Density-Dependent Convective Flow in Closed Basins

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DENSITY-DEPENDENT CONVECTIVE FLOW IN CLOSED BASINS

by

Kim L. McCleary

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil and Environmental Engineering

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Kim L. McCleary
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The Great Basin is a region of the Basin and Range Physiographic Province, which is completely isolated hydrologically from the sea. All precipitation that falls within the Great Basin is lost from the land surface or from the surface of closed inland lakes through evapotranspiration. Playas are often found at the base of these undrained basins. This study focuses on subsurface groundwater flow patterns in closed basins. Because all discharge from the basins occurs via evapotranspiration on and near the playa, the water table of the aquifer beneath the playa is often just below the ground surface. Fluctuations of the water table due to climatic events cause the water table to rise and dissolve the salts on the playa surface. This mass transfer can produce density gradients that in turn cause flow.

This study is an extension of the work performed by Duffy and Al-Hassan (Duffy and Al-Hassan 1988) in which numerical experiments were used to show that the free convection, caused by the variation in density of the bulk fluid properties, appears to play an important role in determining the patterns of groundwater flow beneath the playa. Their
work considered only homogeneous, isotropic porous media in symmetric basins. The effects of anisotropy, periodic stratification, and asymmetric basins on the groundwater flow and transport patterns was studied here. Dimensionless parameters, the Rayleigh number and the salt nose length, $L_0^*$, were defined for each of the systems incorporated and were shown to be linearly related. The implication of this relationship is that the Rayleigh number can be used to predict basin-scale circulation patterns in the hypothetical closed basins studied. It was also determined that an equivalent anisotropic system could be defined for a horizontally stratified aquifer in order to predict basin-scale circulation patterns. An equivalent isotropic system was defined for each anisotropic system in a similar manner.

(99pages)
CHAPTER I

INTRODUCTION

The Great Basin is a region of the Basin and Range Physiographic Province, which is completely isolated hydrologically from the sea. All precipitation that falls within the Great Basin is lost from the land surface or from the surface of closed inland lakes through evapotranspiration. The hydrologic system in the Great Basin typically includes recharge areas in the mountains and discharge areas in the lowlands. This system is modified by local geologic, climatic, and physiographic factors. In 1967 Maxey recognized that most water-supply, contamination, and disposal problems arise from a combination of features superimposed on this hydrologic system by concentration of population and agricultural activity in the discharge areas (Maxey 1967). Even today, most of our hydrologic data from arid lands comes from the lowlands, and little is available from the recharge areas. The hydrology of arid lands must be better understood before one can predict how artificial and natural disturbances will impact the system.

This study will focus on subsurface groundwater flow patterns in closed basins. Playas are often found at the base of these undrained basins. Because all discharge from the basins occurs via evapotranspiration on and near the playa, the water table of the aquifer beneath the playa is often just below the ground surface. Fluctuations of the water table due to climatic events cause the water table to rise and dissolve the salts on the playa surface. This mass transfer can produce density gradients that in turn cause flow. The flow is called free convection (Cussler 1985). Mass transfer also occurs in the aquifer because of forced convection, or advection, which is the transport of the dissolved salts due to the movement of the surrounding fluid.

Duffy and Al-Hassan (Duffy and Al-Hassan 1988) have used numerical experiments
to show that free convection, caused by the variation in density of the bulk fluid properties, appears to play an important role in determining the patterns of groundwater flow beneath the playa. Their simulations demonstrate that vertical and horizontal density gradients may occur in the groundwater that underlies the playa. As a result of evaporation on the playa, the denser fluid often overlies the less dense fluid in the underlying shallow groundwater, resulting in an unstably stratified system. In order for the system to become stably stratified, groundwater flow and mass transfer must occur. Thus, the circulation of groundwater and dissolved salts in closed basins is strongly affected by both advection, or forced convection, and free convection caused by the density gradient of the unstably stratified system.

Duffy and Al-Hassan assumed that the basin hydraulic conductivity was both homogeneous and isotropic and therefore did not incorporate natural heterogeneities introduced by the geology of playas in their conceptual model. The main goal of this study is to extend Duffy and Al-Hassan's work to determine how factors such as anisotropy, horizontal stratification, and basin asymmetry affect basin scale convection and the patterns of groundwater flow and dissolved salts in closed basins.

There are several implications of this study. In the past decade there has been increasing interest in using closed basins as repositories for low- and high-level toxic waste. Obviously, it is preferred that radioactive, industrial, or saline wastes do not come into contact with the hydrologic flow system because of the permanent damage that might result. However, for any operation that requires disposal of fluids into the ground in a closed basin, the subsurface hydrology must be understood, since isolation of wastes cannot be assured. Results from this study could be used to predict how various recharge and playa conditions would affect a contaminant's flow path, given the location of the waste disposal site and concentration of the source.

The results of the numerical experiments performed in this study will also be used to
guide field experiments that will begin later this year in the region studied. By performing the preliminary numerical simulations, understanding of the system will be enhanced, and the field investigations may be performed more efficiently. Results from this study will also increase our knowledge of the hydrology of arid mountain systems. Numerical experiments are used because of the lack of field data and basic understanding of the physical processes that govern groundwater movement in closed basins.

The specific objectives of this study are to 1) determine the effects of basin asymmetry, anisotropy, and certain subsurface heterogeneities on groundwater and solute circulation in undrained closed basins; 2) determine if a modified version of the Rayleigh number is an appropriate measure for the state of hydrogeochemical cycling in asymmetric, anisotropic, and heterogeneous closed basins; and 3) evaluate the effect of various initial conditions of the aquifer on steady-state flow. Pilot Valley, Utah, is a relatively uncomplicated example of the landform, climate, and hydrology of a closed basin. Pilot Valley contains a playa through which precipitation is lost, mainly via evapotranspiration. Some field studies have been performed there (Donahue 1987, Lines 1979, Stephens and Hood 1973) and were used to guide the simulations performed in this study.
CHAPTER II

LITERATURE REVIEW

Description of Study Area

Physiography

Physiography exerts a profound influence on the occurrence and movement of water in the Great Basin (Maxey 1967). The mountains act as catchment areas but have little storage capacity, while the valleys receive little precipitation but have the capacity to store more water than is received. The surrounding mountains and the degree of isolation from neighboring valleys govern the hydrologic conditions in the valleys. The location of the outlet, if one exists, in an arid basin has a strong impact on the hydrology of the basin. If no surface or subsurface outlet exists, the valley is hydrologically "closed," and water accumulates in the valley fill until it reaches the ground surface. Discharge will then occur by evaporation, and the playa on the valley floor will be kept moist.

The playa located in Pilot Valley, Utah, is a typical closed or "dry" lake and will be used as a guide for this study. The Pilot Valley, an area of about 470 square miles, is located in northern Utah and straddles the Nevada border (Figure 2-1). This region is the northeastern part of the Great Basin section of the Basin and Range Physiographic Province (Fenneman 1931). The valley is a trough-shaped depression approximately 45 km (28 miles) long and 22 km (14 miles) wide (Stephens and Hood 1973). It is bounded on the east by the Silver Island Range, with altitudes ranging from 1372 meters to 2134 meters (4500 to 7000 feet), and by the Pilot Range on the west, with altitudes ranging from 2134 meters to 3048 meters (7000 to 10000 feet). The nearly flat valley floor is at an elevation of about 1294 m (4245 feet), giving a total relief of almost 6500 feet. Two-
Figure 2-1. Location of study area and the present-day distribution of closed lakes and their maximum extent during the late Pleistocene (after Morrison, 1965.)
thirds of the bottom of the valley is a barren alkali flat (Stephens and Hood 1973) and is part of the Great Salt Lake Desert. The main physiographic feature in Pilot Valley is this 85000-acre alkali flat. It is covered with salt deposits that inhibit plant growth, resulting in a very flat, featureless salt desert. The topography of the Pilot Valley study area is presented in Figure 2-2.

Geology

The Great Basin is characterized by nearly parallel, north-trending mountain ranges that are separated by alluvial basins. Individual basins are in various states of surface and subsurface closure; however, the physiographic region can be referred to as "closed" hydrologically. The mountain ranges of the Great Basin in western Utah and eastern Nevada are composed of Paleozoic limestone, dolomite, shale, and Precambrian quartzite. The mountain ranges are bordered on one or more sides by block faults that can have several thousand feet of displacement (Lines 1979). The basins, or grabens, are filled with material eroded from the adjacent mountains. Much of the basin fill in the Great Salt Lake Desert was deposited in ancient Lake Bonneville. The basin fill in the deepest part of Pilot Valley is about 5300 feet thick.

A representative geologic cross section of Pilot Valley is shown in Figure 2-3. The geologic formations exposed in Pilot Valley range in age from Precambrian to Permian and from Tertiary to Holocene (Stephens and Hood 1973). Three types of sediment deposits dominate in closed basins at and near Pilot Valley. Alluvial fan deposits of Quaternary age occur along the basin margins. Basin fill deposits accumulate in the graben. The upper part of the basin fill is of Quaternary age and lies beneath sediments from Lake Bonneville. The lower part of the basin fill is of Tertiary age. Near-surface deposits reflect the lacustrine environment of Pleistocene Lake Bonneville.

The upper 20 feet of Lake Bonneville deposits underlying the Pilot Valley playa are
Topography of Study Area

Legend:
- Alluvium and Colluvium
- Lakeshore Deposits
- Lake Bottom Deposits
- Intrusive Igneous Rocks
- Sedimentary Rocks Undivided
- Quartzite and Associated Metamorphic Rocks Undivided
- Halite Perennial Salt Crust

Figure 2-2. Surficial geology and topography of the Pilot Valley study area (after Duffy and Al-Hassan, 1988)
Figure 2-3. Geologic section showing inferred Pilot Valley stratigraphy across the study area. Interpretation by J. McCalpin, Dept. of Geology, Utah State University.
described by Lines (1979) as composed mainly of dark-gray to dark-brown carbonate muds made up of clay-size calcite, aragonite, and dolomites. Interbedded with the carbonate muds are thin stringers of fine-grained quartz oolitic sand. Overlying the carbonate muds are gypsum evaporite deposits and the crystalline salt crust. (p. 27)

Below an altitude of about 5200 feet the surface of the valley has been modified by wave and current action of Lake Bonneville. The shoreline of the highest level of the lake is very apparent in aerial photographs of the area. Wave-cut terraces are also evident on consolidated rocks, and gravel bars and spits are deposited in several locations. The surficial geology is also affected by the high evaporation rates. Thrust polygons are very evident on parts of the Pilot Valley playa and are associated with rapid capillary discharge (Neal 1965). Slower capillary discharge produces the thin salt crust that is also evident on the playa surface. The position of the water table also has a strong effect on the surficial geology of Pilot Valley. If the water table is lowered beneath the surface of the playa, erosion will occur. When the water table is above the land surface, a depositional environment is created. Thus, the fluctuating water table controls which surficial forces dominate and indirectly maintains the balance between erosion and deposition. This dynamic interaction between climate, hydrology, and the surficial salts and sediments makes playas "the flattest of all landforms" (Neal 1975).

Hydrology

The undrained closed valley environment is defined by Snyder as a basin in which the surface water and groundwater drain internally (Snyder 1962). Closed and undrained valleys are both topographically and hydrologically closed and have playas, or "closed" lakes, which remain wet throughout the year or have water just below the surface during dry periods. All closed lakes are saline, and their salinity is greater than the influent
water. However, not all saline lakes are closed lakes as defined above. The one control on the existence of closed lakes is evaporation. The occurrence of closed lakes is restricted to regions where "gross evaporation is appreciably in excess of precipitation" (Langbein 1961, p. 3).

Figure 2-4 illustrates an idealized flow system for a closed basin that consists of three parts: a recharge zone in which movement of water is downward from the surface to the subsurface, a zone of lateral subsurface flow where there is no discharge to the surface, and a zone of discharge where movement of water is upward and springs appear at the ground surface (Maxey 1967). The flow system operates in a geologic framework of varying degrees of heterogeneity, which causes local or perhaps even regional modifications in the diagram. The flow system exhibits varying degrees of heterogeneity, and determining the importance of these heterogeneities is a major thrust of this study.

The hydrologic conditions in Pilot Valley are governed by the precipitation that falls on the surrounding mountains. The climate in the basin is arid to semi-arid, depending on elevation. Past studies (Chang 1969, Peck and Brown 1962) have shown that there is a direct correlation of annual precipitation with elevation in the Great Basin region. That is, more precipitation falls at the higher elevations. For example, the Pilot Valley floor receives less than 6" per year while the upper elevations of the Pilot Range can receive more than 15", mostly in the form of snow. Data for the Wasatch Front actually show a linear relation between precipitation and altitude (Duffy and Al-Hassan 1988). Temperatures often exceed 100°F in July and August and average below 40°F during the winter. A more complete climatic description of Pilot Valley can be found in Stephens and Hood (1973).

Because evaporation exceeds precipitation, streams do not usually reach the playa in Pilot Valley. Ephemeral streams in the Silver Island Mountains and the lower reaches of
Figure 2-4. A conceptual view of the hydrology of a topographically closed and undrained desert basin (after Schumm 1977, Toth 1962, and Maxey 1967).
streams in the Pilot Range flow only as a result of thunderstorms or rapid snowmelt. The main channels of the streams and tributaries are relatively straight as they descend from the mountains and cross the alluvial fans. When stream floods do reach the playa the main channels divide and fan out over the playa surface. However, for most of the year streamflow is lost before it reaches the playa.

The major sources of recharge to the unconsolidated deposits are the runoff from consolidated rocks upslope and spring discharge at altitudes above 5400 feet. Approximately 10% of the precipitation that falls on the mid to upper elevations in Pilot Valley infiltrates directly to the subsurface and becomes recharge. Little recharge originates as precipitation on areas below 5600 feet due to low annual precipitation and surficial soil moisture retention (Stephens and Hood 1973). The water that does infiltrate becomes more concentrated with dissolved solids as it is transported through the subsurface to the playa. Because there is no outlet for water on the playa other than evapotranspiration, the dissolved solids are left behind and concentrate on the playa surface.

There is a close relation between the distribution of these dissolved solids and the sediment regime. Lines described the zonation of the evaporite minerals in Pilot Valley using three categories: the carbonate zone, the sulfate zone, and the chloride zone. The area and location of these zones are transitional due to the seasonal reworking of the surface by the incoming fresh water (Lines 1979). During the wetter season, the water table rises to the surface, and the chloride evaporites near the edge of the playa are dissolved and transported toward the central playa. When evaporation rates rise, the water table lowers beneath the ground surface, and the evaporites are left on the center of the playa.

The result of this seasonal transport process is shown in Figures 2-5a and 2-5b. The contours of the total dissolved solids (TDS) and water table elevation are concentric
Figure 2-5a. Water table elevation contours (feet asl) in the shallow brine aquifer in Pilot Valley (after Lines, 1979).

Figure 2-5b. Total dissolved solids contours (g/l) for the shallow brine aquifer in Pilot Valley (after Lines, 1979).
about the perennial salt crust zone. Lines' figures show the strong interaction between hydrology and salinity. The concentric TDS contours are related to fluid density, as the most concentrated fluid is the most dense. Vertical variations in groundwater salinity (thus density) were measured beneath the playa in Pilot Valley by Donahue (Donahue 1987). He found the most dense fluid overlying the less dense fluid, resulting in an unstably stratified system. The dense fluid of the central playa sinks and may cause an increased depression in the water table. This may induce more flow towards the central playa.

Theoretical Background

Convection

Density-driven flows occur when gravity forces act on denser regions, causing fluid to flow downward relative to the regions that are less dense. This idea of a density gradient causing flow to occur relates to the idea of free convection. An important question to ask is whether free convection occurs in a system. A stable density configuration is one in which the fluid density is constant or increases with depth. For an otherwise static system, if the system is stable, free convection will not occur and the description of mass transfer depends only on advection and hydrodynamic dispersion. If the system is unstable, free convection will occur, and advection and dispersion analyses alone will not describe the mass transfer process. An example of an unstable system is one in which a dense fluid overlies a less dense fluid.
Cussler presents a simple example of free convection in a vertical tube. The apparatus he used is shown schematically in Figure 2-6. The apparatus consists of two well-mixed reservoirs. The upper reservoir contains a dense solution and the lower reservoir contains a less dense fluid. The solute in the upper reservoir diffuses from the upper reservoir into the lower reservoir. Cussler summarized the development of a dimensionless group of parameters that will determine if free convection occurs (Cussler 1985). One of these dimensionless ratios, the Rayleigh number, describes the forces and thus the stability in the vertical tube. If the density difference in the tube is so great that the Rayleigh number exceeds some critical number, then free convection will occur. Cussler uses linear perturbation theory to define the critical Rayleigh number for various systems. Others have also used linear perturbation theory to define critical Rayleigh numbers for thermal and thermohaline convection (Kvernvold and Tyvand 1979, McKibbin 1986, Rubin 1982, Wooding 1978). Cussler describes the Rayleigh number as a ratio of buoyancy forces tending to cause flow to other processes tending to resist flow. Also, the Rayleigh number is described as the ratio of a buoyancy velocity to a diffusion

Figure 2-6. Schematic diagram of Cussler's two-reservoir apparatus.
velocity.

Duffy and Al-Hassan used a modified version of the Rayleigh number to actually predict the position of the salt nose in a homogeneous, isotropic aquifer (Duffy and Al-Hassan 1988). Various simulations were made using different combinations of input parameters such as the average recharge velocity, the concentration of salts on the playa, and the hydraulic conductivity. The distance that the wedge of saline water moved from the playa surface, \( L_\alpha/L \), was shown to be linearly related to the Rayleigh number, as shown in Figure 2-7.

![Figure 2-7. Rayleigh number vs. \( \text{Lo}^* \) for Duffy and Al-Hassan's numerical experiments.](image)

**Numerical experiments**

As mentioned previously, Duffy and Al-Hassan used numerical simulations to study the effects of various orographic precipitation and recharge conditions on the
groundwater and solute circulation in a homogeneous, isotropic aquifer in a closed basin (Duffy and Al-Hassan 1988). Dimensionless parameters were used to apply the results more generally. Others have also found the use of dimensionless parameters convenient when modeling haline convection (Paschke and Hoopes 1984), thermal convection (Kvernvoild and Tyvand 1979, McKibbin 1986, McKibbin and Tyvand 1982), and thermohaline convection (Rubin 1982, Wooding 1978).

Duffy and Al-Hassan found that the location of the springs in Pilot Valley may be explained primarily by the effect of topography. In the past, the existence of springs was thought to be due to the juxtaposition of beds of relatively high permeability with beds of lower permeability (Neal 1965), i.e., the interface between coarse-grained material in alluvial fans and finer-grained clays beneath the playa. But Duffy and Al-Hassan's simulations did not incorporate heterogeneities in their model and found that springflow could occur simply due to the break in slope between the alluvial fans and the playa. This break in slope was termed the "hinge line." Topography has also been shown to be important in the analysis of thermal convection in groundwater (Wooding 1978). The present study incorporates Duffy and Al-Hassan's linear orographic and recharge functions and follows their approach but is extended to test additional representations of the closed basin.

The theory of fluid flow through anisotropic media was summarized by Maasland in 1957. Maasland summarized a proof showing that the net effect of anisotropy in the hydraulic conductivity is equivalent to the effect of shrinkage or expansion of the coordinate of a point in the flow system. One can obtain an equivalent, homogeneous, isotropic system (EIS) by shrinking or expanding the coordinates of an anisotropic medium. The EIS could be formed by either shrinking the horizontal coordinate by the factor \((K_h/K_v)^{1/2}\) or by expanding the vertical coordinate by the factor \((K_h/K_v)^{1/2}\), where the subscripts h and v correspond to the horizontal and vertical directions, and K
can represent either the hydraulic conductivity or the permeability. The equivalent isotropic hydraulic conductivity for the EIS was shown to be

$$K = (K_h K_v)^{1/2}$$  \hspace{1cm} (2-1)

for two-dimensional flow where the coordinates are aligned with the principal directions of the anisotropy. The equivalent isotropic permeability may be defined using the same relationship. Anisotropic hydrodynamic dispersion for the case of uniform flows through porous media was investigated by Moranville (Moranville, Kessler, and Greenkorn 1977).

Maasland explained the connection between anisotropy and fine-scale stratification of sediments and soils. Soils are often stratified due to lacustrine or volcanic depositional origins. Where the horizontal layers are composed of homogeneous isotropic material, the integrated effect of a large number of these layers produces anisotropy. Maasland showed that if a stratified medium consists of regularly alternating, infinitesimally thin layers, which have different thicknesses and different hydraulic conductivities, an apparent directional hydraulic conductivity can be derived. Thus, the layered porous medium can be represented by an anisotropic model medium. The average or effective horizontal and vertical conductivities can be calculated by considering uniform flows parallel and perpendicular to the layering. If the x and z coordinate axes are aligned with the horizontal and vertical layering, these effective parameters are

$$K_x = \sum_i \frac{K_i d_i}{D} \quad K_z = \frac{D}{\sum_i \frac{d_i}{K_i}}$$  \hspace{1cm} (2-2a, 2-2b)

where D is the total depth of the system, \(d_i\) is each individual layer's thickness, and \(K_i\) corresponds to a layer's hydraulic conductivity (or permeability).

This anisotropic representation has been applied in models of thermal and thermohaline convection (McKibbin 1986, McKibbin and Tyvand 1982, Rubin 1982,
Wooding 1978). McKibbin and Tyvand showed that a layered porous medium can be represented by an anisotropic medium for certain circumstances (McKibbin and Tyvand 1982). They determined an effective anisotropy of the stratified system in order to make comparisons with an 'equivalent' homogeneous anisotropic layer. The anisotropic representation was shown to be relevant for curved flow in a layered porous medium when the length scale of the flow is sufficiently large compared to the layer thicknesses. Their results also showed that for the homogeneous-anisotropic model, convection occurs throughout the whole layer, which they termed large-scale convection. Whereas convection may begin in only some layers for large non-homogeneities in permeability or conductivity in stratified systems. This 'local' convection is almost totally confined to the most conductive layers. As the number of layers becomes large, the results from the stratified case converge with those from the anisotropic model. McKibbin and Tyvand concluded that only large-scale convection will occur in the equivalent anisotropic medium.
CHAPTER III

METHODOLOGY

Theory of Groundwater Flow and Transport

Equations of groundwater flow and solute transport

Numerical experiments were used to examine groundwater flow and solute transport patterns using different representations of the subsurface. In order for the numerical simulations to accurately represent the flow and solute transport processes that occur in a closed basin, the model must use governing equations that incorporate a coupling between flow and salinity. Salt transport problems are much more difficult to solve than normal groundwater flow problems because of the nonlinearities involved. The salt dissolves and can change the density of the groundwater to such an extent that variations in density strongly affect the flow (Herbert, Jackson, and Lever 1988).

The computer model that was used to simulate subsurface fluid movement and solute transport is entitled "SUTRA - Saturated and Unsaturated Transport" (Voss 1984). The groundwater flow is simulated through numerical solution of a fluid mass balance equation. Solute transport is simulated through numerical solution of a solute mass balance equation where solute concentration may affect fluid density. The dispersion processes incorporated in the model are molecular diffusion and two types of mechanical dispersion. The longitudinal dispersivity may be designated as being velocity-dependent or velocity-independent. The SUTRA code also introduces a new method for calculating fluid velocities, which minimizes numerical dispersion. This method involves discretizing all terms that include a \( \rho g \) term in a more consistent manner spatially and
The primary variable in the flow model is fluid pressure \((\text{m/lt}^2)\). The effect of gravity acting on fluids with different densities is accounted for in the flow field. The physical properties of the fluid are described by fluid density, \(\rho\); fluid viscosity, \(\mu\); and fluid pressure, \(p\). The fluid density is assumed to be a weak function of pressure and mainly dependent upon the solute concentration by the relation

\[
\rho (c) = \rho_0 + \frac{\partial \rho}{\partial c} \, dc = \rho_0 + \gamma (c - c_0)
\]

(3-1)

where

- \(c\) = solute concentration (mass fraction) \(\{\text{m/m}\}\)
- \(\gamma\) = coefficient of density change with concentration \(\{\text{m/lt}^2\}\)
- \(\rho_0\) = reference density \(\{\text{m/l}^3\}\)
- \(c_0\) = reference concentration \(\{\text{m/m}\}\).

For mixtures of fresh and a near-saturated brine, \(\rho_0\) is equal to 998.2 kg/m\(^3\), and \(\gamma\) is equal to 756 kg/m\(^3\) (Duffy and Al-Hassan 1988). The fluid viscosity mainly depends on the temperature of the fluid and is assumed to be constant and equal to 0.001 kg/ms for this study.

The fluid within the solid matrix is defined by porosity, \(\phi\), and aquifer storativity. The porosity is a dimensionless parameter defined as the ratio of the volume of void space within the matrix to the total volume of the matrix. The porosity was specified to be 0.35 for all simulations performed in this study. The aquifer storativity is defined as the volume of water released from saturated pore storage due to a unit drop in fluid pressure per total solid matrix plus pore volume, or as

\[
\frac{\partial (\phi p)}{\partial p} = \rho \, S_{op}
\]
where

\[ S_{op} = \text{specific pressure storativity (lt}^2/\text{m}) = (1-\phi) \alpha + \beta \phi \]

\[ \alpha = \text{matrix compressibility (lt}^2/\text{m}) \]

\[ \beta = \text{fluid compressibility (lt}^2/\text{m}). \]

It was assumed that both the fluid and solid matrix are incompressible, so the value of the specific pressure storativity is zero.

The fluid movement is driven by differences in the pressure field or by unstable variations in fluid density. Pressure-driven flows are directed from regions of higher than hydrostatic pressure toward regions of lower than hydrostatic pressure. A form of Darcy’s law is used to represent the mechanisms of pressure- and density-driving forces for flow in SUTRA and is presented below.

\[
\nu_i = -\frac{\kappa_{ij}}{\phi \mu} \cdot (\frac{\partial p}{\partial x_i} - \rho ( - g \frac{\partial z}{\partial x_i} ))
\]

Previously undefined terms are

\[ \nu_i = \text{average fluid velocity (l/t)} \]

\[ \kappa_{ij} = \text{permeability tensor of the porous medium (l}^2) \]

\[ g = \text{gravity (l/t}^2) \]

In general, \( \partial f/\partial x_i = \nabla f = \text{grad } f \). In a two-dimensional system both \( i \) and \( j \) can take on the values 1 and 2, as \( x_{i=1} \) refers to the \( x \) direction, and \( x_{i=2} \) refers to the \( z \) direction. The permeability \( \kappa \) is a measure of the ease of fluid movement through the saturated voids and is assumed to be the same forward and backward in the same direction. For reference the Darcy velocity = \( q_i = \phi \nu_i \).

The flow simulation calculates how fluid mass within the void spaces in a control volume changes with time. The fluid mass balance can change due to inflow or outflow, a change in density, or sources and sinks within the control volume. The
mass being conserved is the sum of the pore water mass and the solute mass and is defined as

$$\frac{\partial (\phi \rho)}{\partial t} = -\frac{\partial}{\partial x_i} (\phi \rho v_i) + Q_p \tag{3-3}$$

where the term on the left-hand side describes the total change in fluid mass contained in the void space with time. The first term on the right-hand side describes the changes in mass due to net inflow. The final term on the right is the fluid and solute mass associated with prescribed fluid sources. For saturated conditions, the term on the left-hand side is

$$\frac{\partial (\phi \rho)}{\partial t} = (\rho S_{op}) \frac{\partial \rho}{\partial t} + (\phi \frac{\partial \rho}{\partial c}) \frac{\partial c}{\partial t}$$

For an incompressible fluid and porous medium the value of $S_{op}$ is zero, thus eliminating the first term.

The solute mass is transported by flow of the groundwater and by hydrodynamic dispersion. Hydrodynamic dispersion incorporates both mechanical dispersion and molecular diffusion, which carries mass from areas of high- to low-solute concentration. Mechanical dispersion is the process by which solute mass travels at different velocities due to the different tortuosities of each individual flow path in the porous medium. The average uniform velocity is used to calculate the advective transport even though the actual velocities may vary considerably from point to point within the porous medium.

SUTRA represents solute dispersion in the form of anisotropic dispersivities. The dispersion tensor is defined as

$$D_{ij} = \begin{bmatrix} D_{xx} & D_{xz} \\ D_{zx} & D_{zz} \end{bmatrix}$$
where

\[ D_{xx} = \left( \frac{1}{v^2} \right) \left( \alpha_L v x_x^2 + \alpha_T v x_z^2 \right) \]

\[ D_{zz} = \left( \frac{1}{v^2} \right) \left( \alpha_T v x_x^2 + \alpha_L v x_z^2 \right) \]

\[ D_{xz} = D_{zx} = \left( \frac{1}{v^2} \right) \left( \alpha_L v - \alpha_T v \right) (v_x v_z) \]

\[ \alpha_L = \text{longitudinal dispersivity of solid matrix} \]

\[ \alpha_T = \text{transverse dispersivity of solid matrix} \]

\[ v = \text{magnitude of velocity} \]

\[ v_x = \text{magnitude of x-component of velocity} \]

\[ v_z = \text{magnitude of z-component of velocity} \]

\[ \phi = \text{porosity} \]

\[ \delta = \text{specific dispersivity} \]

\[ D = \text{diffusion coefficient} \]

\[ Q_p = \text{source term} \]

For systems in which the heterogeneities are much smaller than the field transport scale, the parameters \( \alpha_L \) and \( \alpha_T \) can be considered fundamental transport properties of the system. This idea is similar to the permeability being the fundamental property of the porous media through which water flows. The transverse and longitudinal dispersivities are assumed to be independent of the flow direction.

The solute mass balance for a system in which there are no adsorption/desorption, production/decay, or dissolution/precipitation reactions is defined as

\[
\frac{\partial (\phi \rho c)}{\partial t} = -\frac{\partial}{\partial x_i} \left( \phi \rho v_i c \right) + \frac{\partial}{\partial x_i} \left[ \left( \phi \rho \left( D_{xx} + D_{yy} \right) + D_{zz} \right) \frac{\partial c}{\partial x_i} \right] + Q_p c^* \quad (3-4)
\]

where the term on the left-hand side describes the total change of solute mass in the saturated volume with time. The first term on the right-hand side defines the average advection of solute mass into or out of the local volume. The second term is the
contribution of solute mass from diffusion and dispersion processes. The final term defines the solute mass added to the system by prescribed fluid sources with concentration \( c^* \). The term \( D_m \) refers to the molecular diffusion coefficient for a porous medium and has units \( (L^2/t) \). The term \( D_{ij} \) is the mechanical dispersion coefficient.

SUTRA uses the Galerkin weighted residual formulation of the finite element method to numerically approximate the above governing partial differential equations 3-1 through 3-4. Refer to the code documentation for the details of the numerical algorithms used. The subsurface system must be discretized into quadrilateral elements, for which approximations to the governing equations are calculated. Each element has four corresponding nodes, illustrated in Figure 3-1. Some parameters are defined element-wise (\( K \)) while others are assigned node-wise (\( p, c, q, \phi \)).

\[ \text{Figure 3-1. Schematic diagram of quadrilateral element.} \]

SUTRA incorporates the various types of boundary conditions in the following manner. For nodes at which a prescribed pressure is assigned, a fluid flux is applied cellwise through a highly conductive core in order to make \( p_i \) equal to \( p_{BC} \), as in Figure 3-2. The conductivity of the core is required as input to the SUTRA code. For prescribed fluid sources, mass is added cellwise to the designated node in cell \( i \). For nodes at which concentrations are specified, the prescribed value is simply assigned to the node. There is also a specified solute concentration assigned to fluid inflow that occurs at boundary nodes.
The solution sequencing is as follows. The fluid flow mass balance matrix is set up, followed by the formation of the solute mass balance equation. Once these matrices are defined for each element, the fluid pressure field is solved for, followed by the solution of the solute concentration field. An iterative method is used to converge to a solution for each time step. The time discretization is based on a backward finite-difference approximation for the time derivatives in the balance equations. The finite-element grid used for the half-basin simulations is shown in Figure 3-3.

The non-dimensional system

Non-dimensional variables and parameters were used as in Duffy and Al-Hassan's work. This was done to generalize the results to basins of similar shapes and characteristics. All of the field variables and parameters were made dimensionless using either a characteristic length, velocity, density, or pressure. The characteristic length for the system is $L$, the dimension of the basin from the mountain divide to the center of the playa. The characteristic velocity is the mean recharge rate, $\bar{e}$, and the reference density is the density of fresh water, $\rho_0$. The term used to scale the pressure term is $\rho_0 g L$, the
Figure 3-3. Finite-element grid used in half-basin simulations.
product of the reference density, gravity, and the characteristic length. The dimensionless variables are defined as

\[ x^* = \frac{x}{L}, \quad t^* = \frac{t}{\bar{c}}, \quad \bar{q}_i = \frac{q_i}{\bar{c}}, \quad \varepsilon^*(x) = \frac{\varepsilon(x)}{\bar{c}}, \quad c^* = c, \quad p^* = \frac{p}{\rho_o g L}, \quad \rho^* = \frac{\rho}{\rho_o} \]

and the dimensionless parameters are

\[ S_s = \rho_o g L (\alpha + \beta \phi), \quad \phi^* = \phi, \quad \gamma^* = \frac{\gamma}{\rho_o}, \quad K_{ij}^* = \frac{K_{ij}}{\varepsilon}, \quad D_{ij}^* = \frac{D_{ij}}{\varepsilon L} \]

where \( K_{ij} = K_{ij} \rho_o g / \mu \) is the reference hydraulic conductivity. By substituting these parameters into the governing equations presented in equations 3-1 through 3-4 and rearranging terms, the following dimensionless system of governing equations are obtained.

\[ \rho^* = 1 + \gamma^* \left[ c^* - c_0^* \right] \quad (3-5) \]

\[ K_{ij}^* \left[ \frac{\partial p^*}{\partial x_i^*} + \rho^* \frac{\partial z^*}{\partial x_j^*} \right] = q_i^* \quad (3-6) \]

\[ \frac{\partial}{\partial x_i^*} \left[ K_{ij}^* \rho^* \left[ \frac{\partial p^*}{\partial x_j^*} + \rho^* n_j^* \right] \right] = S_s^* \frac{\partial p^*}{\partial t^*} \quad (3-7) \]

\[ \frac{\partial}{\partial x_i^*} \left[ \phi^* \rho^* D_{ij}^* \frac{\partial c^*}{\partial x_j^*} \right] = \rho^* q_i^* \frac{\partial c^*}{\partial x_i^*} + \phi^* \rho^* \frac{\partial c^*}{\partial t^*} \quad (3-8) \]

The output from SUTRA is in terms of dimensionless time, pressure, velocity, and concentration. The range of values used for each parameter will be presented in the Model Application and Results chapter.
Formulation of Numerical Experiments

The conceptual model

The cross section of a conceptual model of playa hydrogeology is illustrated in Figure 3-4a. During the summer, the water table is lowered by direct evaporation, and the ephemeral salt crust forms in the mud flat zone. Evaporation is reduced in the winter, and the water table rises, dissolving the salt crust. The excess water runs off as overland flow to the central playa, forming an ephemeral lake. The most important process in any modeling study is to accurately represent the conceptual model with the mathematical model. Appropriate boundary and initial conditions must be formulated to accurately represent the hydrologic processes that occur in a closed basin. For each simulation that was performed, a conceptual model was formulated based on observed physical characteristics of Pilot Valley. An attempt was made to use reasonable boundary and initial conditions, but it should be emphasized that the study here is conceptual, and the simulations do not represent Pilot Valley except in a very crude way.

An idealized mathematical representation of a conceptual model is shown in Figure 3-4b. The system illustrated is a homogeneous and isotropic porous medium of thickness D and length L. The closed basin was assumed to be symmetric, so only one half of the basins' extent was modeled. The bottom and sides of the system were assumed to be impermeable to water and solutes. The boundary condition at each side represents a groundwater flow divide at the basin's topographic divide and a flow divide in the center of the basin due to symmetry. Contributions from groundwater baseflow were not incorporated due to their small impact on the system's flow regime. All of the system inputs and characteristics remain constant through time.
The recharge zone, corresponding to the zone between the mountain divide and the hinge line, is defined as $\sigma$. An orographic function for the dimensionless recharge was determined by Duffy and Al-Hassan based on the assumption that recharge is linearly related to elevation. The incomplete beta function $I_x (a,b)$ fit data for the Pilot Valley reasonably well (Duffy and Al-Hassan 1988). Since the rectangular boundary of the
mathematical model does not incorporate changes in topography, it is the orographic recharge function that actually represents a change in altitude. A dimensionless recharge distribution must be used and was developed by scaling the recharge at each node by the value at the divide, thus producing recharge values ranging from 0 to 1. A short program for the generation of non-dimensional recharge is included in Appendix 1. Several different recharge conditions were generated for the numerical experiments performed, depending on the desired mean recharge rate, $\bar{e}$, and the length of the recharge zone. The concentration of the recharge was specified to be 0.003 g/g (mass concentration), corresponding to the mean concentration of chemical weathering of source rocks within the basin.

The discharge zone, $L-\sigma$, corresponds to the playa surface. A specified pressure boundary condition was applied for all nodes within the discharge zone. The pressure was prescribed to be zero, representing the top of the water table. This boundary condition allows fluid to move into and out of the system in order to satisfy the mass balance equations at each time step. SUTRA requires the user to specify the concentration of any inflowing fluid at nodes with a specified pressure boundary condition. It was assumed that all inflow along the playa margin was the same concentration as the mountain recharge, 0.003 g/g. Realistically, the inflow along the playa margin probably contains more solutes than the relatively fresh recharge, but this was not incorporated.

The zone labeled $\eta$ represents the half-width of the salt accretion zone in the central playa. It is assumed that there is an infinite supply of salt crust in this zone, which is represented by specifying a constant concentration at the nodes in this region. Different values for the specified concentrations were used for each simulation to represent different amounts of salt deposition on the central playa. The pressures were also specified to be equal to zero at these nodes, which allows fluid to move across the upper boundary. Any fluid that enters the system at these nodes was assumed to have a solute
concentration equal to the specified boundary concentration. This boundary condition does not allow the perennial salt crust to change location. For analysis of long-term, steady-state behavior this boundary condition seems to be appropriate.

Modifications to the mathematical representation presented in Figure 3-4b were made to represent basin asymmetry and anisotropy and horizontal stratification in the porous medium. The details of each simulation are presented in the Model Application and Results chapter, but the general approach is described here. For the asymmetric cases, the full length of the basin was modeled rather than the half-basin, as in Duffy and Al-Hassan's study and as in all other simulations performed in this study. The conceptual model of a basin bounded by two mountain ranges of different maximum heights is illustrated in Figure 3-5a. The length of the discharge and recharge zone is different on each side of the basin, corresponding to a different amount of recharge entering at each side of the basin.

Figure 3-5b illustrates the cross section of a basin in which the volume of recharge on the right-hand side is less than the recharge volume on the left-hand side. The ratio of the length of the recharge zone to the half-length of the basin, $\sigma/L$, is always kept constant and equal to 0.725. The ratio of the length of the recharge zone to the length of the discharge zone, $\sigma/(L-\sigma)$, is kept constant on both sides also and equal to 2.636. The ratio of the salt accretion zone to the half-length of the basin, $\eta/L$, is specified to be 0.06. In general these ratio values are based on the physical characteristics of Pilot Valley. The aspect ratio of the flow field, $L/D$, was specified as 10 for all cases in this study.

The cross section in Figure 3-6 illustrates how horizontal stratification was incorporated into the model. Only half of the basin was modeled because of basin symmetry. The number of layers used in the simulations ranged from 2 to 10 and always consisted of two alternating homogeneous and isotropic layers. The most conductive layer was not always the top layer, and different values for the ratio of the hydraulic con-
Figure 3-5a. Conceptual model of an asymmetric closed basin.

Figure 3-5b. Mathematical representation of conceptual model of an asymmetric closed basin.
Figure 3-6. Schematic diagram of horizontal stratification.

For the anisotropic porous media cases, the horizontal permeability was assumed to always be larger than the vertical permeability, as presented in the diagram in Figure 3-7. Again, it was assumed that the basin is symmetric. Various ratios of $K_z/K_x$ were used. In this study the effect of the magnitude of dispersion on the circulation pattern was not tested.

Figure 3-7. Diagram of anisotropic permeability.

The basin-scale force balance approach

In order to evaluate how each representation of the model relates to basin-scale flow and solute patterns, a force-balance approach was used to represent all possible factors that affect the flow system beneath the playa. The dominant forces that affect solute movement are advection, hydrodynamic dispersion, and buoyancy, which are functions of
the mean recharge rate, structure of the porous matrix, and concentration of salts on the central playa, respectively. The buoyancy force contributes to mass transfer by allowing the possibility of free convection to occur. Free convection results from the density difference between the ambient fluid, which is relatively fresh, and the residual evaporating fluid at the surface of the playa. The concentration of this fluid can reach 1.2 g/cm$^3$, a sodium chloride saturated brine. Gravity forces cause the dense fluid beneath the center of the playa to sink, and it is this movement that causes the solute to be transported toward the playa margin. The advective forces work to impede the transport of these dissolved salts because the water wants to flow in the opposite direction, toward the center of the playa.

A parameter, the Rayleigh number, was defined to determine whether free convection will occur in salt-freshwater solutions. Duffy and Al-Hassan presented the buoyancy and resistant forces in terms of velocities. For an idealized porous medium in a simplified geometry, the Rayleigh number can be expressed as

$$Ra = \frac{\kappa \Delta \rho g/\mu}{\bar{e}} = \frac{\frac{buoyancy \ velocity}{recharge \ velocity}}{\frac{\alpha}{L} \frac{dispersive \ length \ scale}{system \ length \ scale} \frac{[aspect \ ratio]}{}}$$

(3-1)

where

- $\kappa$ = mean intrinsic permeability (l$^2$)
- $\Delta \rho$ = density contrast between recharge and discharge fluids (m/l$^3$)
- $g$ = gravity (l/t$^2$)
- $\mu$ = dynamic viscosity (m/lt)
- $\bar{e}$ = mean recharge rate (l/t)
\( \alpha_T = \text{transverse dispersivity} \) \( \{1\} \)

\( L = \text{length of basin} \) \( \{1\} \)

\( D = \text{depth of basin} \) \( \{1\} \)

Any increase in the buoyancy velocity will increase \( R_a \), or the chance for free convection. Any increase in the recharge velocity, the aspect ratio, or the dispersion term decreases \( R_a \) and impedes the movement of the salt nose toward the basin margin. It should be noted that the Rayleigh number is only useful when analyzing steady-state behavior in a system.

A parameter must also be defined to represent the resulting state of the system. Since it was the basin-scale circulation patterns that were of interest, the distance that the salt nose, or wedge of saline fluid, migrates toward the basin margin was used to measure the equilibrium point or balance between velocity of the brine and the velocity of mountain recharge. Duffy and Al-Hassan defined the dimensionless parameter, \( L_o^* \), as follows:

\[
L_o^* = \frac{L_o}{L} \left( \frac{\sigma}{L - \sigma} \right)^{\frac{1}{2}} \left( \frac{L}{D} \right)^{\frac{3}{2}}
\]  \( \{3-2\} \)

where

\( \sigma = \text{length of recharge zone} \)

\( L - \sigma = \text{length of discharge zone} \)

\( L/D = \text{aspect ratio of system} \)

\( L_o = \text{length of salt nose} \) (shown in Figure 3-1a and Figure 3-2a)

This parameter represents the salt nose length in a homogeneous and isotropic porous medium. Duffy and Al-Hassan argued that the Rayleigh number and \( L_o^* \) are the fundamental parameters controlling hydrogeochemical cycling in closed basins.

For this study, the appropriate \( L_o^* \) had to be defined to represent the salt nose length
for non-homogeneous, anisotropic systems. Also, appropriate terms had to be defined to use in the Rayleigh number to represent the basin-scale force balance for asymmetric basins or heterogeneous/anisotropic porous media. For example, an effective permeability that should be used in the Rayleigh number to represent a horizontally stratified or anisotropic system had to be defined.
node that bounds both the recharge and discharge zones. The length scale, \( L_0/L \), was defined as the distance from the hinge line to the point at which \( c/c^* = 0.5 \) along the bottom of the system. The concentration along the bottom is \( c \), and \( c^* \) is the specified concentration at the central playa. Alternatively, the salt nose is the point where the fluid velocity across the mixing zone between fresh and saline water equals zero. Initially the distance, \( L_0/L \), was estimated at the point where \( q^* = 0 \) along the bottom, but this method was not consistent for all types of simulations. The length of the salt nose, \( L_0/L \), provides a measure of the balance between the recharge velocity and the velocity of the dense plume.

To better understand the implications of each simulation, one should first observe the flow and solute transport patterns in a closed basin in which there are no buoyancy or density gradients. The simulation results are for an aquifer that is a homogeneous, isotropic freshwater system and can be used as a reference for subsequent results. Also, the steady-state pressure condition from this freshwater simulation was used as the initial pressure condition for all other simulations. Figure 4-1 illustrates the flux and flow patterns that occur when there is no accumulation of salts in the ephemeral lake region. Figure 4-1a is the flux velocity across the upper boundary, with recharge into the system defined as a positive flux. The specified recharge distribution is shown on the left, or upland side of the hinge line, and the model-simulated discharge is on the right, or playa side of the hinge. The groundwater discharge is maximum just inside the hinge line on the \( p^* = 0 \) surface and then gradually decreases toward the center of the playa. This corresponds to observations made in Pilot Valley of springflow occurring along the hinge line. Since geologic variability was not accounted for, this result exhibits the effect of topography on margin springs and shows that varying lithology and permeability may not be the only cause of springflow.

The dimensionless hydraulic head distribution (\( h^* = p^* + z^* \)) is presented in Figure 4-1b. The hydraulic head decreases with depth in the recharge zone, representing a
Figure 4-1. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, and (c) normalized velocity field, for the freshwater case.
downward flow component. The hydraulic head increases with depth beneath the center of the playa, showing a vertical upward flow component. At the surface of the playa, where the pressure is specified to be zero, the hydraulic head is equal to the elevation head, which also corresponds to the inverse of the aspect ratio, $D/L = h^* = 0.1$. Note the location of this pressure condition, as it will be used in comparisons with subsequent simulations.

Figure 4-1c illustrates the velocity vectors, which show that the maximum exit velocity occurs at the hinge line. The velocity vectors represent the velocity of the centroid of each element, approximated from the velocities at each node.

**Asymmetry**

Numerical experiments were designed to determine how topographic asymmetry influences the groundwater flow and solute transport patterns in a closed basin. The Pilot Valley is bounded by two mountain ranges of different maximum height and is an example of a topographically asymmetric basin. Since recharge is assumed to be proportional to elevation, the asymmetric topography produces an asymmetry in the recharge conditions. Evidence of the hydrologic asymmetry in the Pilot Valley is found in the large number of fresh springs that occur on the Pilot Range side of the playa, while there is little evidence of margin springs on the Silver Island side. This condition results from the imbalance in the amount of mountain recharge on each side of the basin.

The numerical simulations were designed to test different degrees of asymmetry and different playa evaporation and permeability conditions. A total of six simulations were performed, each requiring from 6 to 18 hours of cpu time on a VAX 8650 machine to reach steady state. The cpu time varied because the time incrementations were not identical. Please refer to Figure 3-5b for an illustration of the input and output parameters. The non-dimensional values of these parameters for each simulation are
presented in Table 4-1, which will be referred to often in the discussion of the results. The grid spacing was modified from the 41 x 11 nodal array used in the half-basin simulations. The horizontal spacing was modified in order to keep the recharge and discharge regions the appropriate lengths. The grid used for the symmetric full-basin simulation is presented in Figure 4-2. The initial pressure condition for all six simulations was the steady-state pressure distribution of a freshwater aquifer. The initial concentration was specified to be zero everywhere in the system.

The first simulation was performed to verify that if exact conditions were applied on both sides of the basin, the resulting pressure and concentration fields would be symmetric. Figure 4-3 illustrates the results of this symmetric, full-basin simulation and will be used for comparison purposes. Figure 4-3a is the fluid flux velocity across the upper boundary. As in the freshwater case, the flux on the outer, or upland, side of the hinge line is the specified symmetric recharge condition. The flux on the inner, or playa, side of the hinge line is model-simulated groundwater discharge, also symmetric for this simulation. The groundwater discharge is maximum just inside the hinge line on the $p^* = 0$ surface and gradually decreases toward the central playa, where there is now additional inflow. This inflow is required to satisfy the mass balance equations. A density gradient develops when gravity acts on the solute at the surface of the playa. When the solute mass begins to sink, fluid enters the system to satisfy the mass balance equations for the elements in the central playa region.

Figure 4-3b is the dimensionless head distribution, which is symmetric as expected. The position of the $h^* = L/D = 0.1$ surface is now beneath the playa surface. The velocity vectors, shown in Figure 4-3c, illustrate the counterflow that developed. Freshwater is flowing toward the central playa, while the brine is flowing toward the basin margin, creating a salt nose. The salt nose migrated a distance, $L_v/L = 0.235$, beneath the hinge on each side of the basin. The presence of the salt nose caused the hydraulic head to
Table 4-1. Non-dimensional input and output parameters for asymmetric basin simulations.

<table>
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<tr>
<th>Simulation</th>
<th># elements</th>
<th>$\bar{c}^*$</th>
<th>$K^*$</th>
<th>$C^*$</th>
<th>$L$ (rhs)</th>
<th>Total cpu(hr)</th>
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<td>277.96</td>
<td>0.265</td>
<td>0.77335</td>
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<tr>
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<td>277.96</td>
<td>0.265</td>
<td>0.9093</td>
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</tr>
<tr>
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<td>0.819948</td>
<td>790.41</td>
<td>0.04</td>
<td>0.77335</td>
<td>10:23</td>
</tr>
<tr>
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<td>0.560121</td>
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Figure 4-2. Finite element grid used for symmetric full-basin simulation.
increase from the freshwater case in the recharge zone (Figure 4-3b).

The other asymmetric basin simulations incorporate different degrees of basin asymmetry and perturbations in the playa conditions, such as permeability and ephemeral lake concentration. Figure 4-4 is the resulting flux and flow pattern for simulation 2, as defined in Table 4-1. The only difference in the boundary conditions from the previous simulation is that the mean velocity on the right side of the basin was specified to be 56% of the mean velocity on the left side of the basin. All other parameters remained the same as in simulation 1. The recharge distribution was generated using the exact same function for both sides, but only the lower section of the distribution was used for the right side, corresponding to a smaller mountain range.

The discharge pattern has changed, shown in Figure 4-4a. There is a larger discharge velocity on the left side of the basin than on the right side due to the larger amount of recharge entering the system on the left. The resulting hydraulic head contours are no longer symmetric, as shown in Figure 4-4b. The hydraulic head distribution on the left side is the same as in the symmetric full-basin simulation, but the head values on the right side of the basin decreased. This occurred because less water enters the system on the right side, so there is less flow to be restricted by the salt nose. The location of the $h^* = D/L = 0.1$ contour is now beneath the playa similar to the previous simulation.

The velocity vectors (Figure 4-4c) show that two convection cells have developed beneath the central playa and that the velocities are lower on the right side. The concentration contours, shown in Figure 4-4d, are asymmetric as expected. The salt nose migrated further toward the right margin due to a less resistant recharge velocity. The length of the salt nose on the left was exactly the same as in the symmetric simulation and was not affected by any of the boundary conditions applied on the right side.

Four other simulations were performed for basins with different degrees of asymmetric recharge distributions, different ephemeral lake concentration conditions, and
Figure 4-4. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for full basin simulation 2.
different aquifer permeabilities. It was originally thought that the conditions on one side
of the basin would affect the salt nose migration on the other side of the basin. But the
results of the simulations show that the salt nose length is dependent only upon the
Rayleigh number for the corresponding side of the basin, defined as

\[ R_a = \frac{\kappa \Delta \rho g / \mu}{\frac{\varepsilon}{\alpha T \frac{L}{L_0 D}}} \]

Figure 4-5 is a plot of the Rayleigh number for each side of the basin versus the
corresponding salt nose length for each side of the basin for all six simulations. The
points lie very close to the curve generated from Duffy and Al-Hassan's results for homo-

![Figure 4-5. Rayleigh number vs. Lo* for asymmetric full-basin simulations.](image-url)
geneous, isotropic, half-basin simulations. There was one simulation in which no salt nose formed on either side of the basin. These results are from simulation 5 and will be discussed in the Anomolies section of this chapter.

Anisotropy

A total of twelve simulations were performed to study the effects of anisotropic permeabilities on the groundwater flow and solute transport patterns in a closed basin. It was assumed that the basin is symmetric for these simulations, so only half of the basin was modeled. The same aspect ratio ($L/D = 10$) and the original 41 x 11 nodal array were used for all twelve simulations. Table 4-2 lists the dimensionless parameters used in each simulation. The same recharge distribution was used for all simulations. The horizontal and vertical permeabilities were varied, as was the ephemeral lake concentration. The horizontal permeability was specified to be from 2 to 100 times as great as the vertical permeability.

A Rayleigh number was calculated for each of the anisotropic simulations and is defined as:

$$R_a = \frac{[\kappa \Delta \rho g/\mu]}{\bar{\varepsilon}} \left[ \begin{array}{c} \alpha \varepsilon \\ L \\ L \end{array} \right]$$

where $\kappa = \kappa_{\text{eff}} = (\kappa_x \kappa_z)^{0.5}$. This definition of the effective permeability is based on the effective permeability (or hydraulic conductivity) used in the equivalent isotropic system (EIS) representation of an anisotropic aquifer, as discussed previously. When analyzing the appropriate salt nose length, the $x$-coordinates were transformed by the ratio $(\kappa_z/\kappa_x)^{0.5}$ in order to define the EIS. Note that the coordinates were not transformed in the actual simulation but only for analysis of the scaling factor to define the appropriate
Table 4-2. Non-dimensional input and output parameters for anisotropy simulations.

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salt nose length term. This transformation produces a system with a new aspect ratio, \( L/D' = (L/D) (\kappa_x/\kappa_z)^{0.5} \). The salt nose length must incorporate this new aspect ratio and must also be rescaled by \((\kappa_x/\kappa_z)^{0.5}\) in order to be on the same scale as the \(L_0^*\) values from Duffy and Al-Hassan's simulations. Thus, the final definition of the salt nose length for anisotropic permeabilities is:

\[
L_0^* = \frac{L_0}{L} \left( \frac{\sigma}{L-D} \right)^{1/2} \left( \frac{L}{D} \right)^{3/4} \left( \frac{\kappa_z}{\kappa_x} \right)^{1/4}
\]

The first set of results presented are from simulation 11, which represents a basin with a relatively low Rayleigh number \((Ra = 1021.3)\). Figure 4-6 exhibits the dimensionless flux velocity, hydraulic head, velocity, and concentration distributions. The flux velocity distribution (Figure 4-6a) appears to have the same general shape as in previous simulations. The location of the \(h^*=D/L=0.1\) contour has dropped beneath the central playa. The hydraulic head contours are slightly elongated in the horizontal direction near the hinge line. The velocity vectors in Figure 4-6c exhibit a convection cell beneath the hinge line and at the very bottom of the aquifer beneath the central playa. The salt nose has migrated a distance of \(L_0/L=0.04167\) toward the playa margin (Figure 4-6d).

Figure 4-7 presents the results from simulation 8, which is for a system with a slightly higher Rayleigh number \((Ra = 1522.4)\). The horizontal permeability was specified to be 25 times greater than the vertical permeability. The effective permeability is about twice as large as that in the previous simulation, and thus the flow is not as restricted. This effect is reflected in the hydraulic head contours shown in Figure 4-7b. Less pressure has built up due to the higher permeability and relatively short salt nose length. The hydraulic head contours are also more elongated in the horizontal direction than in
Figure 4-6. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for anisotropy simulation 11.
Figure 4-7. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for anisotropy simulation 8.
previous simulations with less anisotropy. The higher degree of anisotropy tends to broaden the region at which discharge occurs, shown both in Figures 4-7a and 4-7c, the discharge flux velocity, and the velocity distribution. The discharge is no longer occurring just at the hinge line but has spread toward the central playa. The velocity vectors also show that a convection cell has developed beneath the deep portion of the brine aquifer. As expected, the salt nose migrated further toward the playa margin than in simulation 11 due to the higher Rayleigh number for the system.

Simulation 9 represents a system with an even higher Rayleigh number (Ra=2283.6). The only difference in the boundary conditions of simulation 8 and 9 is that the ephemeral lake concentration is higher for simulation 9, resulting in the larger Rayleigh number. Figure 4-8 exhibits the results from the simulation. Notice that the hydraulic head contours shown in Figure 4-8b are of the same magnitude and shape near the hinge line as in simulation 8. Both simulations have the same permeability structure. Since the salt nose has migrated further in simulation 9, the mountain recharge is more restricted, and the pressures beneath the recharge zone have built up slightly. The discharge flux velocity distribution for simulation 9 is also similar to that in simulation 8. The small perturbation in the ephemeral lake concentration results in a salt nose that has migrated almost completely to the basin divide, shown in Figure 4-8d.

Results for simulation 2 (Ra=4676.9) are presented in Figure 4-9. This Rayleigh number is high enough to allow the salt nose to migrate across the entire basin to the divide, shown in Figure 4-9d. The horizontal permeability was specified to be 100 times as great as the vertical, which produces an even more elongated hydraulic head distribution (Figure 4-9b). The extent of the salt nose reduces the area through which the mountain recharge may flow, and the build-up of pressure is reflected in the higher hydraulic head values. The effective permeability is twice as large as in simulations 8 and 9, but this does not overcome the flow restriction caused by the presence of the salt
Figure 4-8. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for anisotropy simulation 9.
Figure 4-9. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for anisotropy simulation 2.
nose beneath the entire recharge zone.

The final set of results presented in Figure 4-10 are for simulation 12. These results will be used for comparison with a simulation incorporating horizontal stratification. The Rayleigh number for this system is 6670.9, which allows the salt nose to migrate across the entire basin. The ratio of horizontal to vertical permeability is 8, so the hydraulic head contours, Figure 4-10b, are not as elongated as in the previous simulation. The discharge zone in Figure 4-10a is also not as broad as in simulations with higher degrees of anisotropy. The fluid entering the system as recharge is again restricted by the presence of the salt nose. The pressure has built up considerably more than in the previous simulation, due in part to the broader extent of the salt nose and also due to a smaller effective permeability. Note the shape of the dense solute plume, where $c^* = 0.08$, that has sunk from the playa surface, shown in Figure 4-10d.

Figure 4-11 illustrates the linear relation between $L_0^*$, the salt nose length, and the Rayleigh number as defined for the anisotropic simulations. There is more variation from an exact linear relation than was the case for the full-basin and Duffy and Al-Hassan's isotropic half-basin simulations. There is also an outlier, which represents results from simulation 4. Possible causes for this outlier will be discussed in the Anomalies section of this chapter. A possible cause for the variation about the linear regression line is the way in which SUTRA models anisotropy. The dispersivities were assumed to be independent of the flow direction. It is possible that a dispersion tensor that is dependent on flow direction should have been incorporated, but there was no field data on which to formulate the exact relationship.

Another possible cause for the poorer fit is that the methodology used to define $L_0^*$ and the effective permeability for the Rayleigh number is incorrect. The methodology is based on the assumption that an equivalent isotropic system may be defined for both flow
Figure 4-10. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for anisotropy simulation 12.
and solute transport in anisotropic systems. This methodology was used for simulations of energy transport (McKibbin and Tyvand 1982), which uses only slightly modified versions of the governing equations used to simulate solute transport. Numerical dispersion and numerical diffusion, caused by inappropriate grid spacing, might also be a cause for the poorer fit. However, two simulations were performed using finer grids to determine if this was the case. One simulation was performed in which the spacing between nodes in the vertical direction was reduced by half, and a simulation was also performed in which the horizontal spacing was reduced by half. Results from both simulations matched the simulation that used the original 41 x 11 grid. This match was determined by overlaying the plots of hydraulic head, discharge, and concentration.
distributions. There was no visible difference between the plots. Therefore, it is assumed that the grid spacing did not cause the variation seen in the results. It seems likely that the variation was caused by undefined numerical 'errors' resulting from discretizing the highly non-linear system of equations, or possibly by the simplified model for dispersion used here.

**Horizontal stratification**

Ten simulations were performed to study the effects of horizontally stratified sediments on subsurface flow and solute transport patterns in a closed basin. Idealized conceptual models were used to represent periodic stratification. Simulations were performed that incorporated 2, 5, and 10 layers composed of alternating higher and lower conductivity layers. The simulations were designed to study the effect of the number of layers, the importance of the conductivity of the upper-most layer, and the effect of the Rayleigh number on the flow patterns. Table 4-3 presents the dimensionless input and output parameters for all of the simulations.

One of the objectives of this group of simulations was to determine if a horizontally stratified system can be accurately represented by an equivalent anisotropic, homogeneous system. The idea of representing fine-scale stratification as anisotropic was presented in the Literature Review chapter and will be reviewed briefly here. If infinitesimally thin layers are deposited in a regularly alternating fashion, an effective or apparent directional hydraulic conductivity can be defined for the system. Thus, the layered porous medium can be represented by an anisotropic medium. The effective horizontal and vertical conductivities of the equivalent anisotropic medium are defined
Table 4-3. Non-dimensional input and output parameters for horizontal stratification simulations.

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as:

\[ \kappa_x = \sum_i \kappa_i d_i \]
\[ \kappa_z = \frac{D}{\sum_i \kappa_i d_i} \]

where \( D \) is the total depth of the system, \( d_i \) is each individual layer's thickness, and \( \kappa_i \) corresponds to a layer's permeability (or hydraulic conductivity). Some of the horizontal stratification simulations were designed to correspond to anisotropic simulations through the use of these effective conductivities.

An effective horizontal and vertical permeability was calculated for each simulation, and the Rayleigh number was defined using one overall effective permeability, \( \kappa_{\text{eff}} = (\kappa_x \kappa_z)^{0.5} \), where \( \kappa_x \) and \( \kappa_z \) are defined above. The effective permeabilities were also used to determine the appropriate scaling factor used to define the length of the salt nose. The horizontally stratified system can be represented by an equivalent anisotropic system, which can in turn be represented by an equivalent isotropic system as was shown in the previous section. This methodology was used to determine the appropriate scaling factor to define the salt nose length term, \( L_0 \). The salt nose length term, \( L_0^* \), is the same as for the anisotropic cases, since no coordinate transformations were required to define the equivalent anisotropic system for the stratification.

Figures 4-12, 4-13, and 4-14 exhibit the flux velocity and flow patterns for simulations 1, 2, and 3. Each of the systems have approximately the same Rayleigh number, or potential for free convection. The same boundary conditions were applied for simulations 1, 2, and 3, but different layering patterns were incorporated. As shown in Table 4-3, the number of layers used in each simulation were 2, 5, and 10, respectively. The system in simulation 1 was composed of 2 layers of equal thickness. The upper layer is the most conductive and appears to be the layer through which most flow occurs,
Figure 4-12. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 1.
evident in the velocity vectors shown in Figure 4-12c. The flux velocity distribution (Figure 4-12a) is of the same shape as in previous simulations with a relatively narrow discharge zone. The hydraulic head distribution (Figure 4-12b) illustrates that flow is predominantly horizontal between the hinge line and divide. The salt nose has developed in both the high and low conductivity layers (Figure 4-12d) and migrated a distance of \( L_0/L = 0.1861 \) toward the basin divide.

When the system is composed of five thinner layers of equal thickness and the same alternating conductivities as those in simulation 1, the resulting flow conditions change and affect the migration of the salt nose. The top, middle, and bottom layers are the high permeability layers. Because simulation 2 was designed with an odd number of layers, the effective permeabilities vary slightly from those in simulation 1, which produces a slightly higher Rayleigh number for the system. Based on the relationship between the Rayleigh number and the length of the salt nose that has been evident thus far, one would predict that the salt nose would migrate further toward the basin margin than it did in simulation 1 due to the higher Rayleigh number. Figure 4-13 illustrates that this did not occur. Even though the flux velocity distribution (Figure 4-13a) appears identical to that in simulation 1, the hydraulic head and velocity distributions are quite different (Figure 4-13b,c). The velocity vectors show that the high-conductivity layers transport most of the flow, but it is more difficult for the flow to reach these zones because it must first cross one or two low-permeable zones. The pressures have built up more because the recharge fluid must travel through several less-permeable layers near the top of the aquifer. The higher hydraulic head values reflect this flow restriction. The salt nose only migrated \( L_0/L = 0.1528 \) toward the basin margin. It is evident that the bottom layer, which is highly conductive, restricts the migration of the salt nose. That is, the advective force of the recharge velocity impedes the buoyancy force of the salt nose. In simulation 1, low velocities occurred throughout the lower half of the aquifer and did not have the same
Figure 4-13. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 2.
advective force to impede the movement of the salt nose. It is the combination of all these factors that caused the salt nose migration to be impeded more in the five-layer case than in the two-layer case.

Results from simulation 3 are consistent with the previous two simulations in that the advective force increased with the increase in the number of layers. Figure 4-14a presents the results for the ten-layer system. The layers are of equal thickness, and the top layer is a high-permeability layer, as are the third, fifth, seventh, and ninth layers. As the layers decrease in thickness from the previous two cases, the amount of flow that moves through to the bottom portion of the aquifer increases. The hydraulic head values (Figure 4-14b) increase because the flow has to move across more low-permeable layers. The salt nose is impeded even more and migrates only $L_0/L=0.0555$ toward the basin margin. Since the salt nose is smaller, it does not restrict the flow in the discharge zone as much, allowing the discharge to occur over a broader zone. This same correlation between the salt nose length and the shape of the discharge zone was seen in the previous two sets of simulations.

Three more simulations were performed to determine if the resulting flow patterns were affected by the strength of the buoyancy force, represented by the value of the Rayleigh number. The concentration of the ephemeral lake was perturbed to raise the Rayleigh number to 4024 for simulations 4, 5, and 6. All other boundary conditions remained the same as in the first three simulations and the same three layering patterns were used. Results from these three simulations are presented in Figures 4-15, 4-16, and 4-17. The same trends are evident as in the first three simulations. That is, as the number of layers increases, the flow distribution becomes more uniform throughout the depth of the aquifer. As the flow becomes more uniform the advective force near the bottom of the aquifer increases, and the salt nose migration is impeded. The salt nose length decreased from $L_0/L=0.4889$ to $0.4399$ to $0.3467$ for the two-, five-, and ten-layer
Figure 4-14. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 3.
Figure 4-15. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 4.
Figure 4-16. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 5.
Figure 4-17. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 6.
systems, respectively. The hydraulic head values also increased, representing a build-up of pressure from flow crossing the low permeable layers.

The discharge zone did not become more broad as the number of layers increased, as occurred in the first three simulations. In the previous three cases, the motion of the salt nose was not as extensive due to a smaller perturbation in the buoyancy. For the higher Rayleigh number cases, the salt nose has migrated almost all the way across the basin and restricts the flow, keeping the discharge zone fairly narrow.

The relationship between the Rayleigh number and salt nose length for the eight horizontally stratified systems studied is presented in Figure 4-18. The relationship is not linear when all of the layering patterns are considered. The best results are produced when the five- and ten-layer systems were used. This is consistent with the assumption on which the methodology was based. It is stated in the literature that effective permea-

![Figure 4-18. Rayleigh number vs. L0* for simulations of horizontal stratification.](image)
bilities can be defined for a system composed of "infinitesimally thin" regularly alternating layers (Maasland 1957). The results from the numerical experiments show that as the number of layers is increased, the Rayleigh number becomes a more accurate predictor for the length of the salt nose. The layers do not have to be infinitesimally thin, though, for the effective permeabilities to be of use. It is shown that the Rayleigh number can be used to accurately predict the salt nose length for the five-layer systems, which are relatively coarse when considering that the total depth of an aquifer may be as much as a kilometer.

Results from simulations 7 and 8 may be compared with results from two anisotropic simulations, simulations 11 and 12. Simulations 7 and 8 both represent a of ten-layered system, but all other design parameters differ. The permeability structure of the horizontally stratified cases were defined such that the effective horizontal and vertical permeabilities would correspond to the anisotropic permeabilities in simulations 11 and 12. This comparison was made to determine if an equivalent anisotropic system could be used to represent periodic stratification in order to predict basin-scale circulation patterns in closed basins. This simplification would reduce the amount of field data required to define the permeability structure of the aquifer, since only basin-scale effective permeabilities would be needed rather than data for each individual layer.

Figure 4-19 exhibits the relationship between the Rayleigh number and salt nose length, $L_0^*$, for the four simulations. It is evident that for the higher Rayleigh number case, the equivalent anisotropic system could be used to represent the stratification and would predict the same salt nose length. The results for the lower Rayleigh number case were not as good. The equivalent anisotropic system underestimated the distance that the salt nose migrated.
The flux and pressure distributions for the horizontal stratification simulations 7 and 8 are presented in Figures 4-20 and 4-21. The equivalent anisotropic system should not be used to predict the local pressure and velocity distributions for a stratified aquifer. If one compares these distributions in Figure 4-21 and Figure 4-10, it is evident that the pressures and velocities vary due to the stratification. The same is true for the lower Rayleigh number case, exhibited in Figures 4-20 and 4-6. Even though the local pressure and velocity fields may not be able to be predicted using this methodology, the Rayleigh number may be used to predict the basin-scale solute circulation patterns in a closed basin.
Figure 4-20. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 7.
Figure 4-21. The steady-state distribution of (a) recharge-discharge, (b) dimensionless hydraulic head, (c) normalized velocity field, and (d) solute concentration for horizontal stratification simulation 8.
Effect of initial condition

Several simulations were performed to determine if the specified initial conditions affect the simulation results. It is possible for the initial conditions to influence the steady-state results for such non-linear systems. Six simulations were performed to determine the effect of varying initial conditions. Figure 4-22a illustrates that the simulations were performed successively, with the results of simulation 1 used as the initial condition for simulation 2 and so on. The first three simulations might represent step changes in the climate that would reduce the salinity on the playa. The concentration on the playa was the only boundary condition that was perturbed to decrease the Rayleigh number from 4567.2 for simulation 1 to 1522.4 for simulation 3. The last three simulations were designed to represent step changes in climate that would increase the evaporation rates and thus increase salt deposition on the playa.

Figure 4-22b exhibits the temporal patterns of the total mass balance and the location of the salt nose. The length of the salt nose for a given set of boundary conditions was independent of the initial state of the aquifer. Figure 4-23 is another illustration of this same result and shows the path of the simulations performed. The exact same salt nose length resulted for a given Rayleigh number whether the playa was initially more or less saline.
Figure 4-22a. Specified concentration boundary condition and Rayleigh number for successive steady-state simulations.

Figure 4-22b. Salt nose length and total mass balance for successive steady-state simulations.
Figure 4-23. Rayleigh number vs. $L_0^*$ for successive steady-state simulations.

**Anomalies**

There were some simulations in which the salt nose did not develop at all when the Rayleigh number predicted that it would. These results will be examined in this section, and ideas are proposed to explain the cause of the anomalous results. There were two outliers in Figure 4-6, which represents the results from the asymmetric full-basin simulations. These outliers are both from simulation 5, representing the salt nose length for each side of the basin. The Rayleigh number in simulation 5 was 1661.7, for which a salt nose should have developed based on the relationship between the Rayleigh number and $L_0^*$ for evident in the other simulations. However, the concentration specified on the playa was only $c^*=0.02$ g/g, which is the lowest ephemeral lake concentration applied in
any of the anisotropic simulations. The Rayleigh number appears to be within a critical zone for which the development of the salt nose is unpredictable. When the Rayleigh number is this low, the development of the salt nose is unstable and more dependent on exact boundary conditions. Whereas if the Rayleigh number were higher, the salt nose would migrate the same distance regardless of each individual boundary condition.

One of the anisotropic simulations produced a similar outlier due to the absence of a salt nose, as was shown in Figure 4-11. The result is from simulation 4 (Ra=1558.9). The boundary condition on the playa specified a very low concentration, $c^*=0.01 \text{ g/g}$, which is the lowest concentration specified for all twelve simulations studying the effects of anisotropy. It appears that this Rayleigh number is again within a region that can produce unstable results. The development of the salt nose seems to be dependent on the specified ephemeral lake concentration when the Rayleigh number is in this range.

The results from the horizontal stratification simulations produced an outlier, as was shown in Figure 4-18. However, the reason the salt nose didn't develop is not related to the Rayleigh number being in a critical or unstable zone. The result is from simulation 9, which is a two-layered aquifer with the more permeable layer on the bottom. It was shown in the previous section that the Rayleigh number can not be used to predict basin-scale circulation patterns for systems that are defined by only two layers. One of the reasons is illustrated in this simulation. When there are only two layers, most of the flow is transported through the more permeable layer. When this layer is on the bottom, the salt nose is totally impeded by the advective force of the mountain recharge travelling through this lower layer. Therefore, when such a coarse layering pattern is applied, the resulting circulation patterns are very dependent upon the location of the more permeable layer. Simulation 10 was for the same system except the more permeable layer was on top, which allowed the salt nose to develop and migrate in the lower layer.

A linear regression was performed on the resulting Rayleigh numbers and salt nose length values for all of the simulations. The anomolous results were not included in this
analysis. The resulting regression equations and correlation coefficients are presented below. The values used for y were the log of the Rayleigh numbers, and the values used for x were the log of the salt nose length, $L_0^*$. 

Table 4-4. Linear regression results for all types of simulations.

<table>
<thead>
<tr>
<th>Simulation type</th>
<th>Regression equation</th>
<th>$r^2$</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous/isotropic</td>
<td>$y=-3.34+1.233x$</td>
<td>.946</td>
<td>30</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>$y=2.7258+.76821x$</td>
<td>.991</td>
<td>10</td>
</tr>
<tr>
<td>Anisotropy</td>
<td>$y=2.694+.8159x$</td>
<td>.864</td>
<td>11</td>
</tr>
<tr>
<td>Horizontal Stratification</td>
<td>$y=2.6149+.87057x$</td>
<td>.984</td>
<td>6</td>
</tr>
<tr>
<td>(5 and 10 layer cases only)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER V

CONCLUSIONS

Four groups of numerical experiments were performed to determine the effects of basin asymmetry, anisotropy, horizontal stratification, and initial condition on basin-scale groundwater flow and solute transport patterns. The conclusions stated here are based on the results presented in the previous chapter.

**Asymmetry**

1. Asymmetry in the mean recharge rates for mountain ranges bounding a closed basin produces an asymmetric salt nose. The salt nose migrates further toward the side of the basin with the lower mean recharge rate.
2. A Rayleigh number can be defined independently for each side of a closed basin and can be used to predict the extent of the salt nose migration on each side of the basin.

**Anisotropy**

1. An equivalent isotropic system (EIS) can be defined for an anisotropic aquifer in order to evaluate basin-scale circulation patterns.
2. A Rayleigh number and salt nose length term, $L_0^*$, can be defined for an anisotropic system such that the same linear relationship defined by Duffy and Al-Hassan and the asymmetric basin simulations can be applied. Therefore, the Rayleigh
number can be used to predict the basin-scale circulation patterns in closed basins containing an anisotropic porous medium.

**Horizontal stratification**

1. Effective horizontal and vertical permeabilities can be defined for a horizontally stratified system that can be used to define an equivalent anisotropic system.
2. A Rayleigh number and salt nose length term can be defined for a horizontally stratified system such that they are linearly related in the same manner as in all previous simulations.
3. As the number of layers increases, the advective force near the bottom of the aquifer increases and the salt nose migration is impeded.
4. The coarse two-layer structure is not a fine enough layering pattern to be able to use the Rayleigh number for prediction purposes. A five-layer system is adequate.
5. The equivalent anisotropic system can not be used to predict the local velocity and pressure fields for a stratified system.

**Overall**

The following conclusions are based on observations from all of the numerical experiments performed in this study.

1. As the salt nose length increases the mountain recharge is restricted, causing the hydraulic head values in the recharge region to increase.
2. As the salt nose length increases the discharge zone on the playa becomes narrower, and the location of the $h^* = D/L = 0.1$ contour moves beneath the playa surface and toward the basin margin.
3. As the ratio \( \kappa_x / \kappa_z \) increases, the shape of the hydraulic head contours are elongated in the horizontal direction, and the discharge zone on the playa becomes wider.

4. As the effective permeability of the aquifer decreases, the hydraulic head values increase.

5. A Rayleigh number can be defined for each of the systems considered that can be used to predict the length of the salt nose, which represents the basin-scale circulation pattern. Therefore, with relatively little subsurface data one could predict the steady-state subsurface flow and transport patterns. Results from this study would allow one to predict the effects of asymmetric mountain ranges, the degree of anisotropy, and the effects of the dominant layering structure in the basin.
CHAPTER VI

RECOMMENDATIONS FOR FUTURE RESEARCH

Additional field data and numerical experiments are necessary in order to represent the physical system in a more realistic manner. It is recommended that this study be extended to learn more about the response of an aquifer to transient climatic forcing. It is also necessary to determine if the dispersion process is dependent upon flow direction in closed basins and to define this relationship for different types of sediments. Field data are crucial for this type of a study. Different periodic layering patterns should also be investigated to determine if a Rayleigh number could be applied.

There are many studies that could be performed that would increase our understanding of the hydrological processes that occur in closed basins. For example, there may be scaling properties that could be determined based on geologic structure, the distribution of plant types, or drainage patterns to help define the appropriate length scales for a basin ($\sigma, L_,-\sigma, \eta$). Also, the relationship between the degree of salinity in a closed basin and the altitude of the basin should be explored. If the studies focus on specific basins or smaller-scale circulation patterns, the boundary conditions should also become more specific. For example, perhaps the unsaturated zone, subsurface baseflow from adjacent basins, or spatially varying recharge or evaporation might be incorporated. Overall, it is recommended that numerical experiments play an important role along with field experiments in studies of the hydrology of closed basins.
REFERENCES


Donahue, Dave. 1987. Personal communication with Dr. Christopher Duffy.


PROGRAM TEST

This program generates the dimensionless recharge distribution with the incomplete beta function.

REAL GAMMLN, BETAI, BETACF, X(150), Y(150), dx(150)
INTEGER JJ(150)
OPEN(1, FILE = 'TEST.DAT', STATUS = 'NEW')
open(2, file='rechx.dat', status='old')
open(4, file='ebar.in', status='new')

The values of emax and eavg are determined by the user (meter/sec used here)

emax=1.0255e-08
eavg=3.1688e-09
write(*,*)'enter # of rech nodes,dx of 1st elem, and # of rows'
read(*,*)ib, dxl, nr
write(*,*)'enter sigma(length of recharge zone)'
read(*,*)sigma
write(*,*)'enter parameters a and b for incomp. beta fn.'
read(*,*)a,b
x(l)=0.
do i=1,ib
   x(i)=x(i-1)+.0125
   read(2,*) fake, x(i)
enddo
dx(l)=dxl/2.
do i=2,ib
   dx(i)=x(i)-x(i-1)
enddo
DO I = -1,IB
   N = i*nr
   bx=x(i)/sigma
   Y(I) = (1. - BETAI(A,B,bx)) *emax/eavg
   Y(I)=Y(I)*1000.*dx(i)*1.
   IF(I .EQ. 1) Y(I) = Y(I)/2.
   C = 0.0003
   write(4,*) Y(i), dx(i)
   WRITE(1,10) N, Y(I), C
ENDDO

10 FORMAT(I10,2F15.10)
STOP
END

FUNCTION BETAI(A,B,X)
C****RETURNS THE INCOMPLETE BETA FUNCTION /X(A,B)
IF(X .EQ. 0. OR. X .EQ. 1.)THEN
   BT = 0.
ELSE
   BT = EXP(GAMMLN(A+B) - GAMMLN(A) - GAMMLN(B) + A*ALOG(X) + B*ALOG(1.-X))
ENDIF

IF(X .LT. (A+1.)/(A+B+2.))THEN
   BETAI = BT*BETACF(A,B,X)/A
   RETURN
ELSE
   BETAI = 1. - BT*BETACF(B,A,1.-X)/B
RETURN
FUNCTION BETACF(A,B,X)
C***CONTINUED FRACTION FOR INCOMPLETE BETA FUNCTION, USED BY BETAI***
PARAMETER (ITMAX=100,EPS=1.E-7)
AM = 1.
BM = 1.
AZ = 1.
QAB = A+B
QAP = A+1.
QAM = A-1.
BZ = 1. - QAB*X/QAP
DO 11 M = 1,ITMAX
   EM = M
   TEM = EM + EM
   D = EM*(B-M)*X/((QAM+TEM)*(A+TEM))
   AP = AZ + D*AM
   BP = BZ + D*BM
   D = -(A+EM)*(QAB+EM)*X/((A+TEM)*(QAP+TEM))
   APP = AP + D*AZ
   BPP = BP + D*BZ
   AOLD = AZ
   AM = AP/BPP
   BM = BP/BPP
   AZ = APP/BPP
   BZ = 1.
   IF(ABS(AZ-AOLD) .LT. EPS*ABS(AZ))GO TO 1
11 CONTINUE
PAUSE 'A OR B TOO BIG, OR ITMAX TOO SMALL'
1 BETACF = AZ
RETURN
END

FUNCTION GAMMLN(XX)
C**RETURNS THE VALUE OF LN(GAMMA(XX) FOR XX > 0.
REAL COF(6)
DATA COF,STP/76.18009173D0,-86.50532033D0,24.01409822D0,-1.231739516D0,.120858003D-2,-.536382D-5,2.50662827465D0/
DATA HALF,ONE,FPF/0.5D0,1.OD0,5.5D0/
X = XX - ONE
TMP = X + FPF
TMP = (X+HALF)*LOG(TMP)-TMP
SER = ONE
DO 11 J = 1,6
   X = X + ONE
   SER = SER + COF(J)/X
11 CONTINUE
GAMMLN = TMP + LOG(STP*SER)
RETURN
END