flock 1B TLEs is just under 8 hours. Given these large propagation errors of tens of kilometers, the operational use of a TLE is constrained to within a day after its published epoch.

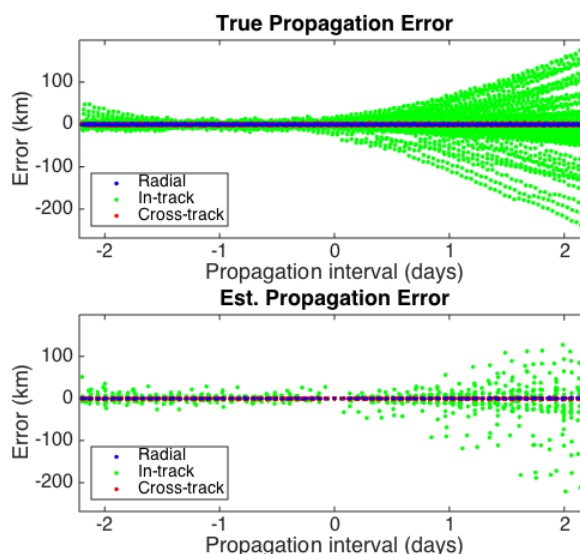
**Table 2: Flock 1B one- and two-day forward propagation errors.**

Sat. ID	1-day 1- $\sigma$ error (km)	2-day 1- $\sigma$ error (km)
1B-1	16.8	29.3
1B-2	11.8	26.0
1B-7	14.5	35.0
1B-8	14.8	37.7
1B-15	25.4	62.7
1B-16	20.4	45.4
1B-23	9.3	18.7
1B-24	16.3	32.1
1B-25	30.3	71.4
1B-26	17.2	42.0

Self-consistency analysis was conducted for the set of Flock 1B satellites. This procedure must be conducted for each satellite individually as the TLEs for each

satellite may vary in statistical parameters. From the ten satellites examined, several important findings emerged. First, none of the satellites’ TLEs showed a significant in-track bias in either the positive or negative direction. This behavior differs from the biases seen in the case of GPS satellites<sup>5</sup>, and unfortunately means that a bias correction technique cannot improve propagation accuracy.

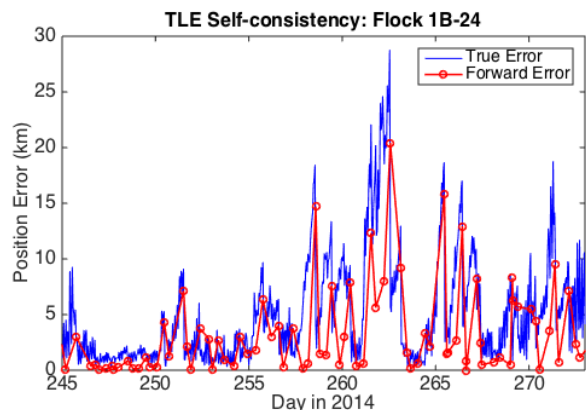
The self-consistency analysis gives a reasonably good approximation of the propagation error. Figure 2 shows the self-consistency analysis compared against the true error. In this case, the standard deviation of error in the 1.5 to 2 day range is 60.3 km, while the self-consistency analysis yields a standard deviation of 58.8 km, accurate to 2.5%. The other satellites show similar self-consistency performance, with standard deviation estimates accurate to 10%.



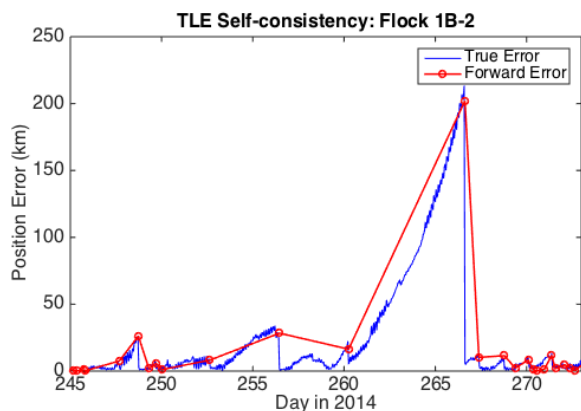
**Figure 2: True propagation error (top) compared to self-consistency (bottom) for Flock 1B-15.**

Finally, a chronological comparison between TLEs can alert the operator when TLEs become somewhat unreliable. Figure 3 shows the true error plotted against forward-propagated chronological TLE comparison for Flock 1B-24, one of the better-tracked satellites of Flock 1B. In this example, the TLEs are quite consistent around day 249, but have significantly degraded around day 263. The chronological self-consistency comparison is a simple way to alert the operator to this change without relying on external measurements. Figure 4 shows the same procedure for Flock 1B-2 which had much less consistent tracking and fewer JSpOC TLE updates. Again, the self-consistency check reliably tracks error between TLE

updates, letting the operator know the severity of propagation errors.



**Figure 3: Forward propagated chronological self-consistency check for Flock 1B-24.**



**Figure 4: Forward propagated chronological self-consistency check for Flock 1B-2.**

Self-consistency analysis can provide the operator with important information on orbital accuracy and expected errors.

## PROPOSED ESTIMATION TECHNIQUES

This section describes the estimation method used to improve current TLEs based on prior TLEs. First, the treatment of TLEs is described, followed by the least squares technique used, and finally the state elements considered for estimation. It is found that in cases of sparse updates, estimation techniques have the potential to greatly improve published TLE accuracy.

### Method

A least squares approach is taken for improving the current TLE based on prior TLEs. Each TLE consists of six orbital elements, an epoch, and a drag term related

to the ballistic coefficient. The ballistic coefficient is defined by:

$$B = C_d A / m \quad (1)$$

Where  $C_d$  is the drag coefficient,  $A$  is the cross-sectional area, and  $m$  is the mass of the object. The drag term provided in a TLE, known as the B-star term, is given by:

$$B^* = B \rho_0 / 2 \quad (2)$$

where  $\rho_0$  is a reference atmospheric density. The resulting  $B^*$  term has units of (Earth radii)<sup>-1</sup>.

The other six orbital elements are the inclination, right ascension of the ascending node (RAAN), eccentricity, argument of perigee, mean anomaly, and mean motion. Combined with the epoch, these elements can be converted to three components of position and three components of velocity in an Earth-centered Cartesian frame. In a perfectly Keplerian orbit, this would fully define the orbit for all time. However, the orbit is affected by many factors such as gravitational perturbations from the Earth and other bodies, solar radiation pressure, and atmospheric drag. SGP4 contains a simplified model of these perturbations, and the  $B^*$  term is used not only for drag but also as a catch-all to account for unmodeled effects. For this reason, it is recommended that TLEs only be used with SGP4<sup>1</sup>.

For the purposes of improving JSpOC TLE updates, the TLEs can be treated as “pseudo-observations” by removing the  $B^*$  term. Multiple observations can be taken from a TLE by propagating it to different times near the published epoch. This approach was shown to be successful in improving orbit determination when used with a high precision orbit propagator<sup>8</sup>. However, since the aim in this work is to generate an improved TLE for operational use, a high precision orbit propagator is not used. SGP4 is used for all fitting, with publicly available code provided by Vallado<sup>9</sup>.

As was seen in Figure 2, TLEs are generally more accurate in backward propagation than in forward propagation, which is intuitive given that they are based on past observations. As a result, the technique for generating pseudo-observations in this work is to propagate each TLE backwards to the epoch of its prior TLE, making pseudo-observations in 1-hour intervals.

Orbit determination is done with a batch least squares approach, generally following the steps laid out in prior work by Vallado<sup>9,10</sup>. Each state consists of the seven elements of a TLE.

The state  $\mathbf{X}$  is defined by:

$$\mathbf{X} = \begin{bmatrix} i \\ \Omega \\ e \\ \omega \\ M \\ n \\ B^* \end{bmatrix} \quad (3)$$

where  $i$  is inclination,  $\Omega$  is RAAN,  $e$  is eccentricity,  $\omega$  is argument of perigee,  $M$  is mean anomaly, and  $n$  is mean motion. The state is initialized to be the most recent TLE. Prior TLEs are used as pseudo-observations and are converted to a set of positions and velocities. Observations are of the form:

$$\mathbf{y}_i = \begin{bmatrix} \mathbf{r}_i \\ \mathbf{v}_i \end{bmatrix} \quad (4)$$

corresponding to a position  $\mathbf{r}_i$  and velocity  $\mathbf{v}_i$  at a given time.

The state dynamics are specified by the orbital propagation routine SGP4. When SGP4 is used to propagate a TLE, the output is the position and velocity vectors in the same form as the observations. This is convenient for direct comparison.

The current TLE is propagated to the observation points and the residuals are formed. The Jacobian matrix is formed using finite differencing. Each element in the state is perturbed slightly, and the state is propagated to each observation. The nominal and perturbed states are then used to form the Jacobian as follows:

$$A = \frac{\delta \text{ observations}}{\delta \hat{\mathbf{X}}_0} \quad (5)$$

$$\delta_i = \hat{\mathbf{X}}_{\text{mod},0} - \hat{\mathbf{X}}_{\text{nom},0} \quad (6)$$

$$\frac{\delta \text{ observations}}{\delta \hat{\mathbf{X}}_0} \approx \frac{\text{obs}_{\text{mod}} - \text{obs}_{\text{nom}}}{\delta_i} \quad (7)$$

where  $A$  is the Jacobian,  $\delta_i$  is the change in initial state, and the states at the observation points are of the form given in Equation 4.

For each observation, the matrices  $A^T W A$  and  $A^T W \mathbf{b}$  are accumulated, where  $\mathbf{b}$  is the residual and  $W$  is the weighting matrix (while a weighting matrix was not used for this estimation, further work can examine if it

may be useful). Once all observations are accounted for, the initial state is corrected by:

$$\delta \hat{\mathbf{X}} = (A^T W A)^{-1} A^T W \mathbf{b} \quad (8)$$

Iterations occur on the initial TLE until the stopping condition is met, which in this case is a threshold on the magnitude of  $\delta \hat{\mathbf{X}}$ .

While the state is given by seven elements, not all elements were modified in all cases. If TLE updates are infrequent and a full state estimate is attempted, the problem rapidly becomes ill posed when using SGP4 and finite differencing. An examination of propagation error in the reference frame of the satellite provides insight into which elements are the most important in minimizing propagation error.

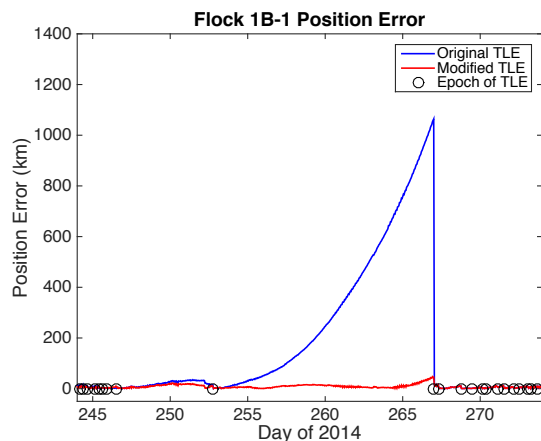
The position error is dominated by the in-track component as was seen in Figure 1 relative to the truth set, which is consistent with prior work<sup>5,7</sup>. This suggests that it is best to focus on elements that have the largest effect on the in-track error. The key element requiring estimation is the  $B^*$  term, as it is directly related to in-track error and is the most poorly defined parameter of the provided TLE.

Two cases are considered based on the published TLEs: the ‘‘poorly-tracked’’ case and the ‘‘well-tracked’’ case. In the well-tracked case, the satellites are tracked consistently and updates occur within a few hours. However, in the poorly-tracked cases, the updates are sparse and it seems that JSpOC is not tracking the satellite consistently. In the case where TLE updates are supplied frequently, there is enough data to fit all of the orbital elements. If there have been at least five updates in the past 36 hours, the satellite is considered well-tracked. If there is sparse data and the above criteria is not satisfied, then not all elements can be estimated. Instead, in an effort to control the in-track error, only the  $B^*$  term is estimated based on the past ten days of data.

## Results

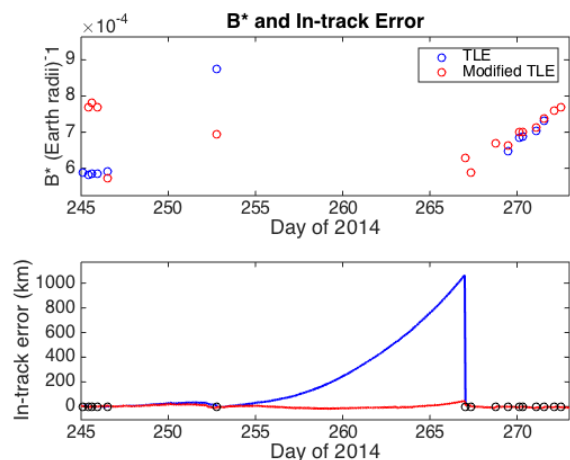
In the case where the data is sparse, estimating the ballistic coefficient can produce a significant improvement. Flock 1B-1 is the ‘‘worst-case’’ TLE error seen on orbit and has a gap of two weeks in which no new TLEs are published. During this time, the error in propagating the last TLE grows to nearly 1100 km. This is well beyond the acceptable error limit and could result in a loss of operations for nearly two weeks. However, if least squares estimation is used to determine the  $B^*$  term, the error is less than 50 km after

two weeks as shown in Figure 5. For this particular case, there is a 95% reduction in error.



**Figure 5: Improvement of modified TLEs over original TLEs in position error for Flock 1B-1.**

To understand why estimation of the  $B^*$  term improves the TLE, it is useful to compare the  $B^*$  term with the in-track error, as shown in Figure 6. It can be seen that on day 253 (i.e. September 10th) there is a published TLE that is followed by a large gap in TLE updates. This gap gives rise to the rapid growth in error. Figure 6 also indicates that the overall position error is almost entirely attributed to the in-track error.

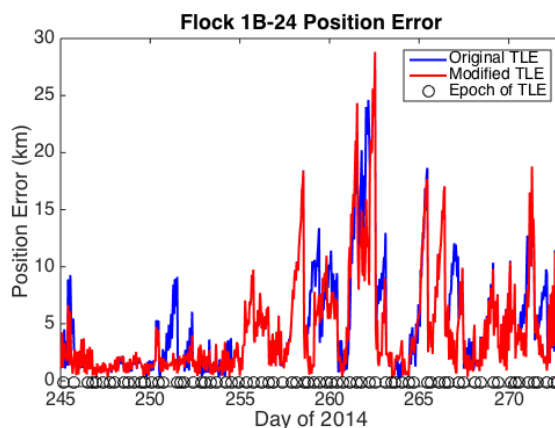


**Figure 6: Comparison of in-track error against  $B^*$  term for original and modified TLEs of Flock 1B-1.**

Looking at the  $B^*$  terms in the original TLEs, it is clear that there is a fairly large increase in that TLE's  $B^*$  term as compared to the prior TLEs. It is unclear why this increase occurs, but based on the propagation error in Figure 5 it appears to be erroneous. Recalling the counterintuitive nature of orbital dynamics, thrusting

backwards will cause a spacecraft to speed up by entering a lower orbit. Similarly, a  $B^*$  term that is too large will result in positive in-track error, such that the estimate tends to overshoot the true location of the satellite. The estimated  $B^*$  for the modified TLE is significantly lower than the original and much closer to the  $B^*$  values of the prior TLEs. This leads to a reduction in in-track error over the propagation period of two weeks.

In the case where the satellite is well-tracked, estimating the current TLE based on prior TLEs over a short period still shows some improvement over the initial TLE. Flock 1B-24 falls under the “well-tracked” criteria for the entire data set, and the errors for the unmodified and modified TLEs are shown in Figure 7. The error reduces by 15% on average for Flock 1B-24 using the estimation technique. While there is some improvement, it is not enough for precision orbit determination, so it is unlikely to have a large effect on operations. The orbit could be better fit by taking into account future TLEs for smoothing, but that problem has not been considered here since the aim was to improve operations.



**Figure 7: Improvement of modified TLEs over original TLEs in position error for Flock 1B-24.**

The methodologies for the “poorly-tracked” and “well-tracked” cases were combined into a single estimation technique. In this technique, a full state estimation is conducted if enough information is available as in the well-tracked case. If not enough information is available, only the  $B^*$  term is estimated as in the poorly-tracked case. This approach allows each technique to be applied to the cases where it is best-suited for each individual satellite TLE. This technique is applied across Planet Labs Flock 1B satellites for each TLE during the month of September 2014, and the mean error is computed for each satellite in Table 3.

**Table 3: Average error in TLE position for original set and modified set.**

Sat. ID	Mean propagation error in position		
	Orig. TLE (km)	Mod. TLE (km)	% Improvement
1B-1	165.2	8.6	95%
1B-2	23.7	32.2	2%
1B-7	11.9	13.3	-11%
1B-8	73.1	43.8	40%
1B-15	8.2	7.3	10%
1B-16	4.3	3.9	9%
1B-23	5.0	4.3	15%
1B-24	5.1	4.3	15%
1B-25	8.1	7.1	13%
1B-26	5.5	5.0	6%

For the ten satellites analyzed, improvement is shown for all but one satellite. The amount of improvement covers a fairly wide range and in general is dependent upon the frequency of TLE updates. Very large errors result from sparse TLE updates, and these cases show the most potential for error reduction. In cases where updates are frequent, estimation produces only small error reductions.

## CONCLUSION

While values reported in prior literature estimate TLE error for CubeSats in LEO of 1 km or less, we found TLEs of CubeSats deployed from the ISS to be much less accurate. Median error in position was found to be 4.52 km, with error greater than 10.6 km for 25% of the time in orbit. The large discrepancy in accuracy is likely the result of several factors. First, prior studies of small satellites in LEO have only covered a few TLE updates due to the unavailability of GPS “truth” data. When TLEs are examined over several weeks, it becomes clear that there are both periods where the TLEs are frequently updated and low in error as well as periods where TLEs are sparse and poorly fit. Second, it is possible that nanosatellites deployed from the ISS uniquely show this problem. This launch technique has only become prevalent in the last two years and has resulted in a large number of nanosatellites placed in very similar orbits. Cross-tagging or misidentification of the satellites may be producing issues with TLE accuracy for satellites deployed from the ISS. A brief analysis of the TLEs of Planet Labs Flock 1C, which is in a sun-synchronous orbit over 600 km, shows results that are comparable to the 1 km error value cited in previous studies.

A self-consistency metric was found to provide a good estimate of TLE propagation error statistics. One-day propagation error ranges from 10-30 km  $1-\sigma$ , while

two-day propagation produces 20-70 km  $1-\sigma$  error. The described self-consistency procedure successfully estimated  $1-\sigma$  error to within 10% for the Flock 1B satellites. Chronological self-consistency checks between TLEs can also alert the operator to when periods of unreliability arise.

Finally, a least squares estimation technique has been presented for improving the accuracy of a satellite's current TLE based on prior, recent TLEs. This estimation technique is intended for operational use, so it does not utilize future TLEs or external data in its estimate. Two cases are considered in which the satellite is treated as “well-tracked” or “poorly-tracked” based on the update frequency. It is shown that in the poorly-tracked case, significant improvement up to 95% is possible with an estimation of the  $B^*$  term from prior TLEs, potentially enabling satellite operations under conditions in which they would otherwise be lost. In the well-tracked case, some improvement is achieved by estimating all elements from prior TLEs, but the improvement is unlikely to have a significant effect on operations. Overall, the estimation technique appears to improve the TLE in the vast majority of cases, and is particularly useful in the case of sparse updates.

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## REFERENCES

1. Hoots, F. and R. Roehrich, “Spacetrack Report No. 3: Models for Propagation of NORAD Element Sets,” U.S. Air Force Aerospace Defense Command, 1980.
2. Vallado, D., Crawford, P., Hujsak, R. and T.S. Kelso, “Revisiting Spacetrack Report #3,” Proc. of AIAA/AAS Astrodynamics Specialist Conference, 2006.
3. Kelso, T.S. and S. Alfano, “Satellite Orbital Conjunction Reports Assessing Threatening Encounters in Space (SOCRATES),” Proc. of the 15th AAS/AIAA Space Flight Mechanics Conference, Paper AAS 05-124, 2005.
4. Vallado, D. and J. Seago, “Covariance Realism,” Proc. of the AAS/AIAA Astrodynamics Specialist Conference, Paper AAS 09-304, 2009.



5. Kelso, T.S., "Validation of SGP4 and IS-GPS-200D against GPS precision ephemerides," Proc. of 17<sup>th</sup> the AAS/AIAA Space Flight Mechanics Conference, 2007.
6. Coffee, B., Cahoy, K. and R. Bishop, "Propagation of CubeSats in LEO using NORAD Two Line Element Sets: Accuracy and Update Frequency," Proc. of AIAA Guidance, Navigation, and Control Conference, 2013.
7. Kahr, E. and K. O'Keefe, "Estimation and Analysis of Two-Line Elements for Small Satellites," Journal of Spacecraft and Rockets, Vol. 50, No. 2, 2013.
8. Levit, C. and W. Marshall, "Improved orbit predictions using two-line elements," Advances in Space Research, Vol. 47, No. 7, 2011.
9. Vallado, D., Fundamentals of Astrodynamics and Applications, Third Edition, Microcosm Press and Springer, 2007.
10. Vallado, D. and P. Crawford, "SGP4 Orbit Determination," Proc. of AAS/AIAA Astrodynamics Specialist Conference, 2008.