Effects of gravitational time dilation in short-duration balloon satellite flights

George Zhang, Aaron Flowers, Saleem Ali
US Military Academy at West Point
606 Thayer Road, West Point, New York 10996, USA; 850.688.0804
ageorge.zhang@westpoint.edu

ABSTRACT

The theory of general relativity stipulates that clocks located in a stronger gravitational field run more slowly than clocks located in a weaker gravitational field. This time dilation is often small and hard to detect. Its measurement requires precisely synchronized clocks accurate to the nanosecond level. An experiment aimed at detecting gravitational time dilation should last long enough to allow for detectable time drift to accumulate within the two clocks. This amount of time depends on the precision of the clocks used in the experiment. We describe the procedures followed to accomplish the task of measuring gravitational time dilation relying on methods and equipment available to undergraduate students. Our experiment employed chip scale atomic clocks on board a high-altitude helium balloon’s payload. We illustrate how, under the constraints of our experiment, we attempted to model and experimentally verify gravitational time dilation. We also describe the method we devised to detect time dilation without constant measurement and communication between the two clocks used in our experiment. Our payload component, aimed at verifying gravitational time dilation, had to conform to balloon satellite payload weight restrictions.

1 INTRODUCTION

Clocks located closer to bodies with large mass tick more slowly than those located in free space because of the effects of general relativity on time. The closer a clock’s location is to a massive object, such as a planet like Earth, the more slowly its “hands” will seem to be moving to an outside observer. For example, clocks read by astronauts on the International Space Station run incrementally faster than clocks read by a person located at sea level on Earth. Time dilation would be even larger in a stronger gravitational field in the proximity of a more massive celestial body.

Clocks orbiting the Earth at different altitudes also travel at slightly different speeds as they are at different distances from the center of the Earth. According to Einstein’s time dilation formula in special relativity, tick rates of identical clocks are also affected by the speed at which the clocks are moving. This type of time dilation is very small for the altitude and speed reached by our balloon satellite payload. Gravitational time dilation has a much larger effect on our atomic clocks than the time dilation due to special relativity, thus the effect of the latter can be neglected.

Though Albert Einstein posited the idea of general relativity in his 1916 paper it was not until the Hafele and Keating experiment, conducted in 1971 on board of airplanes, that the theory was tested with positive results using four cesium atomic beam clocks. We seek to perform a similar experiment with high-altitude balloon flights to further confirm the theory as well as to test the capabilities of chip scale atomic clocks (CSACs).

CSACs are relatively new devices, becoming commercially available in the last decade and a half. They provide the accuracy of an atomic clock, while having extremely reduced size, weight, and power consumption. While time dilation was measured using larger atomic clocks flying on airplanes decades ago, it has not yet been verified using CSACs deployed in high-altitude balloon satellite payloads. Other time dilation experiments have been attempted using high-altitude helium balloons, such as with a University of Minnesota undergraduate project, the process is much harder and has not yielded many positive results.

The goal of this project is to measure the time difference caused by gravitational time dilation between readings on one CSAC that remains stationary on the ground at West Point and another that is launched in a high-altitude balloon satellite. We expect that our balloon will rise to over 30,000 meters and stay in the air from three to five hours.

Figure 1 shows satellite trajectories taken by bal-
Figure 1: Balloon sat launches by West Point cadets.

(a) 12 APR 2012: 2.4 hours  

(b) 25 APR 2014: 5.8 hours  

(c) 18 APR 2015: 1.5 hours  

(d) 16 APR 2016: 1.7 hours  

(e) 22 APR 2017: 1.4 hours  

(f) 21 APR 2018: 2.2 and 3.3 hours, respectively  

(g) 6 APR 2019: 3.3 hours  

(h) 24 APR 2021: 1.7 hours
loons launched by West Point cadets since 2012. Because the balloon satellite is not moving at speeds that approach the speed of light to any significant percentage, we will not account for the influence of special relativity. We will also consider that Earth’s geoid, our planet’s idealized sea-level surface and surface of constant effective gravitational potential, is a perfect sphere. We expect that the altitude reached by our balloon satellite is high enough, the flight time of the balloon is long enough, and the CSAC’s frequency is stable enough to detect time dilation on the order of 10 to 20 ns.

2 THEORY

Einstein’s unconventional explanation of why a stationary observer receives a different number of wave crests than the emitted rate was simple: the number of wave crests counted does not change, but the time unit itself changes. Clocks placed at different locations in a gravitational field run at different rates because the unit of time changes in the presence of a gravitational potential. The equation that relates the rate at which a clock at Earth’s idealized sea-level surface clicks to a clock that operates at a certain height above Earth’s surface is given by:

\[
\frac{\Delta \tau}{\tau_0} = \frac{\tau - \tau_0}{\tau_0} = \frac{g}{c^2} h, \tag{1}
\]

where

- \( \tau \) is the free time, the time elapsed on a clock located at an arbitrary height \( h \) above Earth’s geoid assumed to be a perfect sphere
- \( \tau_0 \) is the proper time, the time measured by an observer located in the same gravitational potential as the clock; we used time transmitted by the balloon satellite’s GPS locator
- \( g \) is the acceleration due to gravity, \( 9.81 \text{ m/s}^2 \) at the surface of Earth’s geoid, but otherwise taking values that vary with height, \( g(h) \)
- \( h \) is the altitude difference between the clocks, and
- \( c \) is the speed of light.

Equation (1) can be derived using the Schwarzschild metric of general relativity, which describes the gravitational field outside of a spherically symmetrical mass. The complete solution incorporates the combined effects of time dilation due to general and special relativity, and it agrees with the Newtonian result in the weak gravitational field limit. Effects due to special relativity can be neglected when clocks are not moving at relativistic speeds, as the impact of general relativity becomes dominant for stationary clocks located near very large and dense celestial bodies.\(^6\)

2.1 Numerical integration method

For our experiment, one CSAC will be stationary on the ground and the other CSAC’s location will be changing in altitude as it travels with the high-altitude balloon satellite. Therefore, \( g \) and \( h \) in Equation (1) will be changing with respect to time.

Figure 2: Variation of the local value of gravitational acceleration, \( g \) (plotted on the vertical axis) as a function of time (plotted on the horizontal axis). From 2014 balloon satellite data.

Height, \( h \), will increase as the balloon rises to its peak altitude and then it decreases as the payload starts to fall after balloon burst. The gravitational acceleration, \( g \), will vary as a function of \( h \) since it will be getting weaker as the balloon’s altitude increases, and then it will get stronger again as the payload falls. We model this variation using Newton’s universal law of gravity:

\[
F_g(h) = mg(h) = m \frac{GM_E}{(R_E + h)^2}, \tag{2}
\]

where

- \( F_g \) is the force of gravity,
- \( m \) is the gravitational mass of an object,
- \( g(h) \) is the altitude-dependent acceleration due to gravity,
- \( G \) is Newton’s universal constant of gravity,
- \( M_E \) is the mass of the Earth, and
- \( R_E \) is the volumetric mean radius of the Earth.

Figure 2 shows variations in the local acceleration due to gravity experienced by a CSAC traveling on board of one of our balloon satellites. As flight altitude changes with time, the local value of the acceleration due to gravity also changes. Based on this graph, a theoretical prediction of gravitational time dilation can be calculated by determining the
area under the curve (dark red in the Figure 2). We use a numerical integration method since direct integration is not possible, unless we approximate the shape of the curve by a time- and altitude-dependent function.

We used balloon satellite data from years past to express numerical values for \( g \) and \( h \). Several trajectory altitude versus time plots of previously-conducted balloon satellite launches are shown in Appendix (6). Based on this historical data, we estimated the numerical values of an on-board CSAC’s gravitational time dilation using Mathematica®. The estimated gravitational time dilation values obtained from balloon satellite flight data from nine previous launches are summarized in Table 1.

<table>
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<tr>
<th>launch year</th>
<th>midpoint (rectangle) summation rule GTD[ns]</th>
<th>left-hand (rectangle) summation rule GTD[ns]</th>
<th>detect with CSAC</th>
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<tr>
<td>2021</td>
<td>3.9</td>
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3 METHODOLOGY

The first step is to synchronize the CSAC by 1 pulse per second (1PPS) disciplining to a reference frequency. In this experiment, a second CSAC’s 1PPS output served as the reference frequency. The experimental setup is shown in Figure 3.

By connecting the reference CSAC to an oscilloscope via a SubMiniature version A (SMA) cable, the 1PPS output of that CSAC can be observed. The second CSAC is connected to the oscilloscope the same way but is also connected to a computer running the CSACdemo software, which allows us to monitor and adjust the time difference between the two CSACs through frequency steering. From there, the CSACdemo software can be used to synchronize the CSAC.

Eventually, the nominal 1PPS time difference is reduced to near 0 ns with an uncertainty of 5 ns. After synchronization, the two clocks will inevitably begin to drift. However, by periodically logging the drift, it was observed that the drift was linear and can be corrected by measuring the slope of the drift and using the frequency steering feature in the CSACdemo software to adjust accordingly, as shown in Figure 5.
Figure 5: An example of adjusting for the clocks’ drift by issuing a steer command near 0 ns phase difference. Time shown in 5-minute increments.

The CSAC was launched in a Styrofoam balloon satellite payload enclosure.

A 5-V lithium polymer battery was used to power the launched CSAC during the balloon flight and during travel to the launch site and back to the lab. Both the battery and the CSAC were placed in a second, additional Styrofoam container, as shown in Figure 7.

The CSAC and the battery pack were surrounded by padding to ensure that the CSAC stays within its operating temperature and it is not damaged by jostling of payload cause by the strong winds experienced by the balloon sat along its trajectory or by impact upon landing. A key piece of equipment that was unavailable for use both during the balloon satellite launch experiment, and prior to launch was a frequency counter.

Consequently, the time drift between the two CSACs was only measured prior to the launch, as shown in Figure 8 and then again after payload recovery, when the payload component CSAC was returned to the laboratory.
4 RESULTS

Due to balloon satellite payload weight restrictions neither a frequency counter, nor an oscilloscope could be launched with the CSAC. Drift data for the period of the experimental flight was impossible to be recorded. We plan to modify our on-board experiment in the future by including a mini-computer (an Atomic Pi) that will allow direct recording and storing of data.

Figure 9: Initially, we have seen an offset that maintained the same slope of systemic drift. After recovery, however, the CSAC’s behavior changed. On the evening of payload recovery, the slope of the launched CSAC did not show a systematic drift.

Our results thus rely on measuring a known slope of systematic drift and comparing against the slope offset as we brought back the chip scale atomic clock from the balloon satellite flight, as shown in Figure 9.

5 RESULTS AND CONCLUSIONS

Our results were inconclusive and were most likely influenced by external error factors. The most probable sources of error were sudden changes in acceleration experienced by the balloon satellite payload, which translated into shocks and vibration of the CSAC. The payload was secured to the balloon by a long rope, meaning that a pendulum motion was experienced by the payload, causing movement and sudden stops. Such random acceleration may have degraded the performance of the launched clock.

The temperature of the Styrofoam box housing the CSAC was also not well-controlled or monitored. The temperature compensated crystal oscillator (TCXO) internal to the CSAC is designed for deployment in field applications, but may not function as desired when exposed to sudden accelerations encountered while tethered to a weather balloon.

In future experiments, we plan to record CSAC data using a mini-computer. We also plan to launch two sensors to accompany the CSAC on the balloon flight. One is a temperature sensor, and the other a triple-axis accelerometer. After we obtain flight data that includes the temperature profile of the enclosure during flight as well as the vibration profile, we can reproduce these conditions experienced by the CSAC in the laboratory. Through such experiments, we can determine by what factor does the performance of the CSAC worsen under field conditions experience during flight.

In addition to these environmental factors, we can only expect to detect gravitational time dilation in a balloon satellite flight that reaches the altitude of and lasts the length of the balloon satellite launched in the year 2014.

Acknowledgments

We would like to acknowledge Mr. Tom Van Baak for his assistance in synchronization and much advice given on our project.

Mandatory disclaimer statement

The views expressed are those of the authors and do not reflect the official policy or position of the United States Military Academy, the US Army, the Department of Defense, or the US Government.

References

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Figure 10: Balloon sat trajectory data. Altitude versus time plots. Years presented 2012-2019.