Children's Mathematical Engagement Based on Their Awareness of Coding Toy Design Features

Abstract

Tangible coding toys are designed to make coding accessible to young children, and because of their tangible and spatial nature, they are also viewed as tools for engaging children in mathematics. To best leverage the mathematics afforded by these toys, research is needed to understand children's awareness of the toys' design features and how they afford engagement with mathematics. We conducted a qualitative study of 106 5-to-6-year-old children completing coding tasks (42 hours of video). Our research questions focused on children's perceptions of design features, mathematical engagement, and how different design features afforded mathematics engagement. Results indicated (a) children perceived a variety of design features across coding toys; (b) children engaged in mathematics while perceiving features; and (c) distinct design features afforded engagement in specific mathematical concepts and skills, importantly, unit construction and coordination as well as spatial thinking. Implications include instructional strategies for making use of design features to elicit mathematical engagement; product design suggestions to forefront mathematical eliciting features; and the development of theoretical relationships between coding toy design features and mathematics.

Introduction

Advancement in early childhood coding technologies provide opportunities for children to engage with mathematics in novel ways. Researchers began to investigate the types of mathematical concepts and skills young children could access through interacting with coding tools such as LOGO in the late 1970s and 1980s (Clements & Battista, 1989; Papert, 1980). While research on the affordances of coding interfaces for mathematical learning did not go away, recent interest in bringing coding into K-5 schooling has led to an influx in coding tools available to classrooms and has rekindled interest in research on the kinds of mathematical learning that is afforded by such tools in early childhood classrooms (e.g., Angeli & Valenides, 2019; Bers et al., 2014, 2019; Murcia & Tang, 2019; Shumway et al., 2023). These coding tools range from screen-free tangible coding toys to screen-based and hybrid interfaces (Hamilton et al., 2020; Yu & Roque, 2019). In particular, the screen-free tangible coding toys (examples in Figure 4) are a promising choice for U.S. preschool and kindergarten classrooms (Bers et al., 2019) due to recommendations on limited daily screen time (American Academy of Pediatrics [AAP], 2016). In addition and more globally, tangible coding toys are viewed as promising tools in integrated STEM and interdisciplinary mathematics education efforts (Goos et al., 2023), 21st-century skills learning (Keane, 2023), and playful learning (Bers et al., 2019; Heljakka & Ihamäki, 2019).

Accordingly, researchers have begun to identify different physical design features of tangible coding toys, which are the components of the coding toy that can be visually perceived or can be physically manipulated during a child-coding toy interaction (Hamilton et al., 2020). However, the design features have not yet been investigated in terms of their affordances for mathematics. Because little is known about coding toy design features and their relationship to mathematics, the present study was developed to examine kindergarten-aged children’s awareness of coding toys design features and to understand how they afforded children’s engagement with mathematics. It is important to elucidate that this study is not about the development of mathematical thinking; it is aimed at understanding how children engage in mathematical concepts and skills while perceiving design features. Three research questions guided this study:

1. What design features do kindergarten-aged children perceive and use when interacting with four different coding toys?
2. What mathematics do kindergarten-aged children engage in when they are perceiving design features of four different coding toys?
3. How do design features of four different coding toys afford kindergarten-aged children's mathematical engagement?

Literature Review

Research on Technology Tools' Affordances for Mathematics Learning

Extensive research on physical and virtual manipulatives identifies affordances for mathematics learning (Carbonneau et al., 2013; Desoete et al., 2016; Guarino et al., 2013; Lesh & Johnson, 1976; Manches & O'Malley, 2016; Moyer-Packenham & Westenskow, 2013; Paek, 2012). For example, Mix (2009) offers four categories of affordances of physical manipulatives: *offloading intelligence, focusing attention, representing conceptual metaphors*, and *generating action*. Moyer-Packenham and Westenskow (2013, 2016) conducted two meta-analyses on the effects of virtual manipulatives on children's achievement and learning and identified five categories of virtual manipulative affordances that consistently supported mathematics learning: *focused constraint, creative variation, simultaneous linking, efficient precision*, and *motivation* (p. 2013, p. 35).

Extending the research on affordances of virtual manipulatives, Moyer-Packenham et al. (2020) conducted studies on the design features of virtual manipulatives within digital games that were hypothesized to afford mathematics learning. They examined 193 elementary children's digital game play on iPads and found that three specific design features (i.e., *providing information, manipulable math objects*, and *focused constraint*) had unique benefits when they were perceived by the children. Bullock et al. (2017) also reported that, in order to take advantage of the potentially beneficial mathematical learning afforded by digital games' design features, children must be aware of the design features. Taken together, this research demonstrates that not all design features are high quality, and children miss the potentially beneficial affordances of high-quality design features if they are unaware of them. Therefore, it is hypothesized that design features of screen-free tangible coding toys – which share tangible and digital characteristics with physical and virtual manipulatives – also play an important role in supporting children's interactions and learning with mathematics when they are aware of the design features of these tools.

The Case for Research on Coding Toys' Design Features

Research on early childhood coding toys indicates potential mathematical benefits for young children (Moore et al., 2020; Murcia & Tang, 2019; Palmér, 2017; Shumway et al. 2019, 2023). However, the research is limited regarding how the coding toys' design features specifically afford mathematics learning, with few exceptions (Clarke-Midura et al., 2019; Hamilton, et al., 2020; Yu & Roque, 2019). Yu and Roque's (2019) examination of current early childhood computer science computational kits (which include coding toys) found that design features varied across kits. Sometimes there were differences in design features across platforms, such as one coding toy being programmed directly from the body and another coding toy being programmed from a separate input mechanism. Sometimes there were similarities in design features across platforms, such as all coding toys incorporating flashing lights or sounds. These findings show the beginnings of identifying different design features of coding toys, yet they lack specificity on how design features afford students' engagement with mathematics.

Early Childhood Mathematics and Coding Toys

Literature highlights that coding toys have the possibility to engage young children with at least three specific mathematical concepts: (a) number concepts, (b) spatial concepts, and (c) measurement concepts.

Early Childhood Research on Number Concepts and Coding Toys

Research suggests that young children use number concepts as they engage in coding toy tasks (e.g., Fessakis et al., 2013; Moore et al., 2020; Nam et al., 2019; Palmér, 2017; Shumway et al., 2023; Sung et al., 2017). Across these studies, certain number concepts emerged in coding toy contexts as children corresponded counting numbers to coding tiles or arrows (coordination), referenced the total quantity of different physical aspects of the coding toy environment (cardinality), or added and subtracted certain codes to accomplish programming goals (operations). Important to the current study, Shumway et al. (2023) implemented coding toy tasks with 36 kindergarten students and qualitatively analyzed the data to document the mathematics that emerged as children participated in the tasks. The number concepts skills that were documented in the results were *counting, counting on, coordinating counts,* and *operations.*

Beyond mathematics just emerging in these coding toy contexts, Nam et al. (2019) suggests that engaging with coding toys can improve the use of numbers in mathematical problem-solving. In a quasi-experimental study with 53 Korean kindergarten children, half of the children participated in 12 typical classroom instructional sessions (control) and the other half participated in 12 coding toy activities with TurtleBot (experimental). The Turtlebot activities involved children doing a range of tasks, including mastering basic functions, directing the coding toy to go here and there, and creating a dance with the TurtleBot. A pre- to post-test number assessment indicated that the experimental TurtleBot group significantly outperformed their peers on the post test.

Early Childhood Research on Spatial Concepts with Coding Toys

The National Research Council (2006) described spatial thinking as physically and mentally orienting oneself in space, and is comprised of space concepts, representational tools, and reasoning processes. On top of the strong correlation between spatial ability and success in STEM careers (Wai et al., 2009), research shows that early spatial skills are a strong predictor of mathematics achievement at age seven (Gilligan et al., 2017) and in later years (Cross et. al., 2009). That being said, there continues to lack attention to spatial development in early years instruction (Pritulsky et al., 2020).

Papert's (1972) LOGO and Turtle Geometry were foundational in the emergence of spatial mathematics and coding toys. A cybernetic turtle in Turtle Geometry represented a virtual point that could be moved around and programmed using directional movement and rotational commands. Papert (1980) described how spatial mathematics through engagement with Turtle Geometry was important for young children:

> A Turtle is at some place–it, too, has a position–but it also faces some direction–its heading. In this, the turtle is like a person–I am *here* and I am facing north—or an animal or a boat. And from these similarities comes the Turtle's special ability to serve as a first representative of formal mathematics for a child. Children can *identify* with the Turtle and are thus able to bring their knowledge about their bodies and how they move into the work of learning formal geometry. (pp. 55-56)

Papert was interested in the ways that young children perceived the different spatial organizations of the Turtle, and how spatial referencing and development was linked to mathematics. Researchers have extended Papert's work to further investigate spatial mathematics in similar environments (Berson et al., 2023; Clark-Midura et al., 2021; Clements & Battista, 1989; Clements et al., 1996; Cittá et al., 2019; Cuneo, 1985).

Recent studies have also demonstrated how spatial mathematics concepts emerge as young children play with coding toys (Berson et al., 2023; McClusky et al., 2023; Moore et al., 2020; Palmér, 2017; Shumway et al., 2019, 2023). For example, Palmér (2017) studied eight, 3- to 5-year-old children by giving them a pretest on basic programming, providing three to four weeks of a ‘body coding’ intervention, and then providing another basic programming posttest. During the intervention phase, children programmed the researcher around the room by saying words to move the person. These intervention activities progressed until children were putting paper arrows on a grid to program a robot to move around. Results indicated that children mentally compared the grid map to the real life-sized map, and associated movements with symbols.

Early Childhood Measurement in Coding Toy Contexts

Researchers have argued that coding toy contexts can be especially supportive of children’s exploration of measurement concepts (Murcia & Tang, 2019; Shumway et al., 2019, 2023; Winters et al., 2020). Winters et al. (2020) created a progression of instructional activities with K-2 children where they had experiences (a) observing and exploring, (b) interpreting, (3) developing and writing, and (d) critiquing and refining, as they played with two coding toys named Ozobot and Bee-Bot. Teacher observations indicated that children engaged with measurement concepts and made length estimations as they tried to figure out the distance Bee-Bot would move. Additionally, children employed the use of metersticks to standardize movements and use units of measurement.

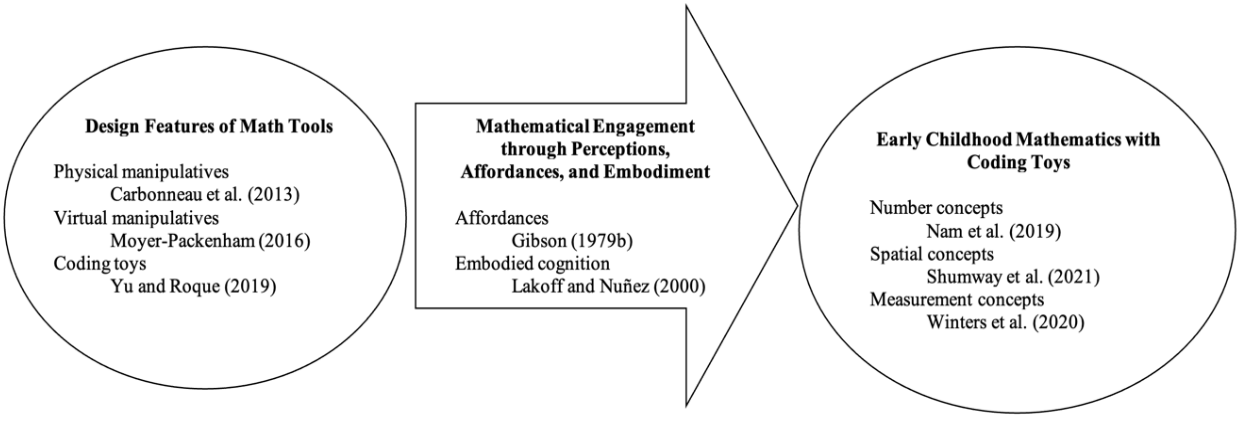
The measurement concept of linear units is a complicated one for young children. Research is mixed on young children’s readiness to understand that a unit can be either a linear measurement or a discrete, countable entity (Friso-van de Bos et al., 2018; Sarama et al., 2022; Smith III et al., 2013; Solomon et al., 2015; Welch et al., 2022). Research has consistently documented the pervasive struggles young children have with understanding that the space between two hash marks can be a unit – rather than the hashmarks themselves (Smith III et al., 2013; Solomon et al., 2015). Although there are known challenges, Sarama et al. (2022) explored tasks to support learning trajectory gains in linear unit understanding with 35 young children and found strategies that were successful such as: focusing on mental and physical iteration, having children take a continuous quantity and deconstruct it into an abutted set of unit pieces, and starting instruction with a standardized unit size to build stable understanding before implementing multiple non-standard units. Similarly, Welch et al. (2022) used a case study approach to understand how a group of four children expressed their emergent understanding of a linear movement with a coding toy. They found that children used gestures to mimic the movement of the toy along a straight path and paired that with verbalizations that aligned to linear measurement units.

Theoretical Lens: Affordance Theory and Embodied Cognition

Expanding on the research above, the present study was theoretically informed by Gibson's notion of affordances (1977, 1979b) and Lakoff and colleague's model of embodied cognition (Lakoff & Johnson, 1999; Lakoff & Nunez, 2000). These theoretical perspectives were used to interpret the design features of coding toys (Figure 1, Oval 1) for early childhood mathematics (Figure 1, Oval 2).

Figure 1

Relationship Between Design Features, Early Childhood Mathematics, and Engagement



When data were analyzed, we viewed affordances as “cues of the potential uses of an artefact by an agent in a given environment” (Burlamaqui & Dong, 2014, p. 13). Affordance Theory is based on the “complementarity of the animal and the environment” (Gibson, 1979b, p. 56). I this conception, environmental objects have an inherent influence on perceptions and actions of an individual. The main way that affordance theory diverges from orthodox psychology is that it relates our classification and understanding of objects not principally to their qualities of properties, but by the affordances they offer the individual. Adopting this theoretical perspective helped us interpret children’s perceptions of design features of the coding toys. It also helped explain how perception precipitated engagement in mathematics.

Due to the dynamic nature of coding toys, children generally employed their bodies in a physical way during coding toy activities in this study. For this reason, theoretical conceptions of embodied cognition allowed us to situate understanding of mathematical engagement through children’s physical interactions with the coding toy environment (e.g., gestures, body turning, body movement). Embodied cognition purports that our sensory-motor interactions are the components and nature of cognition, and challenges a number of amodal symbol systems models. In his work with Logo and a programmable Turtle, Papert (1980) describes activities as syntonic, and because of that, “encourages the conscious, deliberate use of problem-solving and mathematic strategies” (pp. 63-64). Syntonicity, a term coined by Papert, describes learning episodes where children connect to the activities with their bodies, allowing them to form a personal association between sensory-motor perception (i.e., sensory engagement) and cognition. Papert’s conception of syntonicity helps frame thinking about embodied cognition in a coding toy setting.

Method

Research Design

Data for the present study were collected as part of a large NSF-funded Design Based Research (DBR; Cobb et al., 2003; diSessa & Cobb, 2004) project. DBR is characterized by interventions that establish long lasting relationships with participants, iteratively implement and revise intervention designs, and carefully examine data gathered from multiple cycles to document changes in effectiveness, learning, and theory. This study needed to iteratively develop and test new coding toy lessons, and carefully analyze participant interactions during the lessons. Rather than do so in an out-of-classroom environment, this DBR project sought to acknowledge that only in an active classroom setting could we understand participant interactions situated within a classroom context, over time (Confrey & Lachance, 2012; Steffe & Thompson, 2012).

Research Context: The Participants, Sites, Coding Toys, and Tasks

Our participants were 106, 5- and 6-year-old kindergarten children (47 females, 59 males) from six different public and private schools (sites) in the Western United States. We obtained the school district's approval and participants' informed consent according to the university's Institutional Review Board (ethics) guidelines prior to any data collection. Participants interacted with curricular tasks around four coding toys in their kindergarten classrooms: (a) Bee-Bot by Terrapin, (b) Code-a-pillar by Fisher Price, (c) Botley by Learning Resource, and (d) Cubetto by Primo Toys (Figure 2).

Figure 2

Four Coding Toys Used in this Study

	Bee-Bot	Code-a-Pillar	Botley	Cubetto
Coding Toy				

Each coding toy system is commercially available, designed for young children, and is advertised to support problem-solving and coding skills. Additionally, elements within each coding toy system can be schematized such as a body (agent), some way of telling it what to do (input mechanism), specific chunks of information (codes), and a space for it to move (grid). Botley is described below to provide context for how the coding toys were implemented.

Botley by Learning Resources

Botley, by Learning Resources (Figure 3), includes the Botley body (agent), a remote control that programs Botley (input mechanism), and grid squares that provide a path for Botley's movement. Supplementary materials include a goal and a ball, and a researcher-created magnetic program organizer to plan and organize codes. All of these are specific design features of the Botley coding toy system. Some design features of Botley cannot be observed in Figure 3. See Table 2 for the corpus of design features for each coding toy.

Figure 3

Botley with Select Design Features

Botley is controlled using the remote control, which keeps track of button pushes (e.g., codes such as move forward, move forward, rotate left). When the ‘enact’ button is pressed, Botley enacts the sequence of codes stored in the remote control. The grid squares align with Botley movements so that one movement translates the Botley from the center of one grid square to another. In Figure 3, children are attempting to get Botley to drive forward three movements and place the ball (blue) in the goal (orange).

Data Source and Data Collection

The data for this study are video recordings of 84 lessons using the four coding toys described above (42 hours total). Table 1 presents a breakdown of the video dataset.

Table 1

Number of Lessons in Dataset for Each Coding Toy by Site

	Number of lessons						Total	
Coding Toy	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Lessons	Hours
Bee-Bot				8			8	4
Code-a-pillar	8	8					16	8
Botley	2		8	8	8	4	30	15
Cubetto	2		8	8	8	4	30	15
Total (hrs.)	12 (6)	8 (4)	16 (8)	24 (12)	16 (8)	8 (4)	84	42

Note. Each lesson lasted approximately 30 minutes. The total hours of data equaled the total lessons multiplied by .5 hours. For example, total hours of video data from Site 1 totaled 6 hours of video data: 12 lessons x .5 hours = 6 hours of video data.

Two members of the research team were assigned to a group of ~4 children who worked on a coding toy activity. These two researchers worked as a pair in planning, implementing, and refining the coding toy activities for the children. Prior to the lesson, each researcher within a pair was assigned a different role (i.e., teacher-researcher, videographer-researcher). The teacher-researcher presented the task, guided and prompted children’s thinking, and provided collaboration scaffolding (e.g., turn-taking, group work logistics). The teacher-researcher urged problem-solving by asking questions such as “Why do you think that will work?”, “Do you all agree with this strategy?", or "What is another strategy you think is worth trying?" The videographer-researcher made sure the video camera captured participants' verbal and physical interactions and took detailed notes on a design memo about critical events during the teaching episode. The videographer-researcher moved around the activity space to capture the interactions.

The coding toy activities implemented by the researchers varied by year, school, and coding toy, but had some general similarities. The similarities important to the current study are: (a) each group of children participated in two lessons with at least one coding toy – considered introductory lessons; (b) videos involved children learning to use basic codes to program the coding toys (i.e., forward, backward, rotate left, rotate right); (c) videos captured children actively engaged in testing and trying, rather than listening to the facilitator; and (d) each videoed lesson lasted approximately 30-minutes. A common challenge across activities was to have the children start the coding toy on point A and collaborate with one another to get it to land on point B. The small group of children were handed the set of resources (e.g., coding toy, remote, coding tiles, program organizer) and were given necessary autonomy to work together to solve the challenge. Progress in each activity typically involved the children coming up with an initial program (program writing); enacting the program (implementation); seeing that it went off the grid squares or went the wrong direction (evaluation), and then changing their program to try and get it to do something different, possibly more accurate, in their next implementation of the program (debugging).

Data Analysis

We implemented a multi-phased qualitative video analysis (DeCuir-Gunby et al., 2012; Erickson, 2006) using MAXQDA software (VERBI, 2022) with the unit of analysis being specific interactions of individual children working in a small group. What this means in the data is that anytime the small group of children interacted with the coding toy and demonstrated perception of design features or engagement in mathematics, the segment was marked in MAXQDA as a specific interaction. We used descriptive and process coding (Charmaz, 2002; Corbin & Strauss, 2015; Saldaña, 2021) to answer RQ1 about design feature perception, a-priori and open coding (Saldaña, 2021; Shumway et al., 2023) to answer RQ2 about mathematical engagement, and causation coding (Saldaña, 2021) with a variable-oriented strategy (Miles et al., 2020) to answer RQ3 on how design features afforded mathematical engagement. Our theoretical framework directly guided our data analysis by informing our decisions about physical toy design features (RQ1); how coding toys support early childhood mathematics (RQ2); and how inherent affordances of the features may afford mathematical engagement, possibly through unique embodied manifestations (RQ3).

Procedurally, the video data was stored on an encrypted system and uploaded onto MAXQDA software. This software allowed the video data to be analyzed and portions of the videos to be marked with codes. The first step was the descriptive and process coding, which involved (a) creating a comprehensive codebook of design features for each coding toy, and (b) documenting when children perceived the features through verbalization, gestures, use, or visual ques that were directly targeted toward a feature. This codebook of design features was iteratively analyzed and modified by an eight-person research team over three months, where video data segments of participants using the coding toys would be presented on a large screen in front of the entire team for group analysis. This phase of coding was complete when we achieved a comprehensive codebook of design features for each toy, and the video data was coded for instances children perceived those design features (e.g., *tapped* [perception] *_grid squares* [design feature]). The second round of coding was focused on just those portions of previously coded video data, which were then coded with additional mathematical concepts or skills that were observed. In this way, we were able to document the mathematics children engaged in while they perceived design features (e.g., *tapped*[perception]_*grid square*[design feature]_*counting, discrete unit, coordination*[mathematics]). Finally, during the third round of coding, all the portions of video data that had both design feature codes and mathematical engagement codes were analyzed using causation coding to see if there were patterns within and across toys regarding a relationship between design feature and mathematics. For example, a portion of the data that was coded (*tapped*[perception]_*grid square*[design feature]_*counting, discrete unit, coordination*[mathematics]) would be qualitatively analyzed for the relationship between the *tapping on the grid* and the *coordination, counting,* and *discrete unit*. The resulting analysis documentation was "*the discrete grid squares were sequentially touched by the child with a finger, and the child counted '1, 2, 3, 4' with each counting word corresponding to each touch*."

Results

In this results section, we build up to the main analysis of how design features afforded kindergarten children's mathematical engagement (RQ3). First, though, we set the stage for this main analysis by reporting 1) the design features we observed children perceiving and using in their interactions with the coding toys, and 2) the mathematics children engaged with when they perceived the coding toy design features.

Design Feature Perception and Use

Affordances refer to possibilities that the agent has for action (Gibson, 1979), and hence, for this study, it is important to understand what design features children perceived when interacting with the coding toys that could lead to the potential engagement in mathematics. Table 2 organizes the results with the design features for each coding toy sorted into systems of use by columns. Two notable design features that emerged in this table were grid squares (Figure 4) and command arrows (Figure 5). All four coding toys had the design feature “grid square” within the environmental system, and while this afforded spatial movement across all coding toys, the type of movement the feature afforded differed based on the size of the grid square (and toy’s movements on the grid squares).When interacting with Botley, Cubetto, and Bee-Bot (6 in^2 grid squares), children usually engaged with the grid squares through gestures and verbalizations, while Code-a-pillar (3 ft^2 grid squares) often afforded full body use.

Command arrow design features were found across systems of use and these features afforded engagement in spatial mathematics in planning and commanding (i.e., executing/enacting) programs. In other words, Bee-Bot and Botley have two sets of arrows (i.e., for commanding, for planning) and Code-a-pillar and Cubetto have one set of arrows/tiles (i.e., for commanding). Additionally, the command arrows positions and purposes afforded various spatial coordination. Bee-Bot and Botley have arrow cards for planning a program that must be coordinated with pushing codes to enact the plan, while Code-a-pillar's arrows are attached directly to the body and Cubetto's command tiles are placed directly on the programming board (thereby, not needing to coordinate the planning and enactment). Children perceived and used the command arrows/tiles in a variety of ways, including matching command arrows with planning arrows, holding arrows up to see which way they were facing, counting arrows, and describing the directional shape of the arrows. Hence, both the grid squares and command arrows of the coding toys are design features that could be perceived and used, affording potentially varied mathematics embodiment, understanding, and skills.

Table 2

Design Features of the Four Coding Toys

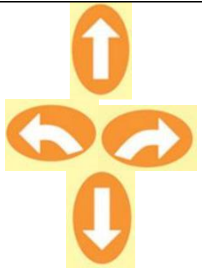
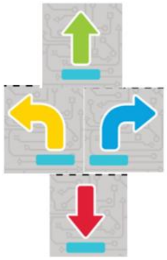
Coding Toy	Body	Separate Controller	Environment	Program Organizer
Bee-Bot	Stops w/ Codes Codes on Body Eyes Flash Light for Code Lights at End Face on Body X Button	N/A	6X6 Grid Map Pictures	Code Cards Seq. Spaces Bottom Line
Code-a-pillar	Coding Arrows Cont. Moving Light for Code Wall Hit Light Face on Body Ending Song Sing w/ Motion Colored Codes Codes on Body	N/A	3X3 Grid Grid Pictures	N/A
Botley	Cont. Moving Light for Code Beep for Code Face on Body On/Off Voice Pause Whistle Say's "WEEE"	Colored Codes Trash Can Flash for Button	Adj. Grid Ball Goal Flags Barriers	Magnet Cod. Seq. Spaces Preset Prog.
Cubetto	Stops w/ Codes Separate Body Beep w/ Codes Beeps at End Face on Body Slow Motion	Cod. Arrow/Tiles Col. Arrow/Tiles Prog. Board Line Con. Holes Back and Forth Flash w/ Code	6X6 Grid Human Loc. Comp. Rose	N/A

Figure 4

The Four Grid Square Design Features

Figure 5

Coding Toy Command Arrows and Tiles

Coding Toy	Bee-Bot	Code-a-Pillar	Botley	Cubetto
Command Arrows/ Tiles				

Engagement with Mathematical Concepts During Coding Toy Interactions

Results from our second research question about mathematical engagement indicated that coding toy interactions prompted children’s engagement with mathematical concepts and skills in five broad mathematical topics: spatial reasoning, geometry, comparison, measurement, and number (Table 3). These concepts and skills are a mix of a-priori codes (Shumway et al., 2023) and open codes from the current analysis (See codebook; Table 4). We focus on two important patterns from Table 3 which involve (a) spatial reasoning and, (b) counting, linear/discrete unit, and coordination.

Table 3

Mathematical Concepts and Skills Children Engaged in During Perception and Use of Design Features

Math concepts and skills	Bee-Bot	Code-a-pillar	Botley	Cubetto
Spatial Reasoning				
Spatial orientation	Observed	Observed	Observed	Observed
Estimation	Observed	Observed	Observed	Observed
Matching symbols	Observed	Observed	Observed	Observed
Visualization: URF[a]	Observed	Observed	Observed	Observed
Geometry				
Describing location	Observed	Observed	Not observed	Observed
Describing shapes	Not observed	Observed	Observed	Observed
Comparison				
Matching movements	Observed	Observed	Observed	Observed
More/less/same	Observed	Observed	Observed	Observed
Coordination	Observed	Observed	Observed	Observed
Patterning	Not observed	Observed	Observed	Observed
Measurement				
Angle	Not observed	Observed	Not observed	Not observed
Linear/discrete unit	Observed	Observed	Observed	Observed
Velocity	Not observed	Not observed	Observed	Observed
Number				
Multipl. reasoning	Not observed	Not observed	Observed	Not observed
Decomposition	Not observed	Observed	Not observed	Observed

Counting on	Observed	Not observed	Observed	Observed
Subitizing/cardinality	Observed	Observed	Observed	Observed
Counting	Observed	Observed	Observed	Observed
Subtraction	Observed	Observed	Observed	Observed
Addition	Observed	Observed	Observed	Observed
Sequencing	Observed	Observed	Observed	Observed

Note. A-priori codes for concepts and skills developed by Shumway et al., 2023. Green is used to highlight mathematical concepts and skills observed with each coding toy.

[a] Updating Reference Frame

Table 4

Open Codes that Emerged from Analysis

Thematized Math Topics	Open Math Codes	Description	Sample of Indicators
Geometry	Location Description	Child verbalizes relativity of the coding toy to another thing using mathematical language such as next to, besides, on top of, underneath, around.	● Child uses mathematical language such as above, besides, around, next to, passing
	Shape Description	Child describes a geometrical shape while engaging in the coding activity	● Child says, "Look, it makes a circle!" ● Child says, "The board is in the shape of an L!"
Comparison	Matching Movements	Child attempts to imitate, mimic, or match individual movements or a path	● Child simulates the coding toys with their hand to show an existing path. ● Child traces a path with their fingers that is supposed to match another existing path.
	Comparing Quantity	Child compares two or more things using mathematical language such as more than, less than, same as	● Child uses mathematical terms such as: more than, less than, same as.
	Patterning	Child acknowledges some sort of pattern; repeating or singular module.	● Child verbally mentions a pattern. ● Child describes a pattern (e.g., forward, right, right! Forward, right, right!)
Measurement	Angle	Child reasons with various angles other than 90 degrees.	● Child says, "it needs to turn this much!" and gestures an angle in the air. ● Child places the coding toy on the mat in a way indicating a non 90-degree angle.
	Velocity	Child perceives some aspect of speed	● Child says, "This toy is fast!" ● Child says, "This toy is slow!"
Number	Multiplicative Reasoning	Child uses numbers and quantity using multiplicative reasoning rather than additive.	● Child says, "I need to use one green forward, three times!"
	Subitizing & Cardinality	Child says the amount of a quantity without explicitly counting, by looking at accumulated sets of objects	● Child says, "it needs four!" without counting. ● Child previously counted an amount, and then verbalizes the whole set.
	Sequencing	Child attends to the order of things, such as	● Child uses mathematical language such as first,

		something being first, next, or last.	next, or last. ● Child sequences command arrows in environment.
	Decomposing	Child explicitly breaks numbers or paths apart.	● Child says “Bee-Bot did this part first, then did this part second!”
Spatial	Visual Estimation	Child estimates ending location of a coding toy without enacting codes or individual movements	● Child points to a grid square when asked where the coding toy will end. ● Child verbally approves or disapproves of whether a program will get the coding toy to a destination.

Children Engaged Spatial Reasoning Concepts and Skills with all Coding Toys

One pattern from Table 3 is that children engaged with all spatial reasoning concepts and skills with all four coding toys. One example was children’s demonstration of “Visualization: Updating Reference Frame (URF)” with all four coding toys, which was observed when children made hand gestures in the air to show they were changing orientation or planning a path using mental images. As children spatially visualized paths and movements, they recreated mental maps – imaginal updating – based on the face that was on the side of Cubetto; they had to visualize a new path based on the new reference frame that the Cubetto’s face was using. For example, Kylee’s (pseudonym) teacher posed the task: “Botley wants to look at the ladybug. Can you get Botley to turn and look at the ladybug?” Kylee made a turn gesture with a cocked hand and bent elbow and declared, “It needs a turn!” Her partner then programed a backwards code, and upon seeing that this motion was not what she wanted, Kylee repeated her previous turn gesture and declared, “No, it needs to do this [gesturing] like turn around!” Kylee’s mental visualized action was communicated with her own body and supported by the face on the Botley body (a design feature of the coding toy). She was using the face on Botley to visualize intended reorientations and changes in position, and engaged her body to communicate ideas. This engagement of her body to communicate ideas allowed us to see spatial visualization, which is inherently an internal concept that is difficult to observe.

Children Engaged in Counting, Linear/Discrete Unit, and Coordination with all Coding Toys

A second pattern from Table 3 was that children engaged in counting, linear/discrete unit, and coordination (e.g., coordinating number words, movements, grid squares (units), button pushes, and command arrows), and typically, children engaged in these three mathematical concepts and skills concurrently. A few examples of children coordinating include counting grid squares and then counting that same number of forward/backward command arrows; moving their bodies on the large Code-a-pillar mat, and saying the specific body code segment (e.g., forward, backward, right rotation) that matched that movement; and counting movements of the coding toy and then acquiring the same number or command arrows/tiles. Within each of these examples, notice that a unit is defined (e.g., one movement or one command arrow) and counted, which implicitly involves some kind of coordination of quantities. Figure 6 highlights an example of engagement in counting, linear/discrete unit, and coordination when a group of children coded Cubetto. The teacher-researcher prompted the children to get the Cubetto to go four spaces backwards to land on the tree. In Pane A, the children touched the squares and verbally counted, “1, 2, 3, 4,” coordinating discrete grid square units with counting words. Then, they sorted through the pile of codes, pulled out four purple backwards codes, and placed them on the programming board (Pane B). After they enacted the program to see if it worked, the child pointed to each tile as she counted, and the child held up counting fingers to match each counting number (Pane C). This shows various instantiations of coordinating, including coordinating counting of the grid squares with the number of purple backward arrows on the programming board, coordinating the tiles with movements (linear units), and coordinating finger counts with verbal counts. Hence, children's interactions with the toy led to engagement in coordination of counting, units, and quantities. This was likely afforded by the design features of the grid squares, command arrows, and toy movements, which we discuss next and was the focus of the third research question.

Figure 6

Children Coordinate Counting of Grid Spaces with Command Arrows with Cubetto

Design Features Affording Mathematical Engagement

The third research question and main analysis for this study focused on how design features afforded children's mathematical engagement. We investigated this question by analyzing the overlapping design feature and mathematics codes. We report on design features that afforded mathematical engagement similarly across all the coding toys as well as less common but distinct design features of coding toys that afforded mathematical engagement in unique ways. We present these results within two main themes that emerged: (a) design features that afforded unit construction and coordination, and (b) design features that afforded spatial thinking.

Design Features that Afforded Unit Construction and Coordination

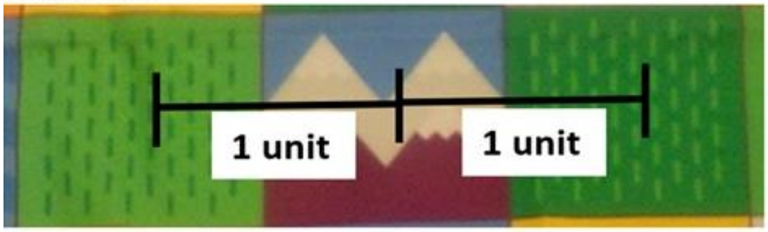
We found that the design features of (a) grid squares, (b) stops between motion, and (c) lights and sounds afforded children's engagement with unit construction and units coordination.

Grid Squares Afforded Linear/Discrete Unit Construction. There was a similar relationship across coding toys in the way grid square design features afforded construction of linear and discrete units. Figure 7 highlights this relationship.

Figure 7

Grid Squares for Discrete and Linear Units Construction

Discrete Units



Linear Units

For example, as a child tried to program Botley to go two squares forward, initially, the child touched each square, including the square that Botley started on and counted “1, 2, 3, it needs three” (see Figure 8). Pane A shows his physical touch of a grid square and then verbalizing a counting number that corresponded to the touch of the discrete grid square (i.e., touch-count). This touch-count demonstrated his use of the grid squares as *discrete units*, each grid square represented a unit of one. After programming the three forwards and watching it go too far over the intended landing path, the child shifted to using *linear units*, or in other words, the distance unit of the toy’s forward movement. The child pointed his finger to the original starting spot of Botley, and instead of counting the point of his finger like in his initial attempt, he slid his finger in the air from the start position to the center of the next square and counted “1” (Pane B) and then he slid his finger from the center of that square to the center of the final square and counted “2, it needs 2.” This sliding of the finger and counting the slide (e.g., slide-count) indicated that the child shifted from counting the discrete squares (i.e., touch-count) to counting the linear movement from the center of one square to the center of another (i.e., slide-count); he constructed a linear unit. This shift showed a more dynamic and embodied understanding of movement in space, which was afforded by the grid squares’ structure, more clearly showing the child the distance of a movement—a length—due to the start and stop of the toy on the grid squares.

Figure 8

A Child Shifted from Counting Discrete Units to Counting Linear Units

This type of grid square-unit construction relationship was common across all four coding toys and was demonstrated by children similarly with the three coding toys that had 6in.^{2} grids. However, with the Code-a-pillar that had large 3ft.^{2} grids, children demonstrated engagement in units differently. Children moved larger parts of their bodies, and often, discrete units were recognized by children slapping the center of squares with their hands, and linear units were recognized by children making big sweeping motions with their arms. On these large grid squares, children also walked around and either said a number on top of each grid square upon arrival (discrete), or dragged out a long counting word as they moved between each grid square (linear).

Bee-Bot and Cubetto: Stops Between Movements Afford Units Coordination. The unique design feature of Bee-Bot and Cubetto's *stop-between movements* (i.e., a stop between each forward rather than a continuous movement of two or more forwards) afforded units coordination. Bee-Bot and Cubetto were designed to stop between enactment of each code, while Botley and Code-a-pillar were designed to continue moving without pausing between codes (though they have flashing lights or sounds). With the discrete movements of Bee-Bot and Cubetto, children coordinated counting words with individual movements, codes with individual movements, and distinct hand gestures with individual movements.

Unit coordination was often observed when children coordinated a counting word with a unit of linear movement. Children counted number words coordinated to one forward movement of the coding toys in real time, moving to the next counting word after the toy stopped and then initiated its next movement. For example, as Bee-Bot moved from one square to the next, children called out "one" and then "two." Additionally, children often dragged out or pronounced the syllables of counting words to match the movement (e.g., oonnee, twwoo) so that the duration of the counting word aligned to the entire unit of movement of the toy. The children continued this coordination between counting word and linear movement until Cubetto stopped moving (i.e., arrived at its ending location).

Lights and Sounds Afforded Coordination when Perceived. Similar across all four coding toys, there were a variety of light and sound design features, though children seldom perceived and used them. During the rare instances where children did perceive and use the lights and sounds, they engaged in mathematics through coordination of the quantities of lights and sounds of the coding toy and the codes they used to create the program. For example, one teacher-researcher prompted a child to look at the programming board while Cubetto was enacting a program. The child took her finger and pointed to the tile on the programming board as it flashed, turned her head quickly to look at Cubetto, and said "It's moving and it's blinking every time it does it!" In the subsequent lessons with this same child, she made various references to the flashing blue light and the movements of Cubetto. One instance was when the Cubetto enacted a program and she called out "The light's blinking, backwards!" Another example is when the Cubetto enacted a longer program of 10 codes, she reached forward in the middle of Cubetto's enactment of the program and started pointing at the code on the programming board that was being enacted. She used the blinking light as a reference to know which code was currently in use. These instances demonstrate how children made an explicit coordination between the light design features, movements, and command arrows, and, while not commonly perceived by children, it afforded engagement in coordination when perceived.

Design Features that Afforded Spatial Thinking

We found that the design features of (a) monochromatic command arrows, (b) command arrows on the coding toy body, and (c) large size of certain grid squares afforded spatial thinking.

Monochromatic Bee-Bot Command Arrows Afforded Spatial Orientation. The variation in colors of command arrows/tiles (codes) afforded spatial orientation concepts and skills in different ways. Three of the four coding toys (i.e., Cubetto, Botley, Code-a-pillar) had command arrows/tiles that were a different color for each command/direction (color-coded toys). Children used this coloration to identify each code. For example, the forward command for Cubetto, Botley, and Code-a-pillar are all green; the rotate right arrow for Botley is blue. In contrast, the arrows on Bee-Bot are all white (i.e., monochromatic-coded toy).

When children planned paths and programmed the color-coded toys, they took advantage of color terms to communicate their reasoning. However, when children did these same activities with the monochromatic-coded toy, they used spatial orientation language and gestures, spatial visualization, and symbols matching. The short transcript below highlights an incident where Tom used exclusively color words to describe the coding tiles to program Cubetto (color-coded toy) to match a specific path. In the excerpt, Tom watched a pre-programmed Cubetto move on the grid area. As it moved, Tom pointed to the Cubetto and called out colors. Then after Cubetto stopped, Tom started grabbing the colored tiles and programming his own Cubetto to match the program.

> Tom: [Presses the go program and watches the pre-programmed Cubetto rotate to the right] Red [watches the Cubetto move forward] Green [then stops talking as the Cubetto finishes by rotating left and moving forward]
>
> Researcher: Do you want to watch again or do you want to try?
>
> Tom: I’ll try.
>
> Researcher: Okay, you can code it when you’re ready. Actually, let’s watch it one more time. [Presses go on the pre-programmed Botley]
>
> Tom: [As the pre-programmed Cubetto is moving, Tom is programming his other Botley, and chants] Red [codes a red] Green [codes a green, and then codes a yellow and green without saying other words]

In this excerpt, Tom named the color of the codes to communicate and reason with the command tiles and the movements of Cubetto, rather than their corresponding directional actions (i.e., more forward, rotate right). In contrast, the following excerpt demonstrates Kyle using spatial language when planning a path with Bee-Bot, the monochromatic-coded toy.

Researcher:	[Points to Kyle] … Do you think this program here [gesturing to the forward, forward, forward program on organizer] is going to get Bee-Bot up to the beehive and back? [sliding finger on grid three forward to beehive and three backwards to starting location].
Kyle:	[Puts thumb down]
Researcher:	Thumb down, why?
Kyle:	[Leans forward and points to the program organizer spaces right after the three forwards] Because there's back, back, one, two, three.

In this excerpt, Kyle discussed the coding arrows and the intended movement of Bee-Bot using spatial orientation language to communicate spatial reasoning. Ultimately, the monochromatic design of the Bee-Bot arrows encouraged children to find other, spatially-based ways to communicate and reason with the directional command arrows because children could not rely on the use of color terms.

Body Arrows Afforded Spatial Orientation. Bee-Bot and Code-a-pillar were distinct from Cubetto and Botley because the coding arrows were directly on the toy's body. This meant the arrows were always aligned with the coding toy's orientation so children did not need to re-coordinate the position and meaning of the code with the orientation of the toy every time it moved. This design feature of having the coding arrows on the agent, more easily afforded engagement with accurate spatial orientation. For example with Bee-Bot, children took advantage of the arrows on the agent to navigate spatial orientation situations through turning their heads, reaching, and touching the arrows directly. Figure 9 highlights one child who demonstrated this relationship between the arrows on the body and spatial orientation. The child tried to get Bee-Bot to land on the grid square with the beehive, but the Bee-Bot had stopped one space in front of it. Earlier in the lesson, she consistently used incorrect codes when trying to rotate or move Bee-Bot, however in this instance, she reached forward, slightly turned her head and body, and touched the one forward arrow on the agent that was facing the hive. In this example, the child used the directions of the arrows on the Bee-Bot agent to select the appropriate next arrow in the sequence.

Figure 9

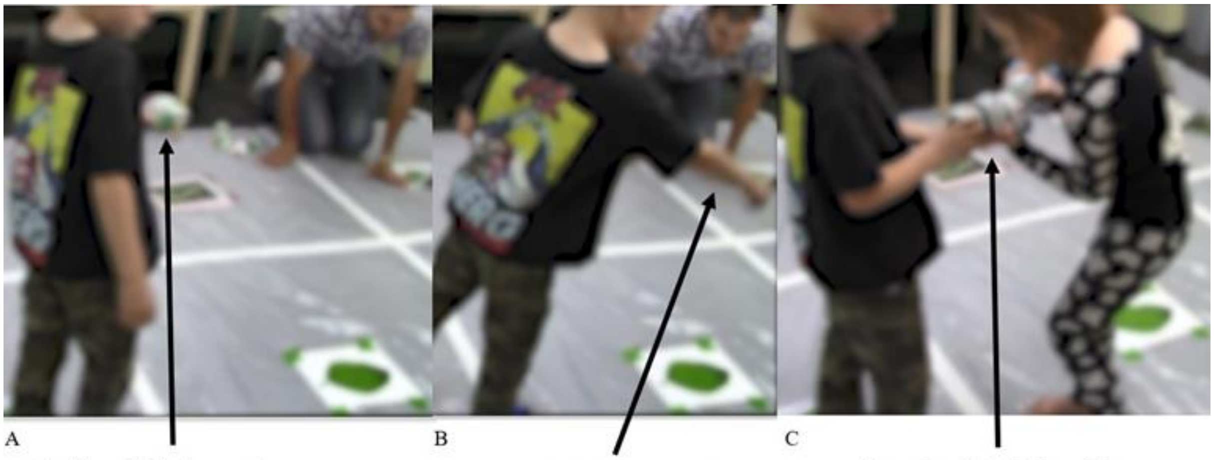
A Child Reached for and Touched the Repositioned Arrows

The child leaned forward, turned her body and head slightly sideways, and pushed the forward arrow because it was pointed at the hive.

Large Grid Squares Afford Spatial Orientation, Spatial Visualizations, and Spatial Estimation. The Code-a-pillar large grid squares, different in size from the other coding toys, afforded spatial orientation, spatial visualization, and spatial estimation. Children partnered their bodies with the removable codes to aid in solving spatial orientation problems. This was often seen as children called out codes that their partner needed to program when they walked their bodies around on the large grid spaces. When children reached a point on the grid where they were unsure of the code that should be called out to their partner, they grabbed a physical code and took it to the last space on the grid where they were calling out codes from. They held the code up in front of them to see if the arrow on the code was facing the correct way or not. If it was, they put that code on the Code-a-pillar. If it was not, they picked up another code to test. In these instances, the children used the detachable arrows as tools to help them solve spatial orientation problems. Figure 10 shows a child who called out codes and used a detached body segment to solve a spatial orientation problem. The child travelled around the large grid with his body and called out codes for his partner to add to the Code-a-pillar. He reached one instance where he aligned his body in the grid square and was deciding how to get to the next square forward. Pane A shows him grabbing a green forward arrow and using it as a tool to identify the needed code. He aligned the code to the grid squares, and then pointed forward with his hand and stated, “one straight” (Pane B). After this use of the discrete body segment code to solve the spatial orientation problem, his partner brought the Code-a-pillar body over and appended the code to the body (Pane C).

Figure 10

A Child Used the Separated Codes to Solve a Spatial Orientation Problem



Holds up the body segment command arrow as a tool to aid in choosing the correct direction.

Points in direction the arrow is pointing and verifies by calling out “one straight.”

Appends code to Code-a-pillar with partner after deciding it is accurate.

This example demonstrates how the design feature of the large grid squares afforded mathematical engagement in spatial orientation. The large grid squares allowed the child to physically engage his body in the coding process when faced with a spatial orientation challenge.

Discussion

Importance of Design Features to Afford Unit Construction and Coordination

An important contribution of this study is that specific design features afforded linear and discrete unit construction, as well as meaningful coordination. Literature is mixed on young children’s developmental readiness to work with continuous linear units versus discrete units (Friso-van de Bos et al., 2018; Smith III et al., 2013; Welch et al., 2022), and we know that coordination—sometimes called one-to-one correspondence and/or action-instruction correspondence—is an important concept young children can learn while playing with coding toys (Bers et al., 2014, 2019; Muñoz-Repiso & Caballero-Gonzalez 2019; Murcia & Tang, 2019). This study contributes implications pertaining to both these mathematical topics for educators, product designers, and researchers.

Linear/Discrete Unit Construction

One result of this study was that the grid square design feature afforded linear and discrete unit construction across all four coding toys. Children counted the squares to construct discrete units and they counted the movements from square to square to construct linear units. This means that children engaged in construction of two different types of units (i.e., linear, discrete) when perceiving and using the grid squares. This is important because it shows that the grid square design features of these coding toys offered children an opportunity to reason with and construct multiple unit types, including a linear unit. This new type of dynamic learning tool offered children an environment where a linear unit was understandable and appropriate. Moreno-Armella et al. (2008) documented a progression of mathematical tools from static-to-dynamic and stated that *continuous dynamic* tools allow in-the-moment re-orientation of the tool and body. There is promising evidence in the current study – which theories on embodied cognition would enthusiastically acknowledge – that kindergarten-aged children constructed linear units in an appropriate manner with these coding toys due to their dynamic nature and the way the children could engage their bodies and re-orient their perspectives in-the-moment.

Future designers may consider making the grid squares with explicit markings to show the linear movements, like an overlayed number line from the center of each square to the center of the next. This may afford children more opportunities to engage in linear unit construction because they could visually see the start and stop point between squares, but also attend to the discrete single squares. Basically, this design would allow children to attend to the linear movement – line running from the center of each square – or the individual discrete squares, based on their developmental readiness. Another design suggestion is to omit the grid squares entirely and create an environment for the coding toys with ticks—like rulers—with the coding toy moving from one tick to the next tick. This design idea would more explicitly highlight the linear unit of movement.

Design Features Afford Meaningful Coordination

The results of this study highlighted a few specific design features that afforded coordination. For example, the coding toys that had the stop-between-movement design feature afforded coordination between code-movement. A possible special link between the stop-between-movement design feature and the engagement in coordination could be an idea called *specification of dynamic movement*. Children at this age mostly work with concrete, static, and discrete units and, therefore, their ability to recognize a unit of dynamic movement may not yet be well developed. Making sure to have a design feature which explicitly specifies each dynamic movement of the coding toy, like Bee-Bot and Cubetto's stop-between-movement feature, may be essential in helping young children coordinate between each dynamic movement and each code. Additionally, literature suggests that design features that include simultaneous linking of representations are beneficial to mathematics learning; however, children must first be aware of the features in order to take advantage of the potentially beneficial affordances (Bullock et al., 2017; Moyer-Packenham et al., 2020).

Instructors using these coding toys for mathematics instruction should be aware that they may have to prompt explicit connections between these coordinating features and should prompt students to math-count movements with codes. To aid children and instructors, designers of toys could make the flashing lights and sounds and stops-between-movement features more explicit. For example, the coding toy could say and count the spatial movements out loud as the toy is enacting codes—*count code calling.* This design feature would look like the coding toy moving around and making auditory noises, "*one movement forward, one movement forward, 90-degree right rotation, 90-degree right rotation*." Additionally, the simultaneous linking features could be directly on the moveable body so the child's gaze is on the design feature and the light at the same instance, making this connection between movement and design feature more visible.

Importance of Design Features to Afford Spatial Thinking

A second important contribution of this study is that specific design features afford a variety of spatial thinking opportunities to young children. Current research has demonstrated that holistically, coding toys support early childhood spatial thinking (Berson et al., 2023; Cittá et al., 2019; Terroba et al., 2021). However, it was unknown to what features of the toys were eliciting such development. Three specific design features of the coding toys in this study were shown to afford spatial thinking.

Monochromatic Codes May Afford Spatial Reasoning

Monochromatic codes afforded spatial orientation language. Due to the monochromatic nature of the codes on Bee-Bot, children used terms like "*turn around, go straight, turn to the right, back up,"* whereas, with the toys that had colored codes (e.g., green for forward, yellow for backward), children took advantage of the colors and used terms like "*green, red, blue*." Because research has demonstrated that technological tools with programmed directionality support the development of spatial reasoning (Cittá et al., 2019; Terroba et al., 2021), it is important to understand how the design features of these coding toys supported different directional and spatial terms and discriminations. Literature on discriminating spatial language, including left and right, varies (Benton, 1959; Harris, 1972; Piaget, 1968). However, big ideas that remain relatively constant are that developing directional discrimination is happening from 4- to 8-years old, and that discriminating left versus right is more challenging for young children than discriminating up versus down or front versus back.

For instructors and designers, coloration of the codes could be used as a differentiating feature to match the developmental ability of users. The current study indicates that the coloration of the codes acted as a differentiating feature that allowed children—still unable to discriminate between some of the more technical spatial language (e.g., left, right)— a simplified cognitive process by supporting communication with color language. On the other hand, for children more advanced in their development of technical spatial terms, monochromatic codes may afford more engagement in spatial concepts due to the restrictions on descriptive terms.

Spatial Orientation through a Shared Perspective from Arrows on Agent

Arrows directly on the Bee-Bot and Code-a-pillar agent allowed children to engage in spatial concepts and skills. We hypothesize that this is due to the way the arrows on the agent maintained a shared perspective with the direction the agent was facing. Research on early childhood coding indicates that young children struggle to understand rotation arrows (Cuneo, 1985), but mental rotational thinking begins developing as early as 3 years old (Krüger et al., 2014). The current study supports previous findings in that children struggled with rotation codes, but did show the ability to use mental rotations (Clarke-Midura et al., 2021). The current findings highlight how the arrows on the agent may aid children in use of rotation codes and mental rotations. The coding arrows being directly on the agent afforded a shared spatial perspective with the child and the way the agent was facing. This allowed children to match the codes with the desired movement without having to mentally recreate a spatial map through imaginal updating (Klatzky, 1998).

Teachers considering using coding toys should evaluate the benefits of the arrows on the agent. If the arrows do not share a perspective with the coding toy agent, the teacher may need to encourage children to do more body movements to orient themselves to the coding toy. The future design of coding toys for this age group could have the codable arrows on top of the agent, which would aid children's ability to match the movements with the codes.

Spatial Skills through Embodied Engagement and Visual-Spatial Correspondence

Another finding in the current study was that the large grid squares afforded spatial orientation, visualization, and estimation, by allowing children to use their body as a spatial tool. This finding supports research which has shown visual-spatial opportunities and children's embodied engagement increases success (Berson et al., 2023; Paek, 2012; Sung et al., 2017) and is linked to mathematics development (Barnes et al., 2011; Gunderson et al., 2012; McCluskey, 2023; Zhang, 2016), ultimately supporting the theoretical position that active bodily engagement strengthens and supports learning connections (Lakoff & Johnson, 1999; Papert, 1980). Furthermore, research has shown that spatial structuring occurs in second and third grade (Battista, 1999; Battista et al., 1998). This suggests that exposing kindergarten-aged children to activities that physically engage them in spatial orientation, visualization, and estimation could support them in their readiness to structure their spatial thoughts more abstractly and formally in the later grades (Siegler & Braithwaite, 2017). Code-a-pillar offers an environment where children engage in spatial visualization and updating of reference frames by using their body as a tool to code. Embodied engagement was essential to success with actualizing spatial visualizations and determining correct codes based on updating reference frames.

To advantage the embodied engagement opportunities of these toys, instructors could implement teaching moves such as *prompting to engage body* which would support the spatial updating of reference frames. Likewise, designers of coding toys for young children could consider increasing length of movements—similar to Code-a-pillar—so children can engage their full body when reasoning with spatial concepts and skills.

Limitations

Appleton et al. (2008) examined 19 definitions of engagement in the psychological literature and found that all definitions of engagement included two similarities: *participation* either *behaviorally* or *cognitively*. The current study used videos as the main data source, which lends itself to analysis of behavioral indicators (e.g., gestures, movements, verbalizations, gaze). Although children sometimes verbalized potential cognitive processes which can be examined, the analysis was primarily limited to the children's' behavioral engagement, not cognitive.

Another similar limitation has to do with perception. It is known that children sometimes perceived design features but chose to ignore them, or not attend to them (e.g., hearing sounds/seeing lights but not mentioning or attending to them). However, we were unable to document this 'non-attention to perception' phenomena, as we relied on physical indications of perception, such as verbal indications, gestures, or use.

Finally, this study focused primarily on children, not teachers. In this research context, there were times when the group facilitator comments led to child action. We did not discriminate these circumstances but did account for them through our methods. Our group facilitators assumed such a role, rather than 'teacher' – they were mostly focused on supporting the children's cooperation and problem solving, rather than telling them what to do.

Conclusion

Considering the rapid advances to digital educational tools, it is critical that such implements are carefully used in the classroom and thoughtfully designed. Tangible coding toys are becoming more and more common in early childhood classrooms, but there has been a lack of attention to how their design leads to affording mathematics. The purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. This study provided targeted analysis of which design features afforded engagement in certain mathematical concepts. Results indicated that different characterizations of the coding toy features lead to engagement in mathematics, and sometimes even mathematics historically difficult for kindergarten-aged children, such as linear unit construction. That being said, there is still wide variation across coding toy products, and the design of such products lacks attention to the possibly beneficial affordances for meaningful mathematics. One of the overall recommendations of this work is that future coding toy designers be aware of the specific design feature-mathematics relationships found in this study and implement targeted design features in future product design (e.g., number line marks between grid squares to afford linear unit construction, stops between motions to highlight one-code-to-one-movement correspondence).

Further, teachers implementing tangible coding toys need more resources in terms of specific instructional strategies to forefront mathematics as children use these toys. Findings of this study offer teachers suggestions on ways to leverage certain design features to help children make more explicit mathematical connections (e.g., prompting children to match-count between codes and movements to afford one-to-one correspondence, encourage children to use specific spatial language when describing the command arrows rather than color terms, suggesting children activate their body in alignment to the coding toy agent to problem solve spatial situations).

Finally, one of the resounding open questions from this study which warrants future in-depth investigation has to do with development of mathematical thinking. This study is limited in scope in the fact that we analyzed the cooccurrence between design feature use and mathematics, and we did not analyze the development of mathematical thinking. Basically, this study does not answer the question "*Do certain design features benefit the development of mathematical thinking?"* Although we found cooccurrences that excite us about these possibilities, such as the dynamic movement from the center of one grid square to the center of the next affording linear unit construction, we are unable to say if there is a set of features that support the development of a measurement learning trajectory. Looking at how features could support a specific mathematical learning trajectory is an exciting possibility for future research stemming from this work.

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