Magnetic Substance Disturbance Torque Caused by Shape Magnetic Anisotropy and its Applications in Small-Sized Satellites

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• Introduction
• Small satellites in the University of Tokyo
• Attitude disturbance caused by magnetic substances
  – Ellipsoid shape
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Nano- and micro-satellites (University of Tokyo)

- **Nano-JASMINE** (ー) (2014)
- **PRISM** (2009)
- **PROCYON** (2014)
- **Cubesat XI-V** (2005)
- **XI-IV** (2003): Technical demonstration satellite, 1 kg
- **Nano remote sensing satellite, 8.5 kg**
- Micro Astronomical satellite, 35 kg
- Micro interplanetary spacecraft

- 10cm × 10cm × 10cm, 1 kg
- Magnetic passive attitude control system
Coarse attitude estimation by thermometers

- Although the z axis is not stabilized, the Y axis is aligned with the geomagnetic field vector and stabilized.
- A strong disturbance may affect the satellite attitude.
The strongest attitude torque is the magnetic torque by the permanent magnet.

An unexpected disturbance affects the satellite attitude.

### Table 1 Attitude disturbances in XI-IV

<table>
<thead>
<tr>
<th>Attitude disturbance</th>
<th>(Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic disturbance</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Aerodynamic disturbance</td>
<td>$2 \times 10^{-10}$</td>
</tr>
<tr>
<td>Solar pressure disturbance</td>
<td>$1 \times 10^{-9}$</td>
</tr>
<tr>
<td>Gravity gradient disturbance</td>
<td>$1 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
Attitude torque caused by a magnetic substance

Perpendicular to magnetic field
Magnetically unstable

Parallel to magnetic field
Magnetically stable

Attitude torque
\[ u = \frac{\mathbf{B} \cdot \mathbf{H}}{2} \]

- The magnetic energies in Fig. 1 and Fig. 2 are not the same.
- The rod rotates to more stable configuration.

→ Magnetic torque by shape magnetic anisotropy

In a uniform magnetic field, a magnetic substance with nonsymmetrical body causes a magnetic torque.

{Attitude disturbance torque caused by shape magnetic anisotropy of on-board magnetic substances}
Magnetic field in an ellipsoid magnetic substance can be solved with the Laplace equation ($\nabla^2 \Phi = 0$) for an ellipsoid coordinate system.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1
\]

\[
\mu
\]

\[
\frac{\partial}{\partial \xi} \left( R_\xi \frac{\partial \Phi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( R_\eta \frac{\partial \Phi}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left( R_\zeta \frac{\partial \Phi}{\partial \zeta} \right) = 0
\]

\[
R_s = \sqrt{(s + a^2)(s + b^2)(s + c^2)} \quad (s = \xi, \eta, \zeta)
\]

### Analytical solution

\[
H_x = \frac{H_{0x}}{1 + X(-1 + \mu_r)}
\]

\[
H_y = \frac{H_{0y}}{1 + Y(-1 + \mu_r)}
\]

\[
H_z = \frac{H_{0z}}{1 + Z(-1 + \mu_r)}
\]

\[
X = \frac{abc}{2} \int_0^\infty \frac{ds}{(a^2 + s)R}
\]

\[
Y = \frac{abc}{2} \int_0^\infty \frac{ds}{(b^2 + s)R}
\]

\[
Z = \frac{abc}{2} \int_0^\infty \frac{ds}{(c^2 + s)R}
\]

\[
R = \sqrt{(a^2 + s)(b^2 + s)(c^2 + s)}
\]
Magnetic substance torque in the ellipsoid

Uniform magnetic field in the x-y plane

\[ H_{0x} = H_0 \cos \theta \]
\[ H_{0y} = H_0 \sin \theta \]
\[ H_{0z} = 0 \]

Magnetic torque:

\[ T = \frac{\partial U}{\partial \theta} \]
\[ T = \frac{V \mu}{\mu_0} \left( \frac{1}{(1 + X(-1 + \mu_r))^2} + \frac{1}{(1 + Y(-1 + \mu_r))^2} \right) B^2 \cos \theta \sin \theta \]

Parameter depending on magnetic permeability and shape Ms.

\[ T = M_s B^2 \cos \theta \sin \theta = M_n B_x B_y \]

① The magnitude is proportional to the square of the geomagnetic field strength.
② The torque line can be expressed with a sine curve with period \( \pi \).
Dipole and magnetic substance attitude disturbances

- Attitude disturbance caused in a magnet.
  - Proportional to an outer magnetic field.
  - Can be expressed as a sine curve with period $2\pi$
    \[ T = M_d B \sin \theta \]
    Residual magnetic moment

- Attitude disturbance caused in magnetic substances (in ellipsoid body)
  - Proportional to the square of an outer magnetic field.
  - Can be expressed as a sine curve with period $\pi$
    \[ T = M_n B^2 \cos \theta \sin \theta \]
    Magnetic substance disturbance coefficient.

- Can we express the torque in arbitrary magnetic substance with the same equation $T = M_s B^2 \sin 2\theta$?
Magnetic substance torque in an arbitrary-shape magnetic substance

A small volume $dV$ (small enough to consider that the magnetization in the volume is constant)

Uniform magnetic field in the $x$-$y$ plane.
(small enough to consider that the B-H curve is linear, and the magnetic permeability is constant)

$B_0 = \begin{pmatrix} B_0 \cos \theta \\ B_0 \sin \theta \\ 0 \end{pmatrix}$
Magnetization in an arbitrary-shape magnetic substance

Magnetization in a small volume

\[ M_i = \chi H_0 dV_i, \]

\[ M_{v_i} = \begin{pmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{pmatrix} v_i \begin{pmatrix} H_{0x} \\ H_{0y} \\ H_{0z} \end{pmatrix} dV_i, \]

\[ B_0 = \begin{pmatrix} B_0 \cos \theta \\ B_0 \sin \theta \\ 0 \end{pmatrix} \]

Magnetic energy in a small volume

\[ dU_{V_i} = \frac{1}{2} M_{v_i} \cdot H_0 \mu_0 \]

Magnetic energy in the whole body

\[ U = \frac{\mu_0}{2} \int_V M \cdot H_0 dV \]

\[ X_{jk} = \mu_0 \int_{V_i} \chi_{jk} dV_i. \]
Magnetic substance torque
in an arbitrary-shape magnetic substance

• Magnetic energy

\[ U = \frac{1}{2} \mu_0 \left( X_{xx} X_{xy} X_{xz} \right) \left( \begin{array}{c} H_{0x} \\ H_{0y} \\ H_{0z} \end{array} \right) \cdot \left( \begin{array}{c} H_{0x} \\ H_{0y} \\ H_{0z} \end{array} \right), \]

\[ U = \frac{1}{2} \mu_0 H_0^2 (A + B \cos 2\theta + C \sin 2\theta) \]

• Magnetic torque

\[ T = \frac{dU}{d\theta} = \mu_0 H_0^2 (-B \sin 2\theta + C \cos 2\theta) = \mu_0 H_0^2 A' \sin(2\theta + \delta). \]
Magnetic substance disturbance in Cubesat XI-V

- Hysteresis damper
  - $5 \times 10^{-8}$ Nm

- Antenna
  - $4 \times 10^{-6}$ Nm

Table 1 Attitude disturbance XI-V

<table>
<thead>
<tr>
<th>Torque (Nm)</th>
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<tbody>
<tr>
<td>Residual magnetic (permanent magnet)</td>
</tr>
<tr>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Aerodynamic</td>
</tr>
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The torque is coarsely calculated using the ellipsoidal model.
FEM analysis

Fig. 1 Antenna

Fig. 2 Hysteresis damper

Fig. 3 Magnetic torque (Antenna)

Fig. 4 Magnetic torque (Hysteresis damper)
Conclusions

- Nonsymmetrical magnetic substances in a uniform magnetic field cause a torque.
- Magnetic substances in a satellite causes attitude disturbance torque.
- The magnitude of the disturbance is almost the same magnitude of the other attitude disturbances in the worst case scenario.