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The Influence of Different Virtual Manipulative Types on Student-Led Techno-Mathematical Discourse

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This exploratory study examined the influence of different virtual manipulative (VM) types on the nature of student pairs’ techno-mathematical discourse (TMD). Three fifth-grade student pairs participated in 9 sessions of mathematics instruction using VMs. The study compared three VM types: linked, pictorial, and tutorial. Students’ levels of discourse in generalization, justification, and collaboration were measured while working with each VM type. One-way ANOVAs indicated statistically significant differences in the quality of student discourse when using the different VM types. When working with linked VMs, students’ discussions reflected consistently higher levels of discourse than when working with pictorial or tutorial VMs. However, pictorial VMs were associated with the largest amount of student to student discussion. The results of this study suggest that in order to encourage meaningful TMD, teachers should choose VMs with features that link multiple representations. The results of this study also indicate that for these pairs, tutorial VMs did not encourage meaningful student to student mathematical discourse. The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom.
Reform efforts in mathematics education have called for classrooms where all students have access to engaging mathematics and high-quality instruction. In particular, the Common Core Standards for Mathematical Practice call for teachers to develop positive mathematical dispositions in their students. Mathematical discourse and technology are key components that mediate how students learn mathematics and promote mathematical practices of “construct[ing] viable arguments” and “us[ing] appropriate tools strategically” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The National Council of Teachers of Mathematics (2007) also highlights the importance of discourse in the Standards for Teaching and Learning Mathematics:

The discourse of the mathematics class reflects messages about what it means to know mathematics, what makes something true or reasonable, and what doing mathematics entails. It is central to both what students learn about mathematics and how they learn it. Therefore, the discourse of the mathematics class should be founded on mathematical ways of knowing and ways of communicating. (p. 54)

Uses of technology are becoming more prevalent in today’s classrooms as compared to years past. Over the past two decades, virtual manipulatives (VMs) have emerged as effective tools for teachers to use in their instruction of mathematics and support of student learning (Moyer-Packenham & Westenskow, 2013). A VM is defined as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard, & Spikell, 2002). Early collections of VMs that still exist today include Shodor Interactivate Activities (originally created in 1994, www.shodor.org/interactivate/activities/), the National Library of Virtual Manipulatives (originally created in 1999, nlvm.usu.edu/), and Illuminations: Activities (originally created in 2000, illuminations.nctm.org). Since their inception, these websites have grown to offer more than 300 online VM tools collectively. In recent years, textbook companies (e.g., McGraw-Hill, Glencoe) have also developed VMs as resources for their most recent editions of elementary mathematics textbooks. These resources contain three different VM types: linked, pictorial, and tutorial.

With this increase of available online resources, questions arise regarding how different VM types influence students’ learning experiences—particularly in the ways that students interact with each other and discuss mathematical ideas when they are using these tools (NCTM, 2007). The purpose of this exploratory research study was to (a) describe and categorize
the nature of students’ mathematical discourse as they worked with various VM types and (b) to develop theory on the interactions among student-led discourse, VMs, and mathematical tasks. The following research question guided the design of the study: How do different VM types influence the levels of generalization, justification, and collaboration in students’ mathematical discourse?

LITERATURE REVIEW

Virtual Manipulatives

With the advancement of computer capabilities, VMs have emerged as cognitive technology tools for use in mathematics classrooms by providing teachers and students with expanded tools for thinking about mathematics concepts. The impact of VMs on conceptual understanding and student achievement has been found to have a moderate effect when compared to other instructional treatments. A meta-analysis of research on the effects of VMs on student achievement synthesized data from 32 unique studies and yielded an average effect size (Cohen’s $d$) of 0.35 (Moyer-Packenham & Westenskow, 2013). This finding suggests that when used for instruction, VMs have a moderately greater impact on student learning than other instructional tools. Overall, research indicates that VMs positively contribute to students’ understanding of mathematics concepts (Bolyard & Moyer-Packenham, 2012; Lee & Tan, 2014; Mendiburo & Hasselbring, 2011; Mendiburo, Hasselbring, & Biswas, 2014; Moyer Packenham et al. 2014; Moyer-Packenham & Suh, 2012; Moyer-Packenham, Ulmer, & Anderson, 2012; Reimer & Moyer, 2005; Sarama & Clements, 2009; Steen, Brooks, & Lyon, 2006; Suh & Moyer-Packenham, 2008; Suh, Moyer, & Heo, 2005; Trespalacios, 2010).

VM tools vary in their type of feedback and mathematical representation (Bolyard & Moyer, 2007). Linked VMs are defined as open-ended VMs that present multiple representations of mathematical concepts (e.g., pictorial images, number line models, and numeric symbols) that change simultaneously as they are manipulated. Linked VMs reflect the user’s actions and choices without dictating solution strategies (Martin & Schwartz, 2005). Pictorial VMs are defined as visual representations of mathematics concepts similar to physical manipulatives. Like the linked VMs, pictorial VMs reflect the user’s actions and choices, but they do not include numeric symbols associated with the visual representation (Zbiek, Heid, Blume, & Dick,
tutorial VMs are defined as structured instructional tools designed to guide students to a conceptual or procedural understanding of mathematics. Tutorial VMs dictate specific solution strategies give direct feedback to users based on their adherence to those strategies. These varying types of feedback and representation in VMs have implications for instructional use.

**Mathematical Discourse**

Students develop understanding as they interact with others through verbal or nonverbal communications or written words (Vygotsky, 1978). Meaningful classroom discourse contributes to students’ understanding by promoting effective communication and articulation of thought (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). One key aspect of small-group interaction is “co-construction,” defined by Mueller (2009) as a “form of collaboration in which an argument is built simultaneously by the learners from conception” (p. 141). This collaboration is characterized by the back and forth nature of its discourse. Students’ solutions become stronger as they integrate the ideas of others.

Multiple studies have examined the process of mathematical explanation and reasoning (e.g., Carpenter, Fennema, & Franke, 1996; Hufferd-Ackles, Fuson, & Sherin, 2004; Zolkower & Shreyar, 2007). Notably, the framework for Robust Mathematical Discussion (RMD) describes components of effective mathematical classroom discourse (Mendez, Sherin, & Louis, 2007). In this framework, robustness refers to the “mathematical and discursive strength of the discourse” (Mendez et al., 2007, p. 42). RMD categorizes students’ comments along two dimensions: mathematics and discussion. The mathematics dimension addresses three aspects of mathematical argumentation: representation, generalization, and justification. The discussion dimension examines three aspects of discourse: engagement, intensity, and building on others’ ideas. Mendez and colleagues (2007) found that discourse was most effective in developing mathematical understanding when students’ discourse was classified as high in each of the RMD dimensions.

To date, extensive research has been conducted on the nature of classroom mathematical discourse (e.g., Herbel-Eisenmann & Wagner, 2010; Iiskala, Vauras, Lehtinen, & Salonen, 2011; Imm & Stylianou, 2012; Wood & Kalinec, 2012). However, limited research exists on the interactions students have with each other when using technology to learn mathematics (e.g., Ares, Stroup, & Schademan, 2008; Evans, Feenstra, Ryon, & McNeill, 2011).
THEORETICAL FRAMEWORK

The Techno-Mathematical Discourse (TMD) framework was developed to provide a means for analyzing and interpreting aspects of social learning with technology during mathematics instruction (Anderson-Pence, 2014). This framework consists of three main elements: technology tools, classroom discourse, and mathematical tasks (see Figure 1). First, technology tools (e.g., VMs) enhance the learning of mathematics concepts by expanding representational possibilities and by amplifying and reorganizing students’ approaches to problem solving. As indicated above, the type and quality of the technology tool influences students’ engagement with mathematics. Second, classroom discourse enhances students’ mathematical understanding as they interact with other individuals through verbal and nonverbal communications or written words. This interaction is influenced by classroom culture, students’ perceptions of self and others, and the teacher’s orchestration of mathematical dialogue. Third, the mathematical tasks presented in a lesson influence the richness of classroom discourse and students’ mathematical understanding. A worthwhile mathematical task is one that engages students’ intellect, calls for problem solving and mathematical reasoning, and promotes communication about mathematics (NCTM, 2007). Altogether, these three elements characterize the TMD framework. Dynamic visual displays from technology tools serve as common experiences for students to engage in meaningful classroom discussions centered on worthwhile mathematical tasks.

Figure 1. Theoretical Framework of Techno-Mathematical Discourse (TMD).
METHODS

This exploratory study aimed to answer the following research question: How do different VM types influence the levels of generalization, justification, and collaboration in students’ mathematical discourse? Different VM types were identified as linked, pictorial, and tutorial.

Participants

The study included 3 pairs of fifth-grade students aged 10–11 years (each pair consisting of one female and one male student). These students were selected based on their ability to verbally process thinking and to get along with their assigned partner. Mathematics achievement was not a deciding factor when selecting students for this study. Classroom teachers assisted the researcher in selecting the students.

Procedures & Data Collection

At the beginning of the study, lessons were designed to promote discourse between student-partners. The structure of lessons designed for this study was based on a widely used three-stage guided-inquiry model (e.g., Hendrickson, Hilton, & Bahr, 2010; Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), which included a 5–10 minute introduction of the learning task, a 20–30 minute small-group exploration of mathematical tasks and concepts, and a 20–30 minute whole-class discussion to examine students’ solution strategies, highlight key mathematical concepts, and conclude the lesson (see Table 1). Tasks for each VM were adapted from tool-specific lesson explorations suggested by the illuminations.nctm.org and nlvm.usu.edu websites. Appendix A shows an example of a lesson plan used for equivalent fraction instruction.
During the study, each pair of participating students shared a laptop computer while they interacted with different VMs: 3 linked, 3 pictorial, and 3 tutorial. Appendix B shows the specific VMs selected for this study. The 3 student pairs participated in 9 lessons using the VMs—a total of 27 lessons. An episode was defined as a student pair’s interaction with one VM during one lesson.

Data collection took place during the 20–30-minute exploration part of each lesson as student pairs worked together on the pre-designed mathematical tasks. Two different video perspectives were recorded as data for analysis. First, a face-capture perspective, recorded the students’ mathematical discussions using the built-in camera located at the top and center of the computer screen. Second, a computer screen-capture perspective, recorded how the students interacted with the VMs on the computer screen. This screen-capture included a record of mouse movement, mouse clicks, and external audio. Both perspectives were recorded simultaneously using Quicktime Player.

Data Analysis

First, speaking turns in each of the 27 episodes were transcribed and coded for levels of discourse according to generalization, justification, and collaboration dimensions (see Table 2). Second, percentages and frequencies of codable speaking turns were calculated to quantify the discourse in each episode. Third, leveled codes were used to calculate composite scores—a measure of the quality of discourse in each episode. The composite scores were calculated by a summation of the codes for each speaking turn within the episode divided by the total number of codable speaking turns, and multiplied by 100. For example, a discussion with 91 total speak-

### Table 1

<table>
<thead>
<tr>
<th>Stage</th>
<th>Duration</th>
<th>Activity</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>5-10 minutes</td>
<td>Demonstration</td>
<td>Introduce learning task</td>
</tr>
<tr>
<td>Exploration</td>
<td>20-30 minutes</td>
<td>Completion of student task sheet</td>
<td>• Orientation to VM</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Concept development and application</td>
</tr>
<tr>
<td>Conclusion</td>
<td>20-30 minutes</td>
<td>Whole-class discussion</td>
<td>Examine students’ solution strategies, highlight key mathematical ideas</td>
</tr>
</tbody>
</table>
ing turns coded for justification—60 as statement (level 1), 13 as explanation (level 2), and 18 as proof (level 3)—would yield a justification composite score of \( \left( \frac{60 \times 1 + (13 \times 2) + (18 \times 3)}{91} \right) \times 100 = 153.85 \).

### Table 2

<table>
<thead>
<tr>
<th>Level of Generalization (^a)</th>
<th>Level of Justification (^a)</th>
<th>Level of Collaboration (^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 Not Codable</td>
<td>0 Not Codable</td>
<td>0 Not Codable</td>
</tr>
<tr>
<td>1 Concrete</td>
<td>1 Statement</td>
<td>1 Unrelated Idea</td>
</tr>
<tr>
<td>2 Comparison</td>
<td>2 Explanation</td>
<td>2 Response</td>
</tr>
<tr>
<td>3 Generalization</td>
<td>3 Proof</td>
<td>3 Build</td>
</tr>
</tbody>
</table>

\(^a\) Adapted from the Robust Mathematics Discussion Framework (Mendez et al., 2007)

A one-way ANOVA on the amount of coded speaking turns per episode was conducted to compare the quantity of discourse for each VM type (i.e., linked, pictorial, and tutorial). One-way ANOVAs on composite scores were also conducted for generalization, for justification, and for collaboration to compare the quality of discourse for each VM type.

**RESULTS**

The results that follow discuss how different VM types influenced the student pairs’ levels of generalization, justification, and collaboration in terms of the quantity and quality of students’ mathematical discourse.

**Quantity of Discourse**

A one-way ANOVA indicated no statistically significant differences among VM types in terms of the number of students’ speaking turns, \( F(2, 24) = 3.258, p = .056 \). However, the number of speaking turns covered a large range—from 47 when using a tutorial VM to 182 when using a pictorial VM (see Figure 2). Discussions associated with the pictorial VMs had the highest number of speaking turns (\( M = 111.22, SD = 42.92 \)), and the tutorial VMs had the lowest number of speaking turns (\( M = 69.67, SD = 19.61 \)).
Quality of Discourse: Generalization

Generalization is defined as the capacity to form connections among related ideas (Mendez et al., 2007). Overall, students engaged in higher levels of generalization when working with linked VMs than with pictorial or tutorial VMs. Figure 3 displays the range and means of generalization composite scores for each VM type. Linked VMs had the highest average composite score ($M = 128.52$, $SD = 15.56$), followed by pictorial ($M = 115.26$, $SD = 5.80$) and tutorial ($M = 107.39$, $SD = 13.37$).
A one-way ANOVA comparison of generalization composite scores indicated a statistically significant overall difference among the VM types at the 95% level, $F(2, 24) = 9.460$, $p = 0.001$. This corresponded to an effect size of $\eta^2 = .44$; that is, about 44% of the variance in generalization composite scores was predictable from the type of VM. This is a moderate effect. Individual post hoc comparisons using Tukey’s HSD (Warner, 2013) indicated a statistically significant difference between the linked and pictorial VM types, $p = 0.033$, and between the linked and tutorial VM types, $p = .001$. There was not a statistically significant difference between the pictorial and tutorial VM types.

Differences in composite scores resulted from the frequency of different levels of generalization in students’ discussions. Table 3 displays the number of speaking turns at each level of generalization for each VM type.

<table>
<thead>
<tr>
<th>Level of Generalization</th>
<th>Linked</th>
<th>Pictorial</th>
<th>Tutorial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalization (highest level)</td>
<td>63 (9.33)</td>
<td>39 (5.09)</td>
<td>10 (1.89)</td>
</tr>
<tr>
<td>Comparison</td>
<td>53 (7.85)</td>
<td>43 (5.61)</td>
<td>18 (3.40)</td>
</tr>
<tr>
<td>Concrete (lowest level)</td>
<td>559 (82.81)</td>
<td>684 (89.30)</td>
<td>502 (94.72)</td>
</tr>
</tbody>
</table>

When working with the linked VM type, 17.18% of students’ speaking turns were at the two higher levels of generalization—nearly twice as much as when working with pictorial VMs (10.7%) and three times as much as when working with tutorial VMs (5.29%). Levels of generalization rarely rose above the concrete level when students worked with tutorial VMs. These results suggest that students typically had higher levels of generalization when working with the linked VM type.
Quality of Discourse: Justification

Justification is defined as a logical, warranted argument or proof of mathematical processes (Mendez et al., 2007). Overall, students engaged in higher levels of justification when working with linked VMs than with pictorial or tutorial VMs. Figure 4 displays the range and means of justification composite scores for each VM type. Linked VMs had the highest average composite score ($M = 135.00, SD = 14.78$), followed by pictorial ($M = 122.20, SD = 6.15$) and tutorial ($M = 113.15, SD = 9.35$).

A one-way ANOVA comparison of justification composite scores indicated a statistically significant overall difference among the VM types at the 95% level, $F(2, 24) = 9.459, p = 0.001$. This corresponded to an effect size of $\eta^2 = .44$; that is, about 44% of the variance in justification composite scores was predictable from the type of VM. This is a moderate effect size. Individual post hoc comparisons using Tukey’s HSD (Warner, 2013) indicated a statistically significant difference between the linked and pictorial VM types, $p = 0.046$, and between the linked and tutorial VM types, $p = .001$. There was not a statistically significant difference between the pictorial and tutorial VM types.

Differences in composite scores resulted from the frequency of different levels of justification in students’ discussions. Table 4 displays the number of speaking turns at each level of justification for each VM type.
Table 4
Frequencies of Speaking Turns at each Justification Discourse Level for VM Types

<table>
<thead>
<tr>
<th>Level of Justification</th>
<th>Linked</th>
<th>Pictorial</th>
<th>Tutorial</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Total</td>
<td>Total</td>
</tr>
<tr>
<td></td>
<td>n = 653</td>
<td>n = 726</td>
<td>n = 536</td>
</tr>
<tr>
<td>Proof (highest level)</td>
<td>47</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>(7.20)</td>
<td>(3.44)</td>
<td>(1.12)</td>
</tr>
<tr>
<td>Explanation (lowest level)</td>
<td>115</td>
<td>109</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>(17.61)</td>
<td>(15.01)</td>
<td>(10.63)</td>
</tr>
<tr>
<td>Statement (lowest level)</td>
<td>491</td>
<td>592</td>
<td>473</td>
</tr>
<tr>
<td></td>
<td>(75.19)</td>
<td>(81.54)</td>
<td>(88.25)</td>
</tr>
<tr>
<td></td>
<td>54.56</td>
<td>65.78</td>
<td>52.56</td>
</tr>
</tbody>
</table>

* N = 9 episodes

When working with the linked VM type, 7.2% of students’ speaking turns were at the highest level of justification, which was nearly twice as much as when working with pictorial VMs (3.44%) and nearly four times as much when working with tutorial VMs (1.12%). These results suggest that students typically had higher levels of justification when working with the linked VM type.

Quality of Discourse: Collaboration

Collaboration is defined in this study as a process in which two or more students exchange ideas and problem-solve together to develop understanding of mathematics concepts (Mendez et al., 2007). Overall, students engaged in higher levels of collaboration when working with linked VMs than with pictorial or tutorial VMs. Figure 5 displays the range and means of collaboration composite scores for each VM type. Linked VMs had the highest average composite score (\( M = 175.99, SD = 16.57 \)), followed by pictorial (\( M = 153.69, SD = 11.62 \)) and tutorial (\( M = 148.40, SD = 9.72 \)).
A one-way ANOVA comparison of collaboration composite scores indicated a statistically significant overall difference among the VM types at the 95% level, $F(2, 24) = 11.476, p < 0.001$. This corresponded to an effect size of $\eta^2 = .49$; that is, about 49% of the variance in collaboration composite scores was predictable from the type of VM. This is a moderate effect size. Individual post hoc comparisons using Tukey’s HSD (Warner, 2013) indicated a statistically significant difference between the linked and pictorial VM types, $p = 0.004$, and between the linked and tutorial VM types, $p < .001$. There was not a statistically significant difference between the pictorial and tutorial VM types.

Differences in composite scores resulted from the frequency of different levels of collaboration in students’ discussions. Table 5 displays the number of speaking turns at each level of collaboration for each VM type.
Table 5
Frequencies of Speaking Turns at each Collaboration Discourse Level for VM Types

<table>
<thead>
<tr>
<th>Level of Collaboration</th>
<th>Linked</th>
<th></th>
<th></th>
<th>Pictorial</th>
<th></th>
<th></th>
<th>Tutorial</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>M(^a)</td>
<td>Total</td>
<td>M(^a)</td>
<td>Total</td>
<td>M(^a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>n = 860</td>
<td></td>
<td>n = 979</td>
<td></td>
<td>n = 625</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Build (highest level)</td>
<td>172</td>
<td>19.11</td>
<td>145</td>
<td>16.11</td>
<td>80</td>
<td>8.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(20.00)</td>
<td></td>
<td>(14.81)</td>
<td></td>
<td>(12.80)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response</td>
<td>295</td>
<td>32.78</td>
<td>251</td>
<td>27.89</td>
<td>145</td>
<td>16.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(34.30)</td>
<td></td>
<td>(25.64)</td>
<td></td>
<td>(23.20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unrelated idea (lowest level)</td>
<td>393</td>
<td>43.67</td>
<td>583</td>
<td>64.78</td>
<td>400</td>
<td>44.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(45.70)</td>
<td></td>
<td>(59.55)</td>
<td></td>
<td>(64.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)N = 9 episodes

When working with the pictorial and tutorial VM types, students’ discourse reflected similar levels of collaboration (14.81% and 12.8% at the highest level). However, students’ collaboration when working with the linked VM was considerably higher (20% at the highest level). These results suggest that students typically had higher levels of collaboration when working with the linked VM type.

Quartile Analysis of Discourse Levels

In the final analysis, data were examined for levels of discourse over the course of the students’ interactions in each lesson type. This analysis indicates differences in the progression of discussions among VM types. In order to compare the discourse progressions of discussions of varying lengths, each discussion was divided into quartiles according to the number of speaking turns. Then, for each quartile, the number of speaking turns coded for each level was calculated. The following sections report the results of this quartile analysis.

**Generalization.** Figures 6, 7, and 8 compare levels of generalization across the three VM types. For linked VMs, the highest level of generalization occurred steadily throughout the course of the discussions (see Figure 6). However, it occurred most frequently in the last quartile of the discussions. The second level of generalization—comparison—occurred in similar proportions in the first and second quartiles (14.1% and 14.2%), and then decreased for the third and fourth quartiles (1.71% and 5.26%). For pictorial VMs, the two highest levels of generalization occurred most during the
last quartile of the discussion (see Figure 7). For tutorial VMs, discussion remained at the most basic level, concrete throughout the discussion (see Figure 8). More statements were coded for the second level, comparison, in the first two quartiles of the discussions than for the last two quartiles of the discussions. Speaking turns coded at the highest level accounted for less than 1% of the first and fourth quartiles of discussions with tutorial VMs.

Figure 6. Quartile analysis of generalization for linked VMs.

Figure 7. Quartile analysis of generalization for pictorial VMs.
Figure 8. Quartile analysis of generalization for tutorial VMs.

**Figure 9.** Quartile analysis of justification for linked VMs.
Collaboration. Figures 12, 13, and 14 compare levels of collaboration across the three VM types. For linked VMs, the overall proportion of levels of collaboration remained steady throughout the course of the discussions (see Figure 12). For pictorial VMs, the percentage of speaking turns coded for build gradually increased until the third quartile (18.65%), and then decreased slightly (see Figure 13). The percentage of speaking turns coded for response remained the same throughout the discussions (approximately 25%). For tutorial VMs, the percentages of speaking turns coded for response and build gradually decreased (27.97% to 21.34% and 13.29% to 9.76%, respectively), while the percentage of speaking turns coded for unrelated ideas rose from 58.74% to 68.9% by the fourth quartile (see Figure 14).
In summary, the quartile analysis confirms that students’ discussions when using the tutorial VM type consistently reflected lower levels of
generalization, justification, and collaboration. In addition, the analysis of discourse from the beginning to the end of each episode shows that when working with the linked and tutorial VM types, the level of generalization in students’ discussions remained constant. However, when working with the pictorial VM type, the level of generalization increased toward the end of students’ discussions. Levels of justification in students’ discussions remained relatively constant when working with the pictorial and tutorial VM types. However, when working with the linked VM types, levels of justification in students’ discussions increased after the first quarter of the episode. Levels of collaboration remained somewhat constant throughout each episode for all VM types.

DISCUSSION

Linked Virtual Manipulatives

When working with the linked VM type, students’ discussions reflected statistically significant higher levels of generalization, justification, and collaboration than when working with the other VM types. This indicated that affordances of the linked VM type enhanced students’ mathematical discourse for these student pairs. The simultaneous linking of representations supports students in connecting ideas and generalizing concepts (Moyer-Packenham & Westenskow, 2013). For example, when students make changes to pictorial fraction images and simultaneously observe how those changes affect the fractions’ number line and numeric representations (see Figure 15), they make comparisons and see patterns more readily.

![Figure 15. Multiple representations of fraction concept.](image)

With respect to developing theory, the findings of this study indicated that the multiple representations displayed by the linked VMs enabled the students to effectively generalize and justify mathematics concepts during problem solving tasks. This is especially evident as levels of justification in
students’ discussions increased from the beginning to the end of each episode when using linked VMs. This pattern is similar to findings of Ares, Stroup, and Schademan (2008) who noted that collective representations encouraged students to interact with each other and comment on each other’s solutions.

**Pictorial Virtual Manipulatives**

When working with the pictorial VM type, students’ discussions reflected slightly greater quantities of discourse than when working with the other VM types, as measured by the number of speaking turns in each episode. This indicates that there was a greater need for the students to communicate the meaning of the representations with each other. The meaning of the representation was not as explicit as with the linked VMs. Therefore, the students had to assume responsibility for making connections for themselves. However, despite the increased quantity of discourse, the quality of collaboration reflected mostly middle to lower levels of discourse. Typically, the quality of students’ generalization and justification were not as high with the pictorial VM type as with the linked VM type.

The quartile analysis revealed that students’ discussions focused on describing their actions with the VMs, and that they rarely commented on why they had chosen a certain method or why that method led to a correct answer. However, levels of generalization in students’ discussions tended to increase toward the end of the episodes. This indicates that a certain amount of time may be required for students to start to make high-level generalizations when working with pictorial VMs.

**Tutorial Virtual Manipulatives**

When working with the tutorial VM type, students’ discussions reflected the lowest levels of discourse as compared with the other VM types. Despite the positive affordances of tutorial VMs when students work alone (Moyer-Packenham & Suh, 2012; Reimer & Moyer, 2005), this study indicated that tutorial VMs discouraged mathematical discourse for these student pairs. One possible theory is that the students assumed a more passive learning role because the tutorial guided them through the mathematical processes. Due to the structured nature of the tutorials, students did not feel the need to generalize, justify, or collaborate as much. Instead, their focus
was on responding to the tutorials’ direct feedback. Interaction with their partner was secondary to their interaction with the VM. Another possible theory is that the tasks presented by the tutorial VMs were not as cognitively demanding as the tasks presented with the other VM types (Smith & Stein, 1998). This means that, theoretically, the type of mathematical task posed and level of feedback given by each VM may be more influential than the affordances it offers when students work in pairs.

**Suggestions for Future Research on TMD**

This was an exploratory study, designed to identify the characteristics of TMD. In order to progress the development of the TMD Framework, more investigations are needed to determine its generalizability and robustness. Similar studies with more diverse populations and larger sample sizes could help to determine if the results of this study were unique to these students or if interactions with these VM types are common in the larger population.

This study represents one possible variation of examining TMD—an in-depth analysis of students’ classroom discourse as influenced by different VM types. Other variations need further investigation. For example, future research could focus on other factors related to technology tools, such as students’ familiarity with the technology tools, students’ perceptions of the technology, differences in platform (e.g., mouse-controlled versus touchscreen devices), or a comparison of students’ discourse with and without technology. This study did not focus on factors of the broader classroom environment related to classroom discourse. Future research could be conducted on these factors, such as the role of the teacher or varying levels of student achievement. This study examined students’ discussions during division, geometry, and fractions units. Future research could examine the influence of variations in mathematical tasks, such as procedural versus conceptual tasks, specific mathematical domains (e.g., fractions, integers, or place value), or lesson formats (e.g., inquiry- versus direct-instruction). Investigation of these factors was beyond the scope of this study. However, their examination could aid in refining the TMD Framework and deepening understanding of how students interact with each other when engaging in mathematical tasks through the use of technology.
CONCLUSION

This study represents an intersection of two aspects of instruction: the nature of mathematical discourse and the use of VMs in the classroom. The purpose of the present study was to describe, categorize, and interpret students’ discussions as they worked with different VM types. This study was built on the premise (a) that mathematics learning occurs when students communicate ideas and discuss mathematics concepts one with another, (b) that VMs offer unique affordances that support students’ learning of mathematics, and (c) that meaningful discourse takes place when students engage in cognitively demanding tasks.

The results of this study indicated statistically significant differences in levels of student discourse when using different VM types. One explanation of these variations is that unique affordances of each VM type had a direct influence on how student pairs discussed and communicated their mathematical ideas with each other. Most notably, the simultaneous linking of representations present in the linked VMs seemed to support students’ ability to generalize concepts and justify solutions when communicating with each other. However, even though the tutorial VM type linked representations simultaneously, its structured manner of presenting learning activities actually discouraged student to student discourse.

Findings from this study suggest that linked VMs are effective tools for engaging students in meaningful TMD. Linked representations in these VMs provide an opportunity for students to generalize, justify, and collaborate while learning mathematical concepts. This study also indicates that the tutorial VMs did not encourage meaningful student to student mathematical discourse for these student pairs. However, tutorial technology tools have been shown to be effective learning instruments. Therefore, tutorial VMs may be better suited for the practice of mathematics concepts or for individual learning rather than group or partner learning.

The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom. While this exploratory study aimed to develop the construct of TMD by examining the interactions among partner discourse, VMs, and mathematical tasks, further studies with broader, more diverse populations will contribute to its generalizability.
The Influence of Different Virtual Manipulative Types

References


APPENDIX A

Example of Lesson Plan Used for Equivalent Fractions Instruction

<table>
<thead>
<tr>
<th>Subject/Strand/Topic</th>
<th>Grade</th>
<th>Common Core Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractions</td>
<td>5th</td>
<td>5.NF.1</td>
</tr>
</tbody>
</table>

**Key Concepts**
The numerators and denominators of equivalent fractions are proportional to each other. Multiplying the numerator and the denominator of any fraction by the same number will result in an equivalent fraction. Equivalent fractions occupy the same location on a number line.

**Virtual Manipulative**
Equivalent Fractions (linked) [http://illuminations.nctm.org/Activity.aspx?id=3510](http://illuminations.nctm.org/Activity.aspx?id=3510)

**Materials**
Student Task Sheet, Pencils, 1 computer per 2 students

**Introduction (10 min)**
Introduce topic
Activate prior knowledge by having students identify fractions from picture models (square and circle)
Define **numerator** (top: how many pieces you have) and **denominator** (bottom: how many equal-sized pieces are in the whole).

**How do you know if two fractions are equivalent?** Invite a few student responses (full understanding not required at this time).
Introduce virtual manipulative.
Provide a brief instruction on features/aspects of the virtual manipulative.
**Goal:** create a blue and a green fraction equivalent to the red fraction.
**Square/Circle:** switches between different models. This lesson starts out on the CIRCLE model.
**Sliders:** change the denominator (number of pieces in the whole), change the numerator (number of pieces selected)—can also click on the sections to select them.
**Checkmark:** if fractions are correct, they will be added to the chart to the right. If they are incorrect, students should fix the error.
**Automatic/Build Your Own:**
Briefly go over the structure of the task sheet. Part A orients the students to the features of the virtual manipulative. Part B guides them through an exploration of the mathematical concepts, and contains partner discussion questions.

**Exploration (20-30 min)**
Students will work in pairs—each pair at one computer.
Instruct students to open Safari and type in the link at the top of the Student Task Sheet OR go to the TMD Study website and navigate to the Equivalent Fractions: Illuminations link.
Students should spend about 10 minutes on Part A and 10-20 minutes on Part B.
Circulate around the room encouraging students to talk with their partners about the mathematics. Remind students of time constraints and encourage them to finish.

**Conclusion (20 min)**
Have a whole-class discussion/sharing of answers on task sheet & what they learned. Discuss relationships in sets of equivalent fractions. Suggested questions to guide the discussion.
**How do you know if two fractions are equivalent?**
**How can you make equivalent fractions?**
Check for understanding: Write a fraction on the board. Have students identify new equivalent fractions.
### APPENDIX B

VMs Selected for Instructional Use

<table>
<thead>
<tr>
<th>VM Name</th>
<th>URL</th>
<th>Mathematical Domain</th>
<th>CCSS-M</th>
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<tbody>
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<td>Division</td>
<td>5.NBT.6</td>
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<td>Fractions</td>
<td>5.NF</td>
</tr>
<tr>
<td>Tutorial Dividing Decimals</td>
<td><a href="http://www.glencoe.com/sites/common_assets/mathematics/im1/concepts_in_motion/interactive_labs/M1_05/M1_05_dev_100.html">http://www.glencoe.com/sites/common_assets/mathematics/im1/concepts_in_motion/interactive_labs/M1_05/M1_05_dev_100.html</a></td>
<td>Division</td>
<td>5.NBT.7</td>
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<tr>
<td>Fractions–Adding</td>
<td><a href="http://nlvm.usu.edu/en/nav/frames_asid_106_g_2_t_1.html?from=topic_t_1.html">http://nlvm.usu.edu/en/nav/frames_asid_106_g_2_t_1.html?from=topic_t_1.html</a></td>
<td>Fractions</td>
<td>5.NF.1,2</td>
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