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Mesoscale Atmospheric Circulation and Diffusion Characteristics

Ronan I. Ellis
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MESOSCALE ATMOSPHERIC CIRCULATION AND
DIFFUSION CHARACTERISTICS

by
Ronan I. Ellis

A thesis submitted in partial fulfillment
of the requirements for the degree

of
MASTER OF SCIENCE

in
Meteorology

UTAH STATE UNIVERSITY
Logan, Utah

1973
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Recognition is given to the Army which through its Civil Schools Program made this study possible. The program allows members of the Army while on active duty, to pursue Army required advanced degrees at civilian universities.

To my wife, Harriet, for her understanding, patience and sacrifice throughout this difficult period, I am truly grateful.

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Ronan I. Ellis
Major, United States Army
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ABSTRACT

Mesoscale Atmospheric Circulation and Diffusion Characteristics

by

Ronan I. Ellis, Master of Science

Utah State University, 1973

Major Professor: Dr. Gene L. Wooldridge
Department: Soil Science and Biometeorology

Constant volume superpressure pillow balloons were flown over a mountain valley downwind from a sharp ridge line. Balloon trajectories revealed and atmospheric soundings confirmed a persistent turbulently mixed adiabatic layer over the valley at approximately ridge top height except in the immediate lee of the ridge where strong vertical motions were observed. Temporary stationarity of the relative mesoscale turbulence was found to exist on a time scale exceeding 4 hours. Power spectrum analysis of component relative velocities showed greater variance in lower wave numbers and anisotropy in mesoscale turbulence. Eddy diffusivity coefficients for turbulence above the ridge height were $K_x = 1 \times 10^7$, $K_y = 5 \times 10^6$, $K_z = 1 \times 10^6$ (cm$^2$ sec$^{-1}$). Below the ridge line $K_z$ was nearly constant at $2 \times 10^5$ cm$^2$/sec$^{-1}$. $K_z$ was found to vary exponentially with $R$, a balloon-ridge ratio; $K_z \propto \exp(3R)$.

(92 pages)
INTRODUCTION

The atmospheric mesoscale constitutes the vital bridge between planetary processes and small-scale diffusion and energy dissipation processes. Furthermore, understanding the role of mesoscale dynamics in the energy budget including kinetic energy of vertical motions and energy transfer and dissipation processes is required for numerical models of the atmosphere to accomplish medium and long range forecasting. Mesoscale motions must also be understood to predict the transport of atmospheric admixtures such as radioactive contaminants or industrial effluents (Reiter, 1969a).

Mesoscale wind patterns are quite complex and little known. These winds are strongly influenced by surface characteristics such as hills, mountains, and cities. While some mesoscale wind patterns are understood qualitatively (sea breezes and mountain valley effects) they are not known quantitatively, and much needs to be done in this area (Panofsky, 1969).

An understanding of long range transport and large scale distribution of air pollutants requires a correct representation of the development of the vertical spread of air pollutants through the full depth of the atmospheric boundary layer (Pasquill, 1971). The main obstacle to significant progress in the calculation of atmospheric dispersion is the proper specification of the diffusivity profile in relation to meteorological conditions.

Mesoscale information on the diffusion of artificial ice nuclei in cloud seeding and vertical motions of the atmosphere transversing
mountains comprises an important factor in successful weather modifica-
tion (Chappell, et al., 1972).
OBJECTIVES

The overall goal of this study was to examine mesoscale circulation characteristics over irregular terrain. This was accomplished by observing air flow over a mountainous ridge and valley using constant volume superpressure balloons to simulate air parcel trajectories. These trajectories describe terrain-caused perturbations in the flow, such as wave structure. Analysis of the component velocities relative to the mean air motion, in Lagrangian form, yields information on the scale of turbulent eddies, kinetic energy dissipation, and diffusion.

Specifically, this study will portray mesoscale circulations in the vicinity of a mountainous ridge and over a mountain valley. The scale of turbulent eddies contributing to the perturbation kinetic energy of the air flow will be identified as will the space distribution of component eddy diffusivity coefficients. Terrain features and synoptic meteorological conditions will be related to the results, thereby providing a relatively comprehensive analysis of atmospheric mesoscale activity in a mountainous region. This analysis provides an original and necessary adjunct to understanding mesoscale meteorological phenomena.
**LITERATURE REVIEW**

**Diffusion Theory**

An understanding of the use of the relative velocities obtained from superpressure balloon flights in determining atmospheric turbulent diffusion characteristics requires a review of classical diffusion theory. Priestly, McCormick and Pasquill (1958) describe the gradient-transfer theory of turbulent diffusion as applying to all treatments which use the linear relationship between flux, $F$, across a surface and the gradient along the normal in the direction of the flux:

$$ F = -A \frac{\partial s}{\partial n} \approx \rho k \frac{\partial s}{\partial n} \tag{1} $$

(A is the "Austausch" coefficient and $K$ the "eddy diffusivity"). The rationale given for the above relationship was provided by analogy with molecular diffusion. This line of reasoning led to Prandtl's mixing length theory which provided a useful first approximation of turbulent processes (Hess, 1959). While the transfer theory has no real physical basis for its use as a description of turbulent transfer, transfer theories have provided a mainstay for atmospheric diffusion over the past forty years (Priestly, McCormich and Pasquill, 1958). These authors describe the most important contribution of the transfer theory as the solution of the differential equation:

$$ \frac{dY}{dx} \frac{\partial}{\partial x} (K \frac{\partial Y}{\partial x}) + \frac{\partial}{\partial y} (K \frac{\partial Y}{\partial y}) + \frac{\partial}{\partial z} (K \frac{\partial Y}{\partial z}) \tag{2} $$
In allowing for differences in the eddy diffusivity along component directions, the solution goes farther than the "Fickian" diffusion (constant K) assumption and allows for anisotropic diffusion.

Much of the difficulty which has been seen to arise in the transfer-theory approach to the diffusion from a continuous source may be resolved by the purely statistical treatment introduced by G. I. Taylor in his 1921 paper. (Pasquill, 1962, p. 86)

Taylor's analysis is explained by Pasquill as a statistical analysis of the properties of a continuously varying quantity.

Taylor's discussion concerned the random fluctuation of particle velocity; he demonstrated that the usual laws of differentiation can be applied to mean values of fluctuating variables and their products. The mean square of a large number of particle deviations due to eddy velocity, \((u')\) can be determined from the time autocorrelation coefficient and the mean square eddy velocity, \((\bar{u}'^2)\). From this, Taylor developed expressions for large and small diffusion times where the time autocorrelation coefficient approaches zero and one, respectively. The main assumptions are that \(\bar{u}'^2\) is representative of the whole field of turbulence, i.e., homogeneous, and that the turbulence is stationary with time. Isotropy or Fickian diffusion is not necessarily assumed. Pasquill explained that eddy diffusivities can be represented by the differential of mean square particle deviations with time, and that the time autocorrelation coefficients can be generalized as an exponential of time and a time constant, \((\varepsilon)\).

\[ R_u (\tau) = e^{-\varepsilon \tau} u \]  

[Bendat and Piersol (1971) describe stationarity as the property of a random process remaining in-variant with time. In the special case]
where the mean value and autocorrelation function do not vary with time, a random process is considered weakly stationary, or stationary in the broad sense. Verification of weak stationarity justifies an assumption of strong stationarity.

One basic approach to establish stationarity as described by Bendat and Piersol states that if the basic physical factors which generate the phenomenon are time invariant, then stationarity of the resulting data can be accepted without further study. Power spectral analysis can be used to establish time invariance since it establishes the frequency composition of the data, which in turn describes the basic characteristics of the physical system involved.

Kao (1962) made an analysis of the Lagrangian equations of motion for turbulent diffusion of marked particles relative to the center of mass in a rotating system. He assumed the turbulent motion relative to the center of mass to be statistically homogeneous and stationary and found that (3) held with the qualifying stipulation that the cross-correlation coefficient equal zero. Using that relationship, Kao (1962) obtained constant level balloon data from Angell (1960) and Angell and Pack (1960) and computed eddy diffusivities of the large scale flow.

Kao and Al-Gain (1968) examined the characteristics of relative velocity correlation functions, power spectra and cross spectra by analysing particle clusters following streamlines. From this study it was concluded that a quasi-stationary process exists in the large scale relative diffusion in the atmosphere. This conclusion was based on the similarity of autocorrelation functions and power spectra. This work also found that the large scale turbulent dispersion was anisotropic.
Isoberic trajectories computed by Kao and Bullock (1954) and analysis of GHOST balloon data by Wooldridge and Reiter (1970) also support the similarity and stationarity of the large scale flow.

Kao and Wendell (1968) examined the characteristics of turbulent diffusion in the atmospheric boundary layer. This study revealed similarity in power spectra of relative velocity and stability of autocorrelation functions establishing that a quasi-stationary process also exists in relative mesoscale turbulence. By applying Taylor's large-scale diffusion time relationships, the authors published component eddy diffusivities.

As recommended by Kao (1968), Kao and Wendell (1968) chose to study dispersion parameters in the along- and cross-wind direction so that

$$X_r(t) Y_r(t) = 0,$$

satisfying the cross-correlation stipulation mentioned above.

Wooldridge and Lester (1969), flying pillow shaped superpressure balloons to study mountain lee waves, also found a semblance of stationarity in the mesoscale flow. Comparison of balloon tracks with computed wave patterns showed that during balloon flight times the wave conditions were unchanging; furthermore soundings and wind information were representative of the conditions described by the balloons. In this report, spectral power peaks were used to identify lee waves and periods of these spectral peaks were converted to actual wave length estimates through the use of average horizontal wind velocities.

As a result of the Kao and Al-Gain (1968) paper on stationarity and from results of earlier studies, it was determined that the relative diffusion between particles released in clusters and particles released serially from a fixed position give similar results (Kao, 1967).
Superpressure Balloons

Much of the recent work on turbulent diffusion has been accomplished using superpressure balloons. Constant volume superpressure balloons have been used to describe approximate air parcel trajectories since the early 1960's. Angell (1960) analyzed a series of constant level 300 mb transosonde balloon flights launched over the Pacific Ocean between 1957 and 1958. Analysis of these flights demonstrated the usefulness of constant volume balloons in the study of meteorological problems. These balloon trajectories described the Lagrangian flow and through the use of power spectrum analysis gave estimates of component wind oscillations, wave trajectories and dispersion.

Angell and Pack (1960), encouraged by the success of constant level balloons approximating air parcel trajectories, investigated the characteristics of small scale (mesoscale) air flow with specially constructed tetrahedron shaped constant volume balloons called tetroons. Mylar, a new material at the time, was used in construction of these balloons because of its low permeability to helium and high tensil strength. Data from those early low level flights recommended the use of constant volume balloons to describe vertical air motions, turbulence and diffusion, and to assist in the solution of air pollution problems.

Detailed information pertaining to the performance of superpressure balloons was presented by Booker and Cooper (1965). Constant volume balloons made of Dupont mylar, for reasons stated above, will seek a density level where the weight of air displaced equals that of the balloon. Superpressure balloons will only be displaced, excluding balloon failure, from this equilibrium level by vertical air motion to
an extent largely governed by the restoring and drag forces of the balloon. To follow vertical air currents closely, the authors recommended the use of a balloon with a large vertical drag coefficient and small restoring force. To accomplish this, they developed a pillow shaped mylar balloon for their study of air flow over mountainous terrain. Results of comparisons of computed simulated air parcels and pillow balloon trajectories showed that the balloons underestimate the amplitude of the air flow by about 15 percent and indicate maxima and minima too far upstream. These authors concluded, however, that pillow-shaped constant volume balloons give reliable and accurate information concerning the movement of air.

Hanna and Hoecker (1971) investigated the response of constant volume superpressure balloons and also found better approximation of air motion using small balloons which have large drag coefficients.

Because of the frequent use of constant volume balloons, Cherry (1971) published general performance characteristics of tetroons with a view towards evaluating their assets and limitations. It is recognized that superpressure balloon ascent rates were relatively slow, making it difficult to place a balloon at the level of interest in a reasonable period of time. After comparing the ascent rates of several systems, Cherry concluded that the tandem system (towing the tetroon by a liftoon) was the most suitable method.

In studying the response of superpressure balloons to vertical air motions, Angell and Pack (1961) and Angel, Pack and Delver (1969) found that air parcel (Lagrangian) oscillations in the vertical within the planetary boundary layer take place preferentially within the Brunt-Väisälä period. Again using tetroons, Angell, Pack and Dickson (1968)
verified the existence of longitudinal roll-vortices in the planetary boundary layer; Dickson and Angell (1968) used a tetron transponder system to examine eddy velocities within this boundary layer. Hoecker and Angell (1969) determined air trajectories for a proposed site of a nuclear reactor from tetron flights and in doing so examined the effect on air flow caused by a change from flat to hilly terrain. They observed that rapid rise in ground height induced an increase in air speed below an inversion layer due to a venturi effect.

Constant volume balloons have also been used to estimate mesoscale dispersion. Angell (1962) showed that tetroons represent a feasible means of estimating mesoscale atmospheric dispersion from a continuous point source. The measure of dispersion used was the mean square separation, obtained from the mean Lagrangian velocity variance calculated for short travel intervals of balloon runs. Pack (1962) showed that the mean square separation was proportional to the second and third power of time. He also observed that eddy energy dissipation rates from spectral analysis of tetron flight data were in good agreement with research done by other investigators. Continuous point source and instantaneous point source dispersion were also estimated by Angell and Pack (1965) by determining the standard deviation of sequentially and simultaneously released tetroons. The authors then plotted the logarithm of these variances versus the log of distance traveled, obtaining power of distance traveled separation figures.

More recently Angell et al., (1972) studied three-dimensional air trajectories in the atmospheric boundary layer of the Los Angeles basin by means of tetron flights. It was felt that data from such flights provide a unique contribution to urban air quality studies.
It has been shown, therefore, that superpressure constant volume balloons do describe the Lagrangian flow reasonably well, providing considerable information pertaining to the large-scale and mesoscale circulation.

**Spectral Analysis**

Much of the constant volume balloon data analysis by Wooldridge, Angell and Kao above, consisted of subjecting a time series of component relative velocities or locations to spectral analysis. Spectral analysis is essentially the application of a type of harmonic analysis to autocorrelation or autocovariance functions (Panofsky and Brier, 1963). The basic theory of spectrum analysis is described by Blackman and Tukey (1958).

The spectrum of a time series is analogous to an optical spectrum which shows the contribution of different wave lengths or frequencies to the energy of a light source. The autocovariance (covariance of a value in the time series and itself at a given time, called lag, later) subjected to harmonic analysis and smoothing when plotted against frequency shows in its peaks where most of the energy or variance of a given eddy velocity component is produced (Panofsky and Brier, 1963).

In the statistical sense, Muller (1968) describes the harmonics of the autocovariance function as a partitioning of the entire power (variance) of the time series among classes of frequencies (treatments).
METHOD OF PROCEDURE

Operation

Air flow characteristics were measured by observing pillow-shaped constant volume superpressure balloons trajectories in the vicinity of the Wellsville mountain range in Cache Valley, using a dual theodolite system. Balloons were ballasted to fly at predetermined pressure altitude essentially by the method described by Booker and Cooper (1965). Nomographs supplied by the balloon manufacturer, Metrodata Systems Inc., Norman, Oklahoma, were used to compute the ballast. The volume of the balloon completely filled at 0° C was 0.3m³ and the weight of each balloon was given by the manufacture. Using the nomograph, the balloon volume was adjusted according to the ambient air temperature at flight altitude and based on this equivalent volume, a gross lift for flight altitude was determined. The difference between gross lift and weight of neutral buoyancy at launch site less 25 grams was removed and the balloon was ballasted for flight. The 25 grams constitutes the superpressure which assures that the balloon achieved full inflation prior to attaining flight altitude and that it maintains a constant volume during any displacements which might occur in flight.

To assist the pillow balloons in reaching flight altitude rapidly, a tandem lift system similar to that described by Wooldridge and Lester (1969) was used. A 100 gram pilot balloon was used as a tow balloon which carried the superpressure balloon aloft at an approximate ascent rate of ~3 m/sec. This tandem system accomplished the average 1 km rise
to equilibrium height in about 5 minutes as opposed to the 20 minutes experienced during preliminary tests when superpressure balloons were launched alone. This slow rise rate of superpressure balloons is supported by data presented by Cherry (1971). Separation of the pillow balloon was accomplished by using a combination of pre-timed slow-burning and fast-burning fuses. The use of the tandem system avoids a slow ascending trend from the data and therefore eliminates the need for detrending the data. Also flight information is maximized, since the balloon arrives at the level of interest with minimum delay.

Position Computation

Balloons were flown sequentially and observed using dual theodolites, instrument readings of azimuth and vertical angle were taken every 20 seconds. Pairs of sequentially-released balloons were flown at several different altitude levels. Balloons were released from various locations in the valley, and balloon positions were located using the scheme in Figure 1. Absolute values were used in the determination to make the azimuth angle at station A the sign-determining angle. Cartesian coordinate locations were computed relative to station A with the abscissa perpendicular to the base line and the ordinate along the base line, which was generally oriented North-South. The length of the base line must be sufficiently long since the vertex angle (angle C) of the triangulation must remain reasonably large to prevent small instrument reading fluctuations from causing large position errors.

This procedure for locating balloon positions has two disadvantages. First the height, Z component, computation measures the distance above the tangent plane thru station A rather than the actual height of the balloon.
1. \( \psi A = 360 - RA \)
2. \( \psi B = RB - 180 \)
3. \( \psi C = RA - RB \)
4. \( \overline{d} = \overline{AB} \times \frac{\sin \frac{\psi B}{\sin \frac{\psi C}{}}}{\sin \frac{\psi A}{}} \)
5. \( X = \overline{d} \cos \frac{\psi A}{\sin \frac{\psi C}{}} \)
6. \( Y = \overline{d} \sin \frac{\psi A}{\sin \frac{\psi C}{}} \)
7. \( Z = \overline{d} \tan VA \)

Figure 1. Balloon location triangulation scheme.
Second, computations tend to "blow up" when the balloon passes over the base line. To solve the first problem, an adjusted height was computed by compensating for the curvature of the earth. This computation provided a correction of ~30 m in 20 km (.1 percent error) which was not considered significant. The second problem normally resulted in the loss of about 2 data points as the balloon approached and transversed the base line; these points were smoothed from the overall plot as were any other single missing points to provide a continuous time series.

One other computation was performed on the raw data which yielded locations relative to the direction of the balloon flight rather than the base line. This was done by rotating the base line to coincide with the orientation of the mean downwind trajectory. The new configuration has components along the wind and cross wind, normal to the direction of motion. The standard rotation-of-axis equation was used. Such an orientation also satisfied Kao's (1962) requirement that the cross-correlation of the along and cross wind components be zero.

Meteorological Information

The synoptic meteorological condition concurrent to balloon flight event days was examined and recorded. Surface, 700 mb and 500 mb, analyses were used in portraying the weather situation for each event day. Additionally, rawinsonde data from either Salt Lake City, Utah or Cache Valley itself was obtained. Thermodynamic plots of these soundings indicated lapse rate and stable layers of the atmosphere which were then used to help explain some features of the balloon trajectories.

Richardson's numbers were calculated for selected levels through the vertical depth traversed by the balloons to describe the flow and to compare atmospheric characteristics between event days.
The Richardson number is a non-dimensional estimator of turbulence; a function of stability and shearing. The numerator \( \frac{g}{\theta} \left( \frac{\partial \theta}{\partial z} \right) \) is a measure of the stability of the atmosphere and the denominator \( \left( \left( \frac{\partial U}{\partial z} \right)^2 \right) \) is a measure of the destabilizing effect of wind shear, (Hines, 1971). It is generally accepted that \( R_i \leq 0.25 \) indicates turbulence. Hines reached the conclusion that this criteria is generally too stringent as a necessary condition for the generation of turbulence. For this study the horizontal wind, \( U \) in [5] was obtained from pilot balloon trajectories. Pilot balloons were flown from the launch site at the onset and conclusion of each event day.

**Analysis Technique**

As stated earlier, Taylor's statistical approach to turbulent diffusion assumes stationarity. In order to get a first approximation of the stationarity of the atmosphere during two sequentially released balloons, trajectories were plotted in the downwind-vertical plane. Sequential runs that exhibited stationarity were then subjected to center of mass and relative velocity computations, yielding component velocities relative to the motion of the center of mass (perturbation or eddy velocities). The component relative velocities were then subjected to spectral analysis by the standard Blackman and Tukey (1958) technique. In spectral analysis, the total variance of the time series, \( u(t) \), is partitioned into power spectra or variance by frequency, by subjecting the autocovariance function to a Fourier series harmonic analysis. The autocovariance function of the time series for lag, \( \tau \), is
\( \text{cov} (\tau) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} u'(t) u'(t + \tau) d\tau \) \[6\]

where \( \text{cov} (0) \) represents the total variance of the time series, \( \bar{u}'^2 \).

Power spectra are related to autocovariance by the one-sided cosine transform

\[ P(f) = 2 \int_{0}^{\infty} \text{cov}(\tau) \cos 2\pi f \tau d\tau \] \[7\]

The normalized autocorrelation function is related to autocovariance by

\[ R_u(\tau) = \frac{\text{cov}(\tau)}{\bar{u}'^2} \] \[8\]

A more detailed explanation of spectral analysis can be found in Appendix A.

Diffusivity Calculation

Diffusion, measured by eddy diffusivity coefficients, \( K_x, K_y, K_z \), can be related to relative velocities by the analysis presented by G.I. Taylor and explained in Pasquill (1962).

If \( X \) is the deviation of a particle due to eddy velocity \( u' \), after time, \( t \), then

\[ \frac{dX^2}{dt} = 2X \frac{dx}{dt} \]

or

\[ \frac{dX^2}{dt} = 2 \int_{0}^{t} u'(\tau) u'(t + \tau) d\tau \] \[9\]
If turbulence is considered homogeneous and stationary, then

\[ \overline{u'}^2 R_u(\tau) = u'(t)u'(t + \tau) \]

where \( R_u(\tau) \) is the normalized autocorrelation coefficient. Integrating [9], then:

\[ \overline{X}^2 = 2\overline{u'}^2 \int_0^T \int_0^T R_u(\tau)d\tau dt \]

\( X \) is now the deviation of the particle at time \( T \).

From the above it is deduced that for small time \( T \), where

\[ R_u(\tau) \approx 1, \text{ then } \overline{X}^2 = \overline{u'}^2 T^2 \; \text{; for large time } \overline{X}^2 = 2 \left( \int_0^T R_u(\tau)d\tau \right) \overline{u'}^2 T \]

where \( \int_0^T R_u(\tau)d\tau \) is the time scale constant. Pasquill shows that

\( K_x \), eddy diffusivity, is related to particle displacement by

\[ K_x = \frac{1}{2} \frac{d\overline{X}^2}{dt} \]

and as \( K \) tends to a constant,

\[ K_x = \overline{u'}^2 \int_0^T R_u(\tau)d\tau \quad [10] \]

The full spectrum of turbulent velocity can be generalized to

\[ R_u(\tau) = \exp(-t/T_{ul}) \]

where

\[ T_{ul} = \int_0^\infty R_u(\tau)d\tau \]

is the Lagrangian time scale.

From the above, it can be seen that a plot of the natural log of the autocorrelation function versus lag (time) yields the time constant as its slope,
\[
\ln \frac{R_{u}(\tau)}{t} = -\frac{1}{T_{ul}} = -\varepsilon_{u}
\]  

and that the component eddy diffusivity can be determined from [10] by

\[
K_{x} = \frac{\bar{u'}^{2}}{\varepsilon_{u}}
\]  

**Cross Spectrum Analysis**

While considerable information can be obtained from the analysis of autocovariance where frequencies are considered as treatments (spectral analysis), cross-spectrum analysis also contains useful information about the time series. Cross-spectrum analysis is handled exactly as spectrum analysis with the exception that cross-covariances of time series are used rather than autocovariances. The cross-spectrum consists of two components, the co-spectrum and the quadrature spectrum. The co-spectrum is the real and quadrature the imaginary parts of the cross-spectrum. The co-spectrum measures the contribution of oscillations of different frequencies to the total cross-covariance at lag zero between two time series, u(t) and v(t) (Panofsky and Brier, 1963). A plot of co-spectra against frequency indicates the momentum transport of the component velocities and the degree of anisotropy of the turbulent field (Kao, 1968).

The ratio of the quadrature spectrum to the co-spectrum is a measure of the phase relationship of the harmonics of u(t) and v(t).

\[
\phi(f) = \tan^{-1}\left[\frac{Q(f)}{C(f)}\right]
\]
Coherence measures the relationship between the variances of the cross-spectrum and is a dimensionless quantity,

\[ \text{Coh} (f) = \frac{(Q^2(f) + C^2(f))}{[P_{su}(f)P_{sv}(f)]} \]  

The coherence can vary from 0 to 1. This is analogous to the square of a correlation coefficient, except that coherence is a function of frequency (Panofsky and Brier, 1963).
All balloons were flown in Cache Valley except those flown in April 1972 which were launched just west of the Wellsville ridge near the town of Deweyville, Utah. As a result, all balloons traveled in the vicinity of Cache Valley and were primarily affected by the terrain upwind, the Wellsville Mountains. Cache Valley, at Figure 2, is a narrow, generally North-South oriented mountainous valley located in northern Utah. It is approximately 75 km long with its northern extremity in Idaho. The valley from ridge to ridge is about 25 km wide and the floor is about 15 km wide with its base at 1.35 km MSL. To the west, the valley is bounded by the Wellsville Mountains which rise generally to 2.4 km MSL with peaks as high as 2.7 km MSL. West of the Wellsville ridge is the Bear River Valley whose base is 1.3 km MSL. The Wellsville Mountains are an especially precipitous ridge being only 8 km from valley to valley while rising over 1 km in height above the valley. On the east, Cache Valley is bounded by the Bear River Range of the Wasatch Mountains. Here, again, the terrain rises sharply, over 2.7 km MSL in 10 km distance from the valley floor. However, unlike the Wellsville Mountains this terrain remains high eastward into the Wasatch Mountains proper. Cache Valley inclines gently upward to the north as the Bear River flows into the valley from the north and exits to the west. The south end of the valley also drains into the Bear River. On the northwest rim of the valley, the Wellsville ridge terminates and the valley is bounded by a low ridge of approximately 1.6 km MSL which then further north rises to 2.2 km MSL with a major peak at 2.5 km MSL; this massif is called the Clarkston Mountain.
Figure 2. Cache Valley and vicinity. Scale is 1:250,000 where \( \sim 4 \text{ mm} = 1 \text{ km} \) and \( \sim 0.25 \text{ in} = 1 \text{ mi} \).
The fetch of the air flow to the west of the Wellsville Mountains varies. Southwest of the mountains the air flow is unobstructed for over 100 km while due west the Promontory Mountains at 50 km and the Blue Spring Hills at 35 km interrupt and perturb the upper air flow. Further to the northwest, mountains rising to 2.1 km form a continuous north-south massif interrupted only by the narrow north-south valley of the Malad River. Finally almost due north is the Clarkston Mountains which form the northern rim of Cache Valley and run north-northwest presenting a long massif to air flow from the north. In all, Cache Valley approximates an ideal mountain valley very closely, being bounded on the west by a sharp narrow ridge which for all intents and purposes can be considered as a ridge line of infinite length.
DATA ANALYSIS

Collection

Superpressure balloons were flown predominantly during the fall of 1972. While two preliminary flights were accomplished in March and April 1972, eleven events were completed from September through November 1972. Of these events, however, only eight yielded sufficient data for complete calculations (circulation and diffusion). The most frequent problem encountered causing the collection of insufficient data was termination of visual contact usually by clouds.

The requirements for visual contact with the balloons therefore placed a significant bias on the data to follow. Balloons were flown during the day, and in mostly clear weather. Furthermore, as a rule, balloons were flown during midday adding an additional bias to the data. This was done because it is during this time that the atmosphere is relatively unchanging and stationarity of its major characteristics is approximated.

To make the diffusion calculations meaningful to cloud seeding requirements, events were usually attempted just prior to the arrival of storms. However, such periods consist of increasing instability and cloud formation which made observations difficult and in some cases impossible.

The dilemma is one where the best data for computing dispersion for cloud seeding experiments could not be collected because the event to be studied obscured the collection of data. This restriction applied only
to cloud seeding as the diffusion information obtained has many other uses such as removal of pollutants from the valley.

In succeeding paragraphs, four case studies of selected events describing the main characteristics of circulation and diffusion are presented. The launch sites for these events are noted on Figure 2. Information derived from all flights will then be included in the final sections concerning circulation and diffusion, respectively.

Case I

Three superpressure balloons were flown sequentially over Cache Valley from the west bench, just south of the village of Mendon on September 15, 1972 (Figure 2). Figure 3 shows the trajectories of these three balloons where the x-axis is the downwind distance oriented west to east. This orientation is approximately perpendicular to the Wellsville ridge. Figure 4 represents the horizontal projection of the flights where grid north is in the positive y direction. These plots and all others presented in this study represent equal flight periods where positions were computed every 20 seconds and smoothed for projection. All computations were made on essentially unsmoothed data. The trajectories start at the separation point where the superpressure pillow shaped balloon was released from the tow balloon. As is noted in Figure 3 and will be seen in subsequent events, the fuzing method used was highly reliable. Once perfected, the system operated on time on every flight. Cherry (1971) describes the system used here as unsatisfactory; the perfect record attained in this study, however, suggests otherwise.

Balloons were flown during the period 1100 hours to 1500 hours MDT under clear skies with light and variable surface winds. The synoptic
Figure 3. Downwind-vertical projection of constant volume balloon trajectories for 15 September 1972.
Figure 4. Horizontal projection of C.V. balloon trajectories for 15 September 1972. Contours in meters MSL.
situation for the period is depicted by the 700 mb analysis for 1200 Z, September 15, 1972 at Figure 5, and by atmospheric soundings bracketing the event at Figure 6 which were taken at Salt Lake City, 100 km to the south of Cache Valley. A stationary low aloft was situated off the Southern California coast while flow was weak and generally northwesterly over the Intermountain region. The 500 mb analysis for the period reflected the offshore low and weak pressure fields over Utah. The 500 mb winds veered from 260° to 270° during the 12-hour period bracketing this event. With Utah located in the col or null point of the upper air trough, the surface chart showed persistent high pressure over the region. While the lower atmosphere was conditionally unstable, no clouds were observed during the event period.

In order to first establish if stationarity of major atmospheric characteristics existed as required by the statistical approach to diffusion calculation, three constant volume balloons were flown on this day over a period of approximately 4 hours. The computation of eddy diffusivity coefficients and the validity of utilizing these coefficients to represent turbulent diffusion for a given time period require that the broad features of the airflow be unchanging with time over the period in question. While it is necessary to establish that a temporary state of stationarity exists, it is also useful to know how long such a state might be maintained. Certainly visual inspection of existing conditions would reveal when there was a gross change in atmospheric features terminating stationarity but such on site observations would not necessarily indicate, however, if stationarity existed in the first place.

Temporary stationarity was assumed to exist during this event since the autocorrelation and power spectra of the component velocities
Figure 5. 700 mb analysis for 1200Z, 15 September 1972.
Figure 6. Salt Lake City, Utah soundings for 1200Z, 15 September 1972 and 0000Z, 16 September, 1972, plotted on a tephigram.
show good agreement from flight to flight. This agreement is shown in Figures 7 and 8 of the autocorrelation and power spectra, respectively, of the vertical relative velocity component between flight pairs. Plots of the other component velocities, not shown here, also exhibit this similarity from flight to flight. By virtue of the similarity of these plots, stationarity was assumed to exist for the duration of the three balloon runs.

The horizontal projection of the trajectories, Figure 4, shows the flow to be almost westerly with some southerly component, perhaps valley flow, affecting constant volume balloon run I (CVI). Winds at ridge top, measured by pilot balloons, were 10 m/sec and the change of wind speed through the layer was also 10 m/sec as the wind shadow of the ridge kept the lower flow still. From Figure 3 it is seen that after release from the tow balloon all superpressure balloons flew just below ridge top height. All trajectories initially exhibited vertical motion from equilibrium level for the first half of the flights. This upward drafting was also noted during the first portion of other superpressure balloons flown from the west bench of the valley. All three trajectories here show "billow" structure along the flow and CV II indicates a "breaking" billow about 3.0 km downwind from launch site. The amplitude of the billows appears to be ~150 meters while the wave length appears to be ~1000 meters. Unstable billows frequently form on the upper boundary of larger lee waves in the presence of strong vertical sheer (Reiter, 1969b). The turbulent mixing following the breaking of waves causes mixing of atmospheric properties and modifies the vertical temperature structure. It would follow then, that a sounding taken in such an atmosphere would reflect an adiabatic mixed layer with a stable layer above. The Salt
Figure 7. Plot of autocorrelation $R_v(\tau)$ of vertical relative velocity versus time, labeled by balloon runs paired for relative velocity computation.
Figure 8. Plot of power spectral density versus frequency for Langrangian vertical velocities. Plots are labeled according to pairing of balloons for relative velocity computation.
Lake sounding doesn't reflect such a stratification but subsequent soundings taken in Cache Valley do show such a mixed layer consistently at ridge top height. After the appearance of the breaking wave in CV II, that section of the trajectory of CV III appears modified. This apparent modification of air flow supports the view that breaking waves did mix and modify the atmosphere.

Since spectral analysis of relative velocities of all combinations of flight pairs on this day yielded similar results (stationarity), the results of CV I versus CV III are presented here as representative of the entire period. Spectral density of the component relative velocities plotted against frequency are shown at Figure 9. The greatest power of variance is seen in the component along the wind with smaller values for the cross wind and vertical components. This is as expected but it does serve to exemplify the anisotropy of turbulence and diffusion. The slope of these spectral curves is steep, close to $k^{-4}$. Kao and Al-Gain (1968) and Wooldridge and Reiter (1970) showed the slope of large scale power spectra to be near $k^{-3}$. The steeper slope here suggests even less energy is contained in the higher frequencies of mesoscale flow. The spectra of the component along the wind peaks sharply while the cross wind and vertical spectral peaks are more broad. In all cases it is clear that the greatest energy is located in lower wave numbers indicating it is by far the larger not smaller eddies which account for turbulent diffusion.

Spectral peaks and periods for the relative velocities are shown in Table 1.
Figure 9. Plot of power spectral density versus frequency for relative component velocities of CV I vs CV III, 15 September 1972. Plots are labeled by component velocity.
Table 1. Spectral peaks and associated periods of spectral density plots for CV I vs CV III, 15 September 1972

<table>
<thead>
<tr>
<th></th>
<th>Along wind</th>
<th>Cross wind</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak ($m^2/sec^2$)</td>
<td>5.6</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Period (sec)</td>
<td>175</td>
<td>85</td>
<td>100</td>
</tr>
</tbody>
</table>

If the mean wind speed, 7 m/sec, is applied over the period of these eddies, the along wind wave length becomes ~1200 M. Also if the square root of the vertical eddy variance is considered to be half the vertical velocity, then the vertical eddy wave length becomes 140 meters. These estimates computed from spectral peaks compare favorably with the estimates made by inspection of the trajectory plots.

In examining the coherence of the cross-spectra of the relative velocities, no coherence is found in the along vs cross wind and cross wind vs vertical cross-spectra, but good coherence is achieved between the along wind vs vertical velocities. A coherence of 0.8 is found at a period of 130 seconds where the cospectra is positive and in phase, indicating that $\overline{u'w'} > 0$. Such a relationship indicates a forward tilting wave. Here again spectral analysis confirms what the trajectory plots indicated, demonstrating the strength of this analysis technique.

Eddy diffusivities were computed using [11] and [12] for each of the flight pairs, those for CV I versus CV III are shown in Table 2.
Table 2. Eddy diffusivity calculation for CV I vs CV III, 15 September 1972

<table>
<thead>
<tr>
<th></th>
<th>Along wind</th>
<th>Cross wind</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy velocity mean square, $\bar{u}'^2 \ (m^2/sec^2)$</td>
<td>16.1</td>
<td>9</td>
<td>49</td>
</tr>
<tr>
<td>Time constant $\varepsilon_x \ (sec^{-1})$</td>
<td>$1.5 \times 10^{-2}$</td>
<td>$6 \times 10^{-2}$</td>
<td>$5 \times 10^{-2}$</td>
</tr>
<tr>
<td>Eddy diffusivity, $K_x \ (cm^2/sec^{-1})$</td>
<td>$1 \times 10^7$</td>
<td>$1 \times 10^6$</td>
<td>$1 \times 10^5$</td>
</tr>
</tbody>
</table>

The mean eddy diffusivities for the three sets of flight pairs were $K_x = 1 \times 10^7$, $K_y = 1 \times 10^6$, $K_z = 2.5 \times 10^5$. The gradient Richardson's number for the event period computed by bracketing the mean flight altitude by $\pm 500$ meters was $R_i = 0.5$.

Finally with regard to significance of the spectral peaks, confidence limits were computed for the sharp along wind peak of $5.6 \ m^2/sec^2$ (Figure 9). The 90 percent confidence limit for this peak was $3.7 < \sigma^2 < 9.3$.

**Case II**

Two superpressure and supporting pilot balloons were flown on 22 September 1972. Figure 10 shows the horizontal projection of the trajectories as in Case I, and Figure 11 describes the vertical, downwind air flow projection. Weather at the launch site was clear with some high cirrus clouds overhead. Surface winds were initially light and variable, later in the event period they turned southerly as the sky became overcast. Lenticular clouds appeared in the southern end of the valley with the change in surface winds. The wave length of these lenticular clouds was visually estimated to be 5 km.
Figure 10. Horizontal projection of CV balloon trajectories for 22 September 1972.
Figure 11. Downwind-vertical projection of CV balloon trajectories for 22 September 1972.
Balloons were flown during the period 1030-1445 MDT. The synoptic situation as portrayed by the 700 mb analysis is shown in Figure 12 and Figure 13 for the time period bracketing the event. Figure 14 shows the Salt Lake City soundings for the period. The 700 mb analysis shows the wind backing and increasing slightly over Salt Lake City as a trough approached from the west. The 500 mb analysis reflects the approaching upper air trough with wind speeds about 40 knots over the Intermountain region. The surface charts for the period showed a surface front impinging on the northwestern corner of Utah. The soundings show decreasing stability in the lower atmosphere for the period with the upper air again conditionally unstable up to 550 mb. Winds at ridge top, as measured by pilot balloons flown from the launch site, were 7.5 m/sec.

The balloons were flown from about mid-valley approximately 5 km due west of Logan City (Figure 2). The balloons were towed to equilibrium height and flew at a mean height of 2600 m MSL which again put them at ridge top height of the up-wind Wellsville range. By the direction of the horizontal trajectories (Figure 10), it is seen that the balloons followed closely the upper level gradient flow. The balloons separated 400 meters apart vertically (Figure 11) but soon reached equilibrium level and followed the flow, which for the reasons stated in Case I is assumed to be stationary. Both balloons experience vertical motion at about 4 km downwind from separation. This vertical motion appears to be caused by wave motion the exact dimension of which cannot be clearly determined since both flights experience a blocking of the flow further downstream. After the similar rise of both balloons, CV I shows the same billow-like structure seen in Case I. Here, the wave length of the
1200Z FRI 22 SEP 1972
700MB ANALYSIS
NATIONAL WEATHER SERVICE

Figure 12. 700 mb analysis for 1200Z 22 September 1972.
Figure 13. 700 mb analysis for 0000Z 23 September 1972.
Figure 14. Salt Lake City soundings for 1200Z 22 September 1972 and 0000Z 23 September 1972, plotted on a tephigram.
billows appears to be about 1-1.5 km. CV II later appears to be fairly smooth through this same space. The hypothesis concerning breaking waves, discussed in Case I, is further supported here. CV II describes a wave of about 7 km in length between 4 and 11 km downwind of launch. Both trajectories, at the end of the flights, show a discontinuity in the flow by describing a vertical roll or rotor, 13 km from launch. The diameter of the rotor is 3 km and is apparently caused by upwind blockage of the flow by the eastern wall of the valley, the Bear River range of the Wasatch Mountains.

In examining the spectral analysis of the relative velocities of these runs, Figure 15, the along wind component has its major peak at a period of 160 sec. This peak is also reflected in the cross wind and vertical plots. Using the mean wind of 7.5 m/sec, this peak reflects a wave length of about 1200 m. Considering the along wind peak to be a broad peak, the wave length of the along wind eddy becomes 900-1200 m, which coincides with the visual observation of the billow structure on the trajectory plots. Again using the square root of the vertical variance as half the vertical velocity, the main peaks of the vertical plot yield amplitudes of 350 meters and of 200 meters.

Lower frequency oscillations associated with the rotor and perhaps the wave are not seen by this analysis since the lag was only 10 percent. For further definition 20 percent and 30 percent lags were computed to illuminate the oscillations of larger periods. The 30 percent spectral analysis did yield a small peak at ~400 seconds which would correspond to a wave length of 3000 m, but as will be seen later, significance of such a peak is small. In marginal cases, spectral peaks can only be considered real and not a function of the analysis technique when they
Figure 15. Plot of power spectral density versus frequency for component relative velocities of CV balloon flights on 22 September 1972.
are validated by visual inspection of trajectory plots or the like. The slope of the spectral density curves (Figure 15) is steeper than $k^3$ but not as steep as $k^{-4}$ indicating a faster dumping of turbulent kinetic energy into higher frequencies than was shown in Case I. This might result from the repeated high frequency oscillations seen in the end of CV II and, therefore, be indicative of the blocked flow.

Coherence of the cross-spectral analysis is strong only in the along wind, vertical plane. A coherence of 0.85 was computed for the broad main peak. This again indicates that when $u' > o$, $w' > o$ and the billows have a forward tilt which is supported by the trajectory plots.

Eddy diffusivity computation for these flights are in Table 3.

Table 3. Eddy diffusivity calculation for 22 September 1972

<table>
<thead>
<tr>
<th></th>
<th>Along wind</th>
<th>Cross wind</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy velocity mean square, $u'^2$, ($m^2/sec^2$)</td>
<td>14.5</td>
<td>6.8</td>
<td>1.3</td>
</tr>
<tr>
<td>Time constant $\varepsilon_x$ (sec$^{-1}$)</td>
<td>1.2x10$^{-2}$</td>
<td>1.6x10$^{-2}$</td>
<td>1.4x10$^{-2}$</td>
</tr>
<tr>
<td>Eddy diffusivity $K_x$ (cm$^2$/sec)</td>
<td>1x10$^7$</td>
<td>4x10$^6$</td>
<td>9x10$^5$</td>
</tr>
</tbody>
</table>

The gradient Richardson's number for the flow at flight altitude based on local wind analysis and the Salt Lake City sounding was 2.6.

Confidence limits for the main along the wind peak for both 10 percent and 30 percent lags illustrate the loss of significance with higher lag spectral analysis. The physical appearance of a phenomenon described in the trajectories, rather than a spectral peak, however, is the real
proof of its presence. The peak used for the confidence limit computation was the $5.5 \text{ m}^2/\text{sec}^2$ peak of the along wind curve.

The 90 percent confidence limits are as follows:

10 percent lag: $3.7 < \sigma^2 < 9.1$

30 percent lag: $2.8 < \sigma^2 < 16.1$

Case III

On November 7th, 1972, two CV constant volume and two pilot balloons were launched from the same site used in Case I. The balloons on this date, however, were not towed to equilibrium height but launched with almost neutral buoyancy; just enough lift to get them off the base line. Figures 16 and 17 show the vertical, west-east projections and horizontal projection of balloon trajectories, respectively. Balloons were launched during the period 0930 and 1115 hours MST. Surface winds at the launch site were calm and the sky was completely overcast with cloud height about 4250 MSL. Scattered low level stratus were also present as the pilot balloon launched at the end of the event was lost in such a cloud at ridge top height. The mean wind in the vicinity of the balloon flights was nearly zero as can be seen from the trajectories.

The synoptic situation portrayed by the 700 mb analysis bracketing this event period is shown at Figures 18 and 19. The sounding at Figure 20 was one of three local soundings obtained during an event day. These soundings were launched from within the valley about 3 km due west of the city of Logan (Figure 2). As can be seen from the analysis, Utah and the Intermountain west were in a southwesterly flow during the period, in advance of a trough entering the west coast of the United States. During this period the trough weakened with a slight lowering
Figure 16. East-west-vertical projection of CV balloon trajectories for 7 November 1972.
Figure 17. Horizontal projection of CV balloon trajectories for 7 November 1972.
Figure 18. 700 mb analysis for 1200Z 7 November 1972.
Figure 19. 700 mb analysis for 0000Z 8 November 1972.
Figure 20. Cache Valley sounding 1100 hrs MST 7 November 1972, plotted on a tephigram.
of pressure over Utah. The 500 mb analysis also reflected the upper air trough but without any loss of intensity; and lower pressures prevailed over Utah. The surface charts showed a weak surface low with attendant front entering Utah. The Cache Valley sounding revealed a nearly moist lapse rate up to 600 mb with a very small super-adiabatic layer (A) just below 700 mb. This layer is hypothesized to be the mixed layer predicted by the billow structure observed at ridge top level in both Case I and Case II.

From Figure 16, it can be seen that strong vertical motions existed in the immediate lee of the ridge. The fact that the balloons were launched with neutral buoyancy emphasizes these motions. CV II describes half of a clockwise roll, moves north, and then completes another clockwise roll. The balloon did not intersect the mountain as suggested by Figure 16 but went behind the knoll shown. CV II initially was caught in a small eddy and then moved northward, up slope. Since the sky was completely overcast throughout this event, the motions observed can for the most part be considered as dynamically induced; the winds at ridge top on this day were 12.5 m/sec. The lack of insolation discounts any major diabatic influences with attendant slope valley effects.

Very similar trajectories were also observed during an event conducted on 30 October 1972. On this day, balloons were launched from the same location, again without tow balloons and with very little lift. While this was a clear day as opposed to 7 November, the recurrence of the strong vertical motion in the lee supports the view that this motion is dynamically induced with perhaps reinforcement from radiative effects.

Spectral analysis of relative velocities is shown in Figure 21. Here the coordinate axis was oriented along and perpendicular to the
Figure 21. Plot of power spectral density versus frequency for component relative velocities of C.V. balloon flights on 7 November 1972.
ridge line. This was done because of the proximity of the flights to the mountain, the lack of any downwind trajectory and the fact that the ridge top flow was from 215° making it close to perpendicular to the ridge. All components of the spectral density plots show weak peaks at a period of about 90 sec. Longer periods show considerable variance as would be expected, since the large roll seen in the trajectories is longer than the analysis can identify. The 90 second period reflects smaller eddies and since the analysis is in the Lagrangian frame it is difficult to identify them physically. The cross wind variance is noticeably greater than the along wind showing greater lateral motion here in the lee of the ridge. The increased lateral turbulence was also reflected in the data taken on 3 October. The slope of these spectral density curves is again steep showing quite clearly that the larger eddies account for almost all of the power and therefore the turbulent kinetic energy of the flow.

Eddy diffusivity computations are shown in Table 4.

Table 4. Eddy diffusivity calculation for 7 November 1972

<table>
<thead>
<tr>
<th></th>
<th>Along wind</th>
<th>Cross wind</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy velocity mean square $\overline{u'^2}$ (m²/sec²)</td>
<td>22.1</td>
<td>75.1</td>
<td>.94</td>
</tr>
<tr>
<td>Time constant $\epsilon_x$ (sec⁻¹)</td>
<td>2.2x10²</td>
<td>2.1x10²</td>
<td>2.5x10²</td>
</tr>
<tr>
<td>Eddy diffusivity $K_x$ (cm²/sec)</td>
<td>1x10⁷</td>
<td>4x10⁷</td>
<td>4x10⁵</td>
</tr>
</tbody>
</table>
In this case we see that the lateral diffusion is greater than the along wind as expected from the distribution of the spectral density curves. This suggests that there is considerable lateral or along ridge dispersion in the lee of the ridge. This was also true for the 3 October event.

The Richardson number computed as before was 0.8. However, the horizontal wind was computed from the sounding taken at mid-valley rather than from pilot balloons launched from the west bench of Cache Valley.

Case IV

Two superpressure and pilot balloons were flown from the west side of the north end of the valley on 30 November 1972. Figures 22 and 23 show the down wind vertical and horizontal trajectories, respectively. The launch site was in the valley, 8.5 km north of the village of Mendon (Figure 2). Balloons were flown during the period 0945-1230 MST. Surface winds were calm and the sky was generally clear. There were some high cirris and intermittent lee wave clouds appeared over the valley. To the south, a large lee rotor was described by stratus clouds forming over the Wellsville ridge. The synoptic situation is described by the 700 mb chart at Figure 24. Flow during the period was almost northerly at 330° and strong, between 25-35 knots. During the period, the trough over the mid U.S. deepened slightly bringing tighter pressure contours over Utah and therefore a stronger wind. The flow was ridged over a large high pressure off the northwestern U.S. coast. The 500 mb analysis reflected the strong northerly flow which increased eastward over the mountains towards colder temperatures clearly describing thermal winds as cold air spilled out of Central Canada. The surface charts for the period placed a high pressure center over Utah. The sounding
Figure 22. Downwind-vertical projection of CV balloon trajectories for 30 November 1972.
Figure 23. Horizontal projection of CV and pilot balloon trajectories for 30 November 1972. Parenthetical figures represent MSL height of pilot balloon.
Figure 24. 700 mb analysis for 1200Z 30 November 1972.
(Figure 25), taken in Cache Valley, showed very stable conditions, isothermal, up through the valley. However at 690 mb, approximately ridge top height, a small adiabatic layer (A) was again observed; such a layer also appeared on 14 November 1972 at 730 mb. In all, three soundings were launched in Cache Valley and each had a thin adiabatic layer at about 700 mb.

Both superpressure balloons were flown at 2.35 m MSL and headed in a northeasterly direction. Air flow at balloon height was at 8 m/sec and the ± 500 meter change in wind speed about balloon height was 11 m/sec. The 700 mb flow was at 15 m/sec and came from 330°. The upper flow is further described in Figure 23 by the horizontal trajectory of a pilot balloon released from the launch site. Parenthetical figures along the pilot balloon trajectory represent MSL height. Here the pilot balloon initially followed the superpressure balloon path but then at 2.8 km MSL veered sharply into the upper flow.

Throughout their flights the constant volume balloons appear to follow a valley circulation even though they flew at 2.35 km MSL which was above the immediate upwind ridge of 1.6 km MSL. However, 10 km upwind to the northwest of this ridge line, Figure 2, the mountains rise to 2.3 km MSL with a peak of 2.5 km at a distance of 20 km distance. This 2.3 km ridge extends upstream over 50 km forming a substantial massif. It is hypothesized that the superpressure balloons followed a valley circulation even though they flew above the nearby ridge line because the separation of flow between upper air and valley circulation was perturbed vertically by the upstream massif. The possibility also exists that the balloons follow an up-valley flow because of downstream blocking similar to what was seen in Case II. This is not believed to be true.
Figure 25. Cache Valley sounding 0900 hrs MST 30 November 1972, plotted on a tephigram.
here because all other flights flown at or above ridge level have flown approximately with the gradient wind. It is only in this case and with a pair of superpressure balloons flown at nearly the same altitude on 28 November 1972 under almost identical synoptic conditions that the balloon trajectories followed a flow different than the predominant upper air flow. Also this was the strongest upper flow measured in all events recorded during data acquisition for this study. Finally it was only in these events that such a large massif was present upstream. Therefore, it is rather conclusive that the separation of flow was perturbed vertically by the massif upstream and the balloons therefore flew in the valley circulation.

There are other interesting facets about this event. CV I is blocked quite completely by the downstream obstruction of Little Mountain which raises to 1.7 km MSL and is only 3 km in depth. The second balloon, CV II, follows a more eastward path and completely misses the deadened flow. Both balloons start rising at 2 km downwind from launch forming a wave, which CV II completes, with a wave length of ~4 km. This wave, in the valley flow, is perpendicular to the high speed upper flow and is not caused by Little Mountain which lies further north, downstream. The only terrain feature which could cause such a perturbation appears to be the Bear River which lies directly below the crest of the wave. The river is broad (1 km) at this point. From the sounding (Figure 25), the surface air temperature was almost freezing (0.8°C). If the water temperature (unknown), is above freezing by 1-2° C then the valley wave might have been thermally induced. The river on this day was unfrozen and flowing freely.
Spectral density plots at Figure 26 show strong peaks in the along and cross wind at a period of 100-80 seconds. Using the mean wind 8 m/sec, the main peak of the along wind spectral curve equates to an eddy with a dimension of 700 meters. The slope of the spectral density curves is again steep, close to $k^{-4}$.

Eddy diffusivity for this period is below in Table 5.

Table 5. Eddy diffusivity calculation for 30 November 1972

<table>
<thead>
<tr>
<th></th>
<th>Along wind</th>
<th>Cross wind</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy velocity mean square $\overline{u'}^2$ (m$^2$/sec$^2$)</td>
<td>19.0</td>
<td>9.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Time constant $\epsilon_x$ (sec$^{-1}$)</td>
<td>2.8x10$^2$</td>
<td>2.8x10$^2$</td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>Diffusivity $K_x$ (cm$^2$/sec)</td>
<td>7x10$^6$</td>
<td>3x10$^6$</td>
<td>4x10$^5$</td>
</tr>
</tbody>
</table>

The Richardson number for the ± 500 meters about the mean flight altitude was 2.6.
Figure 26. Plot of power spectral density versus frequency for component relative velocities of C.V. balloon flights on 30 November 1972.
RESULTS AND DISCUSSION

Circulation

A highly generalized view of the circulation in the vicinity of Cache Valley is at Figure 27. The flow shown incorporates the cases discussed here and other balloon flights conducted during the course of this investigation. The view in Figure 27 is west to east with north oriented into the paper.

Upwind, to the west of the Wellsville ridge the flow impinges on the mountain and separates. Over the top, balloons were ducted through a narrow layer and then rose vertically until out of sight. This ducting was similar to the venturi effect observed by Hoecker and Angell (1969). Over the valley, at ridge top height, the flow was billowy with sharp breaking waves imposed on short lee waves. All atmospheric soundings flown in the valley revealed a mixed adiabatic layer at about 700 mb. The occurrence of these two phenomena at approximately the same height is strong evidence supporting the presence of a mixed layer at approximately ridge top height over the valley. Such a mixed layer would separate the air flow over the valley into essentially upper air and valley flow. In Case IV it was seen that this separation occurred at a higher level. The separation of flow is apparently perturbed and raised downwind from a large massif but the separation occurs approxi-
mately at ridge height downwind from a narrow ridge line such as the Wellsville Mountains.
Figure 27. Generalized circulation in vicinity of Cache Valley.
There is also evidence that this mixed layer is discontinuous at both sides of the valley. On the Wellsville side of Cache Valley above the western bench, strong vertical motions in the lee of the ridge have been found. A rotor has been sighted on several occasions and in Case II outlined by the path of a superpressure balloon. That rotor appeared to be standing 2 km off from the mountain and had a diameter of \( \approx 3.0 \) km. Vertical motion in the immediate lee of the mountains have been suggested for some time (Riehl, 1954). The mixed layer also appears to be discontinuous on the downwind, eastern side of the valley where the flow suddenly deadens. Here a rotor is also formed as the flow is blocked by the mountains to the east. This blocking was quite apparent here and similar blocking was also observed in the valley flow of Case IV where Little Mountain intervened.

The variance of perturbation velocities in the along wind and cross wind direction is analogous to the kinetic energy of turbulent motion or scale of turbulence contained in the longitudinal and transverse circulations. From the case studies presented, it is seen that the along wind flow generally contained twice as much turbulent energy as the cross wind except in the immediate lee of the Wellsville ridge. Here, the cross wind flow variance was three times that of the along wind. This was true in Case III presented here and in the data taken on 3 October 1972. The magnitude of this cross wind turbulence coupled with the strong vertical motions observed recommends the west bench of the valley as an excellent cloud seeding or waste burning site with both good lateral and vertical dispersion. This recommendation is made without taking into consideration downslope winds which commence when the ridge shadows the west bench slopes. Before a cloud seeding site is chosen these slope winds must be
taken into account. It should also be noted that the general picture presented here is drawn from a limited number of cases and a more complete investigation is required to obtain a detailed representation.

**Diffusion**

Eddy diffusivities were computed from eight of the events conducted during the project. The results are plotted by component against the ratio of balloon height above the valley to the ridge height above the valley at Figures 28 and 29. \( R \), the ratio is defined as:

\[
R = \frac{\text{balloon height} - \text{valley height}}{\text{ridge height} - \text{valley height}}
\]

Figure 28 shows the vertical eddy diffusivity against the balloon-ridge ratio. The points plotted closely approximate an exponential curve. A slope of +0.33 was found to have a correlation coefficient of 0.88 for the plot of \( R \) versus the natural logarithm of \( K_z \). Therefore, the vertical eddy diffusivity can be related to ridge height by the following expression:

\[
K_z \propto e^{+3R}
\]  \hspace{1cm} [15]

Such a relationship defines a vertical flux out of the valley. This relationship also appears to hold reasonably well when extended to the macro-scale. The three circled points, not considered in the computation, are from Wooldridge's (1972) work in Camp Hale, Colorado. There appears to be a definite break in the curve above ridge top where the balloon-ridge ratio is greater than 1.0. These points form a straight line whose slope is \( 0.3 \times 10^{-6} \) with a very high correlation of 0.98. Considering then
Figure 28. Plot of vertical eddy diffusivity, $K$, versus the ratio of balloon height above the valley to ridge height above the valley. Circled points represent data from Wooldridge (1972).
Figure 29. Plot of along wind, $K_x$, and cross wind, $K_y$, eddy diffusivity versus the ratio of balloon height above the valley to ridge height above the valley.
two separate regimes of $K_z$, the below the ridge diffusivity appears constant at $\sim 2 \times 10^5$ cm$^2$/sec while the above the ridge $K_z$ follows this relationship:

$$K_z \propto R(3 \times 10^6) \quad [16]$$

The along and cross wind diffusivities, $K_x$ and $K_y$, respectively, do not seem to be as related to the balloon-ridge ratio as strongly (Figure 29). The $K_x$ plots form a straight line with a slope of $+0.5 \times 10^7$, however, the correlation is only 0.73. The cross wind plot reveals no discernable relationship at all. Again, considering the eddy diffusivities as belonging to two separate regimes, the above ridge diffusivities yield the following average values:

$$
\begin{align*}
K_x & = 1 \times 10^7 \\
K_y & = 5 \times 10^6 \\
K_z & = 1 \times 10^6 \text{ (cm}^2/\text{sec})
\end{align*}
$$

Below the ridge $K_x$ and $K_y$ vary over two orders of magnitude while as stated, $K_z$ is fairly constant. Table 6 lists for comparison purposes, mesoscale eddy diffusivities reported by other investigators. These values though computed for varying locations support, at least by order of magnitude and proportion the diffusivities presented above.
Table 6. Mesoscale eddy diffusivity determined by other investigators

<table>
<thead>
<tr>
<th></th>
<th>( K_x )</th>
<th>( K_y )</th>
<th>( K_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5x10^7</td>
<td>10^7</td>
<td>10^6</td>
</tr>
<tr>
<td>2</td>
<td>(5x10^7-5x10^6)</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>4x10^6</td>
<td>4x10^5</td>
<td>5x10^5</td>
</tr>
</tbody>
</table>

Note: 1 = Kao and Wendell (1968)
2 = Angell, Allen and Jessup (1971)
3 = Wooldridge (1972)
SUMMARY AND CONCLUSIONS

A limited climatology of mesoscale circulation and eddy diffusivities for Cache Valley has been presented in the preceding sections.

From the analysis of relative velocities of superpressure balloon flights it is concluded that temporary stationarity does exist in the mesoscale and may last on a time scale exceeding 4 hours. Therefore, diffusivity coefficients should be valid for such a period and could be parameterized in modeling the atmosphere for periods of a few hours.

Evidence is sufficient to conclude that there is a persistent turbulently mixed adiabatic layer over the valley at nearly ridge top height. This layer effectively separates the flow at ridge top into upper air and valley circulation. The separation of flow appears to be perturbed vertically when the upwind flow is interrupted by a long massif as opposed to a narrow ridge line. The mixed layer is discontinuous, however, over the western side of the valley in the immediate lee of the ridge where strong vertical motions were observed. This motion is primarily dynamically induced. The lee of the ridge is also the location of increased lateral diffusion.

Finally, the vertical eddy diffusivity, $K_z$, was found to vary with balloon-ridge ratio exponentially according to the relationship:

$$K_z \propto e^{+3R}$$

A plot of the diffusivity along the wind versus the ratio approximates a straight line but no such relationship is indicated for cross wind diffusion.
LITERATURE CITED


Muller, F. B., 1966. Notes on the Meteorological Application of Power Spectrum Analysis, Meso-Meteorology and Short-Range Forecasting Report #1; Canadian Meteorological Memoirs #24; Department of Transport, Toronto, Ontario; 84 pp.


APPENDIX A

Spectral Analysis

Spectral analysis separates the total variance of a time series, in this study relative velocity, into effects due to treatments (frequency) as in a normal analysis of variance or covariance.

The autocovariance function is designated as

\[ \text{Cov}(\tau) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} u'(t)u'(t + \tau) d\tau \]  \[17\]

where \( \text{Cov}(0) = \bar{u}'^2 \) represents the total variance of the time series \( u(t) \), and \( u'(t) = u(t) - \bar{u}(t) \) is the eddy or perturbation velocity. Autocovariances in the time series are computed up to a maximum time between velocities, \( \tau \), called a maximum lag, (m). In general the maximum lag should not exceed 10 percent of the time series record length. While such practice tends to group the effects of the lower frequencies together and thereby reduce resolution, reasonable statistical stability is obtained. Maximum lag may be increased to 20 percent of the record to provide greater resolution of the lower frequencies but a loss in significance of the spectral estimates is incurred.

The normalized autocorrelation function is defined as

\[ R_u(\tau) = \frac{\text{Cov}(\tau)}{\bar{u}'^2} \]  \[18\]

Power spectra are determined by subjecting the autocovariance function to a harmonic analysis. The analysis used here is the Fourier cosine transformation.
Pasquill (1962), in describing Taylor's analysis, showed when the total variance could be expressed as an integral of the Fourier coefficients $I_1(n)$ and $I_2(n)$, then

$$\overline{u'}^2 = 2\pi^2 \lim_{T \to \infty} \int_0^T \frac{I_1^2(n) + I_2^2(n)}{T} \, dn$$

where

$$2\pi^2 \lim_{T \to \infty} \int_0^T \frac{I_1^2(n) + I_2^2(n)}{T} \, dn$$

was denoted as $\overline{u'}^2 F(n) \, dn$, spectral density and

$$\int_0^\infty F(n) \, dn = 1$$

the spectrum. Pasquill then showed that

$$R(t) = \int_0^\infty F(n) \cos 2\pi nt \, dn \quad [19]$$

and

$$F(n) = 4 \int_0^\infty R(t) \cos 2\pi nt \, dt \quad [20]$$

Expressions [19] and [20] were also utilized by Kao and Wendell (1968), Kao and Bulloch (1964) and Kao (1965) to compute normalized power spectra.

The method used in this study follows the Blackman and Tukey (1958) procedures, which expressed in the exponential form are:

$$\text{Cov}(\tau) = \int_{-\infty}^{\infty} P(f) \exp(i2\pi f \tau) \, df \quad [21]$$
and

\[ P(f) = \int_{-\infty}^{\infty} \text{Cov}(t) \exp(-i2\pi ft) dt \]  \hspace{1cm} [22]

where \( P(f) \) is the power spectral density or sometimes noted as just spectral density and is analogous to:

\[ P(f) = \bar{u}^2 \rho(f). \]

When expressed as even functions [21] and [22] above revert to the two-sided Fourier cosine transform:

\[ \text{Cov}(t) = \int_{-\infty}^{\infty} P(f) \cos 2\pi ft df \]  \hspace{1cm} [23]

\[ P(f) = \int_{-\infty}^{\infty} \text{Cov}(t) \cos 2\pi ft dt \]  \hspace{1cm} [24]

and when treated more simply as a mirror image, a one-sided cosine transform

\[ \text{Cov}(t) = 2 \int_{0}^{\infty} P(f) \cos 2\pi ft df \]  \hspace{1cm} [25]

\[ P(f) = 2 \int_{0}^{\infty} \text{Cov}(t) \cos 2\pi ft dt \]  \hspace{1cm} [26]

These final relationships simplify the expression of the operation of convolution between functions of lag or of frequency related by [19] and [20] used by Kao and Pasquill above. The spectral density is divided by \( 2\pi \) to yield power estimates in terms of cycles per second rather than radians per second.
Autocovariances and therefore spectral densities are computed for the specified number of lags up to the maximum lag. Estimates at these lags are represented in frequency space from \( f = 0 \) to \( f = 1/2 \) oscillation per twice the maximum lag, in steps of \( \frac{1}{2\pi} \) frequency units. The raw spectral estimates at the specified lags or frequencies are then subjected to a smoothing process. The lag or frequency estimates, in effect, are finite windows looking at the entire spectrum. While these spectral windows estimate the power density at the specific frequency, no weight is given to points elsewhere on the curve. For this reason estimates in the spectral window are smoothed to allow power (variance) from either side of the specified frequency to be included in the individual spectral estimate. The smoothing procedure used in this study is called "Hanning" after Austrian meteorologist Julius Von Hahn (Muller, 1966):

\[
\begin{align*}
P_s(f) &= 0.25 (P(f - 1) + P(f + 1)) + 0.5 \ P(f) \\
P_s(0) &= 0.5 \ (P(0) + P(1)) \\
P_s(m) &= 0.5 \ (P(m - 1) + P(m))
\end{align*}
\]

[27]

The plot of spectral density versus frequency yields power spectral peaks which are the estimates of the contribution to the total variance from the effects of certain frequency bands. In considering a spectrum of an observed short time series it is assumed that the spectrum is a random sample of an infinitely long stationary time series. Therefore, spectral peaks yield estimates of actual energy peaks. Since these estimates are variances, confidence limits can be placed on them following standard statistical methods (Dixon and Massey, 1969). The percentiles of the sampling distribution of \( \frac{\chi^2}{\sigma^2} \) are called the percentiles of the \( \frac{\chi^2}{df} \), chi square over degrees of freedom distribution. Using this
distribution one can establish a percentile confidence interval based on the sample spectral density peaks which estimate the true spectral peaks.

$$\frac{s^2}{\chi^2/df(1-\alpha)} < \sigma^2 < \frac{s^2}{\chi^2/df(\alpha)}$$

where $\alpha = \%$.

Significance of the peaks can be tested by null hypothesis of some smooth or known spectral peak. The computations outlined above differ from standard statistical procedures in the determination of the number of degrees of freedom for the spectral estimates. Muller (1969) describes a common approximation for degrees of freedom computation for spectral estimates. The number of degrees of freedom is given by,

$$df = 2 \, W_e \, L,$$

where $W_e$ is equivalent width. The equivalent width for the spectral window, Hanning, used in this study is $W_e = 1.33/m$. At $f = 0$, $m$ these widths are halved. Therefore, for a record length of 100 items with a 10 percent lag; the number of degrees of freedom (df) would be 26.
APPENDIX B

Explanation of Symbols

F . . . . . Flux
A . . . . . Austausch Coefficient
\frac{\partial s}{\partial n} . . . . . Gradient along normal
t, T . . . . . Time
\chi . . . . . Concentration parameter
u . . . . . Component velocity
u' . . . . . Component eddy velocity
\bar{u'}^2 . . . . Mean square of component eddy velocity
R_u(\tau) . . . . Component Lagrangian autocorrelation
e . . . . . Time constant
X_r . . . . . Position relative to mean location
VA . . . . . Vertical angle
RA . . . . . Instrument reading at station A
R_i . . . . . Gradient Richardson's number
\theta . . . . . Potential temperature
U . . . . . Horizontal resultant wind speed
g . . . . . Acceleration of gravity
Cov(\tau) . . . Covariance of time lag
m . . . . . Maximum lag
I(n) . . . . Coefficient of Fourier series
P(f) . . . . Power spectral density of frequency
\chi^2/df . . . Chi square over degrees of freedom
$\sigma^2$ . . . . . Population variance
$s^2$ . . . . . Sample variance
$W_e$ . . . . . Equivalent band width
$L$ . . . . . Length of time series
$\overline{X^2}$ . . . . Particle location mean square
$T_{uL}$ . . . . Component Lagrangian time scale
$\phi(f)$ . . . . Phase angle
$\text{Coh}(f)$ . . . Coherence function
$Q(f)$ . . . . Quadrature spectrum
$C(f)$ . . . . Co-spectrum
$R$ . . . . . Ratio of balloon height above valley to ridge height above valley
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