A Relativistic One Pion Exchange Model of Proton-Neutron Electron-Positron Pair Production

William A. Peterson
A RELATIVISTIC ONE PION EXCHANGE MODEL OF
PROTON-NEUTRON ELECTRON-POSITRON
PAIR PRODUCTION

by

William A. Peterson

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William A. Peterson
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>ix</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I.   INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II.  FORMULATION OF THE PROBLEM</td>
<td>7</td>
</tr>
<tr>
<td>Green's Function Treatment of the Dirac</td>
<td></td>
</tr>
<tr>
<td>Equation</td>
<td>7</td>
</tr>
<tr>
<td>Application of Feynman Graph Rules to</td>
<td></td>
</tr>
<tr>
<td>Pair Production in Neutron-Proton</td>
<td></td>
</tr>
<tr>
<td>Collisions</td>
<td>11</td>
</tr>
<tr>
<td>III. DIFFERENTIAL CROSS SECTION</td>
<td>29</td>
</tr>
<tr>
<td>Symmetric Coplanar Case</td>
<td>29</td>
</tr>
<tr>
<td>Frequency Distributions</td>
<td>44</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>72</td>
</tr>
<tr>
<td>APPENDIXES</td>
<td>75</td>
</tr>
<tr>
<td>Appendix A. Notation and Definitions</td>
<td>76</td>
</tr>
<tr>
<td>Appendix B. Expression for Cross Section</td>
<td>79</td>
</tr>
<tr>
<td>Appendix C. Trace Theorems and $\gamma$</td>
<td></td>
</tr>
<tr>
<td>Identities</td>
<td>84</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix D.</td>
<td>Phase Space Distributions for Neutron-Proton Electron-Positron Pair Production</td>
<td>86</td>
</tr>
<tr>
<td>Appendix E.</td>
<td>Computer Program PHASE</td>
<td>111</td>
</tr>
<tr>
<td>VITA</td>
<td></td>
<td>136</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Glossary of symbols</td>
<td>12</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Typical Feynman diagrams for nucleon-nucleon electron-positron pair production.</td>
<td>5</td>
</tr>
<tr>
<td>2.</td>
<td>Vertices for (a) spinor electrodynamics, (b) electrodynamics of spin-zero boson, and (c) meson-nucleon scattering.</td>
<td>15</td>
</tr>
<tr>
<td>3.</td>
<td>Feynman graphs for process considered in this paper</td>
<td>17</td>
</tr>
<tr>
<td>4.</td>
<td>Symmetrical geometry for cross section</td>
<td>30</td>
</tr>
<tr>
<td>5.</td>
<td>Theoretical curves of differential scattering cross section for the symmetric geometry of Figure 4</td>
<td>34</td>
</tr>
<tr>
<td>6.</td>
<td>Angular distribution of final proton in laboratory system with the direction of incident proton with matrix for weights $&lt; 10^{-8}$</td>
<td>48</td>
</tr>
<tr>
<td>7.</td>
<td>Angular distribution of electron in laboratory with the direction of the incident proton with matrix for weights $&lt; 10^{-8}$</td>
<td>50</td>
</tr>
<tr>
<td>8.</td>
<td>Angular distribution in final proton with the direction of the incident proton in center of momentum system with matrix for weights $&lt; 10^{-8}$</td>
<td>52</td>
</tr>
<tr>
<td>9.</td>
<td>Angular distribution of electron with the direction of the incident proton in center of momentum system with matrix for weights $&lt; 10^{-8}$</td>
<td>54</td>
</tr>
<tr>
<td>10.</td>
<td>Angular distribution between pair in laboratory system with matrix for weights $&lt; 10^{-8}$</td>
<td>56</td>
</tr>
<tr>
<td>11.</td>
<td>Angular distribution between pair in center of momentum system with matrix for weights $&lt; 10^{-8}$</td>
<td>58</td>
</tr>
<tr>
<td>12.</td>
<td>$\Delta^2 = (p_+ + p_-)^2$ distribution of gamma ray with matrix for weights $&lt; 10^{-8}$</td>
<td>60</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>13.</td>
<td>$\Delta^2 = (n_f - n_i)^2$ distribution of $\pi^0$ with matrix for $&lt; 10^{-8}$</td>
<td>62</td>
</tr>
<tr>
<td>14.</td>
<td>Angular distribution between final proton-neutron in the laboratory system with matrix for weights $&lt; 10^{-8}$</td>
<td>64</td>
</tr>
<tr>
<td>15.</td>
<td>Angular distribution between final proton-neutron in center of momentum system with matrix for weights $&lt; 10^{-8}$</td>
<td>66</td>
</tr>
<tr>
<td>16.</td>
<td>Momentum distribution of electron in laboratory system with matrix for weights $&lt; 10^{-8}$</td>
<td>68</td>
</tr>
<tr>
<td>17.</td>
<td>Momentum distribution of electron in center of momentum system with matrix for weights $&lt; 10^{-8}$</td>
<td>70</td>
</tr>
<tr>
<td>18.</td>
<td>Angular distribution of final proton in laboratory with the direction of incident proton without matrix</td>
<td>87</td>
</tr>
<tr>
<td>19.</td>
<td>Angular distribution of electron in laboratory with the direction of the incident proton without matrix</td>
<td>89</td>
</tr>
<tr>
<td>20.</td>
<td>Angular distribution of final proton with the direction of the incident proton in center of momentum system without matrix</td>
<td>93</td>
</tr>
<tr>
<td>21.</td>
<td>Angular distribution of electron with the direction of the incident proton in center of momentum system without matrix</td>
<td>93</td>
</tr>
<tr>
<td>22.</td>
<td>Angular distribution between pair in laboratory system without matrix</td>
<td>95</td>
</tr>
<tr>
<td>23.</td>
<td>Angular distribution between pair in center of momentum system without matrix</td>
<td>97</td>
</tr>
<tr>
<td>24.</td>
<td>$\Delta^2 = (p_- + p_+)^2$ distribution of gamma ray without matrix</td>
<td>99</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>25.</td>
<td>$\Delta^2 = -(n_f - n_i)^2$ distribution of $\pi^0$ without matrix</td>
<td>101</td>
</tr>
<tr>
<td>26.</td>
<td>Angular distribution between final proton-neutron in the laboratory system without matrix</td>
<td>103</td>
</tr>
<tr>
<td>27.</td>
<td>Angular distribution between final proton-neutron in center of momentum system without matrix</td>
<td>105</td>
</tr>
<tr>
<td>28.</td>
<td>Momentum distribution of electron in laboratory system without matrix</td>
<td>107</td>
</tr>
<tr>
<td>29.</td>
<td>Momentum distribution of electron in center of momentum system without matrix</td>
<td>109</td>
</tr>
</tbody>
</table>
ABSTRACT

A Relativistic One Pion Exchange Model of Proton-Neutron Electron-Positron Pair Production

by

William A. Peterson, Doctor of Philosophy

Utah State University, 1973

Major Professor: Dr. Jack E. Chatelain
Department: Physics

Proton-neutron electron-positron pair production cross sections are calculated in the framework of the pseudoscalar one-pion exchange model in a fully relativistic manner.

A computer program has been developed to evaluate invariants and Dirac traces for a given data point.

The cross sections for symmetric coplanar events for laboratory kinetic energies of 10 to 250 MeV were calculated for pair angles of 20° to 120°.

Frequency distributions were also calculated, at a laboratory energy of 200 MeV, using a random number generator to select data points. The frequency distributions are illustrated by curves.

It was noted that the inclusion of heavier bosons will not significantly improve the results at laboratory energies less than 200 MeV.

(145 pages)
CHAPTER I
INTRODUCTION

The early investigations of scattering and absorption of gamma-rays showed that the interaction with heavy elements exceeded that predicted by the Compton and photoelectric effect alone. In 1932 Anderson observed the ejection of electron-positron pairs from lead foil subjected to cosmic ray irradiation; this experiment provided evidence for the Oppenheimer and Plesset (1933) interpretation of pair production on the basis of Dirac's electron theory.

The first studies of the problem of electron-positron pair production were theoretical. Some of the first papers were those of Furry and Carlson (1933) and of Heitler and Nordheim (Heitler, 1960). In these papers a cross section was calculated by the use of the Born-Moller perturbation theory for relativistic particles. They found that for cosmic-ray energies the production of pairs is as important for electrons as it is for gamma-rays. Another early calculation was done by Bhabha (1935). The Bhabha calculation uses the Weizsacker-Williams approximation to calculate the cross-section for pair production by an electron scattering off a fixed electromagnetic field (the trident process). In the Weizsacker-Williams method the incident particle is placed in a rest frame by the Lorentz transformation.
The scattering produced by the nucleus is treated as if its field of virtual photons resemble a flux of photons distributed over a frequency spectrum. Racah (1937) also did a calculation using the 1930 version of perturbation theory. His results did not include exchange of identical particles in the final state.

Experimental evidence for pair production by an electron scattering off a fixed electromagnetic field has been considered by a number of authors and a summary of some of these attempts has been discussed by Johnson (1965).

In 1935 Yukawa first proposed the meson theory of nuclear forces. Since then, a large number of theoretical as well as experimental investigations have been carried out to increase our knowledge about the nature of the nucleon-nucleon interaction.

One way to study the basic problem is to investigate nucleon-nucleon electron-positron pair production. Although this process is more complicated than elastic nucleon-nucleon scattering, it yields more information than can be obtained from the study of the elastic case. In addition it is worthwhile to study the process of pair production partly for its own sake, i.e., as interesting phenomena in itself which must be compared with our theoretical knowledge of elementary processes, and partly because it can be present as part of the "background" when other processes are studied. Only a thorough understanding will permit us to eliminate this background. Also high-energy electron-positron pairs probe the small-distance behavior of
quantum electrodynamics (Bjorken and Drell, 1959). In particular, it offers the possibility of exploring the photon propagator at small distances.

Petiau (1951) and Havas (1952) studied the creation of pairs by collision of particles. They treated the interactions in terms of virtual exchange particles with mass and established the matrix elements for the process to lowest order in perturbation theory. Their papers have been reviewed thoroughly by Parker (1968). Parker calculated a differential cross-section for proton-proton electron-positron pair production. He considered only electromagnetic interactions or photon exchange. Wheeler (1968) made a calculation for neutron-proton electron-positron pair production using only neutral one pion exchange between nucleons. This excluded the possibility of the effect of charge exchange.

In the problem of this thesis, namely proton-neutron electron-positron pair production, the approach taken is that the interaction occurs through one-pion exchange between nucleons. We are aware that the interaction cannot be solely due to one-pion exchange. Exchange of virtual bosons (such as $\rho$, $\omega$, $\mu$, etc.) between two nucleons should be considered as well. Since the pion is the lightest strongly interacting boson, we expect it to be the main contributor to nucleon-nucleon interaction. Furthermore, the nucleons are treated as structureless Dirac particles. Form factors were not introduced to
take into account the fact that the pion is off the mass shell. Another reason for using the one-pion exchange model and for not using form factors is to keep a complicated calculation reasonably simple; and for energies less than a couple hundred MeV, we do not expect an appreciable error for a first calculation.

In this thesis, however, we consider one-pion exchange as a first approximation of nucleon-nucleon interaction and neglect other contributions. There is reason to believe that because the electromagnetic coupling is weak the calculation may normally be treated in lowest order of perturbation theory. The expansion coefficient due to the combined effects of strong and electromagnetic coupling is approximately equal to 0.01. (Figure 1)

One interesting aspect of this approach is that a virtual charged pion can emit a real electron-positron pair (See Figure 1b), a process which can not be explained in the potential model (Simon, 1950).

In our calculation, because the potential for the nucleon-pion will be pseudoscalar and all calculations are done in a fully relativistic manner, it would be a laborious task to square the matrix element represented by Figure 1 and to work out by hand the traces resulting from the spin summations for unpolarized particles. Hearn (1967) has developed a computer code for such problems. This approach was not used because the final answer will contain an exceedingly large number of terms. The difficulty was avoided by eliminating scalar
products of Dirac gamma matrices by hand (Chisholm, 1963). This reduced the matrix to terms involving scalar products of momentum four-vectors and traces of products of gamma matrices.

In Chapter II, a brief discussion of the solution of the wave equation by the Green's function method is presented. Feynman graph rules are applied to proton-neutron electron-positron pair production and the matrix element is displayed. The spin sum averages are then taken of the absolute value square of the matrix element. This reduces the matrix to products of traces involving scalar products of gamma matrices. An example of eliminating scalar products of gamma matrices by hand is then demonstrated.
Because of the tedious task of constructing the differential cross section due to the lengthy algebra involved in working out traces, the problem was programmed to the computer by the author.

In Chapter III, a specific case is first considered in which data points are generated by computer for different energies. These points are substituted into a computer subroutine for the differential cross section and the results are displayed by curves.

An attempt was then made to machine integrate the independent variables by using a random number generator to search for data points. Each data point was given a phase space weight which was then changed to include the dynamic weight of the invariant matrix element. Those weights whose data points fall within a certain specified region of a distribution variable are then summed. Distribution curves are displayed for the neutral theory (non-charge exchange) of the process.

The program used to calculate the traces was written by the author. The phase space subroutines from the computer code PHASE was written at the University of California at Berkeley, California, by Alex Firestone. Due to the complicated nature of the calculation, all aspects of the programs were checked by hand calculations and compared with the value found by the computer.
CHAPTER II

FORMULATION OF THE PROBLEM

When calculating scattering cross sections the equation to be solved is the time dependent wave equation that describes the development of the system with time.

We begin with a brief discussion of the relation of the wave equation to its solution. The Green's function treatment will be used and the scattering matrix or S matrix will be demonstrated. A more rigorous and complete discussion can be found in Schweber (1961).

Finally the Feynman graph rules will be applied to electron-positron pair production in a neutron-proton collision to calculate a scattering matrix for the differential cross section.

**Green's Function Treatment of the Dirac Equation**

The Dirac equation for a particle of mass m in an external field \( \mathcal{A} = A \gamma^\mu \) with \( x = (r, t) \),

\[
(i \nabla - m - eA(x)) \psi(x) = 0 \tag{1}
\]

describes the change in the spinor wave function \( \psi \) with time \( t \). If \( \psi(1) \) is the wave function at \( r_1 \) at time \( t_1 \), the wave function at time \( t_2 > t_1 \)
can always be written as

$$\psi(2) = \int G(2, 1) \psi(1) d^3 r_1,$$

(2)

where $G$ is the Green's function for (1) and we write 1 for $r_1, t_1$ and 2 for $r_2, t_2$.

It can readily been shown that, in general, $G$ can be defined by the solution of

$$i(\nabla_2 - m - e A(2)) G(2, 1) = \delta(2, 1)$$

(3)

where

$$\delta(2, 1) = \delta(2 - 1) \delta(t_2 - t_1);$$

the subscript 2 on $\nabla_2$ means that the operator acts on the variables 2 of $G(2, 1)$.

To derive an expression for $G$ we write (3) in the following form:

$$(i \nabla_2 - m) G(2, 1) = \delta(2, 1) + e A(2) G(2, 1).$$

(4)

the total Green's function $G$ may be written as an integral equation by noting from (3) that $G_0(2, 1)$, the free particle Green's function, satisfies

$$(i \nabla_2 - m) G_0(2, 1) = \delta(2, 1).$$

(5)
Therefore we can write

\[ G(2,1) = G_0(2,1) + e \int G_0(2,3) A(3) G(3,1) d^4 x_3. \] (6)

A Neumann series solution for \( G \) is

\[
G(2,1) = G_0(2,1) + e \int G_0(2,3) A(3) G_0(3,1) d^4 x_3 + e^2 \int \int G_0(2,4) A(4) G_0(4,3) A(3) G_0(3,1) d^4 x_3 d^4 x_4 + \ldots ,
\] (7)

where the integral is extended over all space and time.

\( G(2,1) \) is called the total amplitude for arrival at \( \vec{r}_2, t_2 \),

starting from \( \vec{r}_1, t_1 \). The transition amplitude for finding a particle

in state \( X(\vec{r}_2, t_2) \) at time \( t_2 \), if at \( t_1 \) it was in \( \psi (\vec{r}_1, t_1) \), is

\[
S_{21} = \int \int X^+(2) G(2,1) \psi (1) d^3 \vec{r}_1 d^3 \vec{r}_2 .
\] (8)

This defines the \( S \) matrix when

\( t_2 \rightarrow \infty \) and \( t_1 \rightarrow - \infty \).

We can understand the result from Equation (7) this way. Imagine

that a particle travels as a free particle from point to point but is

scattered by the potential \( A \); thus the total amplitude for arrival at

2 from 1 can be considered as the sum of the amplitudes for various
alternative routes. It may go directly from 1 to 2 (amplitude $G_0(2, 1)$, giving zero order term in (7)), or it may go from 1 to 3 (amplitude $G_0(3, 1)$), become scattered there by the potential and then go from 3 to 2 (amplitude $G_0(2, 3)$). This may occur for any point 3 so that summing over these alternatives gives the first order term in (7).

Again, the particle may be scattered twice by the potential. It goes from 1 to 3 ($G_0(3, 1)$), becomes scattered there then proceeds to some other point, 4, in space time (amplitude $G_0(4, 3)$); it is then scattered and proceeds to 2 ($G_0(2, 4)$). Summing over all possible places and times for 3, 4 gives the second order contribution to the total amplitude. One can in this way write down any of the terms of the expansion (7).

The Fourier amplitude of $G_0$ in momentum space is given by

$$G_0(p) = \frac{-i\frac{m}{2}}{p^2 - m^2} \quad \text{for} \quad p^2 \neq m^2,$$

where $p$ is the momentum and $m$ is the rest mass of the scattered particle. $G_0$, as defined here, is known as the Feynman propagator.

The total transition rate, $W$, from 1 to all 2 is computed by using Fermi’s Golden Rule Number Two

$$W = 2\pi \left| S_{21} \right|^2 \rho \quad \text{(9)}$$

$\rho$ is the phase space available for final states of the system. $\left| S_{21} \right|^2$
in most practical problems is calculated to the lowest non-vanishing order in perturbation theory from the multiple scattering series in equation (7). The terms in the scattering series can be represented by space-time Feynman graphs. This method will now be applied to the problem in this thesis.

**Application of Feynman Graph Rules to Pair Production in Neutron-Proton Collisions**

We will consider the creation of an electron-positron pair in a neutron-proton collision. Feynman graph rules (Feynman, 1949a, 1949b, 1950; Dyson, 1949) will be used to calculate the invariant matrix element $H$, where $H$ is related to the differential cross section by

\[
\frac{d\sigma}{dp_{\perp}dp_{\perp}d\Omega d\Omega} = \frac{m^2 M^4}{(p_i \cdot n_f - M)^{1/2}} \left| \frac{n_f^\circ}{p_f^\circ} \right|^2 \left( \frac{p_f^\circ}{p_f^\circ} - \frac{p_f^\circ}{p_f^\circ} \cdot \frac{n_f^\circ}{p_f^\circ} \right) \left( p_f^\circ \right)^3 H^2
\]

\[
(2\pi)^8 \left( (p_i \cdot n_f - M)^{1/2} \right) \left( \frac{n_f^\circ}{p_f^\circ} \right)^2 \left( \frac{p_f^\circ}{p_f^\circ} \cdot \frac{n_f^\circ}{p_f^\circ} \right) \left( p_f^\circ \right)^3 H^2
\]

The derivation and notation of Equation (10) is presented in Appendix A and B. For definition of the symbols used in the calculation see Table 1.
Table 1. Glossary of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i$</td>
<td>four-momentum of initial proton</td>
</tr>
<tr>
<td>$p_f$</td>
<td>four-momentum of final proton</td>
</tr>
<tr>
<td>$n_i$</td>
<td>four-momentum of initial neutron</td>
</tr>
<tr>
<td>$n_f$</td>
<td>four-momentum of final neutron</td>
</tr>
<tr>
<td>$p_-$</td>
<td>four-momentum of electron</td>
</tr>
<tr>
<td>$p_+$</td>
<td>four-momentum of positron</td>
</tr>
<tr>
<td>$m$</td>
<td>mass of electron</td>
</tr>
<tr>
<td>$M$</td>
<td>mass of proton and neutron</td>
</tr>
<tr>
<td>$U(p)$</td>
<td>the Dirac four-component spinor for a particle of momentum $p$</td>
</tr>
<tr>
<td>$V(p)$</td>
<td>the Dirac four-component spinor for the antiparticle of momentum $p$</td>
</tr>
<tr>
<td>$e$</td>
<td>absolute charge of an electron</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>mass of neutral meson</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>mass of charged meson</td>
</tr>
</tbody>
</table>

The invariant matrix element is computed by drawing all Feynman graphs for the process in question. The Feynman approach allows matrix elements for a given interaction to be formulated from products of appropriate factors, each of which corresponds to a particular segment of the Feynman graph. The interaction probability can be constructed mathematically by simply linking appropriate terms for each step of the process in accordance with certain rules. It
then becomes an easy matter to obtain a formal expression for the cross section.

Those factors needed for the problem in this thesis, using the formalism of Bjorken and Drell (1964), are as follows:

(a) For each external fermion line entering the graph a factor \( U(p) \) or \( V(p) \) denotes whether the line is in the initial or final state; likewise, for each fermion line leaving the graph there is a factor \( \overline{V}(p) \) or \( \overline{U}(p) \).

A Dirac spinor for a particle, characterized by a physical four-momentum \( p \), is denoted by \( U(p) \) while the antiparticle is denoted by \( V(p) \). \( \overline{U}(p) \) and \( \overline{V}(p) \) are the adjoint spinors of the particle and antiparticle, respectively. (See Appendix A)

(b) For each internal fermion line with momentum \( p \) and rest mass \( m \), the factor

\[
\frac{i(p+m)}{2} \frac{1}{p^2 - m^2}
\]

is used, where \( p \) is the inner product of a Dirac \( \gamma \) matrix with a four-vector momentum \( p \), i.e., \( \gamma \, p^\mu = p^\mu \).

(c) For each internal meson line of spin zero with momentum \( q \) and rest mass \( \mu \), the factor

\[
\frac{i}{2} \frac{1}{q^2 - \mu^2}
\]

is used;
(d) For each internal photon line with momentum $k$, the associated factor is

$$\frac{-ig^{\mu\nu}}{k^2}$$

The interactions of the process determine the terms for the vertices of the Feynman graph. The three interactions assumed in this thesis give the following terms:

**Spinor Electrodynamics.** This kind of vertex is shown in Figure 2(a). The rule is a factor $-ie\gamma^\mu$ at each vertex.

**Electrodynamics of a Spin-zero Boson.** The vertex is denoted in Figure 2(b) with the rule that a factor $-ie(p+p')^\mu$, where $p$ and $p'$ are the momenta included in the charged line.

**Pseudoscalar Meson-Nucleon Scattering.** From charge independent theory of nucleon-nucleon interaction, the vertex in Figure 2(c) has a factor $g\gamma^5$ at each meson-nucleon vertex with relative coupling strengths of $\sqrt{2}g$ for charged mesons, $+g$ for neutral to protons, and $-g$ for neutral mesons to neutrons.

In order to write down the matrix element, we do have to pay attention to the order in which the individual processes occur along the fermion line (which may not necessarily be the sequence in time). Due to time reversal invariance of the equation of motion, the end of the graph from which one starts is a matter of convenience. It has, however, become customary to start with the final state and continue down along the fermion lines.
Figure 2. Vertices for (a) spinor electrodynamics, (b) electrodynamics of spin-zero boson, and (c) meson-nucleon scattering. The solid, broken, and wavy lines depict the spinor, meson, and photon respectively.
The lowest order Feynman graphs for pair production by neutron-proton scattering are given in Figure 3. Utilizing the prescription given above the matrix element given by each graph is as follows:

\[ H_a = \frac{-ie g}{k^2 (q_0 - \mu o)(p_a - M^2)} \left\{ \bar{U}(n_f) \gamma_5 U(n_i) \right\} \]

\[ \bar{U}(p_\mu) \gamma^\mu V(p_\mu) \bar{U}(p_f) \gamma_5 [p_a + M] \gamma_5 U(p_i) \left\{ \right\} \]  \hspace{1cm} (11)

\[ H_b = \frac{-ie g}{k^2 (q_b - \mu o)(p_b - M^2)} \left\{ \bar{U}(n_f) \gamma_5 U(n_i) \right\} \]

\[ \bar{U}(p_\mu) \gamma^\mu V(p_\mu) \bar{U}(p_f) \gamma_5 [p_b + M] \gamma_5 U(p_i) \left\{ \right\} \]  \hspace{1cm} (12)

\[ H_c = \frac{2ie g}{k^2 (q_c - \mu c)(p_c - M^2)} \left\{ \bar{U}(p_f) \gamma_5 U(n_i) \right\} \]

\[ \bar{U}(p_\mu) \gamma^\mu V(p_\mu) \bar{U}(n_f) \gamma_5 [p_c + M] \gamma_5 U(p_i) \left\{ \right\} \]  \hspace{1cm} (13)

\[ H_d = \frac{2ie g}{k^2 (q_d - \mu c)(p_d - M^2)} \left\{ \bar{U}(n_f) \gamma_5 U(p_i) \right\} \]

\[ \bar{U}(p_\mu) \gamma^\mu V(p_\mu) \bar{U}(p_f) \gamma_5 [p_d + M] \gamma_5 U(n_i) \left\{ \right\} \]  \hspace{1cm} (14)
Figure 3. Feynman graphs for process considered in this paper.
From conservation of energy-momentum we have

\[ p_a = p_i + n_i - n_f \]  
\[ p_b = p_f - n_i + n_f \]  
\[ p_c = n_f - n_i + p_f \]  
\[ p_d = n_i - n_f + p_i \]

and

\[ q_a = q_b = n_i - n_f \]  
\[ q_c = q_e = n_i - p_f \]  
\[ q_d = q_e' = n_f - p_i \]  
\[ k = p_- - p_+ \]

The numerators of the matrix elements, Equations (11) through (15), can be simplified by using the Dirac operator
\((\not p - m) U(p) = 0,\)

its adjoint

\[ \overline{U}(p)(\not p - m) = 0, \]

and

\[ \gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0. \]

For example Equation (11) has the factor

\[ \overline{U}(p_f) \gamma^5 \mu \gamma_a [\not p + M] \gamma_5 U(p_i) \]

\[ = \overline{U}(p_f) \gamma^5 \mu \gamma_a [-M] U(p_i) \]

\[ = \overline{U}(p_f) \gamma^5 \mu \gamma_a [\not p + \not q - \not q'] - M] U(p_i) \]

\[ = U(p_f) \gamma^5 \mu \gamma_a [\not q - \not q'] U(p_i) \]

\[ = \overline{U}(p_f) \gamma^5 \mu \gamma_5 U(p_i), \]

where

\[ p_i = n_i - n_{\gamma}. \]

Equation (11) can now be written as

\[ H_a = A \overline{U}(n_f) \gamma^5 U(n_i) \overline{U}(p_{-}) \gamma^\mu V(p_{+}) U(p_f) \gamma^5 \mu \gamma_5 U(p_i). \quad (24) \]
Similarly for Equations (12) through (15),

\[ H_b = B \overline{U}(n_f) \gamma_5 U(n_i) \overline{U}(p_-) \gamma_{1\mu} V(p_+) \overline{U}(p_f) \gamma_5 \overline{U}(p_i) \gamma_{1\mu} U(p_i) \]  

\[ H_c = C \overline{U}(p_f) \gamma_5 U(n_i) \overline{U}(p_-) \gamma_{1\mu} V(p_+) \overline{U}(n_f) \gamma_5 \overline{U}(p_i) \gamma_{1\mu} U(p_i) \]  

\[ H_d = D \overline{U}(n_i) \gamma_5 U(p_i) \overline{U}(p_-) \gamma_{1\mu} V(p_+) U(p_f) \gamma_5 \overline{U}(n_i) \gamma_{1\mu} U(p_i) \]  

\[ H_e = E \overline{U}(p_f) \gamma_5 U(n_i) \overline{U}(n_f) \gamma_5 U(p_i) \overline{U}(p_-)(\overline{p}_1 - \overline{p}_2) V(p_+) \]  

denoting

\[ P_2 = n_i - p_f' \]

\[ P_3 = p_i - n_f' \]

and

\[ A = \frac{-i e^{2/2}}{k (P_1 - \mu_o)(P_a - M^2)} \]

\[ B = \frac{i e^{2/2}}{k (P_1 - \mu_o)(P_b - M^2)} \]

\[ C = \frac{-2i e^{2/2}}{k (P_2 - \mu_c)(P_c - M^2)} \]
The total matrix element $H$ is

$$H = H_a + H_b + H_c + H_d + H_e.$$  \hspace{1cm} (29)

The absolute square is

$$|H|^2 = HH^+$$

$$|H|^2 = |H_a|^2 + H_a H_b^+ + H_a H_c^+ + H_a H_d^+ + H_a H_e^+ + H_b H_a^+ + |H_b|^2 + H_b H_c^+ + H_b H_d^+ + H_b H_e^+ + H_c H_a^+ + H_c H_b^+ + |H_c|^2 + H_c H_d^+ + H_c H_e^+ + H_d H_a^+ + H_d H_b^+ + H_d H_c^+ + |H_d|^2 + H_d H_e^+ + H_e H_a^+ + H_e H_b^+ + H_e H_c^+ + H_e H_d^+ + |H_e|^2.$$  \hspace{1cm} (30)

Consider the square of Equation (24)
Equation (31) has factors of the form

\[ |U_{\mathbf{b} \mathbf{a}}|^2 = (U_{\mathbf{b} \mathbf{a}} U_{\mathbf{b} \mathbf{a}})^* (U_{\mathbf{a} \mathbf{a}} U_{\mathbf{a} \mathbf{a}})^* , \]

where \( \bar{Q} = \gamma^0 Q \gamma^0 \), and in particular

\[ \bar{\gamma}^\mu = \gamma^\mu \]
\[ \overline{\gamma}_5 = -\gamma_5 \]

\[ \gamma^\mu \gamma_5 = \gamma^\mu \gamma_5 \]

Therefore,

\[ |H_a|^2 = |A|^2 \overline{U}(n_f)\gamma_5 U(n_i)\overline{U}(n_i)\gamma_5 U(n_f) \]

\[ \overline{U}(p_-)\gamma^\mu V(p_+) \overline{V}(p_+)\gamma^\nu U(p_-)\overline{U}(p_f)\gamma_\mu \frac{1}{2} \gamma_5 U(p_f) \]

\[ \overline{U}(p_i)\gamma_5 \frac{1}{2} \gamma_\nu U(p_f). \]

In general, one does not observe the polarization of the final particles, and one does not know the initial polarization. In absence of such information, one assigns equal a priori probabilities to the different initial polarization states. This means that the actual cross section observed will be a sum over final spin states and an average over initial states. The spin sums can be reduced to traces if we use energy projection operators and closure:

\[ \frac{1}{4} \sum_{\text{spins}} |H_a|^2 = \frac{|A|^2}{4} \text{Tr} \gamma_5 \left( \frac{n_i + M}{2M} \right) \gamma_5 \left( \frac{n_f + M}{2M} \right) \]
\[
\text{Tr} \gamma^\mu \left( \frac{p^+ - m}{2m} \right) \gamma^\nu \left( \frac{p^- + m}{2m} \right)
\]
\[
\text{Tr} \gamma_{\mu 1} \gamma^5 \left( \frac{p^+_i + M}{2M} \right) \gamma_{5 \nu} \left( \frac{p^+_f + M}{2M} \right)
\]
\[
= \frac{|A|^2}{256 m^2 M^4} \text{Tr}(\sigma_i - M)(\sigma_f + M)
\]
\[
\text{Tr} \gamma^\mu (p^+_+ - m) \gamma^\nu (p^- - m)\text{Tr} \gamma_{\mu 1} (\sigma_i - M)\gamma_{1 \nu} (\sigma_f + M).
\] (33)

Using the trace theorems of Appendix C, it can be shown that the spin sums of non-diagonal terms in Equation (30) are equal, i.e.,

\[
\sum_{\text{spins}} H a H^+ b = \sum_{\text{spins}} H b H^+ a,
\]

for example. This leaves fifteen independent terms in (30). Reducing these terms to traces (30) can be written as

\[
\frac{1}{4} \sum_{\text{spins}} |H|^2 = \frac{\text{Tr} \gamma^\mu (p^+_+ - m) \gamma^\nu (p^- - m)}{256 m^2 M^4}
\]
\[
\left| A \right|^2 \text{Tr}(\sigma_i - M)(\sigma_f + M)\text{Tr} \gamma_{\mu 1} (\sigma_i - M)\gamma_{1 \nu} (\sigma_f + M)
\]
\[+ 2AB^+ \text{Tr}(\sigma_i - M)(\sigma_f + M)\text{Tr} \gamma_{\mu_1} (p_{1\mu} - M) \gamma_{\nu_1} (p_{1\nu} + M)\]

\[+ 2AC^+ \text{Tr}(\sigma_i - M)(\sigma_f + M)\gamma_{\mu_1} (p_{1\mu} + M) \gamma_{\nu_2} (p_{2\nu} + M)\]

\[+ 2AD^+ \text{Tr}(\sigma_f - M)(\sigma_i + M)\gamma_{\nu_3} (p_{2\nu} + M) \gamma_{\mu_1} (p_{1\mu} + M)\]

\[+ |B|^2 \text{Tr}(\sigma_i - M)(\sigma_f + M)\gamma_{\nu_1} (p_{1\nu} + M) \gamma_{\mu_1} (p_{1\mu} + M)\]

\[+ |C|^2 \text{Tr}(\sigma_i - M)(\sigma_f + M)\gamma_{\nu_3} (p_{2\nu} + M) \gamma_{\mu_1} (p_{1\mu} + M)\]

\[+ |D|^2 \text{Tr}(\sigma_i - M)(\sigma_f + M)\gamma_{\mu_3} (p_{1\mu} - M) \gamma_{\nu_3} (p_{2\nu} + M)\]

\[+ 2 \text{Tr} \gamma^\mu (p_{1\mu} + m) \gamma (p_{2\nu} + m) \]

\[
\frac{2}{256 m^2 M^4}
\]

\[
\left\{ 2AE^+ \text{Tr}(\sigma_f + M)(\sigma_i - M)(\sigma_f + M)\gamma_{\mu_1} (p_{1\mu} - M) \right. \\
+ 2BE^+ \text{Tr}(\sigma_i - M)(\sigma_f + M)(\sigma_f - M)\gamma_{\mu_1} (p_{1\mu} + M) \\
+ 2CE^+ \text{Tr}(\sigma_i - M)(\sigma_f + M)\text{Tr} \gamma_{\mu_1} (p_{1\mu} + M)(\sigma_f - M) \right. \\
\]
Again utilizing the theorems in Appendix C, the scalar products of $\gamma$ matrices can be eliminated. As an example, consider the first term in Equation (34),

$$\left| \mathbf{A} \right|^2 \text{Tr}(\mathbf{n}_1 \cdot \mathbf{M})(\mathbf{n}_f \cdot \mathbf{M})$$

$$\text{Tr}(\gamma^\mu \gamma^\nu \mathbf{n}_f \cdot \mathbf{M}) \gamma^\mu \gamma^\nu$$

Expanding the above and using theorems 7, 9 and 10 of Appendix C,

$$\left| \mathbf{A} \right|^2 \left\{ 4 \left[ (\mathbf{n}_1 \cdot \mathbf{n}_f) - M^2 \right] \right\}$$

$$\begin{align*}
&= \left| \mathbf{A} \right|^2 \left\{ 4 \left[ (\mathbf{n}_1 \cdot \mathbf{n}_f) - M^2 \right] \right\} \\
&\quad + DM \mu \gamma^\mu \gamma^\nu \text{Tr}(\mathbf{n}_1 \cdot \mathbf{M})(\mathbf{n}_f \cdot \mathbf{M}) \gamma^\mu \gamma^\nu \text{Tr}(\mathbf{n}_1 \cdot \mathbf{M})(\mathbf{n}_f \cdot \mathbf{M})K
\end{align*}$$

(34)
Considering a) and applying theorems 5, 6, and 12 gives

\[
-\frac{2}{\sqrt{2}} \text{Tr} \gamma^\mu \gamma^\nu \text{Tr} \gamma \frac{p_1 \cdot p_1}{\mu_1} \quad \text{c)}
\]

\[
+ \frac{2}{\sqrt{2}} \text{Tr} \gamma^\mu \gamma^\nu \text{Tr} \gamma \frac{p_1 \cdot p_1}{\mu_1} \quad \text{d)}
\]

Applying the needed theorems to b), c), and d) yields

\[
\text{Tr} \gamma^\nu \frac{p_1 \cdot p_1}{\mu_1} \text{Tr} \gamma \frac{p_1 \cdot p_1}{\mu_1}
\]

\[
= 2 \text{Tr}(p_+ \gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu \gamma^\mu \gamma^\nu)
\]

\[
= 8 \left[(p_+ \cdot p_1) \text{Tr} p_+ \frac{p_1 \cdot p_1}{\mu_1} + (p_- \cdot p_1) \text{Tr} p_+ \frac{p_1 \cdot p_1}{\mu_1} \right].
\]

The scalar products were summed without use of the computer for the rest of the terms in Equation (34) in a similar manner. These terms were left in the form of scalar products of four-vectors and traces of products of matrices. The algebraic expression for \( \sum |H|^2 \) resulted in an exceedingly complicated array of terms. In order to
interpret our results, computer subroutines were written to evaluate scalar products of four-vectors and traces of products of $\gamma$ matrices for a given data point. This allowed the evaluation of $\sum |H|^2$. 
CHAPTER III
DIFFERENTIAL CROSS SECTION

The differential cross section for the scattering of particles with pair production was given on page 11 by equation (10). Adopting it we first consider the scattering of a neutron initially at rest \((n_i^0, \overrightarrow{0})\) to the final state \((n_f^0, \overrightarrow{n_f})\) by a proton initially in the state \((p_i^0, \overrightarrow{p_i})\) to a final state \((p_f^0, \overrightarrow{p_f})\). The collision causes the creation of an electron-positron pair in the final state described by \((p_-, \overrightarrow{p_-})\) and \((p_+, \overrightarrow{p_+})\) respectively.

**Symmetric Coplanar Case**

We shall consider a specific case in the symmetrical geometry shown in Figure 4 and graph the differential cross section versus initial proton energies for \(\Theta=10^\circ\) to \(60^\circ\) inclusive.

In order to determine how the energy and momentum in the final state were distributed the following set of equations were solved.

\[
\begin{align*}
\left| p_i^0 \right| &= 2\left| p_f^0 \right| \cos 5^\circ + 2\left| p_-^0 \right| \cos \Theta \tag{1} \\
p_i^0 &= M = 2p_f^0 + 2p_-^0 \tag{2} \\
(p_i^0)^2 &= \left| p_i^0 \right|^2 + M^2 \tag{3}
\end{align*}
\]
Figure 4. Symmetrical geometry for cross section.

\[(p_f^{o})^2 = |p_f|^2 + M^2 \tag{4} \]

\[(p_+^{o})^2 = |p_+|^2 + m^2 \tag{5} \]

Equations (1) and (2) are the conservation of momentum and energy. The energy and momentum of the neutron and proton in the final state are assumed to be equal, and the energy and momentum of the electron and positron are also equal. A Newton-Raphson (Moursund and Duris, 1967) iteration procedure was used to solve the above set of equations, (1) through (5), for a given initial energy. The equations were solved
by the following computer program.

SUBROUTINE ITER

REAL M1, M2
COMMON/S67/ ALPHA, THETA, EBU, PB0, EB1, PB1, El, Pi, R

F1(X, Y)=E*X+B*Y-C
F2(X, Y)=SQRT(X**2+M1**2)+SQRT(Y**2+M2**2)-P
F2X(X)=X/SQRT(X**2+M1**2)
F2Y(Y)=Y/SQRT(Y**2+M2**2)
D(X, Y)=E*Y/SQRT(Y**2+M2**2)-B*X/SQRT(X**2+M1**2)
M1=939.
M2=.511
E=2.*COS(0.017453*ALPHA)
B=2.*COS(0.017453*THETA)
ET=EB0+939.0
C=SQRT(EB0**2-M1**2)
P=ET/2.
X=3.*C/(4.*E)
Y=C/(4.*B)
WRITE (6, 25)
DO 50 K=1,15
T=F2(X, Y)*B-F1(X, Y)*F2Y(Y)
G=X+T/D(X, Y)
T=F1(X, Y)*F2X(X)-F2(X, Y)*E
H=Y+T/D(X, Y)
WRITE(6, 5) K, G, H
IF(ABS(X-G). LT. 0.0001. AND. ABS(Y-H). LT. 0.0001) GO TO 60
X=G
Y=H
50 CONTINUE
60 EB1=SQRT(X**2+M1**2)
F1=SQRT(Y**2+M2**2)
PB0=C
PB1=X
P1=Y
WRITE(6, 15)C, EB1, El
5 FORMAT(1H15, 2F20.10)
15 FORMAT(/1H15, 5HPBO =F12.4, 4X5HEBI =F12.4, 4X, 4HEI =F12.4)
25 FORMAT(/23X, 3HPB1, 18X, 2HPI)
RETURN
END
M1 = mass of proton and neutron
M2 = mass of electron
EBO = initial energy of proton
EBf = final energy of neutron and proton
El = final energy of electron and positron
PBl = final momentum of neutron and proton
Pl = final momentum of electron and positron
PBO = initial momentum of proton
ALPHA = 5°
THETA = θ

An initial guess for the final momenta was to set

\[ PBl = \frac{3}{4} \frac{PBO}{\cos 5°} \]

and

\[ Pl = \frac{1}{4} \frac{PBO}{\cos θ} \]

This avoided unphysical negative solutions in the iteration procedure.

The above computer subroutine was linked to a second program to evaluate the differential cross section for the process. This consists of subroutines, as mentioned before, for calculating inner products of four-vectors and traces of four-vectors contracted into gamma matrices. The subroutines were tested by performing a calculation by hand for a data point and compared with the value found by the computer. Other
aspects of the programs were also checked by hand calculations, to
test each subroutine.

An example of a particular set of output is shown below.

<table>
<thead>
<tr>
<th>EBO</th>
<th>PBO</th>
<th>EB1</th>
<th>PB1</th>
<th>El</th>
<th>Pl</th>
<th>THETA</th>
</tr>
</thead>
<tbody>
<tr>
<td>990</td>
<td>313.65</td>
<td>950.37</td>
<td>146.57</td>
<td>14.13</td>
<td>14.12</td>
<td>40°</td>
</tr>
</tbody>
</table>

SIGMA(1) = $0.2309 \times 10^{-12}$  
SIGMA(2) = $0.4943 \times 10^{-10}$

SIGMA(1) and SIGMA(2) is the differential cross section for noncharge
exchange and charge exchange theory respectively.

The differential cross section versus energy is shown in Figure 5.
Each figure includes two curves to illustrate the effect of charge exchange
and noncharge exchange theories. At low energies and $\theta = 10^o$ the ex-
change theory is about $10^5$ larger than the neutral theory, but the
difference becomes progressively smaller for larger $\theta$, approaching
$10^2$ for $\theta = 60^o$. The peak in the exchange theory appears to shift
toward threshold (941 MeV) for large $\theta$. Extrapolation seems to
indicate that the cross section for the two theories approach one another
at large energies and both tend to zero.

Coplanar proton-proton electron-positron pair production differ-
ential cross section has been calculated by Parker (1968) for laboratory
kinetic energies ranging for 10-100MeV. Parker's corresponding
differential cross section decreases from $10^{-25}$ to $10^{-31}$ cm$^2$/MeV$^2$Sr$^3$.
Comparing Parker's calculation, our calculation gives values of the
order of $10^{-33}$ to $10^{-35}$ cm$^2$/MeV$^2$Sr$^3$ at 10-100 MeV, respectively.
Figure 5. Theoretical curves of differential scattering cross section for the symmetric geometry of Figure 4. The symbols have the following meaning:

N. C. Noncharge Exchange (neutral theory)

C. E. Charge Exchange (symmetric theory)
Figure 5.

Differential scattering cross section in barns /MeV²·steradian³.

(a)

\[ \theta = 10^\circ \]

C.E. \( \times 10^{-10} \)

N.C. \( \times 10^{-13} \)

Incident proton energy in MeV
Figure 5. continued

(b)

\[ \theta = 20^\circ \]

Differential scattering cross section in barns /MeV² steradian⁻³

- C.E. \( \times 10^{-11} \)
- N.C. \( \times 10^{-13} \)

Incident proton energy in MeV
Figure 5. continued
Figure 5. continued
Figure 5. continued

(e)

$\theta = 50^\circ$

Differential scattering cross section in barns/MeV$^2$-steradian

C.E. $x10^{-13}$

N.C. $x10^{-14}$

Incident proton energy in MeV

$10^3$

$10^2$

$10^1$

$10^0$

900 1000 1100 1200 1300 1400
Figure 5. continued

Differential scattering cross section in barns / MeV^2 steradian

- C.E. x10^{-13}
- N.C. x10^{-14}

Incident proton energy in MeV

θ = 60°
We are not aware of any experimental data at the present time with which we can directly compare our results, but it would seem reasonable to assume that the cross section for nucleon-nucleon electron-positron pair production to be no larger than the fine structure constant times that of bremsstrahlung. Therefore, if one uses the experimental measurements for the differential cross section of coplanar events observed in nucleon-nucleon bremsstrahlung, which is a lower order process in comparison to nucleon-nucleon pair production, we have values given by Baier et al. (1969) of the order of $10^{-30}$ cm$^2$/sr$^2$ for proton-proton bremsstrahlung and $0.5 \times 10^{-29}$ cm$^2$/sr$^2$ for proton-neutron bremsstrahlung corresponding to laboratory kinetic energies 10–100 MeV and 200 MeV, respectively. Thus, using the experimental values given by Baier et al., one may expect that the cross section for proton-proton and proton-neutron pair production to be no larger than of the order of $10^{-32}$ cm$^2$/sr$^2$ and $10^{-31}$ cm$^2$/sr$^2$ for kinetic energies of 10–100 and 200 MeV, respectively. Parker's values appear to be too large when compared to bremsstrahlung. From Figure 5 our results does not exceed the upper limit of $10^{-31}$ cm$^2$/sr$^2$ predicted by bremsstrahlung at 200 MeV kinetic energy. It is expected that Parker's values may not be correct since he considered only electromagnetic interaction between nucleons which of course is not the dominant interaction in this energy region.

The pi-meson is believed to be the major contributor to nuclear force at large distances, although heavier bosons such as $\sigma, \eta, \xi, \rho$, and $\omega$ may also play a role for small impact parameter collisions with
large momentum transfer.

To obtain an estimate of the contribution of heavier bosons one might consider the Feynman propagator for a particle of spin zero and mass $\mu$ in the scattering amplitude of two nucleons

$$H = \frac{g^2}{q^2 - \mu^2}.$$  \hspace{1cm} (6)

In writing (6) we have suppressed all factors coming from the vertices at which the particle is absorbed or emitted by the two nucleons. The invariant momentum transfer is $q^2 = (p_1 - p'_1)^2 = (p_2 - p'_2)^2$, where the two nucleons have initial and final momenta $p_1, p_2$ and $p'_1, p'_2$ respectively. In the nonrelativistic limit in which the recoil kinetic energies of the nucleons are neglected relative to their rest energies, $q^2 \approx -|q|^2$ and we may approximate (6) to

$$H \sim \frac{g^2}{|q|^2 + \mu^2}.$$ \hspace{1cm} (7)

Fourier-transforming to coordinate space, we see that (7) corresponds to the scattering in a Yukawa potential

$$V = \frac{g^2 e^{-\mu r}}{r}.$$ \hspace{1cm} (8)

If we consider $\sigma$, the next lightest scalar meson, with rest mass 410 MeV and coupling $g \sim 2$ (Baier, et al., 1969)
\[
\frac{V\sigma}{V\pi} = 0.03.
\]

Where we have taken \( r = 10^{-13} \) cm. as a characteristic distance for strong interactions.

The other bosons such as \( \mu, \epsilon, \rho, \) and \( \omega \) are estimated to contribute about one percent each. From this type of calculation one may expect the contribution of the heavier bosons to the interaction potential to be less than ten percent of the pion contribution.

To arrive at an estimate of the energies at which one may expect to approximate Equation (6) by (7) we write (6), for maximum momentum transfer, as

\[
q^2 = -\left| p_1 \right|^2 \left( 1 - \frac{T_1}{T_1 + 2M} \right).
\]

(9)

Where \( T_1 \) is the kinetic energy of the incident nucleon in the laboratory system and \( M \) is the mass of the nucleon. Equation (9) shows that

\[
q^2 \rightarrow \left| q^2 \right| \text{ for } T_1 \leq 200 \text{ MeV}.
\]

We would like also to mention briefly the possible effects of pionic form factors.

Ueda (1966) investigated proton-proton bremsstrahlung by using a relativistic field theory approach. He adopted the one-pion-exchange model and considered the cases where pionic form factors
were neglected and where they were taken into account.

Ueda used the phenomenological expression of Amaldi and Selleri (1964) for the pionic form factor. His results showed a damping effect of approximately 30 percent for an incident laboratory energy of 200 MeV. If we used Ueda's approach, the same rather large damping effect could be expected in our cross section.

**Frequency Distributions**

In order to find distributions for the cross section the computer code PHASE was adopted for use.

PHASE is composed of two separate computer subroutines. The first is a search program which uses random numbers to generate data points. The final particles are allowed to take on any configuration with the constraint that energy and momentum are conserved across the collision. A statistical phase space weight was then calculated for each data point.

The second program added those weights that fell within a given range of a distribution variable allowing the other variables to take on any value consistent with the conservation laws.

The versatility of PHASE is that the four vector momentum for each final particle is generated and a weight is calculated for each generated particle configuration. This allows one to insert his own weighting modification. In our problem the modification was the invariant matrix element. This follows from
\[ w \sim \sum |H|^2 \rho \]

where \( w \) is the frequency of interaction and \( \rho \) is the phase space factor.

In principle one would have to generate an infinite number of data points, but since this is impossible a rather small finite number of ten thousand data points were generated. This would be consistent with the number one might observe in an experimental situation. For the ten thousand points generated it was found that the matrix varied by a factor of \( 10^8 \). In order to smooth out the graphs for the distributions, weights greater than \( 10^{-8} \) were not included. This excluded 323 data points from the total generated. From the study of scatter plots it was determined that this would not greatly bias the results.

The following Figures 6-17 are only for the noncharge exchange part of the problem. Charge exchange has not been included. Furthermore, no attempt has been made to give an absolute value to the vertical scale; the values represented are relative. The incident kinetic energy of the initial proton in the laboratory is 200 MeV.

Figure 6 shows the angular distribution of the final proton in the laboratory system. The distribution is strongly peaked in the forward direction. Figure 7 shows the angular distribution of the electron in the laboratory system peaked in the forward direction about the incident proton.

The angular distribution of the proton in the center of momentum depicted by Figure 8 shows a maximum around \( 90^\circ \) while, the angular
distribution of the electron in the center of momentum system Figure 9, appears isotropic.

The angular distribution between the pair in the laboratory, Figure 10, and the center of momentum, Figure 11, shows a peak about zero degrees.

The $\Delta^2$ distribution of the exchange gamma ray for the pair, Figure 12, has a maxima near zero, while, the $\Delta^2$ for the pion, Figure 13, has a maxima around $0.125$ (BeV)$^2$.

Figure 14 gives a maxima at about an angle of $90^\circ$ for the angular distribution between the final proton and neutron in the laboratory and in the center of momentum, Figure 15, the maxima is at an angle of $180^\circ$.

Momentum distributions for the electron are given by Figures 16 and 17.

For comparison phase space distributions have been included in Appendix D.

For a test the above method was applied to the elastic scattering of two electrons (Möller scattering). The results agreed very well with previous calculations (Gasiorowicz, 1966).

Because of the extreme variation of the matrix, the conclusion is that any computer evaluation of complicated matrix elements for finding distributions, one can not ignore physical insights about possible singularities. In particular, a program designed to evaluate any (lowest order) process is feasible, but if it is to be useful it will probably have
to include some means of locating and treating carefully the near zeros of denominators.

Because of the greater amount of computer time needed only ten thousand data points were generated. Future work should increase the number of data points by one or two orders of magnitude.
Figure 6. Angular distribution of final proton in laboratory system with the direction of incident proton with matrix for weights < $10^{-8}$. 
Figure 7. Angular distribution of electron in laboratory with the direction of the incident proton with matrix for weights $< 10^{-8}$.
Figure 8. Angular distribution in final proton with the direction of the incident proton in center of momentum system with matrix for weights $< 10^{-8}$.
Figure 9. Angular distribution of electron with the direction of the incident proton in center of momentum system with matrix for weights $< 10^{-8}$. 

Figure 10. Angular distribution between pair in laboratory system with matrix for weights $< 10^{-8}$. 
Figure 11. Angular distribution between pair in center of momentum system with matrix for weights $< 10^{-8}$. 
Figure 12. $\Delta^2 = (p_- + p_+)^2$ distribution of gamma ray with matrix for weights $< 10^{-8}$. 
Figure 13. $\Delta^2 = -(n_f - n_i)^2$ distribution of $\pi^0$ with matrix for weights $< 10^{-8}$. 
Figure 14. Angular distribution between final proton-neutron in the laboratory system with matrix for weights $< 10^{-8}$. 
Figure 15. Angular distribution between final proton-neutron in center of momentum system with matrix for weights < $10^{-8}$. 
Figure 16. Momentum distribution of electron in laboratory system with matrix for weights $< 10^{-8}$. 
Figure 17. Momentum distribution of electron in center of momentum system with matrix for weights $< 10^{-3}$. 
Frequency

Momentum in MeV


APPENDIXES
Appendix A

Notation and Definitions

In our units $c$ and $\hbar$ are set equal to one. With this particular choice of units, time appears to have the dimension of a length; energies, momenta, and masses the dimension of an inverse length; electric charges appear as dimensionless quantities ($e^2 = \frac{e^2}{\hbar c} \approx \frac{1}{137}$).

Momentum four-vectors are defined

$$p^\mu = (p^0, \mathbf{p})$$

and the inner product is

$$p_a \cdot p_b = p_a^\mu p_b^\nu = p_a^0 p_b^0 - \mathbf{p}_a \cdot \mathbf{p}_b.$$  \hspace{1cm} (2)

Three vectors are written as $\mathbf{p}$.

The inner product of a Dirac $\gamma$ matrix with a momentum four-vector is denoted by

$$p_\mu \gamma^\mu = p^0 \gamma^0 - \mathbf{p} \cdot \gamma.$$ \hspace{1cm} (3)

The $\gamma$ matrices are defined through the relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^\mu \nu$$ \hspace{1cm} (4)
with the Hermitian condition

\[ \gamma^\sigma \gamma^\nu = \gamma^\sigma, \quad \gamma^\nu = -\gamma. \]  
(5)

Equation (5) may be written in the condensed form

\[ \gamma^\mu = \gamma^0 \gamma^\mu \gamma^0. \]  
(6)

with

\[ (\gamma^i)^2 = -1, \quad (\gamma^0)^2 = 1, \quad i = 1, 2, 3. \]  
(7)

\[ g^{\mu\nu} \] is a metric tensor used for the lowering and raising of indices, i.e., \( p^\mu = g^{\mu\nu} p^\nu. \)

Spinors \( U \) and \( V \) satisfy the Dirac equation

\[ (\not{\!p} - m) U(p) = 0 \]  
(8)

\[ (\not{\!p} + m) V(p) = 0. \]  
(9)

In terms of the adjoint spinors

\[ \bar{U} = U^+ \gamma^0, \] where \( U^+ \) is the Hermitian conjugate, the Dirac equation becomes

\[ \bar{U}(p) (\not{\!p} - m) = 0 \]  
(10)

\[ \bar{V}(p) (\not{\!p} + m) = 0. \]  
(11)

Summing over spin states lead to energy projection operators:
\[ \frac{p + m}{2m} \text{ for positive energy states} \]

and

\[ \frac{p - m}{2m} \text{ for negative energy states.} \]
Appendix B

Expression for Cross Section

Cross sections can be divided into two parts, the invariant amplitude \( H \) in which the physics lies, and the phase space and kinematical factors. In terms of \( H \) the expression for the differential cross section \( d\sigma \) is, Bjorken and Drell, (1964),

\[
\frac{d^3 p}{E} = \frac{m^2 M^4}{\left[ (p_i \cdot n_i)^2 - m^2 \right]^{1/2}} \left| H \right|^2 \frac{d^3 p_f}{(2\pi)^3 n_f^o} \frac{d^3 p_f}{(2\pi)^3 p_f^o}.
\]

Where

\[
\left[ (p_i \cdot n_i)^2 - n^4 \right]^{-1/2} = \frac{1}{p_i^{o o} \left( p_i - p_f - p_i + p_f \right)}. \quad (1)
\]

is a scalar invariant and \( \vec{V}_1, \vec{V}_2 \) are the collinear velocities of the two initial particles.

If the following identity is used

\[
\frac{d^3 p}{E} = 2 \int d^4 p \, \delta \left( p - p_i - p_f \right) \Theta(p^o). \quad (2)
\]
Where

\[ \Theta(p^0) = \begin{cases} 1 & \text{for } p^0 > 0 \\ 0 & \text{for } p^0 < 0 \end{cases} \]

is the unit step function.

The cross section becomes, when equation (2) is substituted for the final neutron,

\[
d\sigma = \frac{2m^2M^2}{\left[ (p_i \cdot n_i)^2 - M^2 \right]^{1/2}} \frac{\left| H \right|^2}{(2\pi)^8} \frac{d^4n_f \delta(4)(n_i + p_i - n_f - p_f - p_- - p_+)}{d^3p_f \delta(3)_{p_0}^o \delta(3)_{p_-} \delta(3)_{p_+}^o} \]

\[
d\sigma = \int_{M}^{M + n_f^0} \frac{2m^2M^2}{\left[ (p_i \cdot n_i)^2 - M^2 \right]^{1/2}} \frac{\left| M \right|^2}{(2\pi)^8} \delta(n_f \cdot n_f - M^2) \]

\[
\frac{d^3p_f^o}{p_f^o} \frac{d^3p_-^o}{p_-^o} \frac{d^3p_+^o}{p_+^o}. \quad (3)
\]

By energy momentum conservation

\[ n_f = n_i + p_i - p_f - p_+ - p_- \]
\[ n_f \cdot n_f - M^2 = 2 (M^2 + m^2 + n_i \cdot p_i - n_i \cdot p_f) \]
\[- n_i \cdot p_+ - n_i \cdot p_- + p_i \cdot p_f - p_i p_+ - p_i \cdot p_- \]
\[ + p_f \cdot p_+ p_f \cdot p_- + p_- p_+ \]  
(4)

and writing \( d^3 p = |\vec{p}| p^0 \, dp^0 \, d\Omega \) for a final particle to emerge into a given solid angle \( d\Omega \) about an angle \( \theta \), we have

\[
d\sigma = \left\{ \frac{|H|^2}{(2\pi)^4} \right\} \left[ \frac{1}{(p_i \cdot n_i)^2} \right] \begin{vmatrix} |p_f| & |p_-| & |p_+| \\ \lfloor p_i \cdot n_i \rfloor \cdot M & 1 & 1 \end{vmatrix} \]  
(5)

If the argument of the delta function, equation (4) is set equal to \( y \), we have differentiating with respect to \( p^0_f \)

\[
dy = -2 \left\{ \frac{d(n_i \cdot p_f)}{dp^0_f} + \frac{d(p_i \cdot p_f)}{dp^0_f} - \frac{d(p_f \cdot p_+)}{dp^0_f} \right\} \] 
\[
- \frac{d(p_f \cdot p_-)}{dp^0_f} \right\} dp^0_f . \]  
(6)

Substituting the following relation into Equation (6)
\[
\frac{d(p_a \cdot p_+)}{dp_a} = p_b^o - \frac{p_a^o}{|p_a|^2} (p_a \cdot p_b)
\]  

(7)

Therefore by taking the reciprocal of (8)

\[
dy = -\frac{2}{|p_f^o|^2} \left[ \left( p_f^o \frac{1}{|p_f^o|^2} p_f^o \cdot p_f^o - p_f^o \cdot p_f^o \cdot p_f^o \cdot p_f^o \right) - p_f^o p_f^o \cdot p_f^o \cdot p_f^o \right] dp_f^o
\]

(8)

Therefore by taking the reciprocal of (8)

\[
\left| \frac{dy}{dp_f^o} \right|^{-1} = \left| \frac{-p_f^o}{|p_f^o|^2} \right|^{\frac{2}{2 \left[ n_f^o \frac{1}{|p_f^o|^2} n_f^o - p_f^o \cdot p_f^o \cdot n_f^o \right]}}
\]

and using the relation

\[
\int dp_f^o \delta \left[ f(p_f^o) \right] = \left| \frac{df}{dy} \right|^{-1}
\]

The cross section becomes
\[
\frac{d\sigma}{dp_+^0 dp_-^0 d\Omega_p d\Omega_f d\Omega_+}
\]

\[
\frac{m^2 M^4 \mathbf{p} \cdot \mathbf{p}_+ \mathbf{p}_+ \mathbf{p}_- \mathbf{H}^2}{(2\pi)^8 \left( (\mathbf{p}_+ \cdot \mathbf{n}_f)^2 - M^4 \right)^{1/2} \left[ M_f^0 \mathbf{p}_f \mathbf{p}_f - p_f^0 \mathbf{p}_f \cdot \mathbf{n}_f \right]}
\]
Appendix C

Trace Theorems and $\gamma$ Identities

The following trace theorems and properties of Dirac matrices are derived from the commutation algebra of the $\gamma$'s, $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g_{\mu \nu}$, and are valid independently of the choice of representation for the $\gamma$'s.

1. $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \gamma_5$

2. $AB + BA = 2A \cdot B$

3. $A \gamma^5 + \gamma^5 A = 0$

4. $\gamma^\mu A \gamma = -2A^\mu$

5. $\text{Tr } ABC = \text{Tr } CBA = \text{Tr } BCA = \text{Tr } CBA$

6. $\gamma^\mu A \cdot B = 4A \cdot B$

7. $\text{Tr } AB = 4A \cdot B$

8. $\text{Tr } (A + B) = \text{Tr } A + \text{Tr } B$

9. $\text{Tr } 1 = 4$

10. $\text{Tr } (\text{odd number of slashed terms}) = 0$

11. $\text{Tr } ABCD = 4 [(A \cdot B)(C \cdot D) - (A \cdot C)(B \cdot D) + (A \cdot D)(B \cdot C)]$
12. $\text{Tr} (\gamma S) \text{Tr} \gamma^\mu S' = 2 \text{Tr} (S + R)S'$

Where $R$ is $S$ written in reverse and $S$ and $S'$ are "odd strings" or strings containing an odd number of matrices.

If $S = \gamma_1 \gamma_2 \ldots \gamma_n$ then $R = \gamma_n \ldots \gamma_2 \gamma_1$.

13. $\gamma^\mu_\mu R \gamma^\mu = -2R$

Where $S$ and $R$ are defined in 12 above.

14. $\gamma^\mu_\mu S A \gamma^\mu = 2 [A S + R A]$

Theorems 12 and 14 are derived by Chisholm (1963) and Caianiello and Fubini (1952) respectively.
Appendix D

Phase Space Distributions for Neutron-Proton

Electron-Positron Pair Production
Figure 18. Angular distribution of final proton in laboratory with the direction of incident proton without matrix.
Figure 19. Angular distribution of electron in laboratory with the direction of the incident proton without matrix.
Figure 20. Angular distribution of final proton with the direction of the incident proton in center of momentum system without matrix.
Figure 21. Angular distribution of electron with the direction of the incident proton in center of momentum system without matrix.
Figure 22. Angular distribution between pair in laboratory system without matrix.
Figure 23. Angular distribution between pair in center of momentum system without matrix.
Figure 24. $\Delta^2 = (p_- + p_+)^2$ distribution of gamma ray without matrix.
Figure 25. $\Delta^2 = -(n_f - n_i)^2$ distribution of $\pi^0$ without matrix.
Figure 26. Angular distribution between final proton-neutron in the laboratory system without matrix.
Figure 27. Angular distribution between final proton-neutron in center of momentum system without matrix.
Figure 28. Momentum distribution of electron in laboratory system without matrix.
Figure 29. Momentum distribution of electron in center of momentum system without matrix.
Appendix E

Computer Program PHASE
COMMON PLACE(200)  
COMMON/COUNT/ NO1,NO2,NO3  
COMMON/PHASCPC/WEIGHT,WG,WS,EMP,T,TMAX,ECM,NP,NUM,NOTRY,PCM  
DIMENSION EM(10),P(10,3),T(10),TMAX(10),KNUM(50),KPLACE(50),U(50)  
DIMENSION B(50),D(50),HTL(50,20),STL(10,20),KAL(50),PCM(10)  
DIMENSION BIN(50,100),RINS(50,100),WG(50),WS(10)  
DIMENSION KSCAT(10),NX(10),NY(10),XU(10),YU(10),XL(10),YL(10)  
DIMENSION SCA(10,20,20),WTSUMH(50),ASUMH(50),WTUMS(10),ASUMS(10)  
DIMENSION LH(50),LA(50),WA(50),LB(50),WB(50),LC(50),WC(50)  
NO1=0  
NO2=0  
NO3=0  
ZIP=1.0/10**10  
C THIS PROGRAM USES A RANDOM NUMBER AND A WEIGHTING PROCESS TO  
C GENERATE EVENTS.  
C CUTS TO BE MADE. THE RESULTS CAN BE ADDED USING WEIGHTS SUPPLIED BY  
C BY DATA CARDS. THE RESULTS ARE THEN HISTOGRAMMED OR PLOTTED IN CHANNELS  
C OF 1*2*5 BIN WIDTHS. THE INFORMATION IS READ INTO PHASE USING DATA  
C CARDS OF THE FOLLOWING FORMATS.  
C** EVENT INFORMATION E10.0/6110  
C ECM*CENTER OF MASS ENERGY, KNORM*FINAL NORMALIZATION.  
C KHIST*NUMBER OF HISTOGRAMS, KAD*NUMBER OF SUPERIMPOSINGS.  
C KS*NUMBER OF PLOTS, NEVT*NUMBER OF EVENTS GENERATED.  
C** MASS INFORMATION 8E10.0  
C EM(1),=MASSES OF OUTGOING PARTICLES, J=1,NP  
C** HISTOGRAM INFORMATION 3110*3E10.0  
C KNUM*NUMBER OF HIST, KPLACE*PLACE(KPLACE) IN HIST KNUM.  
C KAL*PLACE(KPLACE)+PLACE(KPLACE+1),ECT..KAL  
C B*LOWER LIMIT, U*UPPER LIMIT, D*BIN WIDTH  
C** HTL**TITLE 10A8  
C** SUPERIMPOSING INFORMATION* 2110*E10.0*110*E10.0*110*E10.0  
C LH*NUMBER OF FIRST HIST, LA*LB*LC, HIST TO BE ADDED WITH WTS WA*WR*WC.  
C** PLOTS INFORMATION 3110*4E10.0  
C KSCAT*NUMBER OF PLOT*, NX*NY*NUMBER OF BINS IN X AND Y COORDINATES.  
C XL*YL*LOWER LIMITS, XU*YU*UPPER LIMITS  
C** STL**TITLE 10A8
READ(5*1000) ECM, NP, KNORM, KIST, KAD, KS, NEVT
READ(5*1013) (FM(J), J=1*NP)
WRITE(6*870) (ECM, NP, KNORM, KIST, KAD, KS, NEVT
WRITE(6*871) (FM(J), J=1*NP)
ANORM=KNORM
IF(KHIST) 99, 74, 71
71 DO 1 I=1*KHIST
READ(5*1001) KNUM(I), KPLACE(I), KAL(I), B(I), U(I), D(I)
READ(5*1042) (HTL(I,J), J=1*20)
WRITE(6*888) KNUM(I), KPLACE(I), KAL(I), B(I), U(I), D(I)
WRITE(6*876) (HTL(I,J), J=1*20)
1 CONTINUE
70 IF(KTAD) 99, 72, 73
73 DO 2 I=1*KAD
READ(5*1002) LH(I), LA(I), WA(I), LB(I), WB(I), LC(I), WC(I)
WRITE(6*877) LH(I), LA(I), WA(I), LB(I), WB(I), LC(I), WC(I)
2 CONTINUE
72 IF(KS) 99, 74, 75
75 DO 76 I=1*KS
READ(5*1014) KSCAT(I), NX(I), NY(I), XL(I), YL(I), XU(I), YU(I)
READ(5*1042) (STL(I,J), J=1*20)
WRITE(6*878) KSCAT(I), NX(I), NY(I), XL(I), YL(I), XU(I), YU(I)
WRITE(6*876) (STL(I,J), J=1*20)
76 CONTINUE
74 ECM=SORT(2.*EM(1)*ECM)
NUM=0
NM=0
WTSUM=0.0
EMSUM=0.0
DO 7 I=1*NP
EMSUM=EMSUM+EM(I)
7 CONTINUE
DO 8 I=1*NP
TMAX(I)=(ECM*2+EM(I)**2-(EMSUM-EM(I))**2)/(2.0*ECM)-EM(I)
8 CONTINUE
DO 88 I=1,50
WTSUMH(I)=0.0
ASUMH(I)=0.0
DO 96 J=1,100
BIN(I,J)=0.0
BINSQ(I,J)=0.0
96 CONTINUE
88 CONTINUE
DO 89 I=1,10
WTSUMS(I)=0.0
ASUMS(I)=0.0
DO 97 J=1,20
DO 98 K=1,20
SCAT(I,J,K)=0.0
98 CONTINUE
97 CONTINUE
89 CONTINUE
100 NUM=NUM+1
NOTRY=0
DO 14 I=1,200
PLACE(I)=0.0
14 CONTINUE
CALL EVENT
WTSUM=WTSUM+WGT
DO 25 I=1,50
WGT(I)=WGT
25 CONTINUE
DO 26 I=1,10
WS(I)=1.0
26 CONTINUE
CALL SUB
IF(NUM-10)9,9,87
9 WRITE(6,1003)
DO 11 I=1,25
J=I*8-8
DO 12 K=1,8
L=J+K
IF(PLACE(L))13,12,13
13 N=J+1
NN=J+8
WRITE(6,1004)N,(PLACE(M)+M=N,NN)
GO TO 11
12 CONTINUE
11 CONTINUE
87 IF(KHIST)99,69,10
10 DO 33 I=1,KHIST
KX=KAL(I)
DO 500 J=1,KX
KH=PLACE(I)+J-1
IF(PLACE(KH))39,44,39
39 IF(PLACE(KH)-U(I))34,34,44
34 IF(PLACE(KH)-B(I))44,35,35
35 N=(PLACE(KH)-B(I))/D(I)+1.
IF(N)44,44,38
38 IF(N-100)36,36,44
36 BIN(I,N)=BIN(I,N)+WG(I)
WTSUMH(I)=WTSUMH(I)+WG(I)
ASUMH(I)=ASUMH(I)+1.0
BINSO(I,N)=BINSO(I,N)+WG(I)**2
44 CONTINUE
500 CONTINUE
33 CONTINUE
69 IF(KS)66,66,52
52 DO 53 I=1,KS
J=NX(I)
IF(PLACE(J))54,53,54
54 IF(PLACE(J)-XL(I))55,55,53
55 IF(PLACE(J)-XL(I))53,56,56
56 MX=20.*((PLACE(J)-XL(I))/(XL(I)-XL(I)))+1.
IF(MX)53,53,59
59 IF(MX-20) 57, 57, 53
57 J=NY(I)
   IF(PLACE(J)) 60, 53, 60
60 IF(PLACE(J)-YU(I)) 61, 61, 53
61 IF(PLACE(J)-YL(I)) 53, 62, 62
62 MY=20.*(PLACE(J)-YL(I))/(YU(I)-YL(I))+1.
   IF(MY) 53, 53, 65
65 IF(MY-20) 63, 63, 53
63 SCAT(I,MX,MY) = SCAT(I,MX,MY)+WS(I)

   WTSUMS(I) = WTSUMS(I)+WS(I)
   ASUMS(I) = ASUMS(I)+1.0
53 CONTINUE
66 CONTINUE
   IF((NUM-NM)-1000) 188, 189, 189
189 NM=NUM
   WRITE(6,1050) W G(I), W G(18), W G(34), NUM
188 IF((NUM-NEVT) 100, 15, 15
15 IF(KHIST) 99, 78, 79
79 IF(KAD) 99, 40, 3
3 DO 4 I=1,KAD
   K=H(I)
   L=A(I)
   M=B(I)
   N=L(I)
   UPW T=WTSUMH(K)+WA(I)*WTSUMH(L)+WB(I)*WTSUMH(M)+WC(I)*WTSUMH(N)
   DO 5 J=1,100
   SAME=BIN(K,J)/WTSUMH(K)
   AAD=(WA(I)*BIN(K,J))/WTSUMH(L)
   BAD=(WB(I)*BIN(K,J))/WTSUMH(M)
   CAD=(WC(I)*BIN(K,J))/WTSUMH(N)
   IF(WTSUMH(K)*LT*ZIP) SAME=0.0
   IF(WTSUMH(L)*LT*ZIP) AAD=0.0
   IF(WTSUMH(M)*LT*ZIP) BAD=0.0
   IF(WTSUMH(N)*LT*ZIP) CAD=0.0

116
BIN(K+J) = SAME + AAD + RAD + CAD
BIN(K+J) = BIN(K+J) * UPWT
SAME = BINSO(K,J) / WTSUMH(K) ** 2
AADSO = WA(I) * BINSO(L,J) / WTSUMH(L) ** 2
BADSO = WB(I) * BINSO(M,J) / WTSUMH(M) ** 2
CADSO = WC(I) * BINSO(N,J) / WTSUMH(N) ** 2
IF(WTSUMH(K) * LT * ZIP) SAME = 0.0
IF(WTSUMH(L) * LT * ZIP) AADSO = 0.0
IF(WTSUMH(M) * LT * ZIP) BADSO = 0.0
IF(WTSUMH(N) * LT * ZIP) CADSO = 0.0
BINSO(K,J) = SAME + AADSO + BADSO + CADSO
CONTINUE
WTSUMH(K) = UPWT
ASUMH(K) = ASUMH(K) + ASUMH(L) + ASUMH(M) + ASUMH(N)
CONTINUE
DO 916 I = 1, KNUM
WRITE(6, 1005) KNUM(I), WTSUMH(I)
WRITE(6, 1006) (HTL(I,J), J = 1, 20), ASUMH(I)
WRITE(6, 1007) KPLACE(I), D(I)
DO 17 J = 1, 100
IF(BIN(I,J)) 18, 17, 18
CONTINUE
WRITE(6, 1009)
GO TO 916
18 UH = B(I)
BH = B(I)
DH = D(I)
IB = (UH - BH) / (DH * 250) + 0.99
GO TO (101, 102, 103, 104) * IB
101 WRITE(6, 1030)
EN = 1.0
N = 1
105 EMM = B(I) + (EN - 1.0) * D(I)
AN = ANORM * BINC(I,N) / WTSUMH(I)
IF(BINC(I,N)) 195, 195, 198
195 \text{ANS}0=0.0 \\
\text{GO TO 197}
197 \text{CONTINUE}
\text{WRITE (6, 1031) EMM}
\text{WRITE (6, 1032) AN, ANSQ}
\text{IF(UH-EMM) 16, 15, 106}
106 \text{IF(N-25) 107, 1E, 16}
107 E N= EN+1.0 \\
N=N+1 \\
\text{GO TO 105}
108 \text{WRITE (6, 1033)}
\text{EN1}=1.0 \\
\text{EN2}=26.0 \\
N1=1 \\
N2=26 \\
111 EM1=R(I)+(EN1-1.0)*O(I) \\
EM2=R(I)+(EN2-1.0)*O(I) \\
AN1=ANORM*BIN(I*N1)/WTUMH(I) \\
AN2=ANORM*BIN(I*N2)/WTUMH(I) \\
\text{IF(BIN(I*N1)) 190, 190, 191}
190 \text{ANSQ1}=0.0 \\
\text{GO TO 170}
191 \text{ANSQ1=SORT(BINSQ(I*N1))/BIN(I*N1)}
170 \text{IF(BIN(I*N2)) 192, 192, 193}
192 \text{ANSQ2}=0.0 \\
\text{GO TO 194}
193 \text{ANSQ2=SORT(BINSQ(I*N2))/BIN(I*N2)}
194 \text{CONTINUE}
\text{WRITE (6, 1034) EM1, EM2}
\text{WRITE (6, 1035) AN1, ANSQ1, AN2, ANSQ2}
\text{IF(UH-EM2) 108, 108, 109}
108 E N=EN1+1.0 \\
N=N1+1 \\
\text{GO TO 105}
109 IF(N2-50) 110, 16, 16
110 EN1 = EN1 + 1.0
EN2 = EN2 + 1.0
N1 = N1 + 1
N2 = N2 + 1
GO TO 111
103 WRITE (6, 1036)
   EN1 = 1.0
   EN2 = 26.0
   EN3 = 51.0
   N1 = 1
   N2 = 26
   N3 = 51
115 EM1 = B(I) + (EN1-1.0)*D(I)
EM2 = B(I) + (EN2-1.0)*D(I)
EM3 = B(I) + (EN3-1.0)*D(I)
AN1 = ANORM*BIN(I*N1)/WTSUMH(I)
AN2 = ANORM*BIN(I*N2)/WTSUMH(I)
AN3 = ANORM*BIN(I*N3)/WTSUMH(I)
   ANS01 = SORT(BINS0(I*N1))/BIN(I*N1)
   ANS02 = SORT(BINS0(I*N2))/BIN(I*N2)
   ANS03 = SORT(BINS0(I*N3))/BIN(I*N3)
WRITE (6, 1037) EM1, EM2, EM3
WRITE (6, 1038) AN1, ANS01, AN2, ANS02, AN3, ANS03
   IF(UH=EM3) 112 112, 113
112 EN1 = EN1 + 1.0
EN2 = EN2 + 1.0
N1 = N1 + 1
N2 = N2 + 1
GO TO 111
113 IF(N3-75) 114, 16, 16
114 EN1 = EN1 + 1.0
EN2 = EN2 + 1.0
EN3 = EN3 + 1.0
N1 = N1 + 1
N2 = N2 + 1
N3 = N3 + 1
GO TO 115
104 WRITE (6, 1039)
   EN1 = 1.0
   EN2 = 26.0
   EN3 = 51.0
   EN4 = 76.0
   N1 = 1
   N2 = 26
   N3 = 51
   N4 = 76
119 EM1 = B(I) + (EN1 - 1.0) * D(I)
   EM2 = B(I) + (EN2 - 1.0) * D(I)
   EM3 = B(I) + (EN3 - 1.0) * D(I)
   EM4 = B(I) + (EN4 - 1.0) * D(I)
   AN1 = A * N1
   AN2 = A * N2
   AN3 = A * N3
   AN4 = A * N4
   ANS01 = R(T(BINSQ(I + N1)) / BINSQ(I + N1))
   ANS02 = R(T(BINSQ(I + N2)) / BINSQ(I + N2))
   ANS03 = R(T(BINSQ(I + N3)) / BINSQ(I + N3))
   ANS04 = R(T(BINSQ(I + N4)) / BINSQ(I + N4))
   WRITE (6, 1040) EM1, EM2, EM3, EM4
   WRITE (6, 1041) AN1, ANS01, AN2, ANS02, AN3, ANS03, AN4, ANS04
   IF (UH - EM4) 116, 116, 117
116 EN1 = EN1 + 1.0
   EN2 = EN2 + 1.0
   EN3 = EN3 + 1.0
   N1 = N1 + 1
   N2 = N2 + 1
   N3 = N3 + 1
   GO TO 115
117 IF (N4 - 100) 118, 116, 116
118 EN1 = EN1 + 1.0
EN2=EN2+1.0
EN3=EN3+1.0
EN4=EN4+1.0
N1=N1+1
N2=N2+1
N3=N3+1
N4=N4+1
GO TO 119
16 CALL GRAPH(I,E,U,O,BIN,HTL)
916 CONTINUE
78 IF(KS)80,80,77
77 DO 90 I=1,KS
   DO 94 J=1,20
      DO 95 K=1,20
95 CONTINUE
94 CONTINUE
      WRITE (6,1015) KS,CAT(I),WTUMS(I)
      WRITE (6,1006) (STL(I,JK),JK=1,20),ASUMS(I)
      WRITE (6,1017) NX(I),NY(I)
      WRITE (6,1018) XL(I),NX(I),XU(I)
      WRITE (6,1019) YU(I)
     DO 81 K=1,20
      L=21-K
      WRITE (6,1020) (SCAT(I,J,L),J=1,20)
81 CONTINUE
     WRITE (6,1019) YL(I)
90 CONTINUE
     WRITE (6,1019) YR(I)
80 WRITE (6,1044) N0M,WTUM,N01,N02,N03
    READ(5,1000) ECM,NP,KNORM,KHIST,KAO,KS,NEVT
    READ(5,1013) (EM(J),J=1,NP)
    ANORM=KNORM
    IF(ECM)99,99,74
870 FORMAT(1H,E15.7,6I10)
871 FORMAT(1H,E15.7/)
876 FORMAT(1H,20A4)
877 FORMAT(1H,E15.7,5X)
878 FORMAT(1H,9I10,4E15.7)
888 FORMAT(1H,9I10,3E15.8)
1000 FORMAT(E10.0,E10.0)
1001 FORMAT(3I10,9F10.0)
1002 FORMAT(2I10,E10.0,E10.0,E10.0,E10.0,E10.0)
1003 FORMAT(1H,7H PLACE)
1004 FORMAT(2H,2I4,8E14.4)
1005 FORMAT(1H,9H 'HISTOGRAM NUMBER','I2','79x','WTSUM ','E11.4')
1006 FORMAT(1H,2A4,19x,'EVSUM ',E11.4)
1007 FORMAT(1H,' PLACE ','I4',' BIN SIZE IS ',F11.5)
1009 FORMAT(31H THIS HISTOGRAM HAS NO EVENTS. )
1013 FORMAT(8E10.0)
1014 FORMAT(3I10,6E10.0)
1015 FORMAT(1H,21H 'SCATTERGRAM NUMBER ','I2','79x','8HTSUM ','E11.4')
1017 FORMAT(8H ' PLACE ','I5','10H VS PLACE ','I5')
1018 FORMAT(1H,'10x','E12.4','35x','5PLACE','I5','40x','E12.4')
1019 FORMAT(1H,' E12.4')
1020 FORMAT(18x,'20F5.0')
1030 FORMAT(30H 'VARIABLE FREQUENCY ERROR')
1031 FORMAT(1H,2F12.4)
1032 FORMAT(12X,E11.4,F7.4)
1033 FORMAT(60H 'VARIABLE FREQUENCY ERROR' VARIABLE FREQUENCY F ERROR)
1034 FORMAT(1H,'F12.4','17x','1HI','F12.4')
1035 FORMAT(12X,'E11.4','F7.4','1HI','11X','E11.4','F7.4')
1036 FORMAT(90H 'VARIABLE FREQUENCY ERROR' VARIABLE FREQUENCY F ERROR)
1037 FORMAT(1H,'F12.4','17x','1HI','F12.4','17x','1HI','F12.4')
1038 FORMAT(12X,'E11.4','F7.4','1HI','11X','E11.4','F7.4','1HI','11X','E11.4','F7.4')
1039 FORMAT(120H 'VARIABLE FREQUENCY ERROR' VARIABLE FREQUENCY ERROR)
1040 FORMAT(1H,'F12.4','17x','1HI','F12.4','17x','1HI','F12.4','17x','1HI','F12.4')
1041 FORMAT(12X,'E11.4','F7.4','1HI','11X','E11.4','F7.4','1HI','11X','E11.4','F7.4','1HI','11X','E11.4','F7.4')
1042 FORMAT(20A4)
1044 FORMAT(1H/2H *I10,19H EVENTS GENERATED. *5X,E11.4,18H WTSUM GE
1NERATED. *5X,5HNO1 =16,5X,S5HNO2 =16,5X,5HNO3 =16//)
1050 FORMAT(1H *3E15.8*I8)
99 CONTINUE
END

SUBROUTINE EVENT
GENERATES EVENTS AND TRANSMITS PX, PY, PZ, T, M, WEIGHT INTO PHASE
COMMON PLACE(200)
COMMON/PHASPC WIGHT, WG, WS, EM, P, T, TMAX, ECM, NP, NUM, NOTRY, PCM
DIMENSION EM(10), P(10,3), T(10), TMAX(10), COSTH(10), PHI(10), PCM(10)
DIMENSION SINH(10), SINPH(10), COSPH(10), WG(50), WS(10)
IF(NP-1)*2.3
1 COSS=2.0*(0.5-RAND(D))
NOTRY=NOTRY+1
IF (ABS(COSS).GE.0.96) GO TO 1
SINN= SQRT(1.0-COSS**2)
PHE=6.283185*RAND(D)
T(1)=TMAX(1)
T(2)=TMAX(2)
EA=T(1)+EM(1)
EB=T(2)+EM(2)
E=EA=EM(1)
EMB=EM(2)
PCCM(1)=SQRT(EA*2-EMA*2)
PCCM(2)=SQRT(EM*2-EMB*2)
P(1,1)=PCCM(1)*SINN*COS(PHE)
P(1,2)=PCCM(1)*SINN*SIN(PHE)
P(1,3)=PCCM(1)*COSS
P(2,1)=-P(1,1)
P(2,2)=-P(1,2)
P(2,3)=-P(1,3)
WGHT=1.0
GO TO 30
3 NPA=NP-1
4 NOTRY=NOTRY+1
DO 10 J=1,NPA
\[ T(J) = T_{\text{MAX}}(J) \times \text{RAND}(D) \]

10 CONTINUE
SUM = 0.0
DO 6 J = 1, NPA
SUM = SUM + T(J) * EM(J)
6 CONTINUE
T(NP) = ECM - SUM - EM(NP)
IF(T(NP)) 4, 11, 11

11 DO 12 J = 1, NPA
PCM(J) = SQRT(T(J)) * (T(J) + 2.0 * EM(J))
12 CONTINUE
NPC = NP - 3
DO 13 J = 1, NPC
COSTH(J) = 2.0 * (0.5 - RAND(D))
PHI(J) = 6.283185 * RAND(D)
SINTH(J) = SQRT(1.0 - COSTH(J)**2)
SINPH(J) = SIN(PHI(J))
COSPH(J) = COS(PHI(J))
P(J, 1) = PCM(J) * SINTH(J) * SINPH(J)
P(J, 2) = -PCM(J) * SINTH(J) * COSPH(J)
P(J, 3) = PCM(J) * COSTH(J)

13 CONTINUE
THET2 = 3.141593 * RAND(D)
F3X = 0.0
F3Y = 0.0
F3Z = 0.0
DO 21 J = 1, NPC
F3X = F3X + P(J, 1)
F3Y = F3Y + P(J, 2)
F3Z = F3Z + P(J, 3)
21 CONTINUE
F3 = SQRT(F3X**2 + F3Y**2 + F3Z**2)
NPB = NP - 2
COSTH(NPB) = COS(THET2)
\[
\sinh(NPB) = \sin(\theta_2)
\]
\[
F_2 = \sqrt{F_2^2 + F_3^2 + F_4^2}
\]
\[
\cos(\text{NH}) = \frac{F_1 - \sqrt{F_2^2 + F_3^2}}{F_1}
\]
\[
\sin(\text{NH}) = \frac{F_3 - \sqrt{F_2^2 + F_3^2}}{F_1}
\]
\[
23 \sin(\text{NH}) = \sqrt{1 - \cos(\text{NH})^2}
\]
\[
\Phi_1(NP_1) = 6 + 28.3178F_3 + 3.185F_1
\]
\[
\Phi_1(NP_1) = F_3 + 3.185F_1
\]
\[
\sin(\Phi_1) = \sin(\Phi_1(NP_1))
\]
\[
\cos(\Phi_1) = \cos(\Phi_1(NP_1))
\]
\[
P_{2X} = \sin(\Phi_1) \cdot x + \cos(\Phi_1) \cdot y
\]
\[
P_{2Y} = -\cos(\Phi_1) \cdot x + \sin(\Phi_1) \cdot y
\]
\[
P_{2Z} = \cos(\Phi_1) \cdot z + \sin(\Phi_1) \cdot z
\]
\[
\Phi_2(NP_2) = 6 + 28.3178F_3 + 3.185F_1
\]
\[
\Phi_2(NP_2) = F_3 + 3.185F_1
\]
\[
\sin(\Phi_2) = \sin(\Phi_2(NP_2))
\]
\[
\cos(\Phi_2) = \cos(\Phi_2(NP_2))
\]
\[
P_{3X} = \sin(\Phi_2) \cdot x + \cos(\Phi_2) \cdot y
\]
\[
P_{3Y} = -\cos(\Phi_2) \cdot x + \sin(\Phi_2) \cdot y
\]
\[
P_{3Z} = \cos(\Phi_2) \cdot z + \sin(\Phi_2) \cdot z
\]
\[
\Phi_3(NP_3) = 6 + 28.3178F_3 + 3.185F_1
\]
\[
\Phi_3(NP_3) = F_3 + 3.185F_1
\]
\[
\sin(\Phi_3) = \sin(\Phi_3(NP_3))
\]
\[
\cos(\Phi_3) = \cos(\Phi_3(NP_3))
\]
\[
P_{4X} = \sin(\Phi_3) \cdot x + \cos(\Phi_3) \cdot y
\]
\[
P_{4Y} = -\cos(\Phi_3) \cdot x + \sin(\Phi_3) \cdot y
\]
\[
P_{4Z} = \cos(\Phi_3) \cdot z + \sin(\Phi_3) \cdot z
\]
\[ P(NP, 2) = F2Y - P(NPA, 2) \]
\[ P(NP, 3) = F2Z - P(NPA, 3) \]
\[ PRO = 1.0 \]
\[ DO 25 J = 1, NP8 \]
\[ PROD = PROD * P\text{CM}_1 / 10000.0 \]
\[ 25 \text{ CONTINUE} \]
\[ \text{WGT} = (\text{PROD} * \text{SINTH}(NPB) / F2) * 10000.0 \]
\[ \text{GO TO} \ 30 \]
\[ 2 \text{ NOTRY} = \text{NOTRY} + 1 \]
\[ T(1) = TMAX(1) * \text{RAND}(0) \]
\[ T(2) = TMAX(2) * \text{RAND}(0) \]
\[ T(3) = E\text{CM} - T(1) - T(2) - EM(1) - EM(2) - EM(3) \]
\[ \text{IF}(T(3)) > 50,50 \]
\[ 50 \text{ THEI} = \text{THEI} + \text{THEI} + \text{THEI} \]
\[ \text{PCM}(1) = \text{SORT}(T(1) * (T(1) + 2.0 * \text{EM}(1))) \]
\[ \text{PCM}(2) = \text{SORT}(T(2) * (T(2) + 2.0 * \text{EM}(2))) \]
\[ \text{PCM}(3) = \text{SORT}(T(3) * (T(3) + 2.0 * \text{EM}(3))) \]
\[ \text{COSTH}(2) = (\text{PCM}(3) * 2 - \text{PCM}(2) * 2 - \text{PCM}(1) * 2) / (2.0 * \text{PCM}(1) * \text{PCM}(2)) \]
\[ \text{IF}((\text{COSTH}(2)) > 2, 2.0) \]
\[ 26 \text{ PHI} = 6.283185 * \text{RAND}(0) \]
\[ \text{SINTH}(1) = \text{SINTH}(\text{THEI}) \]
\[ \text{COSTH}(1) = \text{COSTH}(\text{THEI}) \]
\[ \text{SINTH}(2) = \text{SORT}(1.0 - \text{COSTH}(2) * 2) \]
\[ \text{SINPH}(1) = \text{SINPH}(\text{PHI}) \]
\[ \text{COSP}(1) = \text{COSP}(\text{PHI}) \]
\[ \text{SINPH}(2) = \text{SINPH}(\text{PHI}) \]
\[ \text{COSP}(2) = \text{COSP}(\text{PHI}) \]
\[ P(1, 1) = \text{PCM}(1) * \text{SINTH}(1) * \text{SINPH}(1) \]
\[ P(1, 2) = \text{PCM}(1) * \text{SINTH}(1) * \text{COSP}(1) \]
\[ P(1, 3) = \text{PCM}(1) * \text{COSTH}(1) \]
\[ P2X = \text{PCM}(2) * \text{SINTH}(2) * \text{SINPH}(2) \]
\[ P2Y = \text{PCM}(2) * \text{SINTH}(2) * \text{COSP}(2) \]
\[ P2Z = \text{PCM}(2) * \text{COSTH}(2) \]
\[ P(2, 1) = \text{COSP}(1) * P2X - \text{COSTH}(1) * \text{SINPH}(1) * P2Y * \text{SINTH}(1) * \text{SINPH}(1) * P2Z \]
\[ P(2, 2) = \text{SINPH}(1) * P2X * \text{COSTH}(1) * \text{COSP}(1) * P2Y - \text{SINTH}(1) * \text{COSP}(1) * P2Z \]
P2Z = \sin(\theta) \cdot \cos(\theta) \cdot P2Z

P(3,1) = -P(1,1) - P(2,1)
P(3,2) = -P(1,2) - P(2,2)
P(3,3) = -P(1,3) - P(2,3)

WGT = \sin(\theta)

30 IF(NUM-10)31,31,99
31 WRITE(6,1000)ECM,NP
WRITE(6,1001)
DO 32 I=1,NP
WRITE(6,1002)EM(I),TMAX(I),T(I),PCM(I),(P(I,J),J=1,3)
32 CONTINUE
WRITE(6,1003)NOTRY
WRITE(6,1004)WGT

1000 FORMAT(1H1/20H TOTAL CM ENERGY = F12.4/20H NO OF PARTICLES = 1*12/1)
1001 FORMAT(107H MASS T MAX E KIN 1 PCM PX PY PZ1)
1002 FORMAT(2H,7F15.4)
1003 FORMAT(1H/17H NO OF TRIALS = 1*I3)
1004 FORMAT(1H/11H WEIGHT = F12.4)
99 RETURN

SUBROUTINE SUB
COMMON A(200)
COMMON PHASPC,WGT,WS,EM,P,T,TMAX,ECM,NP,NOTRY,PCM
DIMENSION WG(50),EM(10),P(10,3),T(10),TMAX(10),PCM(10)
DIMENSION PIL(10),WS(10)
REAL M1,M2
M1=939.5
M2=0.511
A(1)=0.0
A(2)=0.0
A(3)=SQR((ECM/2.)*+2-EM(1)**2)
A(4)=ECM/2.
A(5)=0.0
A(I)=0.0
A(J)=A(I)
A(J+1)=P(I,1)
A(J+2)=P(I,2)
A(J+3)=E(I)+T(I)
J=J+4
DO 13 I=1,10
A(I+24)=A(I+4)-A(I+12)
13 CONTINUE
CALL UTE(A(5),A(1),A(101),6)
DO 60 I=1,10
60 PIL(I)=0.
M=109
DO 30 I=1,10
PILS=DT(M,M)
PIL(I)=SQRT(PILS)
A(I+60)=A(I+2)/PIL(I)
A(I+70)=P(I,3)/PCM(I)
A(I+80)=PIL(I)
A(I+94)=PCM(I)
IF(NUM-10)50,50,40
50 WRITE(6,2000) PIL(I),A(I+60),A(I+70)
40 M=M+4
30 CONTINUE
A(41)=2.*DOT(17,21)+M**2)/(10**6)
A(42)=2.*DOT(5,10)-M**2)/(10**6)
A(57)=DOT(117,121)/(PIL(3)*PIL(4))
A(58)=DOT(17,21)/(PCM(3)*PCM(4))
A(59)=DOT(109,113)/(PIL(1)*PIL(2))
A(60)=DOT(9,13)/(PCM(3)*PCM(2))
CALL MATRIX
2000 FORMAT(1H17H MOMENTUM IN LAB=F8.3,5X,8HCOS LAB=F7.4,E8.4,5X,1 9HCOS C.M.=F7.4)
RETURN
END
SUBROUTINE MATRIX
DOUBLE PRECISION DOT, TR, TR6, TR8
REAL M1, M2
COMMON A(200)
COMMON PHAS, PGHT, WG, WS, EM, P, T, TMAX, ECH, NP, NUM, NOTRY, PCM
DIMENSION WG(50), EM(10), P(10, 3), T(10), TMAX(10), PCM(10), WS(10)
M1 = 939.0
M2 = 0.511
PP = 2.0 * (DOT(17, 21) + M1 * M2)
XMP1 = 2.0 * (M1 * 2 - DOT(5, 13)) - 135.0 * 2
XMP2 = 2.0 * (M1 * 2 - DOT(5, 9)) - 139.6 * 2
XMP3 = 2.0 * (M1 * 2 - DOT(1, 13)) - 139.6 * 2
TR51 = 4.0 * (DOT(5, 13) - M1 * M2)
TR52 = 4.0 * (DOT(5, 9) - M1 * M2)
TR53 = 4.0 * (DOT(1, 13) - M1 * M2)
XL1 = 2.0 * (DOT(5, 1) - DOT(5, 13) - DOT(1, 13) + M1 * M2)
XL2 = 2.0 * (DOT(9, 13) - DOT(5, 13) - DOT(5, 9) + M1 * M2)
AA = (XL1 * XMP1) ** 2
AB = XL1 * XL2 * (XMP1 ** 2)
AC = XL1 * XL2 * (XMP2 ** 2)
AD = -(XL1 ** 2) * XMP1 * XMP3
BB = (XL2 * XMP1) ** 2
BC = -(XL2 ** 2) * XMP1 * XMP2
BD = XL1 * XL2 * XMP1 * XMP3
CC = (XL2 * XMP2) ** 2
CD = -(XL1 * XL2) * XMP2 * XMP3
DD = (XL1 ** 2) * (XMP3 ** 2)
TRB = TR6(17, 1, 25, 21, 25) + TR6(17, 1, 25, 1, 25) + TR6(17, 1, 25, 1, 25) + TR6(17, 1, 25, 1, 25) + TR6(17, 1, 25, 1, 25) + TR6(17, 1, 25, 1, 25) + TR6(17, 1, 25, 1, 25)
3 2) *(DOT(9,25)*TR(33,5,13,1)-2.*(M2**4)*DOT(25,33))
TRBD=TRBD1+TRB02
TRCC=8.*(DOT(21,1)*TR(17,29,13,29)+DOT(17,1)*TR(21,29,13,29))+8.*(M2**2)*TR(1,29,13,29)+32.*(M1**2)*DOT(29,29)*(DOT(17,21)+2.*(M2**2)
2 2)
TRCD=4.*(TR(5,9,17,33)+TR(1,21,29,13)+TR(5,9,21,33)+TR(1,17,29,13)
1 )-16.*(DOT(17,21)+DOT(5,9)*TR(1,33,29,13)-DOT(9,33)*TR(1,5,29,13
2 3)+DOT(5,33)*TR(1,9,29,13)-16.*(M1**2)*(DOT(17,29)+TR(5,33,21,9
3 1+DOT(21,29)*TR(5,33,17,9)+DOT(17,33)*TR(13,29,21,9)+DOT(21,33)*
4 TR(13,29,17,1)-DOT(17,21)*(TR(5,33,29,9)+TR(1,13,29,33,1)-TR(1,21,29,13
5 *TR(17,29,21,33)-16.*(M2**2)*(DOT(5,33)*TR(13,29,9,1)-DOT(9,33)
6 *TR(13,29,5,1)+DOT(5,9)*TR(1,13,29,33,1)+16.*(M1**2)**2)*TR(5,3
7 3,29,9)+TR(13,29,33,1)-4.*(M1**2)*DOT(29,33)
TRDD=8.*(DOT(17,9)*TR(21,33,5,33)+DOT(21,9)*TR(17,33,5,33))+8.*(M2
1 **2)*TR(9,33,5,33)+32.*(M1**2)*DOT(33,33)*(DOT(17,21)+2.*(M2**2)
1 )
TAC=8.*TR51*TRAA/AA+4.*TRAC/AC+4.*TR52*TRCC/CC
TAD=8.*TR51*TRAA/AA+4.*TRAD/AD+4.*TR53*TRDD/DD
TBC=8.*TR51*TRBB/BB+4.*TRBC/BC+4.*TR52*TRCC/CC
TBD=8.*TR51*TRBB/BB+4.*TRBD/BD+4.*TR53*TRDD/DD
TCD=4.*TR52*TRCC/CC+4.*TRCD/CD+4.*TR53*TRDD/DD
T1=8.*TR51*(TRAA/AA+TRAB/AB+TRBB/BB)
T2=4.*(TRAC/AC+TRAD/AD+TRBC/BC+TRBD/BD+TR52*TRCC/CC+TRCD/CD+
1 TR53*TRDD/DD)
T4=T1/(PP**2)
T5=(T1+T2)/(PP**2)
DO 12 K=1,2
WG(K+2)=WG(1)*T4
12 CONTINUE
IF(NUM=10) 60,60,70
60 WRITE(6,3000) T4,T5
WRITE(6,2000) TAC,TAD,TBC,TBD,TCD
WRITE(6,1000) WG(1),WG(3)
1000 FORMAT(1H,*7HWG(1)=E15.8,1X,8HWG(3)=E15.8)
2000 FORMAT(1H,5HTAC =E15.8,1X,5HTAD =E15.8,1X,5HTBC =E15.8,1X,5HTBD =E15.8,1X,5HTCD =E15.8)
3000 FORMAT(1H,4HT4 =E15.8,1X,4HT5 =E15.8)
70 RETURN
END
SUBROUTINE GRAPH(I*R*U*D*BIN*HTL)  
DIMENSION R(50), U(50), D(50), BLN(50, 100), COL(100), AX(5)  
DIMENSION HTL(50,10)  
DATA X*BLANK/5HXXXXXXXXX:SH /  
IBIN=(U(I)-D(I))/D(I)*O.1  
IF(IBIN<50)1,1+2  
1 IF(IBIN=20)3,3,4  
2 K=100  
   GO TO 5  
3 K=20  
   GO TO 5  
4 K=50  
5 WRITE(6,999)(HTL(I,J),J=1,20)  
   WRITE(6,1000)  
   TOP=0.0  
   DO 15 J=1,K  
      IF(TOP-BLN(I,J))16,15,15  
   16 TOP=BLN(I,J)  
   CONTINUE  
   DO 6 J=1,50  
      IN=51-J  
   6 CONTINUE  
   DO 7 JJ=1,K  
      AJ=J  
      IF(BLN(I, JJ)/ TOP-(1.0-AJ/50.0))8,8,9  
   8 COL(JJ)=BLANK  
      GO TO 7  
   9 COL(JJ)=X  
   7 CONTINUE  
   IF(IBIN<50)10,10,11  
   IF(IBIN=20)12,12,13  
   11 WRITE(6,1001) IN, (COL(JJ),JJ=1,100)  
      GO TO 6  
   12 WRITE(6,1002) IN, (COL(JJ),JJ=1,20)  
      GO TO 6  
   13 WRITE(6,1003) IN, (COL(JJ),JJ=1,50)  
   6 CONTINUE
SUBROUTINE UTE(C,P,Q,N)
UTE LORENTZ TRANSFORMS N 4-VECTORS P TO THE FRAME WHERE THE VECTOR C IS AT REST. THE RESULTS ARE STORED IN Q.*
C P AND Q MAY BE THE SAME VECTOR, BUT C MAY NOT BE IN Q.*
DIMENSION C(4), P(4,N), Q(4,N)
C FORM CM INVARIANTS.
C W IS MASS OF C.
C W = SQRT(C(4)**2 - C(1)**2 - C(2)**2 - C(3)**2)
C Z IS E+M IN LAB (FOR C)
C TRANSFORM VECTORS TO CM.*
C J COUNTS VECTORS.*
DO 20 J=1,N
C D IS SCALAR PRODUCT OF P AND C.*
C D = C(4)*P(4,J) - C(1)*P(1,J) - C(2)*P(2,J) - C(3)*P(3,J)
C E = D/W
C A = (P(4, J) + E) / 2 A IS CONSTANT FOR TRANSFORMING MOMENTA
C 0(4, J) = E STORE ENERGY.
C DO 10 I = 1, 3 NOW TRANSFORM MOMENTA.
10 0(I, J) = P(I, J) - A * C(I)
C 20 CONTINUE LOOP BACK FOR NEXT VECTOR.
C 20 CONTINUE
C RETURN
C END
C FUNCTION DOT(I, J)
C COMMON A(200)
C IF(I /= J) GO TO 3
C DT = A(I) ** 2 + A(I+1) ** 2 + A(I+2) ** 2
C RETURN
C 3 DT = A(I) * A(J) + A(I+1) * A(J+1) + A(I+2) * A(J+2)
C RETURN
C END FUNCTION DOT(I, J)
C COMMON A(200)
C IF(I /= J) GO TO 3
C 1 DOT = A(I+3) ** 2 - A(I) ** 2 - A(I+1) ** 2 - A(I+2) ** 2
C RETURN
C 3 DOT = A(I+3) * A(J+3) - A(I) * A(J) - A(I+1) * A(J+1) - A(I+2) * A(J+2)
C RETURN
C END FUNCTION TR(I, J, K, L)
C COMMON A(200)
C TR = 4. * (DOT(I, J) * DOT(K, L) - DOT(I, K) * DOT(J, L) + DOT(I, L) * DOT(J, K))
FUNCTION TR6(I1, I2, I3, I4, I5, I6)
COMMON A(200)
TR6=DOT(I1, I2)*TR(I3, I4, I5, I6)-DOT(I1, I3)*TR(I2, I4, I5, I6) + DOT(I1, I4)*TR(I2, I3, I5, I6)- DOT(I1, I5)*TR(I2, I3, I4, I6) + DOT(I1, I6)*TR(I2, I3, I4, I5)
RETURN
END

FUNCTION TR8(I1, I2, I3, I4, I5, I6, I7, I8)
COMMON A(200)
RETURN
END

FUNCTION RAND(0)
DATA R/782451/
RAND=RGEN(R)
RETURN
END

FUNCTION PGEN(IARG)
NUMBER GENERATOR PROGRAM OR FUNCTION
DATA IX/0/
IF(IARG.EQ. IX) GO TO 3
IX=IARG
IY=IX
3 IY=IY*262147
IF(IY.LT.0) IY=IY+34359738367+1
RGEN=IY
RGEN=RGEN+.291383E-10
RETURN
END
VITA
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Dr. of Philosophy

Dissertation: A Relativistic One Pion Exchange Model of Proton-Neutron Electron-Positron Pair Production

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