Quatennessee-Based Tracking Control Law Design
For Tracking Mode

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Quat{eion-Based Tracking Control Law Design
For Tracking Mode

Presentation Agenda

• Paper objectives
• Introduction
• Spacecraft combined nonlinear model
• Proposed RW nonlinear attitude tracking controller
• Stability analysis
• Kinematic of fixed ground target tracking
• Design example and simulation results
• Summary and conclusions
Paper objectives

- Introduce a modified robust nonlinear tracking control law used with RW actuator for a tracking complex mode.
- Derive a ground target attitude, rate, and acceleration relative to orbit reference frame in details.
- Demonstrate the validity of the proposed controller and target data generator using MATLAB/SIMULINK environment.
Introduction

- pointing modes:
  - Inertial pointing mode
  - Earth or nadir pointing mode
  - Orbital object pointing mode
Introduction

Tracking a fixed target on the Earth for a certain period of time is one of the most important requirements for Earth-pointing satellites in order to provide improved long-period up-down-link communication, low-distortion imaging and accurate observation.
Introduction

- **open loop schemes**
  - require a predetermined pointing maneuver and are typically determined using optimal control techniques.
  - sensitive to spacecraft parameter uncertainty and unexpected disturbances.

- **closed loop schemes**
  - account for parameter uncertainties and disturbances
  - more robust design methodology.
Introduction

Most of the control algorithms introduced previously in the open literature care about pointing accuracy and ignore the time elapsed to reach these strict requirements.

Actually there is an increasing demand from customers for a fast ground-target tracking even on the expense of pointing accuracy itself.
• Introduction

- When the target locations and satellite ground station are both within the satellite’s footprint, it may be much more important for the ground station to steer the main imager bore sight towards the immediate required target areas before capturing the imaging data.

- Quick satellite response allows for the military intelligence data gathering and downloading in the same communication session.
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- **Spacecraft combined nonlinear model**
  - Dynamic in orbital reference coordinate (ORC)
    \[
    \dot{\mathbf{O}}_{B}^{ORC} = J^{-1}(N_{\infty} + N_{ext} - \Omega(\omega_{B}^{I})(J\omega_{B}^{I} + h) - \dot{h})
    \]  

Where

\[
N_{\infty} = -J \dot{\mathbf{A}}_{SBC}^{SBC} \mathbf{O}_{O}^{I} - J \dot{\mathbf{A}}_{SBC}^{SBC} \mathbf{O}_{I}^{I} 
= -J \Omega(\omega_{B}^{ORC}) A_{SBC}^{SBC} \mathbf{O}_{O}^{I} - J A_{SBC}^{SBC} \mathbf{O}_{I}^{I}
\]

\[
\mathbf{O}_{O}^{I} = [0 - \frac{r_{sat} \times v_{sat}}{\| r_{sat} \|^2} 0]^T
\]

\[
\dot{\mathbf{O}}_{O}^{I} = [0 - \frac{2(r_{sat} \cdot v_{sat}) r_{sat} \times v_{sat}}{\| r_{sat} \|^4} 0]^T
\]
Spacecraft combined nonlinear model

- Kinematic in orbital reference coordinate (ORC)

\[
\begin{align*}
\dot{q} &= -\frac{1}{2} \Omega(q_{\text{ORC}})q + \frac{1}{2} q_4 q_{\text{ORC}} \\
\dot{q}_4 &= -\frac{1}{2} (\Omega(q_{\text{ORC}}))^T q
\end{align*}
\]

Where

\[
q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} e_1 \sin \left( \frac{\varphi}{2} \right) \\ e_2 \sin \left( \frac{\varphi}{2} \right) \\ e_3 \sin \left( \frac{\varphi}{2} \right) \end{bmatrix}, \quad q_4 = \cos \left( \frac{\varphi}{2} \right), \quad \tilde{q} = \begin{bmatrix} q \\ q_4 \end{bmatrix}
\]
Spacecraft combined nonlinear model

\[
\frac{d}{dt} \begin{bmatrix} \omega^\text{ORC}_B \\ q \\ q_4 \\ h \end{bmatrix} = \begin{bmatrix} J^{-1}(N_{\infty} + N_{\text{ext}} - \Omega(\omega^I_B)(J\omega^I_B) + N_{\text{eff}}) \\ -\frac{1}{2} \Omega(\omega^\text{ORC}_B)q + \frac{1}{2} q_4 \omega^\text{ORC}_B \\ -\frac{1}{2} (\omega^\text{ORC}_B)^T q \\ -\Omega(\omega^I_B)(h) - N_{\text{eff}} \end{bmatrix}
\]  

Where

\[N_{\text{eff}}\] is effective RW torque input
Quaternion-Based Tracking Control Law Design
For Tracking Mode

- Proposed RW nonlinear attitude tracking controller

\[ \dot{h} = N_{\omega_0} + N_{\text{ext}} - \Omega(\omega_B^I)(J \omega_B^I + h) \]
\[ -\Omega(\omega_B^{\text{ORC}}) J \omega_B^{\text{ORC}} + \Omega(\omega_c^{\text{ORC}}) J \omega_c^{\text{ORC}} \]
\[ -\Omega(J \omega_c^{\text{ORC}})(\omega_B^{\text{ORC}} - \omega_c^{\text{ORC}}) - J \omega_c^{\text{ORC}} \]
\[ + D (\omega_B^{\text{ORC}} - \omega_c^{\text{ORC}}) + K \delta q \]

Where \( \delta q \) - attitude error quaternion vector between spacecraft attitude \( q \)
and target attitude \( q_c \) w.r.t ORC.

\( D, K \) are constant gains.
\( \omega_c^{\text{ORC}}, \omega_c^{\text{ORC}} \) are target rate and acceleration w.r.t ORC.
Quat Mock|Based Tracking Control Law Design
For Tracking Mode

- Proposed RW nonlinear attitude tracking controller

\[
\delta q = \begin{bmatrix} \delta q \\ \delta q_4 \end{bmatrix} = (q_c^{-1} \cdot q)
\]

\[
\delta q = \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}
\] (5)

Substituting (4) into (2) gives

\[
\dot{\omega}_B^{ORC} - \dot{\omega}_c^{ORC} = J^{-1}(-D(\omega_B^{ORC} - \omega_c^{ORC})) + K \delta q - \Omega(\omega_c^{ORC}) J \omega_c^{ORC} + \Omega(J \omega_c^{ORC})(\omega_B^{ORC} - \omega_c^{ORC}) + \Omega(\omega_B^{ORC}) J \omega_B^{ORC}
\] (6)
Proposed RW nonlinear attitude tracking controller

Eq. (6) and Eq. (2) represent the closed-loop time varying nonlinear dynamic system of the spacecraft attitude with the proposed control law (4)

\[
\begin{align*}
\dot{\omega}_B^{ORC} - \dot{\omega}_c^{ORC} &= J^{-1} \left( -D \left( \omega_B^{ORC} - \omega_c^{ORC} \right) \\
&+ K \delta q - \Omega(\omega_c^{ORC}) J \omega_c^{ORC} \\
&+ \Omega(J \omega_c^{ORC})(\omega_B^{ORC} - \omega_c^{ORC}) \\
&+ \Omega(\omega_B^{ORC}) J \omega_B^{ORC} \right) \\
\dot{q} &= -\frac{1}{2} \Omega(\omega_B^{ORC}) q + \frac{1}{2} q_4 \omega_B^{ORC} \\
\dot{q}_4 &= -\frac{1}{2} (\omega_B^{ORC})^T q
\end{align*}
\]
Stability analysis

- It is clear that the following is a solution of Eq.(2) and Eq.(6)

\[
\begin{align*}
\omega_B^{ORC} &= \omega_c^{ORC} \\
q &= q_c
\end{align*}
\] (7)

- Solution (7) means theoretically that it is possible for the spacecraft to follow the commanded attitude and attitude rate.

- Including the terms \( N_{\omega} , \Omega(\omega_B^I)(J\omega_B^I + h) \) and \( J\omega_c^{ORC} \) ensure that the solution (7) always exists.
Stability analysis

- Generally asymptotic stability analysis is required to ensure that the spacecraft can follow the commanded attitude with commanded attitude rate from any condition of both $\omega^{ORC}_B$ and $q$.

- The stability analysis is started by studying the local asymptotic stability, at which the initial values of both and are near the solution (7). Once this local stability is confirmed, a global asymptotic stability, at which any values of both are assumed, is further searched.
Stability analysis

- The tracking error dynamics are employed for the indirect stability analysis alternatively to the non-convenient direct analysis of solution (7).
- Defining state variable $x$ to represent the tracking error dynamics as follows:

$$
x = \begin{pmatrix}
x_{13} \\
x_{46} \\
x_7
\end{pmatrix} = (x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7)^T \tag{8}
$$
Stability analysis

Where

\[
\begin{align*}
    x_1 &= \omega_{1B}^{ORC} - \omega_{1c}^{ORC} \\
    x_2 &= \omega_{2B}^{ORC} - \omega_{2c}^{ORC} \\
    x_3 &= \omega_{3B}^{ORC} - \omega_{3c}^{ORC} \\
    x_4 &= \Delta q_1 \\
    x_5 &= \Delta q_2 \\
    x_6 &= \Delta q_3 \\
    x_7 &= \Delta q_4 \\
\end{align*}
\]

\[
\begin{align*}
    x_{13} &= (x_1 \quad x_2 \quad x_3)^T \\
    x_{46} &= (x_4 \quad x_5 \quad x_6)^T
\end{align*}
\]
• **Stability analysis**

With

\[
\Delta \overline{q} = \overline{q} - \overline{q}_c = \begin{pmatrix} \Delta q \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \end{pmatrix} = \begin{pmatrix} q_1 - q_{1c} \\ q_2 - q_{2c} \\ q_3 - q_{3c} \\ q_4 - q_{4c} \end{pmatrix}
\]

(10)

- Substituting Eq. (9) and Eq. (10) into Eq.(6) and Eq. (2) gives tracking error dynamics model Eq. (11) which is equivalent to Eq. (2) and Eq. (4).
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- Stability analysis
  Tracking error dynamics model

\[
\begin{align*}
\dot{x}_{13} & = f_{13} = -\bar{D} x_{13} - \bar{K} [\bar{q}_c] \begin{pmatrix} x_{46} \\ x_7 \end{pmatrix} \\
\dot{x}_{46} & = f_{46} = \frac{1}{2} Q_c x_{13} + \frac{1}{2} q_{4c} x_{13} \\
& \quad - \frac{1}{2} \Omega(\omega_c^{ORC}) x_{46} + \frac{1}{2} \omega_c x_7 \\
& \quad - \frac{1}{2} X x_{46} + \frac{1}{2} x_7 x_{13} \\
\dot{x}_7 & = f_7 = -\frac{1}{2} (q_c^T x_{13} + \omega_c^{ORCT} x_{46} + x_{13}^T x_{46})
\end{align*}
\]
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For Tracking Mode

- Stability analysis

where

\[ \bar{D} = J^{-1} D \quad , \quad \bar{K} = J^{-1} K \]

\[ Q_c = \begin{pmatrix}
0 & -q_{3c} & q_{2c} \\
q_{3c} & 0 & -q_{1c} \\
-q_{2c} & q_{1c} & 0
\end{pmatrix} \]

\[ X = \begin{pmatrix}
0 & -x_3 & x_2 \\
x_3 & 0 & -x_1 \\
-x_2 & x_1 & 0
\end{pmatrix} \]

\[ [\bar{q}_c] = \begin{pmatrix}
q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\
-q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\
q_{2c} & -q_{1c} & q_{4c} & -q_{3c}
\end{pmatrix} \]
Stability analysis

- Eq. (11), which represents the tracking error dynamics model, has an equilibrium point, which corresponds to solution (7).
- Therefore, instead of studying the deviation of $\omega_{BORC}$ and $q$ from $\omega_{cORC}$ and $q_c$ for Eq. (2) and Eq. (4), we may simply study the stability of the tracking error Eq. (11) with respect to the equilibrium point $x = 0$.
- A Lyapunov function based global stability analysis of the time-varying non-linear tracking error dynamics Eq. (11) is employed.
Stability analysis

- The candidate Lyapunov function $V$, which is independent of time and is radially unbound, is defined by

$$V(x) = x^T \bar{P} x$$

where

$$\bar{P} = \begin{pmatrix} P_{3 \times 3} & 0_{3 \times 4} \\ 0_{4 \times 3} & I_{4 \times 4} \end{pmatrix}$$

$P$ is a symmetric and positive definite matrix. $I_{4 \times 4}$ is an identity matrix.
**Quaternion-Based Tracking Control Law Design**

For Tracking Mode

- **Stability analysis**
  - Global stability is confirmed when
    \[
    P = \frac{1}{2} \bar{K}^{-1}
    \]  \hspace{1cm} (13)
  - Assuming \( \bar{D} \) and \( \bar{K} \) are diagonal matrices, the dynamic behavior of the nonlinear system (11) can be controlled around the equilibrium point in the direction of the eigenvalues with nonzeros real parts of the Jacobian matrix.
Stability analysis

Expressing the characteristic equation by the desired damping ratio $\xi$ and natural frequency $\omega_n$ for the special case $\omega_{c ORC}^C = 0$ allows to relate the gains matrices directly to the desired response.

\[
\begin{align*}
\bar{d}_1 &= 2\xi_1 \omega_{n1} ;
\bar{k}_1 &= 2\omega_{n1}^2 \\
\bar{d}_2 &= 2\xi_2 \omega_{n2} ;
\bar{k}_2 &= 2\omega_{n2}^2 \\
\bar{d}_3 &= 2\xi_3 \omega_{n3} ;
\bar{k}_3 &= 2\omega_{n3}^2
\end{align*}
\] (14)
Stability analysis

- Since actuator constraints are considered in this paper, these gain matrices figured using Eq. (14) can be used as initial guide values.
- A Matlab/Simulink Optimization tool is used to adjust the dynamic behavior of the nonlinear system Eq. (11) around the equilibrium point.
- Tracking error is the selected parameter to be optimized within short time.
Kinematic of fixed ground target tracking

- Since the nominal mode of earth orbiting satellite usually maintains the satellite body frame aligned with orbital (ORC) frame, it is preferable to describe the kinematics of the ground target with respect to the in-orbit satellite in ORC frame.
- The vector from the satellite to the ground target represented in ORC will specify the tracking direction of the selected pointing axis which may be the positive mounting axis of a camera or antenna.
Kinematic of fixed ground target tracking

The vector from the satellite to the target in orbit frame is

\[ x_{S/T}^{ORC} = A_{EIC}^{ORC} (x_T^{EIC} - r_{sat}^*) \]  

(15)

Where \( A_{EIC}^{ORC} \) is the transformation matrix from EIC frame to ORC frame and defined by

\[ A_{EIC}^{ORC} = [\hat{u} \ \hat{v} \ \hat{w}]^T \]  

(16)

\[ \hat{w} = -\frac{r_{sat}^*}{\|r_{sat}^*\|}, \quad \hat{v} = -\frac{r_{sat}^* \times v_{sat}^*}{\|r_{sat}^* \times v_{sat}^*\|}, \quad \hat{u} = \hat{v} \times \hat{w} \]  

(17)

hence the unit vector must be calculated by normalizing \( x_{S/T}^{ORC} \), thus

\[ u_{S/T}^{ORC} = \frac{x_{S/T}^{ORC}}{\|x_{S/T}^{ORC}\|} \]  

(18)
Kinematic of fixed ground target tracking

- Target tracking task requires that the mounting axis of the commissioned payload being controlled to point towards the direction of $u^{ORC}_{S/T}$.
- In most cases, the commissioned payload, $u^{SBC}_{Com}$ such as a camera, antenna or telescope is usually mounted along the z-axis of the satellite body to allow controlling the rotation around this axis.
- Paper considers the case of constant yaw angle $\psi$ during tracking.
Kinematic of fixed ground target tracking

- The commanded angular rate $\omega_{c}^{ORC}$ is deduced in details and was found

$$\omega_{c}^{ORC} = \left[ -\frac{\dot{u}_{S/T}^{ORC}}{u_{S/T}^{ORC}} \quad \frac{\dot{u}_{S/T}^{ORC}}{u_{S/T}^{ORC}} \quad 0 \right]^{T}$$  \hspace{1cm} (19)

- The commanded angular acceleration $\ddot{\omega}_{c}^{ORC}$ is deduced in details and was found

$$\ddot{\omega}_{c}^{ORC} = \dot{u}_{S/T}^{ORC} \times \dot{u}_{S/T}^{ORC} + u_{S/T}^{ORC} \times \ddot{u}_{S/T}^{ORC}$$

$$= u_{S/T}^{ORC} \times \ddot{u}_{S/T}^{ORC}$$  \hspace{1cm} (20)

Where

$$\dot{u}_{S/T}^{ORC} = \frac{1}{\sqrt{x_{S/T}^{ORC}}} \left[ I_{3} - u_{S/T}^{ORC} (u_{S/T}^{ORC})^{T} \right] x_{S/T}^{ORC}$$  \hspace{1cm} (21)
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• Kinematic of fixed ground target tracking

  ▪ The commanded angular rate $\omega_{c}^{ORC}$ is deduced in details and was found

    $\omega_{c}^{ORC} = \begin{bmatrix}
    -\frac{u_{S/T-x}^{ORC}}{u_{S/T-x}^{ORC}} & \frac{u_{S/T-x}^{ORC}}{u_{S/T-x}^{ORC}} & 0
    \end{bmatrix}^{T}$ (19)

  ▪ The commanded angular acceleration $\dot{\omega}_{c}^{ORC}$ is deduced in details and was found

    $\dot{\omega}_{c}^{ORC} = u_{S/T}^{ORC} \times \dot{u}_{S/T}^{ORC} + u_{S/T}^{ORC} \times \ddot{u}_{S/T}^{ORC}$ (20)

    $= u_{S/T}^{ORC} \times \ddot{u}_{S/T}^{ORC}$

    Where

    $u_{S/T}^{ORC} = \frac{1}{\|\chi_{S/T}^{ORC}\|} [I_{3} - u_{S/T}^{ORC} (u_{S/T}^{ORC})^{T}] \chi_{S/T}^{ORC}$ (21)
Quaternion-Based Tracking Control Law Design
For Tracking Mode

- Kinematic of fixed ground target tracking

\[
\ddot{u}_{S/T}^{ORC} = \vec{\ddot{x}}_{S/T}^{ORC} - \left( \frac{1}{\|x_{S/T}^{ORC}\|^3} \begin{pmatrix} x_{S/T}^{ORC} \\ \dot{x}_{S/T}^{ORC} \end{pmatrix}^T \begin{pmatrix} \dot{x}_{S/T}^{ORC} \end{pmatrix} \right) \]

\[
\dot{x}_{S/T}^{ORC} + \left( \frac{1}{\|x_{S/T}^{ORC}\|^3} \begin{pmatrix} x_{S/T}^{ORC} \\ \dot{x}_{S/T}^{ORC} \end{pmatrix}^T \begin{pmatrix} \dot{x}_{S/T}^{ORC} \end{pmatrix} \right) (u_{S/T}^{ORC})^T \dot{x}_{S/T}^{ORC} (u_{S/T}^{ORC}) - \left( \frac{1}{\|x_{S/T}^{ORC}\|^3} \begin{pmatrix} x_{S/T}^{ORC} \\ \dot{x}_{S/T}^{ORC} \end{pmatrix} \right) (u_{S/T}^{ORC})^T (u_{S/T}^{ORC})^T
\]

\[
\dot{x}_{S/T}^{ORC} + u_{S/T}^{ORC} \left( \begin{pmatrix} \dot{x}_{S/T}^{ORC} \end{pmatrix} + u_{S/T}^{ORC} \begin{pmatrix} \dot{x}_{S/T}^{ORC} \end{pmatrix} \right)
\]

\[
\dot{x}_{S/T}^{ORC} = \dot{A}_{EIC}^{ORC} \begin{pmatrix} A_{EIC}^{EIC} \begin{pmatrix} x_{T}^{EIC} \\ \dot{x}_{T}^{EIC} \end{pmatrix} - v_{sat} \end{pmatrix} + A_{EIC}^{ORC} \begin{pmatrix} \begin{pmatrix} A_{EFC}^{EIC} \begin{pmatrix} x_{T}^{EIC} \\ \dot{x}_{T}^{EIC} \end{pmatrix} - v_{sat} \end{pmatrix} \end{pmatrix}
\]

(22)

(23)
Quatetion-Based Tracking Control Law Design
For Tracking Mode

- Kinematic of fixed ground target tracking

\[
\begin{align*}
\ddot{x}_{S/T}^{\text{ORC}} &= \frac{d}{dt} \left( \dot{A}_{EIC}^{\text{ORC}} \left( \dot{A}_{EFC}^{\text{EIC}} x_T^{\text{EIC}} - r_{\text{sat}} \right) \right) \\
&\quad + \frac{d}{dt} \left( A_{EIC}^{\text{ORC}} \left( A_{EFC}^{\text{EIC}} x_T^{\text{EIC}} - v_{\text{sat}} \right) \right) \\
\dot{A}_{EIC}^{\text{ORC}} &= \begin{bmatrix} \hat{u} & \hat{v} & \hat{w} \end{bmatrix}^T \\
\dot{A}_{EIC}^{\text{ORC}} &= \begin{bmatrix} \hat{u} & \hat{v} & \hat{w} \end{bmatrix}^T \\
\alpha_{\text{sat}} &= \ddot{r}_{\text{sat}} = - \frac{\mu}{\| r_{\text{sat}} \|^3} r_{\text{sat}} 
\end{align*}
\]

Details of target parameters are derived straightforward as explained in the proposed paper.
• Design example and simulation results
  ▪ An imaginary satellite in a low-Earth-eccentric orbit is used as an example during these simulations.
  ▪ In order to investigate the attitude change of the satellite, we define the roll, pitch and yaw angles respectively to represent the rotations of the satellite body x, y and z axes with respect to the ORC coordinates.
  ▪ The simulation parameters are given in the following Table.
Modeling And Simulation of Spacecraft Pointing Modes Using Quaternion-Based Nonlinear Control Laws

- Design example and simulation results

- Assumption
  - perfect attitude knowledge.
  - Perfect measurements of spacecraft position and velocity.

<table>
<thead>
<tr>
<th>Required Tracking accuracy</th>
<th>$(0.3^0)$ within $(1)$ mili of being in satellite FOV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target position</td>
<td>$x_T^{EFC} = [4021.9, -35.1, 4933.6]^T$</td>
</tr>
<tr>
<td>Unit vector of commissioned axis in SBC frame</td>
<td>$u_{COM}^{SBC} = [0, 0, 1]^T$</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$\begin{bmatrix} 40 &amp; 0 &amp; 0 \ 0 &amp; 40 &amp; 0 \ 0 &amp; 0 &amp; 32 \end{bmatrix} \text{ kgm}^2$</td>
</tr>
<tr>
<td>Reaction wheel</td>
<td>Max torque = $(0.02) \text{ Nm}$, Max momentum = $(4) \text{ Nm s}$, Initial Momentum = $[0.4, -0.5, 0.6]^T \text{ Nm s}$</td>
</tr>
<tr>
<td>Orbit parameters</td>
<td>perigee altitude = $(650) \text{ km}$; inclination $(64.5^0)$; eccentricity $(0.3)$; argument of perigee $(0^0)$; right ascension of ascending node $(10^0)$; initial mean anomaly $(0^0)$</td>
</tr>
<tr>
<td>Initial phase</td>
<td>$\alpha = 0^0$</td>
</tr>
</tbody>
</table>
Design example and simulation results

- Matlab/ Simulink Optimization tools is used to search almost best gains to allow the tracking process meets the required rapid pointing to the target with a little relaxed accuracy which yields:

\[
K = \begin{bmatrix}
25.8648 & -0.0050 & 0.0117 \\
-0.0048 & 16.0027 & -0.0022 \\
-0.0018 & -0.0020 & 0.8504
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
18.3484 & 2.2739 & 0.4557 \\
-0.5086 & 17.5204 & 0.4733 \\
0.1048 & 0.1045 & 17.4478
\end{bmatrix}
\]
The controller succeeded to maneuver spacecraft through a large angle to a predetermined attitude.
Design example and simulation results

- The proposed tracking controller succeeded to maneuver satellite through a large angle to a predetermined attitude with the required relaxed accuracy (0.280) within only (40) sec which is well below the requirements.
Both orbit referenced satellite attitude and desired attitude are almost coincide within only (30) sec.
The orbit referenced angular velocity of the spacecraft body does track the desired angular rate command with acceptable limits during the tracking period at which the target is within the FOV of the spacecraft.
The reaction wheels will apply maximum torque (0.02 Nm.) to provide a bang-bang control maneuver to meet the required target tracking within specified limited time with relaxed pointing accuracy.
The wheel momentums are kept well away below the permissible limits (4 Nms).
Robustness against model uncertainties up to ±20% error in the moment of inertia tensor of the satellite.
The pointing accuracy can reach $0.120$ and further enhancement can be carried out.
The enhancement in tracking accuracy will be on the expense of the time elapsed to track the fixed ground target and actuator effort.
• Summary and conclusions

- The simulation results show that RW non-linear controller is capable to achieve a fast ground target tracking maneuver once the target is being in the satellite FOV.

- Quick satellite response allows for the military intelligence data gathering and downloading in the same communication session.
• Summary and conclusions
  - The reaction wheels will apply maximum torque to provide bang-bang control maneuvers to meet the required target tracking within specified limited time with relaxed pointing accuracy.
  - The wheel momentum is kept well away below the permissible limits.
  - The simulation results also show well and considerable robustness behavior of the proposed controller.
Summary and conclusions

- The enhancement in tracking accuracy is possible but it will be on the expense of the time elapsed to track the fixed ground target and actuator effort.
Thank you