Water Decision-Making Under Uncertainty

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WATER DECISION-MAKING UNDER UNCERTAINTY

by

Augustina Yaa Oye Odame

A dissertation submitted in the partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

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UTAH STATE UNIVERSITY
Logan, Utah

2015
ABSTRACT

Water Decision-Making under Uncertainty

by

Augustina Yaa Oye Odame, PhD

Utah State University, 2015

Major Professor: Dr. Charles Sims
Department: Applied Economics

This dissertation comprises three separate studies under the unifying theme of “Water Decision-Making under Uncertainty.” The first study evaluates the decision to invest in water-saving infrastructure in the face of technical, as well as water supply uncertainty. It finds that the price at which one would not produce might be a more important consideration in water-saving investment decisions than previously thought. It also asserts, by the effect of incorporating after-adooption efficiency uncertainty, that a careful identification and consideration of all significant sources of uncertainty and their relative impacts on investment outcomes should be undertaken in determining where to target policy, as well as future research efforts.

The second study, an extension of the first to allow for incremental undertaking of infrastructure investment, finds that the option of gradual increments to an investment as opposed to the all-or-nothing investment story, has significant impact and potential with regards to investments meant to effect behavioral change for improved system outcomes such as more efficient irrigation systems towards regional water conservation goals.

The third and final chapter comprises a spatially explicit hydroeconomic model of water-use behavior in the Cache Valley of Utah and analyzed the impact of individual decisions, actions and interactions on available water supplies in terms of both quantity, as well as the efficiency contribution of an omniscient water master to overall water-use efficiency of actors in the canal.

(233 pages)
PUBLIC ABSTRACT

Water Decision-Making under Uncertainty

by

Augustina Yaa Oye Odame, PhD

Utah State University, 2015

Major Professor: Dr. Charles Sims
Department: Applied Economics

This dissertation is made up of three separate studies under the unifying theme of “Water Decision-Making under Uncertainty.” The first study analyzed a farmer’s decision to invest in a more efficient irrigation system given uncertainty about future water supplies and his post-investment efficiency. It found the price at which farmers would no longer produce to be a bigger consideration in irrigation investment than previously thought. It also found support for a careful identification and consideration of all significant sources of uncertainty in order to create better policy incentives for irrigation technology investments.

The second study extended the first to allow the farmer to gradually update his irrigation system rather than undertake a single, complete overhaul. It found that giving the farmer this option, has significant impact and potential with regards to investments meant to effect behavioral change for improved system outcomes such as more efficient irrigation systems towards regional water conservation goals.

The third and final chapter established a spatially explicit hydroeconomic model of water-use behavior in the Cache Valley of Utah to evaluate the impact of individual decisions, actions and interactions on available water supplies, and whether and how a water master’s privileged information about the behavior of users in his system may be used to improve system outcomes for users in the canal, downstream water requirements and storm-water management.
DEDICATION

To my mother, Eunice Anorkor Larrey
“It is not the quantity of water applied to a crop, it is the quantity of intelligence applied which determines the result - there is more due to intelligence than water in every case”

ALFRED DEAKIN, 1890.
ACKNOWLEDGMENTS

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INTRODUCTION

The continuously intensifying scarcity of water resources is a crucial problem in almost all contemporary societies (Tsegaye and Vairavamoorthy, 2009). In fact, the scarcity, distribution and quality of water resources are highlighted in the literature as factors with the greatest propensities to invoke powerful political pressures and potential conflict (Wolf et al., 2003).

This problem becomes even more pertinent in the face of aridity, and with uncertainty about the ability of future water supplies to meet future demand in the face of changing demographic and hydroclimatic conditions. Utah’s per capita water usage ranks first in the country despite being the second driest state in the country (Frankel, 2010). Given the region’s high water-use population, limited water supply due to its arid nature, and current as well as projected changes in its hydroclimatic conditions, managing water for current and future populations is a pertinent issue in the region.

While agriculture remains the largest user of water, substantial population gains have seen increased adaptation of agricultural land, water infrastructure and management systems for urban use in the region (Barnett, 2008). Concurrently, increasing variability of snowfall and rainfall attributed to climate change has amplified concern and uncertainty about future water supplies in the region. While contending with the changing human and physical (infrastructure) systems; water management in the region has to cope with changing hydroclimatic patterns and its attendant increased uncertainty about future water supplies.

Water management in the region is thus a complex, multi-dimensional system challenge which requires decisions on infrastructure projects highly dependent on more
dynamic human and actions (past and future), environmental parameters and processes, as well as long-term climate change (Taptiklis, 2011). Infrastructural investment decisions of this nature have historically been based on cost-benefit analysis where an investment is undertaken only if its net present value is positive. However, in the presence of human uncertainty about returns to investment and natural uncertainty about future precipitation, the efficiency of these investments depend on more than a cost-benefit rule (Dixit and Pindyck, 1994). Their efficiency depends on individual water-use behaviors which may factor in additional opportunity costs as well as notions of economic efficiency.

This dissertation thus analyzes long-term water-use decisions such as investments in irrigation infrastructure, employing a real options approach to incorporate the value of waiting for more information when making investment decisions in the face of uncertainty. Vano et al. (2014) more recently confirmed the need to understand the uncertainties inherent in climate predictions, and to better communicate these uncertainties to the larger water management community. This study speaks to this need and brings out of the tangle of uncertainty debates, useful information that may be readily used by water decision makers.

It also studies, via a bi-level agent-based modeling approach, how day-to-day (short-term) interactions between individual farmers, residents, water managers and policy makers affect investments towards higher water-use efficiency and conservation, and the ability of the region’s water supply to respond to the future water shocks and constraints. By simulating these behaviors under different scenarios, the study will bring to light the conditions that precipitate different agent responses to the changing hydroclimatic
conditions in the region and what causes them to switch from one behavioral rule to another.

This combination of methods will provide a larger range of short-term human responses to changing environmental conditions, illustrate how these impact over-arching water management goals, and provide a more realistic account of the evolving human water infrastructure in the region. Distinguishing between these two types of decision-making is critical for understanding how water-use behaviors adapt to changes in social and natural contexts and respond to various sources of uncertainty.

This is important because while other approaches may show agent behavior as being inefficient, they may be creating rules based on the best available information. The interesting question then arises of how much uncertainty (from either natural or human variability) is needed to make switching from an economically efficient strategy to a rule of thumb preferable to agents. By incorporating agent response to changing hydroclimatic conditions, the model will provide a framework to help answer this question by exploring how natural- and human-induced variability in things like stream flows and field runoff may trigger less efficient uses of water. It will help determine whether increased uncertainty may make water-users revert to a rule of thumb, to exhibit seemingly ‘inefficient’ behavior.

These three unified studies provide insight into how humans impact and are impacted by water, and identify several links on the bridge between the qualitative social science data on human behavior and quantitative natural science data on water.
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CHAPTER ONE: WATER-SAVING INFRASTRUCTURE INVESTMENT UNDER UNCERTAINTY: A CASE STUDY OF IRRIGATED LANDSCAPES IN THE WESTERN UNITED STATES

ABSTRACT

While research on water-use efficiency typically focuses on improving day-to-day water-use decisions, longer-term investments in new technology have the greatest potential for sustaining water conservation. Employing a real options approach, this study evaluates the decision to invest in water-saving infrastructure in the face of technical, as well as water supply uncertainty. An agent must determine whether and when to invest in water-saving technology in order to maximize his water-use efficiency and subsequent water-savings gains. The study extends previous research on the effect of stochastic water prices and emerging water markets on the adoption of modern irrigation technology by Carey and Zilberman (2002) in three ways. First, it illustrates a way of incorporating water-price uncertainty based on readily available stream flow data. Second, it investigates the influence of a market exit condition on the decision to invest in new technology. Third, it allows for an explicit consideration of uncertainty in technical efficiency. A numerical solution procedure is used to allow for both sources of uncertainty. A key finding of this study is that the price at which one would not produce might be a more important consideration in water-saving investment decisions that

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1 Paper Co-author: Dr. Charles Sims
2 This study acknowledges and builds off the investment model put forward by Carey and Zilberman (2002) and refers amply to their model for comparison and juxtaposition.
previously thought. Relaxing the assumption of a negligible probability of the price of water exceeding the upper bound required to continue production, we find that agents are even less likely to invest when they are provided the option to exit the market in addition to the uncertainties they face. The significant impact of this relaxation suggests that analyses done with the implicit assumption of no market exit may engender spurious conclusions and policy suggestions, as well as suboptimal investment decisions. The study also confirms that uncertainty generally stalls investment in both farm and meso-scale investments, and that the incorporation of a second source of uncertainty compounds this investment inertia in the face of uncertainty. While unsurprising in itself, this result has interesting research and policy implications. It hints at the potential value identification of additional sources of uncertainty and their relative imports on investment decisions may have by way of informing which sources of uncertainty would yield the best bang for policy buck. Thus, a careful identification and consideration of all significant sources of uncertainty and their relative impacts on investment outcomes should be undertaken in determining which particular sources of uncertainty research efforts and policy dollars should prioritize in order to better inform investment decisions to attain desired outcomes.

Key Words: Water-Saving Infrastructure Investment, Real Options, Uncertainty
1 INTRODUCTION

Potentially non-efficient use of reservoirs and water distribution systems in the western U.S. is a growing concern (Frankel, 2010). While water has always been a limited resource in the region, mounting population pressure and the slow but steady manifestation of a changing climate in recent times has made water an even more pertinent issue. Steady population growth, coupled with changing hydroclimatic conditions in the western U.S., generates considerable concern over the ability of water supply to meet future water needs (USBR, 2012; CLCWSMP, 2007). This concern has turned attention to increasing water-use efficiency and water conservation in the region.

An economic sector with great potential for increased water-use efficiency is agriculture (Wallace, 2000). While irrigation research typically focuses on improving day-to-day decisions of water-use, longer-term investments in new technology are essential for improvements in water-use efficiency and conservation over the long-term. However, the large, irreversible capital investment required coupled with slow and uncertain returns spanning many years, hamper adoption of these technologies (Hafi et al., 2006). For example, it has been established in the literature that more efficient irrigation technologies such as drip or center pivot irrigation can produce superior profit margins relative to traditional technologies (McKenry, 1996; Adusumilli and Almas, 2007). Yet, conversations with farm advisors suggest “under-investment” in new irrigation technologies relative to investment levels proposed by positive expected net present values for these technologies (Carey and Zilberman, 2002).

One explanation for this apparent under-investment is the uncertainty in future returns from water-saving investment. This uncertainty is rooted in annual variations in
precipitation that make water supplies difficult to predict. While the region generally experiences cyclical trends in precipitation, climate scientists predict that as temperature increases through the century, it is prospective that more precipitation will occur as rain rather than snow, that the snow accumulation season will get shorter, and that more of the region’s snowpack will be lost through evaporation (BRAC, 2007). These predictions do not only forecast scarce water supplies in the future and more pronounced trade-offs among competing uses, but also uncertainty about the timing and quantity of precipitation received. It has been established in the literature that such uncertainty creates uncertainty about future returns from investments through production risks (Koundouri et al., 2009; Kato et al., 2009), and has the tendency to delay investments in conservation technology.

This paper develops a dynamic technology adoption model to examine the impact of uncertainty considerations on the decision to invest in water-saving infrastructure, such as improved irrigation technology or canal lining to improve water-use efficiency and minimize water leakages and loss. Adoption of the technology requires paying a sunk investment cost (I) to obtain new infrastructure, such as a center pivot irrigation system whose value (V) is dependent upon the monetization of water savings in the water lease market. The optimal time to adopt the technology maximizes the expected present value of returns from the sale/lease of water-saving resulting from investing in more efficient irrigation technology by the agent, to other water-users seeking to supplement their water supply. Uncertainty creates speculation about future water prices, and subsequently about the profitability of water-saving infrastructure investments.

There are two relevant sources of uncertainty influencing water-saving infrastructure investments. The first uncertainty concerns the timing and quantity of future water
supplies generated by changing weather conditions and increasing variability in precipitation patterns. Changing climatic conditions are projected to influence seasonal, year-to-year, and longer time-scale variability in precipitation timing and quantity. Boer (2009) finds that while the increases are generally greater in the tropics, the standard deviation of average yearly precipitation increases across the board with increasing global temperatures\(^3\). This increase in variability is congruous with the prediction of more intense future precipitation events attributable to the ability of a warmer atmosphere to hold moisture (Räisänen, 2002; Stouffer and Wetherald, 2007; Meehl et al., 2007). Water supply uncertainty is thus a pertinent consideration for all, and more so for Utah which is expected to experience greater temperature increases than the global average (BRAC, 2007).

The second is technical uncertainty, also referred to as after-adoption efficiency (AAE) uncertainty, which is uncertainty pertaining to the performance of an adopted technology that lingers after adoption (Azevedo and Paxson, 2012). The realized water savings from center pivot technology adoption can vary from 10% to 35% depending on factors such as what system the center pivot would replace, soil type and water infiltration rates, wind drift, irrigation timing, evapotranspiration, and field gradient (slope). Thus, temporal and spatial variations in these factors may render the assumption that the efficiency of a center pivot irrigation system would equal the performance predicted by the manufacturer or irrigation extension officer, inappropriate for evaluating the investment decision.

\(^3\) Precipitation patterns are quite regional (some areas expected to be wetter, but some – like the American southwest – are anticipated to be much drier). Also predicted precipitation changes have a huge amount of uncertainty, as compared to predicted air temperatures
This assertion is supported by Azevedo and Paxson (2012) who aver that manufacturing, renewable energy, agricultural, and mining sectors are exposed to high levels of technical uncertainty due to the dependence of efficiency after adoption, at least in part, on human and natural resource conditions which restrict full pre-adoption testing. It is noteworthy that this does not entail human-performance variation which may be resolved over time, as in the learning-by-doing literature; but is due to naturally occurring stochasticity which does not resolve by learning over time.

The study extends previous research in three ways. First, it illustrates a way of incorporating water-price uncertainty based on readily available stream flow data. Water-price uncertainty clearly matters for the optimality of water conservation investments. However, time series data needed to estimate this uncertainty rarely exists (Carey and Zilberman 2002). Second, this study investigates the influence of a market exit condition on the decision to invest in new technology. The decision to discontinue irrigated agriculture is a critical consideration as many irrigated landscapes are transitioning from agriculture to residential development. Third, the study allows for an explicit consideration of water supply uncertainty and uncertainty in technical efficiency jointly. A numerical solution procedure is used to allow for both sources of uncertainty simultaneously.

To illustrate our theory’s applicability, we will apply the model to three watersheds in Utah. An average annual precipitation of 13 inches ranks Utah the second driest state in the country (Adams et al., 2010). The state’s agricultural land is slowly being converted to urban use in response to population pressure in the state (Leydsman McGinty, 2009), as well as the relatively lower valued agricultural produce in Utah. About three hundred
thousand acres of the state’s agricultural land was converted to developed land between 1982 and 2007 (Dempsey, 2010), with an additional 120,304 acres developed between 2007 and 2012 (NASS, 2012). Agriculture, however, remains the greatest user of Utah’s developed water withdrawals as the region’s aridity makes its agriculture mainly irrigation-fed. Irrigation remains the largest use of fresh water in Utah (at about 83% of the water withdrawn from all sources in 2008) (UDWR, 2010; ORGC, 2012), underlying the potential importance of water-saving irrigation technology in the region. The Utah State Water plan lists, as one of its conservation recommendations, increased efficiency in agriculture with sprinklers or surge-flow, watertight canals and turnout structures. And yet, more than half of Utah’s 1.3 million irrigated acres are watered using surface methods such as flood, furrow, border, or basin irrigation (Barnhill et al., 2009).

Applying our theoretical model to various Utah case studies, we will consider the influence of uncertainty on the decision to invest on irrigation investment behavior to answer three general questions; 1) What effect does the possibility of market exit have on investment behavior? 2) Do uncertainties in future water supplies and the efficiency of new water-saving technology, respectively; create similar incentives to delay adoption of new water-saving infrastructure investments? 3) How would reductions in, and interaction between the two types of uncertainty influence the adoption of water-saving infrastructure?

The paper proceeds as follows. Section 2 reviews the relevant literature on uncertain investments in resource conservation. Section 3 introduces the theoretical model as well as a characterization of the investment problem and the option of waiting to invest. Section 4 details the empirical case studies. Section 5 concludes.
2 BACKGROUND

Modeling approaches employed in the study of the adoption of conservation and agricultural technology in existing literature include multivariate regression analyses to decipher the factors most influential in adoption decisions (Marenya and Barret, 2007); Logit (Daberkow and McBride, 2003) and Probit (Koundouri, 2006) models which analyze the decision to adopt or not; Tobit models (Fernandez-Cornejo et al., 2001) which help measure likelihood and extent of adoption; duration analysis to capture the temporal dynamics of technology adoption and diffusion (De Souza Filho et al., 1999; Burton et al., 2003; D’Emden et al., 2006); and discounted net present value accounting (Ward and Pulido-Velazquez, 2008) which evaluates the economic profitability of investing in the adopted technology. Marques et al. (2005) employ a two-stage economic production model to investigate how the availability, price, and reliability of water affects economic performance, annual and long-run cropping patterns, and irrigation technology decision of a farm, while Cai and Rosengrant (2004) adopt a two-stage stochastic programming model to study how hydrological uncertainty impacts decisions about irrigation technology.

The decision-centric models tell you who is likely to adopt based on characteristics, but provide no insight on the incentives for adoption. The discounted net present value applications evaluate the economic incentive to invest, but fail to accurately depict observed behavior. The gap between the number of conservation investments that yield positive net present values and the actual investments made is due in part to the fact that missing from the traditional analysis, is an option value which reflects the value of the
possibility or option to delay in order to obtain more information about the payoff from investing in the face of uncertainty. Option value (or Real Options) theory considers this value in evaluating investment decisions.

2.1 Environmental and Natural Resource Applications of Real Options

Environmental and resource economics often face problems which deal with the timing and making of decisions under varied contexts (Balikcioglu, 2008). They arise routinely in the management of renewable resources, such as in problems of optimal fish stock harvesting (Ludwig, 1980), forest rotation (Willassen, 1998; Alvarez, 2004; Forsyth, 2000; Insley, 2002) and invasive-species management (Sims et al., 2010; Sims and Finnoff, 2013). They are also present in the management of non-renewable resources with applications in mining (Brennan and Schwartz, 1985), conservation (Arrow and Fisher, 1974) and pollution abatement (Conrad, 1997; Pindyck, 2000a; Wirl, 2006). Properties common to these problems are the irreversibility of associated decisions, the dynamic nature of the problems and the inherent uncertainties associated with these decisions (Balikcioglu, 2008).

Irreversibility arises from two sources. First, there are the sunk costs usually associated with the specialized nature of the capital equipment, control measures, or technology adoption (Pindyck, 2000b). A second source of irreversibility stems from the nature of the resource itself. As in the case of an old-growth forest where trees once felled require an effectively infinite number of years to grow back, decisions concerning non-renewable resources are generally irreversible. Irreversibility could also stem from the long-lasting effects of an investment decision. An example is the environmental and
health damages that may result from a decision to forgo or delay investment in pollution abatement.

These problems are dynamic because decisions made in one period determine new starting conditions for decisions in subsequent periods. Uncertainty regarding these problems may stem from investment uncertainty where the future pay-offs (or damages) from an investment or decision is uncertain due to uncertain effects of the investment on the market, or uncertain occurrence of the event the investment was supposed to mitigate (Heal and Kristrom, 2002; Peterson, 2006; Pindyck, 2007). It may also stem from policy uncertainty, where decision-making agents are uncertain about the timing and magnitude of policy announcements and regulations which may totally change the nature of the decision-making problem (Fuss et al., 2008).

Early attempts to incorporate uncertainty and irreversibility in environmental investment problems can be found in Kolstad (1992), who developed a three-period model to examine the effects of uncertainty on policy adoption where uncertainty persisting in the first two periods is resolved in the third period. Kolstad (1992) found that when policy is irreversible, it is better to wait for the resolution of uncertainty before policy adoption. Conrad (1992) finds that uncertainty leads to early adoption of abatement policy in the absence of irreversibility but ignores sunk costs in his model. Dixit and Pindyck (1994) supplement Conrad’s finding by showing that when irreversibility is considered, increasing uncertainty delays the emission reduction policy. Pindyck (2002) formalizes and generalizes these results by incorporating both policy and investment uncertainties as well as irreversibility into his model and shows that in all
cases, as uncertainty increases, the value to wait for future information increases and delays policy adoption.

Saphores and Carr (2000) establish the importance of the choice of stochastic processes on model outcomes. Employing a similar framework employed by Pindyck in Dixit and Pindyck (1994), but with a different assumption on the stochastic process of the stock pollutant, they investigated the effects of ecological uncertainty on policy adoption, and found that uncertainty has ambiguous effects on the timing of the policy adoption. At sufficiently high levels, increasing uncertainty leads to more conservative policy, i.e. a delay in and/or reduced extent of adoption. This result is attributable to the fact that expected social cost (damages) in the absence of pollution abatement increases with uncertainty in the Dixit and Pindyck model, while in most other real option models, expected social cost is not a function uncertainty (Balikcioglu, 2008). Since water infrastructure systems are large and long-lasting (about 50 years of useful life); the costs of technological adaptation could be high if water demand fails to evolve as predicted (Schleich and Hillenbrand, 2009).

The literature generally concurs that when 1) a problem is characterized by uncertainty about future costs and benefits of the alternatives under consideration, 2) prospects exist for resolving the uncertainty with the passage of time, and 3) investment in at least one of the alternatives is irreversible, an extra value, an option value, should be attached to the reversible alternative(s) in the decision-making process (Fisher, 2001). In other words, an irreversible (investment) decision or action has a higher hurdle to clear in order to pass a benefit/cost test (Carey and Zilberman, 2002).
2.2 Water-saving Infrastructure Investments

Variables identified as affecting farmers’ technology choice include economic factors such as commodity prices, costs of inputs, and the costs of the irrigation technology; environmental factors such as climate, topography and land quality; and institutional factors such as land-tenure and water rights arrangements. While results from different studies assign different weights to these factors, it is a common theme that low land quality, high crop and input prices, and low costs of switching technology correlate positively with the adoption of improved irrigation technologies (Scheierling, 2004).

The indispensability of efficient agricultural water management in meeting future food needs is widely accepted (Heumesser et al., 2012). Known benefits of improved irrigation technologies include significant improvements in on-farm irrigation efficiencies and irrigation uniformity, as well as reductions in water delivery and withdrawal requirements, deep percolation and runoff. With improved technology, it is possible to attain output gains with no change in quantity of water applied. However, this improvement comes with the price of higher equipment costs (and, in the case of a switch to sprinklers, energy costs). Although research on irrigation technology adoption may be fairly nascent, it is grounded in the accepted development economics and industrial organization literature on general agricultural technology adoption (Caswell, 1991; Scheierling, 2004).

Urbanization-induced increases in water demands and an appreciation of the value of in-stream flows, have increased competition for scarce water supplies in the Western United States (MacDonnel, 2009; Zellmer, 2009; Schaible and Aillery, 2012). As the principal and yet often lowest-priced water-use, irrigated agriculture has garnered
concerted attention and efforts to encourage agricultural water conservation (Huffaker and Whittlesey, 2000), with the objective of shifting some water to higher-valued uses and improving the economic efficiency of overall water allocation. The adoption of more efficient irrigation technologies is often touted as an approach with great potential for agricultural water conservation with minimal losses in farm yield and income (Scheierling, 2004).

Proposed conservation measures target irrigated agriculture because: 1) irrigated agriculture accounts for the bulk of both total water withdrawals and consumptive water-use globally (Johnson et al., 2001; Tiwari and Dinar, 2001), and this demand is expected to grow to meet food needs as populations continue to grow, and 2) irrigation water-use efficiency is generally low world-wide, with national averages generally below 50% of potential efficiency (Tiwari and Dinar, 2001). In fact, more than fifty percent of the water which enters irrigation distribution systems does not make it to the crops due to leakage and evaporation (Postel, 1995; Johnson et al., 2001).

Researchers and policymakers typically focus on the provision of economic incentives, such as higher prices or subsidies, in precipitating investment in more efficient irrigation technologies (Weatherford, 1982; Caswell et al., 1990; Huffaker and Whittlesey, 2003). In real-world applications however, these policy tools may not be available, or practicable (Scheierling, 2004). For example, Huffaker and Whittlesey (2003) establish that higher applied water costs may be more efficient at achieving conservation goals than investment subsidies. However, this is largely not an available policy option under the prior appropriation system prevalent in the western United States. This necessitates the need for a better understanding of water-saving investment
decisions in the west, to design appropriate conservation policies which consider the west’s particular realities and water law.

Ajami et al. (2008) testify to the significance of hydrological uncertainties in the sound management of water resources. Past and on-going research has seen advancement in assessment techniques for the different sources of uncertainty related to hydrological projections (Beven and Binley, 1992; Kuczera and Parent, 1998; Vrugt et al., 2003; Maier and Ascough, 2006; Ajami et al., 2007). However, there still remains room for more extensive exploration of the links between hydrological uncertainties and water resources management such as in (Huang and Loucks, 2000; Zhang et al., 2011). These linkages are necessary given the potential impacts of hydrological uncertainty on the ability of water policies to meet desired management and conservation goals. Ajami et al. (2008) aver that there is a need for proper understanding of the sources and impacts of uncertainty in order to attain reliability and sustainability goals in managing water resources. The same is true with regards to encouraging investment in water-saving technology.

Contrary to the predictions of net present value (NPV) models, Carey and Zilberman (2002) affirm that farms wait until the return on an investment is significantly greater than the cost before adopting modern irrigation technologies. Adopting a stochastic dynamic approach to model farm irrigation investment under uncertainty, they find that farmers’ requirement that the expected benefits of investment outweigh costs by a large margin (hurdle rate) before investing in new technology may be explained by the consideration of uncertainty in their investment decisions. Thus, the conclusion that observed rates of investment in modern farm irrigation technology were suboptimal in
previous assessments was based on the use of traditional NPV models of investment, which ignore issues of uncertainty and irreversibility. Observed investment can only be considered optimal under the NPV framework if farmers have extremely high discount rates. However, factoring in the farmers’ consideration of the value of waiting to invest under uncertain conditions (option value ) seems to explain the farmers’ extremely high discount rates in evaluating irrigation investments (Carey and Zilberman, 2002).

By waiting to invest, a farm can gather more information on the evolution of water prices before committing funds to irreversible investments. Thus, information is essential in decision-making under uncertainty. More information reduces perceived risk and precipitates investment. On the other hand, inadequate information increases perceived risk and delays investment in favor of gathering more information.

Carey and Zilberman also find that, contrary to common belief, water markets can delay the adoption of irrigation technology. This is because the introduction of a water market may make it more profitable for farms with abundant (scarce) water supplies to adopt earlier (later) than they would otherwise due to economies (diseconomies) of scale. The presence of water markets increases opportunities for those with abundant water supplies to recoup investment costs through sold water savings and lowers pressure to invest for those with scarce supplies as they can buy water to supplement supply.

Zilberman et al. (1995) find that drought events had substantial, positive impact on adoption rates of drip irrigation in California. Adoption rates remained low from 1982 to 1986 despite proven cost effectiveness. However, the drought persistence from 1987 to 1991 triggered widespread adoption by increasing returns sufficiently above investment costs. Thus, significant catastrophic events appear to provide a counter balance for
uncertainty and reduce the option value by increasing the cost of waiting, and increasing the value of (thus prompting) investment.

Most recently, Bhaduri and Manna (2014) examine the impacts of water supply uncertainty and storage on efficient irrigation technology in a stochastic dynamic setting, and under a flexible water regime. They find that allowing for flexible water pricing does not guarantee higher technology adoption in all cases. They also demonstrate a complementary relationship between investments in storage capacity and efficient irrigation technology adoption.

While we too focus on the impacts of water supply uncertainty on the adoption of efficient irrigation technology, our model presents differing features with regards to incorporating a second source of uncertainty: technical uncertainty into the real options investment model and commodity market exit conditions which act as an upper bound on the incentive to invest. Existing real options models in the literature generally assume ex-ante knowledge of technology performance and ignore the possibility that agents may terminate production and, in turn, the use of technology. The introduction of technical efficiency uncertainty and market exit conditions introduces an added level of difficulty in the optimization of an investment decision, as it becomes more difficult to know what actual post-adoption revenues would be, and thus the optimal investment decision.

Finally, our case studies which involve three different rivers (the Logan, Bear and Blacksmith Fork) in the Middle Bear-Logan Watershed provide differing water supply parameters allowing for interesting spatial comparative statics and illustrate by their differing investment considerations even within a single watershed, the importance of differing spatial realities on optimal investment decisions.
3: MODEL

An agent is faced with the choice of determining whether and when to invest in water-saving technology given that the efficiency of the technology as well as the future price of conserved water is uncertain. The optimal time to invest maximizes his expected present value of future payoffs. For example, this investment may represent investment in irrigation-efficient technology or the lining of canals and ditches to mitigate water seepage losses. Motivated by farm irrigation investments, the model set-up will defer to the irrigation investment decision where compelled, but will maintain generalizability for canal lining investment evaluations.

The agent is assumed to maximize an instantaneous profit function by choosing the amount of water to apply in production. The agent initially operates under a traditional irrigation technology regime or an unlined canal system where most, if not all, of the canal is unlined and susceptible to considerable seepage losses. The agent has the option to invest in more efficient modern technology. Should the agent decide to make the investment in period $t$, he will incur an irreversible cost of adoption; $I(t) = I$. The model thus essentially lays out a dual-level decision-making process where the agent first has to optimize an instantaneous choice of inputs for production, setting pre-investment benchmarks for productivity and profits. Then, given that he is choosing inputs to maximize profits, he also chooses the timing of investment.

---

4 The cost of investment may also be uncertain (Fuss et al, 2008; Fuss et al, 2009; Eryilmaz and Homans, 2013; Feder et al, 1985). Accounting for stochastic investment costs in this particular model, however, did not make a difference in the optimal investment decision and thus the assumption of non-stochastic costs is upheld.
The theoretical component of the model follows Carey and Zilberman (2002) very closely. However, it generalizes this work by tracing water price uncertainty back to the underlying uncertainty in water supplies, incorporating a market exit condition, and considering a second source of uncertainty. While Carey and Zilberman find that water price uncertainty is a key driver in water conservation investments, they were forced to estimate this uncertainty based on a small number of observations. Our approach allows water price uncertainty to be estimated based on a rich time series of stream flow data provided by the U.S. Geological Survey (USGS). The model solution with the incorporation of the second source of uncertainty and market exit conditions require numerical analysis, and thus their treatment while extending Carey and Zilberman’s theoretical model, is more nested within the empirical section of this study.

3.1 Water Supply and Price Uncertainty

The agent produces a commodity whose price is given exogenously by $p_y$. Following Carey and Zilberman (2002), the agent’s production is represented by a Von-Liebig production function such that:

$$y_i = \begin{cases} a_i B X_i & \text{when } X_i < X_i^* \\ y^* & \text{when } X_i \geq X_i^* \end{cases}$$

where $X_i$ is the amount of water employed in production, with $i = 0$ corresponding to production using pre-investment technology and $i = T$ corresponding to production using the modern technology.\(^5\) $y^*$ is the agent’s optimal level of production and $X_i^*$ is the

---

\(^5\) It is assumed that yield-increasing effects of the new technology on production are negligible (Letey, 1991; Berck and Helfand, 1990; Carey and Zilberman, 2002), leaving the focus of gains from adoption to rest solely on water savings from increased water efficiency. Thus we assume that $y_0^* = y_T^* = y^*$ so that
agent’s optimal water demand under technology $i$ (what he would require for optimal production if unconstrained by water supply). $X_i$ represents the actual amount of water demanded by the agent for use in production. In times of water abundance ($X_i \geq X_i^*$), the agent is able to obtain no less than his optimal water demand for production optimization $X_i^*$, and produces at the optimal level $y^*$. In times of relative water shortage ($X_i < X_i^*$) however, the farmer is constrained by the amount of water available to him, and his production ($a_i B X_i$) is proportional to the amount of applied water ($X_i$), $B$ is a term representing the contribution of the optimal levels of all non-water inputs in production determined holding water constant and $a_i$ is a scalar that reflects the efficiency of technology $i$ with $a_T > a_0$.

In a linear response and plateau (or Von-Liebig) model, production responds to the addition of a limiting input until a different input becomes limiting. This reflects the fact that production inputs are not as readily substitutable as implied by smooth, concave functions. This model thus employs a Von-Liebig production function to show the non-substitutability between an agent’s other production inputs and water. In our model, it is assumed that that agent has applied other production inputs at such a level that water is the limiting input in production (Letey, 1991). Thus, ceteris paribus, production will respond to increasing water availability until another production input, say land or canal size became limiting. It has been shown in the literature (Berck and Helfand, 1990) that by accounting for heterogeneity of input levels in production, Von-Liebig production functions can be reconciled with smooth, differentiable production functions via we do not have a yield-increasing effect associated with adoption if both technologies operate at full potential. Nutrient processing by agricultural crops could be an effect here in the case of the farmer and is an angle that may be explored in subsequent work.
aggregation and so this specification is employed with the knowledge that it is not mutually exclusive to smooth production functions in the aggregate.

The total amount of water available for use by the agent is represented by:

\[
A_t = \begin{cases} 
\bar{A} & \text{if } W_t \geq \bar{W} \\
\theta W(t) & \text{if } W_t < \bar{W} 
\end{cases}
\]

where \( \bar{W} \) is an exogenously determined scarcity threshold and \( W_t \) is the aggregate water supply at time \( t \). This aggregate water supply may reflect the flow of a river or canal or the level of a reservoir supplying water to the agent. \( \bar{W} \) represents the amount of water that has to be in the canal for every water rights holder to be able to divert the full amount of water their water rights entitle them to under normal conditions.

When aggregate water supply is high (\( W_t \geq \bar{W} \)), an agent may use (i.e. has at his disposal) \( \bar{A} \) which represents how much water he is entitled to divert under ‘normal’ conditions based on the agent’s water rights or shares. The model assumes shares have been acquired in the past (sunk costs) and that recurrent expenses associated with holding the share are negligible.

When the aggregate water supply is low, the amount of water available to the agent is restricted to a proportion of the aggregate supply. This proportion \( \theta \) may be a function of his geographical location, seniority of water right, production requirements, etc. This conditional water supply reflects the reality in many parts of the western U.S. where water rights were allocated on historic conditions and often exceed available supply.

The relationship between \( A_t \) and applied water \( (X_t) \) is as follows:

\[
X_t = \begin{cases} 
X_t^* & \text{if } A_t \geq X_t^* \\
X_t < X_t^* & \text{if } A_t < X_t^* 
\end{cases}
\]
Thus, \( A_t - X_t \) is the difference between water the agent has available for production and how much he actually employs in production. If positive (\( A_t > X_t \)), it represents the total amount of water left over after production and available for sale by the agent. If negative (\( A_t < X_t \)), it indicates how much water the agent buys to supplement his available water for production. Investment in efficient water technology helps tilt the difference of the two terms towards a surplus by reducing \( X_t \).

Current annual aggregate water supply (\( W_t \)) is certain but future values are uncertain due to changes in weather. This stochastic supply process is represented by a geometric Brownian motion,

\[
dW = \alpha_w W dt + \sigma_w W dz_w
\]

where \( \alpha_w \) is the instantaneous drift rate of the supply process, \( \sigma_w \) is the instantaneous variance rate, and \( dz_w \) is the increment of a Wiener process (Dixit and Pindyck, 1994). The Geometric Brownian Motion (GBM) assumption is established in the literature as appropriate for describing the evolution of an uncertain/unpredictable process over time (Paddock et al., 1988; Marathe and Ryan, 2007), and specifically for describing the evolution of water supply (Bhaduri and Manal, 2014).

A GBM assumption implies that the logarithm of the randomly evolving quantity (here, water supply) follows a Brownian motion (also called a Wiener process) with drift. A simplifying definition of a Brownian motion would be to see it as the limit of a random walk process, a process which assumes randomness in the evolution of a process such that the observation in one time step contains no information on what will happen in the
next time step. This ‘randomness’ and attendant unpredictability, suits it for describing uncertainty-laden processes such as water supply over time.

The agent can smooth his water supply by buying, leasing, renting or selling water (buy/sell physical quantities of water, lease/rent water rights in the spot market). The water rights can be expressed as physical quantities for price comparison purposes. The agent can trade an acre-foot for use at time \( t \) at price \( P(t) \). However, a spot market in long-term water rights is assumed not to exist. This is because the existence of such a market would imply that expected future values of \( \bar{A} \) could change as more rights are allocated in addition to existing ones in a given watershed. However, as it stands, there is no new allocation of new rights, only transfer and appropriation of already allocated rights.

Periods of low aggregate water supply are assumed to correspond to periods of high water prices in the spot market. For simplicity, we assume (inverse) demand for water on the spot market is isoelastic

\[
(5) \quad P(t) = \left( \frac{W(t)}{\varphi} \right)^{-\frac{1}{\varepsilon}}
\]

where \( \varphi \) is a positive constant and \( \varepsilon > 0 \) is the price elasticity of demand. An isoelastic demand function depicts constant elasticity and is chosen here to demonstrate the relative (short run versus long run) inelasticity of water demand to water price (Olmstead and Stavins, 2009). Following Ito’s Lemma, a result from the Ito Calculus solution of stochastic differential equations such as the Geometric Brownian motion, we find that the market price of a water right also follows a Geometric Brownian Motion.

\[
(6) \quad dP = \frac{\partial P}{\partial t} dt + \frac{\partial P}{\partial W} dW + \frac{1}{2} \frac{\partial^2 P}{\partial W^2} \sigma_w^2 W^2 dt = \alpha_P P dt + \sigma_P P dW
\]
where \( \alpha_p = \left( -\frac{\sigma_w}{\epsilon} + \frac{(1-\epsilon)\sigma_w^2}{2\epsilon^2} \right) \) and \( \sigma_p = \left(-\frac{\sigma_w}{\epsilon}\right) \).

To the extent that changes in a farm’s water supply mirror changes in the aggregate supply, \( W \) and \( P \) will be negatively correlated as expressed in our isoelastic demand function. Due to the presence of a more developed water market in California, and its attendant differences in competitiveness and transaction costs vis-a-vis the water markets in Utah, the model deviates from the (Carey and Zilberman, 2002) assumption that an agent’s investment decision is independent of aggregate water supply, and accounts for the influence of aggregate water supply on agent investment decisions via the posited isoelastic demand function, and established relationship between the stochastic processes for the evolution of water supply and price as above (equations 4 and 5).

The agent’s decision to invest in modern irrigation technology depends on the tradeoff between the expected present value of the investment and the cost of switching technologies. The value of the investment at a given time is the expected increase in the profit flow with the adoption of modern technology.

The agent’s optimal production level is assumed to be determined by a constrained optimization of his profit function subject to his initial water right allocation. This set-up leaves open the possibility of four different cases where aggregate water supply, coupled with policy and environmental constraints, may result in situations where the agent owns more shares than needed or less shares than needed for his desired level of production. The four possible cases arising out of water allocation policy and environmental (water supply) constraints are detailed in figure 1 in the appendix.
If the agent is not policy constrained, his normal water allocation implied by his water rights is sufficient to satisfy his optimal water demand: \( X_t^* \leq A \). Case 4 considers an agent that is not constrained by policy considerations or environmental considerations \((A_t \geq X_t^*)\), the agent will employ \( X_t^* \) in production to produce at the optimal output level \( y^* \). There is no question of choosing water input to maximize profit as he will always produce at the optimum level of output when that option is available to him. This case thus has a trivial solution. We therefore consider the case where \( W(t) < \bar{W} \) and thus \( A_t \leq X_t^* \). This occurs in cases 1, 2, and 3 in figure 1. In these cases, the agent would then decide to supplement his water supply or not, by considering the availability and cost of doing so.

When \( A_t < X_t^* \) (Cases 1, 2, and 3), the agent’s optimized profit function at time \( t \), derived by optimizing his instantaneous profit function, is

\[
(7) \quad \Pi(P, W) = \max_{X_i} P \alpha_i B X_i - P(t)(X_i - \theta W(t)) - C_i
\]

where \( C_i \) represents the day-to-day fixed cost of operating the irrigation system under technology \( i \). The agent incurs this cost whether he invests or not. However, the level of \( C_i \) may increase (decrease) upon adoption of the modern technology due to say higher pressurization costs (greater energy efficiency) with the new system. The change in the value of this profit function \( \Pi(P, W) \) as a result of investment (i.e. \( \Pi_r(P, W) - \Pi_0(P, W) \)), is compared with the cost of investment \( I(t) \) to make the investment decision.

Aggregate water supply determines both the profit to be derived from investment, and the penalty for non-investment. If aggregate supply is expected to be lower in the future, the investment is relatively more desirable as it indicates a greater demand and higher
future price for water savings. However, if aggregate supply is expected to be higher in the future, investments in water-saving technology may appear inferior compared to other investments considered by the agent. Consequently, the future evolution of aggregate water supply is pivotal to the agent’s investment decision and the uncertainty surrounding this process needs to be adequately addressed to enable the agent to make the best investment decision.

3.2 The Investment Decision

The agent’s objective is to choose the timing of investment to maximize the expected discounted flow of profit. The agent’s decision of whether and when to invest depends on the trade-off between the expected present value of the investment, and the cost of switching technologies. The value of the investment at a given time, t, is the increase in profit flow as a result of technology adoption.

\[ v(P) = \Pi_T(P, W) - \Pi_0(P, W) \] (8)

Using equation (7) and simplifying, given that \( X_t = X_t^* \) (i.e. the agent would demand the amount of water needed to produce at his optimal level) and \( y_t = y^* \), the increase in profit flow is

\[ v(P) = PX^* - q, \] (9)

where \( X^* = X_T^* - X_0^* \), and \( C_0 - C_T = q \). \( PX^* \) is the market value of water conserved as a result of technology adoption, and \( q \) is the change in the cost outlays for production. Both technologies generate the same revenue when \( X_t^* \) is applied due to the
yield equivalence assumption. Thus potential profit gains depend solely on the value of conserved water and differences in cost outlays.

From (9), an agent’s investment decision depends on the expected present value of profit gains over all future time periods. Profit gains here is defined as the increase in profit as a result of investment, and is expressed by the difference between revenue from the sale of water savings \((PX^*)\) and increase in operational costs \((q)\) as a result of investment.

\[
V(P) = E\left[\int_0^\infty PX^* e^{-\rho t} dt\right] - \int_0^\infty q e^{-rt} dt
\]

Since \(P\) is stochastic, it is discounted by the risk-adjusted interest rate, \(\rho\). \(q\), on the other hand, is deterministic and thus discounted by the risk-free interest rate \(r\). The expected present value can be written as,

\[
V(P) = \frac{PX^*}{\delta} - \frac{q}{r},
\]

where \(\delta = \rho - \alpha_p\). Given that \(\rho\) is the risk adjusted interest rate, a drift rate larger than the risk adjusted interest rate (i.e. \(\alpha_p > \rho\)) would create a situation where it is always optimal to wait and there would be no determinable optimum in sight (i.e. wait indefinitely). The rate of return on holding the asset (i.e. waiting to invest) in this case would be greater than the rate of return the agent could earn by investing in the conservation technology, and so he would hold on to the asset and wait to invest. \(\alpha_p > \rho\) would imply that our convenience yield, \(\delta = \rho - \alpha_p\), which measures the value of waiting to invest today (and earning the risk-adjusted interest rate \(\rho\) on one’s money) versus investing today and earning the positive drift rate \(\alpha_p\) on water prices, would be
negative, making investment a non-issue. Thus, we assume $\alpha_p < \rho$, and thus $\delta > 0$ for a closed, non-trivial solution.

In the traditional NPV investment model, the agent would invest if $V(P) \geq I$, that is, if the expected present value of the investment is no less than the fixed cost of investment. Thus, in the traditional NPV model, the agent will choose to adopt water-saving technology if the price of water is greater than, or equal to some threshold $\bar{P}$, where

$$
\bar{P} = \frac{\delta}{X^*} \left( I + \frac{q}{r} \right).
$$

Thus, the agent is more likely to adopt water-saving technology as the water-savings associated with technology adoption, $X^*$, increases. He is also less likely to adopt if the discount rate ($\delta$), fixed cost of investment ($I$), or additional operational cost outlays ($\frac{q}{r}$) increase.

3.3 Option Value with Water Supply Uncertainty

To account for irreversibility, uncertainty, and the agent’s option to delay investment until a later date, we extend the above discussion to consider the investment’s option value. Recall that the agent’s profit function is defined as $\Pi(P,W) = P_y \alpha_l BX_l - P(t)(X_l - \theta W(t)) - C_l$. To understand the motivation for the option value, consider the second and third terms, $P(t)(X_l - \theta W(t)) - C_l$. These two terms account for the possibility of the agent buying water on the water market to supplement his water supply in the case of $\theta W(t) < X_l$. When the price of water is low, buying water to supplement agent’s water supply may make more economic sense than investing in water-saving
technology. However, by waiting to invest now the agent has the option to invest in new technology should water prices go up in future. The value of this option to delay investment is further reinforced by the specialized nature of irrigation investments which may constrain the value of scrapping the irrigation technology or its conversion to other uses should one seek to undo or reverse the investment post installation.

Suppose $F(P)$ is the value of the agent’s option to invest in modern water-saving technology defined explicitly below. Given the positive relationship between $P$ and $V$ as seen from (11), the expected value of investing, $V(P)$, rises and falls as water price rises or falls. Over low enough price ranges, $V(P)$ falls lower than the fixed cost of investment, $I$; and the option to switch technologies is not a viable one. When water prices get sufficiently high, however, the option to invest in water-saving technology will become viable, and the agent will exercise his option to invest. The agent will trade off the potential benefits of waiting for more information to invest against the opportunity cost of waiting.

While uncertainty never goes away (due to the geometric Brownian motion), waiting affords the agent more information about the uncertainty through increased observations of the movement of the relevant stochastic processes. In particular, waiting would afford the agent more observations of the movements of $P$ and $W$, and the agent may then incorporate this information in deciding how best to deal with the uncertainty associated with the investment decision.

The agent will choose to invest if water price reaches a critical threshold. Dynamic optimization methods are employed to determine the threshold. Suppose $\hat{P}$ is the price of
water that triggers investment, then in the region \((0, \hat{P})\), in which the agent exercises his option to wait, the Bellman equation is:

\[
(13) \quad \rho F(P) dt = E[dF(P)].
\]

This states that over the interval \(dt\), the return on the investment opportunity \(\rho F(P) dt\) must equal it’s expected rate of capital appreciation, \(E[dF(P)]\) to ensure the agent exercises his option to wait (Carey and Zilberman, 2002).

Using Ito’s Lemma to expand the right-hand side of equation (13), \(F(P)\) can be shown to satisfy the following differential equation (Dixit and Pindyck, 1994; Carey and Zilberman, 2002):

\[
(14) \quad \frac{1}{2} \sigma_p^2 P^2 F''(P) + \alpha_p PF'(P) - \rho F(P) = 0
\]

This states that over any infinitesimal time interval, the loss in the value of the investment due to time decay and gain in value as a result of the impact water supply evolution on water prices must offset each other such that the return is a return at the riskless rate \(\rho\).

Making the substitution \(\alpha_p = \rho - \delta\), we obtain

\[
(15) \quad \frac{1}{2} \sigma_p^2 P^2 F''(P) + (\rho - \delta)PF'(P) - \rho F(P) = 0
\]

subject to the boundary conditions:

\[
(16) \quad F(0) = 0
\]

\[
(17) \quad F(\hat{P}) = V(\hat{P}) - I
\]

\[
(18) \quad F'(\hat{P}) = V'(\hat{P})
\]

Equation (16) states that when the price of water is zero, the option to invest is worthless. The value-matching condition (Dixit and Pindyck, 1994) (17) states that the
value of the investment option should equal the expected present value less the fixed cost of investment at the threshold. The smooth-pasting condition (Dixit and Pindyck, 1994) (18) states that the change in the value of the investment option should equal the change in the expected present value of the investment at the threshold.

Equation (15) must be solved subject to boundary conditions (16)-(18) in order to find $F(P)$. Given that (15) is linear in $F$ and its derivatives, the general solution (Dixit and Pindyck, 1994) for this second-order homogenous differential equation can be expressed as a linear combination of any two independent solutions. By trial and error, the function $AV^\beta$ is shown by substitution to satisfy equation (15) provided $\beta$ is a root of the characteristic quadratic equation,

$$Q(\beta) = \frac{1}{2} \sigma^2 \beta (\beta - 1) + (\rho - \delta)\beta - \rho = 0$$

Thus, the general solution may be expressed as $(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2}$. To satisfy boundary condition (16), $B_2$ must be zero. This is because zero to a negative power is undefined, and we know our quadratic equation in (15) has one positive, and one negative root.

The positive co-efficient ($\frac{1}{2} \sigma^2$) of $\beta^2$ in $Q(\beta)$ depicts an upward facing parabola (Figure 2 in appendix) that approaches infinity as $\beta$ approaches positive or negative infinity. Now, $Q(0) = -\rho$ and $Q(1) = -\delta$. Given $\rho$ and $\delta$ are both positive, this suggests the two roots of $Q(\beta)$ are one to the right side of 1 ($\beta_1 > 1$), and another to the left of zero (i.e. a negative root) because at $Q(0)$, the function is still below the horizontal axis (negative) (Dixit and Pindyck, 1994).

Thus, the general solution for the value of the option must be of the form,
where $\beta_1 > 1$ is the positive root of the fundamental quadratic equation of the differential equation (15), and the constant $B_1$ must be determined as part of the solution. To see why, substitute $F(P) = B_1P^{\beta_1}$, $F'(P) = \beta_1B_1P^{\beta_1-1}$ and $F''(P) = \beta_1(\beta_1 - 1)B_1P^{\beta_1-2}$ into (15) and simplify, to obtain the fundamental quadratic equation $Q(\beta)$

\begin{equation}
\frac{1}{2} \sigma_p^2 \beta (\beta - 1) + (\rho - \delta) \beta - \rho = 0
\end{equation}

$F(P) = B_1P^{\beta_1}$ satisfies the differential equation provided $\beta_1$ is a root of $Q(\beta)$. Using the quadratic formula to solve equation (20), the two roots are;

\[
\beta_1 = \frac{1}{2} - \frac{(\rho - \delta)}{\sigma_p^2} + \sqrt{\left[ \frac{(\rho - \delta)}{\sigma_p^2} - \frac{1}{2} \right]^2 + 2\rho} > 1
\]

and

\[
\beta_2 = \frac{1}{2} - \frac{(\rho - \delta)}{\sigma_p^2} - \sqrt{\left[ \frac{(\rho - \delta)}{\sigma_p^2} - \frac{1}{2} \right]^2 + 2\rho} < 0
\]

Now $b^2 - 4ac > 0$ since the square of any real number is positive, thus the expression in the square brackets must be positive and $\rho > 0$. The whole expression under the square root ($b^2 - 4ac$) is therefore positive, and the two roots are distinct and real.

Combining equations (12) and (20) with the boundary conditions (16-18), the investment threshold is

\begin{equation}
\frac{\dot{p}x^*}{\delta} = \left( \frac{\beta_1}{\beta_1 - 1} \right) \left( \frac{q}{r + 1} \right)
\end{equation}
where $\frac{\beta_1}{\beta_{1-1}}$ is known as the hurdle rate (Carey and Zilberman, 2002). Defining 
\[ \hat{I} = \frac{a}{r} + I \] and 
\[ \hat{V}(\hat{P}) = \frac{P^xP^r}{\delta}, \] the threshold condition can be rewritten as:

\[ (23) \quad \hat{V}(\hat{P}) = \left(\frac{\beta_1}{\beta_{1-1}}\right) \hat{I} \]

Because $\beta_1 > 1$, the condition states that the expected revenue from the investment must be greater than the total investment cost at the threshold.

Rearranging equation (22), the threshold price of investment is

\[ (24) \quad \hat{P} = \left(\frac{\beta_1}{\beta_{1-1}}\right) \frac{\delta}{X^r} \hat{I} \]

For $P < \hat{P}$, the agent holds onto the option to invest, and for $P \geq \hat{P}$ the agent invests in the water-saving infrastructure. Note that $\hat{P} = \left(\frac{\beta_1}{\beta_{1-1}}\right) \bar{P}$, where $\bar{P}$ is the threshold price of the NPV model. Thus, when one accounts for uncertainty, irreversibility and the option to wait, a farmer or institutional agent such as a canal company or a municipality, would require a higher price than is stipulated by a NPV model, before it is willing to invest in water-saving technology. Uncertainty makes the agent more cautious about investment due to the presence of an option value to waiting in the face of an irreversible, uncertain investment decision.

The disparity between the net present value and option value solutions, is the factor 
\[ \left(\frac{\beta_1}{\beta_{1-1}}\right). \] Thus its underlying factors (the variables that constitute $\beta_1$), become useful in determining the response of the agent’s option value and therefore optimal investment decision, to changes in these parameters.

Recall the fundamental quadratic equation $Q(\beta)$, represented by equation (20).
\[ Q(\beta) : \frac{1}{2} \sigma_p^2 \beta (\beta - 1) + (\rho - \delta) \beta - \rho = 0 \]

Taking the total derivative with respect to sigma, we get;

\[
(25) \quad \frac{\partial Q}{\partial \beta} \cdot \frac{\partial \beta}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0
\]

Now,

\[
(26) \quad \frac{\partial Q}{\partial \beta} = \frac{1}{2} \sigma^2 (\beta - 1) + (\rho - \delta)
\]

Evaluated at \( \beta_1 \), \( \frac{\partial Q}{\partial \beta} > 0 \) since \( \beta_1 > 1, \sigma > 0, \text{ and } \rho - \delta > 0 \).

Also, \( \frac{\partial Q}{\partial \sigma} = \sigma \beta (\beta - 1) > 0 \) at \( \beta_1 > 1 \).

Since \( \frac{\partial Q}{\partial \beta} \) and \( \frac{\partial Q}{\partial \sigma} \) are both greater than zero evaluated at \( \beta_1 > 1 \), then \( \frac{\partial \beta}{\partial \sigma} \) must be less than zero for equation (24) to hold.

Thus, \( \beta_1 \) decreases as \( \sigma \) increases implying that the disparity factor \( \left( \frac{\beta_1}{\beta_1 - 1} \right) \) increases as \( \sigma \) increases. Now, \( \sigma \) is the volatility parameter. Hence, the greater the uncertainty, the greater the disparity between the real options and net present value solutions.

Similarly, taking the total derivative of \( Q(\beta) \) with respect to \( \delta \) yields;

\[
(27) \quad \frac{\partial Q}{\partial \beta} \cdot \frac{\partial \beta}{\partial \delta} + \frac{\partial Q}{\partial \delta} = 0
\]

Here, \( \frac{\partial Q}{\partial \delta} = -\beta \), which is less than zero evaluated at \( \beta_1 > 1 \). Thus, given \( \frac{\partial Q}{\partial \beta} > 0 \), \( \frac{\partial \beta}{\partial \delta} \) must be positive for equation (26) to hold. \( \frac{\partial \beta}{\partial \delta} > 0 \) implies that \( \frac{\beta_1}{\beta_1 - 1} \) decreases with a greater \( \delta \). Hence, the greater the convenience yield, \( \delta = \rho - \alpha_p \), which measures the value of owning a good today versus owning the option to acquire the good in future, the smaller the option value of waiting.
Finally, taking the total derivative with respect to $\rho$ yields:

$$(28) \quad \frac{\partial Q}{\partial \beta} \cdot \frac{\partial \beta}{\partial \rho} + \frac{\partial Q}{\partial \rho} = 0$$

$$\frac{\partial Q}{\partial \rho} = \beta,$$ which is greater than zero evaluated at $\beta_1 > 1$. Again, given $\frac{\partial Q}{\partial \beta} > 0$, then $\frac{\partial \beta}{\partial \rho}$ must be negative for equation (27) to hold. Thus, a larger $\rho$ corresponds with a smaller $\beta_1$, and a larger $\frac{\beta_1}{\beta_1 - 1}$. Thus, a larger risk-adjusted interest rate leads to a greater (option) value to waiting.

It is also worthy to note that $\beta_1$ approaches 1 and $V(\hat{P})$ approaches infinity as $\sigma$ approaches infinity. Thus, the agent will never invest in the case of an infinitely large uncertainty. This is because while it increases the potential gains from investment, it also increases the value of waiting for more information ($F(\hat{P}) = V(\hat{P}) - I$) due to the extremely high levels of uncertainty. On the other hand, as $\sigma$ approaches zero, there are two possible cases depending on the value of $\alpha_p$. If $\alpha_p > 0, \beta_1 \to \rho/(\rho - \delta)$ and $V^* \to \left(\frac{\rho}{\delta}\right)I$. If $\alpha_p \leq 0$, on the other hand, $\beta_1 \to \infty$; and $V^* \to I$. Now, $\left(\frac{\rho}{\delta}\right)I > I$ since $\rho = \delta + \alpha_p$, and $\alpha_p > 0$. Thus, even in the absence of uncertainty, there may be an incentive for the agent to require a greater-than-net-present-value premium because waiting to invest means further discounting of the investment cost in terms of today’s dollars (Marglin, 1963; Dixit and Pindyck, 1994). This premium, where present, is further compounded by the introduction of uncertainty.
3.4 Incorporating the Shutdown Price

The critical thresholds in the previous sections assume that water price will approach infinity given a sufficient amount of time. However, it is possible that water will eventually become so expensive that certain farmers will abandon production (shutdown) in favor of leasing their water rights since the amount they obtain from leasing all of the water rights would exceed any profits from production.

We need not look far for examples of this. Hurt by the prolonged drought in 2013, farmers in Eddy county of New Mexico kept bill collectors at bay by selling their water to oil and gas developers\(^6\). In 2014, the drought persisted and transfers continued with the office of the state engineer reporting increased applications, and a significant backlog\(^7\). Frustrated by increasingly uncertain water allocations in the face of maintained payment of water infrastructure maintenance and water transfer costs, in 2010, two farmers in San Joaquin in California proposed to sell 2000 acre-feet of water annually at $5,850 per acre-foot to developers, netting revenue of about $11.7 million. Similar narratives can be found in Santa Fe\(^8\), and given that the current continuing drought in California has driven agricultural water prices to $3000 per acre-foot as against $60 in a normal year\(^9\) and is pricing out many farmers, this trend can be expected to have increasing and far-reaching occurrences.

\(^7\) http://www.currentargus.com/carlsbad-news/ci_25022789/drought-prompts-farmers-sell-off-irrigation-water-oil
\(^8\) http://www.sfrporter.com/santafe/article-6807-death-by-a-thousand-cuts.html
\(^9\) http://www.mercurynews.com/drought/ci_26181042/high-bidding-farmers-battle-water-auctions
A total profit condition is used to determine an upper bound on $P$ below which agents continue production instead of selling their water rights and quitting the industry. The total profit condition is satisfied if:

$$ P_y a_t B X_t^* - P(t)(X_t^* - \theta W(t)) - C_t \geq P(t)\theta W(t) $$

This condition requires that the total profit an agent derives from production should be greater or equal to the profit he stands to gain from selling his entire allocation of water. This condition is necessary to ensure continued agent participation in the market. This establishes an upper limit of $\bar{P}_l = P_y a_t B - \frac{C_t}{X_t^*}$.\(^{10}\)

Including this upper bound on $P$ eliminates an analytic solution for the problem. The previous section proceeds by assuming that the upper bound is so large it is inconsequential. With an upper bound on $P$, there is a positive probability that the agent will choose to terminate production and lease all of his water rights. Since it would no longer be in use, the water-saving technology would provide no value to the agent. This suggests a nonlinear relationship between water price and the value of water-saving investments from the agent’s perspective. We utilize a numerical solution technique that approximates the value function associated with a water-saving investment from the perspective of an agent who may no longer need the water-saving technology.

\(^{10}\)This upper bound on profit also ensures that the marginal cost of buying water must be less than or equal to the marginal profit to be derived by employing that water in production. Similarly, the marginal revenue from selling all available water must be less than, or equal to the marginal profit (Carey and Zilberman, 2002) ensuring agent participation in the market (or an interior equilibrium).
3.5 Investment under Water Supply and Technical Uncertainty

Consider the case where in addition to uncertainty in the agent’s water supply (expressed in this model through its relation to water price) due to changing climatic conditions; there exists uncertainty in the agent’s post-adoption water requirements attributable to uncertainty in the efficiency of the new technology, known as technical uncertainty.

The technical efficiency is denoted by $a_t(t) \in [0,1)$. The lower limit represents complete failure of systems to produce water savings relative to previous technology, and the upper limit, open bound to show that a hundred percent efficiency ($a_t = 1$) is not feasible.

The agent knows the efficiency of the current technology $a_0$ but $a_T$ is unknown until the investment is made. We assume that the uncertainty in modern technology efficiency is governed by a geometric Brownian motion with zero drift described by;

\[
    da_T = ma_T dt + va_T dz_a 
\]

The agent’s optimal water demand under the modern technology is given by

\[
    X^*_T(t) = \frac{y^*}{a_T(t)B} 
\]

where $y^*$ is the agent’s optimal level of production. Following Ito’s Lemma, the agent’s optimal water demand also follows a GBM

\[
    dx^*_T = \alpha_x X^*_T dt + vX^*_T dz_a 
\]

where $\alpha_x = (v^2 - m)$

We assume no trend in the technical uncertainty process here ($m = 0$) and assume that abstracting from changes in technical efficiency due to learning by doing or human
error and given proper maintenance, the irrigation system shows no significant trend in performance.

Thus, (30) reduces to;

\[ da_T = v a_T dz_a \]

and \( \alpha_x = (\nu^2) \) in the evolution of optimal water demand in (31).

The agent’s decision represents an optimal stopping problem where an agent is concerned with choosing the right time to undertake a particular activity, in this case investing in water-saving technology, based on sequentially observed random variables \((P, a_T)\) in order to maximize his expected payoffs from undertaking that particular activity. The solution involves finding the value function and stopping curve that satisfies the agent’s Bellman equation for values of the state variables for which it is optimal to continue.

Numerical techniques are employed in finding a solution to the problem, and the solution procedure is presented in Section 5.
4: A UTAH CASE STUDY

4.1 Study Area

The model developed in the previous section is applied to the Middle Bear-Logan Watershed in northern Utah. Over half of the land in the watershed is used as rangeland for grazing and about one-tenth is irrigated for crop cultivation and other agricultural practices. There are about 113 canal companies in the study region and over 251,550 acres of privately owned farmland in the region. Diversions from the watershed, as well as releases from the Bear Lake facilitate the irrigation of over 485 km² of land by more than 70 irrigation companies in the watershed. To get a sense of the volume of water the canals convey, Last Chance canal, the principal diversion in the watershed, diverts 60,000 acre feet of water annually. Receiving the highest amount of precipitation in the entire Bear River Basin, this watershed’s mean annual precipitation ranges from 43 to 150 centimeters, with most falling as snow in the winter months. This makes it a very crucial watershed in the State (BRWIS, 2011).

The study area consists of three rivers in the watershed - the Logan, Little Bear and Blacksmith Fork Rivers. The Watershed does not include the main stem of the Bear River, and is completely made up of drainage areas for a number of major tributaries of the Bear River. The tributaries flow from the east southwards into the Cutler Reservoir and drain 230,000 hectares. Figure 4 provides a map of the watershed indicating the rivers of interest.

The Logan River, draining the eastern section of the watershed, begins its journey in the high mountains of the Bear River Range in Idaho. From there, it amasses tributary waters from Beaver Creek, Temple Fork and Right Hand Fork; and traverses Logan
Canyon through the Wasatch National Forest. When it gets to the floor of the cache valley, it passes through the city of Logan and agricultural areas on its outskirts, ultimately joining the Blacksmith Fork in the Cutler Reservoir. The Logan River drains about 1,500 km² of land, when you include the Blacksmith Fork and adds the greatest volume of water to the Bear River in the entire Bear Basin.

The Blacksmith Fork also commences up in the Bear River Range and drains the lands south of the Logan River. Traversing the Blacksmith Fork Canyon, it collects waters from Sheep Creek, Curtis Creek, Rock Creek, and Left Hand Fork.

The Little Bear River drainage system caters to the southern end of the watershed. The South Fork Little Bear, which drains the mountains on the southernmost end of the watershed; and the East Fork, which collects water from a large area of forested land in the Bear River Range; constitute its two main sub-drainages.

4.2 Estimating Water Supply Uncertainty

Historical time series data obtained from United States Geological Services (USGS) records on annual stream flow levels \( W_t \) from the Logan (41 years), Bear (69 years), and Blacksmith Fork (88 years) rivers in northern Utah, are used to estimate the drift \( \alpha_w \) and volatility \( \sigma_w \) parameters for the stochastic water supply process.

To test whether the data are consistent with the Geometric Brownian Motion (GBM) assumption, an augmented Dickey Fuller test is performed (Conrad, 1997; Forsyth, 2000; Insley, 2002; Pindyck and Rubinfeld, 1998, Sims and Finoff, 2013). While GBM assumes aggregate water supply is log-normally distributed, logged water supply \( w = \)
\( \ln(W) \) is normally distributed and follows an arithmetic Brownian motion (ABM) 
\[ w = \mu dt + s dz_w \] 
with \( \mu \) and \( s \) as the drift and volatility parameters of the ABM respectively. If \( w \) is consistent with arithmetic Brownian motion, Stratonovich calculus\(^{11}\) ensures \( W \) must be consistent with GBM. Testing that \( w \) is consistent with arithmetic Brownian motion requires running a restricted regression,

\[
(w_t - w_{t-1}) = \beta_0 + \beta_1 (w_{t-1} - w_{t-2}) + \epsilon_t
\]

and unrestricted regression

\[
(w_t - w_{t-1}) = \beta_0 + \beta_1 (w_{t-1} - w_{t-2}) + \beta_2 t + \beta_3 w_{t-1} + \eta_t;
\]

in order to test the null hypothesis for an arithmetic Brownian motion: H0: \( \beta_1 = \beta_2 = 0 \). The F-test results for the regression results are shown below in table 1.1. We fail to reject the null hypothesis that \( w \) follows an arithmetic Brownian motion (H0: \( \beta_1 = \beta_2 = 0 \)) for the dataset for all three rivers (i.e. \( w_{t-1} - w_{t-2} \) and \( w_t - w_{t-1} \) are independent random variables, a characteristic of the Weiner process). Thus, we conclude that GBM is an appropriate assumption. This conclusion is corroborated in the literature by Fisher and Santiago (1997) and Bhaduri and Manna (2014) who assume a GBM for the stochastic evolution of water supply based on the log-normal distribution of water flow.

Performing the restricted and unrestricted regressions on the three data sets, the following F values were obtained.

\(^{11}\) See appendix for explanation of the use of the Stratonovich rather than Ito’s formulation in this case
Given that stream flow levels appear consistent with GBM, we proceed to estimate the drift and water supply parameters of the stochastic water supply process from the mean and standard deviations of our time series data respectively. These \((\alpha_W, \sigma_W)\) are then used to estimate the corresponding drift and volatility parameters for water price \((\alpha_P, \sigma_P)\) using the relationship established between the two processes in equation (5). Drift and volatility parameters are presented in Table 1.2 (and are explained in Section 4.4 below), and plots of the data from the various rivers showing the drift and volatility are shown below in Figures 1.1(a-c).
Figure 1.1a: Stochastic processes for water supply and water price for Logan River.

Figure 1.1b: Stochastic processes for water supply and water price for Little Bear River.
The graphs appear to confirm logged water supply and prices follow ABMs, corroborating the GBM narrative with an obvious downward (upward) trend for water supply (price), and a confirmation of the volatility around these trends.

4.3 Estimating Technical Efficiency Uncertainty

There is a dearth of data on actual on-farm irrigation efficiencies. To obtain a value for the volatility of the technical efficiency process, it is assumed that with 90% probability, the efficiency of the system will remain between 70% and 95% in the next 20 years. This assumption is based off the probability-based volatility estimation in Carey and Zilberman (2002); and informed by the average and high performance efficiencies for center pivot irrigation of 70% and 95% respectively (Rogers et al., 1997).
Given the GBM assumption, changes in \( \ln a_T \) are normally distributed. Using a log transformation, and the 90% confidence interval for the normal distribution, we obtain (via the central limit theorem) the 20-year variance to be 0.416 and obtain by division, an annual variance of \( \sigma^2 = 0.021 \) and standard deviation of \( \sigma = 0.145 \).

4.4 Selecting Parameter Values

The model is general enough to be applied to a wide variety of water-saving infrastructure investments. For illustration, we consider an investment in more efficient on-farm irrigation technology. The parameter values selected in Table 4 are based on this type of investment and drawn from the established values in the literature. To illustrate the generality of the model, select results for an investment in the lining of canals or ditches to mitigate seepage losses are presented in the appendix. The interest rate is set at 6 percent to reflect the interest rate used in evaluating similar investments in the literature (Michailidis and Mattas, 2007). A risk-adjusted interest rate of 12 percent is also chosen based on the literature (Carey and Zilberman, 2002). The convenience yield (which is the agent’s effective discount rate) is calculated as the difference between the risk-adjusted discount rate and the drift in the rate of water supply for each river dataset. The lease
price of water is determined from data from the Utah Water Rights exchange. The expected water savings, $X^*$, is chosen as the mean of the highest and lowest water savings from irrigation investment in semi-arid areas in the western United States (Brown, 2008; Hill, 2000). The values for initial investment costs are obtained from Utah estimates for cost of center pivot irrigation by the Natural Resources Conservation Service of the United States Department of Agriculture (NRCS, 2009). Finally, the commodity price used in determining the shutdown price is derived from using the price of alfalfa in the 2013 Utah Agricultural Statistics to compute the dollar value of alfalfa yield per acre-foot of farm acreage (UAS, 2013).

Table 1.2: Common Parameters for Irrigation Investment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate ($r$)</td>
<td>0.06</td>
</tr>
<tr>
<td>Risk-adjusted discount factor ($\rho$)</td>
<td>0.12</td>
</tr>
<tr>
<td>Water savings ($X^*$)</td>
<td>2.25 AF</td>
</tr>
<tr>
<td>Initial (one-time) Investment cost ($I$)</td>
<td>$575</td>
</tr>
<tr>
<td>Total Investment cost ($\hat{I} = I + \frac{q}{r}$)</td>
<td>$805</td>
</tr>
<tr>
<td>Commodity Price ($P_y$)</td>
<td>$193</td>
</tr>
<tr>
<td>Parameter</td>
<td>Logan</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td><strong>Initial Water Supply ((W_0))</strong></td>
<td>135166.60</td>
</tr>
<tr>
<td><strong>Water Supply Drift ((\alpha_W))</strong></td>
<td>-0.0177</td>
</tr>
<tr>
<td><strong>Water Supply Volatility ((\sigma_W))</strong></td>
<td>0.4686</td>
</tr>
<tr>
<td><strong>Initial Water (Lease) Price ((P_0))</strong></td>
<td>$38/AF</td>
</tr>
<tr>
<td><strong>Water Price Drift ((\alpha_P))</strong></td>
<td>0.2415</td>
</tr>
<tr>
<td><strong>Water Price Volatility ((\sigma_P))</strong></td>
<td>0.9189</td>
</tr>
<tr>
<td><strong>Technical Efficiency Volatility ((\nu))</strong></td>
<td>0.145</td>
</tr>
</tbody>
</table>
5 NUMERICAL SOLUTION

Optimal stopping problems with one state variable are relatively easy to solve. They involve the solution of an Ordinary Differential Equation (ODE) and may even provide closed form solutions under certain conditions depending on the nature of the ODE. The solution of optimal stopping problems with two or more state variables differs from the one state variable case in that one has to solve a partial differential equation (PDE) rather than an ODE. Often, a closed form solution may not exist for the relevant PDE, mandating the use of numerical techniques in finding a solution to the problem.

Brekke and Oksendal (1994) introduced the method of variational inequalities as a numerical technique capable of resolving this conundrum, and this method has been increasingly utilized in the real options literature, notable among which is Balikcioglu et al. (2011), Marten and Moore (2011), and Sims and Finnoff (2013). The method, as presented in Balikcioglu et al. (2011), is based on the collocation approach and complementarity representation of the optimal stopping problem. The collocation method seeks to select a 2-dimensional space of candidate solutions (polynomials of degree 2 in our case) and a number of points in the domain (called collocation points), and choose the solution (polynomial) satisfying the given differential equation at these points (Balikcioglu et al., 2011).

The general methodology of the numerical solution method sets the problem up as follows:

The solution to our optimal stopping problem with two state variables (Price (P) and Technical Efficiency ($a_T$)) is obtained by a numerical approximation of a value function,
$Y(S)$ according to Brekke and Oksendal (1994), where $S$ is a continuous 2-dimensional state process described by a diffusion of the form;

\begin{equation}
(35) \quad dS = \mu(S)dt + \sigma(S)dZ,
\end{equation}

where $\mu$ is a 2x1 drift vector, $\sigma$ is a 2x2 diffusion matrix, and $Z$ is a 2-dimensional vector of independent Wiener processes.

An agent, faced with an optimal stopping problem, chooses between receiving a flow of payments $f(S)$ per unit time, and a one-time reward, $U(S)$ of stopping, after which no further rewards are given (Balikcioglu et al., 2011). The agent thus seeks to maximize his expected discounted flow of payments and his optimal policy involves choosing when to stop his value function;

\begin{equation}
(36) \quad Y(S) = \max_{T} \mathbb{E} \left[ \int_{0}^{T} e^{-rt} f(S_t)dt + e^{-rT} U(S_T) | S_0 = S \right]
\end{equation}

Brekke and Oksendal (1994) show that the optimal value function in this case, $Y(S)$, satisfies the set of complementarity conditions;

\begin{equation}
(37) \quad rY(S) \geq \mathcal{L}Y(S) + f(S)
\end{equation}

where $\mathcal{L}$ is the infinitesimal state process generator, and

\begin{equation}
(38) \quad Y(S) \geq U(S)
\end{equation}

with one of these conditions holding with equality at each value of $S$.

In our water-saving investment model, the state space is two dimensional $(P, a_T)$. Applying the numerical solution procedure as pioneered in Brekke and Oksendal (1994) and further applied in (Brekke and Oksendal, 1994; Judd, 1998, Miranda and Fackler, 2002, and Balikcioglu et al., 2011) to the water-saving investment problem, we find the following pair of complementarity conditions.
(39) \[ \rho Y \geq \Pi_0 + \frac{E[y]}{dt} \equiv \Pi_0 + \alpha_p \frac{dy}{dp} + \frac{1}{2} \sigma_p^2 \frac{d^2Y}{dp^2} + \frac{1}{2} \sigma_a^2 \frac{d^2Y}{da^2} + \]

\[ \sigma_a \alpha_P \sigma_p \frac{d^2Y}{d^2p} \]

(40) \[ Y \geq U + I \]

where the last term in (39), \( \sigma_a \alpha_P \sigma_p \frac{d^2Y}{d^2p} \) goes to zero with uncorrelated \( P \) and \( a_T \).

Equation (39) holding with equality implies a delay in investment (i.e. agent exercises his option to delay investment to obtain better information for possible investment at a later date). When it holds with inequality, however, the investment is not viable (the expected return is less than the required return from investment) and therefore falls in the no-investment zone. (40) holding with equality however, implies the agent should invest.

For our water-saving infrastructure investment model, the optimality conditions are evaluated numerically in MATLAB based on the study area in northern Utah.
6 RESULTS AND DISCUSSION

Using the model set-up and baseline parameter values described above in Table 4, real options investment thresholds are solved for three scenarios:

1) without stochastic efficiency and without farm shutdown price\(^{13}\)
2) without stochastic efficiency and with farm shutdown price
3) with stochastic efficiency and with farm shutdown price

Scenario 1 is directly comparable to Carey and Zilberman (2002) and will be more applicable in cases where the efficiency of the technology is well understood and the current price of water is low relative to the price that would trigger shutdown. Scenario two maintains the same characteristics as scenario 1, with the added characteristic that the current water price has the potential to hit the shutdown price in this scenario. In scenario 3, efficiency uncertainty is added to scenario 2. To illustrate the influence of the option value, the real options thresholds (RO) will be compared to their corresponding benefit-cost thresholds (NPV).

6.1: Scenario 1

The thresholds are solved for solely water supply uncertainty and no shutdown price. These results are will be compared with the model results from the added sources of uncertainty and market exit conditions, which are numerically solved for in the next section.

\(^{13}\) Cannot find real option threshold with stochastic efficiency and without shutdown price
Table 1.3: Irrigation Investment Thresholds under Scenario 1

<table>
<thead>
<tr>
<th></th>
<th>Logan</th>
<th>Bear</th>
<th>Blacksmith-Fork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold P (NPV)</td>
<td>21</td>
<td>54</td>
<td>49</td>
</tr>
<tr>
<td>Threshold P (OV)</td>
<td>271</td>
<td>237</td>
<td>246</td>
</tr>
<tr>
<td>Threshold W (NPV)</td>
<td>1,194,218</td>
<td>739,691</td>
<td>475,009</td>
</tr>
<tr>
<td>Threshold W (OV)</td>
<td>3,234,756</td>
<td>349,406</td>
<td>207,692</td>
</tr>
</tbody>
</table>

The results from the study (Table 1.3) indicate that water supply uncertainty delays investment in farm-scale irrigation investments. This can be seen from the disparity between the real options (OV) and benefit-cost (NPV) thresholds required for investment. The consideration of uncertainty in the real option investment evaluations lead to greater water prices required for investment. We also see that differences in volatility make a difference in the impact of uncertainty consideration on investment. The Bear River, which showed the lowest volatility in water price (and consequently, its derivative water supply process) as can be seen from comparing figures 3a, 3b, and 3c, as well as from the respective volatility parameters in table 4, requires the lowest price threshold for investment under the real options consideration.

Thus, even within a particular geographical area, agents may face different uncertainty conditions depending upon their particular water source. It is interesting to
note, however, that under the benefit-cost criteria, irrigators along the Bear River would require the lowest price thresholds for investment. This makes intuitive sense. The Bear is the largest of the three albeit the most volatile and so considering quantity of water supply without considering the volatility factor would give a low estimate for the value of the water-saving infrastructure investment. Thus leaving out uncertainty considerations in evaluating investment decisions can produce significant distortions in investment analyses.

Of interest, in considering these water supply thresholds, are estimations of how long it would take under current conditions for available water supplies to get to these critical thresholds. To this end, we employ Freeware (Excel Spreadsheet) by Marco Antonio Guimarães Dias\(^\text{14}\) which affords estimates for what it would take to attain the thresholds in question.

### Table 1.4: Hitting Time Simulations

<table>
<thead>
<tr>
<th>Irrigation</th>
<th>NPV (Years)</th>
<th>OV (Years)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logan</td>
<td>N/A</td>
<td>827.98</td>
<td>N/A</td>
</tr>
<tr>
<td>Bear</td>
<td>246.84</td>
<td>1259.53</td>
<td>5X NPV</td>
</tr>
<tr>
<td>Blacksmith</td>
<td>152.56</td>
<td>1159.52</td>
<td>8 X NPV</td>
</tr>
</tbody>
</table>

\(^\text{14}\) Dias, Marco Antonio Guimarães. First Hitting Time and Expected Discount Factor. Available online at http://www.puc-rio.br/marco.ind/hittingt.html [2012, November 16]
We find on the average, that uncertainty in year-to-year water availability in northern Utah causes individuals to delay water conservation investments an average of at least 5X what would be suggested by discounted cash flow analysis for center pivot irrigation technology.

The table shows simulated hitting times for the irrigation investment. It shows how long it would take for the critical price threshold to be hit under current conditions for both net present value and option value considerations. The Logan River’s price threshold for irrigation investment under the net present value consideration is lower than what the lease price of water is today, hence the N/A. It would take 5 times the time under the option value consideration for the lease price Bear River of the Bear River to reach the price threshold all other things remaining the same, and 7.6 times as the time for the Blacksmith Fork River, emphasizing the significant delay uncertainty considerations can cause.

6.2: Scenarios 2 and 3

The thresholds presented here reflect the optimal timing of investments in new technologies whose efficiency is uncertain, and where the shutdown water price is imminent. In these instances, conclusions drawn from Carey and Zilberman (2002) may be misleading due to the omission of technical efficiency uncertainty and market exit condition from their investment evaluation. Comparing the results of scenario 2 to scenario 1 provides insight into the effect of market exit. Comparing the results of scenario 3 to scenario 2 provides insight into the effect of uncertain efficiency improvements.
Figure 1.2 shows the approximated value of the investment option when a market shutdown price is incorporated in the investment decision. The x-axis shows the price, the y-axis shows the range of technical efficiency, and the z-axis shows the approximated value of investment. From the graph, it can be seen that the expected present value is negative for lower levels of price due to the sunk cost of investment, and increases with increasing price. It also shows that the investment then becomes less valuable as the price continue to increase and gets closer to the shut-down price. From the graph (read from left to right), the net present value falls below the zero line for low price values. It then increases as price increases, peaks then falls.

This underscores the importance of the market shutdown condition. An assessment done without considering the option of the agent to leave the market will project an infinitely rising value of investment in response to price increase and this projection would have a gap from observed behavior which shows farmers selling their water at high enough prices. While Utah’s water allocation policies and limited water market may act as a bottleneck to the outright sale of saved water at the moment, there are still the options to lease, sell well water and in the worst case scenario, sell off farmland and ranchland. In support of this, McGinty (2009) showed that agricultural land is increasingly being sold due to rising costs of production, and the call for public comment for the draft of a 50-year water strategy for the state (UWF, 2013) saw many calls for more favorable provisions for the transfer of saved water due to efficiency investments.

It can also be seen from Figure 1.2 (reading from right to left along the y- and z-axes, that the value of investment decreases with higher efficiency levels. This is because the value of investment comes from water savings, and the water savings to be made from
converting a 75% efficient system to a 95% one, is lower than converting from a 40% flood system to a 95% center-pivot system. This also underlies the importance of technical uncertainty considerations in the decision to invest in more efficient water-saving infrastructure. An advertised 55% gain in efficiency (40 to 95%) delivering only a 25% actual efficiency gain, could make the difference between a profitable investment and a non-profitable one.

Figure 1.2: Impact of technical uncertainty on expected net present value of Investment.
Figure 1.3a: Comparison of scenarios 1 and 2

Figure 1.3b: Comparison of scenarios 2 and 3
Figures 1.3a and 1.3b detail the impact of technical uncertainty and a market-exit condition on the agent’s decision to invest. Figure 1.3a is the deterministic technical efficiency case and highlights the effect of imposing the shutdown condition on the agent’s investment decision; whereas 1.3b shows the effect of a stochastic technical uncertainty. The unbroken lines detail the scenarios incorporating the shut-down price consideration, and the broken lines detail their no-shutdown counterparts. From the figure, it can be seen that the introduction of the shut-down price introduces a backward bending effect on both curves in the cases of both the net present value and option value considerations, imposing a bound on the ability of price increases to increase the profitability of investment. After a certain point, the value of investment no longer increases in response to price increases, and it rather becomes more profitable to market one’s water, rather than continue production.

The backward bending we see in the top half of Figure 1.3a can be attributed to the reduction of the desirability of investment as water price approaches the shutdown price. In the lower half of the figure, we note that the cases with the shutdown condition (solid lines) lie above their no exit counterparts (broken lines). This is a result of the fact that the option to invest will be less valuable if the technology may someday be rendered worthless if the agent chooses to exit the market.

Similar to Figure 1.2, the curves confirm the delaying effect of uncertainty consideration on the decision to invest. The curves for the real options scenarios (unbroken line) lie above their net present value counterparts in all cases, detailing a higher price threshold requirement for investment under the option value (uncertainty) consideration. This suggests that uncertainty in future water supplies and the efficiency of
new water-saving technology create similar incentives to delay adoption of new water-saving infrastructure investments.

In Figure 1.3b, the real options case is shown in black and red lines, and in both cases, lie above the benefit-cost lines in green and blue signifying greater price thresholds are required for investment in the option value consideration as opposed to the more traditional net present value evaluation. The higher investment thresholds and consequent fewer cases of investment compared to the benefit-cost evaluation is consistent with the findings of Carey and Zilberman (2002) and suggests that the discrepancy between the benefit-cost recommendations and observed investment in water-saving infrastructure may well be due to uncertainty considerations in the investment decisions.

In addition to uncertainty about future supplies, uncertainty about the realized as opposed to advertised performance of water-saving technology factors into an agent’s decision to invest in said technology. This can be seen from comparing the two figures juxtaposed in Figure 1.3b. We note an increase in the investment threshold when stochastic efficiency is introduced into the model as noted by a higher price requirement for investment shown in the right hand diagram. To confirm the effect is attributable to efficiency uncertainty, we note that the figure maintains the lower benefit cost investment threshold in the left-hand diagram, while indicating a higher threshold for the real options case. This means that research and policy action targeted at reducing or managing technical efficiency has the potential to influence irrigation investment in the desired direction. This is important because while water supply uncertainty stems from inherently uncertain climate conditions, and is therefore likely to continue, efficiency uncertainty may be more easily managed, for instance, by more data-collection and research on the
post-adoption performance of conservation technology. A catalog of the various observed performances under various hydro-geophysical and climatic conditions may diminish the agent’s perceived uncertainty about the benefits of adoption, and help him make a more informed and more immediate investment.\textsuperscript{15}

The question of how reductions in both types of uncertainty would influence the adoption of water-saving infrastructure can best be investigated through sensitivity analysis. The added investment delays seen in the non-zero versus zero uncertainty cases, suggest that reduction in both sources of uncertainty would lead to greater investment. However, to determine which source of uncertainty has greater leverage in influencing investments, we conduct sensitivity analyses at 2\%, 5\% and 10\% levels to investigate the impact of a 2\%, 5\% and 10\% increase in the volatility parameters for water supply and technical efficiency have on the results of the model.\textsuperscript{16} The results suggest that changes in the volatility parameter for water supply have a greater impact on the decision to invest relative to volatility in technical efficiency. Thus, while uncertainty surrounding the performance of the efficient technology does have a significant impact on the decision to invest, concerns surrounding the future availability and predictability of water supplies are greater. This has major repercussions with water supply projected to see more seasonal and inter-annual variability with changing climatic conditions.

Introducing the shutdown price in the analysis maintains higher investment threshold requirements in terms of the price of water under the real option value scenario. In

\textsuperscript{15} Rogers et al, 2005 find that deviations of installed system conditions from design operating conditions can cause system performance to vary as much as three times lower than in systems matching design operating conditions.

\textsuperscript{16} See appendix for results of sensitivity analyses
addition, the introduction of a shutdown price makes the threshold curves backward-bending. When it becomes possible to earn more by selling the farm’s water allocation rather than engaging in production, the agent exits the market. Thus, if the agent perceives that his water rights are going to be more valuable on the water market than any future production, he has no incentive to invest, but rather to wait and lease his water right. Since the main incentive for investment considered here is monetization of water savings or money ‘saved’ by not having to buy more water for production with increased water scarcity, the possibility of not investing should have a depressing effect on the level of investment.

This result is perhaps more notable in the case of agricultural produce, and perhaps in the case of relative geographical mobility of food and people present in the United States which permits food importation from other states, as well as the possibility of migration to other states. In practice, agricultural produce is really not permitted, all other things being equal, to fully reflect the cost of production due to government desires to protect consumers from hunger, as well as producers from production risk, at the expense of state and federal subsidies. The United States Department of Agriculture and the Utah Department of Agriculture can only subsidize production so much before it becomes more economical to import agricultural produce. Further research into this issue could thus prove valuable for planning purposes. Considering the predicted regional water supply conditions, long-term planning could, for instance, make provision for gradually diversifying the state economy away from water-intensive production for a worst-case future water supply scenario, while encouraging investment in conservation technology in the short-term.
The backward bend in figures 5a and 5b highlights the fact that some water-conservation technologies may not be efficient enough to trigger investment regardless of the price they could receive for the conserved water. When he has the option to leave a commodity market, higher water prices have two effects on the incentive to invest. First, they increase the benefit of investment in each period. Second, they decrease the expected time until the farm shuts down which decreases the expected time the technology will be in operation and decreases the expected net present value of the investment. In Figure 6b, the shutdown cases see no investment below technical efficiency about 0.55 for the benefit–cost, and 0.75 for the real options cases respectively. These all highlight the impact ignoring the technical efficiency uncertainty could have on optimal investment.

Thus, when the agent considers the uncertainties in water price and technical efficiency, higher water price thresholds are required for investment. And this investment delay is further aggravated by introducing a shutdown price, answering the question, ‘What effect does market entry and exit have on investment behavior.’

A point of interest to note would also be the relative absence of competition when it comes to agricultural production. Azevedo and Paxson (2012) find in a game-theoretic real options model, that technical uncertainty delayed investment in the new technology, but that it had a greater stalling effect on the follower than the leader who had the incentive to seek first mover advantage. Thus, accounting for competition may be a valuable future exercise.
7 CONCLUSIONS

There is increasing interest in the impact of hydrological and economic uncertainty on water infrastructure investment in the face of changing climatic conditions. The frequently observed under-investment in water-saving infrastructure despite positive net present value estimates is worrisome because delayed water infrastructure investments in the face of changing climatic conditions may result in a future water supply problems in the ‘best’ case, and damaged property and lost lives in the worst cases.

The results from this study confirm the delaying impact of uncertainty on water-efficiency investment behavior in our Utah case study. The empirically estimated threshold values of water supply and price to invest in water-saving suggest that uncertainty may very well explain the observed underinvestment in water-saving infrastructure investment in the region.

The results also confirm the effects of uncertainty across rivers within the watershed. The Logan River, with the highest water supply volatility appropriately requires the highest and lowest water price and corresponding water supply thresholds, respectively as seen in table 1.3.

To answer our underlying research question, the study finds that uncertainty in future water supplies and the efficiency of new water-saving technology create similar incentives to delay adoption of new water-saving infrastructure investments.

The introduction of the shutdown condition results in backward bending investment threshold curves, highlighting the fact that some water-conservation technologies may not be efficient enough to trigger investment regardless of the price they could receive for the conserved water. While increases in water price in response to increasing water
scarcity would make these technologies more profitable than they would be at lower prices, profits from production would have to be high enough to make a case for selling saved water under continued production, and selling all water and exiting the industry.

The evaluation of farm irrigation investments has margins which could be explained by the consideration of uncertainty, as well as the inefficiency of the water markets in the watershed relative to that of neighboring regions. The water rights lease price of $38 is relatively low considering the prices for similar amounts of water in neighboring states. Also the prevailing prior appropriation water rights system stymies the benefits to be obtained from sold water savings and may also contribute to the investment inertia in the region.

Finally, the sensitivity analyses for comparing the relative effects of the two sources of uncertainty reveal that the relationship between these uncertainties and their joint effect on investment in irrigation technology, is worth future study, and holds rich promise for targeted policy and research actions.

These observations provide insight as to what measures may be taken to encourage water-saving infrastructure investments in the face of the associated inherent uncertainty across investment types and geographical areas. While policies that target perceived investment uncertainty may work best for irrigation investments, the fastest results with regards to canal lining investments may be subsidies to reduce present cost of these investments, as well as research and development to develop cheaper alternatives.

Continuing research would extend the model to allow for continuous uncertainty in the future cost of investment that may arise from future technological innovations and policy, as well as sequential investment.
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CHAPTER TWO: INCREMENTAL ADOPTION OF EFFICIENT IRRIGATION TECHNOLOGY: A REAL OPTIONS APPLICATION

ABSTRACT

This study employs an incremental real options approach to evaluating the decision to invest in efficient irrigation technology. It found that allowing for gradual investment can lead to up to 75% of investments occurring earlier than in the case of the same investment evaluation without incremental consideration.
1 INTRODUCTION

As with any economic field of enquiry, the application of Real Option Theory to evaluating capital investment decisions in economics has inherent complexities and limitations that are typically dealt with by making simplifying assumptions to ensure tractability. One of these assumptions is in the setting up of problems such that the decision to extract, invest or develop is an all or nothing decision. With the all-or-nothing set-up, a decision to extract a resource meant choosing between leaving the resource wholly intact or a complete extraction of the resource. Likewise, a decision to invest in energy-efficient technology meant a complete overhaul of existing technology; and a decision to develop an area with a resource such as an old growth forest meant that the entire forest would be lost in the development process.

However, decisions are not so black and white in real life applications. We regularly observe sequential decision-making with regards to the eventual scale and scheduling of capital investments (Mezey and Conrad, 2010). Gupta and Rosenhead (1968) proffer a motivation for this more gradual decision-making. They assert that one way investors guard against the danger of regretting large investments, particularly in the face of appreciable uncertainty about future conditions, is ensuring that their early (and irreversible) steps in the investment sequence keep as many of the ‘good’ options open as possible. Thus, the observed more sequential investment decision-making may be explained by investors choosing the timing and scale of their investments in order to minimize their potential of investor regret; and offers them more courses of (potentially more optimal) action relative to the all-or-nothing scenario where their only recourse to avoiding regret would be delaying the whole investment.
For example, consider the case of a farmer choosing to adopt a new agricultural technology. As Ma and Shi (2015) affirm, agricultural technology is often adopted sequentially, with farmers first adopting a new technology on part of their lands and make later adjustments to their use of the new technology based on their experience of the initial partial adoption. A farmer, seeking to invest in terracing to mitigate erosion and soil losses might decide to terrace the steepest slope on his acreage first should he be unable (due to budget or other constraints) or unwilling (due to cost-effectiveness) to do his entire land area at a time. This decision makes economic sense because the steepest section has the most losses and so has the greatest benefit potential. In the terminology of production theory, marginal benefit of terracing an added acre is greater in the steepest sections and would be more likely to measure up to the marginal cost of investment if we allowed for a breakdown of the cost of investments to permit partial and incremental investment. The formulation of such a farmer’s investment decision as an all or nothing decision, however, might see him not investing at all because the requirement of full investment and attendant higher resource commitment is more difficult to justify.

The incremental investment decision approach advanced by this study provides a clearer picture of the value of flexibility by decomposing the full investment from Chapter 1 into a series of partial investments. By presenting the opportunity for a partial initial investment with potential incremental adjustments in future, this approach offers the opportunity to take advantage of new information at each incremental stage. In the face of unfavorable unfolding of events which increase the cost and/or decrease the value of completing the investment, an incremental approach allows one to cut one’s losses by temporarily or permanently halting future investments. Under favorable conditions, an
incremental approach still preserves the option to undertake additional investments that become economically viable under the new conditions.

Thus, assuming an ‘all or nothing’ stance in evaluating investment decisions is more likely to lead to suboptimal investment decisions under uncertain future conditions because it increases the possibility of perceived over-investment (Woodward et al., 2014) by catering to worst case future scenarios (and thus best case investment return scenarios), ignoring the more probable tempered middle ground scenarios. An agent, having made the decision to invest to avoid a worst case scenario, will perceive his investment as over-investment if the future evolution of the variables he considered, fall short of the unlikely worst case scenario. This relates to the Bernanke (1983) bad news principle which asserts that the willingness to invest in the current period depends only on the severity of bad news that may arrive given a current return. And that it does not matter at all how good the potential for future good news regarding the investment may be (Maoz, 2004).

This effect is attributable to the effect of irreversibility on potential for investor regret. Since an investor’s response to bad news (i.e. partially or fully disinvesting) is constrained after an irreversible investment has been made, he ascribes a disproportionate weight to the possibility of bad news, and this suppresses his perception of the value of the validation post-investment good news may bring. This weight increases the greater the resource commitment and uncertainty associated with an investment since these determine the size of potential loss and thus regret, and probability of this loss, respectively. It may also be amplified in the case of a pessimistic investor, and vice versa.
The value of the real options approach is in its inherent flexibility which allows decision-making to be adaptable to a wide range of future scenarios. Thus, by doing away with the all-or-nothing assumption inherent in traditional real options models, we capitalize on the flexibility benefits of using the real options approach in evaluating investment decisions. By allowing the agent to incrementally invest, we temper this distorted perception since smaller scales of investment lessen the possibility and scale of potential overinvestment.

Employing the incremental formulation thus enables us to capture both flexibility with respect to the timing of investment – (relaxing the now or never assumption) from the traditional real options approach, and flexibility with respect to the scale of investment from the incremental or impulse control formulation gained by relaxing the all or nothing assumption in the traditional model. This is illustrated in the following diagram.
Figure 2.1: Assessing the temporal and scale flexibility of technology adoption investments

Figure 2.1 illustrates the possible combinations of investment flexibility and models employed in capturing them, to situate the models in this chapter and in chapter one. Discounted cash flow methods with fixed costs of investment (A) are used for investments with limited flexibility with regards to timing and scale. This investment class is the most restrictive and formulates the investment decision as an all-or-nothing investment which must be undertaken now or never. The next category (B), relaxes this to allow for incremental investments, but maintains the now or never requirement (i.e. it does not consider the option value). It does this by setting up the investment as consecutive segments with costs varying from one segment to the next.
(C) maintains the all-or-nothing formulation, but allows for timing flexibility beyond now or never by considering the value of the option of waiting to invest. And (D), represents the most flexible of these models by allowing for both flexibility in the timing (now or never) and scale (all or nothing) assumptions.

There are three cases where the more flexible incremental real options investment approach proposed in chapter 2 stands to be more appropriate compared to net present value and non-incremental option value formulation, and captures information otherwise missed. These are;

1) Liquidity- where the agent does not have sufficient funds upfront for the full investment

2) Litigation- where public opposition to a part of the project would stall a full investment, as in the case of protests against canal lining

3) Non-constant marginal productivity - where incremental investment allows the agent to optimize timing and scale of investment to maximize return from his investment dollars.

While the first two are pertinent, this chapter focuses primarily on the third to examine the impact of uncertainty on agent investment scale and schedule.

It is hypothesized that in general, and particularly in the case of investments involving big picture goals such as regional water conservation, the added flexibility presented by the incremental investment formulation would increase the probability of positive outcomes (i.e. more incidence of investment) relative to the all-or-nothing formulation. It is hypothesized that this greater incidence of gradual scaled investment has the potential to be large enough to offset the investment margin losses due to scaling. This has
significant implications for investments such as energy or water conservation (irrigation) or pollution abatement where individual investments while evaluated for profitability to individual agents making the investment decisions, also work in tandem towards bigger picture outcomes such as sustainability of agriculture in a semi-arid environment given uncertain evolution of hydroclimatic conditions.

There are two reasons providing backing for this incremental investment story with regards to irrigation investment. First, variation in soil conditions (e.g., soil composition, nutrient content, drainage) lead to variation in the marginal productivity of irrigation technology. Current economic and climatic conditions may make water-saving technology upgrades feasible on only a portion of a farm. Second, in requiring greater pecuniary commitment, a farm-wide technology upgrade is likely to require the kind of return associated with a worst-case future water supply and price scenario. The substantial investment costs; the discounted, uncertainty-adjusted benefits of investing fully in water-saving technology may only surpass cost requirements if future water prices attain significantly high values – the kind associated with extreme future water scarcity (drought). Given the low probability of this event, positive returns would be out of reach in most cases preventing investment. Thus, in trying to get agents to be protected against the worst-case scenario via the nature of the all or nothing model set up, they may elect to acquire zero protection given the uncertainty associated with said worst-case scenario, preferring to defer investment to a later date where more information about future water supply and water prices are available.

Applying an incremental investment approach to the decision to invest in water-saving infrastructure such as efficient irrigation technology becomes critical when water
supply and water price are highly volatile. Since uncertainty has the effect of spreading the tails of a normal distribution, it increases the likelihood of both very good news (greater water scarcity and elevated value of saved water) and very bad news (less scarcity and lower value of saved water) affecting the value of the investment. The Bernanke bad news principle suggests that the possibility of greater water availability (lower valued of conserved water) than expected has a disproportionate drag on the agent’s willingness to invest.

This chapter thus extends the previous chapter to consider the effect of allowing for impulse-control (where the agent may incrementally upgrade technology, investing where he can make the greatest efficiency-gains first, with possible future expansion) on an agent’s decision to invest in water-saving infrastructure in the face of hydrologic uncertainty in the Wasatch Range Metropolitan Area (WRMA).

Lifting the constraint of total or zero investment will temper some of the impact that irreversibility of the irrigation investment has on the agents’ decision to invest in the face of uncertainty. By allowing for gradual scaled investments, it allows the agent to scale his loss potential in increments, mitigating the potential for investor regret since he can stop at any point before total investment is attained. Specifically, it is hypothesized based on existing literature that agents will have greater incentive to invest in more units of technology in the face of greater uncertainty (Dixit and Pindyck, 1994; Schwartz and Trigeorsis, 2004).

However, this will be counteracted by a corresponding increase in the opportunity cost of investment in consecutive marginal units by requiring each marginal unit to meet profitability requirements by itself rather than under the total investment umbrella,
leading to a reduction in optimal investment levels. It is also expected that while independent agents who do invest may invest in less technology units, more agents will invest as compared to the all or nothing scenario and the net effect of this will depend on the relative magnitude of the extensive (number of agents investing) and intensive (number of acres with upgrade technology).
2 BACKGROUND

The incremental real options approach to evaluating capital investments has been amply applied in the strategic management literature (Krychowski and Quelin, 2010; Leslie and Michaels, 1997). Here, Real Option theory is treated as a rhetorical tool which helps organizations structure their investment decisions in the context of high uncertainty. An application by Leslie and Michael (1997) posits an incremental investment strategy as a management best case practice, confirming the merit of sequencing investments into phases in order to benefit from upside risk, while eliminating the cost of downside risk.

The incremental real options approach has also been applied in the engineering literature, as a strategic decision-making tool (Ramirez, 2002; Miller and Park, 2002). It capitalizes on the arguably greatest benefit of the real options approach by encouraging investment decision-making which preserves the flexibility of choice, as well as future modification of investment in response to economic circumstances (McGrath et al., 2004, Miller and Park (2002).

Other fields where this incremental approach has been amply applied include the capacity choice and asset replacement literature. Zambujal-Oliviera and Duque (2011) analyze operational asset replacement strategy in a real options framework, and confirm that different types of uncertainties produce non-monotonic effects on optimal capital replacement level.

Dangl (1999) investigates simultaneous choice of optimal capacity choice and optimal investment timing under irreversible investment costs and uncertain future demand. He asserts that while a once and for all decision (capacity choice is not all or nothing, but cannot be revised in future once chosen) leads to higher optimal installed
capacity, it also causes investment delay and makes investment so sensitive to uncertainty that even small amounts of uncertainty lead to a wait and see conclusion for large ranges of demand.

Incremental real options can also be used to manage risk. Guma (2008) in a real estate development application asserts that developers can better manage risk associated with a weak market and gain the potential to benefit in a strong one, by considering investments that afford the capability to react appropriately to uncertain future events. The study considers the value of the option of vertical real estate expansion in dense urban centers, and determines that incorporating the real options framework enhances the potential of matching investor preferences to development projects.

Extending this application, Boyer et al. (2004), Kong and Kwok (2006), and Smith and Trigeorsis (2004) confirm the advantages of a joint dynamic real option and strategic competition (games) analysis, revealing the considerable impact of imposing the indivisibility assumption on real options investment analyses.

In economics, this study joins the sparse but steadily evolving literature dedicated to real options applications which consider not only whether (and when) to invest (optimal stopping and timing), but also how much to invest (optimal capacity). Pindyck (1988) applies this to the choice of firm capacity, focusing particularly on marginal investment decision. He posits that a firm’s capacity choice is optimal when the value of the marginal unit of capacity is just equal to the total marginal cost of that unit. In true real options fashion, this total cost includes not only purchase and installation costs, but also the opportunity cost of exercising the option to buy the unit.
Pindyck (1988) suggests two steps to an optimal capacity analysis. The first involves a determination of the value of a marginal unit of capacity given a starting capacity. This value must account for the fact that should demand unexpectedly fall; the new unit of capacity may be unutilized. Next, one must determine the value of the option to invest (which will depend, in part, on the value determined in step one), together with the decision rule for exercising the option. This decision rule, in essence, would solve the optimal capacity problem.

Uncertainty has substantial influence in the optimal capacity problem, as with optimal stopping and timing problems. Since a marginal unit of capacity need not be utilized, it is worth more when demand fluctuates stochastically. This appears to create an incentive for the firm to hold more capacity in the face of uncertain future demand (Pindyck, 1988). However, uncertainty also increases the opportunity cost of exercising the option to invest in a marginal unit. As future uncertainty increases, the value of the marginal unit increases, but the opportunity cost of investing increases even more which produces the net effect of a reduction in the firm’s optimal capacity choice. The opportunity cost here is the so-called user cost of capital in the literature which captures investment reluctance borne out of the anticipation of the irreversibility constraint holding in future, (Abel and Eberly, 1996; Bertola, 1988; Dixit and Pindyck, 1994; Dixit, 1989; Pindyck, 1988). To wit - while uncertainty increases the potential value of increased capacity, it also increases the potential of unused capital stock in future (overinvestment) and thus a drag on optimal capital capacity.\(^{17}\) Thus ignoring such costs could lead to over-investment.

\(^{17}\) This result requires that marginal revenue product be a decreasing function of capital stock, as noted by Pindyck (1993) and Abel and Eberly (1997).
This creates the need to correctly compute and account for the opportunity cost in optimal capacity decisions, and provides motivation for this study. While chapter one considers how the joint impact of uncertainty and irreversibility delays investment relative to the net present value case, this study goes a step further to show how the all or nothing formulation of the investment decision contributes to this delay and how allowing for gradual piecewise investments may help better optimize the timing and level/intensity of investment.

Real options with impulse control has also been applied, in the natural resource economics literature, to the management of invasive species where the presence of the complexity of multiple interacting species and uncertainty in the population dynamics mandates control strategies above the single-species control models. Kotani et al., (2009) apply it in examining the effect of species catchability on optimal invasive species management strategy, and conclude that given the sensitivity of species catchability to current stock size, the optimal control sequence could drastically change in response to changes in stock size as successive units of control is applied. Adopting an all or nothing approach in such an evaluation would thus erroneously lead to no control being applied, where management strategies which maintain some positive economically efficient levels of the invasive species would be optimal. The identification of this optimal strategy is however only possible in an incremental investment framework.

Conrad and Yang (2009) identify the threshold gypsy moth concentrations that would minimize the discounted sum of damage and control costs associated with managing the invasive moths. They do this by incorporating the associated damage and control costs
into a complex biological model describing the evolution of gypsy moth populations and find that the minimization of the sum of discounted damage and spraying costs does not result in a smooth convex function relating damage costs to gypsy moth concentrations. The damage function is instead, discontinuous in relation to gypsy moth concentrations and has the graphical form of a series of line segments with multiple, non-unique, local minima. This would be a treacherous optimization problem for a local solver and inaccurately captured by the imposition of a smooth function. In relation to the focus of this chapter, the maximization of the difference between expected costs and benefits of investment in the real options framework given the evolution of water supply and water price may not result in a smooth, convex threshold curve which is imposed by an all or nothing model assumption, and such an assumption would yield spurious results.

Applying real options theory, Saphores (2000) formulates an optimal stopping model for the application of control measures to a pest population whose concentration varies randomly. The expected marginal cost of reentry for a farmer is analyzed by introducing a delay between successive pesticide applications. Applying this to the pesticide-control of a foliar apple pest and solving numerically for threshold levels, Saphores (2000), finds that threshold pest density needed to trigger pesticide control varied significantly with the uncertainty surrounding pest density. He thus established that incorporating the uncertainty in pest spread evolutions may help better manage the pesticide application to soils and crops by using the threshold pest densities as a guide to optimize timing and scale via an incremental pest control strategy.

In their study of aquatic invasive species, Leung et al. (2002) demonstrate the importance of uncertainty in determining optimal prevention expenditures needed to
attain economically efficient invasion risk levels. Saphores and Shogren (2005) contribute an essential extension to this in their application of a real options framework to account for both uncertainty and managerial flexibility in the traditional dynamic model of non-native invasive species management. In their model, a resource manager is faced with an investment decision to eliminate 90% of an invasive species at a fixed cost. Given the population dynamics of the species which was uncertain with an exponential growth expectation, the solution to the model proved to be a trigger population level which, when reached, will prompt the resource manager to exert control on the invasive species population.

Marten and Moore (2011), following the arguments in Saphores and Shogren (2005), develop a more generalized real options framework to incorporate more complex and realistic management strategies such as a hybrid biological and chemical control strategy. Albeit with increases in complexity, their generalization of the model makes it possible to solve for the optimal timing and flow of the control measures rather than restricting the solution to a fixed level of control as was done in Saphores and Shogren (2005). Marten and Moore (2011) found under early detection and control, biological control agents were sufficient to contain the invasive species populations within acceptable limits. However, a delay of control which allowed invasive species populations to explode mandated a more costly optimal (simultaneous chemical and biological) control strategy.

This chapter will adapt an approach based on Dixit and Pindyck (1994) and applied in Marten and Moore (2011) to extend the model of water-saving investment under economic and hydrologic uncertainty in chapter one. We will analyze the investment strategies for improving water-use efficiency when the inherent all or nothing restriction
in the decision to invest in the chapter one is lifted. We depict a model general enough to be applied to a range of water-saving technology investments such as canal lining investments but will focus on irrigation investments for simplicity.

We begin with a modification of our stochastic dynamic investment model from chapter one, to allow for multiple possibilities of different investment magnitudes and sequential timings. The modified model is then used to examine how relaxing the all-or-never investment assumption impacts the incidence and magnitude of investment in water-saving infrastructure in Utah. Using empirical observations to calibrate the model, we will then compare the results to both results from chapter one and observed investments on water-saving technology, to see which one more accurately explains observed behavior, and how policy can be modified, if need be, to reflect partial investment possibilities and capitalize on that for the attainment of conservation policy objectives.

Juxtaposing the model results with that of chapter one, we consider the relative effects of water supply variability on irrigation investments in areas with greater variability in on-field characteristics which would be more likely to benefit from the incremental investment consideration, and discuss the ramifications of the differences in outcomes of the two models for water conservation and adaptation to changes in agricultural water availability and predictability in the western United States. This contributes to the body of knowledge on how the incremental approach to adopting water-saving infrastructural improvements impacts the intensity (magnitude) and extent (incidence) of adoption in the case of investments which may be broken down into units
with differing contributions to productivity and thus value of investment, and the big picture implications of these impacts.

Key Words: Incremental Real Options, Site-specific Irrigation
3 MODEL

This chapter examines an agent’s decision to invest in water-saving technology in a more general context, by formulating the agent’s problem as making adjustments to his existing technology stock.

In chapter one, technological change is treated as a discrete change in a parameter from \(a_0\) to \(a_t\). Here, technological change is treated as a stock to which additions can be made, and is handled by the introduction of a new variable \(U = f(a)\), that denotes the agent’s current level of investment, represented by acres of farmland converted to the new technology. Thus, while the all or nothing investment in chapter one has investment move from an entire field under old technology to the entire field under new technology, increasing overall farm irrigation efficiency from \(a_0\) to \(a_t\), the incremental investment makes stepwise changes to capital stock \((U)\) with each consecutive addition to \((U)\) contributing its marginal change in water efficiency depending on characteristics of that unit which impact its irrigation efficiency (water-saving) potential.

For comparison purposes, the model in chapter one may analogously be interpreted as an agent with a starting capital stock \(U_0\) who makes a single investment to increase his capital stock from \(U_0\) to \(U_T\). Since the agent’s problem was formulated as an all or nothing question, he required a critical water price level that would make the whole \(U_0\) to \(U_T\) investment maximize his expected present value of farm profits under the option value consideration. Under the incremental story, he now has the ability to optimally choose \(U_T\) and has the option to make additional investments in the future that increase his capital stock to \(U_{T+1} > U_T\). This additional flexibility presents the agent with the
option to cut out the additions to capital stock that make the least contributions to the total benefit derived from investment resulting in a lower price required to make the initial investment. In a way, the lower critical water price required for investment may be seen as the agent’s marginal willingness to pay to avoid investing in the least productive units of his capital stock given current water prices. Being able to delay these least productive units has value because full investment under uncertainty requires that investment yield a larger expected return to compensate for the lesser productive units of investment. This higher expected rate of return caters to the less likely worst case future water supply scenarios (i.e. the higher the water price threshold, the greater the scarcity conditions that would be required to hit that price). While uncertainty increases the occurrence probability of these dire scenarios, the majority of the distribution of our underlying stochastic process remains in the middle ground. The partial investment option enables the agent to minimize his potential for investment remorse given the irreversible nature of the investment.

The agent currently has a profit flow which is a function of his existing technology and variable input (water) applied. He can choose to alter this profit flow by making changes to his existing technology stock through a series of irreversible investments.

We break down the agent’s investment in improved water-saving technology to allow for a graduated technology overhaul. These investment graduations can be viewed as distinct individual projects, and can conceptually be lined up, in order of their marginal productivity, as consecutive units of investment from the unit with the greatest marginal product to that with the lowest. This breakdown is appropriate when the production function shows diminishing returns to scale suggesting that individual units of investment
do not contribute equally to the increased total product and profit considered in a full investment evaluation. For instance, the evaluation of an irrigation investment covering 100 acres of land with an estimated profit of $100,000 would suggest an average profit of $1000 per acre. It is however possible, given diminishing returns to scale, that $75,000 out of this total profit is contributed by fifty out of the 100 acres of farmland, with the remaining 50 contribute increasingly smaller margins to the total profit.

A practical way this may manifest itself is with field level variation in geo-hydrological conditions such as slope, soil type, etc. Kumar et al. 2008 confirms that irrigation system performance with regards to field level water savings varies both across- and on-field based on the geo-hydrological environment comprising factors such as slope and soil type; crop type; and the field-specific agro-climate. Thus field level variations may contribute significantly to the potential effect of improved irrigation efficiency on farm-level water savings within field. In this case, the greater the variability in water savings across the field, the greater the bias resulting from the one-time investment model presented in chapter 1.

For their treatment of sequential and incremental investments, Dixit and Pindyck (1994) consider first, the case of multi-stage investments where the time to completion is the major motivator of the sequential investment story. Based on the fact that many projects, especially large ones, take substantial time to complete and thus can be put on hold at any stage and be discontinued or continued at a later date; investment projects are denoted as having many stages where each dollar spent gives the firm the right, but not the obligation to go ahead and invest the next dollar. The underlying characteristic here is the ability to hold or quit the project in response to evolving economic conditions which
may affect cost of and/or returns on investment. The investment decision then comprises choosing a conditional investment path (conditional on hitting certain determined investment thresholds for the firm at each point on the path) to optimize investment timing and cost/return.

With the sequential investment story, the project does not generate cash flows until completed. An example here may be a large hydroelectric dam project which may be put on hold for investment funds to be invested elsewhere in the face of persistent drought which reduces electricity potential of the underlying river, or be halted if a country found a cheaper alternative source of electricity while the project was ongoing. While the flexibility allows money spent on the project (costs) to be reined in, since the dam produces no electricity and thus yields no cash flow until it is complete making cash flow from even a 75% complete project exactly zero.

They then considered the capacity choice treatment of incremental investments where the decision-maker alters his capital stock by undertaking a number of projects, and retiring or adding projects as unfavorable or favorable conditions develop.

For example, in our above mentioned farm investment scenario, the gains from partial investment (in the first and most productive 50 acres of farmland) comprise hedging against increase in the cost of initial investment, and having the option to hold, scrap or abandon the project should unfavorable conditions develop. However, water savings do occur and thus the farmer gets returns (cash flow) from his investment even with a partial investment. Thus the incremental investment story may be said to be a special case of a multi-stage or sequential investment story where one does not have to make the full
investment to start reaping the benefits of investment and a partial investment still yields some cash flow.

The appeal of the possibility of partial incremental investments here is the flexibility it affords the agent to avoid investor remorse by investing under the worst case future water supply scenario the all or nothing investment story panders to; or to make a partial investment in the acres of farmland with the greatest potential for water savings, and potentially either invest in the remaining acreage or sell it off as future conditions evolve.

As chapter one showed, the option to exit the market is a valuable one and under the incremental story, the agent would have the option to keep his most valuable acres, and divulge the rest in sales\(^1\). Thus the availability of option to exit adds another nuance which may further stack things up against the all-or-nothing formulation. Also, there may be grants, subsidies, and other forms of financial assistance the farmer may have access to which may not cover investment in his total acreage. Under this model, he would have the option to make a partial investment, and then later reinvest some profits to cover the remaining acreage\(^2\).

Given the uncertain future supply and price of water, we find a profitability threshold that justifies investment for each conceptually distinct investment project. McClintock (2009) asserts that the option value of investment becomes increasingly important with larger investment. And it is posited that the usual option-value effect of delaying

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18 With a lesser efficient system, the amount of water he would need to supplement production in scarcity conditions would be even higher eroding profitability margins of arm production and increasing the probability that exiting the market would be more beneficial than continued production.”

19 In a canal lining investment analog, the investor could minimize canal seepage losses by making gradual partial investments in the case where legal dispute over the project (as seen in the recent case of the Logan Canal Restoration project) invokes a stay/hold on total investment). This is a valuable consideration given the potential of legal process for lengthy deliberations.
investment will be tempered by the option to make a partial (smaller) investment, leading to greater incidence of investment, but varying extents of investment.

While analysis in this study considers a single agent, this has implications for larger scale analysis and application. A state, in a bid to meet its water conservation goals, may through extension services assist farmers with plans involving gradual incremental improvements to their irrigation systems. This approach would have the merit of catering to farmers’ overinvestment aversion, as well as putting investment dollars (whether from farmer or government incentive coffers) towards water conservation goals every step along the way. For instance, it may be profitable for only 10% of the farming population to make 100% irrigation investments under the all or nothing formulation, but profitable for at least 60% of farmers to upgrade at least 50% of their total acreage under the incremental investment formulation.

Consecutive units of capital with their consecutively lower marginal products will require higher and higher profitability thresholds for investment. The consecutive thresholds are chosen to maximize potential water savings and thus profits from investment, and by timing the investments when the consecutive thresholds are hit, the potential for ‘overinvestment’ and its attendant investment remorse is minimized, limiting any drag it may have on investment.

This contrasts with the set-up in the non-incremental story in chapter one where each acre of farmland is implicitly assumed to contribute equally to the total value derived from investment in more efficient irrigation technology. We can then establish an investment threshold curve by plotting different agent’s technology stock levels against water price threshold values required to invest up to those levels. The threshold curve,
which relates the agents level of irrigation technology to water price (see figure 2.1), then illustrates how each additional unit of investment requires its own progressively higher required threshold for investment, and is upward sloping as a result of the decreasing returns to scale assumption.

As capital stock increases, the acres of farmland remaining to be invested in have increasingly lower potential for water savings and so would require increasingly higher prices to make their investment worthwhile. No investment is made for water price levels below the threshold curve, and our agent will invest in water-saving infrastructure up to the last acre of irrigated farmland where the value of additional water savings for that unit meets the threshold curve.²⁰

We now consider an agent’s incremental technology stock expansion based on Pindyck (1988), Bertola (1989), and Dixit and Pindyck (1994). The agent has the option to invest in water-saving technology for his farm. The agent’s technology stock, \((U)\) refers to acres of farmland converted to the new technology. The new technology adoption has been broken down into units of investment such that the agent incrementally invests in additional acres of farmland, and each irreversible incremental investment costs \(C\) per acre.

Since the agent first converts the most productive acres of farmland before considering the remaining farmland, the uptake of the new technology can be thought of as a series of investments ordered from most productive to least productive. A farmer

²⁰ The definition of investment in \(U\) as farm acreage has an inherent right bound where the investment level is bound by agent’s total acres of farmland. There is thus the potential in future research here for exploring this bound, for instance, by permitting purchase of additional farmland in response to increased profits given how important the exit barrier proved to chapter 1 model results.
who grows alfalfa and other crops for instance, might find it profitable to invest in the acres for alfalfa production, but not for the others. It is also assumed that the nature of the technology and market is such that scrap value is negligible in the agent’s decision due to the specialized nature of the technology which makes it costly to reverse even if not technically irreversible, with any potential scrap value a small fraction of initial capital cost (Conrad and Kotani, 2005). Also, a more competitive market for the agricultural commodity will tend to decrease the incentive to disinvest or scrap the irrigation technology (Dixit and Pindyck 1994)\(^{21}\).

The relationship between technology stock and the agent’s profit is manifest through the impact of agent’s increase in water efficiency on his total demand for (applied) water \(X(U)\)\(^{22}\). For instance, a farmer who is currently using 200 acre-feet of water on his property upon upgrading his irrigation system on a portion of that property, will need less applied water all other things constant. We maintain the same yield neutral assumption in chapter one such that the investment payoff is independent of productivity gains\(^{23}\). As technological efficiency increases with incremental investments (increasing \(U\)), \(X(U)\) increases (i.e. \(X' > 0\)). Thus, the marginal revenue product of each unit of investment is given by \(X^*P\), where \(X^*(U)\) is the water savings from investment \((X_0 - X_T)\), and \(P\) is the same water price process in chapter one evolving stochastically according to;

\(^{21}\) In a competitive agricultural market where farmers are price takers, if the going rate of alfalfa makes farming under a given water scenario profitable for one farmer, it will all other things being equal be profitable for others like him, and so they will all stay rather than scrap, and conversely, they would all be scrapping all other things being equal and so there would be limited incentive to sell or buy another’s scrap.

\(^{22}\) Water demand is implicitly a function of total acreage. This relationship between acreage and water demand is not an issue for the specific problem you are addressing and can therefore be subsumed in the function for \(X\).

\(^{23}\) Productivity gains may prove to be a major incentive for irrigation investment among farmers in certain regions of the country and with certain commodities. The implications of this, as well as potential for future research, is discussed in the conclusions.
\[ dP = \alpha_p Pdt + \sigma_p Pdz_w \]

where \( \alpha_p = \left( -\frac{\sigma_w}{\varepsilon} + \frac{(1-\varepsilon)^2 \sigma_y}{2\varepsilon^2} \right) \) and \( \sigma_p = \left( -\frac{\sigma_w}{\varepsilon} \right) \)

Simplistically, one could boil down the difference between chapter 1 and chapter 2 by a fixed water savings level in chapter one \( (X_0 - X_T = X^*) \), while in chapter 2 it is dependent on our choice variable \( U \) and (designated by \( X^*(U^*) \)). While water savings on each plot of land are exogenously determined in principle, by choosing the magnitude of the impulse (level of irrigation technology), one chooses one’s level of marginal water savings.

We assume diminishing returns to scale such that marginal revenue product \( X'(U).P \) is decreasing in \( U \) (i.e. \( X''(U) < 0 \)). This means that marginal water savings \( X'(U) \), falls with successive additions to the stock of efficient irrigation technology (i.e. additions to water savings in response to successive units of investment, while positive \( X'(U) > 0 \), get increasingly smaller). This may be attributable to physical diminishing returns due to characteristics of the technology or farm.
2.1 Agent’s Problem

Figure 2.2: One-time full investment versus incremental investment path.

Given the initial capital stock $U_0$, the initial water price $P_0$, and $dP/dt$, the agent chooses the timing and magnitude of a series of investments to maximize the expected discounted stream of agricultural profits

\[ \Pi(P, U) = P_y aB X(U) - P(t)[X(U) - \theta W(t)] - \mathcal{C} \]

net of the cost of irreversible additions to the capital stock ($dU/dt \geq 0$ for all $t$) where the marginal revenue product of irrigation capital is $[P_y aB - P]X'$. The optimal timing and magnitude of a series of investments can also be conceptualized as an optimal but discontinuous time path for the capital stock. Figure 1 illustrates the difference in this optimal time path for the one-time investment and incremental investment cases. The solution for the optimal time path of the capital stock for the incremental investment
problem is found through stochastic dynamic programming with value (Bellman) function given by

\[ V(P, U) = \max_U E_0 \left\{ \int_0^{t_1} \Pi(P, U)e^{-\rho t} \, dt + \left\{ [V(P, U + dU) - C_d U]e^{-\rho t_1} \right\} \right\} \]

where \( \rho \) is the discount rate and \( t_1 \) is the time the current marginal investment is made. The inclusion of \( V(P, U + dU) \) in the value function recognizes that the farmer retains the option to make additional investments in the irrigation capital in the future, accounting for the possibility of multiple-period investments. Equation (3) aggregates return during the pre-investment time period, and the farmer evaluates (3) for each possible incremental investment made.

To examine the agent’s piecewise investment decision-making, we consider the time interval \( dt \). Given the continuous nature of agent decision-making, and the fact that \( dt \) is an arbitrary construct, we consider the limit as \( dt \) approaches zero. Suppose the agent invests in water-saving technology, moving his technology stock from \( U \) to \( U' \) at the end of the interval \( dt \). Water price is assumed to shift over this interval from \( P \) to \( P + dP \). While the agent does not know the exact value of \( dP \), he knows its probability distribution, and is thus able to calculate the expected value of the increase in technology stock as;

\[ d\Pi(U, P)/dt + e^{-\rho dt} \{ E[V(U', P + dP)] - C(U' - U) \} \]

Thus, the agent estimates the value of his increase in capital stock as the present value of all future cash-flows associated with the capital stock increase less its cost. This
includes the immediate profit, if the investment is made $d\Pi(U, P)/dt$,\(^{24}\) plus the expected capital gain from making the investment - $E[V(U', P + dP)]$. Change in profit $(d\Pi(U, P)/dt)$ is zero while the agent is still holding the option (waiting to invest) and positive when he invests due to the increase in water savings from his increased efficiency valued at current water price. This is akin to dividends from investment. Capital gain depends on the instantaneous change in the value of the investment opportunity and is tied to the stochastic process specified for $P$. Thus any increase (decrease) accruing to the value of investment due to an increase (decrease) in $P$ above current $P$ is a capital gain (loss).

The agent will choose $U'$ to maximize $e^{-\rho dt}\{E[V(U', P + dP)] - C(U' - U)\}$, with the irreversibility property of the investment requiring that $U' \geq U$. If $V(U', P + dP) \leq C(U' - U)$ in the initial period, the initial value of the value function $V(P, U)$, will be the resulting maximum and no investment will be made until say $P$ increases by enough to turn the inequality around. The concavity property assumed for the production function ensures that the Bellman function obtained is also concave in $U$, and this makes it possible to employ the Kuhn-Tucker conditions of calculus, in characterizing the maximization of the expression in (4). Differentiating (4) with respect to $U'$, we obtain

5) $e^{-\rho dt}\{E[V(U', P + dP)] - C]\}$

As $dt$ tends to zero, the expression in (5) tends to $V(U', P) - C$.

Now, if $V(U', P) \leq C$, the expression to be maximized as expressed in (4) decreases with respect to $U'$ within the range $U' \geq U$. In this case, the agent will find it optimal to

\(^{24}\) We can ignore the effect of discounting over the interval as it shall be of the order $dt^2$ due to Itô’s lemma.
make no alterations to his current technology stock and maintain $U' = U$. In fact, he may find it optimal to decrease $U$ if that were possible. If, however, $V_U(U', P) > C$ in the initial period, the agent will find it optimal to invest in $(U' - U)$ technology in order to set his new technology stock $U'$ at the level defined by the condition (i.e. make an increment to his capital stock up to a level equating value marginal product from investing, and marginal cost of investment);

6) $V_U(U', P) = C$, which specifies our threshold curve.

Figure 2.3: Investment Threshold Curve under Incremental Investment
The figure above (Figure 2.3) illustrates the choice problem faced by the agent under incremental investment. As P evolves stochastically, the point \((P, U)\) moves up or down the threshold curve. Whenever it rises above the threshold curve, the agent increases his capital stock by just enough to move it horizontally back to the threshold curve. The upward slope of the threshold curve here (positive relationship between \(U\) and \(P\)) illustrates the preceding discussion, that consecutively higher price thresholds would be required for investment in lesser productive units.

To develop the optimal policy argument, we start with the region to the right of the threshold curve where it is optimal to take no action (wait). We substitute \(U' = U\) (i.e. \(dU = U' - U = 0\)) in (3) to obtain the initial value \(V(P, U)\). This substitution gives us,

\[
V(P, U) = d\Pi(P, U)/dt + e^{-\rho dt}E[V(P + dP, U)],
\]

and expresses as in chapter one, the sum of profit accruals over the interval \((t, t + dt)\) (first term on right-hand side) and the continuation value beyond \(t + dt\) (second term on right-hand side).

Applying Ito’s lemma to the right hand side, we obtain

\[
V(P, U) = \Pi(P, U)dt + \left[\alpha_pPV_p(P, U) + \frac{1}{2}\sigma_p^2P^2V_{PP}(P, U)\right]dt + (1 - \rho dt)V(P, U) + 0(dt)
\]

where \(0(dt)\) captures all terms which go to zero faster than \(dt\). Thus,

\[
V(P, U) = \Pi(P, U)dt + (1 - \rho dt)V(P, U) + \left[\alpha_pPV_p(P, U) + \frac{1}{2}\sigma_p^2P^2V_{PP}(P, U)\right]dt
\]

\[
= V(P, U) + \left[\Pi(P, U) - \rho V(P, U) + \alpha_pPV_p(P, U) + \frac{1}{2}\sigma_p^2P^2V_{PP}(P, U)\right]dt
\]

This implies that \(V(U, P)\) satisfies the differential equation.
If after investing in the nth ‘unit’ of irrigation technology, the evolution of water price fails to rise to approach the value of the marginal efficiency of the (n+1)th unit of irrigation technology and get the agent back to the curve, investment stops until it does. This means that from an initial point above the curve, the agent invests to move the point horizontally to the curve. Thereafter, his level of irrigation investment remains constant while $P$ is low enough to keep him on the curve, or responds continuously to small increments in $P$ at the curve. Since any upward move of $P$ at the barrier (threshold curve), leads to a corresponding rightward move of $U$, the time path of $U$ at such locations is not differentiable. This means that the time derivative $dU/dt$ is infinite and thus the impulse (barrier) control policy comprises rather than a finite rate (flow) of investment through time, occasional bursts of investment as the threshold curve is hit. It also eliminates derivatives of the value function with respect to $U$ from (7), and allows us to treat $U$, as a parameter (rather than a variable) which shifts the whole functional relationship. The absence of derivatives with respect to $U$ permits us to treat it as an ordinary differential equation relating $V$ to $P$, and regard $U$ as a parameter that shifts the whole functional relationship (Dixit and Pindyck, 1994). While it is true that farmers typically sign contracts that stabilize their water prices, the market for lease water prices does vary. Thus, the cost (benefit) of leasing (renting water to supplement any shortfalls in supply can vary more readily. This model hold his initial water allocation as exogenous and only considers the impact of water price evolution to his water savings.

25 One property of Brownian motions is that the time path of a Brownian process is, with a probability one, nowhere differentiable. Thus, the time path of $P$ is not differentiable.
Since equation (7) is analogous to equation (15) in chapter one, the general solution of the ordinary differential equation in (7) is then analogously given by;

8) \[ V(P, U) = B_1(U)P^{\beta_1} + B_2(U)P^{\beta_2} + \frac{\Pi(P,U)}{\rho - \alpha}, \]

where the betas are the positive and negative roots of the fundamental quadratic equation, and the B’s are ‘constants’ to be determined.\(^{26}\)

Thus, the fundamental quadratic equation is derived as \[ Q = \frac{1}{2} \sigma_p^2 \beta (\beta - 1) + \alpha \beta - \rho = 0; \] and the ‘wait’ region includes the limit as water price goes to zero. As a result, we must leave out the negative power of the price variable in the solution, in order to keep \( V(P, U) \) finite. To this end, \( B_2(U) \), which is the negative root from the fundamental quadratic equation, is set to zero, and substituting \( \delta \) for \( \rho - \alpha \), the general solution becomes

9) \[ V(P, U) = B_1(U)P^{\beta_1} + \frac{\Pi(P,U)}{\delta}, \]

The first term on the right-hand side in equation (9), is the value of the agent’s optimal future technology investment, and the second term tells us the present value of the agent’s future profits should he maintain his initial technology stock forever. \( B_1(U) \) is then determined by making use of the curve \( V_\delta(P, U) = \mathbb{C} \), which serves as the other boundary of the “wait” or inaction region.

\(^{26}\) The constants of integration are functions of U. This specification of the constants as functions of the capital stock does not suffer from the issues highlighted in Balikcioglu et al (2011) due to the nature of the capital stock here. Specifically, since the value function is not differentiable in U above the threshold curve, U can be treated as a parameter that shifts the functional relationship and not as a second stochastic variable that would necessitate a second set of partial derivatives.
The solution procedure to our impulse control investment problem, follows the same technique as the analytical (single source of uncertainty) section in chapter one; with adaptations made to approximate the curve in figure 1. This is done by taking discrete approximations of $U$ and solving for the optimal threshold at each discretely approximated level of $U$. Subsequently, these derived critical thresholds are linked through interpolation to produce the critical threshold curve.

With every marginal unit of investment the agent makes, he gives up the option value (of waiting) to invest in that unit. Intuitively, since option value captures the value of waiting, this can be thought of as the agent starting with the maximum option value and having each incremental investment erode the option value a little. This illuminates the motivation and benefit of this approach. The option value being calculated for every marginal unit enables the agent to at each step evaluate whether benefit of waiting for more information about the investment in that particular unit is worth the opportunity cost of not making that particular unit of additional investment at that step. With non-constant marginal productivity, some units of investment may never be profitable to make no matter how long one waits and when these are lumped together with the other units, they drag down the value of investing in the all-or-nothing decision framework.

For our water-saving infrastructure investment model, the optimality conditions are evaluated in MATLAB based on the study area in northern Utah.
4 CASE STUDY

We maintain the study area from chapter one, for comparison purposes. The study area thus consists of three rivers - the Logan, Bear and Blacksmith Fork Rivers. We maintain the derived parameter values for water price, and the drift and volatility parameters for their respective water price processes.

<table>
<thead>
<tr>
<th>Table 2.1: Common parameters for Irrigation Investment</th>
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<tbody>
<tr>
<td>Interest rate ((r))</td>
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<tr>
<td>Risk-adjusted discount factor ((\rho))</td>
</tr>
<tr>
<td>Water savings ((X^*))</td>
</tr>
<tr>
<td>Initial (one-time) Investment cost ((I))</td>
</tr>
<tr>
<td>Cost of unit upgrade ((C))</td>
</tr>
<tr>
<td>Commodity Price ((P_y))</td>
</tr>
</tbody>
</table>
| Initial Water Supply \((W_0)\) | Logan: 135166.60  
Bear: 90424.79  
Blacksmith-Fork: 82605.83 |
| Water Supply Drift \((\alpha_W)\) | Logan: -0.0177  
Bear: -0.0059  
Blacksmith-Fork: -0.0060 |
| Water Supply Volatility \((\sigma_W)\) | Logan: 0.4686  
Bear: 0.3806 |
The analytical results from section 5.1 in chapter one, showed the following threshold values for water price and water savings under the all or nothing investment formulation across rivers:

Table 2.2: Analytical Results for Irrigation Investment from Chapter One

<table>
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<tr>
<th></th>
<th>Logan</th>
<th>Bear</th>
<th>Blacksmith-Fork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold P (NPV)</td>
<td>20.93</td>
<td>54.38222</td>
<td>48.58622</td>
</tr>
<tr>
<td>Threshold P (OV)</td>
<td>149.17</td>
<td>134.43</td>
<td>140.00</td>
</tr>
<tr>
<td>Threshold P* (OV)</td>
<td>168.39</td>
<td>152.28</td>
<td>158.59</td>
</tr>
<tr>
<td>Threshold W (NPV)</td>
<td>135166.6036**</td>
<td>75356.72</td>
<td>72391.84</td>
</tr>
<tr>
<td>Threshold W (OV)</td>
<td>66680.55</td>
<td>47356.28</td>
<td>42442.37</td>
</tr>
</tbody>
</table>

Initial Water Price \( (P_0) \) $38/AF

Water Price Drift \( (\alpha_p) \)
  - Logan: 0.2415
  - Bear: 0.1480
  - Blacksmith-Fork: 0.1642

Water Price Volatility \( (\sigma_p) \)
  - Logan: (0.9189)
  - Bear: (0.7462)
  - Blacksmith-Fork: (0.7888)

Technical Efficiency Volatility \( (\nu) \) 0.145
This is the threshold from chapter one adjusted for on-field differences

** returns starting value (depicting immediate investment) since threshold has already been hit.

P*(OV) is calculated to show the effect of forcing an average water savings which caters to the best case scenario on a field with substantial on-field variability of water savings potential. Thus, P*(OV) traces the price thresholds under the all-or-nothing scenario that would correspond to a constant marginal productivity of subsequent units of investment implicitly assumed by lumping all the units of investment together. It normalizes the results from chapter one to serve as a baseline for comparison.

These results indicated the disparity between the real options (OV) and benefit-cost (NPV) thresholds required for investment. The consideration of uncertainty in the real option investment evaluations lead to greater water prices required for investment. We also see that differences in volatility make a difference in the impact of uncertainty consideration on investment. The Bear River, which showed the lowest volatility in water price (and consequently, its derivative water supply process), requires the lowest price threshold for investment under the real options consideration.

We then set the rationale/motivation and parameters for the incremental investment scenario.

As described in the Introduction section, base conditions at the field level contribute significantly to the potential effect of improved irrigation efficiency on farm-level water savings. Irrigation system performance with regards to field level water savings varies
both across- and on-field based on the geo-hydrological environment comprising factors such as slope and soil type; crop type; and the field-specific agro-climate (Kumar et al., 2008).

This has spurred research into and development of techniques and technology to tackle the on-field variability of technology performance. This has engendered site-specific farm management using tools such Variable Rate Irrigation (VRI). VRI allows irrigators to tailor water application on their fields based on soil data maps, yield data, topography maps, and other user-defined information.

Studies on VRI applications tend to focus on yield improvement. For instance, King et al. (2002) in an application divided a 2.9 ha (7.1 ac) field into eighteen arbitrary irrigation management zones under one quadrant of a 4-span 191 m (628 ft) long center pivot irrigation system. They found that under variable rate application based on on-field spatial variation in soil texture gross receipts were $165/ha ($67/ac) greater under the site specific management scenario.

As a result of the yield focus, studies on the water conservation benefits of site-specific sprinkler irrigation for crop production are rather limited, and studies on its cost-effectiveness scarce. With increasing water scarcity and rising interest in environmental flows of water, initial studies focusing on water-saving potential suggest considerable potential. For instance; simulation studies comparing conventional and site-specific irrigation have reported gains of up to 26% water savings for well-watered crop production (Evans and King, 2012).
Given such variation, it is possible to see how lumping together investments which on average produce 26% water savings will incorrectly characterize the decision to adopt water conservation technology.

Our numerical example here will revisit the investment decision from chapter 1 by disaggregating the investment based on heterogeneity in productivity and water savings across the field.

The field size considered under the all-or-nothing formulation in chapter one, corresponds roughly to four times the area needed (160 acres) for the installation of a standard sized (125 acre) center pivot (O’Brien et al., 1997). Thus, we divide the whole field into four units \( U = U_1, U_2, U_3, U_4 \) with varying water savings potential \( (X_1^* > X_2^* > X_3^* > X_4^*) \) based on field characteristics. The total water savings across the 4 fields is set up to average 2.25 AF, which is the water savings consideration \( (X^*) \) for the all or nothing scenario in chapter one: 

\[
X^* = \frac{X_1^* + X_2^* + X_3^* + X_4^*}{4} = 2.25.
\]
Figure 2.3: Variations in water savings from 125 acre center pivot systems installed on a 640 acre field

Thus, the difference between the most productive and least productive portions of his field is 26% based on Evans and King (2012), with a graded progression ($X_1^* = 0\%$, $X_2^* = 5\%$, $X_3^* = 10\%$, and $X_4^* = 26\%$) as shown in Figure 2.3.

To compare the all-or-nothing to the incremental investment case, we maintain the same level of investment and refer to Figure 2.3. The agent is however now able to break up his investment and make an initial partial investment. The agent first invests in the section of his field that is most conducive for the technology (corresponding to water
savings

\[ X_1^* = 2.25 \] and incrementally invests as and when the water price rises to hit his thresholds for the sections with lower marginal irrigation efficiency productivity (\( X_2^* = 2.1375; X_3^* = 2.025, \text{and } X_4^* = 1.665 \))

To demonstrate the impact of differing marginal efficiency productivity of the various sections, we ran a simulation of the total investment scenario.

Figure 2.4: Impact of Varying Potential Water Savings on Investment Threshold

Figure 2.4 illustrates the ‘dragging’ effect of less productive units of investment is seen in the progressively higher price thresholds successive units require for investment and is derived by varying the potential water savings for the real options with
deterministic efficiency and no shutdown from figure 1.3a in chapter one. These threshold curves are negatively sloping here due to the labeling of the axes, but show the relationship between price thresholds and water savings potential impacted by diminishing marginal returns. For any given level of efficiency on the x-axis, increasingly higher price thresholds are needed for investment for increasingly lower marginal water savings potential.

Figure 2.5: All-or-nothing versus Incremental Thresholds

![Blacksmith Fork Incremental Threshold Curve](image.png)
Breaking down the total investment into sections and re-running the simulation, Figure 2.5 is obtained. The figure shows the impulse control timing for investment in the consecutive ‘units’ of irrigation investment with \( U = U_1, U_2, U_3, U_4 \), each unit representing a quarter of the total investment. Using data from the Blacksmith Fork River, we see from Figure 2.5 that investment occurs before the all or nothing price threshold is hit, three out of four times.

The all-or-nothing threshold is represented by the black line, while we see the step-wise graduation of the incremental investments in blue. It demonstrates that the critical price for \( U=U_1 \) (0.25) is less than the critical price from chapter one ($158.59) and the critical price for \( U=U_4 \) is above the $158.59 all-or-nothing threshold in chapter one. In chapter one, the water savings in \( U=U_1 \) is required of all four portions of the field for investment leading to a price threshold higher than required for incremental investment in all but the least productive section of the field, \( U_4 \).

Thus, the burden of proof of investment profitability is reduced under the incremental investment scenario, and has great potential to improve investment incidence as opposed to all or nothing formulations.

To put this in perspective with regards to the length of time it would take to exercise barrier control at the various investment levels, we trace the number of years it would take for the price process to hit the various trigger points on the threshold curve. Doing this for the three rivers, we find the following:
Table 2.3: Inter-River Variability and Time to Barrier Control

<table>
<thead>
<tr>
<th></th>
<th>Logan</th>
<th>Bear</th>
<th>Blacksmith Fork</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water Supply Drift</strong> ($\alpha_W$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water Supply Volatility ($\sigma_W$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barrier 1($)</td>
<td>149.18</td>
<td>134.43</td>
<td>140.00</td>
</tr>
<tr>
<td>number of years</td>
<td>~37</td>
<td>~107</td>
<td>~108</td>
</tr>
<tr>
<td>Barrier 2($)</td>
<td>157.03</td>
<td>14.50</td>
<td>147.37</td>
</tr>
<tr>
<td>number of years</td>
<td>~38</td>
<td>~111</td>
<td>~113</td>
</tr>
<tr>
<td>Barrier 3($)</td>
<td>165.75</td>
<td>149.37</td>
<td>155.56</td>
</tr>
<tr>
<td>number of years</td>
<td>~40</td>
<td>~116</td>
<td>~117</td>
</tr>
<tr>
<td>Barrier 4($)</td>
<td>201.59</td>
<td>181.66</td>
<td>189.19</td>
</tr>
<tr>
<td>number of years</td>
<td>~46</td>
<td>~133</td>
<td>~134</td>
</tr>
</tbody>
</table>

The number of years required to hit threshold points, while not markedly distinct here, do show the difference the incremental consideration makes. The times to hit the adjusted chapter one thresholds are 40 years, 118 years, and 119 years respectively. By these times, the thresholds for investment would have been hit for investment in the third
segment (highlighted in yellow in table 3) of the agent’s acreage whether he is dependent on the Logan, Blacksmith Fork or Bear River observation point. Thus across all three rivers, our agent would invest in about three quarters of his total acreage under the incremental investment story before hitting the corresponding all-or-nothing thresholds.

From table 2, the data from the Logan River shows the highest drift and volatility rates among the three rivers, as well as the highest starting value for its water supply process. Here, it also sees the earliest investments. Having the highest drift rate signifies that the general downward trend in the evolution of water supply is highest in the Logan River and that it is also the most volatile of the three water supply processes. Having the highest volatility (uncertainty) would suggest that the option value would be highest for the Logan River and that it would have the highest price thresholds. This is true as we can see from table 3 that the price thresholds for the Logan river is consistently higher than that for the Bear and Blacksmith fork in all instances. And yet, we see that the Logan River consistently hits its higher thresholds before the other two rivers in all cases.

This offers an interesting peek at the interplay of trend and deviations around a trend where stochastic processes are involved; and has implications for adaptation and resilience of existing systems to changing conditions which are uncertain. While increased volatility gives value to waiting, it also increases the margins of profits (opportunity costs) of investing (or not) and coupled with the higher drift rate signifying a higher rate of decline in water supplies, this appears to produce a joint effect of precipitating investment relative to the other rivers.

With its high drift and volatility rates two random samples from simulations of the stochastic water supply and price processes for the Logan River could present starkly
different pictures of the need and urgency for water conservation investments. This is an important hurdle to be cleared whether at the individual investor level, or in allocating municipal, state or federal funds.
5 DISCUSSIONS AND CONCLUSIONS

While true that an all or nothing, or one-time investment assumption simplifies the analytical model under examination, makes the problem easier to solve, and provides insight about the technology adoption problem, this formulation restricts potential future revisions (abandon or pause project) which may become desirable in response to changes in the economic or environmental state which affect the value of investment over time (Balikcioglu, 2008).

The results from this study confirm the assertion by Gupta and Rosenhead (1968), averring that investors guard against the danger of regretting large investments in the face of appreciable uncertainty about future conditions, and strive to keep as many of their ‘good’ options open as possible, where the good options are the decision to not invest in more marginal areas.

While an irrigator’s only defense against regret is to delay investing in the all-or-nothing investment scenario, the irrigator can employ both investment timing and scaling to help guard against investor-regret in an impulse control framework.

In the face of increased pressure for increased efficiency of agricultural water, it is becoming increasingly important for irrigators to obtain quantitative information on actual post-installation irrigation system performance rather than sticking to manufacturers’ claims regarding water savings. Chapter one shows the importance of this through the technical efficiency uncertainty story. Here, the importance of regional data on hydro-geological factors that influence not just across field, but on-field variability of irrigation system performance is established.
This study also points to the need for improved economic information regarding water-use and conservation informed by data and technical considerations from biophysical sciences, as well as technical studies. Using hydro-geological data to measure the marginal efficiency contribution of subsequent units of conservation technology, including irrigation and canal lining, this study found a significant difference in investment behavior which incorporated on-field hydro-geological variability into evaluating the economic decision to invest in irrigation technology. This study has shown that use of such data has value and may mean the difference between a 75% (three out of four times relative to all-or-nothing formulation) and 0% investment.

This is an area for future study which I intend to collaborate with colleagues from hydrology, irrigation engineering, ecology etc. to explore. One extension would be coupling an extension of this study of an agent to water conservation across a region say the Wasatch Range Metropolitan Area incorporating soil and topographical maps to determine what extending the incremental investment story to the region would dictate as optimal intensity and extent of irrigation investment across areas with variable hydrogeological characteristics across the region.

Additionally, this model may be incorporated with climate projection models to more accurately account for projected changes in hydroclimatic conditions, and how they would affect agricultural activity and investments in the state of Utah and beyond.

This chapter illuminates the gap between levels of investment in irrigation and other conservation technology suggested by net present value evaluations of these investments, and observed investment behavior. As changing hydroclimatic and socioeconomic conditions continue to inject increased variability into decisions once perceived as black
or white, it is important for policy and planning purposes that studies such as this are conducted in order to provide insight into how observed gaps between investment levels suggested by traditional models such as discounted cash flow models and observed investment can be explained and thus influenced by new and/or supplementary theories of economic decision-making.

With regards to intensity versus extent of investment, a comparison of results from the all-or-nothing and incremental set-ups suggests that the relative effect of water supply uncertainty on investment in areas with differing on-field variability is a pertinent consideration concerning water conservation goals and planning. State planners, for instance, would do well to know account for this variability in drawing up water plans and for policy implementation. A slow uptake of prescribed technology may, instead of a perceived problem to be tackled with policy and public funds, be the effective working of the system to ensure only units of investment which at least match their incremental investment costs are made. It is thus necessary to jointly consider water supply variability (uncertainty), sunk costs (irreversibility) and on-field variability (marginal productivity) as factors determining adaptation investments in response to changing hydroclimatic conditions in the western United States.

On-field variability also has important ramifications in the face of limitations to the monetization (sale) of water savings in the west due to the predominant prior appropriation water rights system and related laws. With more efficient water markets (improved individual right to sell water savings, and canal company share prices which are to more readily respond to changing market conditions), the water price would more readily respond to changes in the evolution of the water supply process allowing for
potentially more optimal water allocation. An agent will invest based on the equation of ‘marginal benefit’ and ‘cost,’ meaning that the cost of your next unit of investment would have to be less than what you would pay to buy the water it saves you on the market.

Smith and North (2009) argue that it is not possible to make a blanket “rule of thumb” statement about profitability of center pivot investments, suggesting that evaluations are made with “with” or “without” investment scenario modeling on a case by case basis instead. Our findings support this. While more convenient and most times more cost-effective to effect blanket policy prescriptions, increasingly uncertain conditions may advocate this more case-by-case approach which incorporates a more detailed account of variability in investments across agents, across regions, and in the case of this model – within a single field and it is important to bear these in mind in optimizing policy and planning goals at all levels of governance.

Like in chapter one, this assessment perhaps handicaps itself for profitability by holding constant the most important agronomic consideration for farmers in adopting efficient irrigation technology, which is systematic improvements in crop yield attributable to the adoption of center pivot technology. This comes to bear on the incremental investment story in particular because field variability does tend to inform choice of crops and so one potent motivation for gradual partial investments would be to invest in the sections which hold your highest valued, water-sensitive crops first. This would also take some of burden of proof of investment profitability off how much you could directly convert your water savings for, since increased production and increased crop prices in response to changes in water supplies would contribute to the value of investment.
However, this singling out of the water savings is pertinent. In the context of increasing water scarcity, it becomes more relevant to consider potential decreases in water-use than increases in output (Frija et al., 2009).

Holding productivity constant to consider the water savings effect only has the following benefits:

1) With increasing scarcity, the value of sold water savings will potentially become increasingly important revenue to farmers even if not as physical cash received from sales, as cash they may have had to expend to supplement their own water supply for continued production in the absence of technology adoption. Thus, investing in efficient irrigation investment would gain increased value as a way for farmers to mitigate production risk due to increasingly scarce and uncertain water supplies.

2) With an increasing interest in the environment as a ‘beneficial use,’ it provides some quantitative guidelines as to what a farmer may have to be paid to prohibit conversion of saved water for expanded production, to promote in-stream flows.

3) Given that the water rights system in Utah generally does not encourage sale of water, it hints at the potential this may have in encouraging water conservation. Given that the evaluations in these studies were done using seasonal agricultural water lease prices, it is just the tip of the iceberg as to what a well-functioning water market where prices respond more readily to increasing water scarcity and unpredictability may accomplish.
Real options theory is, certainly, not the sole method available for understanding optimal investing under uncertainty. Alternative modeling frameworks for economic decision-making under uncertainty will only continue to grow in importance.
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CHAPTER THREE: A BI-LEVEL HYDROECONOMIC MODEL OF WATER-USE AND MANAGEMENT IN THE WESTERN UNITED STATES: A UTAH CASE STUDY

ABSTRACT

This study employs spatial dynamic optimization and bi-level programming techniques to analyze water-use behavior in the Cache Valley of Utah. Using a case-study Logan Northwest Field canal system, it explores the impact spatial location of heterogeneous actors has on decision-making hierarchy and outcomes. It also analyzes in particular, 1) the efficiency contribution of an omniscient water master to overall water-use efficiency of actors in the canal system, 2) the impact of varying composition of the system’s actors and their needs (agricultural to residential users ratio) on the decisions and directives of the omniscient water master.

Key Words: Bi-level Programming, Water Management, Spatial Dynamic Optimization
1 INTRODUCTION

The management and use of water in urban areas is a complex and dynamic issue. Water managers must juggle the goals of regulating water demand, with securing and improving access to good quality water for their jurisdiction areas. As demographic and hydroclimatic changes force a shift in the focus of water management from supply expansion to improving management of existing water supply systems (Harou et al., 2010), a holistic understanding of existing systems has become a pertinent need. While past economic studies have produced useful results and emphases, it is becoming increasingly clear that restrictions such as the assumption of a rational representative agent, the assumption that aggregate behavior is based on the summation of individual decisions by clones of this representative agent, and the disregard for the inherent complexity and synergy in human-engineered water systems must be done away with.

Changing precipitation patterns, population growth, urban development and evolving agricultural water demand in response to hydroclimatic conditions are all factors that impact how, when, and where people use water. Effective and efficient adaptation to the evolution of these factors requires enhancements in water-use and management decisions through improved monitoring and prediction of the hydro-climate as well as improvements in how information is used within water-use and management decision processes. To do this, a comprehensive understanding of the current system of water-use and management decision-making and interactions is needed. This study thus develops a spatial-dynamic model of water-use and management to illuminate of the current water-use and management system in Utah.
Water movement on spatial and temporal scales play crucial roles in creating areas of abundance and scarcity across time and space, which then determines the sufficiency or otherwise of existing water resources to meet the needs of current and future populations. For instance, spatial differences with regards to topography can introduce interesting dynamics into the interactions between an upstream agricultural user and downstream residential user with regards to both water quantity and quality. Further, as chapter two confirms, hydro-geophysical differences across spaces can significantly impact optimal water management choices and accounting for spatial differences can improve the outcomes of behavioral adaptations such as the adoption of more efficient irrigation technology.

Indeed, a major general limitation of existing economic studies of water, is the concentration on demand-supply relationships with little, if any, regard to the natural and conveyance (built) systems within which these relationships are nested. The placement of agents and biophysical influences does matter. A farmer located both downstream and downhill on a river system suffers disproportionately from both flood and scarcity. And effluents from a polluting factory located upstream have greater water system consequences than those from a factory located at the river’s mouth.

This consideration is a major contribution of this chapter. Due to the nature of our conveyance system where the omniscient water master determines flows for all canal users, it is possible to model water management in the canal system using bi-level formulation, allowing us to study how this top-down incorporation of residential and agricultural objectives affect both individual and system outcomes. And while the spatial aspects are not as pronounced as they could be due to the scale of the case study and how
quickly water runs through this particular system, space does matter in terms of defining
the user constituents that a water manager must accommodate.

This chapter thus analyzes the water allocation and benefit trade-off decisions of an
omniscient water master who seeks to coordinate the interests of heterogeneous users
under his jurisdiction while prioritizing his own. We study spatial dynamic optimal
decision-making with regards to water-use and management using a case study of the
Logan Northwest Field canal system in the Cache valley of Utah. We develop a model
comprising four different types of actors (water master, storm-water manager,
agricultural user, residential user) each acting to achieve their defined desired objectives
with regards to water in the canal system considering the constraints imposed upon them
by physical attributes of the system, as well as the actions of other actors.

The case-study canal system is modeled as a bi-level problem with the water master
at the topmost level, and the other users at the sublevel. The analysis of the actions and
interaction of these agents is done using spatial dynamic optimization and bi-level
programming techniques, to explore how the spatial location of the different actors in the
system affects decision-making hierarchy and outcomes.

With the bi-level formulation, the optimal choices of the various actors at the
sublevel, determined using dynamic optimization techniques, serve as inputs in the water
master’s problem. Likewise, the solution to the water master’s problem feeds into the
sublevel decisions through the water directives and allocation he makes after he
determines his optimal course of action. The water master – other users hierarchy here
thus reflects a relationship between autonomous and potentially conflicting decision-
makers relatable to the leader-follower scenario in Stackelberg games.
In the Stackelberg (1934) game, the leader acts first, with the follower moving to optimally react to the leader’s move. The leader, aware beforehand that the follower observes his actions before making their move, anticipates and incorporates the follower’s optimal response behavior in optimizing his own. Thus, the follower’s optimization problem (here, the sub-level problem of the agricultural and residential users) is nested within the leader’s problem, which corresponds to the upper level optimization problem in a general bi-level program and the water masters problem in this model.

In this vein, the solution (optimal behavioral rule/strategy) may be likened to a subgame perfect Nash equilibrium (Bruckner and Scheffer, 2011) so that it expresses the strategy that best caters to the objectives of each user in the model, given the behavioral motivations and rules of all other users in the canal system. Each user operates under the scenario where he cannot improve his outcome be changing only his behavior all other things constant (i.e. a Nash equilibrium).

The results are then explored through counterfactual scenario analyses for insights on 1) the impact of individual decisions and actions on available water supplies, 2) the impact these decisions may have on institutional management (i.e., canal companies, state and federal agencies) of water resources, and 3) discrepancies between results from this model and a non-hierarchical one where the objectives of all users are jointly optimized/determined.

In the body of economic literature most related, this study aligns most as a dynamic optimization model incorporating spatial heterogeneity. In particular, it analyzes dynamic
optimal water allocation and use in a canal system with spatially heterogeneous agents with varying objectives.

This study contributes to this literature in two ways: it examines a novel case where spatial location dictates a decision-making hierarchy. The canal manager at the top of the canal decision-making hierarchy makes decisions given the water-use of all individuals along the canal. The use of bi-level programming affords this contribution and it is the first combination of these two techniques that we are aware of. Another contribution is the incorporation of heterogeneous agents. Spatially dynamic optimization models in economics typically assume identical decision makers at each point in the grid, and this study comprises three different user types in the canal system.
2 BACKGROUND

2.1: Water Research in Economics

Traditionally, economics espouses an approach based on the reductionist principle of physics in studying aggregate behavior. According to this principle, the aggregate can be understood simply by considering its individual components. The reductionist framework suggests that “the dynamics of a (linear) model can be decomposed into its constituent parts through the representative agent framework” (Bianchi et al., 2008).

As a result, general equilibrium models are built on the foundational assumption of a rational, omniscient representative agent with optimization goals. While the representative agent theory has support in a complete market scenario (Constantinides, 1982; Guvenen, 2011), most markets are far from that and water markets are no exception. True, aggregation theorems are usually presented as defense for the representative agent approach, with some evidence showing aggregation producing similar results for representative and heterogeneous agent scenarios in even incomplete markets (Krussell and Smith, 1998; Rios-Rul, 1996). However, aggregating heterogeneous agents into a representative agent can produce preferences that do not reflect those of the model underlying the aggregation (Hansen, 1985; Romero, 1988; Browning, Hansen and Heckman, 1999) and heterogeneous agent models can explain aggregate evidence a single representative agent model grapples with.

Indeed, this weakness has been recognized in the literature and the representative agent model assumption where water economics is concerned has been relaxed using tools such as game theory and agent-based modeling.
In addition to the proliferation of representative agent assumptions in existing water economics research, there is a distinct paucity of attention given to the natural and built systems which govern the transportation and delivery of the water under study.

Thus while existing studies on water-use and management cover such topics as the economics of water-use in agriculture (Carlson et al., 1993; Zilberman, 2008); the economics of water management in agriculture (Molden et al., 2007; Bournaris et al., 2014); the economic value of water in irrigation/agriculture (Sunding et al., 2002; Ward and Michelsen, 2002; Rigby et al., 2010); the economic impacts of restricting water supply (Berrittella et al., 2006; Berrittella et al., 2007); the use of pricing policies (Gaudin, 2006; Olmstead et al., 2007) and water markets in regulating water demand and promoting conservation (Olmstead and Stavins, 2009); water supply security and the willingness to pay to avoid drought (Hensher et al., 2006); among others.

These studies are done with no consideration of how the conveyance systems and order of water-use and management decisions, actions and interactions affect individual, as well as system-wide water outcomes. And yet, the systems and conditions governing the transport of water can be incredibly important in water quantity and quality management. A storm-water management decision by one upstream and uphill agent can mean financial destitution for a downstream and downhill farmer depending on the drainage and conveyance systems present.

Finally, existing studies in water economics reveal a knowledge gap with regards to the motivations behind observed water-use and management behavior. There is ample information of which factors influence greater water-use and irrigation technology adoption, but there is a dearth of information on why those factors have said influence. In
Hoekstra and Chapagain’s (2007) study of the water footprint of nations considering water-use as a function of consumption patterns, they find the major direct determinants of a country’s water footprint to be gross national income; pattern of consumption (e.g. high versus low meat consumption); climate (growth conditions); and agricultural practices (water-use efficiency). Household meat consumption and similar behaviors, for instance, is not readily accounted for in traditional economic water-use models. Knowledge of these underlying motivations is invaluable in policy-making. Knowing why people use water the way they do is crucial in determining how to incentivize them to modify their water-use behavior.

Bockstael et al. (1987) establish the sensitivity of measurements of recreation benefits obtained from models of recreational behavior to people’s perception of their decision problems. Goodman and Howe (1997) investigate the determinants of ditch company share prices in the South Platte river basin and find that while characteristics of the ditch company shares and price fluctuations appear to explain share prices well, there were considerable inter-city differences which remained unexplained and mayhap be explained by variances in geographical factors, risk preferences and consumer expectations about the future.

This hints subtly at the synergy and complexity in water-use this study seeks to address at a hydroeconomic system scale and also suggests the importance of not just how much water and at what price people use water, but why (purpose/motivation), when (timing) and where (spatial explicitness) when it comes to water-use and management decisions.
2.2 Multi-Agent Models

Game theory provides an excellent tool for explaining behavioral outcomes and makes a compelling case for detailed study of heterogeneous rather than representative economic agent interactions. Bogardi and Szidarovszky (1976) demonstrate the merit of the game-theoretic approach in analyzing protection of water ecosystems, irrigation system management (Rosen and Sexton, 1993), water quality management, and joint management of water systems for multiple objectives such as storm-water management, recreation, biodiversity, water supply, etc. They find varying motivations for cooperation, and numerical justification for joint optimization. Thus while all actors in a water system may benefit from co-operation, disproportionate distribution of this benefit often leads to non-cooperation and therefore sub-optimal outcomes for all.

Dinar and Howitt (1997), Raquel et al. (2007) and Wei et al. (2010) corroborate this in their application of game theory to water conflict and co-operation among heterogeneous agents. They find that those who would benefit the least (or lose the most) pushed back against co-operation and engendered the non-cooperative sub-optimality embodied in the prisoner’s dilemma. Game theory has also been applied to the allotment of joint water project costs (Giglio and Wrightington, 1972; Suzuki and Nakayana, 1976; Young et al., 1980; Straffin et al., 1981; Dinar and Howitt, 1997); and transboundary water management and cooperation (Bardhan, 1993; Becker and Easter, 1995; Adams et al., 1996; Fisvold and Casswell, 2000, Fernandez, 2002; Ambec and Ehlers, 2008; Fernandez, 2009) with identical conclusions.

Game theory, in allowing for heterogeneous actors, permits analysis of behavior that does not assume everyone is equally committed towards a joint goal. This allows detailed
study of cooperation and non-cooperation among players and how such behavior impacts players’ welfare both individually and jointly.

Cooperative game theory is better than a traditional market model where agents only interact through price when it comes to models involving a relatively small number of players who can make decisions together and individually (Dinar, Ratner and Yaron, 1992). However, it requires agents to be suitably well-informed and this is not always feasible.

(Madani, 2010) employs a series of non-cooperative games to investigate the applicability of game theory to water resource management and conflict resolution. The results from the study demonstrate the inherent dynamism in water resource problems and the need to consider the evolution path of linked/dependent decisions in the study of such problems.

The predicted outcomes from such game theory analyses, however, often differ from results from traditional economic optimization approaches which assume all parties are willing to work towards attaining the best outcome for the system (Madani, 2010). System management with regards to water resource management often involves conflicts. Stakeholders, who might be able to reach mutually beneficial situations through cooperation, often exhibit (non-cooperative) behavior which produces mutually unfavorable results. The model in this chapter considers an omniscient water master in a non-cooperative game set-up comparable to the Stackelberg leadership model where a better informed leader acts first taking the followers motivations and possible reactions into account to optimize welfare. This is a suitable set up for water management in many canal systems in Utah and beyond, as well as in many decision-making scenarios with
different levels of decision-making power. There may be some observed ‘cooperation’ between lower level decision makers, but this is usually in one way or another dictated by the priorities of the higher-level decision maker(s). For instance, water masters may give directives to protect downstream agricultural users in the interest of minimizing the impact of drought on agriculture in the region and subsequently the economy as a whole.

A number of modeling approaches have developed in response to the clarion call for models better able to capture the complexity, heterogeneity and synergies inherent in coupled human-natural systems such as the water system, and also able to accommodate suboptimal choices resulting from the interaction of economic agents with their environment and each other.

One such model touted by many as the standard, is the agent-based modeling approach (ABM). ABM may be defined as “a computerized simulation of a number of decision-makers (agents) and institutions which interact through prescribed rules” (Farmer and Foley, 2009; An, 2012). These agents are usually set in a dynamic environment, and interact with the environment, which includes other agents. Agents are usually given the ability to evolve in their perceptions and behaviors, in response to the behavior of other agents and the evolution of the environment. In social science applications, ABMs typically deduce assumptions from observed conditions in society and nature, and based off these assumptions and simulation, produces data for analysis (An, 2012), successfully marrying the traditional inductive and deductive approaches in scientific research (Axelrod, 1997).

Agent-based modeling applications in water economics allow researchers to account for the complexity and synergistic relationships in the studied water systems, as well as
the heterogeneity of agents. (Troost and Berger, 2014; Berger and Troost, 2014; and Berger, 2001) capitalize on this strength to explicitly capture agent interactions across social and spatial dimensions, as well as capturing response and adaptation of water-users to changes in hydroclimatic and socioeconomic conditions.

In economics, ABM is especially employed in ecological economics whose domain comprises interconnected social and environmental systems. Agent-based models can be used to explore feedbacks and adaptations integral to these systems as it permits representation of autonomous, dynamic and heterogeneous entities and has the capacity to handle interactions among these entities. The interactions between agents and their environments then produce macro-scale results employable as data in the quantitative analysis of complex systems (Heckbert et al., 2010).

ABM applications in ecological economics are seen in the area of natural resource management where it has seen applications to invasive species management (Atallah et al., 2013), water resource management (Schlüter and Pahl-Wostl, 2007), and energy investment decisions (Wittman, 2008). Applications are also seen in studies of land-use change (Parker et al., 2003), urban systems modeling, changes in consumer attitudes, innovation and diffusion of technology and management practices (Berger and Troost, 2012), and psychological aspects to human decision-making and behavior change. Developing research using ABM in ecological economics seeks not just to expand its applications, but also build its capacity for testing specific hypotheses about agent decision-making, through the calibration and validation of models (Heckbert et al., 2010). ABMs can determine and analyze the decisions of parties to water resource conflicts and
delineate how interacting agents who make decisions based on self-interest rather than system-wide interest impact a system’s evolution.

While the ABM approach may very well be the ideal approach in many natural resource applications, the data intensity and time requirements of developing, populating and parameterizing Agent-Based models can be considerable. Thus, there is the need to continue developing other approaches and models which may more readily and generally be employed in the economic analysis of complex water systems.

This study presents one such option, by taking into consideration and marrying spatial dynamic optimization techniques from economics, and bi-level programming techniques imported from mathematical programming. Spatial dynamic optimization allows for incorporation of spatial heterogeneity of agents with respect to hydro-geophysical characteristics and economic characteristics such as agent objectives, costs and benefits. The bi-level formulation allows incorporation of the institutional hierarchy in water decision-making where policies and regulations from irrigation managers, municipalities, cities, states and federal institutions exert an influence a step or two above the other agents in the system.

The following sections examine these two techniques, as well as the merits of coupling them.
2.3 Spatial Dynamic Optimization

The use of optimization methods is widespread in economics - from the analysis of one-time choices using static optimization methods, to optimal decision-making over time using dynamic optimization methods. Further, we see the extension of these dynamic optimization models to account for uncertain variables (stochastic dynamic optimization) and spatial variation (spatial dynamic optimization).

In the resource economics literature, spatial dynamic optimization has seen widespread application, with applications examining the dynamics of spillover effects of individual resource use (Irwin and Bockstael, 2002), designation of use-no use areas like in the case of wildlife reserves (Polasky et al., 2008; Benitez et al., 2006) highlighting questions about the difference between individual resource use and optimal resource use over the whole spatial grid, and the dynamics and spillover effects of managing adverse biological (invasive species, infectious disease, etc.) and physical (wildfire, erosion, etc.) events (Atallah et al., 2013; Conrad and Rondeau, 2014).

The economics of spatial dynamic optimization as it pertains to natural resources differ significantly from separate considerations of spatial and dynamic problems in resource economics. The joint consideration of the space and time components often introduces new challenges birthed from the interaction of hydro-geo-physical, bio-ecological and economic systems across spatio-temporal space. Prevalent examples of such problems include invasive species management, wildfire control and infectious disease spread, which all have the unifying influence of underlying biophysical dispersion or diffusion processes (Smith et al., 2009). From an optimization standpoint,
we now have to consider not just when and how much of an optimal control to apply, but also where to apply this control to optimize impact of said control.

Separately, the spatial and dynamic aspects of resource use have received ample address in existing literature. Most economic investigations considering the optimal policies regarding extraction, development and conservation of resources such as timber (Faustmann, 1995; Conrad, 2000), minerals (Hoteling, 1931), and fishery (Crutchfield and Zellner, 1962; Schaeffer, 1987) thoroughly treat the dynamic aspects of these problems.

On the spatial side, Von Thunen’s exploration of the spatial impacts of autonomous economic decision-making has spurred spatially focused research on how patterns of production and consumption, urban structure and emergence of cities vary across geographical space (Fujita et al., 1999) and how variables such as availability of environmental amenities affect spatial development patterns in urban and rural settings (Wu, 2006; Wu and Plantinga, 2003; Irwin and Bockstael, 2002; Wu, 2001).

The extent of spatial consideration in spatial models in economics varies. We have spatially explicit models which describe a discrete or continuous non-uniform geographical space and spatially-implicit models which, in lieu of geographical space, integrate assumptions about the spatial composition of biological interactions without using geographical space. Due to spatial analyses being centered on an economic resource or activity, spatial analyses was traditionally done along concentric circles of influence. Currently, grid systems populated with cellular automata interacting with neighboring cells are more used, with neighborhoods defined based on expert knowledge
as fine as specific GIS data, to established empirical economic relationships (Verburg et al., 2002).

Joint considerations of spatio-temporal aspects of resource use in economics vary in complexity. At one end are fully integrated models of spatial-dynamic processes such as a spatial bioeconomic optimization model of (migratory) fish populations (Smith et al., 2009, Marten and Moore, 2010), invasive species management (Sims and Finoff, 2013, Aadland et al., 2013), and control of wildfire (Spring and Kennedy, 2005; Thompson and Calkin, 2011). At the other end, we find optimal resource allocation models with considerations of spatial heterogeneity for example the one-time optimal choice of nature conservation reserve sites based on habitat productivity (Polasky et al., 2001; Ando et al., 1998) and its related reserve site selection problems which examine optimal choice of conservation activities for optimal design of protected area networks (Williams et al., 2005; Tischendorf et al., 2000; Margules et al., 1982, Diamond, 1975) and models which incorporate spatial heterogeneity with dynamics but no spatial-dynamic processes - such as dynamic optimization of spatially heterogeneous agricultural runoff (Goetz and Zilberman, 2000) and stock pollutants (Xabadia et al., 2006).

2.4: Bi-Level Programming

Optimization problems which have some or all of their variables constrained to be an optimal solution of other optimization problems are known as multi-level optimization problems. Commonly found in applications with decision-making hierarchies, these problems have their feasible set which satisfies all constraints and optimality conditions
implicitly determined by a series of nested optimization problems which may all be on a single lower level (bi-level problems) or on multiple levels.

These nested problems are represented by mathematical programs parameterized by the non-constrained variables. Thus, when solving the problems at level a, variables at all other levels are held as fixed parameters.

Multi-level formulations of decision-making models are suitable for situations which present a hierarchical structure and generally have the following characteristics (Fortuny-Amat and McCarl, 1981):

a) Two or more decision-making agents with independent, sometimes conflicting goals at the same, or different levels of the decision-making ladder

b) Each agent is limited in the model variables they have direct control over.

c) The decision-making process is undertaken in a sequential manner for each pair of adjacent levels of the decision tree thus – the agent at the higher level first announces his plan of action or policy, and this is taken as exogenous input by the agent(s) at lower levels in deciding their own optimal plan of action or policy.

d) The agent at the highest level, whose optimization is the master problem, then chooses a course of action to optimize its objective function, incorporating information from how the lower level agent(s) reacted to his announcement in c) above.

e) The agent solving the master problem knows the objective function and constraints of the lower-level problem(s) with certainty.

When such a multi-level decision-making hierarchy comprises two decision-making levels which may be represented as pure mathematical programs (i.e. not dependent on or
constrained by other programs), the arising problems are known as bi-level programming problems (Vicente and Calamai, 2004; Dempe et al., 2006) and (Bard 1998).

Bi-level programming techniques present a way to model decentralized hierarchical decision-making comprising a ‘leader’s’ objective (master problem) at the first level, and the follower’s objective (sub problem) at the second level (Matroud and Sadeghi, 2013).

Colson et al. (2007) provides a general overview of bi-level programming applications, which reveals its most common application to transportation analysis, particularly traffic flow problems (Dowling and Skabardonis, 2008; Akgungor and Bullen, 2007; Akcelik, 2003; Daganzo, 1994) and network design (Farvaresh and Sepheri, 2013; Ban et al., 2012; Farvaresh and Sepheri, 2011; Ban et al., 2006).

It has also been applied to the transportation and marketing of natural gas, with particular emphasis on the default payments for late or interrupted delivery (Kalashnikov et al., 2010; Kalashnikov and Peres-Valdes, 2010; Kalashnikov and Rios-Mercado, 2006; Midthun et al., 2009; Dempe et al., 2006; Kalashnikov et al., 2006; Dempe et al., 2005).

Closer to home, bi-level programming has been applied to water resource allocation with existing studies emphasizing the multiplicity of objectives. Xu et al. (2013), for instance, considers water allocation among irrigation, industrial, residential and ecological benefits in a Chinese river basin. Fang et al. (2013) and Xu and Wei (2012) also apply bi-level programming to water allocation and distribution across users. Lv et al. (2010) introduces Fuzzy bi-level programming for water allocation problems which allows for consideration of different users and scenarios. This study will add to this fairly nascent body of literature, with the added contribution of spatial considerations and how it might affect water decision-making, as well as how an omniscient water master’s
interventions may help or derail overall welfare. Bi-level problems differ from traditional optimization problems in their containment of a nested optimization task within the constraints of another. The outer optimization problem is referred as the upper level task and the inner optimization problem is referred as the lower level task. The solution of bi-level optimization problems can have considerable difficulty. This is due to the nested structure of the overall problem which requires that any solution to the upper level problem may be feasible only if it is an optimal solution to the lower level problem.

Our model can be represented as a two-level optimization model where the water master’s optimization problem is the master problem, and the sub-problems are the optimization problems of the residential users, agricultural users, and storm-water management whose optimal decisions are dependent on their objectives and limitations; with the water master having the highest level of influence.

The model set-up is described below, followed by the mathematical programming set-up, solution, and discussion.
3 BI-LEVEL MODEL

The goal is to build a model of water-use in a spatial human-hydrologic system and identify optimal water usage in the system using bi-level programming techniques (Fortuny-Amat and McCarl, 1981). Although we focus on a case study of the Logan Northwest Canal in this model, it maintains a generic enough structure that can be broadly applied to other (canal) systems.

Human Hydrologic System

The human hydrologic system considers a primary canal (Crockett Canal) which is fed by a river (Logan River) and services a number of secondary canals which in turn service a variety of end-use agents. The model considers 4 different categories of decision-making agents each with different motivations, values, and objectives:

1. **Water Master (one for each canal (k) with a total of K canals):**

   Chooses

   1. the amount of water diverted from the Logan River $\varphi^k_t$
   2. the degree to which residential water-use is restricted – rationing calls $\bar{h}^{(i,j)}_t$

   to minimize

   1. maintenance costs
   2. residential service costs
   3. agricultural service costs
subject to

1. downstream flow requirements
2. the flow capacity in the canal (proxied by size of head-gates)

2. Agricultural Users (total of $M$ agricultural users each indexed by $m$):

Choose water withdrawals $h_{m,t}^{(i,j)}$ to maximize profits from agricultural production subject to

1. Agricultural production (this proxies crop type and other non-water inputs)
2. Pumping costs and irrigation capacity constraints (proxies irrigation technology)

3. Residential Users (total of $N$ residential user each indexed by $n$):

Choose water withdrawals to maximize household utility which is a function of

1. Water withdrawals
2. Canal flows for aesthetic motivations: think people who enjoy water running by their property. (These values can be scaled up and down if an individual does not have much value for canal flows)

subject to

1. The flow in the canal $Q_t^k$
2. The level of rationing call $h_t^{(i,j)}$ currently imposed by water master
An important difference between the residential and agricultural users is that residential users recognize that their withdrawals decrease the flow in the canal whereas agricultural users treat canal flow as exogenous to their water withdrawal decision. In other words, agricultural water-users act as if increasing their water-use decisions will have no impact on the current flow in the canal. This reflects the reality in certain areas where residential use restrictions are imposed to supply water to agricultural users.

4. Logan City Storm-water Management:

This agent chooses the percentage of storm-water diverted to the canal to minimize cost of storm-water disposal. Some decisions will be influenced by the decision of other agents. An obvious example is the diversion and rationing call decision made by the water master. Residential users must also predict how likely they are to have their usage limited by the water master. Because of these interactions between agents, the solution to the model will be characterized by a sub-game perfect Nash equilibrium in which each agent makes a decision based on the best response of all other agents. In other words, each agent understands the water-use decisions of all other agents and expects each agent to make rational water-use decisions. Initially we will start with a simple model with six agents – one water master ($K=1$), two residential users ($N=2$), two agricultural users ($M=2$), and one storm-water manager. We will also initially focus on only water quantity.
Space and time

Time is set in discrete daily increments and indexed by $t = 1, \ldots, T$ where $T$ is the length of the irrigation season. The human-hyrdrologic system (NWFIC stakeholders) is divided into a grid with rows indexed by $i = 1, \ldots, I$ and columns indexed by $j = 1, \ldots, J$. Each cell on the grid is represented by the pair $(i, j)$. As shown in figure 2, each cell $(i, j)$ is assigned to one of three categories:

1. Conveyance: These cells move water across the grid. No water-use decisions are made on these cells. There are two different components to the conveyance system
   i. Crockett Canal – links Logan River to field canals
   ii. Northwest canal – one of many field canals connected to Crockett

   These two components are managed jointly in our model (i.e., diversions from the Logan River to Crockett Canal are chosen based on planned diversions by all FIC’s operating along Crockett).

2. Diversion: These cells represent junctions between different conveyance components.

3. Use: These cells represent land parcels where the diverted water is used.

   The Logan River, which supplies water to the Crockett Canal, is dynamically modeled off the grid.

   Cells that fall in the conveyance and diversion categories constitute the hydrologic system that delivers water to the end user. The hydrologic system is represented by a water quantity state variable $Q^k_t$ measured in cubic feet per day and represents the amount
of water that flowed through canal \( k = 1, \ldots, K \) at time \( t \). The actual movement of water through the system is governed by a QUAL2K model representation of the system. The residence time in the canal system is about 6 hours. Water-users are assumed to make water-use decision on a less frequent time step (12-24 hours). As a result, each agent’s decision problem is largely static since current water-use decision will have little or no impact on canal flows when the next decision is made. An implication of this is that no water-users have any incentive to conserve water at this spatial scale. Any water left in the canal for conservation purposes will have flowed out of the canal by the time the agent makes a subsequent decision. The incentive to conserve may increase as the spatial consideration of decision makers increase (and the residence time of water in the system increase) but will still be limited by the presence/absence of storage in the system.

The model for the perceived water system represents an agent’s simplified perception of the more complex hydro system. This model is created using difference equations and simple proportional flow from one canal in the hydro system to another. The quantity of water in the Logan River at the Crockett Canal diversion is a function of the amount of snowpack and \( \epsilon_t \sim i.i.d. (\mu, \sigma^2) \) is a disturbance term with mean \( \mu \) and standard deviation \( \sigma \) that captures natural fluctuations (weather, climate, and hydrologic processes) and variation in anthropogenic diversions upstream of the Crockett Canal diversion.\(^{27}\) Specifically, the snowpack declines over the course of the season according to

\[
S_{t+1} = (1 - \mu)S_t
\]

(1)

\(^{27}\) By treating other canal company diversions as a error term assumes no coordination among canal company operators. This could be relaxed with a modeling effort that encompasses the larger spatial extent.
which implies $S_t = (1 - \mu)^t S_0$ where $\mu$ is the rate the snowpack melts and $S_0$ is the snowpack (in cubic feet of water equivalent) at the beginning of the season in the Logan River watershed. The amount of water upstream of the Crockett Canal diversion is

$$L^U_t = \rho \mu (1 - \mu)^t S_0 + \epsilon_t$$

(2)

while the amount of water in the Logan River below the Crockett Canal diversion becomes

$$L^D_t = L^U_t - \sum_{k=1}^K \varphi^k_t L^U_t \geq \bar{L}$$

(iii)

where $\rho$ captures the proportion of the melted snowpack that makes its way to the Crockett Canal diversion, $\varphi^k_t$ is the rate of withdrawal from the Logan by canal company $k$ with $K$ total canal companies along the Crockett Canal, and $\bar{L}$ represents any downstream flow requirements that may be implicitly or explicitly tied to Logan River flows.

The quantity of water at the bottom of canal $k$ at time $t$ is a function of the flow into that canal (set by the $k^{th}$ water master), the withdrawals by the $N+M$ shareholders along the canal, and the delivery requirement for the canal $\bar{Q}^k$:

$$Q^k_t = \frac{\varphi^k_t L^U_t}{\text{inflow}} - \sum_{n=1}^N h_{n,t}^{(i,j)} - \sum_{m=1}^M h_{m,t}^{(i,j)} + \gamma_t \geq \bar{Q}^k$$

(iv)

where $h_{n,t}^{(i,j)}$ is the quantity withdrawn by residential user $n$, $h_{m,t}^{(i,j)}$ is the quantity withdrawn by agricultural user $m$ and $\gamma_t^{(i,j)}$ is the storm-water return flow. Inflow is
proportional to the flow in the Logan River with $\varphi^k_t \leq \bar{\varphi}^k < 1$ where $\varphi^k_t = \bar{\varphi}^k$ where canal k’s head-gate is fully open.

Human Decision-Making

At the spatial scale considered in this model, no agent has an incentive to conserve canal water assuming no instream storage. Each agent is faced with a static optimization problem since there is no future cost of current actions. At this spatial scale, relative location within the canal system plays a less refined role in decision-making. Agents have to consider the decision of other agents when making a current decision but spatial location matters only in that agent’s only consider decisions of other agent’s using the same canal – being upstream or downstream doesn’t matter.

The static nature of the agent’s decision problem is due to the fact that the time step for decision-making is longer than the residence time in the canal that the person is drawing from. With a larger coordinating body with a larger spatial jurisdiction, effective residence times would be larger but it’s not clear that the time-step for decision-making would stay the same size since larger coordinating bodies often engage in longer-term planning. But if you did have an effective residence time longer than 24 hours, and a decision that was made daily, then agents recognize their relative location to others.

Human decision-making is undertaken by four different types of agents. These agents make daily decisions about the amount of water to withdraw/divert in the system. However, each agent type has different motivations, values, and objectives. Water
decisions on cell \((i,j)\) in period \(t\) are given by \(h_{t}^{(i,j)} \geq 0\) for withdrawal and \(y_{t}^{(i,j)} \geq 0\) for storm-water return flow.

**Residential User Decision Problem**

Multiple agents will fall in the residential water-use category and will be indexed by the cell which they own or are charged with managing. For these agents, water quantity provides value from its withdrawal and usage or *in situ*. The use/withdrawal of the water quantity state variable on cell \((i,j)\) in period \(t\) by residential user \(n\) is given by \(h_{n,t}^{(i,j)}\). Values from *in situ* use might represent riparian benefits from having water in the river, canal, or ditch. A residential user’s preference for these two ecosystem services are captured by the utility function \(U\left(h_{n,t}^{(i,j)}, Q_{t}^{k}; \alpha\right)\) \(0 \leq \alpha \leq 1\) represents relative preferences for water withdrawal. For exposition assume a log-linear utility function making the agent \(n\)’s problem

\[
\max_{h_{n,t}^{(i,j)}} U\left(h_{n,t}^{(i,j)}, Q_{t}^{k}; \alpha\right) = \alpha \ln h_{n,t}^{(i,j)} + (1 - \alpha) \ln Q_{t}^{k}
\]

(v.1)

subject to

\[
Q_{t}^{k} = \varphi_{t}^{k} L_{t} - \sum_{n=1}^{N} h_{n,t}^{(i,j)} - \sum_{m=1}^{M} h_{m,t}^{(i,j)} + y_{t}
\]

(v.2)

\[
h_{n,t}^{(i,j)} \leq \bar{h}_{t}^{(i,j)}
\]

(v.3)
where \( \hat{h}_{n,t}^{(i,j)} \), \( \hat{h}_{m,t}^{(i,j)} \), and \( \hat{y}_t \) represent the best responses by the residential users, agricultural users, and storm-water managers in the canal system. The residential water-user’s problem is similar to the common pool resource problem. The Lagrangian function associated with this agent is:

\[
L = \left\{ \alpha \ln h_{n,t}^{(i,j)} + (1 - \alpha) \ln \left[ \hat{\phi}_t^{k} L_t^{U} - \sum_{n=1}^{N} \hat{h}_{n,t}^{(i,j)} - \sum_{m=1}^{M} \hat{h}_{m,t}^{(i,j)} + \hat{y}_t \right] \right\} + \lambda_{n,t}^{(i,j)} \left[ \hat{h}_{t}^{(i,j)} - h_{n,t}^{(i,j)} \right]
\]

(vi)

with first-order condition

\[
\left\{ \frac{\alpha}{h_{n,t}^{(i,j)}} - \frac{1-\alpha}{Q_t^{k} - \hat{h}_{n,t}^{(i,j)}} \right\} = \lambda_{n,t}^{(i,j)}
\]

(vii)

where \( \hat{h}_{n,t}^{(i,j)} \) is the best response by agent \( n \).

\[
\hat{h}_{n,t}^{(i,j)} = \begin{cases} \alpha Q_t^{k} & \text{if } Q_t^{k} \leq \frac{\bar{h}_{t}^{(i,j)}}{\alpha} \\ \bar{h}_{t}^{(i,j)} & \text{if } Q_t^{k} > \frac{\bar{h}_{t}^{(i,j)}}{\alpha} \end{cases}
\]

(viii)

If the water rationing constraint is not binding, the water master incurs no service damages from this agent \( (\lambda_{n,t}^{(i,j)} = 0) \) and

\[
\hat{h}_{n,t}^{(i,j)} = \alpha Q_t^{k} < \bar{h}_{t}^{(i,j)}
\]

If the water rationing constraint is binding \( (\bar{h}_{t}^{(i,j)} = \hat{h}_{n,t}^{(i,j)}) \), the water master’s service damages from this agent are given by

\[
\lambda_{n,t}^{(i,j)} \left( \bar{h}_{n,t}^{(i,j)} \right) = \left\{ \frac{\alpha}{\bar{h}_{n,t}^{(i,j)}} - \frac{1 - \alpha}{Q_t^{k}} \right\}
\]
**Storm-water Manager Decision Problem**

The second type of agent is the city storm-water manager. To simplify notation, we will assume all storm-water inflow takes place at one point in the canal system (i.e., we rule out multiple intake points). This agent must minimize the cost of disposing of storm-water by selecting the percentage of storm-water that flows to the canal \( \theta_t \). The total amount of storm-water that a city manager must dispose of is \( Q_{St} \). The city manager’s cost function is given by \( w(Q_t^k) \theta_t Q_{S,t} + z(1 - \theta_t)Q_{S,t} \) where \( w(Q_t^k) \) is the marginal cost of dumping storm-water in the canal and \( z \) is the cost of disposing of storm-water under the next best alternative (e.g., retaining ponds). The marginal cost of disposing of storm-water in the canal is higher when the canal flow is near capacity: \( \frac{\partial w}{\partial Q_t^k} > 0 \). For simplicity assume \( w(Q_t^k) = wQ_t^k \). The city manager chooses storm-water inflow to minimize costs

\[
\min_{\theta_t} \{wQ_t^k \theta_t Q_{S,t} + z(1 - \theta_t)Q_{S,t}\}
\]

(ix)

The first-order condition for this problem is

\[ wQ_t^k = z \]

(x)

This implies that the city manager’s problem is a bang-bang solution. When \( Q_t^k > \frac{z}{w} \), the marginal cost of dumping storm-water in the canal is higher than the alternative approach and the manager chooses \( \theta_t = 0 \). When \( Q_t^k \leq \frac{z}{w} \), it is relatively cheap to dump the storm-water in the canal and the manager chooses \( \theta_t = 1 \) and the inflow is equal to
the total amount of storm-water. This leads to a critical threshold for canal flow \( \frac{z}{w} \) below which the city manager begins to dump storm-water in the canal and the best response from the storm-water manager is given by

\[
\hat{y}_t = \begin{cases} 
0 & \text{if } Q^k_t > \frac{z}{w} \\
Q_{s,t} & \text{if } Q^k_t \leq \frac{z}{w}
\end{cases}
\]  

(xi)

**Agricultural Water-user Decision Problem**

The third type of agent is agricultural water-users. Multiple agents will fall in this category and will be indexed by the cell which they own. These agents use water to irrigate agricultural fields which produce a valuable agricultural commodity which can be sold for price \( p \) when harvested. The water master will not restrict the amount of water agricultural users may withdraw and as such they do not consider how their individual withdrawal impacts the flow remaining in the canal (problem not solved subject to the equation of motion for \( Q^k_t \)). Agricultural users choose withdrawals to maximize instantaneous profit

\[
\max_{h_{m,t}} \pi^{(i,j)}_{t} = \left\{ pf \left( h_{m,t}^{(i,j)} \right) - c \left( Q^k_t \right) h_{m,t}^{(i,j)} \right\}
\]  

(xii)

subject to commodity specific production function \( f \left( h_{m,t}^{(i,j)} \right) = \omega h_{m,t}^{(i,j)} \), marginal cost of production \( c \left( Q^k_t \right) = \frac{c}{Q^k_t} \), and \( h_{m,t}^{(i,j)} \leq H_m^{(i,j)} \) where \( H_m^{(i,j)} \) is a capacity constraint from the farmer’s irrigation technology. Since canal flow will change over time the
farmer’s withdrawal may be periodically constrained by the irrigation technology. The Kuhn-Tucker condition for this myopic problem is

\[ p y \omega h_{m,t}^{(i,j)} y^{-1} - \frac{c}{q_t^k} \geq 0 \]

(xiii)

\[ \left[ p y \omega h_{m,t}^{(i,j)} y^{-1} - \frac{c}{q_t^k} \right] (H_m^{(i,j)} - h_{m,t}^{(i,j)}) = 0 \]

(xiv)

The best response for the agricultural user can be described by

\[ \hat{h}_{m,t}^{(i,j)} = \begin{cases} H_m^{(i,j)} & \text{if } Q_t^k \geq \frac{c}{py\omega H_m^{(i,j)} y^{-1}} \\ \left( \frac{c}{q_t^k py\omega} \right)^{\frac{1}{y-1}} & \text{if } Q_t^k < \frac{c}{py\omega H_m^{(i,j)} y^{-1}} \end{cases} \]

(xv)

If canal flow exceeds \( \frac{c}{py\omega H_m^{(i,j)} y^{-1}} \), the farmer will choose to operate at full capacity and when the canal flow is lower than this threshold, the farmer will operate at less than full capacity. Service losses to the agricultural user are equal to the loss in profits from operating at less than full capacity:

\[ \Pi_{m,t}^{(i,j)} = \pi_{m,t}^{(i,j)} \left( H_m^{(i,j)} \right) - \pi_{m,t}^{(i,j)} \left( \hat{h}_{m,t}^{(i,j)} \right) \]

(xvi)

**Water Master Decision Problem**

The water master makes two primary decisions: 1) when to open and close the head-gate and 2) how much residential shareholder usage should be limited during times of water shortage – rationing calls \( \hat{h}_{t}^{(i,j)} \). The decision to initially open the head-gate at the
beginning of the season is determined outside the model. When the flow in the Logan River is high and the head-gate is open, the water master risks flooding areas adjacent to the canal. These high flows in the canal also exacerbate maintenance costs.

The water master chooses what percentage of water in the Logan River should be diverted to the canal $\varphi^k_t$ in order to minimize maintenance costs and the losses to canal users (service damages). This decision must ensure that various demands are met. First, the canal company operator must ensure that a sufficient amount of water remains in the canal to satisfy the demands of downstream users of the primary canal. These demands are assumed to remain constant and given by $\bar{Q}^k$. Second, the canal company operator must also ensure that the diversion also meets the water demands of downstream users of the canal $h^{(i,j)}_t$ and the return flow demands $y^{(i,j)}_t$.

Maintenance costs are a quadratic function of the flow in the canal

$$y[\varphi^k_t L^U_t]^2$$

Service damages only arise when canal users’ demands are limited by the water master. The residential service damage on cell $(i,j)$ is $\lambda^{(i,j)}_{n,t} (\bar{h}^{(i,j)}_t)$ and represents the individuals shadow value for instream canal flows. The agricultural service damage on cell $(i,j)$ is $\Pi^{(i,j)}_{m,t}$ and represents lost profits from not operating at maximum irrigating capacity. The water master problem is:

$$\min_{\varphi^k_t, h_t} y[\varphi^k_t L^U_t]^2 + \sum_{n=1}^{N} \lambda^{(i,j)}_{n,t} + \sum_{m=1}^{M} \Pi^{(i,j)}_{m,t}$$

(xvii)
subject to the best response of all residential $\hat{h}_{n,t}^{(i,j)}$, agricultural $\hat{h}_{m,t}^{(i,j)}$ and storm-water $\hat{y}_t^{(i,j)}$ users and supply constraints

$$Q_t^k = \varphi^k_t L_t^U - \sum_{n=1}^N \hat{h}_{n,t}^{(i,j)} - \sum_{m=1}^M \hat{h}_{m,t}^{(i,j)} + \hat{y}_t^{(i,j)} \geq \bar{Q}^k$$

(xviii)

$$L_t^U = \rho \mu (1 - \mu)^t S_0 + \varepsilon_t$$

(ix)

$$L_t^D = L_t^U - \sum_{k=1}^K \varphi^k_t L_t^U \geq \bar{L}$$

(xx)

When the head-gates are fully open, $\varphi^k_t = \bar{\varphi}^k$. If this flow meets downstream flow requirement demands and the demands of residential and agricultural users, the water master may lower the flow $\varphi^k_t < \bar{\varphi}^k$ to minimize maintenance costs. If the fully open head-gate flow does not meet all demand, the water master may restrict the usage of residential water-users by choosing a lower level of $\hat{h}_{t}^{(i,j)}$ on all residential cells. This rationing call allows downstream flow and agricultural requirements to be met but also incurs residential service damages. Based on their respective decision problems, the water master is able to form a “best guess” for how each agent will act and the service damage that will arise from a given canal flow:

$$\lambda_{n,t}^{(i,j)} = \begin{cases} 0 & \text{if } Q_t^k \leq \frac{\bar{h}_t^{(i,j)}}{\alpha} \\ \alpha \frac{1 - \alpha}{\bar{h}_t^{(i,j)}} Q_t^k & \text{if } Q_t^k > \frac{\bar{h}_t^{(i,j)}}{\alpha} \end{cases}$$

and
\[ \Pi_{m,t}^{(i,j)} = \begin{cases} 
0 & \text{if } Q^k_t \geq \frac{c}{p\gamma \omega H_m^{(i,j)} y^{-1}} \\
\left[ pf\left( H_m^{(i,j)} \right) - c(Q^k_t)H_m^{(i,j)} \right] - \left[ pf\left( h_m^{(i,j)} \right) - c(Q^k_t)h_m^{(i,j)} \right] & \text{if } Q^k_t < \frac{c}{p\gamma \omega H_m^{(i,j)} y^{-1}}
\end{cases} \]
4 BI-LEVEL MODEL SOLUTION

The bi-level program looks at optimal water management in a single canal ($K=1$) and is modeled after the Logan Northwest Field Irrigation Company (NWFIC, see Figures 5.1 and 5.2). For simplicity we will drop the $k$ notation from the model description. The bi-level programming approach presumes a master problem and sub-problem whose optimal decisions are dependent covertly or overtly on input from each other.

This study formulates a bi-level spatial dynamic optimization problem to derive optimal water-use behavior for a canal company manager (water master); and presents an initial mathematical solution and analysis. The first level of our bi-level problem denotes a strategically acting omniscient water master comparable to a Stackelberg leader, while the second level comprises residential and agricultural users analogous to the Stackelberg follower(s).

Our omniscient water master considered in this model is the water master of the Logan North West Irrigation company. The area under his jurisdiction is situated on the west side of Logan city, running south and north along 400 west and 200 west, with an irrigated acreage of 1062.07 acres. This canal system is located in Utah water right area 25, with a period of use from April 1 to October 31.

The large shareholders in this canal system are located on the northern end of the system and are agricultural users whom the water master tries to save from crop failure with his water directives in time of scarcity. The residential users generally use the water for lawn irrigation with minimal, if any culinary use. This can be relaxed, especially when incorporating water quality considerations to account for both culinary and outdoor users.
The mathematical model is defined as follows by equations xvii-xx;

$$\min_{\tilde{r}_t, \tilde{r}_t} \gamma [\tilde{r}_t, \tilde{r}_t]^2 + \sum_{n=1}^{2} \lambda_{n,t}^{(i,j)} + \sum_{m=1}^{2} \Pi_{m,t}^{(i,j)}$$

(xvii)

such that

$$Q_t^k = \tilde{r}_t^k L_t^U - \sum_{n=1}^{2} \tilde{r}_n^{(i,j)} - \sum_{m=1}^{2} \tilde{r}_m^{(i,j)} + \tilde{y}_t^{(i,j)} \geq \bar{Q}^k$$

(xviii)

$$L_t^U = \rho (1 - \mu)^t S_0 + \epsilon_t$$

(xix)

$$L_t^D = L_t^U - \sum_{k=1}^{K} \phi_t^k L_t^U \geq \bar{L}$$

(xx)

Thus, the optimal choices of residential and agricultural users are incorporated into the water masters problem through his choice of $\lambda_{n,t}^{(i,j)}$ and $\Pi_{m,t}^{(i,j)}$ in his objective function, and in his constraint in xviii ($\tilde{r}_n^{(i,j)}, \tilde{r}_n^{(i,j)}$). In turn, when the water master sets $\phi_t^k$ at his optimal choice, this influences the constraints of residential users through their constraints in equation (v.2). Since we assume protection of agricultural users and myopic decision-making, the agricultural user is not directly constrained in this fashion. However, the water masters choice affects both users through the term $Q_t^k$ in both their objective functions.

To solve the bi-level optimization problem, the objective functions at the second decision-making level are substituted with the KKT (Karush-Kuhn-Tucker) optimality conditions of the residential and agricultural user problems ($\tilde{r}_m^{(i,j)}, \tilde{r}_n^{(i,j)}$), resulting in a Mathematical Program with Equilibrium Constraints (MPEC) depicted above (xvii-xx).
This MPEC is then solved in MATLAB using a direct mixed integer quadratic programming approach as promulgated by YALMIP.

The direct mixed integer quadratic program has some of its decision variables constrained to be integer values at the optimal solution. The discrete nature of integers allows us to model decision variables that take yes or no values, and to capture similar bang-bang solutions as observed in the KKT optimality conditions we derived. However, this possibility comes at a price as integers introduce non-convexity into the optimization problems, and thus make it more difficult to solve. Both computer memory demand and time to convergence can increase exponentially with the introduction of integer variables.

Our outer problem is a quadratic programming problem in the variables \( \varphi^k_t, \ h^{(i,j)}_{m,t} \) and \( h^{(i,j)}_{n,t} \).

\[
\begin{align*}
\min_{\varphi^k_t, h^{(i,j)}_{n,t}} & \quad \gamma \varphi^k_t L^U_t \varphi^k_t + c^T h_m + d^T h_n \\
\text{subject to} & \quad Q^k_t = \varphi^k_t L^U_t - c^T h_m - d^T h_n + e^T \hat{y}_t \geq \bar{Q}^k
\end{align*}
\]

where the variables \( h_m \) and \( h_n \) constrained to be the optimal solutions of our lower level optimization (quadratic programming) problems.

That is, \( h_n = h^{(i,j)}_{n,t} = \arg \min \alpha^T \ln h_n + (1 - \alpha)^T \ln Q^k_t \),

and \( h_m = h^{(i,j)}_{m,t} = \arg \min p^T \omega h_m^\gamma - \frac{c^T}{Q^k_t} h_m \)

subject to \( h^{(i,j)}_{m,t} \leq H^{(i,j)}_m \) where \( H^{(i,j)}_m \) is a capacity constraint from the farmer’s irrigation technology.

The solution is derived numerically using the following parameters and variables.
Table 3.1: Parameter Values

<table>
<thead>
<tr>
<th>Variable/Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0= 160.3</td>
<td>Initial snowpack level (SWE) in mm (annual snowfall in Logan)</td>
</tr>
<tr>
<td>gamma=0.5</td>
<td>Water master’s cost parameter</td>
</tr>
<tr>
<td>mu=1.41;</td>
<td>rate of snowpack melt (per day) during peak summer</td>
</tr>
<tr>
<td>rho= 0.38</td>
<td>portion of S which goes to canal</td>
</tr>
<tr>
<td>epsilonkt = sdpvar(n)</td>
<td>rate of water withdrawal by canal company k</td>
</tr>
<tr>
<td>alpha = 0.6</td>
<td>utility preference for consumptive use</td>
</tr>
<tr>
<td>n=4</td>
<td>number of decision variables</td>
</tr>
<tr>
<td>m=3</td>
<td>number of constraints</td>
</tr>
<tr>
<td>St= 0.8<em>S0-d</em>mu</td>
<td>% snow pack equation during peak operating season</td>
</tr>
<tr>
<td>Lut=rho<em>500</em>St</td>
<td>amount of water upstream crockett canal diversion</td>
</tr>
</tbody>
</table>
5 RESULTS AND DISCUSSION

The numerical results are obtained using the YALMIP toolbox in MATLAB. The solver in YALMIP is based on a simple branching strategy which performs branching on the complementary slackness conditions. This is an application of the branch and bound strategy promulgated by Fortuny-Amat and McCarl. The KKT conditions of the sub-problem are stated and then integer programming is employed in accommodating the complementary slackness conditions. The branch and bound scheme searches a defined state space for a list of candidate solutions (branches), and then tests these solutions branch by branch against the bounds of the optimal solution to find the best-performing solution.

Table 3.2 below presents the results obtained by solving for optimal agricultural water demand, residential water demand and optimal permissible storm-water flow using the bi-level formulation in the numerical model, juxtaposed against the same variables optimized with the KKT-conditions without considering the hierarchy of decision-making.
Table 3.2: Bi-level versus Non-hierarchical “Everyman for Himself” scenario

<table>
<thead>
<tr>
<th>Variable/Method</th>
<th>$\hat{h}_{1,t}^{(1,1)}$</th>
<th>$\hat{h}_{2,t}^{(2,1)}$</th>
<th>$\hat{h}_{1,t}^{(2,1)}$</th>
<th>$\hat{h}_{2,t}^{(2,2)}$</th>
<th>$\hat{y}_t^{(3,1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi-level</td>
<td>5000</td>
<td>10000</td>
<td>10000</td>
<td>20000</td>
<td>200</td>
</tr>
<tr>
<td>Non-Hierarchical</td>
<td>11,786</td>
<td>235713</td>
<td>4801</td>
<td>9601</td>
<td>539</td>
</tr>
</tbody>
</table>

These results are estimated averages during the peak summer season from June 1-August 31. The superscripts show the user’s location on the grid while the subscripts show the type of user they are, and the time marker.

Initial results show that the water master will always choose $q^E_t$ to divert the full allowable diversion the upper bound imposed by his maintenance costs and water right allow. While his maintenance and operation costs exert a downward pull, this downward pull, he is also required to ‘use it or lose it’ and so takes that into consideration. Initial results also suggest some improved overall system outcomes in the hierarchical model relative to individual optimization. This is attributable to the downstream agricultural users having a higher value for water in this system set up. In a case where the residential users had a high enough value for water, the benevolence of the water master towards agricultural users could produce worse system outcomes for all involved.
In particular, the omniscient water master is able to optimize outcomes for all users within his system, and also for the purposes of meeting downstream requirements. He elects to divert exactly the amount of water required to fulfill the needs of his agricultural and residential users, and tolerates the least amount of storm-water in a bid to minimize his costs.

He is able to provide an acceptable and desirable level of benefit for his residential users, while taking advantage of water they may have otherwise dumped in the quest to keep their water rights to protect the downstream agricultural users from crop failure. In the absence of the water master, the upstream residential users, as a result of their location and/or seniority of water rights, tend to withdraw more water as seen in the every man for himself scenario. This result is appropriate and observed in the locale as some residential water-users flood their neighboring lots to preserve their water rights.

When the level of water drops further so that the value of water increases for both users, both agricultural and residential users have their use limited, and the water master schedules the timing and allowance in favor of the agricultural users as much as possible taking the needs of the residential users into consideration.

The downstream agricultural users obtain less water in the nonhierarchical instance, leaving them open to crop failure, increased costs when compelled to lease water from upstream users to meet their needs, etc. The benevolence of the water master is not altogether altruistic. The Agricultural users comprise his largest shareholders with some holding thousands of shares and so he does have motivation to keep them satisfied. Secondly, given the importance of agriculture to Utah’s economy, the actions of a benevolent water master who protects the agricultural users, preserves statewide
economic interest and keeps out state agencies which may be as informed as a water master would be, from stepping in to protect Agricultural users and the economy and potentially creating less favorable system outcomes due to their information deficiency.

Where storm-water is concerned, the water master in the current model set-up recognizes the storm-water return flow in the equation of motion for $Q_k^e$. So when the canal flow falls below $z/w$ the WM recognizes more water will enter the system. The benefits of the storm-water inflow are incorporated in $w$. While he does not currently receive much financial benefit from storm-water flows, they do have the potential to help meet downstream flow requirements during reduced canal flow input from the Logan River.

This state of limited financial benefit from storm-water allowance has two ready motivations for relaxation in subsequent post-degree study. Firstly, increasing Environmental Protection Agency (EPA) regulation of storm-water management with regards to water quality creates a potential leverage for water masters of irrigation companies to charge the municipalities for their storm-water allowance. Secondly, as more precipitation falls as rain (a change that current systems are not appropriately equipped to handle), the fee(s) charged could be used to invest in storm-water storage and/or treatment infrastructure so that the water may be used within the canal system, or later released to meet downstream requirements.

In sum, initial results from the model suggest that an omniscient conveyance system manager does tend to improve overall system outcomes in particular when it comes to system benefits or costs which may not be readily considered by all agents (storm-water
management, prioritizing agricultural users, downstream user and in-stream requirements, etc.)

The imports of these benefits shift in response to increased water scarcity. For instance, the value of storm-water flows for meeting downstream requirements increase in times of low canal input from the Logan river where there may be storm-water inflow from rain, etc.

However, further parameterization is needed to strengthen the conclusiveness of those results as they tend to be highly sensitive to constraint values, particularly bounds on the water master’s choice variable, $\varphi^k_t$. Also, further research is needed to improve parameterization and model robustness, and this will be the focus of future work.
6 CONCLUSIONS AND FUTURE RESEARCH

The model confirms its merit for consideration of a water infrastructure-dictated hierarchical decision-making model with heterogeneous users. The model set up and bi-level formulation allow thorough examination of how the differences in water-users and a decision-making hierarchy determined by natural and built components of conveyance systems affect water allocation.

In particular, it shows how differing agent objectives as well as topographical structure of existing conveyance systems impact water diversions within a canal system. It shows that the incorporation of a hierarchical structure with an omniscient water master, allows system inefficiencies (e.g. dumping due to use it or lose it requirement) to be exploited for improved system-wide benefit, and allows water to be moved towards higher valued use.

It also shows the benefit of information in achieving overall system benefit even in a non-cooperative setting. The water master is able to use the information he has on the other users in the model, to meet downstream water requirements in an efficient manner. Should it be left up to myopic individual users, there is the potential to fall short of this consideration and risk stringent regulation. The information benefit is also seen in the agricultural user protection as noted above.

Subsequent research efforts will seek to improve computational performance by fine-tuning system parameters; improving the variable constraints and optimality bounds taking advantage of similar studies in the literature, as well as come primary data collection from interviews with canal manager in other systems, and reducing symmetries.
in the model. Lessons from the model will also be adapted towards extensions to more complicated relationships between water decision makers in a canal system.

There is also room for expansion to explicitly incorporate uncertainty and ecosystem benefits and payment systems in the model, and to consider multiple levels of decision-making to investigate the immediate and trickle down effects of state and federal regulations or directives on water decision-making at the canal and individual levels.

Limitations of the model include difficulty setting bounds and weights for the different objectives in the model. One reason traditional programming favors the representative agent approach is the increased computational difficulty when heterogeneity of agent characteristics and objectives are introduced. There are potential trade-offs in speed and sometimes exactitude when seeking to satisfy multiple constraints within a solution space for global and local optimality. However, this calls for increased research efforts to improve the solution process rather than avoidance as the heterogeneity and hierarchical structure have proven themselves relevant in this and related studies.
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GENERAL SUMMARY AND CONCLUSIONS

As water continues to grow increasingly scarce and less predictable, it is more imperative than ever to better understand the motivations and considerations of water decision-makers in choosing how and when to use and invest in water.

This dissertation, contributes to the body of knowledge on how farm agents, canal users and water masters interact with each other, as well as with the environment.

In chapter one, it is established that uncertainty in future water supplies and the efficiency of new water-saving technology create similar incentives to delay adoption of new water-saving infrastructure investments. It is also affirmed that some water-conservation technologies may not be efficient enough to trigger investment regardless of the price they could receive for the conserved water. While increases in water price in response to increasing water scarcity would make these technologies more profitable than they would be at lower prices, profits from production would have to be high enough to make a case for selling saved water under continued production, and selling all water and exiting the industry.

These observations provide insight as to what measures may be taken to encourage water-saving infrastructure investments in the face of the associated inherent uncertainty across investment types and geographical areas.

In chapter two, it is discovered that on-field variability of irrigation system performance can have significant impact on the economic incentive to invest in irrigation technology. The study found a significant difference in investment behavior which incorporated on-field hydro-geological variability into evaluating the economic decision to invest in irrigation technology. This study has shown that use of such data has value.
and may mean the difference between a 75% (three out of four times relative to all-or-nothing formulation) and 0% investment.

These two chapters illuminate the disparity between levels of investment in irrigation and other conservation technology suggested by net present value evaluations of these investments, and observed investment behavior. It is worth noting, that the investment thresholds obtained in these models are illustrative, rather than prescriptive, of the effect of uncertainty on irrigation investments. Given actual survey data on when individuals purchased water conservation technology, it is possible to parameterize these models to look at how changes in the model parameters lead to deviations from actual behavior. Here, we are only loosely trying to match actual behavior given the data available. The models are thus consistent with individual behavior, but may not be representative of average behavior.

As changing hydroclimatic and socioeconomic conditions continue to inject increased variability into decisions once perceived as black or white, it is important for policy and planning purposes that studies such as this are conducted in order to provide insight into how observed gaps between investment levels suggested by traditional models such as discounted cash flow models and observed investment can be explained and thus influenced by new and/or supplementary theories of economic decision-making.

With regards to intensity versus extent of investment, a comparison of results from the all-or-nothing and incremental set-ups suggests that the relative effect of water supply uncertainty on investment in areas with differing on-field variability is a pertinent consideration concerning water conservation goals and planning. State planners, for instance, would do well to acknowledge this variability in drawing up water plans and for
policy implementation. A slow uptake of prescribed technology may (instead of a perceived problem to be tackled with policy and public funds) be the effective working of the system to ensure only units of investment which whose benefits offset their incremental investment costs are made. It is thus necessary to jointly consider water supply variability (uncertainty), sunk costs (irreversibility) and on-field variability (marginal productivity) as factors determining adaptation investments in response to changing hydroclimatic conditions in the western United States.

Chapter three considered a water infrastructure-dictated hierarchical decision-making model with heterogeneous users. It examines how differences in water-users and a decision-making hierarchy determined by natural and built components of conveyance systems affect water allocation.

In particular, it shows how differing agent objectives as well as topographical structure of existing conveyance systems impact water diversions within a canal system. It shows that the incorporation of a hierarchical structure with an omniscient water master, allows system inefficiencies (e.g. dumping due to use it or lose it requirement) to be exploited for improved system-wide benefit, and allows water to be moved towards higher valued use.

It also shows the benefit of information in achieving overall system benefit even in a non-cooperative setting. The water master is able to use the information he has on the other users in the model, to meet downstream water requirements in an efficient manner. Should it be left up to myopic individual users, there is the potential to fall short of this consideration and risk stringent regulation. The information benefit is also seen in the agricultural user protection as noted above.
Subsequent research efforts will seek to improve computational performance by fine-tuning system parameters; improving the variable constraints and optimality bounds taking advantage of similar studies in the literature, as well as come primary data collection from interviews with canal manager in other systems, and reducing symmetries in the model. Lessons from the model will also be adapted towards extensions to more complicated relationships between water decision makers in a canal system.
APPENDICES
Canal lining investment results

The main distinction between an irrigation investment and a canal lining investment in our model comes from differences in water prices, costs, and water savings. Because water saved by canal lining is often sold rather than leased, the stochastic price process is assumed to convey sale prices rather than lease prices. The prices for water rights are obtained from the Utah water rights exchange\textsuperscript{28} and the cost estimates from the project evaluation documents of the Logan canal reconstruction project (NRCS, 2011). Analogous results for the investment model without uncertainty are presented in tables 1.5 and 1.6 below for reference.

Table A.1: Common parameters for Canal Lining Investment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate ((r))</td>
<td>0.06</td>
</tr>
<tr>
<td>Risk-adjusted discount factor ((\rho))</td>
<td>0.12</td>
</tr>
<tr>
<td>Water savings ((X^*))</td>
<td>40AF</td>
</tr>
<tr>
<td>Initial (one-time) Investment cost ((I))</td>
<td>$575</td>
</tr>
<tr>
<td>Total Investment cost ((\hat{I} = I + \frac{q}{r}))</td>
<td>$135,036</td>
</tr>
<tr>
<td>Initial Water Price ((P_0))</td>
<td>$1500/AF</td>
</tr>
</tbody>
</table>

\textsuperscript{28} Waterrightexchange.com/old/
Table A.2: Canal Lining Investment Thresholds

<table>
<thead>
<tr>
<th></th>
<th>Logan</th>
<th>Bear</th>
<th>Blacksmith-Fork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold P (NPV)</td>
<td>3510.936</td>
<td>9122.432</td>
<td>8150.173</td>
</tr>
<tr>
<td>Threshold P (OV)</td>
<td>45463.38</td>
<td>39698.77</td>
<td>41270.49</td>
</tr>
<tr>
<td>Threshold W (NPV)</td>
<td>87601.25</td>
<td>54259.64</td>
<td>34844.03</td>
</tr>
<tr>
<td>Threshold W (OV)</td>
<td>23728.4</td>
<td>25630.5</td>
<td>15235.17</td>
</tr>
</tbody>
</table>

*Stratonovich versus Ito’s Calculus*

The study begins with the standard initial assumption of a Geometric Brownian Motion (GBM) evolution for the stochastic processes of Water Supply and subsequently water price. Initial unit-root tests performed to investigate the validity of this assumption initially appear to corroborate the assumption. However, using the derived variance and drift parameters to predict the future evolution of the two stochastic processes suggested the processes may not be correctly solved under the Ito formulation.

Clarke and Reed (1989) establish the inappropriateness of the Ito’s calculus approach to solving for the drift and variance parameters of a GBM in the description of stochastic processes based on size (density) dependent growth rather than the age-dependent growth inherently assumed in an Ito calculus application. They aver that while age-dependent
growth may be appropriate for the growth of husbanded biological agents such as herds of livestock and intensively cultivated tree stands where environmental factors play no limiting role, it may be less appropriate for assets of a more wild nature such as unintended forest stands, or natural populations of fish and wildlife.

They posit that in the case of such ‘wild’ agents, growth is more likely to be determined by the size/density of the agent in relation to environmental factors such as the availability of food, soil nutrients, etc. rather than by the age structure of the biological agent population. Given the import of environmental factors on water supply, water supply is more likely to follow density-dependent growth, where density in this case would be factors like water demand (and thus factors that affect water demand such as population growth) and environmental factors such as changing hydroclimatic conditions.

In the case of a density-dependent growth, a Stratonovich calculus approach is more appropriate for solving for the drift and variance parameters in the stochastic evolution of the resource or biological agent. While estimates from Stratonovich and Ito’s calculus are equivalent from an operational point of view, with both providing maximum likelihood estimations of parameters, the distinction becomes important for analytic purposes, specifically with respect to the uncertainty (variance) parameter, which is pivotal in this study. Thus, we employ the Stratonovich calculus approach in our model.

Reed and Clarke (1990) establish that to correctly determine the effects of price uncertainty, it is necessary to establish whether the Ito or Stratonovich form of the GBM best described the price of the commodity in question. This question is not easy to resolve but is essential in this study.
Physical and biological models are typically modeled in terms of an input (driving) white noise process, by a stochastic differential equation of the form:

\[ dX(t) = \mu(X(t), t)dt + \sigma(X(t), t)dB(t) \]

(1)

This formulation has two main advantages. The first is its amenability to mathematical analysis. Solving (1) results in a diffusion process with infinitesimal drift \( \mu(x, t) \) and infinitesimal variance \( \sigma^2(x, t) \). The second merit is that while the inherent white noise assumption represents a mathematical abstraction, it is a good approximation of a variety of noise or other background and environmental random input processes that occur naturally in physical and biological contexts (Karlin and Taylor).

The solution of (1) is in terms of a stochastic integral \( \int \sigma(X(\tau), \tau)dB(\tau) \). Two prominent versions of this stochastic integral exist – Ito’s integral and Stratonovich integral. Ito’s calculus is an analogue for stochastic processes of the ordinary calculus of Leibnitz and Newton and the Ito’s integral plays the role in stochastic calculus that the fundamental theorem of calculus plays in ordinary calculus.

The concept and manipulations of the Stratonovich integral are reduced to calculations of related Ito’s processes, and the Stratonovich integral differs from the Ito’s integral by a corrective term. In particular, the solution from the Stratonovich calculus has the same variance, but a drift co-efficient of \( r_0 = \mu(x, t) \) in contrast to \( r_0 = \mu(x, t) + \frac{1}{2} \sigma^2 \) for Ito’s calculus.

Thus, the Stratonovich formulation is used in the paper, to account for this corrective term.
Figure A.1: Agent water conditions dependent on water allocation policy and environmental constraints

\[ \tilde{A} < X_i^* \text{: Policy Constrained} \]

\[ X_i^* < \tilde{A} \text{: Not Policy Constrained} \]

**Case 1:**
\[ A_t < \tilde{A} < X_i^* \]
Further Constrained by environmental condition
\[ A_t = \theta W_t \text{ and } y_i = \alpha_i \beta X_i \]

**Case 2:**
\[ \tilde{A} = A_t < X_i^* \]
No additional environmental constraints
\[ A_t = \tilde{A} \text{ and } y_i = \alpha_i \beta X_i \]

**Case 3:**
\[ A_t < X_i^* < \tilde{A} \]
Constrained by environmental conditions
\[ A_t = \theta W_t \text{ and } y_i = \alpha_i \beta X_i \]

**Case 4:**
\[ X_i^* \leq A_t \leq \tilde{A} \]
No constraint on production.
\[ y_i = y^* = \alpha_i \beta X_i^* \]
Figure A.2: Illustration of $\beta>1$ condition for positive root of $Q(\beta)$
Figure A.3: Maps of the Middle Bear-Logan watershed indicating the rivers of interest (Source: Bear River Watershed Information System, BRWIS, 2011)
Sensitivity Analyses Results

Figure A.4a: 2% sensitivity analysis for water supply (above) and technical (below) uncertainty.
Figure A.4b: 5% sensitivity analysis for water supply (above) and technical (below) uncertainty.
Figure A.4c: 10% sensitivity analysis for water supply (above) and technical (below) uncertainty
APPENDIX B: CHAPTER THREE SUPPLEMENTARY MATERIAL

Figure 5.1: Logan Northwest Field Canal
Figure 5.2: Stylized Model of Logan North-West Canal System.

(1,1) (1,2) (1,3)

Canal Company Diversion

8000m

Crockett Canal

7500m ----------- Upstream Boundary

7000m

→ Residential Diversion 1

6500m

(2,1) (2,2) (2,3)

Reach 1

(3,1) (3,2) (3,3)

Reach 2 Residential Diversion

(4,1) (4,2) (4,3)

Reach 3
Reach 4: 6000m

Reach 5: 5500m

Reach 6: 5000m

Reach 7: 4500m

Reach 8: 4000m

Residential Diversion 2

North Logan City Storm-water drain
Reach 9
3500m

Reach 10
3000m

Reach 11
2500m

Reach 12
2000m

Reach 13
1500m

Agricultural Diversion 1

Agricultural Diversion 2
1000m
Each cell is 500 m$^2$. Water moves along the conveyance system (Dark Blue = River, Light Blue = Canal). Water quantity on these cells is determined by a QUAL2K model. Diversion decisions are made on pink cells and storm-water inflow decisions are made on the orange cell. No decisions are made and no water flows to the brown cell.
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