1990

Development and application of a state-wide empirical growth and yield model for natural aspen stands

D.K. Walters
A.R. AK

Follow this and additional works at: https://digitalcommons.usu.edu/aspen_bib

Part of the Forest Sciences Commons

Recommended Citation
DEVELOPMENT AND APPLICATION OF A STATE-WIDE
EMPIRICAL GROWTH AND YIELD MODEL FOR
NATURAL ASPEN STANDS

David K. Walters and Alan R. Ek

ABSTRACT.--Forest growth and yield modelling capability has progressed rapidly over the past few years, but few models are in regular use by practitioners. The STEMS model for the Lake States is one such tool with many possible uses. However, it requires tree list detail or the approximation of that to begin projections and, being a physically large package, it can require considerable effort to integrate this model into data base management systems. Simpler empirical yield models and PC-based implementation packages can enable many users to assess future forest resource conditions cost effectively now and at the same time facilitate eventual use of the STEMS model. This paper describes methodology and empirical equations developed for predicting basal area and multi-product yields by species for aspen stands in Minnesota. Independent variables are stand age and site quality, and, optionally, basal area. These equations facilitate rapid stand level projections. A computer implementation package is also being developed to assist forest managers and others in using these equations. Regeneration components and comparisons to existing models are also described.

INTRODUCTION

Empirical yield tables have been described in numerous forestry texts and articles (e.g., Husch et al. 1982, Bruce and Schumacher 1950, and Schumacher 1939). However, few empirical yield models exist except for those constructed from research data. Fewer still are used operationally in the Lake States region. Additionally, none we know of describe mixed species stand yields in detail. Several possible reasons for this void is that researchers have tended to ignore such approaches because they are too simple and management agencies have not had the technical support to develop them on their own (e.g., Ek (1987)). However, empirical yield models can provide an important tool for rapid yield projections with minimal software and hardware requirements. Previous work with an inventory data set in Wisconsin (Walters et al. 1989) also suggested that standard inventory data could be used to develop these empirical yield equations. Therefore, the methodology developed in these earlier studies was extended to the development of a statewide set of equations using USFS Forest Inventory and Analysis (FIA) data.

THE STUDY

A typical forest management operation in the Lake States might include ten to fifteen important forest cover types. This paper describes the development of a system of yield models for the aspen cover type in the state of Minnesota. The aspen cover type is described from 3487 FIA plots. These plots consist of 10-point clusters covering a 1 acre sample area. A 37.5 basal area factor prism plot is

1David K. Walters, Research Specialist and Alan R. Ek, Professor and Head, Department of Forest Resources, College of Natural Resources, University of Minnesota, St. Paul, Minnesota 55113
established at each point and three (3) 1/300 acre fixed radius plots are established on a subset of the points. The fixed radius plot is the basis for information on regeneration. For a detailed description of the plot establishment and measurement procedures, see Hahn and Hansen (1985). Stand-level volumes were computed from the individual tree information according to the procedure outlined by Hahn (1984). Each plot was randomly assigned to be used in either model fitting and development or in model validation. Statistical analysis of the data was conducted with the SYSTAT microcomputer package (Wilkinson 1988) and consisted of extensive correlation and graphical analysis and eventual linear and nonlinear regression model fitting. The data are described in Table 1.

The initial goal was to develop empirical yield model system for these data involving the following system of models.

\[
B = f(A, S)
\]

\[
V = f(A, S, B)
\]

where,

- \(B\) = basal area of all species (\(\text{ft}^2/\text{acre}\) in trees \(\Rightarrow 1.0''\) DBH)
- \(S\) = site index
- \(A\) = stand age class
- \(V\) = total stand volume of all species (\(\text{ft}^3/\text{acre}\))

This system was designed to describe the yield over age relationships for various sites on an ownership. The first equation estimates basal area per acre for stands which are essentially undisturbed, that is with no history of thinning or other treatments. Such conditions are typical of aspen in the Lake States. The second expression estimates pulpwod volume as a function of site and age and the basal area from the first expression. Empirical yield equations typically describe stands of average basal area or stocking and the second expression does exactly that, except that the average stocking is represented by the first expression. Initially, models contained an intercept term because many stands still have some residual volume after clearcutting. Thus, even very young stands might have measurable merchantable volume. After initial screening of different models, the intercept term was found to be insignificant. Also, as aspen stands deteriorate at older ages, yields per acre can decrease. Again, the data often does not support models which allowed for this possibility.

| Table 1.--Summary statistics for aspen data sets. |
|---------------------------------|----------|----------|----------|
| Variable                        | Minimum Value | Mean Value | Maximum Value |
| Model Fitting Data Set (1056 observations) | | | |
| Age                             | 5.0       | 37.2      | 130.0      |
| Site Index                      | 36.0      | 66.0      | 99.0       |
| Basal Area(ft²)                 | 1.2       | 74.7      | 212.2      |
| Total Volume(ft³)               | 0.0       | 1481.0    | 5550.3     |
| Model Validation Data Set (2431 observations) | | | |
| Age                             | 5.0       | 38.1      | 250.0      |
| Site Index                      | 20.0      | 66.7      | 99.0       |
| Basal Area(ft²)                 | 0.6       | 75.6      | 212.7      |
| Total Volume(ft³)               | 0.0       | 1523.4    | 6747.3     |
Of the different model forms which were postulated and examined, the most promising basal area models were the following.

\[
B = a_i S^{a_1} e^{b_1/A} \\
B = a_i S^{a_2} A^{b_2} e^{b_1/A} \\
B = a_i S^{a_3} e^{b_1/A}
\]  

Similarly, several volume models were examined with three models being presented here which are quite similar to basal area equations [1], [2], and [3].

\[
V = b_i S^{b_1} B^{b_3} A^{b_4} e^{b_1/A} \\
V = b_i S^{b_2} B^{b_3} A^{b_4} \\
V = b_i S^{b_2} B^{b_3} e^{b_1/A}
\]

In addition to these three models, a segmented model was examined which effectively considers the yield (volume) relationship for stands less than or equal to 15 years old to be a linear function of age and the relationship for stands older than 15 years old to be adequately modeled by equation [5].

Thus, the model can be expressed as:

\[
V = V_{<15} + I(V_{>15} - V_{<15}),
\]

where,

\[
V_{>15} = b_i S^{b_2} B^{b_3} A^{b_4} \\
V_{<15} = c_0 + c_1 A
\]

I=0 for A <= 15 and I=1 for A > 15

and the model is conditioned such that

\[
V_{<15} = V_{>15} \text{ at } A = 15
\]

This condition enables \(c_0\) to be eliminated and [7b] to be re-expressed as

\[
V_{<15} = b_i S^{b_2} B^{b_3} A^{b_4} + c_i(A - 15)
\]

A fourth equation for basal area growth (B) equation could be developed and applied if sufficient remeasured permanent plot data was available to construct it. The form of that model might be:

\[
B = b_o S^{b_2} (B^{b_3} - b_i B)
\]

Such data were not available in this case. The FIA permanent plots are currently being remeasured and when these additional data are available, the fitting of models such as [8] will be pursued.

Empirical yield models like [1]-[7] represent a smoothing of the average yields by age and site class represented in a compilation of an inventory. The assumptions in the fitting and use of such models are: 1) the plots have no history of management disturbance since the date of stand establishment and 2) the distribution of site quality for each age class is similar for all age classes. To the extent that
the data included in the analysis meets the above stated assumptions, the equations estimate or approximate the natural development of stands. In effect, the average of stands now in a particular site class at age 40 provide an estimate of what the stands of that site class at age 30 will look like in another 10 years. In fact, such simple yield tables are very appropriate for many young stands prior to management treatments, say before age 20 in the Lake States region.

The inventory in this case contained other variables such as stand density and size class characterizations, but these were not deemed very useful in predicting future growth because of the likely change in density and size class with such aging. However, such variables are useful in an inventory context.

MODEL FITTING AND EVALUATION

Models [1]-[7] and other models were fitted to the data described in Table 1 using nonlinear least squares estimation procedures. To provide for an independent validation data set, each stand was randomly assigned to either a model fitting data set or a model validation data set. The model fit statistics used to evaluate the different levels of aggregation were mean residual, mean absolute residual, and mean squared residual in real (untransformed) units. Together, these three statistics and graphical inspection of residuals provided a good picture of the accuracy and precision of the models. The results for the basal area and pulpwood volume models are summarized in Table 2. After examining the results, model [2] and model [7] appeared to be the best predictors of basal area and total volume, respectively. In addition to the results in Table 2, residual and absolute residuals were compared across age classes. This comparison supported the selection of models [2] and [7]. The parameter estimates for these two models are presented in Table 3. Further analysis is continuing to examine the robustness of this selection for different cover types. Figure 1 presents projected total stand volumes using models.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Statistics¹</th>
<th>Statistics⁴</th>
<th>Statistics⁴²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R²</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>[1]</td>
<td>0.867</td>
<td>0.463</td>
<td>23.749</td>
</tr>
<tr>
<td>[2]</td>
<td>.867</td>
<td>.376</td>
<td>23.723</td>
</tr>
<tr>
<td>[4]</td>
<td>0.947</td>
<td>-1.305</td>
<td>264.194</td>
</tr>
<tr>
<td>[5]</td>
<td>.945</td>
<td>-10.970</td>
<td>270.460</td>
</tr>
<tr>
<td>[7]</td>
<td>.947</td>
<td>0.230</td>
<td>262.440</td>
</tr>
</tbody>
</table>

¹R² = coefficient of variation (percent of variation explained by model).

²D = average difference between estimate and true value.

³|D| = average absolute value difference between estimate and true value.

⁴²(D²)¹²³ = square root of the average squared difference between estimate and true value.
Table 3.--Parameter estimates for basal area model [2] and volume model [7].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model [2]</td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>1.0810</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.6277</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.4577</td>
</tr>
<tr>
<td>Model [7]</td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.1581</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.8792</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.8753</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.4522</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-6.4618</td>
</tr>
</tbody>
</table>

[2] and [7]. One interesting point about these curves is that projected total stand volume does not decline at older ages. This is possibly due to the fact that gross (including cull) volume is currently being used as the dependent variable. It is expected that by subtracting the cull volume, the curve will peak and then begin to decline as rot and mortality become increasingly severe.

Figure 1.--Total cubic foot volume predicted for site indices 60, 70, and 80 using models [2] and [7] for basal area and volume projection, respectively.
IMPLEMENTATION AND FUTURE WORK

In practice, the basal area of stands following harvesting and lacking management disturbance is predicted with fitted model [2]. Note that this would estimate average basal area with respect to age and site quality. As many managers know, such averages may underestimate or overestimate basal area for some observed stands. That is expected. Where localized, stand yield information is available, predictions from, say age 20 to age 50, would be developed by forming a ratio of the predicted basal area relative to the known initial stand basal area. For example, a stand initially having 20 percent more basal area than the basal area model would be anticipated as having 20 percent more basal area than the model in the future. Total volume would be estimated using model [7] and the predicted basal area from fitted model [2] (after adjusting for any known initial basal areas). Alternatively, one could use known total volume to adjust the system following a similar algorithm as just described. However, there seems little basis for much deviation from the approach taken other than generalities about approaches to normality in the literature.

For handling mixed species stand conditions we would also proportion the yields such that stands initially noted to have 70 percent aspen, 20 percent softwoods and 10 percent other hardwoods would maintain these yield fractions in the future. After inspecting these data and the yield tables by Hahn and Raile (1982) that appears to be a tenable starting assumption. However, since the proportion of yields in various species groups may change as a function of basal area, age, number of stems, or other variables, the hypothesis that the proportion, \( P_{mp} \), is a function rather than a constant is being tested. Work to date on this model indicates that there is little trend across site quality but that the number of trees per acre or quadratic mean diameter may be important indicator variables. If possible it would be desirable to avoid using either of these two variables because implicit in using them is the recognition that mortality or survival would have to be both estimated and projected.

Also important is the ability to break total stand volume into various merchantability classes. The approach being examined at the present is similar to that being proposed above to handle mixed species stand conditions. A proportion, \( P_{Merchant} \), will be estimated as a function of basal area, age, number of stems per acre, top diameter, and other pertinent variables.

DISCUSSION

The approach to developing a stand level set of equations predicting basal area, total volume, merchantable volume, and volume by species groups is based on the assumption that what should drive such equations are variables which are readily available. Variables such as stand age, stand site index, and initial stand basal area are available in most inventory systems. Traditionally, these are the variables which were used to access tables of yield and basal area. By developing equations instead of tables, the results will be more conducive to application in inventory update programs and forest planning models. From a scientific viewpoint, such simple equations as we are proposing can provide reasonable starting points to examine questions about local differences in yield, and when these simple equations are made for several points in time, they provide a basis for addressing questions about changing climate and other global or regional concerns.
LITERATURE CITED


