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# Using Virtual Manipulatives to Generalize and Justify through Discourse

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**Abstract**

This study examined the influence of different virtual manipulative types on the nature of students' discourse related to generalizing and justifying mathematical concepts. During 27 episodes, students worked on mathematics tasks using three different virtual manipulative types: linked, pictorial, and tutorial. The level of students' discourse in generalization and justification was coded and analyzed for each episode and compared across virtual manipulative types. A one-way ANOVA indicated statistically significant differences in the quality of generalizations and justifications among the different virtual manipulative types. Other patterns indicate that certain virtual manipulative types may be more suited than others for encouraging meaningful mathematical discourse. The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom.

## Purpose

The purpose of this research study was to describe and categorize the nature of students' mathematical discourse as they worked with various virtual manipulative types. As the use of technology in mathematics instruction becomes ubiquitous, questions arise regarding the role of different virtual manipulative types in students' learning experiences—particularly in the ways that students interact with each other and discuss mathematical ideas (Gray, Thomas, & Lewis, 2010; NCTM, 2007, 2014). The larger study from which this paper is taken employed a mixed methods case study design utilizing both qualitative and quantitative methods to analyze students' mathematical discussions. The quantitative results provide the focus of this paper. Full qualitative results are described in other publications (see Anderson-Pence, 2014).

## Theoretical Framework

### Virtual Manipulatives

With the advancement of computer capabilities, virtual manipulatives have emerged as cognitive technology tools for use in mathematics classrooms. A virtual manipulative is defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer-Bolyard, & Spikell, 2002, p. 373). Virtual manipulatives provide teachers and students with expanded tools for thinking about mathematics concepts, and have been found to have a moderate effect size (0.35) when compared to other instructional methods (Moyer-Packenham & Westenskow, 2013). Overall, research indicates that virtual manipulatives positively contribute to students' learning of mathematics concepts (e.g., Bolyard & Moyer-Packenham, 2012; Mendiburo & Hasselbring, 2011; Moyer-Packenham et al., 2014; Moyer-Packenham et al., 2013; Moyer-Packenham et al., 2015; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2008; Suh, Moyer, & Heo, 2005).

Virtual manipulative tools vary in the type of feedback they provide and the type of mathematical representation included (Bolyard & Moyer, 2007). Some tools offer manipulatives that truly reflect the user's actions and choices without dictating solution paths. These open-ended tools provide indirect feedback and may present *linked* representations (e.g., pictorial image, number line model, and numeric symbols presented dynamically together) or simply provide *pictorial* representations for manipulation, such as pattern blocks or base-10 blocks (Clark & Paivio, 1991; Paivio, 2007; Sfard, 1991). Other virtual manipulative tools use direct feedback in structured concept *tutorials* to guide students through a pre-determined pathway to a conceptual or procedural understanding of the mathematics.

**Mathematical Discourse**

Students develop understanding as they interact with others through verbal or nonverbal communications or written word (Vygotsky, 1978). Meaningful classroom discourse contributes to students' understanding by promoting effective communication and articulation of thought (Piccolo et al., 2008). Multiple studies have examined the process of mathematical explanation and reasoning (e.g., Carpenter, Fennema, & Franke, 1996; Hufferd-Ackles, Fuson, & Sherin, 2004). Notably, the framework for Robust Mathematical Discussion describes components of effective mathematical classroom discourse (Mendez, Sherin, & Louis, 2007). Robust Mathematical Discussion categorizes students' comments along two dimensions: mathematics and discussion. The mathematics dimension addresses three aspects of mathematical argumentation: representation, *generalization*, and *justification*. The discussion dimension examines three aspects of discourse: engagement, intensity, and building on others' ideas.

Discourse is most effective in promoting understanding when students' discourse is ranked high in each of the Robust Mathematical Discussion dimensions.

To date, extensive research has been conducted on the nature of classroom mathematical discourse (e.g., Gee, 2005; Herbel-Eisenmann & Wagner, 2010; Iiskala, Vauras, Lehtinen, & Salonen, 2011; Imm & Stylianou, 2012; Nathan & Knuth, 2003; Wood & Kalinec, 2012). However, few studies exist on the interactions students have with each other when using technology to learn mathematics (e.g., Ares, Stroup, & Schademan, 2008; Evans, Feenstra, Ryon, & McNeill, 2011; Sinclair, 2005).

## Methods

This study aimed to answer the following research question: How do different virtual manipulative types influence the levels of generalization and justification in students' mathematical discourse?

### Participants

The study included 3 pairs of fifth-grade students ages 10–11 years (each pair consisting of one female and one male student). Classroom teachers assisted the researcher in selecting the students based on ability to verbally process thinking. Mathematics achievement was not a deciding factor when selecting students for this study.

### Procedures & Data Collection

Each pair of participating students shared a laptop computer while they interacted with nine different virtual manipulatives: 3 linked, 3 pictorial, and 3 tutorial. Over four months, the 3 students pairs participated in 9 lessons using the virtual manipulatives—a total of 27 episodes.

Data collection took place during 20–30-minute episodes as students worked together through assigned tasks. Two different video perspectives were recorded as data for further analysis. First, a face-capture perspective recorded the students' mathematical discussions using the built-in camera located at the top and center of the computer screen. Second, a screen-capture perspective recorded what the students did with the virtual manipulatives. This screen-capture included a record of mouse movement, mouse clicks, and external audio.

**Data Analysis**

The first stage of analysis focused on quantitizing the video data (Tashakkori & Teddlie, 2010). Speaking turns in each of the 27 episodes were transcribed and coded for levels of discourse according to the generalization and justification dimensions of the Robust Mathematical Discussion Framework (see Table 1). The number of codable speaking turns was tabulated to provide a measure of the quantity of discourse in each episode. Next, leveled codes were used to calculate composite scores—a measure of the quality of generalization and justification in each episode. Composite scores were calculated by a summation of the codes for each speaking turn within the episode divided by the total number of codable speaking turns, and multiplied by 100. For example, a discussion with 100 coded speaking turns coded for justification—60 as statement (level 1), 30 as explanation (level 2), and 10 as proof (level 3)—would yield a justification composite score of $\left(\frac{(60 \times 1) + (30 \times 2) + (10 \times 3)}{100}\right) 100 = 150$.

One-way ANOVAs on the composite scores and on the amount of coded speaking turns per episode were conducted to compare the quality and quantity, respectively, of students' discourse when using each virtual manipulative type (i.e., linked, pictorial, and tutorial).

Table 1

| Level of Generalization* | | Level of Justification* | |
|---|---|---|---|
| 0 | Not Codable | 0 | Not Codable |
| 1 | Concrete | 1 | Statement |
| 2 | Comparison | 2 | Explanation |
| 3 | Generalization | 3 | Proof |

*Adapted from the Robust Mathematics Discussion Framework (Mendez et al., 2007)

In the final quantitative analysis, the data were examined for levels of discourse over the course of the students' interactions. This analysis indicated differences in the progression of discussions among virtual manipulative types. In order to compare the discourse progressions of discussions of varying lengths, each discussion was divided into quartiles according to the number of speaking turns. Then, for each quartile, the number of speaking turns coded for each level of generalization and justification was calculated.

## Results

A one-way ANOVA indicated no statistically significant differences among virtual manipulative types in the quantity of discourse. Discussions associated with pictorial virtual manipulatives averaged the highest number of speaking turns, and the tutorial virtual manipulatives averaged the lowest number of speaking turns.

### Generalization

Overall, students engaged in higher levels of generalization when working with linked virtual manipulatives than with pictorial or tutorial virtual manipulatives. Linked virtual manipulatives had the highest average composite score ($M$ = 128.52, $SD$ = 15.56), followed by pictorial ($M$ = 115.26, $SD$ = 5.80) and tutorial ($M$ = 107.39, $SD$ = 13.37). The one-way ANOVA comparison of generalization composite scores indicated a statistically significant overall difference among the virtual manipulative types at the 95% level, $F\ (2, 24) = 9.460$, $p = 0.001$. This corresponded to an effect size of $\eta^2 = .44$. Individual post hoc comparisons using Tukey's HSD indicated a statistically significant difference between the linked and pictorial virtual manipulative types, $p = 0.033$, and between the linked and tutorial virtual manipulative types, $p = .001$. There was not a statistically significant difference between the pictorial and tutorial virtual manipulative types.

Figures 1, 2, and 3 compare levels of generalization across the three virtual manipulative types over the course of each discussion. For linked virtual manipulatives, the highest level of generalization occurred steadily throughout the discussions (see Figure 1). However, it occurred most frequently in the last quartile of the discussions. The second level of generalization—comparison—occurred in similar proportions in the first and second quartiles (14.10% and 14.20%), and then decreased in the third and fourth quartiles (1.71% and 5.26%). For pictorial virtual manipulatives, the two highest levels of generalization occurred most during the last quartile of the discussion (see Figure 2). For tutorial virtual manipulatives, discussion remained at the most basic level—concrete—throughout the discussion (see Figure 3). More statements were coded for the second level—comparison—in the first two quartiles of the discussions than in the last two quartiles of the discussions. Speaking turns coded at the highest level accounted for less than 1% of the first and fourth quartiles of discussions with tutorial virtual manipulatives.

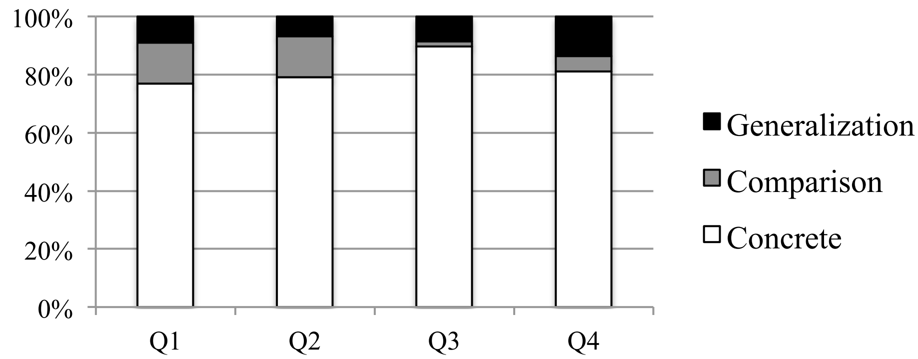


*Figure 1.* Quartile analysis of generalization for linked virtual manipulatives.

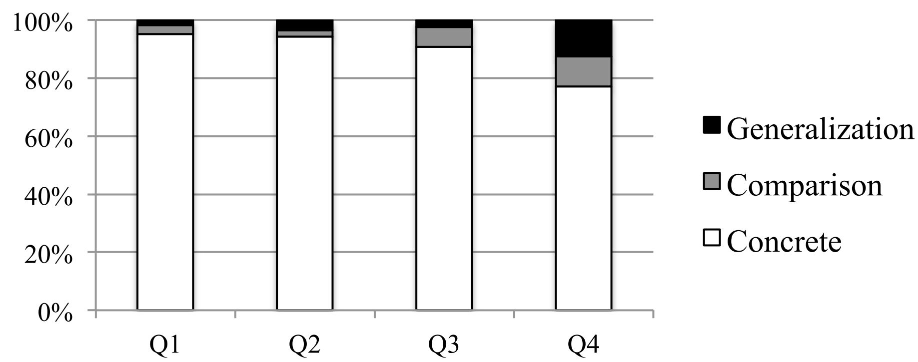


*Figure 2.* Quartile analysis of generalization for pictorial virtual manipulatives.

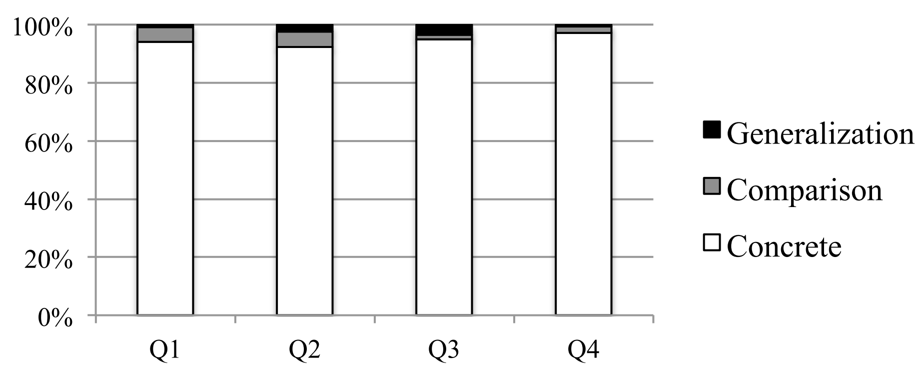


*Figure 3.* Quartile analysis of generalization for tutorial virtual manipulatives.

**Justification**

Overall, students engaged in higher levels of justification when working with linked virtual manipulatives than with pictorial or tutorial virtual manipulatives. Linked virtual manipulatives had the highest average composite score ($M$ = 135.00, $SD$ = 14.78), followed by pictorial ($M$ = 122.20, $SD$ = 6.15) and tutorial ($M$ = 113.15, $SD$ = 9.35). The one-way ANOVA comparison of justification composite scores indicated a statistically significant overall difference among the virtual manipulative types at the 95% level, $F$ (2, 24) = 9.459, $p$ = 0.001. This corresponded to an effect size of $\eta^2$ = .44. Individual post hoc comparisons using Tukey's HSD indicated a statistically significant difference between the linked and pictorial virtual manipulative types, $p$ = 0.046, and between the linked and tutorial virtual manipulative types, $p$ = .001. There was not a statistically significant difference between the pictorial and tutorial virtual manipulative types.

Figures 4, 5, and 6 compare levels of justification across the three virtual manipulative types over the course of each discussion. For linked virtual manipulatives, levels of justification increased as the discussions progressed (see Figure 4). The percentage of speaking turns coded for explanation and for proof increased considerably after the first quartile (4.55% to 20.13% and 0.65% to 5.03%, respectively). For pictorial virtual manipulatives, the levels of justification also increased as the discussions progressed (see Figure 5), but not to the same extent as the linked virtual manipulatives. The percentage of speaking turns coded for explanation and for proof increased after the first quartile (9.76% to 15.93% and 1.83% to 3.85%, respectively). For tutorial virtual manipulatives, the most frequent occurrence of proof happened in the first quartile of the discussions (2.52%). However, for the rest of the discussion, proof accounted for less than 1% of the speaking turns (see Figure 6). The most frequent occurrence of explanation also happened in the first quartile of the discussions (22.69%). Thereafter, the percentage of explanations dwindled to 10% or less for the remaining portion of the discussions.

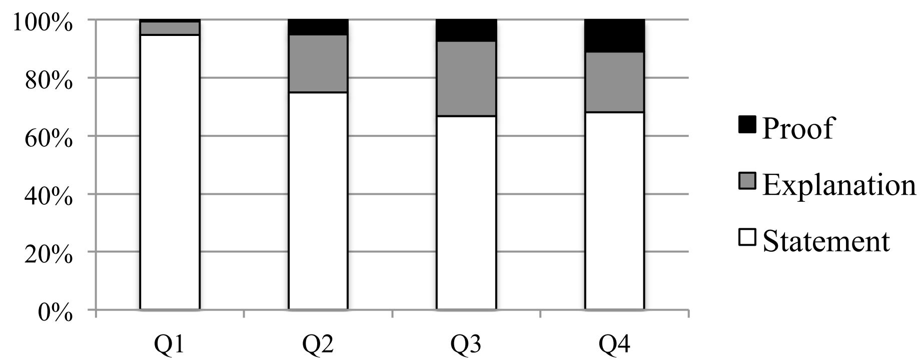


*Figure 4.* Quartile analysis of justification for linked virtual manipulatives.

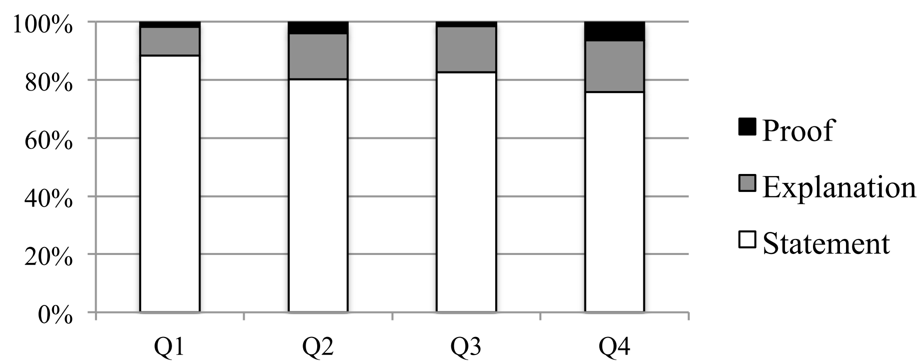


*Figure 5.* Quartile analysis of justification for pictorial virtual manipulatives.

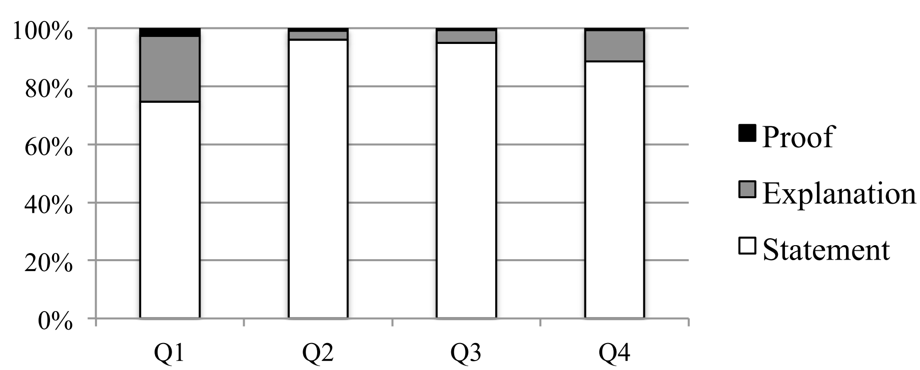


*Figure 6.* Quartile analysis of justification for tutorial virtual manipulatives.

## Educational Importance

Findings from this study suggest ways that teachers may effectively incorporate virtual manipulative types into mathematics instruction to match students' learning paths. First, *pictorial* and *linked* virtual manipulatives may be more useful as students are developing their understanding of mathematics concepts. The flexibility of these virtual manipulative types lends itself to an open exploration of mathematical ideas—guided either by the students themselves or by the teacher. Further the linked virtual manipulatives assist students in making connections between mathematics concepts and representations. In this study, students' discussions when using this linked virtual manipulatives typically reflected higher levels of generalization and justification. Through such robust discussion, students are more likely to learn mathematics in a meaningful way.

This study also suggests that the use of *tutorial* virtual manipulatives may not be an effective instructional strategy for engaging students in mathematical discourse. Tutorial virtual manipulatives are designed to walk an individual student through a concept at his or her own pace using focused feedback on performance. In this study, although the structured feedback included in the tutorial virtual manipulative type effectively guided the students to a mathematical understanding, it did not encourage meaningful discussion between students. Students' interaction with their partners was secondary to responding to the tutorials' direct feedback. Due to the extremely structured nature of the tutorials, students did not feel the need to generalize or justify their answers with each other. Therefore, students' discussions when using tutorial virtual manipulatives typically reflected lower levels of generalization and justification.

## Scholarly Significance

The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom environment. More and more classrooms are using technology, and students are learning mathematics as they interact with the technology and with each other. However, we know very little about the interactions students have with each other when also interacting with technology to complete mathematical tasks. This study represents an intersection of the two research fields of virtual manipulatives and classroom discourse and adds to the research literature on the impact of technology on classroom mathematical discourse.

**References**

Anderson-Pence, K. L. (2014). *Examining the impact of different virtual manipulative types on the nature of students' small-group discussions: An exploratory mixed-methods case study of techno-mathematical discourse* (Doctoral dissertation). ProQuest Dissertations and Theses database. (UMI No. 3683422).

Ares, N., Stroup, W. M., & Schademan, A. R. (2008). The power of mediating artifacts in group-level development of mathematical discourses. *Cognition and Instruction*, *27*(1), 1–24. doi:10.1080/07370000802584497

Bolyard, J. J., & Moyer, P. S. (2007). *Selecting Dynamic Technology Representations for Mathematics Teaching*. Research Presentation, 85th Annual Meeting of the National Council of Teachers of Mathematics (NCTM), Atlanta, GA.

Bolyard, J. J., & Moyer-Packenham, P. S. (2012). Making sense of integer arithmetic: The effect of using virtual manipulatives on students' representational fluency. *Journal of Computers in Mathematics & Science Teaching*, *31*(2), 93–113.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, *97*(1), 3–20.

Clark, J. M., & Paivio, A. (1991). Dual coding theory and education. *Educational Psychology Review*, *3*(3), 149–210. doi:10.1007/BF01320076

Evans, M. A., Feenstra, E., Ryon, E., & McNeill, D. (2011). A multimodal approach to coding discourse: Collaboration, distributed cognition, and geometric reasoning. *International Journal of Computer-Supported Collaborative Learning*, *6*(2), 253–278. doi:10.1007/s11412-011-9113-0

Gee, J. P. (2005). *An introduction to discourse analysis: Theory and method* (2nd ed.). London: Routledge.

Gray, L., Thomas, N., & Lewis, L. (2010). Teachers' Use of Educational Technology in US Public Schools: 2009. First Look. NCES 2010-040. *National Center for Education Statistics*.

Herbel-Eisenmann, B., & Wagner, D. (2010). Appraising lexical bundles in mathematics classroom discourse: Obligation and choice. *Educational Studies in Mathematics*, *75*(1), 43–63.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81–116. doi:10.2307/30034933

Iiskala, T., Vauras, M., Lehtinen, E., & Salonen, P. (2011). Socially shared metacognition of dyads of pupils in collaborative mathematical problem-solving processes. *Learning and Instruction*, *21*(3), 379–393. doi:10.1016/j.learninstruc.2010.05.002

Imm, K., & Stylianou, D. A. (2012). Talking mathematically: An analysis of discourse communities. *The Journal of Mathematical Behavior*, *31*(1), 130–148. doi:10.1016/j.jmathb.2011.10.001

Mendez, E. P., Sherin, M. G., & Louis, D. A. (2007). Multiple perspectives on the development of an eighth‑grade mathematical discourse community. *The Elementary School Journal*, *108*(1), 41–61.

Mendiburo, M., & Hasselbring, T. (2011). *Technology's impact on fraction learning: an experimental comparison of virtual and physical manipulatives*. Society for Research on Educational Effectiveness.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics*, *8*(6), 372–377.

Moyer-Packenham, P. S., Baker, J., Westenskow, A., Anderson-Pence, K. L., Shumway, J. F., & Jordan, K. (2014). Predictors of achievement when virtual manipulatives are used for mathematics instruction. *REDIMAT, 3*(2), 121–150. doi: 10.4471/redimat.2014.46

Moyer-Packenham, P., Baker, J., Westenskow, A., Anderson, K., Shumway, J., Rodzon, K., & Jordan, K. (2013). A study comparing virtual manipulatives with other instructional treatments in third- and fourth-grade classrooms. *Journal of Education*, *193*(2), 25-39.

Moyer-Packenham, P. S., Shumway, J. F., Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Westenskow, A., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K. (2015). Young children's learning performance and efficiency when using virtual manipulative mathematics iPad apps. *Journal of Computers in Mathematics and Science Teaching, 34*(1), 41-69.

Moyer-Packenham, P. S., & Westenskow, A. (2013). Effects of virtual manipulatives on student achievement and mathematics learning. *International Journal of Virtual and Personal Learning Environments, 4*(3), 35–50.

Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction*, *21*(2), 175–207.

National Council of Teachers of Mathematics. (2007). *Mathematics teaching today*. (T. S. Martin & W. R. Spear, Eds.). Reston, VA: Author.

National Council of Teachers of Mathematics, (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Paivio, A. (2007). *Mind and its evolution: A dual coding theoretical approach*. Mahwah, NJ: Erlbaum.

Piccolo, D. L., Harbaugh, A. P., Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics*, *19*(3), 376–410.

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *The Journal of Computers in Mathematics and Science Teaching*, *24*(1), 5–25.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*, 1–36. doi:10.1007/BF00302715

Sinclair, M. P. (2005). Peer interactions in a computer lab: Reflections on results of a case study involving web-based dynamic geometry sketches. *The Journal of Mathematical Behavior*, *24*(1), 89–107. doi:10.1016/j.jmathb.2004.12.003

Suh, J. M., & Moyer-Packenham, P. S. (2008). Scaffolding special needs students' learning of fraction equivalence using virtual manipulatives. *Proceedings of the International Group for the Psychology of Mathematics Education* (pp. 297–304). PME.

Suh, J. M., Moyer, P. S., & Heo, H.-J. (2005). Examining technology uses in the classroom: Students Developing fraction sense by using virtual manipulative concept tutorials. *Journal of Interactive Online Learning*, *3*(4), 1–21.

Tashakkori, A., & Teddlie, C. (Eds.). (2010). *Sage handbook of mixed methods in social & behavioral research*. (2nd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Cambridge, MA: Harvard University Press.

Wood, M. B., & Kalinec, C. A. (2012). Student talk and opportunities for mathematical learning in small group interactions. *International Journal of Educational Research*, *51–52*(0), 109–127. doi:10.1016/j.ijer.2011.12.008