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Insulation of Small Open Economics in the Presence of External Disturbances Under Alternative Exchange Rate Systems

Hamid Reza Azad

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INSULATION OF SMALL OPEN ECONOMIES IN THE PRESENCE OF EXTERNAL DISTURBANCES UNDER ALTERNATIVE EXCHANGE RATE SYSTEMS

by

Hamid Reza Azad

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

Approved by:

UTAH STATE UNIVERSITY
Logan, Utah
1988
To my wife, Vida, and my parents, Behjat and Nazar Azad.
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Hamid Reza Azad
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ABSTRACT

Insulation of Small Open Economies in the Presence of External Disturbances Under Alternative Exchange Rate Systems

by

Hamid Reza Azad, Doctor of Philosophy
Utah State University, 1988

Major Professor: Terrence Glover
Department: Economics

This study analyzes the determination of the exchange rate system in a small economy when external real and monetary disturbances occur. Choice of exchange rate policy is investigated using a model assuming rational expectations and a loss function expressing the squared deviations of the small country output from desired output. The distinguishing feature of the analysis is the emphasis on real as well as monetary disturbances which originate abroad but are a source of domestic output variation. The link between foreign monetary and real disturbances and variance in output is traced using the theoretical model and the loss function assumed.

The emphasis of the analysis is on a three country (one small and two large) trading situation, whereby the small country trades with two major large country trading partners. It is assumed throughout that there is perfect commodity arbitrage between two large countries. The small
country imports an intermediate good from one of the large countries and exports a finished good. The small country does not import goods for consumption. There is perfect capital movement between two large countries, but capital is immobile between the small and these two large countries.

The analysis indicates that occurrence of purely nominal shocks abroad are not transmitted to the small country under floating exchange rate system. The presence of real disturbances in large countries induce lower prices for the goods they produce, but the effect on the exchange rate is ambiguous. This study concludes that in general the adoption of a flexible exchange rate system by a small country is preferred and results in lower loss in most cases of external disturbance.

(94 pages)
CHAPTER I

INTRODUCTION

This study analyzes the determination of the exchange rate in a small open economy. The purpose is to investigate the choice of exchange rate (the domestic currency price of one unit of foreign currency) and its relationship to the insulative properties of exchange rate policy for a small country when external real and monetary disturbances occur. Choice of exchange rate policy is investigated using a model assuming rational expectations and a loss function expressing the squared deviations of the small country output from desired output (the level of output when labor market is cleared). The distinguishing feature of the analysis is the emphasis on real as well as monetary disturbances which originate abroad but are a source of domestic output variation. The link between foreign monetary and real disturbances and variance in output is traced using the theoretical model and the loss function assumed.

The main focus of the analysis is on examining the international transmission of disturbances in a three-country trading situation, whereby the small country (price-taker, less-developed country) trades with two major large country trading partners. This emphasis is a departure from similar work by Flood (1979), Flood and Marion (1982), and
Daniel (1981). Other trading arrangements and subsequent analysis of the impacts of alternative exchange rate regimes on small country insulation from external real and monetary disturbances for the two-country trading situation can be derived using this basic theory.

It is assumed throughout that there is perfect commodity arbitrage between two large countries. A small country imports an intermediate good from one of the large countries and exports a finished good. The small country does not import goods for consumption. There is perfect capital movement between two large countries but capital is immobile between the small and these two large countries. The only financial asset in the market is money. The problem for the small country is to choose an exchange rate policy which minimizes domestic output variation assuming a trading regime in which trade is necessary to production and consumption. Modification of domestic output variation via alternative choice of exchange rate system constitutes a policy to fully or partially insulate against disturbances.

This work, then, is an attempt to determine the conditions under which a small country is fully or partially insulated (if insulated at all) from foreign real and monetary disturbances. The policies from which the small country can derive some form of insulation include alternative fixed, and flexible exchange rate regimes. The criterion used to compare alternative exchange rate systems
is a loss function expressing the expected squared gap between actual and desired output in the small country. The impacts of alternative exchange rate regime are traced using the loss function.

OBJECTIVES

Since trade is usually between the small country and one or two major large country trading partners, the focus of the study is on a three-country trading model. In this model foreign generated monetary and real disturbances affect the level of output in a small country under floating and/or fixed exchange rates. A fixed, or pegged exchange rate, means the domestic country chooses a "par value" for its currency in terms of the foreign currency and the domestic central bank attempts to keep the exchange rate within a stated margin above and below that par value. Under a floating exchange rate system, the exchange rate is allowed to respond to market influences and thus to settle at whatever level it takes to equate the demand and supply of foreign exchange. Usually a flexible exchange rate (as opposed to a fixed link to an external standard) is used when the countries desire greater flexibility and choose "managed" floating with various degree of intervention. The country may choose a rule for officially controlling the movement in the exchange rate according to objective indicators. The study first examines the international
transmission of disturbances in a three-country, rational expectations world. An appropriate criterion, the minimization of domestic output variance is assumed for choosing among different types of exchange rate regimes. Specifically, the objectives of the study are:

1. To review the history of the international monetary system, especially after the breakdown of the Bretton Woods and Smithsonian agreements.

2. To build a theoretical framework which can be used to analyze the effects of alternative exchange rate regimes on insulation from foreign real and monetary disturbances.

3. To examine the international transmission of economic disturbances under various exchange rate systems and to determine to what extent each type of exchange rate system insulates a small, open economy from these foreign generated disturbances.

ORGANIZATION

The plan of the study is as follows: Chapter II contains a historical background of the international monetary system and a discussion of related literature. Chapter III presents the three country-two good rational expectations model. The solution of the model for each exchange rate system and the transmission of foreign disturbances to the small country are theoretically derived and described. The insulation properties of the system are
also compared. Chapter IV discusses the three country-three
good trading generalization of the basic model, and chapter
V details the two country-two good model. Chapter VI
contains concluding remarks and suggestions for further
research.

Examination of the disturbance transmission process
derived from the theoretical model yields the general
results that the small country cannot be fully insulated
because monetary and real disturbances are transmitted via
the price channel and the exchange rate. Briefly, the
analysis shows that the effect of purely nominal
disturbances under a flexible exchange rate system are fully
offset through the change in the exchange rate. External
monetary disturbances do impose losses on the small country
under other regimes such as fixing domestic currency to the
currency of large country trading partners. In the presence
of real disturbances, the flexible exchange rate regime
appears to insulate the small country more than other regime
choices.
CHAPTER II

BACKGROUND AND REVIEW OF LITERATURE

BACKGROUND

Political boundaries tend to separate markets. This can lead to price differences, which provide an incentive for trade. Modern developments in transportation and communication have greatly increased the degree of economic interdependence among nations. Each nation has its own national money and because no generally accepted international currency exists in the modern world, international economic transactions must be financed through the respective national currencies. The use of national currencies for international transactions makes it necessary to convert money from one national currency into another. An exchange rate (the price of one currency in terms of another) is important because it connects the price systems of two countries, and the selection of any type of exchange rate regime is viewed as a sequence of choices and considerations that affect all aspects of the economy.

The international monetary system changes to coincide with the needs of the world. Before World War I, most of the leading industrial nations were on the gold standard that emerged in the latter part of the nineteenth century. Each country defined its currency in terms of gold, and its central bank was required by law to buy and sell gold without limit at a stated price.
With the outbreak of World War I in 1914, the international gold standard ceased to function. After the war, European countries struggled with a legacy of inflation and political instability. Substantial divergence in economic circumstances had developed in several countries and prewar exchange rates could not easily be restored. The interwar period, especially the first few years after World War I (1919-23) and the currency crisis at the depth of the Great Depression (1931-34), were chaotic times in exchange rate history when most countries went on and off the gold standard.

The interwar experience brought delegates from 44 members of the United Nations together at Bretton Woods, New Hampshire, in 1944, to negotiate an agreement on the structure and operation of the international monetary system. They created two institutions: the International Monetary Fund (IMF) and the International Bank for Reconstruction and Development (now called the World Bank). Each member nation agreed to specify a "par value" for its currency in terms of the U.S. dollar (or its equivalent in gold, i.e. 1/35 of an ounce). A narrow band of fluctuation, ±1 percent, was permitted under the agreements made at Bretton Woods.

The Articles of Agreement of the International Monetary Fund provided the basis for the international monetary system that existed from 1945 to 1971. The fundamental flaw
in the IMF system was that it did not provide any mechanism for orderly growth in world monetary reserves. Such growth was needed as world production and trade expanded, but it could come only in gold or dollars. Gold mining contributed very little, therefore dollars had to maintain the reserves. The United States had begun to run a balance of payments deficit as early as 1950, and in the early 1950s the deficits were welcomed because they increased the world supply of foreign exchange reserves. The persistent U.S. deficit was the focus of intense discussion all through the 1960s. The U.S. overall payments deficits continued and had to be financed by increasing sales of U.S. foreign exchange reserves. Eventually the United States chose to change the rules, and on August 15, 1971, the U.S. terminated its commitment to buy and sell gold at the $35 per ounce rate and set the dollar afloat.

After a four month period of intense negotiation and discussion, in December of 1971 a conference was held in Washington, D.C., at which the Smithsonian Agreement was created. This agreement established a new set of exchange rates, termed "central rates" instead of "par value," with a permitted band of ± 2 1/4 percent around the new central rates. Finally, depreciation of the dollar against other major currencies and failure to restore confidence in the dollar caused industrial nations to let their currencies float more or less freely in the foreign exchange market.
With the collapse of the Bretton Woods and Smithsonian agreements between 1971 and 1973, the system of fixed "par value" was abandoned. Several types of exchange arrangements emerged, and they were formally legalized as possible choices in the second amendment of the Fund's Articles of Agreement in April, 1978. Exchange rate policy was left open to choice. Major industrial countries tended to adopt systems of floating exchange rates, either singly or jointly wherein the exchange market takes care of the exchange rate. For less developed countries with inadequate development of domestic financial markets, lack of integration with world markets, limited capital market, restriction on capital flow, and other features which differentiate them from developed countries, independent floating seemed either infeasible or undesirable.

It was expected that flexible rates would lead to smooth working of the external adjustment process without the need for controls on trade and capital flows. It was also widely held that flexible rates would help to achieve stable growth. However, it was also expected that the new system would increase the uncertainty associated with

1. Of the 151 members of the IMF, on December 31, 1986, 91 were pegging their currencies (31 to the U.S. dollar, 14 to the French Franc, 5 to other currencies, 10 to the SDR and 31 to other currency composites), 8 were operating a joint float (cooperative arrangement among European community nations), 19 were independently floating, 21 were attempting a "managed" floating and 12 currencies were otherwise determined (including Democratic Kampuchea for which no current information is available).
international transactions and would therefore discourage both foreign trade and international investment. Developed countries considered flexible exchange rate systems because of the benefits expected. However, limited capital markets, non-diversified economies, dependency of economic growth upon the foreign sector, and rudimentary financial markets, ruled out the system as a feasible choice.

Pegging to a single major currency seems to be the option most favored by less-developed countries at this time. It reduces the fluctuations of the exchange rate between a less-developed country and a developed-country, seemingly reducing uncertainties and facilitating trade between them. For the same reason, capital flow for investment from the developed-country and rest of the world may increase. However, single-currency pegging involves potential disadvantages, such as increasing the need for reserves since movements in the pegged exchange rate of the less-developed country will not reflect actual developments in its balance of payments. Fluctuations in the exchange rate are exogeneous and independent of government policy and may interfere with internal policy objectives.

An alternative approach, which attempts to retain the advantages of pegging while ostensibly minimizing the disadvantages, is the strategy of pegging to a basket of currencies. This strategy has been adopted by some developing countries. To avoid large fluctuations in their
exchange rates, an increasing number of developing countries have abandoned single-peg policies and have started experimenting with composite pegs.

There are qualifications to the various arguments about exchange rate policies, and it is not possible to clearly weigh the benefits against the costs. The ratio of disadvantages to advantages depends upon the conditions of the country in question, and each case must be analyzed in terms of resource costs, institutional costs, instability costs, and political costs. In this study responses to various disturbances under different exchange rate systems, as well as their insulative properties for a small country are discussed in order to delineate the alternative regimes which assist in economic stabilization.

REVIEW OF LITERATURE

The relative advantages of fixed and flexible exchange rates have been debated at length in the literature. Since the early 1970s, several studies have focused on the problems that small open economies have in choosing their regimes. Black (1976), Crockett and Nsouli (1977), Lipschitz (1978), Branson and Katseli-Papaefstratiou (1981) ruled out the floating exchange rate regime as a feasible or realistic option. However, significant differences in the inflation rates in small countries emphasized to the important issue of monetary and fiscal policy independence.
and the adoption of some form of flexible exchange rate regime.

One of the most important aspects of the debate between fixed and flexible exchange rates centers on the relative stability of the two systems in the face of random disturbances. For a small country, increased fluctuations in exchange rates undermine the domestic currency and obstruct its monetary functions. Increasing uncertainty in the trade sector decreases foreign investment and affects economic growth and income. Therefore, the main factor to be considered in choosing an exchange rate regime is whether or not it provides insulation from such foreign disturbances.

The formal examination of the effects of exchange rate systems on insulation of open economies is not new. Several authors have examined the stabilizing features and insulative power of various exchange rate regimes. Mundell (1963), Turnovsky (1976), and Fischer (1977) are among the contributors.

Turnovsky (1976) and Fischer (1977) examined the response of a small open economy subject to various random disturbances under both fixed and flexible exchange rate systems. Turnovsky assumed capital mobility and minimization of domestic output variance as an appropriate criterion for choosing between two types of exchange rate regimes. Fischer does not allow capital mobility and,
because output is fixed in his framework, he uses the minimization of domestic consumption variance as his criterion of choice. Turnovsky concludes that the relative stability of output depends crucially upon the source of random disturbance. In the short run, output will always be more stable under flexible rates if the stochastic disturbances are either in foreign trade or in foreign output prices. By contrast, greater stability is achieved under a fixed rate system if the disturbances occur in the domestic monetary sector. Fluctuation in capital flows tends to favor fixed exchange rates, especially if the domestic monetary authorities are able to sterilize them. However, Fischer found that if disturbances are monetary, under floating rates price level changes absorb the shocks completely, while the shocks are transmitted to consumption under fixed rates. If disturbances are real, floating prevents the transmission of shocks abroad and results in greater instability of consumption than would occur under fixed rates. If monetary shocks produce real effects, the superiority of floating rates in the face of monetary shocks becomes less certain.

Daniel (1981) examined economists' prediction that flexible exchange rates would reduce business fluctuations caused by monetary disturbances. She incorporated rational expectation into her model of a two commodity-two country world. She found that purely nominal shocks, whether
temporary or permanent, are not transmitted. However, when nominal disturbances are accompanied by real disturbances, the latter are transmitted. Monetary shocks, which increase foreign prices, are transmitted only if they have real effects. Inflation itself is not transmitted, but the accompanying real effects are.

Gray (1976) focused on the impacts of real monetary disturbances on price and output variation. He incorporated short term wage rigidities and uncertainty to his model and investigated the role of indexation in dampening macroeconomic fluctuations. His results show that the real sector is insulated from the effects of monetary shocks in a fully-indexed economy. The real effects of real disturbances are more pronounced in an indexed economy than in a nonindexed economy. In an economy subject to both real and monetary disturbances, there must exist a degree of indexing between zero and one which is in some sense optimal.

Flanders and Helpman (1978) considered exchange rate policy for an economy that is a price-taker for tradable goods in the presence of wage and nontradable price rigidity. They showed that a flexible exchange rate regime is always as good as or better than the fixed exchange rate strategy. When all domestic prices (and wages) are flexible, the two regimes produce identical results. In the case of downward rigidity in domestic wages and in prices of
nontradables, exchange rate flexibility counters the rigidity in the domestic price of nontradables, thereby maintaining equilibrium of relative price ratio and in the labor market. Flood (1979) also showed that mobile capital provides an open channel for foreign real and nominal disturbances to influence a small country under flexible exchange rates.

The choice of exchange rate systems is a very delicate decision and depends upon specific parameter values. As Lipschitz (1979) concluded, optimal exchange rate policy should depend on the transitory disturbances likely to be encountered by the developing country. In this study attention is directed to an examination of the stabilizing features of alternative exchange rate systems in a three country-two good rational expectations world. Other trading-regime cases are derived theoretically as additional alternatives with which a small open economy is faced.
CHAPTER III

THE THREE COUNTRY-TWO GOOD MODEL

The first model consists of three countries and two goods. Two of the three countries are assumed to be "large" in that each can affect its own as well as world prices. The third country is "small" in the sense that it cannot affect world prices. There are two goods, one is assumed to be a finished good and the other is an intermediate good. One of the large countries (country A) produces a finished good (good 1) and the other large country (country B) produces an intermediate good (good 2). The small country imports good 2 from country B to produce good 1 and exports good 1 to countries A and B. The small country does not import good 1 from either of the large countries. The countries issue their own currencies which are not internationally tradable. Local currency is the only financial asset in the model. It is assumed that the small country faces foreign real and monetary disturbances through the process of importing good 2 for the production of good 1.

The following assumptions are also made:

1. Expectations are assumed to be rational.

2. Perfect capital mobility exists between the two large countries, but capital is immobile between the small country and the large countries.

3. Goods arbitrage is assumed to be perfect and all goods are traded, but the small country does not import good 1.
4. The model is assumed to be logarithmically linear in all variables except the interest rate.

5. Each country's economy consists of two markets, i.e. the goods market and the money market. No bonds or paper issues exist.

THE MODEL

Notation

Lower case letters denote logarithms of variables while Greek letters represent parameters and they are all assumed to be positive. Subscripts refer to the time dimension.

\( \bar{Y} \)
Small country-real output

\( \bar{Y}^* \)
Country A-real output

\( \bar{Y}^{**} \)
Country B-real output

\( Y \)
Small country-normal (desired) real output

\( Y^* \)
Country A-normal (desired) real output

\( Y^{**} \)
Country B-normal (desired) real output

\( p_{1s} \)
Price of good 1 in small country

\( p_{2s} \)
Price of good 2 in small country

\( p_{1A} \)
Price of good 1 in country A

\( p_{2A} \)
Price of good 2 in country A

\( p_{1B} \)
Price of good 1 in country B

\( p_{2B} \)
Price of good 2 in country B

\( s_A \)
Small country- currency price of country A-currency
Small country-currency price of country B-currency

Country A-currency price of country B-currency

Small country-nominal stock of money balance

Country A-nominal stock of money balance

Country B-nominal stock of money balance

Small country-interest rate

Country A-interest rate

Country B-interest rate

Small country-stochastic disturbance in output supply with mean zero and finite variance $6u^2$

Country A-stochastic disturbance in output supply with zero mean and finite variance $6u^2$

Country B-stochastic disturbance in output supply with zero mean and finite variance $6u^2$

Small country-stochastic disturbance in money market with zero mean and finite variance $6v^2$

Country A-stochastic disturbance in money market with zero mean and finite variance $6v^2$

Country B-stochastic disturbance in money market with zero mean and finite variance $6v^2$

Mathematical expectation operator

Expected value of $P^{1s}$ at time $t$, conditional on information available at time $t-j$

The cost-of-living index for country A which contains prices of both goods, (measured in
domestic currency), expressed in logarithms,

The cost-of-living index for country B which contains prices of both goods, (measured in domestic currency), expressed in logarithms,

\[ \theta^*, 1 - \theta^* \]

Expenditure shares on each commodity for country A

\[ \theta^{**}, 1 - \theta^{**} \]

Expenditure shares on each commodity for country B

**Small Country**

(1) \[ Y_t = \bar{Y}_t + \gamma_1 (P_t^{1s} - E^1 P_t^{1s}) + \gamma_2 \left( (P_t^{1s} - P_t^{2B} - S_t^B) - E(P_t^{1s} - P_t^{2B} - S_t^B) \right) + u_t \]

(Goods supply)

(2) \[ Y_t^d = \alpha_0 + \alpha_1 (m_t - P_t) + \alpha_2 Y_t \]

(Goods demand)

(3) \[ m_t^d - P_t = \beta_0 + \beta_1 Y_t - \beta_2 E (P_t+1^{1s} - P_t^{1s}) \]

(Money demand)

(4) \[ m_t = m_{t-1} + V_t \]

(Money supply)

**Country A**

(5) \[ Y_t^* = \gamma_1^* (C_t^* - EC_t^*) + \gamma_2^* \left( (P_t^{1A} - C_t^*) - E \right) (P_t^{1A} - C_t^*) + u_t^* \]

(Goods supply)

(6) \[ Y_t^{d*} = \alpha_0^* + \alpha_1^* (P_t^{2A} - P_t^{1A}) + \alpha_2^* Y_t^* \]

(Goods demand)

(7) \[ m_t^{d*} - C_t^* = \beta_0^* + \beta_1^* Y_t^* - \beta_2^* i_t^* \]

(Money demand)

(8) \[ m_t^* = m_{t-1}^* + V_t^* \]

(Money supply)

(9) \[ C_t^* = \theta^* P_t^{1A} + (1 - \theta^*) (P_t^{2B} + S_t^{AB}) \]

(Cost-of-living Index)

---

1. \( E = E_{t-1} \)
Equation (1) is the small country supply function. Current output, expressed in terms of its deviation from the normal level $\bar{Y}$, is a function of domestic price prediction error and relative price prediction errors, and $u_t$ is a stochastic disturbance term having zero mean and constant variance. The supply of good 1 is a positive function of the price of finished goods ($P_{t1B}$) and a negative function of input (import) price ($P_{t2B}$). Equations (1) and (2) are dependent on the underlying production function in the small country and have important implications for how effectively various exchange rate regimes insulate domestic output from unanticipated disturbances.

Equation (2) represents the expenditure function for the small country which is assumed to depend positively on real money balances and domestic income. Equation (3) states
the demand for real money by residents of the domestic country and is assumed to be dependent on real output positively and expected inflation negatively. Equations (7) and (12) are demands for real balance by residents of the large countries and are assumed to be dependent on real output positively and nominal interest rate negatively.

Equation (4) specifies the money supply process in the small country. The small country supplies its own money when it elects to adopt flexible exchange rates. Equations (8) and (13) represent the money supply for country A and country B respectively. Equations (9) and (14) define the cost-of-living index which contains the prices of both goods in terms of domestic currency, where \( \theta^*, (1-\theta)^* \) and \( \theta^{**}, (1-\theta)^{**} \) are the expenditure shares on each good for country A and country B respectively.

Equation (5) represents the supply function of country A. Supply is a function of the cost-of-living prediction errors, the domestic real price prediction errors, and a stochastic disturbance term \( (u_t^*) \). Equation (10) is similar. Equations (6) and (11) are expenditure functions for the large countries and are positively related to national income and relative prices.

The assumption of perfect capital mobility between the two large countries implies the following interest rate parity relation.

\[
i^* = i^{**} + E_t S_{t+1}^{AB} - S_t^{AB}
\]
Equation (15), states that the nominal interest rate in country A is equal to the nominal interest rate in country B plus the expected rate of depreciation of country B's currency relative to country A's currency.

The following equations, (16) and (17) represent the purchasing power parity relation (law or one price) and they state that the logarithm of domestic price is equal to the logarithm of foreign price plus the logarithm of the exchange rate between the two countries.

\[ 16) \quad P_t^{1s} = P_t^{1A} + S_t^A = P_t^{1B} + S_t^B \]
\[ 17) \quad P_t^{1A} = P_t^{1B} + S_t^{AB} \]

**THE LOSS FUNCTION**

The appropriate criterion to be used for comparing one exchange rate regime with another is the loss function, \( L \), which is expressed in terms of deviations of the logarithm of actual output from the logarithm of normal (desired) level of output.

\[ 18) \quad L = E \left[ (Y_t - \bar{Y}_t)^2 \right] \]

One of the assumptions of the rational expectations theory is that markets are always in equilibrium and economic agents set wages and prices, given the information they have, so as to achieve maximum profit and utility. However, there are shocks and disturbances that cannot be predicted (imperfect information) and cause disequilibrium and uncertainty in different sectors of the economy.
Disequilibrium and variation result in production inefficiency and distortion of output from desired output, causing loss to the society.

Tobin (1958) and Samuelson (1983), assuming a general quadratic and concave utility function, discussed the effect of risk-and-return variation in portfolio selection. They concluded that, assuming diminishing marginal utility, the gain in utility from using a specific amount of a thing is less than the loss of utility from not using the same thing. That is, the increase in the mean increases utility, but increases in the variance with preserved mean decreases the utility. Therefore, variation decreases utility (welfare) and is viewed as a loss to the society. Under the assumption that expectations are formed rationally, workers and producers make decisions using all available information which depends significantly on the future consequences of their decisions. People tend to avoid repeating previous errors and the economic environment rewards those who make good forecasts and penalize those who make poor forecasts by using available information. Therefore, it is the surprises in the economic environment which cause employment and production decisions to deviate from their optimum and bring about losses to society.

The proposed criterion for selection of an exchange rate system is the minimization of the expected squared difference between actual output, $Y_t$, and desired output,
\( Y_t \), with \( L \) denoting loss. Desired output is defined as full information output, when all markets including the labor market are cleared and the level of employment is derived from equilibrium in the labor market.

In order to derive the loss function for the small country in the form specified by (18), the supply function (equation (1)) is used as:

\[
Y_t = \bar{Y}_t + \gamma_1 (P_t^{1S} - E_{t-1} P_t^{1S}) + \gamma_2 ((P_t^{1S} - P_t^{2B} - S_t^B) - E (P_t^{1S} - P_t^{2B} - S_t^B)) + u_t
\]

By subtracting normal output, \( \bar{Y} \), from both sides, and squaring the expectation of both sides of the equation, the following is obtained,

\[
E [(Y_t - \bar{Y}_t)^2] = E [ \gamma_1 (P_t^{1S} - E P_t^{1S}) + \gamma_2 ((P_t^{1S} - P_t^{2B} - S_t^B) - E (P_t^{1S} - P_t^{2B} - S_t^B)) + u_t ]^2
\]

The result is that the left side of the above equation is the same as the right side of equation (19). Therefore, the following relation results as representative of the loss function:

\[
(19) \quad L = E [(Y_t - \bar{Y}_t)^2] = E [ \gamma_1 (P_t^{1S} - E P_t^{1S}) + \gamma_2 ((P_t^{1S} - P_t^{2B} - S_t^B) - E (P_t^{1S} - P_t^{2B} - S_t^B)) + u_t ]^2
\]

**PROCEDURE**

The method of this study was to work through the following steps:

1. Derive the loss functions for the small country under the following cases:
a- The small country fixes its exchange rate to country A's currency and floats against country B's currency;

b- The small country fixes its exchange rate to country B's currency and floats against country A;

c- The small country adopts a flexible exchange rate system against both countries.

2- Examine the effects of the transmission of various foreign generated disturbances on the loss function.

3- Compare exchange rate regimes for the cases mentioned above, emphasizing their insulative properties.

The following steps for the three cases have been developed.

1. Case Ia:

The small country fixes its exchange rate to country A's currency and floats against country B's currency. In order to derive the loss function in this case, equation (16) and (17) are used to derive:

\[ S_t^B = S_t^{AB} + S_t^A \quad \text{OR} \quad S_t^{AB} = S_t^B - S_t^A \]

When the small country chooses to fix its currency against country A, the domestic price becomes exogenous and is affected by random disturbances which affect the price level in country A. The exchange rate between the small country and country B is also exogenous and changes proportionally to the changes in exchange rate between the currency of country A and the currency of country B. Equation (16) and (20) for this case are as follows:
The domestic price solution and exchange rate solution can be derived from the above equations. The loss function, (19), depends on the domestic price prediction error and the relative price prediction error. Using equation (16'), then
\[
20) \quad P_{t}^{1s} - E P_{t}^{1s} = (P_{t}^{1A} + S_{t}^{A}) - E (P_{t}^{1A} + S_{t}^{A})
\]
Where $S_{t}^{A}$ is now fixed. So the domestic price prediction error is equal to the domestic errors in predicting the price of good 1 in country A.

The relative price prediction error can be found through the following steps:
\[
16') \quad P_{t}^{1s} = P_{t}^{1A} + S_{t}^{A}
\]
\[
20') \quad S_{t}^{B} = S_{t}^{AB} + S_{t}^{A} \quad \text{OR} \quad S_{t}^{AB} = S_{t}^{B} - S_{t}^{A}
\]

Therefore,
\[
22) \quad (P_{t}^{1s} - P_{t}^{2B} - S_{t}^{B}) - E (P_{t}^{1s} - P_{t}^{2B} - S_{t}^{B}) = (P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) - E (P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB})
\]
By substituting equations (21) and (22) into (19) the loss function for this case becomes,
\[
23) \quad L = E [ \gamma_1 ((P_{t}^{1A} + S_{t}^{A}) - E (P_{t}^{1A} + S_{t}^{A})) + \gamma_2 ((P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) - E (P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB})) + u_{t} ]^2
\]
The above loss function is dependent on the domestic errors in predicting the price of good 1 in country A. The
relative price prediction error is equal to the error in predicting the price of good 1 in country A, the price of good 2 in country B, and the exchange rate between countries A and B. Random disturbances in the small country do not affect the loss function.

To find the fluctuations in the loss function, foreign price solutions and the exchange rate solution need to be determined. Determination of these values requires solving the model for current and expected prices, $P_t^A$, $P_t^B$, and exchange rate, $St_{AB}$. The assumption that expectations are rational implies that they are formulated using all available information. The method which is used to solve such a model is to postulate a solution form for each of the endogenous variables - $P_t^A$, $P_t^B$, $P_t^S$, $St^A$, $St^B$, $St_{AB}$, $i$, $i^*$, $i^{**}$, $m_t$, $m_t^*$, $m_t^{**}$, in terms of exogenous variables - $m_{t-1}$, $m_{t-1}^*$, $m_{t-1}^{**}$, $V_t$, $V_t^*$, $V_t^{**}$, $u_t$, $u_t^*$, $u_t^{**}$. The solution for the endogenous variables is log linear in the exogenous variables. Solution of the model requires determining values for each of the price solution (reduced form) coefficients ($\pi s$) of the exogenous variables, where the $\pi_{ij}$ are functions of structural parameters. Taking advantage of the model's linearity in logarithms in order to write foreign prices, $P_t^A$, $P_t^B$, and exchange rate, $St_{AB}$, as linear function of the system's predetermined variables and disturbances, the reduced form equations for $P_t^A$, $P_t^B$ and $St_{AB}$ are,
In this case, the small country pegs its currency to the currency of country A; therefore $S_t^A$ is fixed and $E_{t-1}^A = S_t^A$. The domestic money market is not always cleared since the domestic price is exogenous. The domestic money supply becomes endogenous and is influenced by the balance of payments. Because of the "large" country assumption, country A sets the world price of good 1. The price of good 1 in the small country is set by country A and the loss function is not affected by $P_t^{1S}$. Monetary and real disturbances in the small country do not have any effect on domestic and foreign prices; therefore,

$$
\pi_{11} = \pi_{14} = \pi_{17} = 0
$$
$$
\pi_{21} = \pi_{24} = \pi_{27} = 0
$$
$$
\pi_{31} = \pi_{34} = \pi_{37} = 0
$$

Because the exchange rate between country A and country B is flexible, any monetary disturbance in either of the two
countries will be absorbed by the exchange rate and will not be transmitted to the other. Because the price of good 1 is set by country A and the price of good 2 is set by country B, the following results are derived,

\[ \pi_{13} = \pi_{16} = 0 \]
\[ \pi_{19} = 0 \]
\[ \pi_{22} = \pi_{25} = 0 \]
\[ \pi_{28} = 0 \]
\[ \pi_{12} > 0 \]
\[ \pi_{15} > 0 \]
\[ \pi_{18} < 0 \]
\[ \pi_{23} > 0 \]
\[ \pi_{26} > 0 \]
\[ \pi_{29} < 0 \]
\[ \pi_{31} = \pi_{34} = \pi_{37} = 0 \]
\[ \pi_{32} > 0 \]
\[ \pi_{35} > 0 \]
\[ \pi_{38} = 0 \]
\[ \pi_{33} < 0 \]
\[ \pi_{36} < 0 \]
\[ \pi_{39} = 0 \]

The rational expectation assumption is that the expectation of a variable is its expected value conditioned on available information and using this information efficiently. Since the mean of all random disturbances is zero, the expected value of all random shocks at time \( t \) is zero at time \( t-1 \), allowing the expected foreign prices and exchange rate to be expressed as:

\[ E_{t-1} P_t^1A = \pi_{11} m_{t-1} + \pi_{12} m_{t-1}^* + \pi_{13} m_{t-1}^{**} \]
\[ E_{t-1} P_t^2B = \pi_{21} m_{t-1} + \pi_{22} m_{t-1}^* + \pi_{23} m_{t-1}^{**} \]
\[ E_{t-1} S_{t}^{AB} = \pi_{31} m_{t-1} + \pi_{32} m_{t-1}^* + \pi_{33} m_{t-1}^{**} \]

The unexpected part of domestic and relative price are

(27) \( (P_t^1A + S_t^A) - E (P_t^1A + S_t^A) = P_t^1A - E P_t^1A \)
(28) \( P_t^1A - E P_t^1A = \pi_{14} V_t + \pi_{15} V_t^* + \pi_{16} V_t^{**} + \pi_{17} u_t + \pi_{18} u_t^* + \pi_{19} u_t^{**} \)
Some of the coefficients values and signs were found earlier. These coefficients are restricted to be zero; therefore the new expressions for domestic and relative price prediction error are,

\[ (30) \quad P_t^{1A} - E P_t^{1A} = \pi_15 V_t^* + \pi_18 u_t^* + \pi_19 u_t^{**} \]

\[ (31) \quad (P_t^{1A} - P_t^{2B} - S_t^{AB}) - E (P_t^{1A} - P_t^{2B} - S_t^{AB}) = (\pi_15 - \pi_35) V_t^* + (-\pi_26 - \pi_36) V_t^{**} + (\pi_18 - \pi_28 - \pi_38) u_t^* + (\pi_19 - \pi_29 - \pi_39) u_t^{**} \]

The above equations can be substituted into (23) to derive the loss function for case 1a,

\[ (32) \quad L_{1a} = E \{ \gamma_1((\pi_15 V_t^* + \pi_18 u_t^* + \pi_19 u_t^{**})) + \gamma_2((\pi_15 - \pi_35) V_t^* + (-\pi_26 - \pi_36) V_t^{**} + (\pi_18 - \pi_28 - \pi_38) u_t^* + (\pi_19 - \pi_29 - \pi_39) u_t^{**}) + u_t \}^2 \]

2. Case 1b:

The small country fixes its exchange rate to country B's currency and floats against country A. Using equations (16) and (20) the following is obtained:

\[ (33) \quad P_t^{1S} = P_t^{1A} - S_t^{AB} + S_t^B \]

In this case, \( S_t^B \), the exchange rate between the small country and country B is fixed. Equation (23) indicates that, because \( P_t^{1A} \) and \( S_t^{AB} \) are given, the domestic price is exogenous. So the price level in the small country is
affected by disturbances that affect the price level of good 1 in country A and the exchange rate between country A and country B. The exchange rate between the small country and country A changes inversely with the exchange rate between countries A and B.

The loss function, equation (19), depends on the domestic and relative price prediction errors. Using equation (33), then,

\[ P_t^{1S} - EP_t^{1S} = (P_t^{1A} - S_{tAB} + S_t^B) - E (P_t^{1A} - S_{tAB} + S_t^B) \]

So the domestic price prediction error is equal to the domestic errors in predicting the price of good 1 in country A and the errors in predicting the exchange rate between country A and country B. To find the relative price prediction error for this case, equation (23) is used and substituted for \( P_t^{1S} \) in the relative price to get,

\[ P_t^{1S} - P_t^{2B} - S_t^B = P_t^{1A} - S_{tAB} - P_t^{2B} = P_t^{1A} - P_t^{2B} - S_{tAB} \]

Therefore, the relative price prediction error is given by

\[ (P_t^{1S} - P_t^{2B} - S_t^B) - E (P_t^{1S} - P_t^{2B} - S_t^B) = (P_t^{1A} - P_t^{2B} - S_{tAB}) - E (P_t^{1A} - P_t^{2B} - S_{tAB}) \]

By substituting equations (34) and (35) into (19) the loss function for this case becomes,

\[ L = E [ \gamma_1 ((P_t^{1A} - S_{tAB} + S_t^B) - E(P_t^{1A} - S_{tAB} + S_t^B)) + \gamma_2 ((P_t^{1A} - P_t^{2B} - S_{tAB}) - E (P_t^{1A} - P_t^{2B} - S_{tAB})) + u_t ]^2 \]
The loss function in this case is dependent on the domestic errors in predicting the price of good 1 in country A and the exchange rate between countries A and B. The relative price prediction error is equal to the error in predicting the price of good 1 in country A and the price of good 2 in country B, and the exchange rate between countries A and B. The loss function is again unaffected by random disturbances occurring in the small country.

The foreign price and exchange rate solutions can also be obtained. The solutions for $P_{t1A}$, $P_{t2B}$ and $S_{tAB}$ are as expressed for case 1a. The expected value of $P_{t1A}$, $P_{t2B}$ and $S_{tAB}$ are:

$$E_{t-1} P_{t1A} = \pi_{11} m_{t-1} + \pi_{12} m_{t-1}^* + \pi_{13} m_{t-1}$$
$$E_{t-1} P_{t2B} = \pi_{21} m_{t-1} + \pi_{22} m_{t-1}^* + \pi_{23} m_{t-1}$$
$$E_{t-1} S_{tAB} = \pi_{31} m_{t-1} + \pi_{32} m_{t-1}^* + \pi_{33} m_{t-1}$$

The exchange rate between the small country and country B is fixed, $S_{tB}$, therefore $E_{t-1} S_{tB} = S_{tB}$. The unexpected part of domestic price prediction is,

$$E_{t-1} (P_{t1A} - S_{tAB} + S_{tB}) - E (P_{t1A} - S_{tAB} + S_{tB}) =$$

$$E (P_{t1A} - S_{tAB}) - E (P_{t1A} - S_{tAB})$$

$$E (P_{t1A} - S_{tAB}) - E (P_{t1A} - S_{tAB}) = (\pi_{15} - \pi_{35}) V_t^* +$$

$$(\pi_{16} - \pi_{36}) V_t^{**} + (\pi_{18} - \pi_{38}) u_t^* + (\pi_{19} - \pi_{39}) u_t^{**}$$

As for case 1a, the value for some of the coefficients are zero. Then, the domestic and relative price prediction errors become, respectively,
(39) \((P_{t}^{1A} - S_{t}^{AB}) - E(P_{t}^{1A} - S_{t}^{AB}) = (\pi_{15} - \pi_{35})V_{t}^{*} + (-\pi_{36})V_{t}^{**} + (\pi_{18} - \pi_{38})u_{t}^{*} + (-\pi_{39})u_{t}^{**}\)

(40) \((P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) - E(P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) = (\pi_{15} - \pi_{35})V_{t}^{*} + (-\pi_{26} - \pi_{36})V_{t}^{**} + (\pi_{18} - \pi_{38})u_{t}^{*} + (-\pi_{29} - \pi_{39})u_{t}^{**}\)

Substituting the above expressions for domestic and relative price prediction errors into (36) enables the loss function to be derived for this case as,

\[
L_{1b} = E[\gamma_{1}(\pi_{15} - \pi_{35})V_{t}^{*} + (-\pi_{36})V_{t}^{**} + (\pi_{18} - \pi_{38})u_{t}^{*} + (-\pi_{39})u_{t}^{**}] + \gamma_{2}((\pi_{15} - \pi_{35})V_{t}^{*} + (-\pi_{26} - \pi_{36})V_{t}^{**} + (\pi_{18} - \pi_{28} - \pi_{38})u_{t}^{*} + (\pi_{19} - \pi_{29} - \pi_{39})u_{t}^{**}) + u_{t}^{*}]^{2}
\]

3. Case lc:

The small country adopts a flexible exchange rate system against both countries. If the small country's currency floats against country A and country B's currencies, any disequilibrium in the money market disappears. Price changes to guarantee money market clearing. The balance of payments is in equilibrium, so there is no change in domestic money supply coming from changes in balance of payments. Using equations (16) and (20), and substituting for \(P_{t}^{1s}\) in relative price we get,

\[
P_{t}^{1s} - P_{t}^{2B} - S_{t}^{B} = P_{t}^{1A} + S_{t}^{A} - P_{t}^{2B} - S_{t}^{B} = P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}
\]

Subsequently, the relative price prediction error becomes;
The loss function in this case becomes

\[
L = E \left[ \gamma_1 \left( \left( P_t^{1s} - EP_t^{1s} \right) \right) + \gamma_2 \left( \left( P_t^{1A} - P_t^{2B} - S_t^{AB} \right) - E (P_t^{1A} - P_t^{2B} - S_t^{AB}) \right) + \mu_t \right]^2
\]

Under the flexible exchange rate regime domestic price is endogenous and the relative price of good 2 in terms of the domestic price of good 1 is exogenously determined. The domestic price prediction error is affected by real and monetary random disturbances occurring in the small country as well as in country A and country B. However, the domestic random disturbances do not affect the relative price. The relative price prediction error depends on domestic errors in predicting \( P_t^{1A} \), \( P_t^{2B} \) and \( S_t^{AB} \).

The relative price prediction error in this case is the same as in cases 1a and 1b. Using the small country money and goods market, the domestic price solution can be expressed as

\[
P_t^{1s} = \pi_{51} m_{t-1} + \pi_{52} m_{t-1}^* + \pi_{53} m_{t-1}^{**} + \pi_{54} V_t + \pi_{55} V_t^* + \pi_{56} V_t^{**} + \pi_{57} u_t + \pi_{58} u_t^* + \pi_{59} u_t^{**}
\]

The expected value of domestic price, \( P_t^{1s} \), is,

\[
E t^{-1} P_t^{1s} = \pi_{51} m_{t-1} + \pi_{52} m_{t-1}^* + \pi_{53} m_{t-1}^{**}
\]

And the unexpected part of domestic price becomes,

\[
P_t^{1s} - E t^{-1} P_t^{1s} = \pi_{54} V_t + \pi_{55} V_t^* + \pi_{56} V_t^{**} + \pi_{57} u_t + \pi_{58} u_t^* + \pi_{59} u_t^{**}
\]
Because domestic price is exogenous and the price of good 1 is set by country A, domestic disturbances do not have any effect on $P_t^{L1}$. Therefore $\pi_51 = \pi_54 = \pi_57 = 0$.

If the small country floats its exchange rate against the two large countries, monetary disturbances occurring in country A or country B will not be transmitted. They are absorbed by the exchange rate; and $\pi_52 = \pi_53 = \pi_55 = \pi_56 = 0$ then relation (46) becomes,

$$P_t^{L1} - E_{t-1}P_t^{L1} = \pi_58 u_t^* + \pi_59 u_t^{**}$$

In order to derive the loss function for this case, the price prediction error, (47), and the relative price prediction error, (40), are substituted in, (44), to get,

$$L_{1c} = E [ \gamma_1 (\pi_58 u_t^* + \pi_59 u_t^{**}) + \gamma_2 ((\pi_15 - \pi_35)V_t^* + (-\pi_26 - \pi_36)V_t^{**} + (\pi_18 - \pi_28 - \pi_38)u_t^* + (\pi_19 - \pi_29 - \pi_39)u_t^{**} + u_t)^2]$$

Relations (23), (36) and (44) represent the loss functions for cases 1a, 1b, and 1c respectively. The loss functions depend on domestic price and relative price prediction errors. The domestic and relative price prediction errors are obtained from domestic and foreign price solutions and goods arbitrage conditions. Solve the model for three types of exchange rate regimes and then substitute these price and exchange rate solutions into (24), (36), and (44), respectively relations (32), (41), and (48) are derived. The latter show the loss functions in terms of monetary and real disturbances.
TRANSMISSION OF FOREIGN DISTURBANCES

The loss functions indicate that the prediction error depends on domestic errors in predicting $P_{t}^{1A}$, $P_{t}^{2B}$ and $S_{t}^{AB}$. Random shocks to the stock of money (monetary disturbances) and supply of output (real disturbances) in countries A and B have direct effects on the small country via the price channel and relative price direction. The small country cannot fully insulate itself from real and monetary disturbances in countries A and B under any type of exchange rate regime in a two good-three country model where the small country is importing one good to produce a finished good which is exported to large countries.

**Monetary Shock**

Unexpected monetary expansion in country A or B depreciates its currency relative to other country's currency ($S_{t}^{AB}$ is higher or lower than expected) and causes the price of good 1 (produced by country A) or the price of good 2 (produced by country B) to exceed expected prices. A permanent unexpected monetary disturbance is unlikely to cause exchange rate overshooting, but a temporary disturbance does cause overshooting. Assuming a rational expectation world in which people understand a policy and assume it will be long-lasting, there is no overshooting. Also, in a rational expectation world, following the purchasing power parity theory, the change in $P_{t}^{1A}$ is equal
to change in \( S_{t}^{AB} \) and when supply increases in country B, the change in \( P_{t}^{2B} \) is equal to change in \( S_{t}^{AB} \). With these assumptions as a base, different cases can be discussed.

1. Money Supply Increases In Country A: If country A's money supply is larger than expected, then \( P_{t}^{1A} \) and \( S_{t}^{AB} \) increase more than anticipated. Higher \( P_{t}^{1A} \) and \( S_{t}^{AB} \) lead to fluctuations in domestic price and relative price. The effect of this change on the different loss functions is as follows:

1.1 Case 1a: The loss function relation (23), for this case is:

\[
L = E \left[ \gamma_{1}(P_{t}^{1A} + S_{t}^{A}) - E(P_{t}^{1A} + S_{t}^{A}) + \gamma_{2}(P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) - E(P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) + \mu_{t} \right]^{2}
\]

Higher \( P_{t}^{1A} \) causes the domestic price prediction error to increase, but the effect of higher \( P_{t}^{1A} \) and \( S_{t}^{AB} \) on the relative price prediction error, considering equal change and opposite signs for \( P_{t}^{1A} \) and \( S_{t}^{AB} \), offset each other. The only effect of increased money supply in country A on the loss function is to increase in \( P_{t}^{1A} \) which results in a higher domestic price prediction error and an increase in the loss.

1.2 Case 1b: Recall relation (36),

\[
L = E \left[ \gamma_{1}(P_{t}^{1A} - S_{t}^{AB} + S_{t}^{B}) - E(P_{t}^{1A} - S_{t}^{AB} + S_{t}^{B}) + \gamma_{2}(P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) - E(P_{t}^{1A} - P_{t}^{2B} - S_{t}^{AB}) + \mu_{t} \right]^{2}
\]

Higher \( P_{t}^{1A} \) and \( S_{t}^{AB} \) than expected affect both domestic and relative price prediction errors. An equal magnitude of
change and opposite sign of $P_{t1A}$ and $S_{tAB}$ offset each effect and results in zero change in loss.

1.3. Case 1c: The following relation represents the loss function for this case:

$$L = E \left[ \gamma_1 \left( (P_{t1S} - EP_{t1S}) \right) + \gamma_2 \left( (P_{t1A} - P_{t2B} - S_{tAB}) - E (P_{t1A} - P_{t2B} - S_{tAB}) \right) + u_t \right]^2$$

Expansionary monetary policy in country A increases $P_{t1A}$ and $S_{tAB}$ more than expected. Higher $P_{t1A}$ and $S_{tAB}$ affect the relative price prediction, but the opposite direction of the effects offset each other, thereby leaving the relative price prediction unchanged. The domestic price prediction error is not dependent on change in either of these factors. The currency of country A relative to that of small country depreciates in proportion to the increase in the price level in country A. Thus a monetary shock under flexible exchange rates is not transmitted to small country.

2. Money Supply Increases In Country B: Money supply increases in country B lead to higher $P_{t2B}$ (good 2 produced by country B) and lower $S_{tAB}$ (country B's currency depreciation). The effect of these changes in $P_{t2B}$ and $S_{tAB}$ in different loss functions are as follows:

2.1. Case 1a:

Changes in $P_{t2B}$ and $S_{tAB}$ do not change the domestic price prediction error. The relative price prediction error will be affected by these changes, but the opposite
direction of change in $P_{t}^{2B}$ relative to $S_{t}^{AB}$ offsets any effects, and there is not a change in loss.

2.2. Case 1b: Unanticipated increases in $P_{t}^{2B}$ and $S_{t}^{AB}$ affect domestic and relative price prediction errors. Lower than expected $S_{t}^{AB}$ increases domestic prices but changes in $P_{t}^{2B}$ and $S_{t}^{AB}$ offset each other, and the relative price prediction stays the same. The final result is higher loss to the small-country.

2.3. Case 1c: In this case the domestic price is again not dependent on $P_{t}^{2B}$ or $S_{t}^{AB}$ and opposite direction of changes in $P_{t}^{2B}$ and $S_{t}^{AB}$ leave the relative price unchanged. Depreciation of country B's currency relative to the small country currency absorbs the increase in prices of country B.

Supply Shock

A temporary disturbance increasing the output in country A or B (a real effect) causes the price of good 1 ($P_{t}^{1A}$) or good 2 ($P_{t}^{2B}$) to decrease more than expected, but the change in the exchange rate ($S_{t}^{AB}$) is ambiguous. The effect of changes in $P_{t}^{1A}$, $P_{t}^{2B}$ and $S_{t}^{AB}$ for different cases follow.

1. Output In Country A Increases:

1.1. Case 1a: Changes in $P_{t}^{1A}$ and $S_{t}^{AB}$ cause fluctuations in domestic as well as relative price prediction errors. Lower $P_{t}^{1A}$ than expected decreases the domestic price prediction error, but the change in relative
price prediction error depends on the direction of change in $S_{t}^{AB}$. If the supply shock in country A causes attending $S_{t}^{AB}$ to increase (country A's currency depreciation), then the decline in $P_{t}^{1A}$ and increase in $S_{t}^{AB}$ also cause a lower relative price prediction and a larger loss to follow. If $S_{t}^{AB}$ decreases, it offsets the fall in $P_{t}^{1A}$ and the final effect on relative price, and the loss function depends on the magnitude of changes in $P_{t}^{1A}$ and $S_{t}^{AB}$ and also $\gamma_1$ and $\gamma_2$.

1.2. Case 1b: In this case the effect of changes in $P_{t}^{1A}$ and $S_{t}^{AB}$ on the loss function depend completely on the magnitude and direction of change in both factors. A larger $S_{t}^{AB}$ than expected adds to the lower $P_{t}^{1A}$ and lowers both the domestic and relative price prediction errors, causing a larger loss. However, if an increase in supply in country A lowers $S_{t}^{AB}$ because of its offsetting effect, the final change in loss depends on the new magnitudes of $P_{t}^{1A}$, $S_{t}^{AB}$ and $\gamma_1$ and $\gamma_2$.

1.3. Case 1c: Higher output in country A does not have any effect on the domestic price prediction in this case, but causes variation in the relative price. Higher $S_{t}^{AB}$ adds to an existing lower $P_{t}^{1A}$ and results in higher loss. However, lower $S_{t}^{AB}$ offsets the lower $P_{t}^{1A}$ and the final effect on the loss function is ambiguous depending on the magnitude of changes in $P_{t}^{1A}$ and $S_{t}^{AB}$. 
2. Supply Increases In Country B:

2.1. Case 1a: Real disturbances in country B cause $P_t^{2B}$ to fall and an ambiguous change in $S_t^{AB}$. The domestic price prediction error is not affected by this change, but the change in relative price depends on the direction of change in $S_t^{AB}$. If $S_t^{AB}$ decreases, the resulting lower $P_t^{2B}$ and $S_t^{AB}$ cause a larger relative price prediction error and loss to the society. Higher $S_t^{AB}$ than expected offsets the lower $P_t^{2B}$ and the final effect depends on the magnitude of change in $P_t^{2B}$ and $S_t^{AB}$.

2.2. Case 1b: The direction of change in $S_t^{AB}$ has an important effect on the loss function. If $S_t^{AB}$ decreases, then a higher domestic and relative price prediction error results and a large loss is experienced. However higher $S_t^{AB}$ offsets lower $P_t^{2B}$ in the relative price prediction error and the magnitude of change in $P_t^{2B}$ and $S_t^{AB}$ determine the change in relative price prediction. $\gamma_1$ and $\gamma_2$ also affect the final loss result.

2.3. Case 1c: The domestic price prediction error is not dependent on $P_t^{2B}$ or $S_t^{AB}$ and changes effect only the relative price prediction error. Lower $S_t^{AB}$ with lower $P_t^{2B}$ results in large loss, but higher $S_t^{AB}$ offsets the change in $P_t^{2B}$ and the loss depends on the relative movements of each variable.
COMPARISON OF THE TRANSMISSION OF DISTURBANCES UNDER ALTERNATIVE EXCHANGE RATE SYSTEMS

The loss functions derived indicate that relative price prediction errors are the same under all exchange rate characterizations. So exchange rate systems can be compared based on movement in the domestic price prediction errors and by examining whether this error offsets or increases the relative price prediction error.

Monetary disturbances in country A, causing money supply to be larger than expected, cause no losses to the small country under cases 1b (fixed rates to B and floating rates against A) and 1c (flexible rates against A and B), but result in higher price prediction errors and larger losses under case 1a (fixed rates to A and floating rates against B). However, monetary disturbances in country B do not cause any change in loss to the small country in cases 1a and 1c, but losses do occur in case 1b. Therefore, the small country should choose its exchange rate system depending on the country in which disturbances originate. Pegging to the currency of the country in which the monetary disturbances originate does cause a loss to occur in small country. However, in the presence of monetary disturbances in country A or country B, selection of the flexible exchange rate by the small country is preferred in order to modify this loss because the disturbances are absorbed through depreciation or appreciation of the small country.
currency relative to the large countries’ currencies. Real disturbances in the large countries’ which increase foreign output has transmission effects on the small country. A supply shock results in an excess of foreign goods, causing their prices to fall and exchange rates to change ambiguously. Supply shocks are therefore responsible for short-run deflation abroad. The effects of the occurrence of a supply shock on the small country under different situations are summarized in tables I-VI.

A real disturbance increasing output in country A causes $p_{t}^{1A}$ to decrease, but the effect on $S_{t}^{AB}$ is ambiguous. If the currency of country A appreciates relative to country B’s currency, the following cases, shown in tables I and II, may occur.
TABLE I
REAL DISTURBANCE OCCURS IN COUNTRY A CAUSING LOWER $P_{t}^{1A}$ & $S_{t}^{AB}$ and $|\Delta P_{t}^{1A}| > |\Delta S_{t}^{AB}|$

<table>
<thead>
<tr>
<th>Case</th>
<th>Effect on domestic &amp; relative price prediction error</th>
<th>Effect on loss function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>DPPE ↓</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE ↓</td>
<td></td>
</tr>
<tr>
<td>1b</td>
<td>DPPE ↓</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE ↓</td>
<td></td>
</tr>
<tr>
<td>1c</td>
<td>DPPE  No change</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE ↓</td>
<td></td>
</tr>
</tbody>
</table>

- Order of Loss -

$L_{1a} > L_{1b} > L_{1c}$

DPPE = Domestic Price Prediction Error
RPPE = Relative Price Prediction Error
### TABLE II

REAL DISTURBANCE OCCURS IN COUNTRY A CAUSING LOWER $P_{t}^{1A}$ & $S_{t}^{AB}$ and $|\Delta P_{t}^{1A}| < |\Delta S_{t}^{AB}|$

<table>
<thead>
<tr>
<th>Case</th>
<th>Effect on domestic &amp; relative price prediction error</th>
<th>Effect on loss function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>DPPE $\downarrow$</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>RPPE $\uparrow$</td>
<td></td>
</tr>
<tr>
<td>1b</td>
<td>DPPE $\uparrow$</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE $\uparrow$</td>
<td></td>
</tr>
<tr>
<td>1c</td>
<td>DPPE No change</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE $\uparrow$</td>
<td></td>
</tr>
</tbody>
</table>

Order of Loss:

$L_{1b} > L_{1c} > L_{1a}$
Occurrence of a real disturbance in country A causes its output to increase, price of its product to decrease, and its currency to depreciate relative to foreign currencies. The effect of a real disturbance in country A on the loss function of the small country is shown in table III.

**TABLE III**
REAL DISTURBANCE OCCURS IN COUNTRY A CAUSING LOWER $p_t^{1A}$ & HIGHER $S_t^{AB}$

<table>
<thead>
<tr>
<th>Case</th>
<th>Effect on domestic &amp; relative price prediction error</th>
<th>Effect on loss function</th>
</tr>
</thead>
</table>
| 1a   | DPPE ↑  
      | RPPE ↓  | Increase |
| 1b   | DPPE ↑  | Increase |
| 1c   | DPPE No change  
      | RPPE ↓  | Increase |
      | - Order of Loss - |
      | $L_{1b} > L_{1a} > L_{1c}$ |


Occurrence of real disturbances in country B decrease \( P^2_{tB} \), but the effect on \( S^AB_t \) is ambiguous. The effects on the loss function under different circumstances are summarized in tables IV-VI.

**TABLE IV**

**THE CURRENCY OF COUNTRY B RELATIVE TO COUNTRY A DEPRECIATES, LOWER \( S^AB_t \) THAN EXPECTED**

<table>
<thead>
<tr>
<th>Case</th>
<th>Effect on domestic &amp; relative price prediction error</th>
<th>Effect on loss function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>DPPE No change ( \uparrow )</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE</td>
<td></td>
</tr>
<tr>
<td>1b</td>
<td>DPPE ( \uparrow )</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE ( \uparrow )</td>
<td></td>
</tr>
<tr>
<td>1c</td>
<td>DPPE No change ( \uparrow )</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE</td>
<td></td>
</tr>
</tbody>
</table>

- Order of Loss -

\( L_{1b} > L_{1a} = L_{1c} \)
<table>
<thead>
<tr>
<th>Case</th>
<th>Effect on domestic &amp; relative price prediction error</th>
<th>Effect on loss function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>DPPE: No change, RPPE: ↑</td>
<td>Increase</td>
</tr>
<tr>
<td>1b</td>
<td>DPPE: ↓, RPPE: ↑</td>
<td>?</td>
</tr>
<tr>
<td>1c</td>
<td>DPPE: No change, RPPE: ↑</td>
<td>Increase</td>
</tr>
</tbody>
</table>

- Order of Loss -

L1a = L1c > L1b
TABLE VI
REAL DISTURBANCE IN COUNTRY B CAUSING LOWER $P_t^{2B}$ and HIGHER $S_t^{AB}$ THAN EXPECTED AND $| \Delta P_t^{2B} | < | \Delta S_t^{AB} |

<table>
<thead>
<tr>
<th>Case</th>
<th>Effect on domestic &amp; relative price prediction error</th>
<th>Effect on loss function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>DPPE No change</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE $\downarrow$</td>
<td></td>
</tr>
<tr>
<td>1b</td>
<td>DPPE $\downarrow$</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE $\downarrow$</td>
<td></td>
</tr>
<tr>
<td>1c</td>
<td>DPPE No change</td>
<td>Increase</td>
</tr>
<tr>
<td></td>
<td>RPPE $\downarrow$</td>
<td></td>
</tr>
</tbody>
</table>

- Order of Loss -

$L_{1b} > L_{1a} = L_{1c}$
Ambiguity in direction and magnitude of change in exchange rate and prices in the large countries results in different effects on the loss function. Of the six different cases previously discussed, adoption of the flexible exchange rate system is preferred and results in the least loss in four of the cases. However, adoption of an exchange rate regime depends on the specific circumstances, particularly, the source of the disturbances.

When the small country chooses to fix its exchange rate to country A and float against country B, or to fix to country B and float against country A, its prices are determined exogenously. Price is determined endogenously under the flexible exchange rate system. The loss functions for the three different options have been derived which indicates that the relative price prediction errors are the same under all exchange rate setups. Exchange rate systems were compared based on domestic price prediction error and by examining whether domestic price prediction error offset or expanded relative price prediction error.

Monetary disturbances in country A, causing the money supply to be larger than expected, cause no loss to the small country under cases 1b and 1c, but resulted in higher price prediction errors and larger loss under case 1a (fixed rates to A and floating rates against B). Monetary disturbances in country B did not cause any change in loss to the small country in cases 1a and 1c, but increased the
loss in case 1b (fixed rates to B and floating rates against A). Therefore, the small country should choose its exchange rate system depending on the country in which disturbances originate. Pegging to the currency of the country in which the monetary disturbances originate causes loss to the small country. In the presence of monetary disturbances in country A or country B, selection of a flexible exchange rate by the small country is preferred, since such disturbances are absorbed through depreciation or appreciation of the small country currency relative to the large countries currencies.

Real disturbances in country A or in country B caused lower $P_t^{1A}$ and $P_t^{2B}$ respectively, but the effect on $S_t^{AB}$ was ambiguous. If $S_t^{AB}$ increases as a result of a supply shock in country A or decreases because of a supply shock in country B, adoption of a flexible exchange rate by the small country is preferred. However, if $S_t^{AB}$ decreases or increases because of the occurrence of real disturbances in country A or country B, the result is totally dependent on the magnitude of change in $P_t^{1A}$ and $S_t^{AB}$ as well as values of $\gamma_1$ and $\gamma_2$. The order of loss for each case is shown in tables I-VI.
CHAPTER IV

THE THREE COUNTRY - THREE GOOD MODEL

The world is assumed to consist of three countries and three goods. Two of the three countries are assumed to be "large" and the third country is "small". Of the three goods, one is assumed to be a finished good and the others are intermediate goods. Large countries A and B produce intermediate goods (goods 2 and 3 respectively) and the small country produces a finished good (good 1), using imported goods 2 and 3, and exports to the large countries. The small country does not import good 2 or good 3 from any of the large countries for consumption. The countries issue their own currency which is not internationally tradable. It is assumed that the small country faces foreign real and monetary disturbances through the process of importing goods 2 and 3 for production of domestic output. The following assumptions are also made:

1. Expectations are assumed to be rational.
2. Perfect capital mobility exists between the two large countries (country A and country B), but capital is immobile between the small country and the large countries.
3. Goods arbitrage is assumed to be perfect and all goods are traded.
4. The model is assumed to be logarithmically linear in all variables except the interest rate.
5. Each country's economy consists of two markets, i.e. the goods market and the money market. No bonds or paper issues exist.

THE MODEL

Notation

Lower case letters denote logarithms of variables while Greek letters represent parameters and they are all assumed to be positive. Subscripts refer to the time dimension.

\[ \begin{align*}
Y & \quad \text{Small country-real output} \\
Y^* & \quad \text{Country A real output} \\
Y^{**} & \quad \text{Country B real output} \\
\bar{Y} & \quad \text{Small country-normal (desired) real output} \\
\bar{Y}^* & \quad \text{Country A-normal (desired) real output} \\
\bar{Y}^{**} & \quad \text{Country B-normal (desired) real output} \\
p_{1s} & \quad \text{Price of good 1 in small country} \\
p_{2s} & \quad \text{Price of good 2 in small country} \\
p_{3s} & \quad \text{Price of good 3 in small country} \\
p_{1A} & \quad \text{Price of good 1 in country A} \\
p_{2A} & \quad \text{Price of good 2 in country A} \\
p_{3A} & \quad \text{Price of good 3 in country A} \\
p_{1B} & \quad \text{Price of good 1 in country B} \\
p_{2B} & \quad \text{Price of good 2 in country B} \\
p_{3B} & \quad \text{Price of good 3 in country B} \\
S_{cA} & \quad \text{Small country- currency price of country A-}
\end{align*} \]
currency

$S_t^B$  Small country-currency price of country B-currency

$S_{AB}$  Country A-currency price of country B-currency

$m$  Small country-nominal stock of money balance

$m^*$  Country A-nominal stock of money balance

$m^{**}$  Country B-nominal stock of money balance

$i$  Small country-interest rate

$i^*$  Country A-interest rate

$i^{**}$  Country B-interest rate

$u$  Small country-stochastic disturbance in output supply with mean zero and finite variance $6u^2$

$u^*$  Country A-stochastic disturbance in output supply with zero and finite variance $6u^2$

$u^{**}$  Country B-stochastic disturbance in output supply with zero and finite variance $6u^2$

$V$  Small country-stochastic disturbance in money market with zero mean and finite variance $6v^2$

$V^*$  Country A-stochastic disturbance in money market with zero mean and finite variance $6v^2$

$V^{**}$  Country B-stochastic disturbance in money market with zero mean and finite variance $6v^2$

$E$  Mathematical expectation operator

$t-jE\ P_t^{1s}$  Expected value of $P_t^{1s}$ at time $t$, conditional on information available at time $t-j$

$C^*$  Country A cost-of-living index which contains
prices of both goods, (measured in domestic currency), expressed in logarithms,

Country B cost-of-living index which contains
prices of both goods (measured in domestic currency) expressed in logarithms,

\[ \theta^*, \delta^*, \lambda^* \] Expenditure shares on each commodity for country A

\[ \theta^{**}, \delta^{**}, \lambda^{**} \] Expenditure shares on each commodity for country B

Small Country
1) \[ Y_t = \bar{Y}_t + \gamma_1 (P_t^{1s} - E P_t^{1s}) + \gamma_2 ((P_t^{1s} - P_t^{2A} - S_t^A) - E (P_t^{1s} - P_t^{2A} - S_t^A)) + \gamma_3 ((P_t^{1s} - P_t^{3B} - S_t^B) - E (P_t^{1s} - P_t^{3B} - S_t^B)) + u_t \] (Goods supply)
2) \[ Y_t^d = \alpha_0 + \alpha_1 (m_t - P_t) + \alpha_2 Y_t \] (Goods demand)
3) \[ m_t^d - p_t = \beta_0 + \beta_1 Y_t - \beta_2 E_t (P_t^{1s} - P_t^{1s}) \] (Money demand)
4) \[ m_t = m_{t-1} + V_t \] (Money supply)

Country A
5) \[ Y_t^* = \gamma_1^* (C_t^* - EC_t^*) + \gamma_2^* ((P_t^{1A} - C_t^*) - E (P_t^{1A} - C_t^*)) + u_t^* \] (Goods supply)
6) \[ Y_t^{d*} = \alpha_0^* + \alpha_1^* (P_t^{2A} - P_t^{1A}) + \alpha_2^* Y_t^* \] (Goods demand)
7) \[ m_t^{d*} - C_t^* = \beta_0^* + \beta_1 Y_t^* - \beta_2^* i_t^* \] (Money demand)
8) \( m_t^* = m_{t-1}^* + V_t^* \)  
(Money supply)

9) \( C^* = \alpha^*P_t^2A + \delta^*(P_t^1S - S_t^A) + \lambda^*(P_t^3B + S_t^{AB}) \)  
(Cost-of-living Index)

**Country B**

10) \( Y_t^* = \gamma_1^*(C_t^* - E C_t^*) + \gamma_2^*((P_t^2B - C_t^*) - E (P_t^3B - C_t^*)) + u_t^* \)  
(Goods supply)

11) \( Y_t^* = \alpha_0^* + \alpha_1^*(P_t^1B - P_t^3B) + \alpha_2^*Y_t^* \)  
(Goods demand)

12) \( m_t^* - C_t^* = \beta_0^* + \beta_1^*Y_t^* - \beta_2^*i_t^* \)  
(Money demand)

13) \( m_t^* = m_{t-1}^* + V_t^* \)  
(Money supply)

14) \( C_t^* = \theta^*P_t^3B + \delta^*(P_t^1S - S_t^B) + \lambda^*(P_t^2A - S_t^{AB}) \)  
(Cost of living Index)

Equation (1) is the supply function. Current output, expressed in terms of its deviation from the normal level \( \bar{Y} \), is a function of domestic price prediction error and relative price prediction errors, and \( u_t \) is a stochastic disturbance term having zero mean and constant variance. Supply is a positive function of the price of finished goods (produced by the small country), \( P_t^1S \), and is a negative function of input prices, \( P_t^2A \) and \( P_t^3B \), (produced by country A and country B). Equations (1) and (2) are dependent on the underlying production function in the small country and have important implications for how effectively various exchange rate regimes insulate domestic output from
unanticipated disturbances.

Equation (2) represents the demand for the domestic good which is assumed to depend positively on real money balances, domestic income, and foreign countries' income. Equations (3), (7), and (12) state the demand for real money by residents of the domestic country and are assumed to be dependent on real output positively and nominal interest rate negatively.

Equation (4) specifies the money supply process in the small country when it elects to adopt flexible exchange rates. Equations (8) and (13) represent the money supply for country A and country B respectively. Equations (9) and (14) define the cost-of-living index which contains the prices of both goods in terms of domestic currency, where $\theta^*, \delta^*, \lambda^*$, and $\theta^{**}, \delta^{**}, \lambda^{**}$ are the expenditure shares on each good for country A and country B respectively.

Equation (5) represents the supply function of country A. Supply is a function of the cost-of-living prediction errors, and domestic real price prediction errors and a stochastic disturbance term ($u_t^*$). Equation (10) is similar. Equations (6) and (11) are demands for goods and are positively related to national income and relative prices. As in the three country-two good model, the interest rate parity relation (perfect capital mobility between two large countries) and purchasing power parity relation (law of one price) exist.
The loss function is used as the criterion for comparing different exchange rate regimes.

\[(15) \quad L = E \left[ (Y_t - \bar{Y}_t)^2 \right] \]

In order to derive the loss function for the small country in the form specified by (15), the supply function (1) is used as,

\[ Y_t - \bar{Y}_t + \gamma_1 (P_t^{1S} - EP_t^{1S}) + \gamma_2 ((P_t^{1S} - P_t^{2A} - S_t^A) - E (P_t^{1S} - P_t^{2A} - S_t^A)) + \gamma_3 ((P_t^{1S} - P_t^{3B} - S_t^B) - E (P_t^{1S} - P_t^{3B} - S_t^B)) + \eta_t \]

Subtract normal output, $\bar{Y}$, from both sides and square the expectation of both sides of the equation to get the following:

\[ E [(Y_t - \bar{Y}_t)^2] = E \left[ \gamma_1 (P_t^{1S} - EP_t^{1S}) + \gamma_2 ((P_t^{1S} - P_t^{2A} - S_t^A) - E (P_t^{1S} - P_t^{2A} - S_t^A)) + \gamma_3 ((P_t^{1S} - P_t^{3B} - S_t^B) - E (P_t^{1S} - P_t^{3B} - S_t^B)) + \eta_t \right]^2 \]

The left side of the above equation is the same as the right side of equation (15). Therefore, the following relation is representative of the loss function:

\[(16) \quad L = E \left[ \gamma_1 (P_t^{1S} - EP_t^{1S}) + \gamma_2 ((P_t^{1S} - P_t^{2A} - S_t^A) - E (P_t^{1S} - P_t^{2A} - S_t^A)) + \gamma_3 ((P_t^{1S} - P_t^{3B} - S_t^B) - E (P_t^{1S} - P_t^{3B} - S_t^B)) + \eta_t \right]^2 \]

The above loss function is dependent on domestic and foreign prices, $P_t^{1S}$, $P_t^{2A}$, and $P_t^{3B}$, and the exchange rates between the small country and the two large countries. The loss function is affected by domestic as well as foreign random disturbances occurring in the large countries.
Again the social objective of the small country is to minimize loss. In the following pages the effect of external disturbances on the loss function under alternative exchange rate regimes will be examined, the results will be compared, and alternative regimes ranked according to their insulative properties.

**MONETARY SHOCK**

Unexpected monetary expansion in country A or B depreciates its currency relative to the small country's and causes the prices of good 2 (produced by country A) good 3 (produced by country B) to exceed expectations.

1. **Case 1:**

The small country pegs its currency to country A's and floats against country B's currency. The exchange rate between the small country and country A is fixed, and expansionary monetary policy in country A causes higher \( P_{t}^{A} \), which lowers the relative price prediction error and loss to the society.

2. **Case 2:**

The small country pegs its currency to country B's and floats against country A's currency. In this case, \( S_{t}^{B} \) is fixed and monetary disturbance in country B does not affect the exchange rate but increases the price of good 3 more than expected, which results in lower relative price
prediction error and higher loss.

3. Case 3:

Currency of the small country floats against both large countries' currencies. Monetary expansion in country A or in country B causes higher prices and a lower exchange rate than expected. This expansion is not transmitted to the small country in this case because higher prices in the large countries will be absorbed by the exchange rates. Assuming a rational expectations world and the purchasing power parity theory, change in $P_t^{2A}$ is equal to change in $S_t^A$, and when the money supply increases in country B, the change in $P_t^{3B}$ is equal to change in $S_t^B$. Relation (16) shows that an increase in $P_t^{2A}$ is offset by decrease in $S_t^A$, and an increase in $P_t^{3B}$ is offset by a decrease in $S_t^B$; therefore monetary expansion does not affect the loss function.

SUPPLY SHOCK

A temporary disturbance increasing the output in country A or B (a real effect) causes the price of good 2 ($P_t^{2B}$) or good 3 ($P_t^{3B}$) to decrease more than expected, but the change in the exchange rates ($S_t^A$, $S_t^B$) is ambiguous. Different cases are developed in the following pages:

1. Output Increases in Country A:

Higher output than expected results in a lower price for good 2, $P_t^{2A} +$, but the exchange rate of country A
relative to the small country may appreciate or depreciate. If the large country's currency appreciates, $S_t^A \uparrow$, adoption of flexible exchange rates or pegging to country B is preferred because decreases in $P_t^{2A}$ offset the increases in $S_t^A$, and selection of pegging to country A's currency causes a higher loss. In the case of the large country's currency depreciation, $S_t^A \downarrow$, case 1 (peg to country A and float against country B) is preferred to modify the loss. Under cases 2 and 3 the small country experiences higher loss.

2. Output Increases in Country B:

Higher output causes lower prices for $P_t^{3B}$ and ambiguous changes in the exchange rate. The result is similar to the case when output increases in country A. Depreciation of country B's currency, $S_t^B \downarrow$, as a result of this real effect, makes case 2 (peg to country B's currency and float against country A's currency) the preferred choice. However, appreciation of country B's currency, $S_t^B$, changes the result and makes case 2 the least preferred choice. The other two choices (cases 1 and 3) cause loss to the small country.

In this model again the similar results to those of chapter III are reached. The small country could insulate itself against the country in which the monetary disturbances originate, because the exchange rate change absorbs external nominal disturbances. However, occurrence of real disturbances create different situations and its
CHAPTER V

THE TWO COUNTRY - TWO GOOD MODEL

An open economy model of a small country that faces real and monetary disturbances from a large country (assumed to be rest of the world) is developed. There are two goods, one is assumed to be a finished good (good 1) and the other is an intermediate good (good 2). The small country imports good 2 from the large country to produce good 1 and exports it to the large country. Goods arbitrage is assumed to be perfect, and all goods are traded, except that the small country imports good 2 only for production. Again expectations are assumed to be rational.

THE MODEL

Notation

Lower case letters denote logarithms of variables for the small country while an asterisk indicates the large country. Greek letters represent parameters and they are all assumed to be positive. Subscripts refer to the time dimension.

$Y$  Real output

$\bar{Y}$  Normal (desired) output

$p^1$  Price of good 1

$p^2$  Price of good 2

$S_t$  Small country-currency price of large country-currency

$m$  Nominal stock of money balance
Interest rate
Stochastic disturbance in output supply with mean zero and finite variance $\sigma_u^2$
Stochastic disturbance in money market with zero mean and finite variance $\sigma_v^2$
Mathematical expectation operator
Expected value of $P_t^1$ at time $t$, conditional on information available at time $t-j$
The cost of living index which contains prices of both goods, (measured in domestic currency), expressed in logarithms
Expenditure shares on each commodity

Small Country
(1) $Y_t = \bar{Y}_t + \gamma_1 (P_t^1 - EP_t^1) + \gamma_2 ((P_t^1 - P_t^{2*} - S_t)^2 - E(P_t^1 - P_t^{2*} - S_t)) + u_t$ (Goods Supply)
(2) $Y_t^d = \alpha_0 + \alpha_1 (m_t - P_t^1) + \alpha_2 Y_t$ (Goods demand)
(3) $m_t^d - P_t^1 = \beta_0 + \beta_1 Y_t - \beta_2 E_t (P_{t+1}^1 - P_t^1)$ (Money demand)
(4) $m_t = m_{t-1} + V_t$ (Money Supply)

Large Country
(5) $Y_t^* = \gamma_1^* (C_t^* - E C_t^*) + \gamma_2^* ((P_t^{2*} - C_t^*) - E (P_t^{2*} - C_t^*)) + u_t^*$ (Goods supply)
(6) $Y_t^d^* = \alpha_0^* + \alpha_1^* (P_t^1 - P_t^{2*}) + \alpha_2^* Y_t^*$ (Goods demand)
Equation (1) is the supply function. Current output, expressed in terms of its deviation from the normal level, $Y$, is a function of domestic price prediction error and relative price prediction error, and $u_t$ is a stochastic disturbance term having zero mean and constant variance.

Equation (2) represents the demand for the domestic good which is assumed to depend positively on real money balance of domestic and foreign income. Equations (3) and (7) state the demand for real money by residents of the small and the large countries and are assumed to be dependent on real output positively and nominal interest rate negatively.

Equation (4) specifies the money supply process in the small country when it elects to adopt flexible exchange rates. Equation (8) represent the money supply for the large country.

Equation (5) represents the supply function of the large country A. Supply is a function of the cost-of-living prediction errors and domestic real price prediction errors and a stochastic disturbance term ($u_t^*$). Equation (6) is demand for goods and is positively related to
national income and relative prices. Equation (9) defines the cost-of-living index which contains the prices of both goods in terms of domestic currency, where \(0\) and \((1-0)\) are the expenditure shares on each good.

There is a link between the small and the large countries purchasing power parity.

\[
(10) \quad P_t^1 = P_t^{1*} + S_t, \quad P_t^{2*} = P_t^* - S_t
\]

Equation (10) is the purchasing power parity relation and it states that the logarithm of domestic price is equal to the logarithm of foreign price plus (minus) the logarithm of exchange rate.

Again, a quadratic loss function which provides a measure of the loss is used,

\[
(11) \quad L = E_{t-1} [ (Y_t - \bar{Y}_t)^2 ]
\]

The supply function for the small country, (1), is used to derive the loss function in the form specified by (11), following the same process as for the three country-three good model, the loss function is:

\[
(12) \quad L = E \left[ (Y_t - \bar{Y}_t)^2 \right] = E \left[ \gamma_1 \left( P_t^1 - E P_t^1 \right) + \gamma_2 \left( P_t^{2*} - S_t \right) - E \left( P_t^1 - P_t^{2*} - S_t \right) \right] + u_t \left( Y_t - \bar{Y}_t \right)^2
\]

The above loss function is dependent on domestic error in predicting the prices of good 1 and good 2 in the small country. In the following pages the effects of various disturbances originating in the large country are examined.
MONETARY SHOCK

Expansionary monetary policy in the large country depreciates its currency relative to the small country’s currency, $S_t$, and causes the price of good 2 to increase more than expected. In a rational expectations world and with the existence of the purchasing power parity theory, the change in $P_t^{2*}$ is equal to the change in $S_t$. If the small country adopts a flexible exchange rate system inflation in the large country is absorbed through currency depreciation and is not transmitted to the small country. Selection of a fixed exchange rate regime (the small country pegs its currency to the large country), fixed $S_t$, causes higher relative price prediction error and loss to the small country.

SUPPLY SHOCK

Occurrence of real disturbances in the large country cause lower prices for good 2 than expected, $P_t^{2*}$, but the effect on the exchange rate, $S_t$, is ambiguous. Adoption of flexible exchange rate system by the small country and appreciation of the large country’s currency, $S_t$, offsets the effect of lower $P_t^{2*}$ and results in less loss, however, depreciation of the currency, $S_t$, expands the effect of lower $P_t^{2*}$ and the small country experiences a higher loss. With a fixed exchange rate (the small country pegs its currency to the large country), $S_t$ is fixed and the loss to
the small country is equal to the change in $P_t^{2*}$ and is not affected by appreciation or depreciation of the currency.

Similar conclusions as for other previous cases are reached. Nominal disturbances are not transmitted to the small country under a flexible exchange rate system, but they do cause loss under other systems. Real disturbances cause more losses under fixed than flexible exchange rate systems if they cause the large country's currency to appreciate, $S_t$. If real disturbances result in depreciation of the large country's currency, adoption of a fixed exchange rate system is preferred.
CHAPTER VI

CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

CONCLUSION

This study has focused on the choice of exchange rate regimes for a small country when external real and monetary disturbances occur. The choice of exchange rate was investigated using a loss function expressing the squared deviations of the small country output from desired output. To examine the transmission of economic disturbances, the loss function for the small country's economy was calculated under three different exchange rate regimes for each of the three models. The emphasis of the analysis was on a three-country (one small and two large) trading situation, whereby the small country trades with two major large country trading partners. In the first model, the small country imports an intermediate good from one of the large countries and produces a finished good to export. The same finished good is produced by one other large country. In the second model both large countries produce intermediate goods which are imported by the small country to produce a finished good which is exported to the large countries. The analysis incorporated rational expectations as well as perfect capital mobility between the two large countries. It is assumed throughout that there is perfect commodity arbitrage between two large countries. A third model analyzed the impact of exchange rate regime on the small country in a
world consisting of only two countries (one small and one large). The small country imports an intermediate good from the large country to produce a finished good and exports it to the large country. The small country does not import goods for consumption and it is assumed throughout that trade is necessary for production.

Independent floating is often considered infeasible by developing countries because of underdevelopment of domestic financial markets. However, ruling out some form of "clean" or "managed" floating does not remove the possibility of adopting other types of flexible exchange rate arrangements. Hence, in this study the flexible exchange rate system is found to generally be the superior choice for the small country under the assumptions of the model.

As writers in the 1950s and 1960s predicted and this analysis confirms, under the flexible exchange rate system, occurrence of purely nominal shocks abroad are not transmitted to the small country. Expansionary monetary policy of large country trading partners increases prices more than expected. Increasing the money supply also results in depreciation of the large country's currency relative to the small country's in proportion to the increase in the price level. Exchange rate changes fully offset any nominal disturbances. However, nominal disturbances occurring in the large country do affect the small country when the small country pegs its currency to
the currency of the country in which nominal disturbances originate.

The presence of real disturbances in large countries induces lower prices for the goods they produce, but the effect on the exchange rate is ambiguous. Ambiguity in direction and magnitude of change in exchange rate, as well as undetermined magnitude of change in a large country's prices, results in different effects on the loss function assumed in this study. In the three country-two good situation, if lower prices in the large countries are accompanied by depreciation of the currency of the country in which real disturbances occur, adoption of a flexible exchange rate is preferred. However, appreciation of the large countries' currencies results in four different cases. In the case of a supply shock in, say, country A (a large country), if change in $P_{t}^{1A}$ (price of good 1 in country A) is greater than change in $S_{t}^{AB}$ (appreciation of country A's currency relative to country B's currency), selection of a flexible exchange rate regime is advised, otherwise it is better to peg to country A's currency. The occurrence of supply shock in country B causing greater change in $S_{t}^{AB}$ than in $P_{t}^{2B}$ (price of good 2 in country B) results in lower losses to the small country under case 1c (a strategy to float against two large countries). Pegging to country B's currency is preferred when a real shock in country B results in greater change in $P_{t}^{2B}$ than in $S_{t}^{AB}$. This analysis
indicates in general that adoption of a flexible exchange rate system by a small country results in lower loss in most cases if the small country faces external disturbances.

The analysis concludes that the choice of an exchange rate system is a very delicate decision. A theoretical consideration of different exchange rate regimes indicates that an optimal choice of regime should depend upon specific conditions and parameter values. The United States, Japan, and Korea, or the United States, Japan, and Taiwan are examples of three-country trading situations. The United States and Japan are large countries and Korea and Taiwan could be considered small countries which trade with these large nations. The currency of the U.S. and Japan are independently floating and selection of an exchange rate regime for Korea or Taiwan is open to choice. The new Taiwan dollar has been continuously pegged to the U.S. dollar, though historically with many changes. Nominal and real disturbances originating in the U.S. affect Taiwan even with the adjusting peg system that has been implemented. Taiwan gains or loses its competitiveness against another country (Japan) when the dollar depreciates or appreciates respectively. Appreciation or depreciation of its currency is not really the result of the balance of payment situation. Taiwan has a large surplus and its economy is affected by changes in the U.S. economy. Korea is using a managed floating (crawling peg) system and attempts to
stabilize its currency with respect to changes in the U.S. dollar.

According to this study, these are not the appropriate regime choices and other thing being equal, could result in losses to these countries. The analysis indicates that in most cases the flexible exchange rate regime is preferred and adoption of the flexible regime would modify the effect of real and monetary disturbances occurring in the large countries which are trading partners with these less developed nations.

SUGGESTIONS FOR FUTURE EMPIRICAL STUDIES

This study has provided theoretical direction for making choices regarding exchange rate policy in a small, or less-developed country. Empirical work would involve deriving the magnitude of the loss function for many different trading situations of interest. For example, there are many issues of exchange rate, trade, and growth in the European-African trade situation, the Near East trading patterns, and the Pacific Rim trading conditions, but these trading situations and the choices of exchange rates which are associated with them are beyond the scope of this study. However, a projection of work that could be done to extend the theoretical directions of this study can be briefly outlined.

In the preceding chapters loss functions under
different assumptions for a small country were derived and the effects of external nominal and real disturbances were examined qualitatively. Empirical determination of the loss functions and their fluctuations, involving estimation of domestic and foreign price and exchange rate solutions, need to be carried out. The three country-two good model is an example of empirical work that can be done. Other models follow a similar pattern. This study analyzed the theoretical direction of three-country models. Future empirical work could involve a look at many different representative cases in order to completely describe trading regimes and the insulation policies that would be involved using the foregoing theoretical direction.

Determination of values for prices and exchange rates requires clearing conditions for the money and goods markets. The assumption that expectations are rational implies that prices and exchange rates (and other endogenous variables) are formulated using all available information at a particular time. The following are postulated solution forms for each of the endogenous variables in terms of exogenous variables. The solution is log linear in the exogenous variables.

1) \[ P_t^{1S} = \pi_{11} m_t - 1 + \pi_{12} m_t - 1^* + \pi_{13} m_t - 1** + \pi_{14} V_t + \pi_{15} V_t^* + \pi_{16} V_t** + \pi_{17} u_t + \pi_{18} u_t^* + \pi_{19} u_t** \]

2) \[ P_t^{1A} = \pi_{21} m_t - 1 + \pi_{22} m_t - 1^* + \pi_{23} m_t - 1** + \pi_{24} V_t + \pi_{25} V_t^* + \pi_{26} V_t** + \pi_{27} u_t + \pi_{28} u_t^* + \pi_{29} u_t** \]
3) \[ P_t^{2B} = \pi_{31} m_{t-1} + \pi_{32} m_{t-1}^* + \pi_{33} m_{t-1}^{**} + \pi_{34} V_t + \pi_{35} V_t^* + \pi_{36} V_t^{**} + \pi_{37} u_t + \pi_{38} u_t^* + \pi_{39} u_t^{**} \]

4) \[ S_t^{AB} = \pi_{41} m_{t-1} + \pi_{42} m_{t-1}^* + \pi_{43} m_{t-1}^{**} + \pi_{44} V_t + \pi_{45} V_t^* + \pi_{46} V_t^{**} + \pi_{47} u_t + \pi_{48} u_t^* + \pi_{49} u_t^{**} \]

5) \[ S_t^{B} = \pi_{51} m_{t-1} + \pi_{52} m_{t-1}^* + \pi_{53} m_{t-1}^{**} + \pi_{54} V_t + \pi_{55} V_t^* + \pi_{56} V_t^{**} + \pi_{57} u_t + \pi_{58} u_t^* + \pi_{59} u_t^{**} \]

6) \[ Y_t^* = \pi_{61} m_{t-1} + \pi_{62} m_{t-1}^* + \pi_{63} m_{t-1}^{**} + \pi_{64} V_t + \pi_{65} V_t^* + \pi_{66} V_t^{**} + \pi_{67} u_t + \pi_{68} u_t^* + \pi_{69} u_t^{**} \]

The above solutions are samples and the solution for the other endogenous variables: \( i, i^*, i^{**}, Y, Y^{**}, m, m^*, m^{**} \) should be found in terms of exogenous variables. The form used in the above solutions is based on the assumption that the small country adopts a flexible exchange rate system. If the small country pegs its currency to the currency of a large country the form for the small country's endogenous variables is different. For example, \( P_t^{1S} \) (price of good 1 in small country) is,

7) \[ P_t^{1S} = \pi_{10} r_{t-1} + \pi_{11} d_{t-1} + \pi_{12} m_{t-1}^* + \pi_{13} m_{t-1}^{**} + \pi_{14} V_t + \pi_{15} V_t^* + \pi_{16} V_t^{**} + \pi_{17} u_t + \pi_{18} u_t^* + \pi_{19} u_t^{**} \]

Where \( r_{t-1} \) is the foreign reserve and \( d_{t-1} \) is the domestic credit in the last period.

Finding solutions for the model requires determining values for each of the reduced form coefficients (\( \pi_{ij} \)). This could be done by substituting the postulated solutions for each endogenous variable, in terms of the exogenous
variables, into the demand and supply equations of the model. The interest rate parity and purchasing power parity relations could be used to extend the equation system and to find unknowns ($\pi_{ij}$s). For example, write the supply equation for country A.

8) \[ Y_t^* = \gamma_1^* (C_t^* - E C_t^*) - \gamma_2^* \left( (P_t 1S - P_t 2B - S_t B) - E (P_t 1S - P_t 2B - S_t B) \right) + u_t = 0 \]

Then substitute the postulated solutions for each of the endogenous variables. To find $C_t^*$ (Cost-of-living index for country A):

\[ C_t^* = \theta^* P_t 1A + (1 - \theta^*) (P_t 2B + S_t AB) = \theta^* \left( \pi_{21} m_{t-1} + \pi_{22} m_{t-1}^* + \pi_{23} m_{t-1}^{**} + \pi_{24} V_t + \pi_{25} V_t^* + \pi_{26} V_t^{**} + \pi_{27} u_t + \pi_{28} u_t^* + \pi_{29} u_t^{**} \right) + (1 - \theta^*) \left( \pi_{31} m_{t-1} + \pi_{32} m_{t-1}^* + \pi_{33} m_{t-1}^{**} + \pi_{34} V_t + \pi_{35} V_t^* + \pi_{36} V_t^{**} + \pi_{37} u_t + \pi_{38} u_t^* + \pi_{39} u_t^{**} + \pi_{51} m_{t-1} + \pi_{52} m_{t-1}^* + \pi_{53} m_{t-1}^{**} + \pi_{54} V_t + \pi_{55} V_t^* + \pi_{56} V_t^{**} + \pi_{57} u_t + \pi_{58} u_t^* + \pi_{59} u_t^{**} \right) \]

Simplifying the above relation would result in,

9) \[ C_t^* = \theta^* \left[ (\pi_{21} - \pi_{31} - \pi_{51}) m_{t-1} + (\pi_{22} - \pi_{32} - \pi_{52}) m_{t-1}^* + (\pi_{23} - \pi_{33} - \pi_{53}) m_{t-1}^{**} + (\pi_{24} - \pi_{34} - \pi_{54}) V_t + (\pi_{25} - \pi_{35} - \pi_{55}) V_t^* + (\pi_{26} - \pi_{36} - \pi_{56}) V_t^{**} + (\pi_{27} - \pi_{37} - \pi_{57}) u_t + (\pi_{28} - \pi_{38} - \pi_{58}) u_t^* + (\pi_{29} - \pi_{39} - \pi_{59}) u_t^{**} \right] + (\pi_{31} + \pi_{51}) m_{t-1} + (\pi_{32} + \pi_{52}) m_{t-1}^* + (\pi_{33} + \pi_{53}) m_{t-1}^{**} + (\pi_{34} + \pi_{54}) V_t + (\pi_{35} + \pi_{55} V_t^* + (\pi_{36} + \pi_{56}) V_t^{**} + (\pi_{37} + \pi_{57}) u_t + (\pi_{38} + \pi_{58}) u_t^* + (\pi_{39} + \pi_{59}) u_t^{**} \]

The rational expectation assumption requires that the
expectation of a variable is its expected value conditioned on available information. The expectation at time \( t-1 \) of the domestic price to prevail at time \( t \) is determined according to:

\[
E_{t-1}P_t^1 = \pi_{11} m_{t-1} + \pi_{12} m_{t-1}^* + \pi_{13} m_{t-1}^{**}
\]

Once the excess supply equation is expressed only in terms of exogenous variables, the coefficients of each exogenous variable are constrained to be zero.

The unexpected part of the cost-of-living index and relative price prediction error are determined as:

10) \( C_t^* - EC_t^* = \theta^* (\pi_{24} - \pi_{34} - \pi_{54}) V_t + (\pi_{25} - \pi_{35} - \pi_{55}) V_t^* + (\pi_{26} - \pi_{36} - \pi_{56}) V_t^{**} + (\pi_{27} - \pi_{37} - \pi_{57}) u_t + (\pi_{28} - \pi_{38} - \pi_{58}) u_t^* + (\pi_{29} - \pi_{39} - \pi_{59}) u_t^{**} + (\pi_{34} + \pi_{54}) V_t + (\pi_{35} - \pi_{55}) V_t^* + (\pi_{36} + \pi_{56}) V_t^{**} + (\pi_{37} + \pi_{57}) u_t + (\pi_{38} + \pi_{58}) u_t^* + (\pi_{39} + \pi_{59}) u_t^{**}

11) \( (P_t^1S - P_t^2B - S_t^B) - E (P_t^1S - P_t^2B - S_t^B) = (\pi_{14} - \pi_{34} - \pi_{54}) V_t + (\pi_{15} - \pi_{35} - \pi_{55}) V_t^* + (\pi_{16} - \pi_{36} - \pi_{56}) V_t^{**} + (\pi_{17} - \pi_{37} - \pi_{57}) u_t + (\pi_{18} - \pi_{38} - \pi_{58}) u_t^* + (\pi_{19} - \pi_{39} - \pi_{59}) u_t^{**}

substitute equations (10) and (11) into (8),

12) \( \pi_{61} m_{t-1} + \pi_{62} m_{t-1}^* + \pi_{63} m_{t-1}^{**} + \pi_{64} V_t + \pi_{65} V_t^* + \pi_{66} V_t^{**} + \pi_{67} u_t + \pi_{68} u_t^* + \pi_{69} u_t^{**} - \gamma_1^* (\theta^* [\pi_{24} - \pi_{34} - \pi_{54}) V_t + (\pi_{25} - \pi_{35} - \pi_{55}) V_t^* + (\pi_{26} - \pi_{36} - \pi_{56}) V_t^{**} + (\pi_{27} - \pi_{37} - \pi_{57}) u_t + (\pi_{28} - \pi_{38} - \pi_{58}) u_t^* + (\pi_{29} - \pi_{39} - \pi_{59}) u_t^{**}] + (\pi_{34} + \pi_{54}) V_t^+
\)
\[(\pi^{35} + \pi^{55}) V_t^* + (\pi^{36} + \pi^{56}) V_t^{**} + (\pi^{37} + \pi^{57}) u_t + (\pi^{38} + \pi^{58}) u_t^* + (\pi^{39} + \pi^{59}) u_t^{**}) - \gamma_2^* (\pi^{14} - \pi^{34} - \pi^{54}) V_t + (\pi^{15} - \pi^{35} - \pi^{55}) V_t^* + (\pi^{16} - \pi^{36} - \pi^{56}) V_t^{**} + (\pi^{17} - \pi^{37} - \pi^{57}) u_t + (\pi^{18} - \pi^{38} - \pi^{58}) u_t^* + (\pi^{19} - \pi^{39} - \pi^{59}) u_t^{**}) + u_t = 0\]

Simplify equation (12) to get,

\[\pi^{61} m_t-1 + \pi^{62} m_t-1^* + \pi^{63} m_t-1^{**} + \left[ \pi^{64} - \gamma_1^{*0}^* \right. \]
\[
\pi^{24} - \pi^{34} - \pi^{54} - \gamma_2^* (\pi^{14} - \pi^{34} - \pi^{54}) V_t + \left[ \pi^{65} - \gamma_1^{*0} (\pi^{25} - \pi^{35} - \pi^{55}) - \gamma_2^* (\pi^{15} - \pi^{35} - \pi^{55}) ] V_t^* + \left[ \pi^{66} - \gamma_1^{*0} (\pi^{26} - \pi^{36} - \pi^{56}) - \gamma_2^* (\pi^{16} - \pi^{36} - \pi^{56}) ] V_t^{**} + \left[ \pi^{67} - \gamma_1^{*0} (\pi^{27} - \pi^{37} - \pi^{57}) - \gamma_2^* (\pi^{17} - \pi^{37} - \pi^{57}) + 1] u_t + \left[ \pi^{68} - \gamma_1^{*0} (\pi^{28} - \pi^{38} - \pi^{58}) - \gamma_2^* (\pi^{18} - \pi^{38} - \pi^{58}) ] u_t^* + \left[ \pi^{69} - \gamma_1^{*0} (\pi^{29} - \pi^{39} - \pi^{59}) - \gamma_2^* (\pi^{19} - \pi^{39} - \pi^{59}) ] u_t^{**} = 0\]

The coefficients of each variable could be constrained to zero. Equating the coefficients of the nine exogenous variables in the country A supply equation to zero yields 9 constraints on the \(ij\) variables.

\[\pi^{61} = 0\]
\[\pi^{62} = 0\]
\[\pi^{63} = 0\]
\[\pi^{64} - \gamma_1^{*0} (\pi^{24} - \pi^{34} - \pi^{54}) - \gamma_2^* (\pi^{14} - \pi^{34} - \pi^{54}) = 0\]
\[\pi^{65} - \gamma_1^{*0} (\pi^{25} - \pi^{35} - \pi^{55}) - \gamma_2^* (\pi^{15} - \pi^{35} - \pi^{55}) = 0\]
\[\pi^{66} - \gamma_1^{*0} (\pi^{26} - \pi^{36} - \pi^{56}) - \gamma_2^* (\pi^{16} - \pi^{36} - \pi^{56}) = 0\]
\[\pi^{67} - \gamma_1^{*0} (\pi^{27} - \pi^{37} - \pi^{57}) - \gamma_2^* (\pi^{17} - \pi^{37} - \pi^{57}) + 1 = 0\]
Imposing the same procedure on the rest of the model yields other equations used to determine the reduced form coefficients (π_{ij}').

The values of the reduced form coefficients in the postulated solutions are functions of the parameters of the goods and money markets of the small as well as the large countries. The output supply values can be outlined from the parameters of the production functions of the small and the two large countries. The production function parameters could be estimated. Assuming the Cobb-Douglas production function, for example, the production elasticities are the shares of each input and perhaps could be obtained from secondary data. For the small country the two inputs could be labor and imports.

One then needs to estimate expenditures on goods. The expenditures in the small country depend on real balances and domestic income. The large countries' expenditure functions depend on relative price and income; therefore data on GNP, real balance, and prices gives the information needed to estimate the parameters. The demand for real balance in the small country is

\[ m_t - C_t = \beta_0 + \beta_1 Y_t - \beta_2 E_t \left( P_{t+1} - P_t \right) \]

and \( \beta_0, \beta_1 \) and \( \beta_2 \) in the small and similar parameters in the large countries should be estimated.
Once the structural coefficients have been estimated for these relationships, the reduced form coefficients can be determined and they can be used to examine the loss function as impacted by the occurrence of nominal and real disturbances originating abroad.

Other research that could be done by empirical work beyond the scope of this study, involve the analysis of effects of fluctuations in real income and output price in one large country on the economy of another large-country trading partner. If, for example, the elasticities of demand for goods of a particular large country with respect to relative price and real income of another large country trading partner are small compared to the same elasticities obtained by reversing the large country of interest, then trade deficit patterns may be explained. It may be that fluctuations in relative price and real income in the U.S., for example, do impact the economy, but the converse impact is weak. Hence a partial explanation of the huge trade deficit in the U.S. could be derived.
REFERENCES


APPENDIX


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