Using Virtual Manipulatives to Enhance Collaborative Discourse in Mathematics Instruction

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Abstract

This study examined the influence of different virtual manipulative (VM) types on the nature of students’ collaborative mathematical discourse. During 27 episodes, students’ worked on mathematics tasks using three different VM types: linked, pictorial, and tutorial. The level of students’ collaborative discourse was coded and analyzed for each episode and compared across VM types. A one-way ANOVA indicated statistically significant differences in the quality of collaborative discourse among the different VM types. Other patterns indicate that certain VM types may be more suited than others for encouraging meaningful collaborative discourse. The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom.

Purpose

The purpose of this research study was to (a) describe and categorize the nature of students’ mathematical discourse as they worked with various virtual manipulative (VM) types and (b) to develop theory on the relationships between student-led discourse and VMs. As the use of technology in mathematics instruction becomes ubiquitous, questions arise regarding the role of different VM types in classroom environments and students’ learning experiences—particularly in how students interact with each other and discuss mathematical ideas (National Council of Teachers of Mathematics, 2007, 2014). The larger study from which this paper is taken employed a mixed methods case study design to analyze students’ mathematical discussions. A portion of the data from the larger study provides the focus for this paper. Full results are described in other papers (e.g., Anderson-Pence, 2014).

Theoretical Framework

Virtual Manipulatives

With the advancement of computer capabilities, VMs have emerged as cognitive technology tools for use in mathematics classrooms. A VM is defined as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard, & Spikell, 2002, p. 373). VMs provide teachers and students with expanded tools for thinking about mathematics, and VMs have been found to have a moderate effect size (0.35) when compared to other instructional methods (Moyer-Packenham &
Overall, research indicates that VMs positively contribute to students’ understanding of mathematics concepts (e.g., Bolyard & Moyer-Packenham, 2012; Mendiburo & Hasselbring, 2011; Moyer-Packenham et al., 2014; Moyer-Packenham et al., 2013; Reimer & Moyer, 2005; Suh & Moyer-Packenham, 2008).

VMs vary in the type of feedback they provide and in the type of mathematical representation included (Bolyard & Moyer, 2007). Some tools offer manipulatives that truly reflect the user’s actions and choices without dictating solution paths. These open-ended tools provide indirect feedback and may present *linked* representations (e.g., pictorial image, number line model, and numeric symbols presented dynamically together) or simply provide *pictorial* representations for manipulation. Simultaneously linked representations are particularly helpful in assisting students to make connections among mathematical concepts (Clark & Paivio, 1991; Paivio, 2007). Other VMs use direct feedback in structured *tutorials* to guide students to a conceptual or procedural understanding of mathematics.

**Mathematical Discourse**

Students develop understanding as they interact with others through verbal or nonverbal communications or written word (Vygotsky, 1978). Meaningful classroom discourse contributes to students’ understanding by promoting effective communication and articulation of thought (Piccolo et al., 2008). One key aspect of small-group interaction is “co-construction,” defined by Mueller as a “form of collaboration in which an argument is built simultaneously by the learners from conception” (2009, p. 141). This collaboration is characterized by the back and forth nature of its discourse. Students’ solutions become stronger as they integrate the ideas of others.

Multiple studies have examined the process of mathematical explanation and reasoning (e.g., Carpenter, Fennema, & Franke, 1996; Hufferd-Ackles, Fuson, & Sherin, 2004). Notably, the framework for Robust Mathematical Discussion (RMD) describes components of effective mathematical classroom discourse (Mendez, Sherin, & Louis, 2007). RMD categorizes classroom discourse along two dimensions: mathematics and discussion. A main part of the discussion dimension involves how students respond to and build on others’ ideas. Discourse is most effective in promoting mathematical understanding when students’ discourse is ranked high in both dimensions of RMD.

Extensive research has been conducted on the nature of classroom mathematical discourse (e.g., Gee, 2005; Herbel-Eisenmann & Wagner, 2010; Iiskala, Vauras, Lehtinen, & Salonen, 2011; Imm & Stylianou, 2012; Nathan & Knuth, 2003; Wood & Kalinec, 2012). However, few studies exist on the interactions students have with each other when using technology to learn mathematics (e.g., Ares, Stroup, & Schademan, 2008; Evans, Feenstra, Ryon, & McNeill, 2011; Sinclair, 2005).

**Methods**

This study employed a mixed method case study design to answer the following research question: How do different VM types influence the level of collaboration in students’ mathematical discourse? The mixed method case study utilized a concurrent data collection design using identical samples (Onwuegbuzie & Collins, 2007).
Participants
The study included 3 pairs of fifth-grade students ages 10–11 years (each pair consisting of one female and one male student). Classroom teachers assisted the researcher in selecting the students based on ability to verbally process thinking. Mathematics achievement was not a deciding factor when selecting students for this study.

Procedures & Data Collection
Each pair of participating students shared a laptop computer while they interacted with nine different VMs: 3 linked, 3 pictorial, and 3 tutorial. Over four months, the 3 students pairs participated in 9 lessons using the VMs—a total of 27 episodes.

Data collection took place during 20–30-minute episodes as students worked together through assigned tasks. Two different video perspectives were recorded as data for further analysis. First, a face-capture perspective, recorded the students’ mathematical discussions using the built-in camera located at the top and center of the computer screen. Second, a screen-capture perspective, recorded what the students did with the VMs. This screen-capture included a record of mouse movement, mouse clicks, and external audio.

Data Analysis
The first stage of analysis focused on quantitizing the video data (Tashakkori & Teddlie, 2010). Speaking turns in each of the 27 episodes were transcribed and coded for levels of collaborative discourse (0: not codable; 1: unrelated idea; 2: response; 3: build) adapted from the Robust Mathematics Discussion Framework (Mendez et al., 2007). The number of codable speaking turns was tabulated to provide a measure of the quantity of discourse in each episode. Next, leveled codes were used to calculate composite scores—a measure of the quality of collaboration in each episode. Composite scores were calculated by a summation of the codes for each speaking turn within the episode divided by the total number of codable speaking turns, and multiplied by 100. For example, a discussion with 100 coded speaking turns—60 as unrelated idea (level 1), 30 as response (level 2), and 10 as build (level 3)—would yield a composite score of \( \left( \frac{60 \times 1 + 30 \times 2 + 10 \times 3}{100} \right) = 150 \). One-way ANOVAs on the composite scores and on the amount of coded speaking turns per episode were conducted to compare the quality and quantity, respectively, of students’ discourse when using each VM type (i.e., linked, pictorial, and tutorial).

The second stage of analysis focused on examining the video and transcribed data to identify patterns and categories in students’ discourse using open and axial coding (Stake, 1995; Strauss & Corbin, 1998). A side-by-side comparison (Creswell & Plano Clark, 2011) of students’ discourse with different VM types was displayed numerically and graphically. Significant events were identified as illustrative of each category. This analysis provided an additional perspective to the prior quantitative analysis and contributed to the formulation of a developing theory.

Data Sources & Evidence
A one-way ANOVA indicated no statistically significant differences among VM types in the quantity of discourse. Discussions associated with pictorial VMs averaged the highest number of speaking turns, and tutorial VMs averaged the lowest number of speaking turns.
Overall, students engaged in higher levels of collaboration when working with linked VMs than with pictorial or tutorial VMs. Linked VMs had the highest average composite score ($M = 175.99$, $SD = 16.57$), followed by pictorial ($M = 153.69$, $SD = 11.62$) and tutorial ($M = 148.40$, $SD = 9.72$). The one-way ANOVA comparison of collaboration composite scores indicated a statistically significant overall difference among the VM types at the 95% level, $F (2, 24) = 11.476$, $p < 0.001$. This corresponded to an effect size of $\eta^2 = .49$. Individual post hoc comparisons indicated a statistically significant difference between the linked and pictorial VM types and between the linked and tutorial VM types. There was not a statistically significant difference between the pictorial and tutorial VM types.

The following excerpts illustrate students’ levels of collaboration when working with each of the VM types.

Example 1 illustrates the high-level collaborative discourse associated with linked VMs (see Table 1). In this example, Colton and Callie were working with a linked VM to determine the volume of a 3x5x7 box, a 5x7x3 box, and a 7x3x5 box (see Figure 1).

Table 1

<table>
<thead>
<tr>
<th>Line</th>
<th>Student</th>
<th>Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>Callie</td>
<td>Ok, so how many unit blocks to fill the box?</td>
</tr>
<tr>
<td>66</td>
<td>Colton</td>
<td>20…</td>
</tr>
<tr>
<td>67</td>
<td>Callie</td>
<td>…So 6, no that’s 5 blocks….</td>
</tr>
<tr>
<td>68</td>
<td>Colton</td>
<td>(counting layers of blocks on the screen) 1, 2, 3, 4, 5, 6, 7. 20 times 7 is 140. So it should be 140 blocks.</td>
</tr>
<tr>
<td>69</td>
<td>Callie</td>
<td>…For the whole thing, yeah. 140 blocks.</td>
</tr>
</tbody>
</table>
Colton: 20 times 7 is 140. I’ll just write 140 blocks…. So change it to 5. What’s the next number?

Callie: Width 5, depth 7, height 3. So 3 times….

Colton: Wait. Hold on. Just a second. Let’s go back to that one. It didn’t have 20 on the bottom. We need to go back.

Callie: Yeah it did.

Colton: 3…5…7. Look, (counting width of layer) 1, 2, 3, 4, 5 times 3. So it’s…

Callie: …15…

Colton: …times 7…. Not 20.

Callie: 15…105.

Colton: Ok. All right. Now we do this….

Callie: So the next one is width 5, depth 7, height 3. So….

Both: (counting and placing blocks on screen) 1, 2, 3, 4, 5, 6, 7…

Callie: …times 5.

Colton: …times (counting) 1, 2, 3, 4, 5. Yep.

Callie: So 35 times…

Colton: …3. I got…

Both: …105.

Callie: Again! …Ok “What is the volume of a box with width 7, depth 3,…”

Colton: It’s just changing the numbers up. So I think it will be 105.

Callie: “…height, 5.” Let’s double check to see if it is 105 blocks. So…

Colton: 1, 2, 3.

Callie: 7 times 3. 21 times…

Colton: …5?

Callie: 5. So…105.

Colton: Yep…. Ok

As shown in this example, Colton and Callie collaborated well and were efficient in their problem solving strategies. Their language (e.g., let’s, we, etc.) reflected a joint effort (lines 72, 78, 88) as they built on each other’s ideas (lines, 83–85). They quickly recognized errors in their work and made the necessary adjustments (line 76).

Example 2 illustrates collaborative discourse associated with pictorial VMs (see Table 2). Aaron and Abbie are working with a pictorial VM to solve a word problem.
Table 2
Excerpt of Students’ Discussion While Working With a Pictorial VM

<table>
<thead>
<tr>
<th>Line</th>
<th>Student</th>
<th>Dialogue</th>
</tr>
</thead>
<tbody>
<tr>
<td>82</td>
<td>Abbie</td>
<td>Ok. (reading) “Nancy’s poster for the school council election covers a space of 6.4 square meters. She wants to divide the poster into 4 equal sections for her slogan. How space will be in each section? Hint: one 10 by 10 square represents 1 square meter”</td>
</tr>
<tr>
<td>83</td>
<td>Aaron</td>
<td>Ok. Let’s use these base ten block things.</td>
</tr>
<tr>
<td>84</td>
<td>Abbie</td>
<td>Well, now 6…</td>
</tr>
<tr>
<td>85</td>
<td>Aaron</td>
<td>So 6 point 4.</td>
</tr>
<tr>
<td>86</td>
<td>Abbie</td>
<td>So 6 of those. (Aaron places four 10x10 blocks on the screen)</td>
</tr>
<tr>
<td>87</td>
<td>Aaron</td>
<td>And then you can do the 4 tenths if you want. Wait! We have to make this look cool.</td>
</tr>
<tr>
<td>88</td>
<td>Abbie</td>
<td>No you don’t.</td>
</tr>
<tr>
<td>89</td>
<td>Aaron</td>
<td>I want to. (arranges blocks in a pattern) Then we have to connect them. Then 4 little teeny… Oh wait, no!</td>
</tr>
<tr>
<td>90</td>
<td>Abbie</td>
<td>It’s 6. (places two more 10x10 grids on the screen)</td>
</tr>
<tr>
<td>91</td>
<td>Aaron</td>
<td>And then put 2… put 1 in between each one and put 2 in there…. Fine you don’t have to make it look cool.</td>
</tr>
<tr>
<td>92</td>
<td>Abbie</td>
<td>No…. So that is 6 point 4?</td>
</tr>
<tr>
<td>93</td>
<td>Aaron</td>
<td>Yep, that is what it wanted us to do.</td>
</tr>
<tr>
<td>94</td>
<td>Abbie</td>
<td>(reading) “She wants to divide the poster into 4 equal sections for slogans.” (Aaron starts to move 10x10 grids on top of one another) Just wait a minute…. I don’t get what you’re doing.</td>
</tr>
</tbody>
</table>
| 95   | Aaron   | (arranges blocks) That’s not gonna work. Let’s break up these 2….
Although the students received instruction on the functionality of the VM (e.g., dragging blocks together to connect them, clicking on a hammer icon to break blocks into smaller pieces), explicit instructions were not given on how to set up and organize a solution. As seen in Aaron and Abbie’s discussion, students sometimes had difficulties in communicating to their partner what they thought should be done when they did not have control of the mouse (lines 104–106).

Example 3 illustrates the low-level collaborative discourse associated with tutorial VMs (see Table 3). Aaron and Callie are working with a tutorial VM to solve fraction addition problems (see Figure 3).

![Figure 3. Aaron and Callie working with a tutorial VM, Adding Fractions.](image)

Table 3

Excerpt of Students’ Discussion While Working With a Tutorial VM
<table>
<thead>
<tr>
<th>Line</th>
<th>Student</th>
<th>Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>131</td>
<td>Aaron</td>
<td>How many are we supposed to do? … 20. No, 30.</td>
</tr>
<tr>
<td>132</td>
<td>Callie</td>
<td>It says until teacher says it’s time.</td>
</tr>
<tr>
<td>133</td>
<td>Aaron</td>
<td>That’s a bunch! Ok. Let’s just quickly do this. <em>(clicks arrow repeatedly to change denominator)</em> … This should make it easier. This should make it easy because it’s really six and this one’s five <em>(1/6 + 1/5)</em>.</td>
</tr>
<tr>
<td>134</td>
<td>Callie</td>
<td>Six and five.</td>
</tr>
<tr>
<td>135</td>
<td>Aaron</td>
<td>That’s nice! So we already know the answer. Like, off the bat, is 11. Eleven-thirtieths. <em>(VM feedback: correct)</em></td>
</tr>
<tr>
<td>136</td>
<td>Aaron</td>
<td><em>(clicks ’Next’)</em> Whoa! We haven’t dragged the circle ones. Oh, that’s cool. Wait, does it go on the… Ok. So one-sixth plus one-fifth equals… Oh, we need to write… <em>(VM feedback: correct)</em></td>
</tr>
<tr>
<td>158</td>
<td>Callie</td>
<td>So…</td>
</tr>
<tr>
<td>159</td>
<td>Aaron</td>
<td>I’ll start on this real quick. I don't really see the point of us needing to talk through this ‘cause it’s kind of… simple.</td>
</tr>
</tbody>
</table>

As shown in this example, Aaron did not “see the point” of discussing these problems with his partner (line 159). This attitude was evident in his desire to do the problem on his own. This particular tutorial VM gave frequent feedback to students as it guided them through each step of the process of addition fractions with unlike denominators. Therefore, most of the students’ interactions occurred with the VM itself, and not with each other.

**Results & Conclusions**

Overall, when working with the linked VM type, students’ discussions reflected a statistically significant higher level of collaboration than when working with the other VM types. This indicated that affordances of the linked VM type enhanced students’ mathematical discourse. With respect to developing theory, the findings of this study indicated that the multiple representations displayed by the linked VMs enabled the students to effectively collaborate during problem solving tasks. This pattern is similar to findings of Ares, Stroup, and Schademan (2008) who noted that collective representations encouraged students to interact with each other and comment on each other’s solutions.

Overall, when working with the pictorial VM type, students’ discussions reflected slightly greater quantities of discourse than when working with the other VM types. This indicates that there was a greater need for the students to communicate the meaning of the representations with each other. The meaning of the representation was not as explicit as with the linked VMs. Therefore, the students had to assume responsibility for making connections for themselves.
However, despite the increased quantity of discourse, the quality of collaboration reflected mostly middle to lower levels of discourse.

Overall, when working with the tutorial VM type, students’ discussions reflected the lowest level of collaboration as compared to the other VM types. Despite the positive affordances of tutorial VMs when students work alone (Reimer & Moyer, 2005), this study indicated that tutorial VMs discouraged student collaboration. One possible theory is that due to the structured nature of the tutorials, students did not feel the need to collaborate and build on each other’s ideas as much as when working with other VM types. Instead, their focus was on responding to the tutorials’ direct feedback. Interaction with their partner was secondary to their interaction with the VM.

**Scholarly Significance**

The patterns and trends identified in this study contribute to the existing literature on the complex issues that surround mathematical discourse and the use of technology in the classroom environment. More and more classrooms are using technology (Gray et al., 2010), and students are learning mathematics as they interact with the technology and with each other. However, we know very little about the interactions students have with each other when also interacting with technology to complete mathematical tasks. This study represents an intersection of the two research fields of VMs and classroom discourse and adds to the research literature on the impact of technology on classroom mathematical discourse.
References


