Spacecraft System Options for Best Data Rate

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Abstract

Many spacecraft have brief ground station contact time. Consequently, high data rate is the key to mission success. Since data transmission rate is proportional to transmitter power, the reliable generation of power enters the return on investment (ROI) equation. Therefore, most spacecraft trade analyses consider several solar array designs. This paper presents an approach for evaluating the data rate potential of five solar array system options. A CubeSat (10 cm on edge) example is used to demonstrate the approach. The CubeSat is fixed to an LVLH reference frame with five sides available for solar cells. The sixth side is Earth oriented and dedicated to the payload and data antennas. Realistic electrical power and communication system models provide a data rate coefficient as a function of solar direction cosines. Relationships are developed for the direction cosines as a function of solar beta angle and panel deployment angles. Data rates are estimated for five solar array options. It is shown that data rate increases with respect to body fixed panels of 55% and 60% are achievable. The impact of higher data rates on other subsystems is discussed. Selection of the best configuration requires customer/designer evaluation of total ROI considering cost and reliability.

Nomenclature

- A\textsubscript{i} = side i area
- C\textsubscript{ci} = cosine of (.)
- \text{dc}_i = solar direction cosine of surface i
- \text{dc}_{sun} = surface i sunlight ave. solar direction cosine
- f\textsubscript{p} = solar panel cell packing factor
- i = orbit inclination
- I\textsubscript{d} = initial solar cell inherent degradation
- K\textsubscript{r} = data rate coefficient
- K\textsubscript{s} = communication subsystem power coefficient
- L\textsubscript{d} = 5 yr life solar cell degradation
- P\textsubscript{com} = communication subsystem power allocation
- P\textsubscript{dav} = daylight average power requirement
- P\textsubscript{eav} = eclipse average power requirement
- P\textsubscript{T} = transmitter output power
- R = data rate
- S = solar flux intensity
- SC = spacecraft
- SSP = subsolar point
- S\textsubscript{si} = sine of (.)
- T\textsubscript{d} = time in daylight
- T\textsubscript{e} = time in eclipse
- X\textsubscript{d} = daylight power transfer efficiency
- X\textsubscript{e} = eclipse power transfer efficiency
- \beta = angle from orbit plane to Sun\textsuperscript{[3]}
- \delta = solar panel deployment angle
- \theta = spacecraft angular distance from Sun
- \eta = solar cell efficiency
- \lambda = half of total sunlight orbit angle
- \nu = spacecraft angular distance from SSP
- \nu\textsubscript{1} = upper integration limit for Surface 1
- \nu\textsubscript{3} = upper integration limit for Surface 3
- \rho = Earth’s apparent angular radius
Introduction

Data rate depends on available solar array power. The required solar array power is given by

\[ P_{SA} = \frac{P_d}{X_d} + P_e T_e X_e X_{d} \]  
\[ (1) \]

The solar array power available is given by†

\[ P_{SA} = \eta I_d L_d \sum f_p A_{dc_{isa}} \]  
\[ (2) \]

The CubeSat geometry of interest is shown in Fig. 1. Surfaces 1, 3, 4, 5 and 6 are covered with solar cells. Surface 2 (nadir pointing) is reserved for the payload and antennas. Assuming constant areas and packing factors with \( P_e \) equal \( P_d \), Eqs. (1) and (2) yields

\[ P_d = \eta I_d A_{dc_{isa}} \sum (1/X_d + P_e T_e/P_d T_d X_e) \]  
\[ (3) \]

![Figure 1. CubeSat Geometry](image)

Table 1. Parameter Values (S, I_d, L_d, X_d, X_e)\(^{(1)}\)

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>S (W/m(^2))</th>
<th>( \eta )</th>
<th>I_d (A/m(^2))</th>
<th>( L_d ) (A/m(^2))</th>
<th>X_d (A/m(^2))</th>
<th>X_e (A/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1367</td>
<td>0.25</td>
<td>0.77</td>
<td>0.83</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.6</td>
<td>0.01</td>
</tr>
</tbody>
</table>

From Eq. 3 and Table 1\(^{(1)}\) with \( P_e = P_d \):

\[ P_d = 182.4 \Sigma dc_{isa} \]  
\[ (4) \]

Assuming 20\% of this power can be allocated to the communication subsystem

\[ P_{com} = 36.52 \Sigma dc_{isa} \]  
\[ (5) \]

A reasonable value for solid state transmitter efficiency\(^{(2)}\) is 30\%. The resultant transmitter power is:

\[ P_t = 11.0 \Sigma dc_{isa} \]  
\[ (6) \]

Average Solar Direction Cosines, \( \Sigma dc_{isa} \)

The average direction cosine summation is a function of altitude, deployment angle \( \delta \) and solar angle \( \beta \) (see Appendix A). Plots of this function are shown in Fig. 2 for an altitude of 700 km. The angle \( \beta \)\(^{(3)}\) varies with time and orbit inclination as shown in Fig. 3. The peak value of \( \beta \) is given by the sum of the orbit inclination angle and the solar declination angle (23.45\(^{\circ}\))\(^{(4)}\).

Data Rate

It can be shown that data rate is proportional to transmitter power for a given communication link.\(^{(2)}\)

\[ R = K_s P_t \]  
\[ (7) \]

The value of \( K_s \) is a function of link parameters. Using a typical set of parameters for a spacecraft at 700 km with a 0.1 m antenna diameter\(^{(2)}\)

\[ K_s = 425 \text{ kbps/w} \]  
\[ (8) \]

Combining (6) and (8), data rate is given by:

\[ R = K_s \Sigma dc_{isa} \]  
\[ (9) \]

Consequently, the Fig. 2 ordinate can be viewed as relative data rate as a function of panel deployment angle delta and solar angle beta.

![Figure 2. Sunlight Average Direction Cosine Sum](image)

![Figure 3. Beta vs. Time for i=0\(^{\circ}\) and i=65\(^{\circ}\)](image)
**Solar Array Options**

A comparison of five solar array options is summarized in Table 2. The $\Delta dc_{\text{sa}}$ (relative data rate) is shown for inclinations of $0^\circ$ and $65^\circ$. The data rate variation is driven by the beta excursions (Figs. 2, 3, 4). Option 1 covers all sides with fixed solar panels except side 2. It is the least expensive and most reliable but provides the least power for data transmission. Option 2 deploys Panels 1 and 3 to equal fixed angles. The equal angles provide beta symmetry. (Deployment of Panels 5 and 6 did not increase orbit average power). Option 4 adjusts the wings continuously about the $X_o$ axis to increase relative data rate as shown in Fig. 4. Option 5 tracks the Sun in two axes to maximize power. Option 3 deploys Panel 1 parallel to Panel 3 as indicated in Table 2. The deployment angles in Option 3 are fixed to an angle which maximizes power. However as shown in Appendix B, a $180^\circ$ Yaw maneuver is required each time beta changes sign (Fig. 3). This yaw maneuver might increase the attitude control system (ACS) and ground control system cost beyond that of Option 4. Options 4, and 5 result in ACS disturbance torques. Options 2, 3, 4 and 5 require more thermal control system (TCS) analysis, testing and possibly hardware due to wing shadowing. The control law for Option 4 follows the upper envelope of the curves in Fig. 4.

### Table 2. Solar array options and relative merits at 700 km for $\lambda=0$ and $65^\circ$.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Merit</td>
<td>Cost</td>
<td>Reliability</td>
<td>$i=0^\circ$</td>
<td>$d_i, d_j \text{deg}$</td>
<td>R ~ Mbps</td>
</tr>
<tr>
<td></td>
<td>Lowest</td>
<td>Low</td>
<td>Medium</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>$d_i, d_j$</td>
<td>0.0</td>
<td>6.4</td>
<td>90, 90</td>
<td>70.70</td>
<td>9.9</td>
</tr>
<tr>
<td>R ~ Mbps</td>
<td>6.4</td>
<td>30, 30</td>
<td>20,160</td>
<td>Variable</td>
<td>6.7</td>
</tr>
<tr>
<td>Other</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>

*Product of data rate coefficient (Eq. 10) and average of $\Delta dc_{\text{sa}}$ over inclination beta range from Figs. 2, 3, 4.
†180° Yaw maneuver required for beta sign change (Fig. 3)

**Conclusions**

The approximate data rates achievable are proportional to the direction cosine summations shown in Figs. 2 & 4. Equal fixed wing deployment (Option 2) of about $90^\circ$ at low inclinations provides a significant gain in average data rate, (55%) but a negligible gain at high inclinations.

Unequal deployment ($20^\circ$ & $160^\circ$ Option 3) with a $180^\circ$ Yaw maneuver provides significant data rate increase at all inclinations (about 60%). Variable wing tracking (Option 4), following the upper envelope in Fig. 5, offers little data rate improvement compared to Option 3. The Option 5 data rate improvement is limited only by the solar array area and the power dissipation that can be handled by the TCS.

Selection of the best SA option depends on mission orbit inclination and data rate requirements tempered by cost and total ROI.

![Sunlight Average Direction Cosine Sum vs. Beta and $\delta_1$ at 700 km, for $\delta_2 = 180^\circ - \delta_1$](image)

**References**

Appendix A. Derivation of Sunlight Average Direction Cosines

The angle $\beta$ (Fig. 1) locates the sun with respect to the orbit plane, $\theta$ locates the sun with respect to the spacecraft (SC) and $\nu$ locates the SC with respect to the sub-solar point (SSP). All the angles are functions of time; however beta varies so slowly that it is considered constant for a given orbit.

The Surface 1 solar direction cosine (Table A1 & Fig. A2) is given by

$$dc_1 = C_\beta C_\nu S_{\beta 1} - S_\beta C_{\beta 1}$$  \hspace{1cm} (A1)

The average during sunlight is

$$dc_{1sa} = 2 \int_{0}^{\nu_1} (C_\beta C_\nu S_{\beta 1} - S_\beta C_{\beta 1}) \, dv / 2\lambda$$ \hspace{1cm} (A2)

$$dc_{1sa} = (S_{\beta 1} S_{\nu 1} - \nu_1 T_{\beta 1} C_{\beta 1}) C_\beta / \lambda$$ \hspace{1cm} (A3)

where $\nu_1 = \begin{cases} \text{acos} \left( \frac{T_\beta}{T_{\beta 1}} \right) ; & \text{if } \delta_1 > \beta \\ 0 ; & \text{if } \delta_1 \leq \beta \end{cases}$ \hspace{1cm} (A4)

From Table A1, the Surface 3 solar direction cosine is given by

$$dc_3 = C_\beta C_\nu S_{\beta 3} + S_\beta C_{\beta 3}$$ \hspace{1cm} (A5)

Similar to Eq. A3

$$dc_{3sa} = (S_{\beta 3} S_{\nu 3} + \nu_3 T_{\beta 3} C_{\beta 3}) C_\beta / \lambda$$ \hspace{1cm} (A6)

where $\nu_3 = \begin{cases} \text{acos} \left( - \frac{T_\beta}{T_{\beta 3}} \right) ; & \text{if } \delta_3 > \beta \\ \pi ; & \text{if } \delta_3 \leq \beta \end{cases}$ \hspace{1cm} (A7)

The Surface 4 solar direction cosine is given by

$$dc_4 = C_\beta C_\nu$$ \hspace{1cm} (A8)

$$dc_{4sa} = 2 \int_{0}^{\nu_2} C_\beta C_\nu \, dv / 2\lambda$$ \hspace{1cm} (A9)

$$dc_{4sa} = C_\beta / \lambda$$ \hspace{1cm} (A10)

The Surface 5 solar direction cosine is given by

$$dc_5 = -C_\beta S_\nu$$ \hspace{1cm} (A11)

$$dc_{5sa} = -\int_{0}^{\delta_5} C_\beta S_\nu \, dv / 2\lambda$$ \hspace{1cm} (A12)

$$dc_{5sa} = C_\beta (1-C_\nu) / 2\lambda$$ \hspace{1cm} (A13)

The Surface 6 average is given by symmetry:

$$dc_{6sa} = dc_{5sa}$$ \hspace{1cm} (A14)

Appendix B. Effective Wing Area Increase

Deploying Wings 1 & 3 to the same angle drives the “shady-side” wing (Wing 1) power to zero when $\beta > \delta$ (Figs. A1, A2, B1). The solar power can be increased by deploying Wing 1 to an angle of 180°- $\delta$ as shown in Fig. B1. This effectively doubles the area of the sunny side wing. However, either the wings must be reversed or a Yaw maneuver of 180° is required when beta changes sign.

### Table A1. Unit Vector Components and dc Averages

<table>
<thead>
<tr>
<th>Surface Number</th>
<th>Unit Vector Component</th>
<th>Solar Direction*</th>
<th>Sunlight Average Direction Cosine</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0, C_{\beta 1} - S_{\beta 1}]</td>
<td>[C_\nu, S_{\beta 1}]</td>
<td>(S_{\beta 1} S_{\nu 1} - \nu_1 T_{\beta 1} C_{\beta 1}) C_\beta / \lambda</td>
</tr>
<tr>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>3</td>
<td>[0 - C_{\beta 3} - S_{\beta 3}]</td>
<td>[C_\nu, S_{\beta 3}]</td>
<td>(S_{\beta 3} S_{\nu 3} + \nu_3 T_{\beta 3} C_{\beta 3}) C_\beta / \lambda</td>
</tr>
<tr>
<td>4</td>
<td>[0 0 -1]</td>
<td>C_{\nu}</td>
<td>C_{\beta} / \lambda</td>
</tr>
<tr>
<td>5</td>
<td>[1 0 0]</td>
<td>-C_\beta S_\nu</td>
<td>C_\beta (1-C_\nu)/2 \lambda</td>
</tr>
<tr>
<td>6</td>
<td>[-1 0 0]</td>
<td>C_\beta S_\nu</td>
<td>C_\beta (1-C_\nu)/2 \lambda</td>
</tr>
</tbody>
</table>

*The unit sun vector (Fig.A1) is given by: \( \hat{S} = [C_\beta S_\nu, S_\beta, C_\beta C_\nu] \)