# An Analytical Approach to Modeling Spacecraft Potentials in SPICE

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#### ABSTRACT

As the use of Langmuir probes on small satellites increases, so does the need to understand how spacecraft charging affects Langmuir Probe measurements. This paper explores the important relationship between probe and spacecraft surface area, and how it relates to the floating potential (or charge) on the spacecraft. Further, this paper discusses how a charged body's interaction with the ionosphere can be modeled using custom circuit elements designed in SPICE. The SPICE models introduced are demonstrated with the SPORT spacecraft, a 6U mission that makes Langmuir probe measurements in concert with two electric field probes used to account for the floating potential of the spacecraft.

## **INTRODUCTION**

The Langmuir probe is a technique used to measure parameters of the Earth's ionosphere such as electron,  $n_e$ , or ion,  $n_i$ , density, electron temperature, or charging of the spacecraft relative to the surrounding ionospheric plasma. These parameters are measured by applying a potential to the Langmuir probe sensor (typically a conductive sphere or cylinder),  $V_p$ , relative to the space environment. The probe will collect a current from the ionosphere depending on the applied potential. By sweeping the probe potential a current-voltage relationship is formed. Figure 1 presents a representation of the I-V relationship for a Langmuir probe or more generally a conductor held at a voltage potential relative to the surrounding ionospheric plasma.

The parameters of the ionospheric plasma, including ion density, electron density, and temperature, and the charging of the spacecraft are determined by fitting these



Figure 1: Typical I-V curve for a Langmuir Probe

free parameters to the acquired I-V curve using analytic equations for current collection. Current collection equations for different shaped bodies that are both stationary and moving at orbital velocities have been developed by Langmuir and Blot, and Hoegy and Warton.<sup>1,2</sup> In this process, it is essential to understand the difference between  $V_p$ , the probe voltage as referenced to the distant space plasma environment, and the voltage bias,  $V_b$ , the voltage applied to the sensor by the measurement instrumentation. The bias voltage is necessarily referenced to the ground of the spacecraft power system which must be tied to the spacecraft structure. The current collected from the ionosphere by a Langmuir probe flows through the instrument to the ground of the power system and is then returned to the ionosphere via the surface of the spacecraft. If the surface area of the spacecraft is ~1000 times larger than then probe's surface, as is typical for large spacecraft, then  $V_p \approx V_b$ .

Due to the limited surface area of CubeSats, a positive potential on a Langmuir probe can result in more current from electrons than can be returned through the surface of a spacecraft by ion collection. As a result, the whole spacecraft charges negative relative to the space environment. This is physically required to maintain a continuity between the current collected on the probe and the current returned by the spacecraft surface. Electrons have higher mobility than ions resulting in spacecraft typically charging a few volts negative relative to the surrounding environment. The charging of the spacecraft can be observed in a Langmuir I-V curve as the difference between the floating potential, where electron and ion currents balance and the plasma potential, or the



Figure 2: LTspice graphical circuits elements for modeling the collection of currents from a plasma

potential at which electrons change from being attracted to being repelled as illustrated in Figure 2.

Understanding how the probe/spacecraft surface area ratios affect Langmuir probe operation on CubeSats has been the subject of multiple studies.<sup>4,5,6</sup> In this work we develop an electrical lumped element model of a spacecraft and solve it numerically using a well-known electronics simulation language, SPICE.<sup>7</sup> The models are used to understand the charging and electrical coupling effects of the Langmuir probe as it interacts with the spacecraft body and other electrical probes operating on the same CubeSat. Of concern is the potential for various instruments to charge the shared spacecraft body relative to the surrounding ionosphere resulting in conflicting interference in instrument readings. This was a specific concern for instruments from Utah State University (USU) and The University of Texas at Dallas (UTD) flying on the SPORT mission.

## SPICE MODEL OF CURRENT COLLECTION

SPICE ("Simulation Program with Integrated Circuit Emphasis") is a general-purpose analog electronic circuit simulator. It is a program used in integrated circuit and board-level design to predict circuit behavior and verify circuit designs.<sup>7</sup> USU previously used SPICE models for sounding rocket probes in ionospheric plasma<sup>3</sup> and has revisited this approach for the SPORT CubeSat mission.<sup>3</sup> These models consist of SPICE macro circuits implemented using voltage controlled current sources. We have encapsulated these models into LTspice's schematic capture front end as graphical elements, just like a standard electronic part. Figure 2 shows circuit models of devices encapsulated in LTspice. Shown, are models used to describe both spheres and cylinders as they interact with various conditions within the ionosphere. The device is used by applying a voltage potential between the ionosphere (top node) and the collecting surface (bottom node) and measuring the current through the device. The symbol used is similar to that of a diode

## **OML** Collection Currents

The current a charged body will collect from the ionosphere is dependent on the geometrical properties of the charged body as well as the properties of the ionosphere. The equations used to model the voltage controlled current relationship of a charged probe immersed in the ionosphere can be found in Langmuir and Blot for stationary objects, and Hoegy and Warton's work for objects at orbital velocities.<sup>1,2</sup> Presented here, are just the equations used to model the current collected by stationary cylinders and spheres.

The current, *I*, collected by a body is the sum of the electrons and all ion types in the plasma. The current for each species, *j*, depends upon the saturation current for that species,  $J_{\text{sat}j}$  as well as the probe area,  $A_p$ , which is then scaled by a factor,  $F(\Phi_{sj})$ , that accounts for the shape of the probe and the thickness of the plasma sheath around the object as well as the potential applied to the object,  $\Phi_{sj}$  as given by equation (**3**) the plasma with each species density,  $n_j$ , charge,  $q_j$ , mass ,  $m_j$ , and temperature,  $T_j$ , is defined as:

$$I = \sum_{j} J_{\text{sat}_{j}} A_{p} F(\Phi_{s_{j}}).$$
(1)

Where

$$J_{\rm sat} \equiv q n_j \sqrt{\frac{k_b T_j}{2\pi m_j}} \tag{2}$$

$$\Phi_{sj} \equiv \frac{-q_j \phi_{sj}}{k_b T_j} \tag{3}$$

$$\varepsilon \equiv \frac{s}{r_p} \tag{4}$$

$$\widetilde{\Phi}_{s_j} \equiv \sqrt{\frac{\Phi_{s_j}}{\varepsilon^2 - 1}} \tag{5}$$

$$F\left(\Phi_{s_{j}}\right) = \begin{cases} \exp\left(\Phi_{s_{j}}\right) & \text{, for } \Phi_{s_{j}} \leq 0 \\ \varepsilon^{2}\left(1 - \exp\left(-\tilde{\Phi}_{s_{j}}^{2}\right)\right) + \exp\left(-\tilde{\Phi}_{s_{j}}^{2}\right), \text{ for } \Phi_{s_{j}} > 0 \end{cases}$$

$$F\left(\Phi_{s_{j}}\right) = \begin{cases} \exp\left(\Phi_{s_{j}}\right) &, \text{ for } \Phi_{s_{j}} \leq 0 \\ \varepsilon \operatorname{erf}(\widetilde{\Phi}_{s_{j}}) + \exp\left(\Phi_{s_{j}}\right) \operatorname{erfc}\left(\varepsilon \widetilde{\Phi}_{s_{j}}\right), \text{ for } \Phi_{s_{j}} > 0 \end{cases}$$
(7)

The unique collection for different shaped collectors,  $F(\Phi_{s_j})$ , is shown in Equation 6 for a sphere and in Equation 7 for a cylinder. The function  $\operatorname{erf}(x)$  is the error function and,  $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ , is known as the complementary error function.

The charged body is not in direct contact with undisturbed plasma. Instead, the body collects current within a modified potential structure, called the sheath region, where the object's potential either enhances or reduces ionospheric density near the surface. The amount of current a probe can collect in the saturation region of the I-V curve is dependent upon the sheath to probe radius ratio. Typical I-V curves for a stationary cylindrical probe with varying sheath ratios,  $\varepsilon$ , is shown in Figure 4. As the sheath ratio increases, the effective area of the probe also increases, allowing the probe to collect more current in the saturation region.



Figure 4: Stationary Cylindrical OML Model

## **SPICE** simulation

The validity of these SPICE models is demonstrated by modeling a charged sphere within a typical ionospheric environment containing both electrons and oxygen ions. Figure 3 shows the I-V curve for the current collection of these species with various sheath ratios. The top subplots represent the electron and ion currents separately. The bottom subplots represent the sum current of both species leaving the sphere. For purposes of this discussion, positive current will represent current leaving the probes.

The floating potential of the sphere (shown in the bottom right subplot of Figure 3 - inset of bottom left subplot) is when the sphere has no voltage bias relative to the space environment. At this point the probe will not attract

electrons or ions, and no current is collected. As the sheath distance expands relative to the sphere radius, the floating potential of the sphere diminishes.



#### Figure 3: OML Model of an Electrically Biased Sphere Computed Using SPICE

To demonstrate how these circuits can be used to model a Langmuir probe in space. A simple circuit is created as shown in Figure 5. In this model, the spacecraft is modeled as a sphere, and the Langmuir probe as a cylinder. The drifting models for objects in low Earth orbit are used to represent the collection of current from the space environment, containing both oxygen ions and electrons. This simple model uses an ideal voltage source to sweep the voltage applied to the current collecting body of the Langmuir probe. The current is measured using the tools built into LTspice. Figure 6 summarizes the Langmuir probe's ability to collect current when a saw-tooth voltage is applied to the probe, over varying spacecraft sizes.



Figure 5: LTspice Model of a Langmuir Probe

We expanded the Langmuir probe model to represent the interaction between different science instruments on the SPORT mission. Figure 7 shows a simplified electrical lumped element model of the various sensors on the SPORT 6U CubeSat interacting with the ionosphere.

We were specifically interested in modeling the interaction between the USU Langmuir probe and the ion drift and retarding potential analyzer (IVM) provided by UTD. This circuit model represents the probe's interaction with the ionosphere as voltage controlled current sources and are represented using USU's SPICE macro models. The other voltage supplies used in this circuit are modeled as ideal sources.

The wake caused by the spacecraft is not directly modeled using LTspice components. However, by restricting the collection area to reflect just the ram facing collecting area of the spacecraft, the wake can approximately be accounted for. The two extremes of spacecraft area are used to model the wake effects of the satellite. The first extreme being where ion currents are only collected on the ram face of the spacecraft  $(0.02m^2)$ , and the second where ions can be collected by the entire surface area of the spacecraft  $(0.2m^2)$ . A more reasonable middle ground expectation, where ion current is collected on the ram face, as well as partially on the side of the spacecraft  $(0.07m^2)$  is also presented. Figure 8 shows the expected I-V curve with the spacecraft area varying to represent the different possible collection areas. Figure 9 demonstrates the expected floating potential response for the spacecraft when a saw-tooth voltage sweep is applied. The plot shown is for an ion collection area of  $0.02m^2$ , the worst-case scenario.

## DISCUSSION

The OML models discussed successfully represent the I-V current curves expected by theory. Including modeling



Figure 6- Simulation Results of LP Model

the floating potential of a charged body. We are especially pleased with LTspice's ability to handle the floating nodes which are part of these models. These can be difficult for some version of SPICE to accurately simulate but LTspice's heritage from simulating switched power supplies has improved its ability to compute floating nodes.

The simple Langmuir probe model demonstrates the important relationship between spacecraft size and the Langmuir probe's ability to correctly characterize density and temperature factors. While larger spacecraft can collect enough positive ions to balance the number of electrons collected by the positively charged Langmuir probe. As spacecraft area decreases relative to probe area, the ion collection on the spacecraft body slows, dramatically decreasing the floating potential of the spacecraft. Under this condition, the Langmuir probe cannot be sufficiently charged positive relative to the



**Figure 7: Simplified Circuit Model of SPORT** 



Figure 8: Expected I-V Curve for SPORT LP



Figure 9: Floating Potential Response of Spacecraft During LP Sweep

ionosphere to collect current in the electron saturation region, and even into the electron retardation region.

From modeling the SPORT spacecraft mission, it can be observed that UTD's IVM did not adversely affect the performance of the USU Langmuir probe. We were also able to observe that the Langmuir probe measurements will be affected by the floating potential of the spacecraft for low density and small ram collection areas. Therefore, we expect that the instrument will work well for more dense ionospheric conditions, but the instrument will struggle to correctly measure less dense ionospheric conditions.

The electric potential meters, sampled simultaneously with the USU Langmuir probe, measure the spacecraft's floating potential changes at every voltage point of the Langmuir probe sweep. The simulation suggests that as the floating potential of the spacecraft starts to increase, the floating potential meters were not simulated as expected. We look forward to comparing these models to the actual data collected by SPORT after a late 2021/ early 2022 launch date.

## Alternative Modeling Approaches

Other modeling software exists for understanding spacecraft charging, such as NASCAP, which makes use of a 3D model of the spacecraft and performs a particle in cell type simulation. These models have been shown to be very accurate at matching observations and are useful for predicting the physical interactions of a spacecraft body with the space environment; however, they are limited in their capacity to model the electrical interactions of the spacecraft analog circuits and instrumentation with the space environment. The approach we have taken of integrating these components into the SPICE language allows one to easily explore the coupled probe relationships between instruments and their analog circuits. In the case of SPORT, we were able to model the IVM's feedback voltage controller, and its associated time constant, and its effect on the USU Langmuir probe.

## Future Work

Additional work is in progress to model new analog circuits for a next generation Langmuir and electric field probes. Integrating these models into LTspice will allow us to observe how the new analog compents will interact with the probe surfaces and space environment.

## CONCLUSIONS

The approach presented here uses simple geometrical shapes to approximate the properties of a spacecraft and associated probes. These shapes can be quickly placed into an integrated LTspice model. This allows for quick production of working probe/plasma interaction models for many small satellite and CubeSat electrical engineers without the complexity of other spacecraft charging models.

We were successful in the basic modeling of the SPORT spacecraft and bounding the coupling effects between various science instruments which are simultaneously operating. We and look forward to comparing the SPORT flight data to the SPICE model results. Additional work involving more complex circuit and spacecraft models, including new analog components and circuit architectures, is ongoing.

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