GNC for Commercial and Scientific Applications of Formation Flying

Frank J. Redd Student Competition

Nathan Cole & Starla Talbot
Small Satellite Formation Flying

On-Orbit Data

Time (UTCG): 2 Nov 2014 09:24:50
Range (km): 0.0551
Formation GNC
**Mission Overview**

**Deployment & Commissioning**
- Spacecraft deployed separately
- Drift apart during commissioning

**Formation Initialization**
- Spacecraft brought into appropriate range/configuration for payload operations

**Payload Operations**
- Station-keeping for formation maintenance

≈100 km
System Architecture

**Chief**

**Deputy**

**Navigation**
- Process raw GPS measurements
- Output position/velocity state estimate

**GPS Receiver Files**
- Attitude Files
- Thrust Logs

**Guidance & Control**
- Input relative orbital elements
- Output time tagged thrust commands

**Time Tagged Thrust Commands**

Convert pos/vel into orbital elements
Formation Navigation
Navigation

• Two approaches:
  • Determine absolute state of Chief and Deputy and difference to obtain relative state
  • Estimate relative state $\Delta r$ directly

\[ \Delta r = R_d - R_c \]
\[ \Delta \dot{r} = \dot{R}_d - \dot{R}_c \]
Navigation Techniques

• Comparison to those used on CanX-4/-5
  • Onboard relative Extended Kalman Filter (EKF)
  • Onboard absolute EKF
  • Offline absolute EKF

• For a ground-based architecture
  • Offline relative EKF
  • Commercial software solution (AGI ODTK)
Orbit Determination ToolKit

GPS observables
GPS satellite ephemeris

Initial Orbit Determination

Initial orbit state

Least Squares

Refined orbit
State covariance

Filter/
Smooother

Refined orbit
State covariance
CanX-5: ODTK To Fine EKF
Comparison of Relative Solutions: ODTK to Fine EKF
Implication on Formation

• State estimation errors have an impact on the error in the relative semi-major axis $\sigma_{\Delta a}$
• Larger errors lead to larger along-track drift rates
• $\sigma_{\Delta a} \approx 1$ m for relative solution
• $\approx 10$ m/orbit along track drift rate

$$\sigma_{\Delta a} = 2\sqrt{4\sigma_{r R}^2 + \frac{4}{n} \rho_{r R} v_I \sigma_{r R} \sigma_{v I} + \frac{1}{n^2} \sigma_{v I}^2}$$
Formation Guidance
Relative State & Motion Model

Quasi-Nonsingular Relative Orbital Elements:

\[ \delta \alpha(a, e, i, \Omega, \omega, M) = \alpha - \alpha_{ref} \]

\[
\begin{bmatrix}
\delta a \\
\delta \lambda \\
\delta e_x \\
\delta e_y \\
\delta i_x \\
\delta i_y
\end{bmatrix}^T
\]

- Semi-Major Axis
- Eccentricity Vector
- Mean Longitude
- Inclination Vector
Formation Geometry

\[ \text{sat}_{\text{ref}} \]

\[ a\delta e \]

\[ a\delta \lambda \]

\[ 2a\delta e \]

\[ \text{sat}_{\text{rel}} \]

\[ a\delta i \]
Reconfiguration Guidance

Mean Radial Offset

\[ \delta a \text{ [km]} \]

\[ \delta e_x \text{ [km]} \]

\[ \delta e_y \text{ [km]} \]

Time From Reconfiguration Epoch [orbits]

Mean Along-Track Offset

\[ \delta \lambda \text{ [km]} \]

\[ \delta i_x \text{ [km]} \]

Cross-Track Oscillation Amplitude

\[ \delta i_y \text{ [km]} \]

\[ \delta \text{ [km]} \]
Station-Keeping Guidance
Formation Control
Impulsive Maneuvers

Control input from Gauss’ variational equations:

\[
\Delta \delta \alpha_k = \Gamma_k \Delta v_k = \frac{1}{na} \\
\begin{bmatrix}
0 & 2 \\
-2 & 0 \\
\sin(u_k) & 2 \cos(u_k) \\
-\cos(u_k) & 2 \sin(u_k)
\end{bmatrix}
\]

Impulsive Control Input Matrix

Velocity Vector of Impulsive Thrust

In-Plane

Out-of-Plane
Control Thrusts

SK $\Delta V$ for Satellite 3: $\Delta V_{\text{tot}} = 9.552e+00$ m/s, $\Delta V_{\text{avg}} = 2.274e-01$ [(m/s)/day]

SK $\Delta V$ for Satellite 1: $\Delta V_{\text{tot}} = 5.349e+00$ m/s, $\Delta V_{\text{avg}} = 7.419e-03$ [(m/s)/day], $\Delta V_{\text{year}} = 2.708e+00$ [(m/s)/year]
Reconfiguration Maneuver
Station-Keeping Maneuver
Accomplishments & Next Steps

✓ Extending the success and capabilities from CanX-4/5

✓ Achieved better than required control authority

✓ GNC system validation

➢ HawkEye 360 Pathfinder launches late 2018
Thank you for listening!
Coarse Mode EKF

• Estimate absolute states of Chief and Deputy
• Position/Velocity measurements extracted from GPS receiver logs

**system state vector:** \[ \mathbf{x} = [\mathbf{R}^T \ \dot{\mathbf{R}}^T]^T_k \]

**system model:** \[ \dot{\mathbf{x}} = f(\mathbf{x}, t) \]

**system dynamics:** \[ \ddot{\mathbf{R}} = -\mu \frac{\mathbf{R}}{||\mathbf{R}||^3} + \mathbf{f}_p(\mathbf{R}) + \mathbf{u} - 2\omega_\oplus \times \dot{\mathbf{R}} - \omega_\oplus \times \omega_\oplus \times \mathbf{R} \]

**measurement model:** \[ y_k = H_k x_k + v_k \]
Fine Mode EKF

- Estimate relative state between Chief and Deputy
- Raw GPS measurements (pseudorange and carrier-phase) are filtered

**system state vector:** \( \mathbf{x}_k = [\Delta \mathbf{r}^\top \ \Delta \mathbf{r}'^\top \ \Delta b \ \Delta \dot{b} \ \Delta N_1 \ldots \Delta N_n]^\top_k \)

**system model:** \( \dot{\mathbf{x}} = f(\mathbf{x}, t) \)

**system dynamics:** \( \ddot{\mathbf{R}} = -\mu \frac{\mathbf{R}}{||\mathbf{R}||^3} + f_p(\mathbf{R}) + \mathbf{u} - 2\omega_\oplus \times \dot{\mathbf{R}} - \omega_\oplus \times \omega_\oplus \times \mathbf{R} \)

**measurement model:** \( \mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k \)

\[
y_k = h(\mathbf{x}_k) + \mathbf{v}_k = \left[ \Delta P_1 \ \cdots \ \Delta P_n, \ \Delta \Phi_1 \ \cdots \ \Delta \Phi_n, \ \Delta \dot{\Phi}_1 \ \cdots \ \Delta \dot{\Phi}_n \right]^\top + \mathbf{v}_k
\]
Extended Kalman Filter

System Model \[ \mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}), \]

Measurement Model \[ \mathbf{y}_k = h(\mathbf{x}_k, \mathbf{v}_k) \]

Time-update

\[
\begin{align*}
\hat{x}_k^- &= f(\hat{x}_{k-1}^+, \mathbf{u}_{k-1}, 0), \\
\hat{P}_k^- &= \Phi(t_k, t_{k-1})\hat{P}_{k-1}^+ \Phi(t_k, t_{k-1})^T + Q_k
\end{align*}
\]

\[
\Phi(t_k, t_{k-1}) = \frac{\partial \mathbf{x}_k}{\partial \mathbf{x}_{k-1}}
\]

Measurement-update

\[
\begin{align*}
\hat{x}_k^+ &= \hat{x}_{k-1}^+ + K_k(y_k - h(\hat{x}_k^-, 0)), \\
\hat{P}_k^+ &= (I - K_kH_{x,k})\hat{P}_k^- \\
K_k &= \hat{P}_k^-H_k(R_k + H_k\hat{P}_k^{-1}H_k^T)^{-1} \\
H_k &= \frac{\partial f}{\partial \mathbf{x}_k} \bigg|_{\hat{x}_k^-, 0}
\end{align*}
\]
Filter Residuals

L1 Pseudorange [m]

L1 GRAPHIC [m]

Time [Epoch hours from 22 Oct 2014 17:00:00]
CanX-4: ODTK - MATLAB

Radial [m]

In-Track [m]

Cross-Track [m]

Time [Hours Since 2014-10-22 16:51:54]
Orbit Determination Comparison: Results

- The ODTK and ground-based course EKF agree to within 1 m.
- The ODTK absolute state with absolute state determined from relative filter agree to within 1 m except in radial direction, which is likely due to ionospheric effects.
- Relative solution determined with ODTK agrees to Fine EKF relative solution to within 1 m
# Orbit Determination Comparison: Results

<table>
<thead>
<tr>
<th>Method</th>
<th>R</th>
<th>I</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute Ground Coarse EKF</td>
<td>0.9</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Fine EKF</td>
<td>7.4</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Relative Fine EKF</td>
<td>0.5</td>
<td>1.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Detailed QNS ROE Parameterization

\[
\delta \alpha = \begin{bmatrix} \delta a \\ \delta \lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{bmatrix} = \begin{bmatrix} \delta a \\ \delta \lambda \\ \delta e \cos \phi \\ \delta e \sin \phi \\ \delta i \cos \theta \\ \delta i \sin \theta \end{bmatrix} = \begin{bmatrix} (a - a_{\text{ref}})/a_{\text{ref}} \\ (u - u_{\text{ref}}) + (\Omega - \Omega_{\text{ref}}) \cos i_{\text{ref}} \\ e \cos \omega - e_{\text{ref}} \cos \omega_{\text{ref}} \\ e \sin \omega - e_{\text{ref}} \sin \omega_{\text{ref}} \\ i - i_{\text{ref}} \\ (\Omega - \Omega_{\text{ref}}) \sin i_{\text{ref}} \end{bmatrix}
\]
Relative Motion Model
Detailed Formation Geometry
Detailed Guidance Plan

Optimize for:

\[ \min_{\Delta \delta \alpha_k} J_{\text{guid}} = \sum_{k=1}^{m} (\Delta \delta \alpha_k)^2 + \sum_{k=1}^{m} (\Delta \delta \lambda_k)^2 + \sum_{k=1}^{m} (\|\Delta \delta e\|_k)^2 + \sum_{k=1}^{m} (\|\Delta \delta i\|_k)^2 \]

Subject to constraint:

\[ \delta \alpha_f = \Phi(t_f, t_0)\delta \alpha_0 + \sum_{k=1}^{m} \Phi(t_f, t_k)\Delta \delta \alpha_k \]

Reconstruct trajectory with:

\[ \delta \alpha_k = \Phi(t_k, t_{k-1})\delta \alpha_{k-1} + \Delta \delta \alpha_k, \quad \text{where } k = 1, \ldots, m \]
Reconfiguration Control

SK $\Delta V$ for Satellite 3: $\Delta V_{\text{tot}} = 9.552e+00$ m/s, $\Delta V_{\text{avg}} = 2.274e-01$ [$(\text{m/s})$/day]

- $||\Delta V||$
- $\Delta V_R$
- $\Delta V_T$
- $\Delta V_N$

$\Delta V$ [m/s]

Time [Orbits]

270 275 280 285 290 295 300

Smaller Satellites, Bigger Return

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Station-Keeping Control

SK $\Delta V$ for Satellite 1: $\Delta V_{tot} = 5.349e+00$ m/s, $\Delta V_{avg} = 7.419e-03$ [(m/s)/day], $\Delta V_{year} = 2.708e+00$ [(m/s)/year]

![Graph showing SK Delta-V for Satellite 1 with time in orbits on the x-axis and Delta-V in m/s on the y-axis.](image)
Station-Keeping Control: Transition
Reconfiguration Performance

Along-Track Control Authority During Initialization

- Guidance waypoint
- Simulated trajectory

Error in $a\delta\lambda$ [km]
- $\epsilon$ at waypoint
- $\langle \epsilon \rangle = 0.94196$ km
- $1\sigma = \pm 1.023$ km

Time from Initialization Epoch [days]