1993

A jack pine bark factor equation for Michigan

G.W. Fowler

Recommended Citation
Fowler, G. W. 1993. A jack pine bark factor equation for Michigan. Northern Journal of Applied Forestry 10 (2): 86-89.

A Jack Pine Bark Factor Equation for Michigan

Gary W. Fowler, *School of Natural Resources, The University of Michigan, Ann Arbor, MI 48109-1115.*

ABSTRACT. *A multiple linear regression equation was developed to predict bark factor for jack pine in Michigan as a function of tree height. The equation was validated on independent data sets. The prediction equation yielded average relative errors from –2.9 to 0.4% for all tree heights above stump height. At stump height the average relative errors varied from –5.3 to –2.3%. The jack pine equation was compared with red pine and aspen bark factor equations. The new equation can be used to more accurately estimate tree and log wood volumes than when using a constant bark factor determined at breast height, which, in general, leads to underestimates of wood volume. North. J. Appl. For. 10(2):86–89.*

Bark factor (*F*) at a given tree height is the ratio of diameter inside bark (dib) to diameter outside bark (dob). Bark factors vary with species, age, site, and tree height. Bark factors at stump or breast height usually vary from 0.87 to 0.93. Even though much of the variation in bark factor is related to species, bark factor does increase with tree height for many species. In spite of this relationship, a constant bark factor has been assumed for many tree species for all tree heights. The use of a constant bark factor, determined at breast height, for all tree heights, will, in general, lead to underestimates of most tree and log volumes and overestimates of bark volume.

Multiple linear regression equations have been developed to predict bark factor as a function of various independent variables such as tree height and associated diameter outside bark. Such equations have not been developed for many species because data have been lacking for the independent variables, or the use of a constant bark factor has been considered adequate. As forest management becomes more intensive, the use of such equations should be considered so that more accurate estimates of wood volume can be obtained. Wood values should be estimated as accurately as possible in order to more accurately assess timber values in multiple-use forest management. See Husch et al. (1982) for a detailed discussion on bark factors, and specifically refer to Fowler and Damschroder (1988), who developed a red pine bark factor equation for Michigan and discussed the various uses of bark factors.

The objective of this study was to develop a bark factor prediction equation for jack pine in Michigan and compare this equation with contemporary red pine (Fowler and Damschroder 1988) and aspen (Fowler 1991) bark factor prediction equations.

NOTE: I would like to thank my field research assistants, Mr. David J. Cohen and Mr. Chad A. Robinson, for collecting the data on the six state forests in Michigan.

Procedures

The data set used to develop the prediction equation consisted of felled tree measurements on a total of 454 jack pine trees from 10 jack pine stands in Michigan (one stand in each of the Copper Country, Escanaba River, Mackinaw, Au Sable, and Pere Marquette State Forests, and 5 stands in the Superior State Forest). Trees were measured as they were felled and bucked on active logging sites until at least 10 trees, if available, were sampled in each 1 in. dbh class for each stand. Diameter inside bark (dib) and diameter outside bark (dob) were measured to the nearest 0.1 in. with a metal cased tape at stump height and at the top of each 8.3 ft. bolt (100 in. stick) cut out of each tree to an approximate 3.6 in. diameter top limit. Each dib and dob measurement was based on two readings taken at right angles to each other with the average diameter being determined. Dbh was measured to the nearest 0.1 in. with a D-tape, and the bark thickness to the nearest 0.05 in. at 4.5 ft above the ground was determined using a hatchet and a ruler. Bark thickness was based on the average of two measurements at right angles to each other. The number of trees and average, minimum, and maximum values of dbh in inches and merchantable height in 8.3 ft. bolts are shown in Table 1.

The data set used to validate the prediction equation consisted of 100 jack pine trees from 2 jack pine stands in the Superior State Forest. The number of trees and average, minimum, and maximum values of dbh in inches and merchantable height in feet are shown in Table 2. Merchantable heights are given in feet because variable bolt lengths were cut from these two stands. The same measurements were made on these trees as were made on the prediction data set trees.

For the prediction data set, the bark factor at each tree height was determined using all of the trees with measurements at that height with the formula:

$$k = \frac{\Sigma\, dib}{\Sigma\, dob}$$

Table 1. Number of trees, average $\bar{x}$ and minimum and maximum values of dbh in in. and merchantable height (M.Ht.) in 100 in. sticks for the 10 data sets used to construct the prediction equation.

				Dbh		M.ht.	
Region	State forest	Stand	No. of trees	$\bar{x}$	Min.–max.	$\bar{x}$	Min.–max.
U P.	Copper Country	1	42	9.5	7.0–11.8	3.1	2–4
	Escanaba River	1	51	10.0	6.8–14.4	4.5	3–5
	Mackinaw	1	50	9.5	7.1–12.9	3.0	2–4
L P	Au Sable	1	48	8.4	5.6–11.3	3.8	2–6
	Pere Marquette	1	50	8.8	4.7–14.0	4.7	2–6
L P	Superior	1	45	10.1	7.3–12.3	4.0	2–5
		2	50	8.0	5.9–13.2	4.1	3–6
		3	50	9.8	5.2–16.5	4.0	2–7
		4	50	9.5	5.0–13.7	4.6	2–6
		5	18	8.0	6.0–10.9	4.7	3–5

A good discussion on equations to determine bark factor is presented in Husch et al. (1982).

Results and Discussion

The variation of average k at a given height among the 10 stands in the prediction data set was relatively small, justifying pooling of all of the data for a given height (Table 3). For each height, average ks for the 10 stands were within 0.009 to 0.038 of each other except for 0.33+ ft where they varied by 0.075 (6 stands varied by 0.051). Average k was 0.8583, 0.9292, and 0 9436 at tree heights of 0.33+, 4.5, and 8.7 ft, respectively, increased to 0.9697 at 33.7 ft, and decreased to 0.9632 at 58.7 ft.

Bark factor was plotted against tree height for the pooled data, indicating that bark factor *(Y)* would be very closely predicted by some combination of the following forms of tree height *(X)*: *X*, 1/*X*, and ln *X*. A set of prediction equations (i.e., all combinations of *X*, 1/*X*, and ln *X*) was constructed using weighted multiple linear regression with weights based on the number of trees with measurements at that height for 9 heights (Table 3). The best prediction equation, i.e., that equation that yielded the smallest standard error of the estimate, smallest standard errors of the regression coefficients ($S_{\hat{\beta}_0}$, $S_{\hat{\beta}_1}$, and $S_{\hat{\beta}_2}$) and the largest coefficient of multiple determination (R^2), was:

$$\hat{Y} = 0.889250 - 0.000566X + 0.028297 \ln X \quad (1)$$

$$R^2 = 0.999 \qquad s_{y \bullet x} = 0.027132$$

$$s_{\hat{\beta}_0} = 0.000861 \qquad s_{\hat{\beta}_2} = 0.000592$$

$$s_{\hat{\beta}_1} = 0.000071$$

where $\hat{Y}$ is estimated k and X is tree height in feet. This regression equation is highly significant ($P < 0.001$). The skewness and kurtosis coefficients of the residuals were 0.273 and –1.060, respectively, indicating no serious departures from normality given only $n = 9$ tree heights. Predicted ks from the prediction equation for the 9 tree heights are shown in Table 3.

Another approach to predicting bark factor is to regress bark factor at a given height of an individual tree as a function of height to this measurement, total tree height, and dbh. This approach considers individual tree variability. For this study, the 454 trees yielded 2,736 bark factor values. Multiple linear regression yielded the following best prediction equation:

$$\hat{Y} = 0.889634 - 0.000580X + 0.028359 \ln X \quad (2)$$

$$R^2 = 0.757 \qquad s_{y \bullet x} = 0.021792$$

Adding dbh or total height as independent variables did not increase R^2 or decrease $s_{y.x}$ from the values for Equation (2). There is little difference between the estimated parameters of prediction equations 1 and 2. Thus, both approaches are acceptable for estimating bark factor and would yield essentially the same predictions. If it were important to use R^2 and $s_{y.x}$ on an individual tree basis, then Equation (2) should be used. Equation (1) will be used for validation and comparison with bark factor equations for other species.

Prediction Equation 1 was validated on the two independent data sets (Tables 2 and 4) for various heights and height classes in feet. Average relative errors as percentages ($\overline{RE}$) were calculated for each height or height class for each stand using the formula:

$$\overline{RE} = \sum_{i=1}^{n} RE_i / n$$

Table 2. Number of trees, average $\bar{x}$ and minimum and maximum values of dbh in in. and merchantable height (M.Ht.) in ft for the two data sets used to validate the prediction equations.

				Dbh		M.ht.	
Region	State forest	Stand	No. of trees	$\bar{x}$	Min.–max.	$\bar{x}$	Min.–max.
U P.	Superior	1	50	10.3	7.0–13.0	40.7	25.3–54.5
		2	50	9.0	6.6–14.1	31.8	25.3–42.0

Note Bolt lengths of the first bolt was 12 ft 6 in. for 22 trees and 12 ft 8 in. for 5 trees in stands 1 and 2, respectively. All other bolt lengths were 8 ft 4 in.

Table 3. Number of trees, pooled average bark factors based on tree dib and dob measurements, and predicted bark factors from the prediction equation for nine tree heights in ft.

Tree height	No. of trees	Pooled bark factor	Predicted bark factor
0.33+	454	0.8583	0.8580
4.5	454	0.9292	0.9293
8.7	454	0.9436	0.9455
17.0	454	0.9606	0.9598
25.3	416	0.9682	0.9664
33.7	318	0.9697	0.9697
42.0	157	0.9696	0.9712
50.3	26	0.9671	0.9716
58.7	3	0.9632	0.9712

where

$$RE_i = \frac{\hat{Y}_i - Y_i}{Y_i}(100)$$

$\hat{Y}_i$ = predicted k for the ith tree

Y_i = observed k for the ith tree

n = number of trees with measurements in the specified height class

For the sample from each of the two stands $\overline{RE}$ was less than ±1.5% for each height class except for 0.33+ ft for Stand 1 and 0.33+ and 4.5 ft for Stand 2. Table 4 shows $\overline{RE}$, the minimum and maximum relative errors, and the number of trees that had heights in each height class. RE_i varies from –4.2 to 5.7% for each height class except for 0.33+ ft. Note that the prediction equation tended to underestimate bark factor for the two validation stands. Validation results for other stands would vary from these results, depending on stand characteristics.

Confidence intervals could be used to indirectly evaluate the accuracy of predictions. The validation results give a more direct evaluation of prediction accuracy. In evaluating the accuracy of the prediction equation, it must be remembered that sample sizes decrease greatly as tree height increases (Table 3). For discussions of weighted multiple linear regression, see Brownlee (1965), Draper and Smith (1981), Neter et al. (1990), and Steel and Torrie (1960).

Fowler and Damschroder (1988) developed a bark factor prediction equation for red pine in Michigan of the form

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \frac{1}{X} + \hat{\beta}_2 \ln X$$

Fowler (1991) developed bark factor prediction equations for bigtooth aspen, trembling aspen, and the 2 aspen species pooled using the form used for jack pine in this study. He also used this form for red pine. These prediction equations are as follows:

Bigtooth aspen:

$$\hat{Y} = 0.9219 - 0.000684X + 0.007556 \ln X$$

Trembling aspen:

$$\hat{Y} = 0.9214 - 0.000842X + 0.007588 \ln X$$

Aspen pooled:

$$\hat{Y} = 0.9217 - 0.000749X + 0.007564 \ln X$$

Red pine:

$$\hat{Y} = 0.9167 - 0.000575X + 0.022208 \ln X$$

Figure 1 compares the bark factor prediction equations for aspen pooled, jack pine, and red pine. The maximum heights found in the prediction data sets for aspen pooled, jack pine, and red pine were 67.0, 58.7, and 75.3 ft, respectively. Figure 2 compares the bark factor prediction equations for bigtooth aspen, trembling aspen, and aspen pooled. The maximum height found in the prediction data sets for both bigtooth and trembling aspen was 67.0 ft.

Figures 1 and 2 show distinct species differences. β_0, β_1, and β_2 were significantly different ($P < 0.01$ in each case) for the pooled aspen and red pine equations, b_0 and b_2 were significantly different ($P < 0.01$ in each case) for the jack pine and aspen equations as well as for the jack pine and red pine equations, and

Table 4. Average relative errors $\overline{RE}$, minimum and maximum relative errors, and number of trees for each height class in ft *(n)* for the two stands (validation data sets). All *RE* values are percentages.

	Stand 1			Stand 2		
Height class	$\overline{RE}$	Min.–max.	n	$\overline{RE}$	Min.–max.	n
0.33	-5.2	-11.7, 3.4	50	-2.3	-7.0, 7.8	50
4.5	-0.1	-3.2, 5.7	50	-2.9	-5.6, 2.1	50
8.7	-1.2	-3.4, 1.9	28	-1.5	-3.5, 0.4	45
12.8–13.0	-1.2	-4.2, 2.3	22	-0.5	-1.2, 1.5	5
17.0	-1.0	-2.8, 1.5	28	-0.8	-2.4, 1.3	45
21.2–21.3	-0.6	-2.3, 4.8	22	-0.9	-1.7, 0.9	5
25.3	-1.1	-2.5, 0.7	28	-0.3	-1.9, 2.7	45
29.5–29.7	-0.7	-2.4, 3.3	22	-0.02	-0.9, 1.2	5
33.7	-0.9	-2.2, 0.6	26	0.4	-1.4, 5.5	26
37.8–38.0	-1.0	-1.7, 0.3	21	0.3	-0.7, 1.0	4
42.0	-0.8	-1.8, 1.0	16	0.3	-1.3, 2.5	6
46.2	-1.0	-1.8, 0.9	12	---	-------	---
50.3	-0.5	-1.6, 0.2	4	---	-------	---
54.5	-0.8	-------	1	---	-------	---

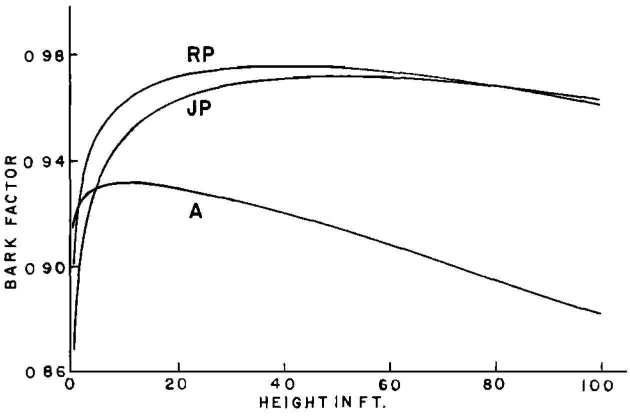

Figure 1. Bark factor as a function of height for the aspen pooled (A), jack pine (JP), and red pine (RP) bark factor prediction equations.

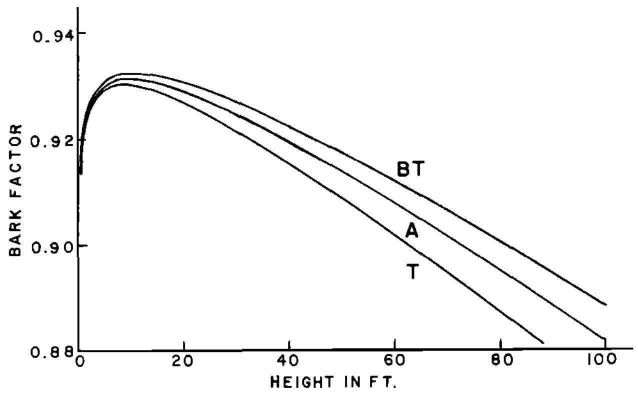

Figure 2. Bark factor as a function of height for the bigtooth (BT), trembling (T), and pooled (A) aspen bark factor prediction equations.

β_0, β_1, and β_2 were were not significantly different ($P > 0.05$ in each case) for the bigtooth and trembling aspen equations using two-sample t-tests. Each prediction equation reaches a maximum bark factor value and then decreases with height, with this decrease being more distinct for aspen. The maximum value is reached at approximately 10, 40, and 50 ft for aspen pooled, red pine, and jack pine, respectively. The decrease in bark factor at greater heights could be due to (1) the relationship between bark thickness and diameter as tree height increases and/or (2) the lack of accuracy in measuring diameter inside and outside bark at greater tree heights where bark thickness is quite small.

Comments

There are distinct species differences in bark factor prediction equations, and separate equations should be used where species specific equations are available. The equations should definitely be used when accurate estimates of bark factors are desired, even in the case of bigtooth and trembling aspen. In most situations, however, the pooled aspen equation appears adequate. For good approximate results, the following constants could be used for bark factors:

	Stump ht	dbh ht	Rest of tree
Aspen	0.91	0.93	0.92
Jack pine	0.86	0.93	0.96
Red pine	0.90	0.95	0.97

Bark factors toward the top of the tree need to be studied in more detail. Fowler and Damschroder (1988) discuss specific uses of bark factor prediction equations.

Literature Cited

BROWNLEE, K.A. 1965. Statistical theory and methodology in science and engineering. Wiley,New York. 590 p.

DRAPER, N.R., AND H. SMITH. 1981. Applied regression analysis. Wiley, New York. 709 p.

FOWLER, G.W. 1991. An aspen bark factor equation for Michigan. North. J. Appl. For. 8(1): 12–15.

FOWLER, G.W., and L.J. DAMSCHRODER. 1988. A red pine bark factor equation for Michigan. North. J. Appl. For. 5(1):28–30.

HUSCH, B., C.I. MILLER, and T.W. BEERS. 1982. Forest mensuration. Ed. 3. Wiley, New York. 402 p.

NETER, J., W. WASSERMAN, and M.H. KUTNER. 1990. Applied linear statistical models. Irwin, Homewood, IL. 1181 p.

STEEL, R.G.D., and J.H. TORRIE. 1960. Principles and procedures of statistics. McGraw-Hill, New York. 481 p.