Design and Evaluation of Stepped Spillways for High Dams

Jeffrey Scott Rau

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DESIGN AND EVALUATION OF STEPPED SPILLWAYS FOR HIGH DAMS

by

Jeffrey Scott Rau

A thesis submitted in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE
in
Civil and Environmental Engineering

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1994
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Many people have given their time and support to make this project a success. I wish to thank my major professor, Dr. J. Paul Tullis, for his advice, insight, and guidance in helping me to complete this research. His financial support for my project was greatly appreciated. I also would like to thank the members of my committee for taking the time to read and comment on my research.

This thesis is dedicated to my wife, Jill, for without her love and support, none of this would have been possible. She has been a constant source of love and support throughout.

Lastly and most importantly, may all glory in this endeavor go to the most important person and guiding hand in my life, Jesus Christ.

Jeffrey Scott Rau
CONTENTS

ACKNOWLEDGMENTS .................................................................................. ii

LIST OF TABLES ....................................................................................... v

LIST OF FIGURES ..................................................................................... vi

ABSTRACT .................................................................................................. viii

INTRODUCTION ......................................................................................... 1

  Statement of Problem ........................................................................ 1
  Purpose of Study .............................................................................. 2
  Method Used ................................................................................... 2
  Difficulties Encountered .............................................................. 5

LITERATURE REVIEW ............................................................................. 8

  Pre-History ....................................................................................... 8

    New Croton Dam ........................................................................... 8
    Essery and Horner ....................................................................... 8

  Model Studies ............................................................................... 9

    Upper Stillwater Dam ............................................................... 9
    Monksville Dam ....................................................................... 13
    De Mist Kraal ............................................................................ 18

  Related Articles ............................................................................ 20

    Rajaratnam ................................................................................. 24
    D. Stephenson .......................................................................... 26
    Diez-Cascon et al. ....................................................................... 27
    Frizell .......................................................................................... 31
    Christodoulou ............................................................................ 31

HYDRAULIC PRINCIPLES ..................................................................... 35

  Energy Equation ............................................................................. 35
  Specific Energy ............................................................................. 37
  Critical Flow .................................................................................. 39
  Energy Dissipators ....................................................................... 40

    Concrete Apron Dissipator ....................................................... 41
    Stilling Basins ............................................................................ 41
    Flip Bucket Dissipators ........................................................... 41
    Roller Bucket Dissipators ......................................................... 41
Spillway Crest Shape .............................................................. 42
Stepped Spillway Crest Shapes ............................................... 46
The Momentum Principle in Open Channel Flow ....................... 46

TESTING PROCEDURES ......................................................... 50
Model Specifics ........................................................................ 50
Froude similitude ................................................................. 50
Model scale ratio ................................................................. 51
Crest design ......................................................................... 51
Step design ........................................................................... 52

Testing Apparatus .................................................................. 53
Test flume .............................................................................. 53
Depth measurements ............................................................ 53
Velocity measurements with pitot tube .................................. 55
Flow measurement with weigh tanks ...................................... 56

USBR Method for Designing Type II Stilling Basins ................... 56

RESULTS AND DISCUSSION .................................................. 62
Spillway Models and Test Results ............................................. 62
Model A ................................................................................ 62
Model B ................................................................................ 63
Model C ................................................................................ 65
Model D ................................................................................ 67

Analysis of Model Data .......................................................... 67
Design Procedure for Stepped Spillways ................................. 72
Design Procedure for Stepped vs. Smooth Spillway Basins ......... 76
Data Comparison to Other Researcher Methods ....................... 85
Rajaratnam ............................................................................. 85
Stephenson .......................................................................... 89
Diez-Cascon et al. ............................................................... 89
Christodoulou ................................................................. 93

CONCLUSIONS AND RECOMMENDATIONS ............................. 96

REFERENCES ........................................................................ 100

APPENDIX ............................................................................. 102
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Model configurations</td>
<td>53</td>
</tr>
<tr>
<td>2</td>
<td>Test data for Model A</td>
<td>64</td>
</tr>
<tr>
<td>3</td>
<td>Test data for Model B</td>
<td>66</td>
</tr>
<tr>
<td>4</td>
<td>Test data for Model C</td>
<td>68</td>
</tr>
<tr>
<td>5</td>
<td>Test data for Model D</td>
<td>69</td>
</tr>
<tr>
<td>6</td>
<td>Spreadsheet for comparing stilling basins for smooth to stepped spillways</td>
<td>78</td>
</tr>
<tr>
<td>7</td>
<td>Comparison table for 0.7H:1.0V slope</td>
<td>81</td>
</tr>
<tr>
<td>8</td>
<td>Limits of model testing on 0.7:H:1.0V slope</td>
<td>81</td>
</tr>
<tr>
<td>9</td>
<td>Spreadsheet for comparing stilling basins for smooth to stepped spillways</td>
<td>82</td>
</tr>
<tr>
<td>10</td>
<td>Comparison table for 0.7H:1.0V slope</td>
<td>83</td>
</tr>
<tr>
<td>11</td>
<td>Limits of model testing on 0.5H:1.0V slope</td>
<td>83</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Model A</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Model B</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Model C</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Model D</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>Model crest design</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>Cross section of Upper Stillwater Dam</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>Upper 20 feet of Upper Stillwater Dam</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>Design for entrance corner on Upper Stillwater Dam</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>Cross section of Monksville Spillway Crest</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>Modified Monksville Crest - final design</td>
<td>16</td>
</tr>
<tr>
<td>11</td>
<td>Results from Sorensen's study on Monksville Dam</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>De Mist Kraal Crest profile</td>
<td>19</td>
</tr>
<tr>
<td>13</td>
<td>Model results from De Mist Kraal study</td>
<td>21</td>
</tr>
<tr>
<td>14</td>
<td>Depth profile for De Mist Kraal study - low flow</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>Depth profile for De Mist Kraal study - high flow</td>
<td>23</td>
</tr>
<tr>
<td>16</td>
<td>Skimming flow on stepped spillways</td>
<td>25</td>
</tr>
<tr>
<td>17</td>
<td>Stephenson's energy loss graph</td>
<td>28</td>
</tr>
<tr>
<td>18</td>
<td>Diez-Cascon test model</td>
<td>29</td>
</tr>
<tr>
<td>19</td>
<td>Results from Diez-Cascon study</td>
<td>29</td>
</tr>
<tr>
<td>20</td>
<td>Christodoulou model - lengths in cm</td>
<td>33</td>
</tr>
<tr>
<td>21</td>
<td>Christodoulou's first energy loss graph</td>
<td>34</td>
</tr>
<tr>
<td>22</td>
<td>Christodoulou's second energy loss graph</td>
<td>34</td>
</tr>
<tr>
<td>23</td>
<td>Energy loss in an open channel</td>
<td>36</td>
</tr>
</tbody>
</table>
24  Specific energy diagram .......................................................... 38
25  USBR - elements of nappe-shaped profiles ................................. 43
26  USBR - nappe-shaped profile design with compound curves .......... 44
27  USBR graphs for determining crest coefficient ......................... 45
28  Momentum across a hydraulic jump ........................................... 47
29  Test flume used in study ......................................................... 54
30  USBR graph #15 for estimating toe velocities ............................ 58
31  USBR graph #12 for estimating stilling basin length ................. 59
32  USBR figure #14 - design of chute blocks and end sill diagram ... 60
33  Test data - energy loss on model ............................................ 70
34  Test data - toe velocities on model ......................................... 73
35  Test data - depths downstream of hydraulic jump ..................... 74
36  Test data - toe velocity vs. flowrate for 0.7H:1.0V slope ............. 80
37  Test data - toe velocity vs. flowrate for 0.5H:1.0V slope ............. 84
38  Unit discharge vs. computed toe velocity for spreadsheet design ... 86
39  Unit discharge vs. computed stilling basin length for spreadsheet design ... 87
40  Unit discharge vs. computed stilling basin volume for spreadsheet design ... 88
41  Test data according to Rajaratnam theory ................................ 90
42  Test data - energy loss according to stephenson theory ............... 91
43  Test data - depth at toe according to Diez-Cascon theory ............ 92
44  Test data - data according to Christodoulou theory ................... 94
45  Test data - data according to Christodoulou theory ................... 95
ABSTRACT

Design and Evaluation of Stepped Spillways for High Dams

by

Jeffrey Scott Rau, Master of Science

Utah State University, 1994

Major Professor: Dr. J. Paul Tullis
Department: Civil and Environmental Engineering

The purpose of this research was to investigate the hydraulic performance of stepped spillways. A thorough investigation was made of all printed material on stepped spillways, and a summary of this material is presented in the text. Data from experiments in the laboratory were used to develop a design procedure for stepped spillways and hydraulic jump stilling basins. The experimental study was conducted at Utah State University - Utah Water Research Laboratory in Logan, Utah. Four models were built and tested in the laboratory under various flowrates.

The crest of the model spillway was constructed in the shape of a standard USBR nappe-shaped crest. Small steps were fitted to the crest so that the envelope of their tips just intersected the crest profile. These small steps allowed a smooth transition of the flow from the nappe-shaped crest to the constant slope region. Two slopes were tested in the research: 0.7H:1.0V and 0.5H:1.0V. At each of these slopes two sizes of steps were tested. Steps did not vary in size down the face of the model, although step sizes varied on the different models. The model height and steps tested were for a dam with a prototype height of over 67 feet and steps in the 1-2 foot range. Diagrams and pictures of the four models tested are included in the text and appendix.

Findings from the research showed that given a ratio of step height over critical depth (0.120 < hs/yc < 1.897), the different slopes and step sizes performed very similarly. Energy loss between the models varied only slightly at the same flowrates. Step size and slope were not a critical factor in
energy loss on the models over the limits of the testing. Energy loss on all four models ranged from 95% at low flows to 65% at the higher flowrates tested. For a value of \( \frac{h_s}{yc} < 0.120 \), the energy dissipation will markedly decrease and eventually match the energy dissipation characteristics of a smooth spillway. For a value of \( \frac{h_s}{yc} > 1.897 \), the energy dissipation will remain in the 90% region, although there might be problems with the flow leaping away from the structure. Data on the models were slightly scattered, but all of the data from the four models agree to one energy loss graph. Support for these data showed a nearly perfect correlation in the downstream conjugate depths on all four models at similar flowrates.

By using data obtained from the models and literature, a design process with guidelines for designing a stepped spillway is presented. This process includes crest design and step placement in the transition region, approximate step size, and approximate slope necessary for adequate operation of the stepped spillway.

By taking data collected from the models, and data from USBR design manuals for smooth spillways, a spreadsheet design process was created that compared the size of stilling basins required using either a smooth spillway or a stepped one. Results showed that given a unit flowrate range of 15 cfs/ft to 140 cfs/ft and spillway height of approximately 100 ft, the stilling basin volume was reduced by 62% to 43%, respectively. This size reduction can translate to a considerable cost savings in prototype construction.

This study, along with data from other researchers, has proven that a stepped spillway can greatly increase the amount of energy dissipation over that achieved on a standard smooth face spillway. The stepped concept can be used as an excellent energy dissipator and in some cases can totally remove the need for any type of dissipator at the toe of the spillway.
INTRODUCTION

Statement of Problem

A common problem encountered in the design of a spillway is the dissipation of the energy contained in the flow. This energy is a result of the highly accelerated water that rushes down the face of the structure, threatening to erode and undermine the structure. Classical design theory calls for a standard hydraulic jump stilling basin or other dissipator at the base of the structure. This basin can be very large and expensive to construct, especially if much excavation is needed. In addition to damage at the base of the structure, the high velocity flow can begin to cavitate and erode the spillway face.

To reduce these problems, energy must be removed from the water as it flows down the face of the structure. If the flow energy can be reduced, the size of the stilling basin necessary to contain the hydraulic jump can be reduced at a considerable cost savings.

One such method in reducing the flow energy on the face of a spillway has been achieved by placing steps in the spillway which act as roughening agents to decrease the velocity of the flow. This type of spillway is known as a stepped spillway. At very low flows, the water plunges from step to step, dissipating much of its available energy. At higher flows, the water skims the steps, creating turbulent eddies in the pockets of the steps. This reduces the velocity of the flow and therefore reduces the size of the stilling basin necessary to contain the hydraulic jump. In addition, the turbulent eddies in the pockets of the steps tend to trap large amounts of air that are entrained in the flow. This causes the flow to become highly saturated with air and a bulking of the depth is achieved. This also helps reduce the average velocity of the flow, which helps in reducing the size of the stilling basin.

Step spillways have been proven from laboratory studies throughout the world to increase the amount of energy dissipation achieved over the face of the structure. Much of the data has been site specific and little information exists to aid in the design of a stepped spillway. Up to this point, many
designs have been sent to hydraulic research centers for model study and evaluation. This can be costly and time consuming to the designer. There is a need for general design criteria for stepped spillways that would enable designers to estimate parameters like step height, energy dissipation, toe velocities, and size of stilling basin required.

Purpose of Study

This study investigated the hydraulic performance of a stepped spillway over a varying amount of flow. By varying the slope, step height, and flowrate, we have gained a better understanding of the hydraulic performance and the benefits acquired in using the stepped concept. By using the information gathered in the literature review along with data gathered in the models, a design process for designing stepped spillways and stilling basins was created. The design process will allow a design engineer to estimate parameters like step height, crest shape, and stilling basin size necessary to contain the hydraulic jump.

Method Used

An in-depth search of all material concerning stepped spillways was completed. All important information was then gleaned from this material and brought together as a summary of existing information. Since the existing information did not contain many design criteria, model studies were completed in a laboratory flume at the Utah Water Research Laboratory.

Four models were built and tested under varying conditions.

Model A: 
Slope = 0.7H:1.0V Steps .75"H x 1.07"V 
Figure 1

Model B: 
Slope = 0.7H:1.0V Steps .375"H x .536"V 
Figure 2

Model C: 
Slope = 0.5H:1.0V Steps .375"H x .75"V 
Figure 3

Model D: 
Slope = 0.5H:1.0V Steps .188"H x .375"V 
Figure 4
Figure 1. Model A.

Figure 2. Model B.
Figure 3. Model C.

Figure 4. Model D.
All four models used the same crest with only minor modifications in the transition region. The crest was fitted with smaller steps near the crest to prevent the flow from springing away from the structure. The crest was not studied to determine the most efficient step configuration since little dissipation occurs at this point. The crest profile used in the study can be found in Figure 5. The small steps near the crest were adequate to keep the flow attached to the face of the structure.

 Preliminary design considerations included:

1. Flow velocities at the toe of the structure over a varying unit flowrate.
2. Energy dissipation at the toe of the structure and residual energy.
3. Geometry considerations:
   a. step height
   b. step thickness
   c. number of steps.

**Difficulties Encountered**

In order to determine the energy in the flow, velocities had to be calculated. The flow was supercritical and aerated, which made accurate measurements difficult. Velocities were originally measured using a standard pitot tube. This method was inaccurate and generally underestimated the theoretical velocities calculated using USBR Monograph #25 (Peterka, 1964). In addition, the highly aerated flow on the stepped face created air locks in the pitot tube, rendering it inaccurate. Depth measurements used with flowrate measurements from the weigh tanks were used to calculate velocities in the models.

An additional difficulty encountered was the ability to get accurate depth measurements, especially near the toe of the model due to the high surface turbulence caused by the aeration and supercritical flow. To reduce measurement error, depth measurements were taken at 1/4 points across
Figure 5. Model crest design.
the flow, resulting in three measurements at each step measured. This depth was averaged and used for the calculations.
LITERATURE REVIEW

An extensive literature review was completed which resulted in the acquisition of over 20 articles and design reports. In addition to published materials found, personal communication was made with two of the leading researchers in stepped spillways. The following is a summary of what can be found in the literature on stepped spillways.

Pre-History

New Croton Dam

Very little material can be found dated prior to 1978. In 1907, a report was filed by R. Wegman to the ASCE, describing the design of the New Croton Dam in New York. The New Croton Dam contained a 1000-foot long waste weir along its north face. The weir face was formed with steps from the crest down to the base. The report contains much about the design of the cross section of the dam but only mentions the spillway and its stepped face without going into any detail of its design or performance. Since it was probably known that the step concept worked, it was simply built without any type of study or calculations.

Essery and Horner

In 1978, a report out of London, England titled "The Hydraulic Design of Stepped Spillways" was published by two researchers, I. T. S. Essery and M. W. Horner. The research entailed hundreds of tests on many different stepped spillway faces and step configurations, including sloped steps and steps with end sills to create small plunge pools. Slopes studied were generally flat with the largest slope tested at 1.0H:1.0V (Essery and Horner, 1978). Since these slopes are mostly found on low embankment earthen dams and are unfeasible for high dams, the research data could not be used for our study. The report is one of the first resources that categorizes the flow pattern down the face of the structure into nappe and skimming flow regimes.
Model Studies

After the 1978 report by Essery and Horner little research was done on designing stepped spillways. Without adequate design criteria, designs were sent to large hydraulic laboratories for testing and modification. The following are some examples of some of the major models tested during this period.

**Upper Stillwater Dam**

One of the first stepped spillways to be constructed in the United States was the spillway on the Upper Stillwater Dam. The Upper Stillwater Dam is approximately 80 miles east of Salt Lake City, Utah and was completed in 1986. The United States Bureau of Reclamation (USBR) in Denver was commissioned to investigate the performance of the stepped design for the uncontrolled overflow spillway.

The Upper Stillwater Dam spillway is 600 feet long with a drop of about 200 feet. The downstream slope is 0.32H:1.0V for the upper 72 feet. The remaining 130 feet are sloped at 0.6H:1.0V. Except for the smaller steps near the crest, all of the steps are 2 feet high. A cross section of the Upper Stillwater Dam is found in Figure 6. The design unit discharge for the dam is 123 cfs/ft (Houston, 1987a).

The original design called for a constant slope of 0.6H:1.0V. This was later changed to allow for a top width of 30 feet instead of 15 feet. Keeping a top width of 30 feet and still having a downstream slope of 0.6H:1.0V would greatly increase the amount of concrete needed therefore the slope was changed for the upper 1/3 of the spillway.

As with most stepped spillway designs, the crest tends to be the most critical aspect of design. A poorly designed crest will cause the water to hit the first step in the transition and spring away from the structure, landing several steps down the face (Houston, 1987a). A common solution has been to design
Figure 6. Cross section of Upper Stillwater Dam (Houston, 1987a).
the crest in a nappe-shaped pattern and fit smaller steps to that pattern. The USBR used the following equation found in *Design of Small Dams* (U.S. Bureau of Reclamation, 1977):

\[
\frac{y}{H_o} = -K \left( \frac{x}{H_o} \right)^n
\]

where \(y\) and \(x\) designate points on the curve from the crest down, \(H_o\) = design head, \(K\) and \(n\) = inch-pound unit constants found on design curves which are based on approach velocity and design head (Young, 1982). In the Upper Stillwater case, the design head used in the equation was purposefully underestimated to allow for better performance at the lower flows, since this is where the spillway would operate more frequently. Because the top of the dam is nearly 30 feet wide, it acted much like a broad-crested weir, accelerating the flow through critical depth long before coming in contact with the ogee crest. This caused the flow to leap away from the structure, and getting it to cling to the steep 0.32H:1.0V face was nearly impossible. The approach velocity was greatly reduced by lowering the floor of the approach to the crest by almost 6 feet to elevation 8166 (Figure 7). The figure also shows how the steps were fitted to the nappe-shaped crest beginning just below the crest down to the intersection of the constant slope (Houston, 1987a).

A 1:15 scale model of the crest and spillway was constructed and installed in the 4-foot wide flume at the USBR testing facility. Tests on the model study included: (1) water surface profiles, (2) pressure measurements on areas of expected low and high pressures, (3) energy dissipation on the spillway face, and (4) size and adequacy of stilling basin (Houston, 1987a).

Training walls had to be used on the stepped face to prevent the flow from spreading. Without training walls, the flow spreads laterally approximately 50 feet in the prototype, greatly increasing the width of the stilling basin. The largest pressures recorded in the model were at the change of slope 72 feet down the face of the structure. The pressure, however, was not great enough to
Figure 7. Upper 20 feet of Upper Stillwater Dam (Houston, 1987b).
damage the concrete and was ignored. Small negative pressures developed on the downstream corners of the steps, but the negative pressures were not great enough to cause cavitation (Houston, 1987a).

During high flows in the model, large surface waves were seen traveling across the face of the spillway. It was thought that the side entrances to the spillways were affecting the flow pattern and generating this wave pattern. In order to correct this, the entrance corners to the spillway were modified from the original 1.5 foot radius, to a 1/4 ellipse with a short radius of 6 feet and a long radius of 12 feet. This helped the flow transition into the spillway and decreased the number of surface waves at high flows. The entrance corner design can be found in Figure 8.

Velocities were measured using high speed video and a pitot tube. The video was indexed every 1/20 second and paper squares were entered into the flow (Houston, 1987a). Using a time-distance relationship, velocities were computed. Average spillway velocities across the face were approximately 35 ft/sec prototype. The estimated energy reduction at moderate flows based on velocities of a conventional smooth spillway was approximately 75%. Originally, a 50 foot stilling basin was used. From observations in the model study, it was observed that the roller in the hydraulic jump remained close to the face of the dam. Because of this, the stilling basin was shortened to 25 feet which was a 50% reduction in size and an enormous cost savings. The final stilling basin was 30 feet long with a 7-foot high end sill (Houston, 1987a).

**Monksville Dam**

A model study of the Monksville Dam near Wanaque, New Jersey was tested in 1983 by Robert M. Sorensen of Lehigh University for O'Brien and Gere Engineers Inc. The model study was conducted to evaluate the performance of the stepped spillway. The stepped spillway for Monksville Dam is 200 feet long and has a maximum drop of approximately 120 feet. The downstream slope of the structure is 0.78H:1.0V. Although the probable maximum discharge for the dam is 100 cfs/ft, the spillway was designed for 65 cfs/ft. Small 1-foot steps help make up the ogee crest profile, with the rest of the steps being 2 feet high (Sorensen, 1983).
A. Original Design for Entrance Corner

B. Recommended Design for Entrance Corner

Figure 8. Design for Entrance Corner on Upper Stillwater Dam (Houston, 1987a).
Two models were constructed and tested. A 1:10 scale model was built of the upper section of the spillway to study the transition of the water profile from the smooth ogee crest to the stepped face. A 1:25 scale model was constructed of the complete spillway profile to study the energy dissipation characteristics of the design and estimate required training wall heights.

Model A consisted of a 1:10 scale model of the upper 22.75 feet of the spillway. The spillway crest shown in Figure 9 followed a standard WES spillway profile using the equation:

$$\frac{y}{12.46} = x^{1.85}$$

The first model was fit with two 1.5-foot steps and three 1-foot steps in the transition area. This configuration worked well for all flowrates except for the low flowrate of 1.8 cfs/ft prototype. At this flow, the thin film of water would hit the first step and leap away from the next few steps. Since this is an average summer flow over the spillway, additional steps were added near the crest to alleviate this problem. Four additional steps in decreasing height were added further up the face as shown in Figure 10. The new profile operated well at all flowrates (Sorensen, 1983).

Two types of models were built of the complete spillway profile. Built first was a traditional smooth spillway. Measurements taken from the flow conditions on it acted as a baseline against tests done on the stepped model. Sorensen used scales and point gauges to measure depths of the flow along with volumetric tanks to measure the different flowrates on all the models. Having a depth and accurate flowrate, he was able to calculate velocities using continuity. As commented in the article, the depths are extremely difficult to obtain due to the highly supercritical flow and bulking due to aeration. When checked against stagnation tube measurements, the values calculated yielded results within 10 to 20% of those calculated with continuity (Sorensen, 1983). This error was contributed partly due to the inability to get extremely accurate depth or velocity measurements.

Figure 11 shows the results from the stepped and unstepped models in prototype velocities. It can be easily seen that the velocities on the unstepped profile are much higher than those recorded on the stepped model. A third curve by Bradley and Peterka was added that used velocities obtained from
Figure 9. Cross section of Monksville Spillway Crest (Sorensen, 1983).

Figure 10. Modified Monksville Crest - final design (Sorensen, 1983).
Figure 11. Results from Sorensen's study on Monksville Dam (Sorensen, 1983).
prototype velocity values from Shasta and Grand Coulee Dams. The prototype measurements are consistently lower than the velocities found on the unstepped model in Sorensen's study. It was stated that a model is unable to replicate the air entrainment on a smooth spillway. This introduces a scaling effect that is likely to be the primary cause for the velocity differences (Sorensen, 1983).

The Monksville Dam study estimated that the toe velocities on the prototype would be around 30 fps at the probable maximum discharge, versus the 75 fps expected on a smooth profile. According to Sorensen, this represented a kinetic energy dissipation of 84% (Sorensen, 1983). Due to the dissipative qualities of the stepped spillway and the excellent material at the toe of the dam, no stilling basin was constructed.

De Mist Kraal

In May 1986 a report was published out of South Africa by J. M. Jordaan, Chief Engineer. The De Mist Kraal weir has a crest length of 689 feet and a hydraulic height of 71 feet. The downstream slope beyond the crest transition is 0.6H:1.0V. The steps on the face of the dam are 3 feet high with decreasing height for the seven steps near the crest. Unlike most designs in which the steps decrease to around 1 foot, the smallest step on the De Mist Kraal is 1.83 feet (Jordaan, 1986). The De Mist Kraal crest profile can be found in Figure 12.

Flows tested in their model study ranged from 40.5 cfs/ft to over 105 cfs/ft prototype. Above 105 cfs/ft, severe oscillations of the water surface were experienced, caused by too narrow of a model and inadequate stilling basin in the approach flume (Jordaan, 1986). The design flood for the weir is 103 cfs/ft while the probable maximum flood is over 350 cfs/ft (Jordaan, 1986).

A 1:20 scale model was constructed and tested in their flume facility. Velocity measurements were attempted with depth and flow measurements, propeller meters, and pitot tubes. The propeller meter needs water of unit relative density and therefore could not be used on the highly aerated flow encountered in Jordaan's study. The aerated flow also prevented the use of the pitot tube due to air locks in the tubing. Both the propeller meter and the pitot tube could, however, be used on the smooth
Figure 12. De Mist Kraal Crest profile (Jordaan, 1986).
profile and both closely agreed with the theoretical curve given from Chow (Chow, 1959). The depth measurements, however, greatly overpredicted velocities on the smooth profile but sufficiently predicted velocities on the stepped profile. This discrepancy was attributed to the greater depths measured on the stepped profile and the more-room-for-measurement error (Jordaan, 1986).

Shown in Figure 13 are the results from the model study. Results give prototype velocities in m/s with varying discharges. In addition to actual measured data, a theoretical smooth curve from Chow has been plotted. Velocities at the base of the stepped spillway (RL 533) ranged from 35 ft/s to 27 ft/s prototype. The theoretical smooth spillway ranged from 60 ft/s to 52 ft/s (prototype) at the same flowrates. Jordaan quoted reductions of kinetic energy in the range of 64% to 78% (Jordaan, 1986). No attempt was given to extrapolate the data to the apron level, but it was expected that energy losses could be greater than 80% (Jordaan, 1986). It was noted that as the flowrate increases, there is a decrease in the kinetic energy reduction. At the Probable Maximum Flood (PMF) it was estimated that the kinetic energy dissipation would drop to less than 15% (Jordaan, 1986).

Studying the velocity profiles given in the appendix of Jordaan's report shows some interesting characteristics. At lower flows, the velocity increases near the top as the flow passes through critical depth, decreases slightly near the midpoint, and then increases again near the toe of the structure. As the unit discharge increases, the S-shaped velocity profile flattens and begins to match more of the smooth spillway velocity profile. A low flow profile and a high flow profile from the De Mist Kraal study are given in Figures 14 and 15 (Jordaan, 1986).

Related Articles

Due to the expense of model studies and the lack of general design criteria, there has been much more research done on stepped spillways in the past 5 years. Following is a summary of the main body of research found.
TOE VELOCITY VARIATION WITH DISCHARGE

Figure 13. Model results from De Mist Kraal study (Jordaan, 1986).
Figure 14. Depth profile for De Mist Kraal study - low flow (Jordaan, 1986).
Figure 15. Depth profile for De Mist Kraal study - high flow (Jordaan, 1986).
Rajaratnam

In 1988 N. Rajaratnam published an article titled "Skimming Flow in Stepped Spillways" in which he attempted to predict the shear stress between the flow and steps. Estimating the shear stress could in turn estimate the energy loss in the skimming flow (Rajaratnam, 1988). Figure 16 illustrates the skimming flow and also the shear stress occurring along the boundary between the skimming flow and the steps. Rajaratnam used a coefficient $c_f$ to represent the coefficient of fluid friction, equal to $f/4$, where $f$ = the Darcy-Weisbach friction factor. From his tests he devised the following equation:

$$c_f = \frac{2y_0^3g\sin \alpha}{q^2}$$

(3)

where:

- $y_0$ = the depth of flow normal to the stepped face
- $g$ = acceleration due to gravity
- $\alpha$ = slope of stepped face in degrees
- $q$ = discharge per unit width of the spillway (Rajaratnam, 1988)

This equation has been used in developing equations for fishways. Average value computed for pool and weir fishways was 0.09. Using data from Sorensen’s tests on the Monksville Dam Spillway, Rajaratnam calculated values for $c_f$ from 0.11 to 0.2 with an average value of 0.18 (Rajaratnam, 1988). At very small flows, $c_f$ tends to become larger due to the flow approaching nappe flow conditions.

Plugging Equation 3 into the energy equation and rearranging gave the following equation:

$$E = \left( \frac{c_f q^2}{2g \sin \alpha} \right)^{1/3} + \left( \frac{q \sin \alpha}{c_f \sqrt{2g}} \right)^{2/3}$$

(4)

This can be used to predict energy on both the smooth and the stepped faces (Rajaratnam, 1988). Combining both the smooth and stepped energy equations and creating an energy loss ratio resulted in the following equation to predict total energy loss in skimming flow on stepped spillways.
Figure 16. Skimming flow on stepped spillways (Rajaratnam, 1988).
\[ \frac{\Delta E}{E'} = \frac{(1-A) + \frac{F_0^2}{2} \left( A^2 - 1 \right)}{1 + \frac{F_0^2}{2}} \]  

(5)

where:

\[ A = \left( \frac{c_f}{c_f'} \right)^{\frac{1}{3}} \]

\[ F_0' = \text{the Froude number at the toe of the smooth spillway} \]

\[ c_f' = \text{coefficient of skin friction for the smooth spillway} \]

By using Sorensen's data for a stepped spillway: \( c_f = 0.0065 \), \( c_f' = 0.018 \), \( A = 3 \), and a large value of \( F_0' \), the previous equation simplifies to \( (A^2 - 1)/A^2 \). This reduces to \( 8/9 \), which is very close to Sorensen's findings (Rajaratnam, 1988).

D. Stephenson

In 1991, D. Stephenson published an article in Water Power & Dam Construction titled "Energy Dissipation Down Stepped Spillways." In this article, Stephenson commented on the increased use of stepped spillways and provided several equations to predict the energy loss. Stephenson believes that model tests overpredict the energy loss due to lower Reynolds number and larger Weber number effects. In addition, since model studies cannot model air entrainment with accuracy, energy loss cannot be deduced with any degree of accuracy (Stephenson, 1991).

Stephenson based his energy loss equations on assuming that the flow reaches uniform depth. Using the Darcy Equation and the general Energy Equation, he formulated the following Energy Loss Equation (Stephenson, 1991):

\[ \frac{\Delta E}{H} = 1 - \left( \frac{4S}{\lambda} + 1 \right) \left( \frac{\lambda}{8S} \right)^{\frac{1}{2}} \left( \frac{y_e}{H} \right) \]  

(6)

where:

\[ S = \text{energy slope } q^2/8gy \]

\[ q = \text{discharge per unit width} \]
The Darcy Coefficient $\lambda$ can be found by solving the turbulent rough boundary layer equation below:

$$\lambda = \frac{1}{\left[1.14 + 2\log\left(\frac{y_c}{k}\right)\left(\frac{8\lambda}{S}\right)^{1/3}\right]^2}$$

Using these equations, Stephenson was able to create the graph shown in Figure 17. Also plotted on the figure are data from two other researchers. From this graph, it is seen that the energy loss ratio increases with the height of dam relative to critical flow depth.

**Diez-Cascon et al.**

A study out of Spain by J. Diez-Cascon and other researchers in 1991 titled "Studies on the Hydraulic Behavior of Stepped Spillways" shed some interesting light into estimating flow depths at the base of the stepped spillway. A 1:10 scale model was constructed in the laboratory as shown in Figure 18. The model had a downstream slope of 0.75H:1.0V. At each flowrate, the end wall (#5 in figure) was raised or lowered to place the hydraulic jump immediately at the toe of the spillway. The conjugate depth $y_2$ was then measured during each flowrate to determine the upstream depth $y_1$ (Diez-Cascon et al., 1991).

Since the densities of the flow are different above and below the hydraulic jump, classical momentum theory does not apply in finding alternate depths. Alternate depths computed are...
Figure 17. Stephenson's energy loss graph (Stephenson, 1991).
Figure 18. Diez-Cascon test model (Diez-Cascon et al., 1991).

Figure 19. Results from Diez-Cascon study (Diez-Cascon et al., 1991).
commonly much less than measured values since the upstream flow is highly aerated. As a potential solution, Diez-Cascon et al. proposed a solution using an equation published in a 1961 ASCE paper. The ASCE paper claims that air concentration can be estimated using the following formula (ASCE, 1961):

\[
c = 0.743 \log \left( \frac{s}{q^{1/5}} \right) + 0.876 \tag{8}
\]

where:
- \( c \) = air volume/(air volume + water volume)
- \( s \) = the sine of the slope
- \( q \) = discharge per unit width

Diez-Cascon et al. then used the momentum equation and the continuity equation combining them to form the following equation relating alternative depths:

\[
F_2^2 \left( 1 - \left( \frac{1}{h\delta} \right) \right) = \frac{1}{2} (h\delta^2 - 1) \tag{9}
\]

where:
- \( h = d_1/d_2 \)
- \( d_1 \) = density before hydraulic jump
- \( d_2 \) = density after hydraulic jump
- \( \delta = y_1/y_2 \)
- \( F_2 \) = Froude number below hydraulic jump = \( q^2/(gy_2^3) \)
- \( h = 1-c \)

Using data obtained from their model study, they created the graph shown in Figure 19. A theoretical curve is plotted showing the results from the equations above, and the experimental values are plotted showing prototype depths measured on the model. The theoretical curve closely estimated depths at the toe of the spillway (Diez-Cascon et al., 1991).
One of the leading researchers on stepped spillways in the United States is Kathleen H. Frizell, a hydraulic engineer for the U. S. Bureau of Reclamation (USBR) in Denver. Frizell can be credited with work on the Upper Stillwater Dam and many other research projects done on stepped spillways. The USBR has spent much time and money over the past decade studying the performance of stepped spillways. Much emphasis of the research in the past 2 years has been given to stepped overlays on low embankment dams. These embankment dams are typically flatter slopes, ranging from 2H:1V to 4H:1V, and data from them are not useful in the slopes studied in this research. There are many factors from Frizell's research that can be helpful in designing stepped spillways. Frizell has studied pressure profiles along the step tread to estimate possible cavitation damage on the steps due to any localized low pressure areas. Frizell has concluded that cavitation on the steps will not be a problem due to the following:

1. Research results suggest that a uniformly rough surface can have a lower cavitation potential than an isolated roughness of the same geometry due to reduced velocities and wake effects.
2. Large surface roughnesses promote self aeration of the flow.
3. Steps form large offsets away from the flow direction. This inhibits its cavitation from residing on the boundary.
4. Step geometries can be designed to prevent sub atmospheric pressures on the surface. (Frizell and Mefford, 1991, p.62)

Frizell and other researchers are continuing their work on flat, embankment-type stepped slopes on large testing facilities both at the USBR lab in Denver and at Colorado State University in Fort Collins, Colorado.

The most recent article published at the time of this writing was an article published in May 1993 in the ASCE Journal of Hydraulic Engineering by George C. Christodoulou. Christodoulou attempted to take previous research done on stepped spillways along with research conducted by him
to compile a design curve for estimating energy loss on a stepped face. Christodoulou's model and sketches can be seen in Figure 20.

Christodoulou plotted his results from the model study, along with Sorensen’s data from the Monksville Dam study, on a graph which related relative head loss against critical depth and step height. As seen in Figure 21, the number of steps greatly affects the measured energy loss in the flow.

Going further, Christodoulou divided the x-coordinate of each data point by the respective number of steps for that study and plotted them to come up with the graph seen in Figure 22. This gives a very nice curvilinear relationship for energy loss. Christodoulou proposed that "this curve could be used in design practice for straightforward preliminary estimates of the energy loss" (Christodoulou, 1993, p.649). He added that this curve would only be applicable for slopes of 0.7H:1.0V and Yc/h ranges of 1-4.
Figure 20. Christodoulou model - lengths in cm (Christodoulou, 1993).
Figure 21. Christodoulou's first energy loss graph (Christodoulou, 1993).

Figure 22. Christodoulou's second energy loss graph (Christodoulou, 1993).
Energy Equation

Energy in an open channel can be expressed as total head in feet of water. It is comprised of three elements that account for the total energy: elevation above a datum, pressure head, and the velocity head. Figure 23 represents the energy in open channel flow. The energy at section A can be represented by the following equation:

\[ H = z_A + \frac{P_A}{\gamma} + \alpha \frac{V_A^2}{2g} \]  

(10)

where:
- \( H \) = total energy in an open channel
- \( z_A \) = the elevation above a horizontal datum to the channel floor
- \( P_A/\gamma \) = pressure head usually equal to the depth of flow
- \( \alpha \) = the kinetic energy correction coefficient
- \( V_A \) = the average velocity of the flow

For straight prismatic channels, the kinetic energy correction coefficient is commonly accepted as unity (Chow, 1959).

When the bed slope is very steep as in most spillways, the pressure is no longer hydrostatic. The pressure at point A is equal to the component of the weight of a small element normal to the bed. If the depth is measured perpendicular to the channel floor, the pressure component in the energy equation is given by:

\[ \frac{P_A}{\gamma} = d_A \cos \theta \]  

(11)

If the depth is measured vertically, the pressure head is then calculated as:

\[ \frac{P_A}{\gamma} = y_A \cos^2 \theta \]  

(12)
Figure 23. Energy in an open channel.
Specific Energy

Specific energy is the energy per pound of water at any section of a channel measured with respect to the channel bottom. Using Equations 10 and 11 and dropping the elevation term gives the general specific energy equation:

\[ E = dA \cos \theta + \frac{V^2}{2g} \]  

(13)

From this equation it can be seen that specific energy is equal to the sum of the pressure head and the velocity head. Since \( V = Q/A \), Equation 13 can be written:

\[ E = dA \cos \theta + \frac{Q^2}{2gA} \]  

(14)

From this equation it can be seen that given a specific flowrate, the specific energy is totally dependent on the depth of flow. When the depth of flow is plotted against the specific energy for a given channel and flowrate, the specific energy curve seen in Figure 24 is obtained. Each curve has two limbs. The upper limb approaches a line at 45 degrees to the horizontal and the lower limb approaches asymptotically to the horizontal axis. At any point on the curve, the ordinate represents the depth and the abscissa represents the sum of the pressure and velocity heads.

For each value of specific energy there can be two depths of flow known as conjugate depths. The greater depth is conjugate to the smaller depth and vice versa. There is one point on the curve where specific energy is a minimum and only one depth can occur. This depth is known as critical depth and will be explained in detail later. Depths greater than critical produce velocities less than critical velocity and the flow is known as subcritical. Conversely, depths less than critical produce velocities greater than critical and the flow is known as supercritical. Subcritical flows carry the bulk of their energy in potential energy while supercritical flows carry the bulk of their energy in kinetic energy.
$y_1$ and $y_2$ are alternate depths since $E_1 = E_2$

$E_1 = y_1 + \frac{V_1^2}{2g}$

$E_2 = y_2 + \frac{V_2^2}{2g}$

Figure 24. Specific energy diagram (Jeppson and Flammer, 1983).
Flow over a spillway is an excellent example of changing specific energy. Flow behind a spillway is highly subcritical. As the flow approaches the spillway, the depth decreases. Somewhere near the crest of the spillway, the depth passes through critical depth and continues to decrease until uniform flow is achieved. This flow is highly supercritical and the bulk of the energy has been changed from potential to kinetic energy.

**Critical Flow**

It was shown before that critical depth occurs when specific energy is a minimum for a given discharge. Using the general energy equation $E = y + \frac{q^2}{2gy}$ where $q$ is the flow per unit width of channel ($q = \frac{Q}{b}$), and setting the derivative equal to zero gives the general form of the critical flow equation:

$$\frac{dE}{dy} = 1 + \frac{2q^2}{2gy_c^3} = 0$$  \hspace{1cm} (15)

From which can be obtained:

$$\frac{q^2}{gy_c^3} = 1$$  \hspace{1cm} (16)

or

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$  \hspace{1cm} (17)

A relationship between critical depth and the Froude number can be obtained. Since $q = Vy$ this can be substituted into Equation 16 to give:

$$\frac{V^2}{gy_c} = 1 \quad \text{or} \quad Fr = \frac{V}{\left(gy_c\right)^{\frac{1}{3}}} = 1$$  \hspace{1cm} (18)
As can be seen from Equation 18, critical flow occurs with a Froude number equal to unity. A Froude number greater than unity confirms supercritical flow and a Froude number less than unity confirms subcritical flow.

If Equation 18 is multiplied by \( y_c \) and divided by 2, it can be seen that at critical depth, the velocity head is equal to one half the critical depth:

\[
\frac{V^2}{2g} = \frac{y_c}{2}
\]  

(19)

In addition, by using the general energy equation, the value of specific energy is equal to 1.5 times the critical depth:

\[
E_c = y_c + \frac{V^2}{2g} = y_c + \frac{y_c}{2} = 1.5y_c
\]

(20)

Energy Dissipators

Energy dissipators have been used in many different applications to limit the effects of the release of kinetic energy. A well designed energy dissipator must remove the excess kinetic energy without damage to the dissipator or the channel downstream.

There are generally two types of energy dissipators; impact-type energy dissipators and dissipators that use a hydraulic jump to dissipate the energy. Impact-type energy dissipators cause the flow to spread in many directions, inducing heavy mixing and energy dissipation. Hydraulic jump dissipators convert highly supercritical flow into subcritical flow by causing a hydraulic jump to form in the dissipator (Peterka, 1964).

Although impact-type energy dissipators generally dissipate more energy than hydraulic jump dissipators, they are usually not practical for high dams. For such situations where large flowrates and very high velocities are encountered, a hydraulic jump dissipator is practical. Following are several different types of energy dissipators that can be used on spillways.
Concrete apron dissipator

This type of hydraulic jump dissipator consists of a large concrete apron at the base of the spillway. This type of dissipator does little to create a hydraulic jump on its own. It can only be used where an adequate tailwater already exists that can contain the hydraulic jump over the apron.

Stilling basins

Stilling basins, also hydraulic jump dissipators, are like the apron dissipator described previously. Stilling basins have been modified with end sills and chute blocks to aid in the formation of the hydraulic jump. Stilling basins are constructed with the floor of the stilling basin low enough so that the tailwater plus the depth of the stilling basin causes the hydraulic jump to remain near the toe of the structure.

Flip bucket dissipators

Flip bucket dissipators are a type of impact-type dissipator. Looking much like a ski jump, the curved lip at the end of the spillway projects the water outward away from the structure. This commonly causes a large scour hole to develop away from the toe of the structure. The scour hole creates a deep pool of water which effectively dissipates much of the energy.

Roller bucket dissipators

The roller bucket dissipator is a combination impact-type/hydraulic jump-type dissipator. The roller bucket dissipator is much like the flip bucket dissipator, but is totally submerged by the tailwater. The jet from the flip bucket does not spring free from the bucket and a large hydraulic jump is formed (Davis, 1969)

Energy dissipators can be damaged by any material in the flow. The high velocities encountered can cause material in the flow to impact the walls, chute blocks, end sills, etc. In addition, high velocities can generate pockets of water that are subatmospheric. The pockets generate cavitation bubbles which upon collapse can erode away the concrete face of the spillway.
Spillway Crest Shape

Spillway crest shapes are commonly built to approximate the profile of the underside of a free overflow nappe. This profile shape depends on the design head and approach conditions. The United States Bureau of Reclamation (USBR) has done extensive research into the design of spillway crests and has produced the following equation to calculate the profile (U.S. Bureau of Reclamation, 1977). See Figure 25.

\[
\frac{y}{H_o} = -K\left(\frac{x}{H_o}\right)^n
\]  

(21)

The coefficients \(x\) and \(y\) are coordinates relative to spillway crest. \(K\) and \(n\) are constants defined by design curves shown in Figure 25. The design head is also a constant and is represented by \(H_o\).

For a design condition where the crest has a vertical upstream face and negligible approach velocity, the USBR has simplified the design by producing a crest shape constructed of compound curves. Figure 26 shows this design. This figure eliminates the need to solve an exponential equation.

Discharge over an uncontrolled oggee crest spillway can be calculated by using the following equation:

\[
Q = CLH_o^{3/2}
\]

(22)

where:

\(Q\) = discharge

\(C\) = variable coefficient of discharge

\(L\) = effective weir length

\(H_o\) = total head on crest, including velocity of approach head (Figure 25)

The discharge coefficient can vary due to different approach conditions, heads different than the design head, relation of actual crest to crest of ideal nappe shape, and upstream face slope. Given a crest with vertical upstream slope, the discharge coefficient can be calculated using USBR graphs in Figure 27. The graphs also allow for a correction in the discharge coefficient for heads greater or less than the design head used to design the crest shape.
Figure 25. USBR - elements of nappe-shaped profiles (U.S. Bureau of Reclamation, 1977).
Figure 26. USBR - nappe-shaped profile design with compound curves (U.S. Bureau of Reclamation, 1977).
Figure 27. USBR graphs for determining crest coefficient (U.S. Bureau of Reclamation, 1977).
Stepped Spillway Crest Shapes

Stepped spillway crest shapes can follow the same profile as defined in the USBR manuals. Small steps are fitted near the crest so that the envelope of their tips just intersect the profile of the ideal nappe shape. The smallest step is slightly beyond the top of the crest with increasing size of steps continuing down the face until the desired step size for the stepped face is achieved. This has proven to work very well on many designs, although little research has been done in finding the most hydraulically efficient step configuration. As a rule of thumb, the USBR crest shape will generally keep the flow adhered to the face of the spillway with little problem of leaping away if enough small steps are placed near the top of the spillway and the steps are not too large.

The Momentum Principle in Open Channel Flow

According to Jeppson and Flammer (1983), one of the primary applications of the momentum principle is the analysis of the open channel phenomenon known as the hydraulic jump. In a hydraulic jump, the water surface rises abruptly as the flow changes from supercritical to subcritical. In this transition, a large amount of energy loss is achieved and can be seen as a large turbulent boil.

The energy loss (hJ) cannot be calculated using the energy equation alone, so momentum must be used to determine the depths both upstream and downstream of the hydraulic jump, known as conjugate depths. Momentum is only concerned with forces acting on and momentum entering or leaving the boundary of flow (Jeppson and Flammer, 1983). The evaluation of the hydraulic jump can be analyzed using Figure 28. The forces acting in the x-direction are \( F_{p1} \) and \( F_{p2} \) and the momentums entering and leaving the control volume are \( QpV_1 \) and \( QpV_2 \). The streamlines before and after the jump are straight and therefore the pressure distribution in \( F_{p1} \) and \( F_{p2} \) is hydrostatic and equal to \( F_p = \gamma h_cA \). Applying the momentum equation across the hydraulic jump results in the following equation:
Figure 28. Momentum across a hydraulic jump (Jeppson and Flanner, 1983).
\[
F_{p_1} - F_{p_2} = \gamma h_{c_1} A_1 - \gamma h_{c_2} A_2 = \frac{\gamma}{g} Q^2 \left( \frac{1}{A_2} - \frac{1}{A_1} \right)
\]

This equation can be rearranged by dividing by \(\gamma\) and collecting terms with the same subscript to the same side as seen in Equation 24:

\[
A_1 h_{c_1} + \frac{Q^2}{gA_1} = A_2 h_{c_2} + \frac{Q^2}{gA_2} = M
\]

In this equation it can be seen that the momentum above and below the hydraulic jump is equal. The equations can be simplified further using several assumptions. By assuming a rectangular channel per unit width, \(A = 1 \times Y\), \(q = Q/b\), and \(h_c = Y/2\). Using these simplifications results in the following equation:

\[
\frac{Y_1^2}{2} + \frac{q^2}{gY_1} = \frac{Y_2^2}{2} + \frac{q^2}{gY_2}
\]

Rearranging these terms yields:

\[
\frac{1}{2}(Y_1^2 - Y_2^2) = \frac{q^2}{g} \left( \frac{1}{Y_2} - \frac{1}{Y_1} \right)
\]

which is the same as:

\[
\frac{1}{2}(Y_1 + Y_2)(Y_1 - Y_2) = \frac{q^2}{g} \left( \frac{Y_1 - Y_2}{Y_2Y_1} \right)
\]

Dividing both sides by \((Y_1 - Y_2)\) and rearranging the terms gives:

\[
Y_2^2 + \frac{2q^2}{gY_1} = 0
\]

This can be solved for the one possible root using the quadratic equation to get:

\[
Y_2 = \frac{1}{2} \left( -1 + \sqrt{1 + \frac{8q^2}{gY_1^3}} \right)
\]

Since \(\text{Fr}_1^2 = q^2/gY_1^3\):
\[ Y_2 = \frac{Y_1}{2}(-1 + \sqrt{1 + 8Fr_1^2}) \]  

(30)

This equation is symmetrical in \(Y_1\) and \(Y_2\) and both can be changed to get the other conjugate depth.

\[ Y_1 = \frac{Y_2}{2}(-1 + \sqrt{1 + 8Fr_2^2}) \]  

(31)
MODEL SPECIFICS

Froude similitude

For hydraulic studies of spillways and stilling basins, gravity forces are predominant. Because of this, models are built to undistorted scale and follow the rules of Froude similitude. The Froude number (Equation 18) is a dimensionless number proportional to the ratio of inertia forces to gravity forces. Data obtained from models can be scaled to prototype values using the following relationships:

Length ratio: \[ L_r = \frac{L_p}{L_m} \]
( subscripts r, p, and m refer to ratio, prototype, and model)

Discharge: \[ Q_p = L_r^{2.5} Q_m \]

Velocity: \[ V_p = L_r^{0.5} V_m \]

Head: \[ H_p = L_r H_m \]

Pressure: \[ P_p = L_r P_m \]

Depth or wave height: \[ L_p = L_r L_m \]

Time: \[ T_p = L_r^{0.5} T_m \]

No attempt to study cavitation was made in our models. Because of the research done by Kathleen Frizell at the USBR lab in Denver, it was decided that cavitation would not be a problem. In all of the literature, cavitation is never a problem with stepped spillways due to the lower velocities on the face of the spillway, and the fact that any cavitation will occur in the circulating flow in the step cavity. This flow will cause any cavitation to occur away from the surface of the steps where it will not be able to damage anything. For this reason, there is no scaling parameter given with the cavitation value sigma.
Although Froude similitude is used in this study, it should be used cautiously. The models used in this study are fairly small and may not exactly model the performance of the larger prototype. There is a possibility for scaling effects with all models due to the inability to model aeration accurately. It is possible for viscous effects to affect the modeling. This could possibly be corrected using a Reynolds scaling parameter. This was not attempted in this study. Froude similitude is used, but the reader should be cautioned that scaling effects may be present. Without prototype data, the scaling effects could not be studied.

Model scale ratio

Since the models in the study were not built to a specific scale, the data can be applied at any scale ratio desired. The scale ratio will determine test parameters like step size, model spillway height, and flowrates tested in the study. Two scale ratios were used in the analysis of the data presented. From the literature review, it was apparent that the common step size is about 1-2 feet. Only on De Mist Kraal did the step size go to 3 feet. With this in mind, scale ratios were calculated so that the models would mimic the 2-foot steps on Models A and C and 1-foot steps on Models B and D. These scale ratios were 22.4:1 for the 0.7H:1.0V slope and 32.0:1 for the 0.5H:1.0V slope.

Crest design

The crest of the spillway is designed in the shape of the underside of a free overflow nappe. By using an estimate of 30.0:1.0 for a scale ratio and an approximate unit discharge of 100 cfs/ft prototype, the model overtopping head was calculated to be 4.3 inches. Given this design head and flowrate, the coefficients for the USBR equation were found. This equation will compute the nappe-shaped crest in coordinates from the crest down.

$$\frac{y}{H_o} = -1.858\left(\frac{x}{H_o}\right)^{0.505}$$ (32)
The results from this equation as well as values computed using the USBR compound curve chart were compared. Both methods gave very similar results. Coordinates from the USBR compound curve chart are much easier to obtain.

Once the crest shape is determined, small steps were fitted to the crest profile so that the envelope of the step tips just intersect the profile of the computed nappe shape. The small steps near the crest introduce the flow to the steps without causing the flow to spring away from the crest surface. These small steps near the crest are extremely important in the performance of the stepped spillway. Too large of steps will cause the flow to hit the first step and spring away from the spillway in a leap frog pattern. Too small of steps will cause an uneven transition of the flow to the larger steps and could cause the leap frog effect to move further down the face of the spillway. Generally, the steps should start fairly small and increase in size until the constant step height is achieved near the transition from the nappe-shaped portion to the constant slope portion.

By using this information along with other crest designs mentioned in the Literature Review section, we designed the crest for our model. This crest design seen in Figure 5 was used for both models. For the steeper slope, the crest was slightly modified to accommodate a smooth transition from the nappe to the constant slope. The crest was raised slightly by adding a 1/4-inch thick plywood panel to the base of the crest model (hatched area in Figure 5). This modification along with an additional smaller step at the base of the crest proved to work very well on the model.

**Step design**

Prototype step heights are commonly 1 to 2 feet. Most design flows are commonly about 100 cfs/ft. Using this information, we were able to estimate our model step sizes so that they would fall in this range given a reasonable scale ratio. Lumber size and construction techniques helped determine the final step sizes in the models.

Two different model slopes were tested, each with two different step sizes. Table 1 describes the model number, slope, step size, and number of steps.
Table 1. Model configurations

<table>
<thead>
<tr>
<th>Model</th>
<th>Slope</th>
<th>Step Size</th>
<th>Number of Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.7H:1.0V</td>
<td>.75&quot;H x 1.07&quot;V</td>
<td>36</td>
</tr>
<tr>
<td>B</td>
<td>0.7H:1.0V</td>
<td>.375&quot;H x .536&quot;V</td>
<td>65</td>
</tr>
<tr>
<td>C</td>
<td>0.5H:1.0V</td>
<td>.375&quot;H x .75&quot;V</td>
<td>49</td>
</tr>
<tr>
<td>D</td>
<td>0.5H:1.0V</td>
<td>.188&quot;H x .375&quot;V</td>
<td>90</td>
</tr>
</tbody>
</table>

Testing Apparatus

Test flume

All models were built at the Utah Water Research Laboratory and tested in a 3-foot wide flume. The original flume had side walls of 2 feet. In order to allow for a model greater than 2 feet high, the side walls were extended 2 feet higher using Plexiglas and plywood. The extended walls allowed a model height of approximately 3 feet, allowing the crest to be overtopped by approximately 6 - 8 inches. The flume was gravity fed by water from a 12-inch line controlled by a valve. Water in the head box was stilled by passing through two sets of baffles. The model was placed approximately 4.5 feet from the head box. Below the model was a 6.5-foot piece of plywood set horizontal. This painted surface provided a uniform surface to model the hydraulic jump characteristics. At the end of the flume there was a variable height end sill which was raised or lowered to place the hydraulic jump at the base of the model. The water flowed out of the flume and into channels where it was routed to weigh tanks for flow measurements. A diagram of the flume facility can be found in Figure 29.

Depth measurements

Depth of water above the crest was measured using a standard hook gage located 3 feet upstream from the crest. This was far enough to ensure that the reading was not in the draw down region of the crest and far enough from the turbulent water in the head box. Readings from the hook gage could be read to the nearest 0.001 foot. The reference elevation for the hook gage was checked frequently to correct for any warping that could have occurred on the model.
Figure 29. Test flume used in study.
Originally, depth data on the steps were obtained using a point gage mounted on a movable frame. This method measured a vertical depth. The vertical depth multiplied by the cosine of the spillway slope equaled the actual depth. This method was very time consuming and did not yield good results. It was very difficult to determine when the point gage actually contacted the water surface due to the small amount of flow leaping away in the turbulent flow.

Depth measurements on the face of the spillway were instead taken with a sharp-edged ruler that, when placed in the flow, could record the depth in millimeters. The ruler did not disturb the flow noticeably and could be read in the nonaerated portion of the spillway by looking through the Plexiglas window under the water surface. In highly aerated regions, as encountered at the toe, the depth was read from above the surface. Accuracy of the ruler method was estimated at ±1 millimeter at most flows. Depth measurements were taken at 1/4 points across the face of the spillway. When averaged, this gave an acceptable value for depth. Any uncertainty in the data could be the result of the highly turbulent flow and the small amount of spray that contacts the ruler while taking a measurement.

Depths below the hydraulic jump were measured using a standard point gage mounted on a rolling carriage. Accuracy of the point gage was 0.001 feet. Depth measurements were taken as close to the hydraulic jump as permitted. At very low flows, the depth measurements were taken about 1 foot from the toe of the model. At higher flows the hydraulic jump was very large and the depth measurement was taken at 6 feet from the toe of the model. To insure that the plywood floor did not settle and create inaccurate depths, both the floor and the water surface were measured at every data point. As with the depth measurements on the face of the spillway, depth measurements were taken at 1/4 points and averaged. This was felt to be accurate and acceptable for our study.

Velocity measurements with pitot tube

The pitot tube can be a very useful device to measure velocities in different flows. The pitot tube can measure the difference between the static head and the specific energy. This difference is the velocity head, which can easily be converted to velocity.
The pitot tube is not useful in estimating velocities on a stepped face due to the highly aerated flow. The air bubbles create air locks in the tubing, which gives inaccurate measurements of velocities. The only successful use of pitot tubes on stepped faces has been with back-pressure pitot tubes. This prevents the air from entering the pitot tube and an accurate measurement can be taken. This method was not attempted in the study so an estimation of accuracy is not available.

Flow measurement with weigh tanks

The Utah Water Research Laboratory is equipped with two large weight tanks capable of weighing 30,000 lbs each with an accuracy of ±5 lbs. The system can weigh and add successive tanks continuously, measuring the elapsed time to the nearest 0.01 second. Flowrate measurements can be achieved within 1/4 of 1 percent.

By using flowrate measurements from the weigh tanks along with depth measurements taken on the model, average velocities in the model can be calculated using continuity.

\[ V = \frac{Q}{A} = \frac{Q}{(y \times w)} \]  \hspace{1cm} (33)

where:

- \( V \) = calculated velocity
- \( Q \) = flowrate from weigh tank measurements
- \( A \) = flow area
- \( y \) = depth of flow
- \( w \) = width of test facility (36.5" in flume)

**USBR Method for Designing Type II Stilling Basins**

For high dam situations where toe velocities are very high and Froude numbers are greater than 5, the USBR recommends using a Type II stilling basin. The USBR has produced a design procedure for estimating toe velocities and designing a Type II stilling basin in its publication "Hydraulic Design of Stilling Basins and Energy Dissipators" (Peterka, 1964). The USBR has done
extensive work on estimating toe velocities at the base of smooth spillways. They have produced the
chart in Figure 30 to determine the toe velocity on a smooth spillway on steep slopes for a given design
situation. The theoretical velocity can be determined from the following equation:

\[ V_T = \sqrt{2g \left( Z - \frac{H}{2} \right)} \] (34)

where:
- \( V_T \) = Theoretical toe velocity
- \( g \) = acceleration due to gravity
- \( Z \) = elevation from stilling basin to upper water surface
- \( H \) = head of water over crest

By using \( Z \) and \( H \), the ratio \( \frac{V_A}{V_T} \) can be found from the USBR chart. Multiplying \( V_T \) by
the ratio obtained in the chart gives the actual velocity at the toe of the structure.

Given the actual velocity, a basin can be designed using additional charts from the USBR.
The depth at the toe can be calculated by dividing the unit flowrate by the actual velocity. The
conjugate depth can be calculated using the following equation:

\[ \frac{D_2}{D_1} = \frac{1}{2} \left( \sqrt{1 + 8F_1^2} - 1 \right) \] (35)

and

\[ F_1 = \frac{V_1}{\sqrt{gD_1}} \] (36)

The value \( D_2 \) is the required tailwater necessary for the hydraulic jump to remain in the
stilling basin. The length of the stilling basin is calculated using the USBR chart in Figure 31. The
remaining items in the stilling basin like the chute blocks and end sill height can be calculated using
the USBR figure in Figure 32.

A large part of this research has been using the measured toe velocities from the models and scaling
those use in the USBR design process to design a Type II stilling basin for a stepped spillway.
Figure 30. USBR graph #15 for estimating toe velocities (Peterka, 1964).
Figure 31. USBR chart #12 for estimating stilling basin length (Peterka, 1964).
Figure 32. USBR figure #14 - design of chute blocks and end sill diagram (Peterka, 1964)
The stepped spillway concept should in theory reduce the velocities and therefore reduce the size of the stilling basin necessary to contain the hydraulic jump.
RESULTS AND DISCUSSION

Spillway Models and Test Results

Four different models were operated and tested in this study.

Model A

Model A (Figure 1) consisted of a stepped spillway of slope 0.7H:1.0V with steps of constant height 0.75"H x 1.07"V. The crest was a standard nappe-shaped crest as described previously. The transition from the smaller steps on the spillway crest to the larger steps of constant height occurred at coordinates (6.07", -3.69"). A total of 36 steps including the smaller steps near the crest comprised this model. Model flowrates tested ranged from 0.175 cfs to 2.511 cfs.

At the very low flowrates, there was a small problem in the transition region from the nappe-shaped portion to the constant slope area. Water was hitting the first step of the constant size steps and leaping away, hitting about 3-4 steps farther down the face. This was easily corrected by adding one more smaller step in the transition region where the flow was leaping away. After this smaller step was attached, there were no more problems like this with this model.

The flow over the steps creates a rotating vortice in the step. This can be easily seen on the steps that are fully aerated, but is not visible to the naked eye on the steps with no aeration. Dye injected into the flow directly in the tread of a step showed the fluid rotation and also fluid translation along the step. Pictures of this can be seen in the appendix. Further down the slope, the boundary layer has contacted the upper water surface and fully turbulent flow occurs with highly aerated vortices visible in the step tread.

The distance down the slope of this aeration pattern varied with flowrate. At the smallest flowrate, the distance to full aeration occurred around steps 6-7. At the highest flowrate of 2.511 cfs tested, the aeration was not fully apparent until steps 32-33 in the model.
Total available energy in the models was measured according to a variable datum which was always the water surface downstream of the hydraulic jump (Y2 in Tables 2-5). The overflow weir at the end of the flume was raised and lowered to allow for the hydraulic jump to form right at the toe of the stepped spillway.

The reservoir energy was calculated 3 feet back from the crest and is labeled Ec. This included the elevation difference from the downstream water surface to the crest, the depth over the crest measured 3 feet back from the crest, and any velocity head. Velocity head was only a factor on the highest flowrates where its value was about 0.001 feet.

Energy at the toe of the model was equal to the elevation from the downstream water surface to the elevation of the last step on the spillway, the depth of flow measured parallel to the face multiplied by the cosine of the slope angle, and the velocity head. As expected, velocity head played a major role in the energy at the toe of the model. The difference in energies divided by the total energy available was used to determine the percent energy loss as seen in Equation 38. The data from the tests on Model A can be found in Table 2.

\[
\Delta E = \frac{E_c - E_t}{E_c}
\]  
(37)

where: 
\( \Delta E \) = energy loss at toe of model  
\( E_c \) = Energy at crest using depth below hydraulic jump as the datum  
\( E_t \) = Energy at toe of model using depth below hydraulic jump as the datum

Model B

Model B (Figure 2) consisted of a stepped spillway of slope 0.7H:1.0V with steps of constant height 0.375"H x 0.536"V. The model was the same model as Model A with half size steps attached between the larger steps of the previous model. There was a total of 65 steps on Model B. Model flowrates tested ranged from 0.124 cfs to 2.398 cfs.
Table 2. Test data for Model A

<table>
<thead>
<tr>
<th>Test #</th>
<th>He</th>
<th>q</th>
<th>Yc</th>
<th>C</th>
<th>Y1</th>
<th>V1</th>
<th>Y2</th>
<th>V2</th>
<th>F2&quot;2</th>
<th>Ec</th>
<th>E1</th>
<th>E Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.062</td>
<td>0.058</td>
<td>0.047</td>
<td>3.730</td>
<td>0.013</td>
<td>4.388</td>
<td>0.124</td>
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<td>2.852</td>
<td>0.307</td>
<td>89.23</td>
</tr>
<tr>
<td>2</td>
<td>0.084</td>
<td>0.090</td>
<td>0.063</td>
<td>3.686</td>
<td>0.016</td>
<td>5.470</td>
<td>0.168</td>
<td>0.536</td>
<td>0.053</td>
<td>2.830</td>
<td>0.431</td>
<td>84.76</td>
</tr>
<tr>
<td>3</td>
<td>0.126</td>
<td>0.169</td>
<td>0.096</td>
<td>3.778</td>
<td>0.033</td>
<td>5.150</td>
<td>0.247</td>
<td>0.686</td>
<td>0.059</td>
<td>2.793</td>
<td>0.309</td>
<td>88.94</td>
</tr>
<tr>
<td>4</td>
<td>0.165</td>
<td>0.256</td>
<td>0.127</td>
<td>3.823</td>
<td>0.039</td>
<td>6.509</td>
<td>0.326</td>
<td>0.786</td>
<td>0.059</td>
<td>2.753</td>
<td>0.479</td>
<td>82.59</td>
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<tr>
<td>5</td>
<td>0.209</td>
<td>0.373</td>
<td>0.163</td>
<td>3.902</td>
<td>0.051</td>
<td>7.331</td>
<td>0.422</td>
<td>0.885</td>
<td>0.058</td>
<td>2.702</td>
<td>0.567</td>
<td>79.01</td>
</tr>
<tr>
<td>6</td>
<td>0.251</td>
<td>0.500</td>
<td>0.198</td>
<td>3.975</td>
<td>0.069</td>
<td>7.255</td>
<td>0.498</td>
<td>1.005</td>
<td>0.063</td>
<td>2.668</td>
<td>0.484</td>
<td>81.85</td>
</tr>
<tr>
<td>7</td>
<td>0.295</td>
<td>0.651</td>
<td>0.236</td>
<td>4.065</td>
<td>0.073</td>
<td>8.889</td>
<td>0.582</td>
<td>1.119</td>
<td>0.067</td>
<td>2.628</td>
<td>0.812</td>
<td>69.10</td>
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<tr>
<td>8</td>
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<td>0.662</td>
<td>1.247</td>
<td>0.073</td>
<td>2.594</td>
<td>1.083</td>
<td>58.24</td>
</tr>
</tbody>
</table>

He = Head of water over crest  
Q = Model flowrate  
Yc = Critical depth  
C = Crest Coefficient at that flowrate  
Y1 = Depth of flow at toe of spillway  
V1 = Velocity of flow at toe of spillway  
Y2 = Depth below hydraulic jump  
V2 = Velocity below hydraulic jump  
F2 = Froude number below hydraulic jump  
Ec = Energy of flow at crest  
E1 = Energy of flow at toe of spillway  
E Loss = Energy Lost = (Ec-E1)/Ec
Since the problem with the transition region was solved with the previous model, there were no additional steps needed in the transition region to keep the flow attached to the face of the spillway. Flows remain attached to the structure over the whole range of flows tested.

The turbulent aeration occurred lower on the structure than measured on the larger step model. Aeration on the average moved approximately 3-4 inches further downstream at similar flowrates. This can be attributed to the smaller steps and the less affect they have on the flow in the transition from the smooth to turbulent region. Data for Model B are found in Table 3.

Model C

Model C (Figure 3) consisted of a stepped spillway of slope 0.5H:1.0V with steps of constant height 0.357"H x 0.75"V. The crest was slightly modified for the steeper slope by adding a thin 1/4-inch sheet of plywood to the base of the spillway and an additional smaller step in the transition. This modified crest is shown in Figure 5 where the modification is the hatched area. The transition from the smaller steps on the spillway crest to the large steps of constant height occurred at coordinates (6.07",-3.69"). A total of 49 steps comprised this model. Model flowrates ranged from 0.113 cfs to 2.324 cfs.

Originally a model with step heights of 0.75"H x 1.5"V was built. Preliminary tests on this model showed that the flow leaped away from the face of the model at low to medium flowrates. At very high flowrates, the flow remained attached to the face of the structure. Five half steps were attached to the upper portion of the constant slope area and the performance was visually inspected again. Adding the five smaller steps moved the problem further down the face of the model to where the larger steps were. It was decided that although the larger steps are working in dissipating energy, we would not be able to test the model at the very high flowrates with accuracy. Smaller steps were then cut and attached to the remaining larger steps to create a constant height of 0.375"H x 0.75"V on the face of the spillway.
Table 3. Test data for Model B

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ho (ft)</th>
<th>q (cfs/ft)</th>
<th>Ye (ft)</th>
<th>C</th>
<th>Y1 (ft)</th>
<th>V1 (ft/sec)</th>
<th>Y2 (ft)</th>
<th>V2 (ft/sec)</th>
<th>F2*2</th>
<th>Ec (ft)</th>
<th>E1 (ft)</th>
<th>E Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.051</td>
<td>0.041</td>
<td>0.037</td>
<td>3.532</td>
<td>0.013</td>
<td>3.100</td>
<td>0.098</td>
<td>0.418</td>
<td>0.055</td>
<td>2.868</td>
<td>0.133</td>
<td>95.35</td>
</tr>
<tr>
<td>2</td>
<td>0.088</td>
<td>0.090</td>
<td>0.063</td>
<td>3.438</td>
<td>0.020</td>
<td>4.559</td>
<td>0.167</td>
<td>0.538</td>
<td>0.054</td>
<td>2.835</td>
<td>0.241</td>
<td>91.50</td>
</tr>
<tr>
<td>3</td>
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<td>0.159</td>
<td>0.092</td>
<td>3.604</td>
<td>0.026</td>
<td>6.068</td>
<td>0.245</td>
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<td>0.054</td>
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<td>0.416</td>
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<td>0.129</td>
<td>3.733</td>
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<td>6.297</td>
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<td>0.055</td>
<td>2.745</td>
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<tr>
<td>5</td>
<td>0.210</td>
<td>0.368</td>
<td>0.161</td>
<td>3.823</td>
<td>0.049</td>
<td>7.477</td>
<td>0.423</td>
<td>0.870</td>
<td>0.056</td>
<td>2.701</td>
<td>0.547</td>
<td>79.74</td>
</tr>
<tr>
<td>6</td>
<td>0.252</td>
<td>0.493</td>
<td>0.196</td>
<td>3.898</td>
<td>0.059</td>
<td>8.349</td>
<td>0.506</td>
<td>0.974</td>
<td>0.058</td>
<td>2.660</td>
<td>0.684</td>
<td>74.29</td>
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<tr>
<td>7</td>
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<td>0.229</td>
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<td>9.495</td>
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<td>0.059</td>
<td>2.620</td>
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<td>64.75</td>
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<td>0.788</td>
<td>0.268</td>
<td>4.030</td>
<td>0.080</td>
<td>9.875</td>
<td>0.663</td>
<td>1.189</td>
<td>0.066</td>
<td>2.588</td>
<td>0.971</td>
<td>62.50</td>
</tr>
</tbody>
</table>

Ho = Head of water over crest  
Q = Model flowrate  
Ye = Critical depth  
C = Crest Coefficient at that flowrate  
Y1 = Depth of flow at toe of spillway  
V1 = Velocity of flow at toe of spillway  
Y2 = Depth below hydraulic jump  
V2 = Velocity below hydraulic jump  
F2 = Froude number below hydraulic jump  
Ec = Energy of flow at crest  
E1 = Energy of flow at toe of spillway  
E Loss = Energy Lost = (Ec-E1)/Ec
Data were obtained on this model in the same way for the previous models. The data collected on this model can be found in Table 4.

**Model D**

Model D consisted of a stepped spillway of slope 0.5H:1.0V with steps of constant height 0.188"H x 0.375"V. The crest remained the same as in Model C. A total of 90 steps comprised this model. Model flowrates tested ranged from 0.108 cfs to 2.322 cfs.

As in Models A and B, the aeration on the smaller steps occurred lower at similar flowrates when compared to Model C. The data collected on the model can be found in Table 5.

**Analysis of Model Data**

Energy loss was evaluated as stated previously on all four models and was presented as total energy lost prior to entering the hydraulic jump in the stilling basin. The results from our analysis resulted in some peculiar findings.

Total energy loss should depend on the step size and number of steps. Decreasing the step size is in effect decreasing the surface roughness and should decrease the amount of energy loss experienced in the model. If the steps become infinitely small, the surface becomes perfectly smooth and the energy dissipation should equal that of a standard smooth spillway. In the other extreme, if there were only one large step, the flow would follow the pattern of a free fall jet with no dissipation. When the step size was decreased 1/2, it was predicted that at the same flowrate, there would be a lower energy loss. As can be seen in Figure 33, this did not happen. In fact, the smaller steps tended to work exactly the same as the steps that were twice as large. Apparently there is a region of step sizes that perform much alike. Although it was not researched in this study, there must be a lower limit to the step size where the energy dissipation will markedly decrease and the spillway will operate
Table 4. Test data for Model C

<table>
<thead>
<tr>
<th>Test #</th>
<th>Hc</th>
<th>q</th>
<th>Ye</th>
<th>C</th>
<th>Y1</th>
<th>V1</th>
<th>Y2</th>
<th>V2</th>
<th>F2/2</th>
<th>Ec</th>
<th>E1</th>
<th>E Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ft)</td>
<td>(cfs/ft)</td>
<td>(ft)</td>
<td>(ft)</td>
<td>(ft/sec)</td>
<td>(ft/sec)</td>
<td>(ft)</td>
<td>(ft)</td>
<td>(ft)</td>
<td>(ft)</td>
<td>(ft)</td>
<td>(%)</td>
</tr>
<tr>
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<td>0.037</td>
<td>0.035</td>
<td>2.889</td>
<td>0.013</td>
<td>2.840</td>
<td>0.085</td>
<td>0.439</td>
<td>0.070</td>
<td>2.925</td>
<td>0.138</td>
<td>95.28</td>
</tr>
<tr>
<td>2</td>
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<td>0.135</td>
<td>0.083</td>
<td>3.141</td>
<td>0.023</td>
<td>5.900</td>
<td>0.198</td>
<td>0.684</td>
<td>0.073</td>
<td>2.880</td>
<td>0.445</td>
<td>84.56</td>
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<tr>
<td>3</td>
<td>0.168</td>
<td>0.231</td>
<td>0.118</td>
<td>3.361</td>
<td>0.033</td>
<td>7.055</td>
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<td>0.764</td>
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<td>9.192</td>
<td>0.627</td>
<td>1.218</td>
<td>0.073</td>
<td>2.666</td>
<td>0.814</td>
<td>69.47</td>
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</table>

Hc = Head of water over crest
Q = Model flowrate
Ye = Critical depth
C = Crest Coefficient at that flowrate
Y1 = Depth of flow at toe of spillway
V1 = Velocity of flow at toe of spillway
Y2 = Depth below hydraulic jump
V2 = Velocity below hydraulic jump
F2 = Froude number below hydraulic jump
Ec = Energy of flow at crest
E1 = Energy of flow at toe of spillway
E Loss = Energy Lost = (Ec - E1)/Ec
<table>
<thead>
<tr>
<th>Test #</th>
<th>Hc</th>
<th>q</th>
<th>Ye</th>
<th>C</th>
<th>Y1</th>
<th>V1</th>
<th>Y2</th>
<th>V2</th>
<th>F2/E2</th>
<th>Ec</th>
<th>E1</th>
<th>E Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ft)</td>
<td>(cfs/ft)</td>
<td>(ft)</td>
<td></td>
<td>(ft)</td>
<td>(ft/sec)</td>
<td>(ft)</td>
<td>(ft/sec)</td>
<td></td>
<td>(ft)</td>
<td>(ft)</td>
<td>(%)</td>
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<tr>
<td>1</td>
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<td>0.036</td>
<td>0.034</td>
<td>3.382</td>
<td>0.010</td>
<td>3.614</td>
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<td>0.087</td>
<td>0.062</td>
<td>3.381</td>
<td>0.016</td>
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<td>0.074</td>
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<td>7.172</td>
<td>0.398</td>
<td>0.888</td>
<td>0.062</td>
<td>2.770</td>
<td>0.518</td>
<td>81.30</td>
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<td>0.250</td>
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<td>0.192</td>
<td>3.833</td>
<td>0.057</td>
<td>8.426</td>
<td>0.483</td>
<td>0.992</td>
<td>0.063</td>
<td>2.727</td>
<td>0.740</td>
<td>72.87</td>
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<td>0.290</td>
<td>0.604</td>
<td>0.225</td>
<td>3.870</td>
<td>0.065</td>
<td>9.367</td>
<td>0.566</td>
<td>1.068</td>
<td>0.063</td>
<td>2.684</td>
<td>0.921</td>
<td>65.70</td>
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<td>8</td>
<td>0.332</td>
<td>0.763</td>
<td>0.263</td>
<td>3.991</td>
<td>0.075</td>
<td>10.117</td>
<td>0.651</td>
<td>1.173</td>
<td>0.066</td>
<td>2.641</td>
<td>1.067</td>
<td>59.59</td>
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</table>

Hc = Head of water over crest
Q = Model flowrate
Ye = Critical depth
C = Crest Coefficient at that flowrate
Y1 = Depth of flow at toe of spillway
V1 = Velocity of flow at toe of spillway
Y2 = Depth below hydraulic jump
V2 = Velocity below hydraulic jump
F2 = Froude number below hydraulic jump
Ec = Energy of flow at crest
E1 = Energy of flow at toe of spillway
E Loss = Energy Lost = (Ec-E1)/Ec
Test Data - Energy Loss on Model

Figure 33. Test data - energy loss on model.
like a smooth spillway. Conversely, a very large step will cause the flow to leap away as seen in the 0.5H:1.0V model with step sizes 0.75"H:1.5"V, which was not tested.

An increase in the spillway slope will produce a greater vertical component of velocity and therefore should decrease the amount of dissipation found on the flatter slope at the same flowrate. As can be seen in Figure 33, the difference between energy loss on the 0.7H:1.0V and the 0.5H:1.0V slope is virtually nonexistent. Part of this could be attributed to the inability to get highly accurate depth measurements at the toe of the model. Since depth measurements were taken at 1/4 points and averaged, the depth measurements could not have been that far off. It is the author's belief that hydraulically, there is a range of slopes, say 0.5H:1.0V to 0.8H:1.0V, at which the spillways will operate hydraulically similarly. The USBR graph shown in Figure 30 supports this theory, stating that this graph is accurate for slopes ranging from 0.6H:1.0V to 0.8H:1.0V (U.S. Bureau of Reclamation, 1977).

The graph of the energy loss shows a good correlation between the different models (Figure 33). All of the data from the models agree within about 10% to a single curve representing the energy loss. Energy loss on the model can be seen to range from about 95% at very low flows to 65% at the higher flows tested. The pattern of this curve is acceptable since at much higher flows (greater than we tested), the energy dissipation performance would approach that of a smooth spillway since the steps are not affecting the flow as much. On many prototypes, at the flowrates where dissipation has decreased to that of a smooth spillway (say at the PMF), there is commonly a large increase of the tailwater, which aids in keeping the hydraulic jump in the stilling basin. Dave Campbell, an Engineer with O'Brien and Gere Engineers, wrote in a personal letter that his company was designing a stepped spillway with an overtopping head of approximately 40 feet. At a depth of only 10 feet over the spillway, the tailwater was expected to be over 50 feet, which was ample to prevent scour at the toe of the spillway (Campbell, 1993). No mention was made as to the depth of tailwater at the higher overtopping heads.
Flowrates tested ranged from approximately 15cfs/ft to 140cfs/ft at the scale ratios specified. There are several reasons that higher flowrates were not tested in the models. First, since many design applications rarely state unit flowrates much higher than 125 cfs/ft, it was not necessary to test higher. Second, at the higher flows, large waves appear on the surface. These waves travel side to side across the spillway, affecting depth measurements. Thirdly, at the high flows, the flow is still accelerating at the base of the spillway. In our measurements it was very important to have achieved a uniform flow with a uniform velocity. If we were to work in a region of nonuniform flow, our velocities at the toe of the structure would not be representative of the velocities encountered on a higher model.

As supporting evidence that there is little difference in dissipation between the two slopes and step sizes tested are the graphs in Figures 34 and 35. As can be seen, the relationship between the depth downstream of the hydraulic jump and model flowrate is almost perfectly curvilinear. These data have not been modified to match a curve of any type and are exact measured data. A smaller energy loss would result in more energy at the toe of the spillway, which would lead to a greater conjugate depth downstream to contain the hydraulic jump. Since the conjugate depth is almost the same at each flowrate for all four models, the energy dissipation must be very near equal on all four.

The toe velocities also support the idea of similar dissipation between the models. Toe velocities although slightly more scattered in this graph also agree with the curvilinear relationship seen in Figure 35. The scatter can be attributed to the difficulty in getting accurate measurements in the highly supercritical flow at the toe of the spillway.

Design Procedure for Stepped Spillways

There are several critical design factors in designing a stepped face. These are the crest of the spillway, the step size for the steps on the constant slope region, and the overall slope of the structure.

The crest is extremely important in getting the flow to pass over the spillway and onto the stepped face without causing it to leap away from the structure. If the flow leaps away near the crest, a
Test Data - Toe Velocity on Model

Figure 34. Test data - toe velocities on model.
Figure 35. Test data - depths downstream of hydraulic jump.
leap frog pattern will develop, which is aesthetically not acceptable for good design. Commonly, the crest is designed in the shape of the underside of a free overflow nappe. There are several design procedures that outline this shape. The procedure used in this study was researched by the USBR and is presented in pages 42 to 47. Given a design discharge and different approach conditions, the nappe-shaped profile can be designed using either the crest equation given by the USBR or the compound curve chart in Figure 26.

The flow over the spillway must be introduced gradually to the steps on the face of the structure. Small steps should be placed starting just beyond the crest of the spillway. There is no set size for these steps although a starting step size of .5 foot is usually adequate. The step size will generally increase in size down the crest profile until the size of the main steps on the face is achieved. Each step is designed so that the tip of the step just intersects the profile of the nappe shape. The model crest shown in Figure 5 is a good example of this. The crest transition region will continue until the desired spillway slope is reached. At this point, the slope is no longer changing and the constant size step region begins. At this point, the step heights are no longer changing and are constant.

Nowhere in the literature are there any instances where the crest to the stepped spillway is gated. Water from below a gate would have a fairly high velocity and there is little chance of getting that type of flow to turn down the face of the spillway without leaping away. The effect of gates on a stepped structure has not been tested and the performance of the spillway with gates is not known. More research would have to be completed to determine the effect of gates on the flow down the stepped face.

The steps in the constant slope region tend to be 1 to 2 feet in height. The tread length varies depending on the slope of the spillway. On the De Mist Kraal Spillway the step height was 3 feet, which worked satisfactorily in their model study. If steps are too small, they are unable to affect the flow as much and a smaller energy loss would be expected. If the steps are too large, the flow will leap
away from the structure, missing several steps. Energy dissipation has not been studied in this situation and it is unknown if the spillway is still operating like one that does not have the leaping flow. Commonly the design discharge for stepped spillways is around 100 cfs/ft. At this flow, the step sizes that perform satisfactorily are 1 to 2 feet in height. This step height will allow for adequate dissipation at all flows up to about 130 cfs/ft. Low flows will not be forced to leap away at this step height and about 95% of the energy will be dissipated. At the high flows, the steps will still be dissipating about 65% of the energy in the flow.

The slope of the spillway is totally dependent on the downstream slope of the dam. The slope will affect the dissipation slightly with the greater amount of dissipation occurring on the flatter slopes. On the slopes tested in the study, the 0.7:1.0V slope dissipated slightly more than the 0.5H:1.0V slope. Flatter slopes in the 0.8H:1.0V to 1.0H1.0V range should dissipate more energy although these were not studied in this research.

The stepped face commonly extends all the way to the toe of the structure where the stilling basin is placed. With the stepped spillway, there is no need to transition the slope from the steep slope to the stilling basin floor as found in some smooth spillway applications. The steps should run continuously to the floor of the stilling basin.

**Design Procedure for Stepped vs. Smooth Spillway Basins**

As explained previously in the last section, the USBR has developed a design procedure for designing a standard USBR Type II stilling basin given a unit flowrate and spillway height. Entrance velocities are computed for a smooth spillway from a USBR chart and used to determine the stilling basin parameters like length, depth, and chute block size. A good comparison would be to design a stilling basin for both a smooth spillway and a stepped spillway to determine the amount that the stepped face will reduce the size of the stilling basin necessary to contain the hydraulic jump. Since model toe velocity data were available from our tests, it was easy to use Froude similitude to scale the
data to prototype velocities and design a Type II basin for the stepped face. This stilling basin can then be compared to a basin designed for a smooth spillway face.

A design procedure incorporating a spreadsheet was developed for designing a USBR Type II stilling basin for both a smooth spillway and also a stepped spillway. This spreadsheet design for a slope of 0.7H:1.0V can be seen in Table 6. As noted earlier, there was little difference between the different slopes and therefore the design could be combined for all steep slopes. The difference, although slight, was calculated and showed small differences in the stilling basin parameters. Although this is not a significant change, the slopes are kept separate in the analysis strictly for design purposes.

The only input parameters needed are the spillway height (Hs) and the design unit flowrate (qd). The spillway width is entered to calculate the volume of the stilling basin in the comparison table. The maximum transition height (Th) is dependent on the scale ratio. The model tested in the flume had a height of approximately 3 feet. On all of the tests, uniform flow was achieved prior to the flow approaching the toe of the spillway. The uniform flow is not accelerating and is not changing with distance. As can be seen in the spreadsheet, Th is equal to 3 times the scale ratio. This means that the flow on a spillway with height less than the maximum transition height could still be in the nonuniform region and still accelerating. If the actual spillway height was less than the maximum transition height, velocities on the stepped spillway would be overpredicted. The design procedure could still be used with the understanding that the results would be fairly conservative.

The second box in Table 6 computes the necessary parameters for a standard USBR Type II stilling basin with a smooth spillway. The spreadsheet uses the USBR graphs shown in Figures 30 and 31 to determine the values Va/Vt and L/D2. An easier approach to the design process is to create regression equations of the data in the charts and use lookup tables in the spreadsheet to calculate the values. Regression equations were computed for the USBR graph in Figure 30. These equations were
Table 6. Spreadsheet for comparing stilling basins for smooth to stepped spillways
Slope = 0.7H:1.0V

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
<th>Units</th>
<th>Source/Equations/Notes</th>
</tr>
</thead>
<tbody>
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<td>Design Flow</td>
<td>( q_d = 100 )</td>
<td>cfs/ft</td>
<td>Input</td>
</tr>
<tr>
<td>Spillway Height</td>
<td>( H_s = 100 )</td>
<td>ft</td>
<td>Input</td>
</tr>
<tr>
<td>Spillway Width</td>
<td>( W = 100.0 )</td>
<td>ft</td>
<td>Input</td>
</tr>
<tr>
<td>Max. Transition Height</td>
<td>( Th = 67.2 )</td>
<td>ft</td>
<td>( Th = 3 \times SR )</td>
</tr>
</tbody>
</table>

### Smooth Spillway Calculations

<table>
<thead>
<tr>
<th>Calculated Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Depth</td>
<td>( Y_c = 6.77 )</td>
<td>ft</td>
</tr>
<tr>
<td>Critical Velocity</td>
<td>( V_c = 14.77 )</td>
<td>ft/s</td>
</tr>
<tr>
<td>Approx. Crest Head</td>
<td>( H = 10.16 )</td>
<td>ft</td>
</tr>
<tr>
<td>Theoretical Toe Velocity</td>
<td>( V_t = 82.26 )</td>
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</tr>
<tr>
<td>Va/Vt</td>
<td>0.884</td>
<td></td>
</tr>
<tr>
<td>Actual Toe Velocity</td>
<td>( V_a = 72.69 )</td>
<td>ft/s</td>
</tr>
<tr>
<td>Depth at Toe</td>
<td>( D_1 = 1.38 )</td>
<td>ft</td>
</tr>
<tr>
<td>Froude Number at Toe</td>
<td>( Fr_1 = 10.92 )</td>
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<tr>
<td>D2/D1</td>
<td>14.95</td>
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<tr>
<td>Conjugate Depth</td>
<td>( D_2 = 20.57 )</td>
<td>ft</td>
</tr>
<tr>
<td>L/D2</td>
<td>4.33</td>
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</tr>
<tr>
<td>Stilling Basin Length</td>
<td>( L = 88.98 )</td>
<td>ft</td>
</tr>
<tr>
<td>End Sill Height</td>
<td>( h_2 = 4.11 )</td>
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</tr>
<tr>
<td>Chute Block Height</td>
<td>( h_1 = 1.38 )</td>
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### Stepped Spillway Calculations

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<td>( SR = 22.4:1 )</td>
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<td>Model Unit Flowrate</td>
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<td>cfs/ft</td>
</tr>
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<td>Model Flowrate</td>
<td>( Q_m = 2.869 )</td>
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<tr>
<td>Measured Toe Velocity</td>
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<td>ft/s</td>
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<tr>
<td>Prototype Toe Velocity</td>
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<td>ft/s</td>
</tr>
<tr>
<td>Depth at Toe</td>
<td>( D_1 = 2.05 )</td>
<td>ft</td>
</tr>
<tr>
<td>Froude Number at Toe</td>
<td>( Fr_1 = 6.02 )</td>
<td></td>
</tr>
<tr>
<td>D2/D1</td>
<td>8.03</td>
<td></td>
</tr>
<tr>
<td>Conjugate Depth</td>
<td>( D_2 = 16.43 )</td>
<td>ft</td>
</tr>
<tr>
<td>L/D2</td>
<td>4.02</td>
<td></td>
</tr>
<tr>
<td>Stilling Basin Length</td>
<td>( L = 66.06 )</td>
<td>ft</td>
</tr>
<tr>
<td>End Sill Height</td>
<td>( h_2 = 3.29 )</td>
<td>ft</td>
</tr>
<tr>
<td>Chute Block Height</td>
<td>( h_1 = 2.05 )</td>
<td>ft</td>
</tr>
</tbody>
</table>
entered into the lookup table and vary with the input spillway height. This procedure calculates required tailwater depth, stilling basin length, end sill height, and chute block height.

The third box in Table 6 computes the necessary parameters for a USBR Type II stilling basin with a stepped spillway. The scale ratio of 22.4:1 for the 0.7H:1.0V slope was computed to model a 1-to 2-foot step which is a standard sized step for most applications. Modifying the scale ratio will affect the step height, maximum transition height, and flowrates tested, according to Froude Similitude.

The spreadsheet uses the design unit flowrate (qd) and computes the model unit flowrate using the scale ratio. This model unit flowrate (qd) is then converted to a model flowrate (Qm) which is used to estimate the velocity at the model toe using either Figure 36 or a regression equation. Figure 36 is a graph of all the velocity data on the 0.7H:1.0V slope over the flowrates tested in the model. A regression equation was computed for the data and is shown on the graph and spreadsheet. The model velocity is scaled to the prototype toe velocity using the scale ratio and Froude Similitude. Once a prototype toe velocity is computed, the design process can follow the same as the smooth basin stilling basin design. Like the smooth basin, computed are conjugate depth, stilling basin length, end sill height, and chute block height.

The results from Box 2 and Box 3 can then be compared to see the reduction in stilling basin length, conjugate depth, toe velocity, and stilling basin volume. Stilling basin size reduction can be very important on a large-scale project. A small reduction in size could be an enormous cost savings in the construction process. The comparison table for the 0.7H:1.0V slope spreadsheet can be seen in Table 7. Table 8 gives prototype ranges over which the model was tested.

A second spreadsheet was constructed which used data for designing a stepped spillway on a 0.5H:1.0V slope. This spreadsheet and comparison table can be found in Tables 9 and 10. Changes to this spreadsheet included a different scale ratio to design for steps 1 to 2 feet high, and a different model velocity equation based on the velocities from the 0.5H:1.0V slope (Figure 37).
Figure 36. Test data - toe velocity vs. flowrate for 0.7H:1.0V slope.

\[ \text{Vel} = -0.525 \cdot Q_m^2 + 3.834 \cdot Q_m + 3.643 \]
Table 7. Comparison table for 0.7H:1.0V slope

<table>
<thead>
<tr>
<th></th>
<th>Smooth Basin Length</th>
<th>Stepped Basin Length</th>
<th>% Reduction of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth Basin Length</td>
<td>88.98 (ft)</td>
<td>66.06 (ft)</td>
<td>25.75%</td>
</tr>
<tr>
<td>Stepped Basin Length</td>
<td>66.06 (ft)</td>
<td>53.33 (ft)</td>
<td>25.75%</td>
</tr>
<tr>
<td>% Reduction of Length</td>
<td>25.75%</td>
<td>25.75%</td>
<td></td>
</tr>
<tr>
<td>Smooth Basin D2</td>
<td>20.57 (ft)</td>
<td>16.43 (ft)</td>
<td>20.15%</td>
</tr>
<tr>
<td>Stepped Basin D2</td>
<td>16.43 (ft)</td>
<td>13.27 (ft)</td>
<td>20.15%</td>
</tr>
<tr>
<td>% Reduction of D2</td>
<td>20.15%</td>
<td>20.15%</td>
<td></td>
</tr>
<tr>
<td>Smooth Basin Vt</td>
<td>72.69 (ft/s)</td>
<td>54.81 (ft/s)</td>
<td>32.78%</td>
</tr>
<tr>
<td>Stepped Basin Vt</td>
<td>54.81 (ft/s)</td>
<td>42.57 (ft/s)</td>
<td>32.78%</td>
</tr>
<tr>
<td>% Reduction of Vt</td>
<td>32.78%</td>
<td>32.78%</td>
<td></td>
</tr>
<tr>
<td>Smooth Basin Volume</td>
<td>183,029 (ft^3)</td>
<td>108,519 (ft^3)</td>
<td>40.71%</td>
</tr>
<tr>
<td>Stepped Basin Volume</td>
<td>108,519 (ft^3)</td>
<td>65,839 (ft^3)</td>
<td>40.71%</td>
</tr>
<tr>
<td>% Reduction of V</td>
<td>40.71%</td>
<td>40.71%</td>
<td></td>
</tr>
</tbody>
</table>

Table 8. Limits of model testing on 0.7H:1.0V slope

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step Sizes H (prototype)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min H</td>
<td>1.00</td>
<td>(ft)</td>
<td>Min H = .0446 * SR</td>
</tr>
<tr>
<td>Max H</td>
<td>2.00</td>
<td>(ft)</td>
<td>Max H = .0893 * SR</td>
</tr>
<tr>
<td>Flow Rates q (prototype)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min q</td>
<td>4.38</td>
<td>(cfs/ft)</td>
<td>Min q = .0413 * SR * 1.5</td>
</tr>
<tr>
<td>Max q</td>
<td>88.73</td>
<td>(cfs/ft)</td>
<td>Max q = .8370 * SR * 1.5</td>
</tr>
</tbody>
</table>
Table 9. Spreadsheet for comparing stilling basins for smooth to stepped spillways
Slope = 0.5H:1.0V

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Value</th>
<th>Units</th>
<th>Source/Equations/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Flow</td>
<td>qd</td>
<td>100</td>
<td>(cfs/ft)</td>
</tr>
<tr>
<td>Spillway Height</td>
<td>Hs</td>
<td>100</td>
<td>(ft)</td>
</tr>
<tr>
<td>Spillway Width</td>
<td>W</td>
<td>100.0</td>
<td>(ft)</td>
</tr>
<tr>
<td>Max. Transition Height</td>
<td>Th</td>
<td>96.0</td>
<td>(ft)</td>
</tr>
</tbody>
</table>

Smooth Spillway Calculations

<table>
<thead>
<tr>
<th>Calculated Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Depth</td>
<td>Yc</td>
<td>6.77</td>
</tr>
<tr>
<td>Critical Velocity</td>
<td>Vc</td>
<td>14.77</td>
</tr>
<tr>
<td>Approx. Crest Head</td>
<td>H</td>
<td>10.16</td>
</tr>
<tr>
<td>Theoretical Toe Velocity</td>
<td>Vt</td>
<td>82.26</td>
</tr>
<tr>
<td>Actual Toe Velocity</td>
<td>Va/Vt</td>
<td>0.884</td>
</tr>
<tr>
<td>Depth at Toe</td>
<td>D1</td>
<td>1.38</td>
</tr>
<tr>
<td>Froude Number at Toe</td>
<td>Fr1</td>
<td>10.92</td>
</tr>
<tr>
<td>Conjugate Depth</td>
<td>D2</td>
<td>20.57</td>
</tr>
<tr>
<td>L/D2</td>
<td>4.33</td>
<td></td>
</tr>
<tr>
<td>Stilling Basin Length</td>
<td>L</td>
<td>88.98</td>
</tr>
<tr>
<td>End Sill Height</td>
<td>h2</td>
<td>4.11</td>
</tr>
<tr>
<td>Chute Block Height</td>
<td>h1</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Stepped Spillway Calculations

<table>
<thead>
<tr>
<th>Calculated Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Scale Ratio</td>
<td>SR</td>
<td>32.0:1</td>
</tr>
<tr>
<td>Model Unit Flowrate</td>
<td>qm</td>
<td>0.552</td>
</tr>
<tr>
<td>Model Flowrate</td>
<td>Qm</td>
<td>1.680</td>
</tr>
<tr>
<td>Measured Toe Velocity</td>
<td>Vtm</td>
<td>9.02</td>
</tr>
<tr>
<td>Prototype Toe Velocity</td>
<td>Vtp</td>
<td>51.04</td>
</tr>
<tr>
<td>Depth at Toe</td>
<td>D1</td>
<td>1.96</td>
</tr>
<tr>
<td>Froude Number at</td>
<td>Fr1</td>
<td>6.43</td>
</tr>
<tr>
<td>D2/D1</td>
<td>8.60</td>
<td></td>
</tr>
<tr>
<td>Conjugate Depth</td>
<td>D2</td>
<td>16.85</td>
</tr>
<tr>
<td>L/D2</td>
<td>4.08</td>
<td></td>
</tr>
<tr>
<td>Stilling Basin Length</td>
<td>L</td>
<td>68.74</td>
</tr>
<tr>
<td>End Sill Height</td>
<td>h2</td>
<td>3.37</td>
</tr>
<tr>
<td>Chute Block Height</td>
<td>h1</td>
<td>1.96</td>
</tr>
</tbody>
</table>
### Table 10. Comparison table for 0.5H:1.0V slope

<table>
<thead>
<tr>
<th></th>
<th>Smooth Basin Length</th>
<th>Stepped Basin Length</th>
<th>% Reduction of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth Basin D2</td>
<td>20.57 (ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stepped Basin D2</td>
<td>16.85 (ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Reduction of D2</td>
<td>18.08%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Smooth Basin Vt</th>
<th>Stepped Basin Vt</th>
<th>% Reduction Vt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth Basin Volume</td>
<td>72.69 (ft/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stepped Basin Volume</td>
<td>51.04 (ft/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Reduction Vt</td>
<td>29.78%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Smooth Basin Volume</th>
<th>Stepped Basin Volume</th>
<th>% Reduction V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth Basin Volume</td>
<td>183,029 (ft³)</td>
<td></td>
<td>36.71%</td>
</tr>
<tr>
<td>Stepped Basin Volume</td>
<td>115,838 (ft³)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 11. Limits of model testing on 0.5H:1.0V slope

<table>
<thead>
<tr>
<th>Step Sizes H (prototype)</th>
<th>Min H</th>
<th>Max H</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Unit</td>
<td>Source</td>
<td></td>
</tr>
<tr>
<td>Min H</td>
<td>1.00</td>
<td>(ft)</td>
<td>Min H = .0313 * SR</td>
</tr>
<tr>
<td>Max H</td>
<td>2.00</td>
<td>(ft)</td>
<td>Max H = .0625 * SR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flow Rates q (prototype)</th>
<th>Min q</th>
<th>Max q</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Unit</td>
<td>Source</td>
<td></td>
</tr>
<tr>
<td>Min q</td>
<td>6.52</td>
<td>(cfs/ft)</td>
<td>Min q = .0360 * SR * 1.5</td>
</tr>
<tr>
<td>Max q</td>
<td>140.23</td>
<td>(cfs/ft)</td>
<td>Max q = .7747 * SR * 1.5</td>
</tr>
</tbody>
</table>
Figure 37. Test data - toe velocity vs. flowrate for 0.5:1.0V slope.

\[ \text{Vel} = -1.121 \text{Qm}^2 + 5.201 \text{Qm} + 3.448 \]
A note on the scale ratios is in order. Both scale ratios seen in the spreadsheet designs were computed to model steps in the 1- to 2-foot range. The scale ratio can be increased to model very large steps if necessary, but it should be understood that the tested flowrate range will also increase. With much larger steps, the stepped face will operate satisfactorily at the very high flows, but excessive leaping and nappe flow will be experienced at the lower flows. If the leaping and nappe action are not a problem, then larger steps can be used. Table 11 calculates the range of the test data and the limits of which the model operated satisfactorily.

As can be seen from Tables 7 and 10, the reduction in values like toe velocity, stilling basin length, and stilling basin volume is very substantial. As a visual example, multiple flowrates were run through both spreadsheets with a spillway height of 100 feet. The values in the comparison tables were then graphed. Figures 38, 39, and 40 show the results of this test. Toe velocity decreased by almost 54% at the low flows, and 33% at the higher flows. The stilling basin length was shortened by 42% to 28%, respectively, and the stilling basin volume was reduced by 62% to 43%. As can be seen, the two stepped faces are very close in comparison on all three graphs. This supports the claim earlier that the slopes and step sizes are not as critical a factor as previously thought.

**Data Comparison to Other Researcher Methods**

From the literature review it was seen that there have been other studies done to determine the energy dissipation on the stepped face. The following section will take the data measured from our models and apply them to other researchers' methods.

**Rajaratnam**

N. Rajaratnam was interested in studying the shear stress between the skimming flow and the stepped interface. He proposed a friction coefficient $c_f$ and a way to estimate the energy loss if this
Figure 38. Unit discharge vs. computed toe velocity for spreadsheet design.
Figure 39. Unit discharge vs. computed stilling basin length for spreadsheet design.
**Figure 40.** Unit discharge vs. computed stilling basin volume for spreadsheet design.
value was known. The values for $c_f$ on our models ranged from 0.042 to 0.048. When our data were entered into the energy loss equation given in his paper, the graph in Figure 41 was obtained. The data look good on the lower end of the unit discharge scale, but the curve seems to flatten at the higher flowrates. As was seen in our evaluation, the energy loss should continue to decrease at the higher flows to a point where the spillway is operating much like a smooth spillway. From our data, Rajaratnam's theory would apply only at smaller flows.

**Stephenson**

Stephenson attempted to create an Energy Loss Ratio for flow down a long flight of steps. Assuming the flow reached uniform depth, he formulated the equations found in the Literature Review based on a turbulent rough boundary layer equation to calculate the Darcy friction coefficient of the steps. The equations he used are difficult to follow and the source of them is not revealed. Users of this method should be careful.

Our data were used to calculate the Energy Loss Ratio proposed by Stephenson. As can be seen in Figure 42, there is a very close resemblance to what the theory has predicted and what was actually measured in the lab. This supports Stephenson's findings but should be used cautiously.

**Diez-Cascon et al.**

Diez-Cascon and researchers attempted to combine the continuity equation and the momentum equation to determine the depth at the toe of the stepped spillway. In their evaluation process they used an equation from an ASCE paper in 1961 that predicted air concentrations in a supercritical flow.

When the data from our model are analyzed using the theory by Diez-Cascon, the theory well underpredicts the depth at the toe of the spillway. As seen in Figure 43, the measured depth is commonly one third that predicted by the theory. This should not totally discount this theory. Other researchers have noted that depths and aeration are not correctly modeled in small laboratory models.
Figure 41. Test data according to Rajaratnam theory.
Figure 42. Test data - energy loss according to Stephenson theory.
Figure 43. Test data - depth at toe according to Diez-Cascon theory.
Phillip Burgi from the USBR in Denver mentioned that in similar tests they had done, the actual depths measured in the prototype were commonly 1.5 times or greater than that predicted by the model. If this is so, the theory is not as far off as the graph in Figure 43 shows.

Christodoulou

George C. Christodoulou proposed, with model studies and other researchers data on stepped spillways, that a relationship could be made between the energy loss and a combination of critical depth, step height, and number of steps. His first graph, which compared energy loss to critical depth over step height, showed much scatter as seen previously in the literature review. Much the same can be seen with the results from our data shown in Figure 44. One interesting aspect of this analysis shows a greater amount of energy dissipation on the smaller steps. This is contrary to what would be expected.

When Christodoulou divided each data set by the number of steps on the face of the structure, the data in his analysis fit a very nice curvilinear relationship. When this same idea was applied to our data, the graph in Figure 45 was produced. The data are nicely grouped and show a nice downward trend of energy loss at increasing flowrates. The data closely match that found by Christodoulou using data from Sorensen's study. Our data did not extend as far as the data by Christodoulou, so we are unable to say if the same pattern as seen in Figure 22 of the Literature Review will occur. The data by Christodoulou are obviously in the nonuniform region, an area that we did not study. Further research should be done to examine if this pattern is correct.
Figure 44. Test data - data according to Christodoulou theory.
Data According to Christodoulou Theory

Figure 45. Test data - data according to Christodoulou theory.
CONCLUSIONS AND RECOMMENDATIONS

A stepped spillway is a good way to reduce energy on the face of a high dam. This reduction of energy can lead to lower velocities on the face of the structure and a smaller stilling basin required to contain the hydraulic jump. The smaller stilling basin results in a considerable cost savings over the stilling basin constructed for a smooth spillway. The cavitation potential for the steps is greatly reduced with the reduction of velocities on the stepped faces. In addition, the steps create a highly aerated and turbulent flow for which there is little or no cavitation damage to the steps.

The literature on stepped spillways is comprised of model studies and theoretical papers on flow over stepped spillways. The model studies are very site specific and many of the theoretical ideas have not been tested using models or prototype data. There is little information for a designer to design a steep stepped spillway and stilling basin. There is considerable information on stepped spillways for flat embankment-type installations, but this material is not beneficial in the design of steep-sloped, stepped spillways. The literature review has brought together all of this information and condensed it for background knowledge on the subject.

Laboratory tests were conducted to determine the hydraulic performance of the stepped spillway. Tests were done on two steep slopes of 0.7H:1.0V and 0.5H:1.0V. On each of these models, two different step sizes were tested for a total of four models. Results from these tests can be seen in Tables 2-5. The range of data covers a ratio of step height over critical depth of 0.120 < ht/yc < 1.897, and a unit flowrate range of q = 0.036 cfs/ft to q = 0.826 cfs/ft. The models were given scale ratios to model step sizes of 1 to 2 feet and hydraulic heights of over 60 feet. Over these parameters, the models operated successfully, dissipating a large amount of the available energy.

Tests showed that the difference between the performance of all four models was very slight. All four models showed energy losses ranging from 95% at low flows to about 65% at the higher flows as seen in Figure 33. Figures 34 and 35 show the toe velocity and conjugate depths associated with the different models.
Design guidelines are presented in designing a steep-sloped, stepped spillway. The crest of the spillway is commonly built in the shape of the underside of a free overflow jet. Small steps are fitted to this profile so that the tips of the steps just intersect the computed nappe-shaped profile. These small steps begin just beyond the crest and increase in size until the desired step size for the constant slope region is attained. If these steps are too large, they can cause the flow to hit the tread of the step and leap away from the spillway in a leap frog pattern. If the steps in this transition region are too small, they can cause the leap frog pattern to move further down the face of the spillway in the region of the constant size steps. A good starting height for an average unit flow of 20cfs/ft to >120cfs/ft is about 0.5 foot high with increasing height down the crest transition.

Nowhere in the literature is any mention of a gated, stepped spillway application. Flow from under the gate would be highly supercritical and it is the author's belief that it would be impossible to get the flow to remain attached to the face of the spillway at the crest. This has not been studied in this research but could be studied if further research was completed.

For most applications where the design unit flowrate is not greater than 140 cfs/ft, a step size of 1 to 2 feet is adequate. Larger step sizes may be used but may induce leaping at the lower flowrates. Steps in the 1- to 2- foot range are easier to construct since the layers of concrete are placed in 1-foot increments. In the model testing, there was very little difference in energy dissipation between the different size steps. More research needs to be completed to study both larger and smaller step sizes to determine the energy dissipation characteristics of the different step sizes.

Constant size steps of 1 to 2 feet are commonly placed on the face of the stepped spillway and extend from the transition area beyond the crest all the way to the base of the spillway at the stilling basin. A transition from the steps to the stilling basin is not necessary. In some instances where the material at the base of the spillway is very resistant and the tailwater is adequate, the need for a stilling basin is totally removed.
The slope of the spillway is totally dependent on the downstream slope of the dam. The slope will affect the dissipation slightly with the greater amount of dissipation occurring on the flatter slopes. On the slopes tested in the study, the 0.7:1.0V slope dissipated slightly more than the 0.5H:1.0V slope. Flatter slopes in the 0.8H:1.0V to 1.0H1.0V range should dissipate more energy although these were not studied in this research.

For most high dam situations, the USBR recommends using a Type II stilling basin. Type II basins are used where high velocity and high discharge flows are encountered and the Froude number is greater than 5. The Type II stilling basin is a hydraulic jump energy dissipator that causes a hydraulic jump to form at the base of the spillway. This hydraulic jump dissipates much of the kinetic energy in the flow and makes the flow safe to enter the downstream channel. For smooth spillways, the Type II stilling basin can be very large and costly to construct. With reduced velocities on the stepped spillway, the Type II stilling basin for a stepped spillway will be considerably smaller at a considerable cost savings.

To evaluate the performance of a stepped spillway compared to that of a smooth spillway, a design procedure adapted from the USBR was used. Based on velocities at the toe of both a smooth spillway and a stepped spillway, Type II stilling basin dimensions were calculated in a spreadsheet application. The dimensions of both stilling basins are then compared in tables to determine the reduction in size of the stilling basin using the stepped concept. This procedure showed a reduction of 42% to 28% in the stilling basin length and 62% to 43% in the total stilling basin volume over a range of unit flowrates from 15cfs/ft < q < 140 cfs/ft. This reduction in size will greatly reduce the material and construction costs in the prototype.

Test data were also applied to theoretical procedures proposed by other researchers in papers explained in the literature review. The results from these comparisons can be found on pages 86 to 96. None of the procedures matched exactly the data measured on the models. The methods proposed by the researchers should be used with caution.
Although the model data can be used to design a stepped face and stilling basin, it is still highly recommended that any large spillway project be modeled in a laboratory to insure adequate performance of both the steps and the stilling basin. Models tested in the laboratory were fairly small and scale effects could be present. In addition, since Froude similitude was used, there are possibilities that viscous effects could be greater than anticipated by the model. Reynolds modeling can be used to study viscous effects, but was not studied in this research. The data contained in this thesis can be used as a guide in determining the initial design sent to the laboratory.

There is still much to be learned from stepped spillways. The study only tested two step sizes that are common to most applications and found that there was little difference between the performance of these steps. Additional research must be done to determine the limit at which the step size either increases or decreases the energy dissipation. Larger models should be built to limit the viscous effects sometimes encountered on smaller models.

Additionally, more research could be done on slopes flatter than those tested to determine the point where the slope of the spillway does become a large factor in the energy dissipation. The study found that on the two slopes tested, there were only slight differences in the performance of the different sloped faces. The flatter slope of those tested showed a slightly greater energy dissipation potential. Testing a whole range of slopes could form a family of energy loss curves for a variety of stepped spillways.

A better way of measuring depths in the supercritical region would be very helpful for additional research. An aid that was not used in this study but has been used at the USBR facilities in Denver is a probability probe. The probability probe calculates the depth at which the probe is contacting the water 50% of the time. This depth is an accurate estimate of the true depth. This would be very helpful in the areas where highly supercritical and turbulent flow make depth measurements very difficult.
REFERENCES


Campbell, David B. 1993. Vice President of O'Brien and Gere Engineers. Personal letter, June 23.


Houston, Kathleen L. 1987a. Hydraulic model studies of Upper Stillwater Dam stepped spillway and outlet works. REC-ERC-87-6, USBR, Denver, Colorado.


APPENDIX
Model A - Slope 0.7:1.0 - Steps .75" x 1.07"

Model B - Slope 0.7:1.0 - Steps .375" x .536"
Model C - Slope 0.5:1.0 - Steps .375" x .75"  

Model D - Slope 0.5:1.0 - Steps .188" x .375"
Model A - aeration starts around Step #20

Model A - increased flow - aeration almost at toe of spillway
Model showing crest design with small steps near the crest.

Model operating at low flowrate - notice the flow rotation in the step tread.