Groundwater Flow Systems and Thermal Regimes Near Cooling Igneous Plutons: Influence of Surface Topography

Mark U. Birch
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GROUNDWATER FLOW SYSTEMS AND THERMAL REGIMES NEAR COOLING IGNEOUS PLUTONS: INFLUENCE OF SURFACE TOPOGRAPHY

by

Mark U. Birch

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Geology

UTAH STATE UNIVERSITY
Logan, Utah
1989
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Mark U. Birch
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<td>A</td>
<td>Area (m²)</td>
</tr>
<tr>
<td>a</td>
<td>Horizontal dimension of simulated domain (m)</td>
</tr>
<tr>
<td>aₜ</td>
<td>Transverse dispersivity (m)</td>
</tr>
<tr>
<td>aₗ</td>
<td>Longitudinal dispersivity (m)</td>
</tr>
<tr>
<td>b</td>
<td>Vertical dimension of simulated domain (m)</td>
</tr>
<tr>
<td>Cᵢ</td>
<td>Specific heat of fluid (J kg⁻¹ °C⁻¹)</td>
</tr>
<tr>
<td>Cₛ</td>
<td>Specific heat of solid medium (J kg⁻¹ °C⁻¹)</td>
</tr>
<tr>
<td>Dᵣ</td>
<td>Fluid conduction-dispersion tensor (W m⁻¹ °C⁻¹)</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity (m s⁻²)</td>
</tr>
<tr>
<td>h</td>
<td>Equivalent freshwater head (m)</td>
</tr>
<tr>
<td>Hₛ</td>
<td>Basal heat flux (mW m²)</td>
</tr>
<tr>
<td>hᵢ</td>
<td>Initial equivalent freshwater head prior to pluton injection (m)</td>
</tr>
<tr>
<td>kₛ</td>
<td>Permeability of Basal unit (m²)</td>
</tr>
<tr>
<td>kᵢᵢ</td>
<td>Permeability tensor (m²)</td>
</tr>
<tr>
<td>kᵤ</td>
<td>Permeability of Upper unit (m²)</td>
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<tr>
<td>L</td>
<td>Characteristic length of elements within the finite element mesh (m)</td>
</tr>
<tr>
<td>Lᵣₐ</td>
<td>Characteristic length scale of the hydrothermal system (m)</td>
</tr>
<tr>
<td>P</td>
<td>Fluid pressure (Pa)</td>
</tr>
<tr>
<td>Pe</td>
<td>Grid peclet number (dimensionless)</td>
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<tr>
<td>qᵢ</td>
<td>Fluid flux vector (m s⁻¹)</td>
</tr>
<tr>
<td>q̄</td>
<td>Total magnitude of fluid flux (m s⁻¹)</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number (dimensionless)</td>
</tr>
<tr>
<td>T</td>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>Target</td>
<td>Specified Target %T (dimensionless)</td>
</tr>
<tr>
<td>Tᵢ</td>
<td>Initial temperature (°C)</td>
</tr>
<tr>
<td>Tₛ</td>
<td>Surface thermal lapse rate (°C km⁻¹)</td>
</tr>
<tr>
<td>Tₒ</td>
<td>Temperature applied to boundary (°C)</td>
</tr>
<tr>
<td>Tol</td>
<td>Specified %T allowed during simulations (dimensionless)</td>
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$T_p$  Initial pluton temperature ($^\circ$C)
$T_r$  Reference surface temperature ($^\circ$C)
$T_1$  Temperature applied to upper or left boundary ($^\circ$C)
$T_2$  Temperature applied to lower or right boundary ($^\circ$C)
$z_x$  Elevation of the water table (m)
$\alpha$  Thermal diffusivity ($m^2 s^{-1}$)
$\Delta H$  Latent heat of crystallization (J kg$^{-1}$)
$\Delta t$  Time interval (sec)
$\Delta T_H$  Additional temperature used to account for latent heat ($^\circ$C)
$\theta$  Time weighting factor (dimensionless)
$\eta$  Porosity (dimensionless)
$\lambda_{ij}^*$  Effective thermal conductivity tensor (W m$^{-1}$ $^\circ$C$^{-1}$)
$\lambda^f$  Thermal conductivity of fluid (W m$^{-1}$ $^\circ$C$^{-1}$)
$\lambda^s$  Thermal conductivity of solid medium (W m$^{-1}$ $^\circ$C$^{-1}$)
$\mu$  Fluid dynamic viscosity (Pa s)
$\rho_f$  Fluid density (kg m$^{-3}$)
$\rho_0$  Reference fluid density (kg m$^{-3}$)
$\rho_r$  Relative fluid density (kg m$^{-3}$)
$\rho_s$  Density of solid medium (kg m$^{-3}$)
$\sigma$  Fluid thermal expansivity ($^\circ$C$^{-1}$)
ABSTRACT

Groundwater Flow Systems and Thermal Regimes Near Cooling Igneous Plutons: Influence of Surface Topography

by

Mark U. Birch, Master of Science
Utah State University, 1989

Major Professor: Dr. Craig B. Forster
Department: Geology

Previous studies of cooling igneous plutons did not consider the possible influence of sloping surface topography. Topographically-driven fluids in high relief terrain, however, are thought to interact with deep buoyancy-driven fluids to produce large lateral-flow systems up to 5 km long and 20 km long in silicic and andesitic volcanic terrain, respectively. In this study, a quantitative investigation of the interaction of topographically-driven and buoyancy-driven fluid flow is conducted through the use of a finite element numerical model to simulate the fluid flow and thermal regimes associated with a cooling igneous pluton in the presence of significant topographic relief. The system considered in this study is that of a pluton with dimensions 2 km by 3 km and an initial temperature of 980 °C centered beneath a mountain having relief of 1 km over a horizontal distance of 3 km. Simulation results indicate that the topographic component of flow interacts with buoyancy to produce two separate flow systems, a shallow topographically-driven flow system and a deeper convecting system. The resulting hydrothermal system evolves in a more complicated fashion than in flat topography cases. In addition, the existence of the shallow topographically-driven flow system partially masks the presence of the heat source by preventing fluids having the chemical signature of the deeper, hotter environment from reaching the surface. Cooling rate of the pluton is also
increased and boiling is inhibited. These effects, however, are primarily a result of the pluton being injected into a cooler host rock. The host rock is cooler in the sloping topography case due to advective cooling prior to pluton injection. Model results also indicate that temperature beneath the mountain and the position of the zone of mixing remain relatively constant for almost 50,000 years. The stability of the temperature conditions and the position of the zone of mixing may increase the likelihood for the deposition of epithermal ore bodies in this region. (98 pages)
INTRODUCTION

Hydrothermal systems produced by the emplacement of hot igneous plutons into the Earth’s upper crust have long been of interest, in part due to their assumed role in the genesis of ore deposits. Heat produced by a cooling igneous pluton drives fluid convection which, in turn, transports ore constituents and influences the thermal regime. Previous numerical studies (e.g. Cathles, 1977; Norton and Knight, 1977; Torrance and Sheu, 1978; Hardee, 1982; and Carrigan, 1986) have contributed to the understanding of the evolution of the patterns of fluid flow and temperature distributions near cooling igneous plutons under simplified conditions. Recent workers, however, have recognized that the current understanding of the hydrology of ore-depositing hydrothermal systems is inadequate in many respects and that further investigation is warranted (Berger and Bethke, 1985; Rye, 1985).

One aspect that has received little attention is the possible influence of surface topography on the fluid flow and thermal regimes of convecting hydrothermal systems. The interaction of topographically-driven fluid flow with deeper convecting fluids, however, is thought to produce extensive lateral-flow systems, such as that depicted in Figure 1, adapted from Henley and Ellis (1983). Large lateral-flow systems, for example, are inferred to exist near the andesitic volcanos of El Tatio, Chile and Ahuachapan, El Salvador (Healy and Hochstein, 1973). Evidence also suggests that lateral-flow systems exist at Wairakei, New Zealand (Healy and Hochstein, 1973) and Mokai, New Zealand (Henley, 1985). Henley (1985) further argues that such systems are common and may extend over 5 km in silicic volcanic terrain and over 20 km in andesitic volcanic terrain.

Hanoaka (1980) used a numerical approach to investigate the influence of surface topography on a steady-state, freely-convecting hydrothermal system. His results suggest that topography and buoyancy interact to produce two flow systems, a shallow topographically-controlled flow system and a deep convective flow system. The patterns of fluid flow thus differ from the flat topography case, and as a result, the corresponding thermal regime differs markedly. More recently, Sammel et al. (1988) used a transient
numerical approach to model the fluid flow and thermal regimes of Newberry Volcano, Oregon. Simulation results obtained after 10,000 years of cooling of an inferred magma chamber at depth compare well with bore hole temperature data. The bore hole temperature data indicate that the shallow thermal regime beneath the caldera and the surrounding flanks of the volcano is cooler than might be expected. This phenomenon, often referred to as the Rain Curtain effect, results from vertical flow of cool meteoric fluids infiltrating into the higher topographic features of volcanic terrain. The Rain Curtain effect is commonly inferred to exist in the high Cascades and in the High Lava Plains northwest of Newberry Volcano (Blackwell et al., 1982).

Fig. 1. Conceptual model of a large lateral fluid flow system. Shaded area indicates region of lateral flow.

Steady-state numerical simulations are useful for studying the interaction of topographically-driven and buoyancy-driven fluid flow; however, a transient approach will provide more insight into the evolution of the hydrothermal systems caused by cooling plutons. Although Sammel et al. (1988) used a transient numerical approach, their primary
goal was to reproduce the current fluid flow and the thermal regimes thought to exist at Newberry Volcano. They did not investigate the evolution of the fluid flow and thermal regimes as the pluton cooled.

The overall objective of this thesis is to provide the foundation for a more in-depth study of the influence of surface topography on the evolution of the fluid flow and thermal regimes near cooling igneous plutons. To accomplish this goal, a steady-state numerical model of Forster and Smith (1988a) was modified and converted to a transient model to provide a quantitative basis for evaluating the influence of surface topography on a pluton-associated hydrothermal system under idealized conditions. A more in-depth study would include additional modeling and field verification of model results. Specific goals of this thesis include evaluating 1) the need to consider surface topography when studying pluton-associated hydrothermal systems, and 2) the possible impact of surface topography on the genesis of epithermal ore deposits.

The approach used to accomplish these specific goals and the results of this study are presented in this thesis as follows. The conceptual model is developed, followed by a description of the mathematical model and the numerical procedure used in solving the boundary value problem. Results of steady-state simulations are then presented to illustrate the basic concepts of convecting hydrothermal systems and to demonstrate the possible impact of sloping surface topography. The impact of sloping surface topography on the evolution of the pluton-associated hydrothermal systems is then evaluated by comparing simulation results of cooling plutons beneath sloping and flat surface topographies. Important aspects of the simulation results are discussed and the possible impacts for the genesis of epithermal ore deposits are explored. The final section summarizes the findings of the study and makes recommendations for further study.
MODELING APPROACH

Conceptual Model

The conceptual model used in this study is based on the ancient ore-forming hydrothermal system which once operated within the collapsed caldera structure at Creede, Colorado. At Creede, an igneous pluton at depth produced a convecting hydrothermal system while topographic relief, caused by the collapse of the caldera, set in motion a shallow groundwater flow system. The fluids of the shallow groundwater flow system are thought to have mixed with the deep hydrothermal fluids, resulting in ore deposition (Steven and Eaton, 1975). General features of the Creede hydrothermal system, adopted from Steven and Eaton (1975), are shown in Figure 2. The intention in this study is not to model the system at Creede specifically, but rather to use the general character and the scale of Creede as a real world example of a cooling igneous pluton in the presence of sloping surface topography.

Fig. 2. Conceptual model of ore deposition at Creede Colorado, adapted from Steven and Eaton (1975).
The conceptual model is based on a cross-section taken perpendicular to the rim of a hypothetical caldera. A pluton is assumed to be injected directly beneath the mountain peak and is likened to a large ring dike that has formed within the fracture zone created during caldera collapse. The rim of the caldera and the pluton are assumed to extend infinitely in a direction perpendicular to the plane of the modeled cross-section, which is a reasonable approximation for a caldera that is several tens of kilometers in circumference. Furthermore, mirror symmetry at the vertical boundaries is assumed, and identical systems are assumed to occur on each side of the modeled system in a periodic fashion.

Boundary Value Problem

The boundary value problem based on the conceptual model discussed above is shown in Figure 3. The domain comprises a cross-sectional area 7 km wide by 7 km deep with a mountain slope centered on the left boundary. The pluton, measuring 2 km wide by 3 km high, rests at a depth of 4 km below the valley floor and is also centered on the left boundary. The host rock in the upper 6 km of the system is considered to be permeable, due to fracturing, while the lower 1 km is assumed to be relatively impermeable basement rock. The pluton extends to the basal boundary, and in all cases is assumed be comprised of the same rock type as the surrounding host rock. The scale of the fractures in comparison to the scale of the system is considered sufficiently small such that the rock mass can be adequately represented as an equivalent porous medium. The assumption of symmetrically identical systems existing in a periodic fashion on each side of the modeled system allows the lateral boundaries to be specified as insulated and impermeable, because horizontal components of heat and fluid flow are non-existent along the lateral boundaries in this case.

Temperatures at the ground surface are assumed to be equivalent to near-surface atmospheric temperatures that remain constant throughout the cooling history of the pluton. The temperatures are described using a reference temperature \( T_0 \), defined at the valley floor elevation, together with a thermal lapse rate \( T_L \). An inherent assumption in defining a constant temperature at the ground surface is that the temperatures at shallow depths are
strongly influenced by atmospheric temperatures. This approach can be somewhat inhibiting because it cannot account for warming of the fluids at the water table, nor does it consider the possible existence of thermal springs (Smith and Chapman, 1983). In this study, however, near-surface heat fluxes are relatively low (< 0.7 W m\(^2\)) in comparison to many geothermal systems (1-2 W m\(^2\), Sass et al., 1981), and the area of elevated heat flux is restricted to a small region near the break in slope. The impact of specifying a constant temperature upper boundary condition on the solution obtained throughout the bulk of the modeled region is, therefore, thought to be insignificant.

\[ T = T_r + T_L z_x \]

\[ h = z_x \]

Fig. 3. Boundary value problem based on the conceptual model.
The thin layer of unconsolidated surficial deposits, which normally overlies the bedrock surface, is ignored, and the water table is assumed to lie everywhere at the bedrock surface. Precipitation rates are assumed to be sufficiently high to maintain the water table at the bedrock surface regardless of the bulk permeability. This is checked by comparing the calculated recharge rates to realistic rates and ensuring that they fall within reasonable limits.

Following the example of Cathles (1977), Parmentier (1981), and Parmentier and Schedl (1981), a constant basal heat flux boundary condition is chosen for this study. Others have used insulated (Norton and Knight, 1977) or isothermal basal boundaries (Giberti et al., 1984). Because heat generated by the hot igneous body is far greater than the regional heat flux over most of the cooling history, there is little difference between the insulated and constant flux cases except at later times when the heat generated by the pluton approaches that of the regional heat flux. The use of a constant heat flux boundary condition is, therefore, preferred. The choice of a constant basal heat flux boundary is also favored over that of an isothermal boundary because, based on field data, regional heat fluxes are easier to estimate than are temperatures at a specified depth (Forster and Smith, 1988a).

Temperature-pressure Properties of the Fluid

Many fluid properties vary as a function of pressure and temperature. In systems in which the temperatures do not exceed 250 °C, however, most of the properties of water remain relatively constant and thus can be regarded as fixed; an exception to this is fluid viscosity (μ) which changes approximately an order of magnitude from 0 to 250 °C. When temperatures exceed 250 °C, the fluid properties of concern in this study (ρ, μ, λ, and Cp) vary considerably, as shown in Figure 4. Notice also in Figure 4 the increasing effect of pressure on the fluid properties as temperatures increase. A sensitivity analysis conducted by Straus and Schubert (1977) demonstrates that the temperature-pressure properties of water significantly affect the thermal regimes and patterns of fluid flow, even in a system with lower temperature-pressure conditions than might be expected in plutonic systems. In
Fig. 4. Temperature-pressure properties of pure water.
this study, the temperature-pressure properties of water are, therefore, incorporated into the model.

Non-boiling temperature-pressure conditions must be maintained throughout the simulation because the numerical model does not account for phase changes or two-phase flow. Although this presents a significant modeling limitation, most geothermal systems are classified according to White et al. (1971) as "liquid dominated", so the assumption of a single-phase liquid hydrothermal system is not unreasonable. A pressure/enthalpy diagram (Figure 5) adapted from Sammel et al. (1988), illustrates the compressed, two-phase, and super-critical temperature-pressure regions of pure water. As long as the temperature-pressure conditions during the simulations remain outside of the two-phase region, phase changes will not occur. Conditions are allowed to change from super-critical to compressed or vice versa as long as the fluid temperature or pressure remains above its respective critical value (374 °C and 22.1 MPa) while passing between the two regions. Figure 5 also shows a curve of temperature versus depth of boiling for pure water, calculated using the formulation of Keenan et al. (1978). This curve illustrates the depth of boiling assuming cold water hydrostatic pressures. Cold water hydrostatic pressures are the pressures that result from the weight of an overlying isothermal column of water. In contrast, hot water hydrostatic pressures result when vertical temperature gradients are present. Vertical temperature gradients lead to vertical changes in fluid density, and thus the weight of the vertical fluid column will differ from the isothermal case. Notice in Figure 5 that boiling cannot occur at depths greater than 2.25 km under cold water hydrostatic conditions. Under hot water hydrostatic conditions (not shown), however, boiling may occur at somewhat greater depths since pressures at corresponding depths are slightly less for non-isothermal conditions than for isothermal conditions.

As suggested in the two previous paragraphs, the fluid within the modeled system is assumed to be pure water. This assumption is reasonable as a first approximation because the total salt concentrations measured in active geothermal systems are usually less than about 1 wt% NaCl, although concentrations can be as high as 30 wt% NaCl (Henley and
Fig. 5. Pressure/enthalpy diagram (A) and temperature versus depth of boiling for pure water (B).
Ellis, 1983). According to Henley and Ellis (1983), McNabb (1975) has suggested the possibility of dense brine in the root zones of geothermal systems. The source of the postulated brine is thought to be either magmatic fluids, the content of which may vary from 2 to 9 wt% of the magma (Burnham, 1979) with salinity of 3 to 4 wt % NaCl or greater (Whitney, 1975), or deep connate/metamorphic fluids. Although the possible existence of dense brine in the root zones of geothermal systems is not accounted for in this study, it should be considered in a complete study of ore genesis since this brine may be an important source of ore-forming minerals.

Properties of the Solid Medium

In this study, all properties of the medium \( (k, \lambda^*, \eta, C_v, \rho_v) \) are held constant through time. The thermal properties of the medium \( (\lambda^* \text{ and } C_v) \) vary somewhat with temperature and pressure. Giberti et al. (1984) compared numerical simulation results of a cooling pluton using variable medium parameters versus average medium parameters and found that average medium parameters represented the system well. Permeability of both the host rock and the pluton also varies throughout the cooling history of the pluton (Parmentier, 1981) due to episodes of fracturing caused by thermal and tectonic forces and chemical reactions. The fracture history associated with a cooling pluton, however, is poorly understood (Knapp and Norton, 1981). As a consequence, permeability is assumed to remain constant throughout the cooling history. Using this approach, the pluton is assumed to be permeable from the time of emplacement, which is obviously inaccurate since the pluton is molten during emplacement. Others have attempted to overcome this problem by assuming the pluton is impermeable until it has cooled below a specified temperature (e.g. Torrance and Sheu, 1978). Some evidence suggests that igneous rocks will fracture at temperatures slightly below their solidus temperatures (Norton and Taylor, 1979) which lends credibility to this approach. Other studies, however, indicate that plutons are partially molten only in the early stages of cooling (less than 10% of the cooling history, Norton and Knight, 1977), and therefore, the assumption of constant
permeability throughout the cooling history should not significantly affect model results.

**Pluton Emplacement**

The igneous pluton is assumed to be injected instantaneously into the host rock. This assumption seems reasonable because upward migration rates of the temperature maxima, based on numerical results of Norton and Knight (1977), are lower than the inferred rates of pluton injection. Shaw (1985) suggests that the average extrusion rate of silicic calderas is $10^7$ km$^3$ yr$^{-1}$ and that the ratio of intrusive volume to extrusive volume is approximately 10/1, which corresponds to an intrusive volume rate of $10^3$ km$^3$ yr$^{-1}$. A conservative estimate of the upward migration rate of the pluton considered in this study can be calculated, based on the intrusive volume rate suggested by Shaw, by assuming that the pluton extends a great distance in the direction perpendicular to the modeled cross-section. The volume of the pluton, in this case, would be large, and thus, the upward migration rate would be low. Assuming the pluton is 80 km long in the direction perpendicular to the modeled cross-section (other dimensions of the pluton are 2 km by 3 km), the upward rate of migration would be 6.25 cm yr$^{-1}$, over 3 times faster than the upward rate of migration of the temperature maxima calculated numerically by Norton and Knight (1977) (2 cm yr$^{-1}$).

The pluton is assumed to be injected at a constant temperature which is somewhat less than the liquidus temperature (Carrigan, 1986), plus an additional quantity ($\Delta T_H$) to account for latent heat of crystallization. Studies by Giberti et al. (1984) indicate that the latent heat of crystallization can best be accounted for with an increase in the initial temperature of the pluton. An alternative approach is to increase the specific heat of the pluton, as was done by Sammel et al. (1988). One disadvantage of using an increased temperature to account for latent heat is that the early history of the cooling pluton is poorly represented in the simulations. Assuming an exaggerated pluton temperature causes large thermal gradients at early time that, in turn, cause accelerated rates of migration of the thermal front. The advantage of using temperature to account for latent heat rather than specifying a higher specific heat, however, is that the later cooling history is better
represented, since crystallization of the pluton occurs early in the cooling history. A higher temperature assigned to the pluton at time zero makes available instantaneously all of the heat generated from crystallization. In contrast, assigning a higher specific heat to the pluton allows the release of the generated heat over the entire cooling history of the pluton, and thus does not account for the latent heat of crystallization as well.

Mathematical Model

The mathematical model consists of the coupled differential equations describing fluid flow and heat transfer, equations of state for fluid properties, and boundary and initial conditions for the thermal and fluid flow regimes. Fluid flow and heat transfer are coupled through the influence of temperature on the properties of the fluid. The differential equations are, thus, non-linear, and an iterative numerical procedure is required, as is discussed in a subsequent section. The following discussion of the mathematical model follows that of Smith and Chapman (1983) and Forster and Smith (1988a).

The steady-state equation for the conservation of fluid mass is:

\[
\frac{\partial}{\partial x} \left[ \rho_f \cdot q_x \right] + \frac{\partial}{\partial z} \left[ \rho_f \cdot q_z \right] = 0
\]  

(1)

where \( \rho_f \) is fluid density, and \( q_x, q_z \) are the horizontal and vertical components of fluid flux, respectively.

Fluid flux is given as:

\[
q_i = -\frac{k_{ij}}{\mu} \left[ \rho_o g \frac{\partial h}{\partial x_j} + (\rho_f - \rho_o) g \frac{\partial z}{\partial x_j} \right]
\]  

(2)

where \( \mu \) is viscosity, \( k_{ij} \) is the permeability tensor, \( \rho_o \) is a reference fluid density defined at a specified temperature, \( g \) is gravity, and \( h \) is the equivalent freshwater head defined as:

\[
q = \frac{P}{\rho_o g} + z
\]  

(3)
where $P$ is fluid pressure and $z$ is elevation. Using equivalent freshwater head, rather than fluid pressure, reduces the possibility of numerical inaccuracies that may result when calculating gradients using large numbers (pressures) with small differences as opposed to smaller numbers (equivalent freshwater heads) with relatively large differences (Frind, 1982).

Using a relative density defined as:

$$\rho_r = \frac{\rho_f}{\rho_o} - 1$$

(4)

Equation 2 can be rewritten in a simpler form:

$$q_i = -k_{ij} \frac{\rho_o g}{\mu} \left[ \frac{\partial h}{\partial x_j} + \rho_r \frac{\partial z}{\partial x_j} \right]$$

(5)

The equation governing steady-state fluid flow through a porous medium is obtained by substituting Equation 5 into Equation 1 and is written:

$$\frac{\partial}{\partial x} \left[ k_{xx} \rho_f \frac{\rho_o g}{\mu} \frac{\partial h}{\partial x} + k_{xz} \rho_f \frac{\rho_o g}{\mu} \frac{\partial h}{\partial z} \right] + \frac{\partial}{\partial z} \left[ k_{xz} \rho_f \frac{\rho_o g}{\mu} \frac{\partial h}{\partial x} + k_{zz} \rho_f \frac{\rho_o g}{\mu} \left( \frac{\partial h}{\partial z} + \rho_r \right) \right] = 0$$

(6)

The numerical expression for the fluid flow boundary conditions are written mathematically as:

$$\frac{\partial h}{\partial z} = 0 \quad \text{for } 0 \leq x \leq -7 \text{ km}$$

at $z = -7$ km

(7a)

$$\frac{\partial h}{\partial x} = 0 \quad \text{for } -7 \text{ km} \leq z \leq z_x$$

at $x = 0$ and $x = 7$ km

(7b)

$$h = z_x \quad \text{for } 0 \leq x \leq 7 \text{ km}$$

at $z = z_x$

(7c)
where \( z_a \) is the elevation of the water table, which coincides with the ground surface.

The transient equation governing heat transfer in a porous saturated media is expressed as:

\[
\frac{\partial}{\partial x} \left[ \lambda_{xx}^e \frac{\partial T}{\partial x} + \lambda_{xz}^e \frac{\partial T}{\partial z} \right] + \frac{\partial}{\partial z} \left[ \lambda_{zx}^e \frac{\partial T}{\partial x} + \lambda_{zz}^e \frac{\partial T}{\partial z} \right] - \rho_f C_f \left( q_x \frac{\partial T}{\partial x} + q_z \frac{\partial T}{\partial z} \right) = \left( \rho_f C_f \eta + \rho_s C_s (\eta - 1) \right) \frac{\partial T}{\partial t}
\]

where \( \lambda_{ij}^e \) is the effective thermal conductivity tensor for the fluid-solid composite, \( C_r \) is specific heat of fluid, \( T \) is temperature, \( \rho_s \) is density of the solid, \( C_s \) is specific heat of the solid, and \( \eta \) is porosity. \( \lambda_{ij}^e \) can be expressed as the geometric mean of the fluid and media thermal conductivities (Woodside and Messmer, 1961) as:

\[
\lambda_{ij}^e = D_{ij}^{(\eta)} \cdot \lambda_{ij}^s (^{1-\eta})
\]

where \( D_{ij} \) is the conduction-dispersion tensor for the fluid, and \( \lambda_{ij}^s \) is the thermal conductivity tensor for the solid. \( D_{ij} \) can be expanded for an isotropic media (Sauty et al., 1982) and expressed as:

\[
D_{xx} = \frac{\rho_f C_f}{\eta} \left( a_L q_x^2 / \bar{q} + a_T q_z^2 / \bar{q} \right) + \lambda^f \tag{10a}
\]

\[
D_{zz} = \frac{\rho_f C_f}{\eta} \left( a_T q_x^2 / \bar{q} + a_L q_z^2 / \bar{q} \right) + \lambda^f \tag{10b}
\]

\[
D_{xz} = D_{zx} = \frac{\rho_f C_f}{\eta} \left( a_L - a_T \right) q_x q_z / \bar{q} \tag{10c}
\]

where \( a_L \) is the longitudinal dispersivity, \( a_T \) is the transverse dispersivity, \( \bar{q} \) is the total specific discharge, and \( \lambda^f \) is the thermal conductivity of fluid.

The numerical expression of the thermal boundary conditions are written as:

\[
\frac{\partial T}{\partial x} = 0 \quad \text{for } -7 \text{ km} < z < z_x \quad \text{at } x = 0 \text{ and } x = 7 \text{ km}
\]

\[
\lambda_{zz}^e \frac{\partial T}{\partial z} = H_b \quad \text{for } 0 < x < 7 \text{ km} \quad \text{at } z = -7 \text{ km}
\]
for $0 < x < 7$ km at $z = z_x$

where $T_r$ is the reference temperature, defined at the valley floor elevation, $T_L$ is the thermal lapse rate, and $H_b$ is the basal heat flux.

The equations of state for the fluid include equations describing $\rho$, $\mu$, $\lambda'$, and $C_r$ as functions of temperature and pressure. In all cases, the fluid is assumed to be pure water as discussed earlier. The fluid density relationship is adopted from Myer et al. (1967) for temperatures below the critical temperature and from Keenan et al. (1978) for temperatures above the critical temperature. The fluid viscosity relationship is adopted from Watson et al. (1980), the thermal conductivity relationship from Kestin (1978), and the specific heat from Keenan et al. (1978). The above equations were coded and supplied by C.W. Mase of the University of Waterloo.

The final components needed to complete the mathematical model are the initial conditions. The transient simulation begins at time zero when the anomalous temperature of the pluton is superimposed onto the pre-existing thermal regime. The initial conditions are represented mathematically as:

\[ T = T_i(x,z) \text{ for } x > 1 \text{ km or } z > -4 \text{ km} \]

\[ T = T_{pi} \text{ for } 0 \leq x \leq 1 \text{ km and } -7 \text{ km} \leq x \leq -4 \text{ km} \]

\[ h = h_i(x,z) \]

where $h_i(x,z)$ and $T_i(x,z)$ describe the fluid flow and thermal regime prior to pluton injection, and $T_{pi}$ is the initial temperature of the pluton.
Numerical Method

The mathematical problem is solved numerically. The cross-sectional region is first discretized into triangular elements and then a Galerkin-based finite element method is used to solve the coupled equations of heat and fluid flow. Details of the formulation are discussed in Appendix A. A transient simulation involves two steps; 1) the initial temperature and head distributions are obtained, and 2) the transient problem is initiated. The initial temperature and pressure distributions are obtained through an iterative process described as follows. First, the steady-state form of Equation 8 is solved to provide an initial temperature distribution required to begin the iterative process. These temperatures, combined with an initial hydrostatic pressure distribution, are used to calculate the temperature- and pressure-dependent properties of water which are then used in Equation 6 to solve for the equivalent freshwater head, and subsequently, the fluid velocity field. The computed fluid velocities and fluid properties \((\rho, \mu, \lambda)\) are then used in solving the steady-state form of Equation 8 to update the temperature distribution. This procedure continues until the temperature difference between successive iterations is below some specified tolerance level (1 °C in this study). Less than seven iterations are normally required in this process. The results define the initial temperature distribution and the initial pattern and magnitude of fluid flow that exists prior to pluton emplacement.

The transient simulation is initiated by specifying the pluton temperature within the pre-emplacement hydrothermal system and solving Equations 6 and 8 for successive time steps. An iterative procedure, identical to that used in solving for the steady state initial conditions, is conducted at each time step using the transient form of Equation 8 instead of its steady-state equivalent. The time-stepping procedure is continued until 1) a specified time is attained, 2) a specified number of time steps has been exceeded, or 3) a final steady-state condition is obtained.

Time steps are calculated automatically using the method presented in Appendix A. The size of the time steps required to obtain an accurate solution is determined by repeating a simulation a number of times, each time using smaller time steps. The size of
the time steps required for accurate results is judged to be adequate once identical solutions are obtained between successive runs.

The cross-section is discretized into triangular elements using the mesh generator developed by Forster and Smith (1988a). The accuracy of the mesh is estimated by solving the isothermal fluid flow problem and computing the balance of fluid flux across the top surface and the balance of fluid flux across all the boundaries combined. For sloping surfaces, the flux calculated normal to the surface may be inaccurate if the mesh is too coarse. The mesh is considered adequate if the flux balance across the upper surface is within 3% and the total flux balance is within 5%. A sample mesh is presented in Figure 6.

![Sample finite element mesh](image)

Fig. 6. Sample finite element mesh.

Mesh peclet numbers provide another useful means of determining how finely discretized the modeled region must be in order to produce accurate solutions. The mesh
Peclet number is defined as:

\[ Pe = \frac{C_p \rho_f \bar{v} L}{\lambda_s} \]  \hspace{1cm} (13)

where \( L \) is the characteristic length of a triangular element. Huyakorn and Pinder (1983) state that accurate results for transport problems can be obtained when mesh peclet numbers are less than 2.0. Mesh peclet numbers greater than 2.0 indicate that the parameter of interest (\( T \)) is changing too rapidly within the region represented by the triangular element to be adequately represented by a linear basis function. One solution to this problem is to decrease the size of the individual triangular elements by more finely discretizing the domain. Peclet numbers are plotted as part of the simulation output to insure that they do not exceed 2.0.
Both steady-state and transient simulations were conducted in this study. Results of steady-state simulations are presented primarily to illustrate specific characteristics of convecting hydrothermal systems and how they may be affected by surface topography. Steady-state results may also provide insight into quasi-steady-state systems that result from periodic pluton injection or convective overturn of magma (Henley and Ellis, 1983). The longevity of many geothermal systems (over millions of years, Silberman et al., 1979) suggests that quasi-steady-state conditions may be common. Cathles (1981), however, suggests that an "average" sized pluton may cool in as little as 10,000 to 100,000 years in the absence of magma reinjection. As a consequence, many hydrothermal systems may be better represented as transient events.

The transient behavior of pluton-associated hydrothermal systems is the focus of this study overall, and is specifically addressed in the second half of this section. In each approach (steady-state and transient), average media parameters taken from the literature are assumed, the values of which are presented in Table 1 along with other model parameters. Parameter values are chosen assuming that the basal unit is granitic and the upper unit is composed of volcanic and sedimentary rocks. In all cases, the properties of the pluton are taken to be identical with the properties of the intruded host rock. The values of $\lambda^*$ and $C_\alpha$ are chosen assuming that the average temperature of the system throughout the cooling history of the pluton is roughly 400 °C. This method of determining appropriate rock parameters follows that of Giberti et al. (1984). Giberti et al. note that since the properties of rock change little with respect to temperature and pressure, a rough estimate of the average temperature is adequate.

Steady-State Numerical Results

Two series of steady-state simulations were conducted. The first series illustrates specific characteristics of hydrothermal systems associated with igneous plutons in the absence of sloping surface topography. The second series illustrates the influence of
surface topography on pluton-associated hydrothermal systems. A constant pluton
temperature of 350 °C and a regional heat flux of 60 mW m$^{-2}$ are chosen for the
steady-state simulations. A regional heat flux of 60 mW m$^{-2}$ is less than might be expected
in volcanic terrane (Sass et al., 1981). Preliminary simulations conducted with higher
pluton temperatures and basal heat fluxes, however, indicate a large region of boiling,
which cannot be accounted for by the numerical model.

**TABLE 1. Simulation Parameters**

<table>
<thead>
<tr>
<th>Thermal Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_f$, Fluid reference density @ 25 °C</td>
<td>998.2 kg m$^{-3}$</td>
</tr>
<tr>
<td>$T_r$, Reference surface temperature</td>
<td>10 °C</td>
</tr>
<tr>
<td>$T_s$, Surface thermal lapse rate</td>
<td>7.5 °C km$^{-1}$</td>
</tr>
<tr>
<td>$H_b$, Basal heat flux (Steady-state simulations)</td>
<td>60 mW m$^{-2}$</td>
</tr>
<tr>
<td>$H_b$, Basal heat flux (Transient simulations)</td>
<td>90 mW m$^{-2}$</td>
</tr>
<tr>
<td>$a_l$, Longitudinal dispersivity</td>
<td>100.0 m</td>
</tr>
<tr>
<td>$a_t$, Transverse dispersivity</td>
<td>10.0 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Media Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Unit</td>
<td></td>
</tr>
<tr>
<td>$k_u$, Permeability</td>
<td>10$^{-18}$ m$^2$</td>
</tr>
<tr>
<td>$\lambda'$, Thermal conductivity</td>
<td>2.0 W m$^{-1}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$\rho_u$, Density</td>
<td>2,700 kg m$^{-3}$</td>
</tr>
<tr>
<td>$C_u$, Specific heat</td>
<td>950 J kg$^{-1}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$\eta_u$, Porosity</td>
<td>0.01</td>
</tr>
<tr>
<td>Lower Unit</td>
<td></td>
</tr>
<tr>
<td>$k_u$, Permeability (Steady-state simulations)</td>
<td>10$^{-18}$ to 10$^{-16}$ m$^2$</td>
</tr>
<tr>
<td>$k_u$, Permeability (Transient simulations)</td>
<td>2 × 10$^{-16}$ m$^2$</td>
</tr>
<tr>
<td>$\lambda'$, Thermal conductivity</td>
<td>1.8 W m$^{-1}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$\rho_u$, Density</td>
<td>2,700 kg m$^{-3}$</td>
</tr>
<tr>
<td>$C_u$, Specific heat</td>
<td>1000 J kg$^{-1}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$\eta_u$, Porosity</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Convection Near an Igneous Pluton**

Figures 7-9 show simulation results of a pluton-associated hydrothermal system
with permeability of the upper unit ($k_u$) ranging from 10$^{-18}$ m$^2$ to 10$^{-16}$ m$^2$ (Recall that in all
cases, the pluton is assumed to have the same properties as the encompassing host rock).
The permeability of the lower unit ($k_u$) is maintained at 10$^{-18}$ m$^2$ for all simulations. Fluid
motion is illustrated by vectors representing the direction and magnitude of fluid flow. The length of the vectors is logarithmically scaled with respect to the maximum fluid flux rate occurring in each case. In later simulations, the vectors are scaled with respect to a specified fluid flux rate so that comparisons between the various cases can be made. Fluid fluxes that are three orders of magnitude or less than the maximum fluid flux rate (or the specified fluid flux rate for later simulations) are plotted as "+".

Figure 7 illustrates a system with \( k_s \) equal to \( 10^{-18} \) m². Note that a convection cell develops as buoyancy forces drive fluids upward near the pluton while being replaced by fluids from the side. Fluid convection is often quantified through the use of a calculated Rayleigh number \( (Ra) \), which relates buoyancy forces to viscous forces. For this geometry, \( Ra \) is defined by Sorey (1978) as:

\[
Ra = \frac{\sigma g k (T_1 - T_0)L_{Ra}}{(\lambda/\rho_o C_p)(\mu/\rho_o)}
\]

where \( \sigma \) is thermal expansivity of water, \( T_1 \) and \( T_0 \) are the lower and upper boundary temperatures respectively, and \( L_{Ra} \) is the characteristic length scale taken as the distance between the upper and lower boundaries. In this study, \( Ra \) is not used to quantify steady-state or transient results. In the steady-state case, \( Ra \) is not used because the choice of appropriate parameter values in Equation 14 is complicated by the relatively complex geometry and boundary conditions of the system being studied. Also, \( Ra \) cannot be calculated for transient systems because the parameters in Equation 14 change through time.

Although fluid convection exists in the case shown in Figure 7, the thermal regime is perturbed only slightly. The solid isotherms represent the thermal regime that would result from conductive heat transfer alone, while the dotted isotherms represent the thermal regime that results from conductive and convective heat transfer. Note that the dotted isotherms, however, are slightly down-warped on the down-flow side of the convection cell. This illustrates that downward-flowing fluids are transporting enough heat to slightly perturb the otherwise conductive thermal regime in this region. The thermal regime is
controlled predominantly by conduction because the fluid flow rates are low, the maximum throughout the system being $7.9 \times 10^{-12}$ m s$^{-1}$.

Fig. 7. Steady-state pluton-associated hydrothermal system with $k_u$ equal to $10^{18}$ m$^2$. Solid and dotted isotherms represent the purely conductive thermal regime and the combined conductive and convective thermal regime, respectively. Isotherm contour interval is 50 °C. Vectors are logarithmically scaled to the maximum fluid flux rate of $7.93 \times 10^{12}$ m s$^{-1}$. Solid dot represents the center of convection.

Figure 8 illustrates a system with $k_u$ equal to $10^{11}$ m$^2$. Note that the center of convection, indicated by the solid dot, is somewhat closer to the ground surface than in the case shown in Figure 7. The center of convection occurs closer to the surface because the fluid flux rates are greater (maximum of $7.61 \times 10^{11}$ m s$^{-1}$) as a result of the increased permeability. With increased permeability, greater quantities of heat are transferred upwards and as a consequence, the net buoyancy force moves upwards, corresponding with the higher temperatures at shallow depth.

The thermal regime shown in Figure 8 is slightly more perturbed than the case shown in Figure 7 because the fluid flux rates have increased. Cool downward-flowing fluids on the down-flow side of the convection cell cool off the surrounding medium while
upward flowing fluids warm the region above the pluton. The maximum fluid flux rate of $7.61 \times 10^{11} \text{ m s}^{-1}$ is close to the threshold values suggested by Norton and Knight (1977) and Forster and Smith (1988b), $10^{10} \text{ m s}^{-1}$ and $10^{11} \text{ m s}^{-1}$ respectively, which causes a significant perturbation of the conductive thermal regime.

![Diagram](image)

Fig. 8. Steady-state pluton-associated hydrothermal system with $k_a$ equal to $10^{-17} \text{ m}^2$. Solid and dotted isotherms represent the purely conductive thermal regime and the combined conductive and convective thermal regime, respectively. Isotherm contour interval is 50 °C. Vectors are logarithmically scaled to the maximum fluid flux rate of $7.61 \times 10^{11} \text{ m s}^{-1}$. Solid dot represents the center of convection.

Smith and Chapman (1983) note that the transition between a conductively dominated system and an advectively or convectively (advection and convection are defined as hydraulically- and buoyancy-driven fluid flow, respectively) dominated system is sharp, occurring over about a magnitude change in permeability. If $k_a$ is increased from $10^{-17} \text{ m}^2$ (Figure 8) to $10^{-16} \text{ m}^2$ (Figure 9) the thermal regime becomes strongly influenced by fluid flow, demonstrating the sharp transition between conductively and advectively or convectively dominated systems reported by Smith and Chapman. In this case, the
maximum fluid flux rate is $1.57 \times 10^9$ m s$^{-1}$, which is above the fluid flux threshold values suggested by Norton and Knight (1977) and Forster and Smith (1988b). The isotherms are severely up-warped on the up-flow side of the convection cell and correspondingly down-warped on the down-flow side. Note also, that the center of convection has again moved upwards within the domain.

Fig. 9. Steady-state pluton-associated hydrothermal system with $k_s$ equal to $10^{-16}$ m$^2$. Solid and dotted isotherms represent the purely conductive thermal regime and the combined conductive and convective thermal regime, respectively. Isotherm contour interval is 50 °C. Vectors are logarithmically scaled to the maximum fluid flux rate of $1.57 \times 10^9$ m s$^{-1}$. Solid dot represents the center of convection.

Simulations presented in Figures 7-9 illustrate the basic characteristics of convecting hydrothermal systems. For systems having low media permeabilities, heat transport is primarily by conduction. The characteristic convective motion of the fluid, however, may still exist in such systems. With increasing permeability, the thermal regime is increasingly influenced by convective heat transfer. Once the system's specific permeability threshold is exceeded, the character of the thermal regime is strongly influenced by fluid flow.
Influence of Surface Topography

It has long been recognized that topographically-driven fluid flow can significantly alter conductive thermal regimes. Topography also alters convecting hydrothermal systems by introducing another fluid flow driving force to the system. Early numerical studies by Elder (1967) indicate that free convection cells can be modified or obliterated by introducing an external component of fluid flow such as that caused by sloping surface topography. Numerical results of Forster and Smith (1988b) also indicate that free convection in mountainous terrain, in the absence of a localized heat source (i.e. pluton), may not exist unless special conditions are present, such as a low permeability horizon separating the free convection cell from topographically-driven flow. A numerical study by Hanoaka (1980), however, indicates that under higher heat flow conditions (regional heat flow approximately 200 to 380 mW m\(^2\) as compared to Forster and Smith’s 120 mW m\(^2\)) free convection cells can exist in the presence of significant topographic relief. The thermal regime and the patterns of fluid flow, however, are significantly influenced by the interaction between the topographic and buoyancy components of fluid flow.

In this section, the interaction of the two fluid flow driving forces is evaluated by varying the topographic relief on a system identical to that shown in Figure 9. The flat topography case shown in Figure 9 is repeated in Figure 10A and is used as a base case against which to compare the patterns of fluid flow and the thermal regimes of the sloping topography cases. This exercise differs from that of Hanoaka (1980) in three main respects: first, the heat source (pluton) is explicitly defined within the modeled region; second, constant basal heat flux is assumed as opposed to constant basal temperature, and third, fluid properties are assumed to vary with temperature and pressure.

Figure 10 shows the fluid flow and thermal regimes for a series of simulated systems having various degrees of sloping surface topography. The left-most plot (Figure 10A) illustrates the flat topography base case whereas the right-most plot (Figure 10D) illustrates the steepest sloping case considered in this study, having vertical relief of 1 km in a horizontal distance of 3 km. The plots between Figure 10A and 10D have topographic
Fig. 10. Steady-state pluton-associated hydrothermal systems with varying surface topographic relief: (A) flat topography base case, (B) 0.3 km, (C) 0.6 km, and (D) 1.0 km relief in a horizontal distance of 3.0 km. Profiles A1-D1 and A2-D2 represent surface heat and fluid flux, respectively. Isotherm contour interval is 50 °C. All vectors are logarithmically scaled to the fluid flux rate of $1.57 \times 10^9$ m s$^{-1}$, which corresponds to a length of 0.9 cm. Dashed lines indicate the interface between the two flow systems. Solid dots represent the center of convection.
relief intermediate between the two extremes. Shown above each plot of the hydrothermal system are histograms of the fluid recharge/discharge profile (Figures 10A2-10D2) and the surface heat flux profile (Figures 10A1-10D1). The recharge/discharge profiles illustrate the spatial distribution of the recharge and discharge areas and the mass fluid flux in kg sec\(^{-1}\) m\(^{-2}\) normal to the surface. The surface heat flux profiles depict the spatial distribution and the magnitude of heat flux in W m\(^{-2}\) normal to the surface.

As in Hanoaka's (1980) study, the two driving mechanisms, topography and buoyancy, interact to produce two flow systems. With increasing topographic relief, the topographically-driven flow system forces the center of convection deeper into the domain. The center of convection in the steepest topography case (Figure 10D) is approximately 1 km deeper than in the flat topography case (Figure 10A). In addition, the center of convection is shifted laterally slightly.

The character and position of the interface between the two flow systems also changes with increasing topographic relief. For the gently sloping system depicted in Figure 10B (0.3 km relief in a horizontal distance of 3 km), the interface (dashed line) occurs about 1 km beneath the mountain, slopes upwards towards the valley and intersects the valley floor just to the right of the break-in-slope. As the slope is increased (0.6 km in 3 km, Figure 10C), the interface is forced deeper below the mountain and intersects the valley floor farther out in the valley floor. Further increase in the slope (Figure 10D) results in the interface intersecting the right lateral boundary, thus isolating the buoyancy-driven fluids from the ground surface.

The chemical character of the topographically-driven fluids is likely to differ from that of the buoyancy-driven fluids due to their contrasting temperature-pressure histories, origins, and ages. The intersection of the interface with the ground surface thus marks a point of transition between fluids of different chemistry. The possible existence of such a chemical transition of near-surface fluids provides motivation for collecting isotope samples in active hydrothermal systems to help delineate patterns of flow. In addition, isotope data obtained from fluid inclusions may help delineate patterns of flow in paleo-hydrothermal
systems. Note in Figure 10 that with increasing topographic relief, the discharge of buoyancy-driven fluids occurs progressively farther out in the valley floor or may not reach the ground surface at all (Figure 10D). The chemical character of shallow groundwater above the heat source may thus not indicate the presence of the hotter environment at depth, assuming the rate of chemical diffusion across the interface is not large.

The maximum mass fluid flux across the surface, \(4.2 \times 10^{-7}\) kg sec\(^{-1}\) m\(^{-2}\), occurs in the steepest topography case (Figure 10D). This corresponds to a maximum recharge rate of about 1.3 cm yr\(^{-1}\), which is low for even arid conditions. The assumption that the water table exists at the ground surface is, thus, reasonable since precipitation available for recharge is most likely more than adequate to maintain the water table at the ground surface, even after accounting for evapotranspiration and surface run off.

The recharge/discharge profiles (Figures 10A2-10D2) reveal that with increasing slope, the profile becomes similar to the profile of a non-plutonic hydrothermal system (Figure 11). The system shown in Figure 11, with the exception of the absence of a pluton, is identical to that shown in Figure 10D. Note also that the patterns of fluid flow at shallow depth are very similar in Figures 10D and 11. The character of the recharge/discharge profile and the patterns of flow at shallow depth shown in Figure 10d, illustrate how steep topography can mask the existence of the hot buoyancy-driven flow system caused by the pluton.

With increasing slope, the maximum fluid flux within the simulated domain decreases from \(1.57 \times 10^{-9}\) m s\(^{-1}\) for the flat topography case to \(9.2 \times 10^{-10}\) m s\(^{-1}\) for the steepest sloping case. Note that the vectors in Figures 10A-10D are scaled to a fluid flux rate of \(1.57 \times 10^{-9}\) m s\(^{-1}\) for purposes of comparing flux rates between the various cases. Not only is the maximum fluid flux rate reduced as a result of increasing slope, the fluid fluxes throughout the domain are substantially influenced, particularly between the mountain and the underlying pluton. Figure 12 illustrates the influence of sloping topography on the fluid fluxes throughout the system. The plots in Figure 12 were constructed by contouring the differences between the fluid fluxes of the sloping
Fig. 11. Steady-state non-pluton-associated hydrothermal system with topographic relief of 1 km in a horizontal distance of 3 km. Profiles (A) and (B) represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $1.57 \times 10^9$ m s$^{-1}$, which corresponds with a length of 0.9 cm.

Topography cases and the flat topography base case. This enables one to clearly see the magnitudes and the locations of fluid flux differences between each sloping case and the flat topography base case. Figure 12C, which shows the fluid flux differences between the steepest topography case (Figure 10D) and the flat topography case, for example, shows that the fluid flux in the region directly between the pluton and the overlying mountain decreases by as much as a factor of 10. In the region near the break-in-slope, the flux rate is as much 3.2 times greater than in the flat topography case. The region between the
Fig. 12. Fluid flux differences between the steady-state sloping topography cases and the flat topography base case: (A) 0.3 km, (B) 0.6 km, and (C) 1.0 km relief in a horizontal distance of 3 km. Contours represent the factor of the fluid flux differences of the sloping topography cases relative to the flat topography base case.
break-in-slope extending down towards the side of the pluton, however, is the only region where fluid flux is greater than in the flat topography case.

The reduced fluid flow rates between the pluton and the overlying mountain, illustrated in Figure 12, are caused by topographically-induced hydraulic gradients which drive fluid downwards, opposite the direction of the buoyancy component of flow. The net force driving fluid flow is thus reduced. The increased flow rates near the break-in-slope result from the complementary interaction of the two driving mechanisms: both the topographically-induced hydraulic gradients and the buoyancy force are directed upwards in this region. Lower fluid fluxes exist throughout the remaining region because the rate of rotation of the convection cell is reduced by the downward hydraulic component of flow acting on the up-flow side of the convection cell.

The thermal regimes presented in Figures 10B-10D illustrate how topographically-driven fluid flow alters the otherwise convectively and conductively controlled thermal regime of a flat topography system (Figure 10A). The thermal regime of the sloping topography cases differs from that of the flat topography case primarily for two reasons. First, the patterns of fluid flow differ, as seen in comparing Figures 10B-10D with Figure 10A. Second, the rates of fluid flow in the region between the mountain slope and the pluton, are significantly lower in the sloping topography cases than in the flat topography case (Figure 12). The change in the pattern of fluid flow results in cool recharge fluids migrating downward below the mountain and cooling off the surrounding medium. As the fluids migrate laterally, heat is absorbed and transported towards the discharge area, warming the region below the valley floor. The fluid flux rates between the mountain and the pluton for the sloping cases are as much 10 times lower than for the flat topography base case (Figures 12C). The reduced fluid flow rates above the pluton reduce the amount of heat transferred vertically away from the pluton. The combined effect of the change in the patterns of fluid flow and the reduced convective cooling of the pluton is that the region below the mountain becomes significantly cooler and the region below the valley floor becomes warmer than in the flat topography case.
The change in the character of the thermal regime with increasing topographic relief is further illustrated by the change in the surface heat flux profiles shown in Figures 10A1-10D1. For the flat topography case, the surface heat flux is maximum above the pluton, coinciding with the maximum fluid discharge. Farther to the right, recharging fluids transport heat downwards, opposite the direction of conductive heat transfer, and thus the net upward heat flux is reduced. Over the entire ground surface, the net heat flux is directed upwards, because the upward component of conductive heat transport is greater than the amount of heat transported by the fluids. With increasing slope, the effect of the changing recharge/discharge profiles (Figures 10A2-10D2) is illustrated by the corresponding change of the heat flux profiles (Figures 10A1-10D1). Note that as the slope increases, the heat flux profile appears more like that of the non-pluton-associated hydrothermal system (Figure 11). This illustrates further how surface topography can mask the presence of the hotter environment at depth. Note that the overall surface heat flux is somewhat greater for the pluton-associated system (Figure 10D) than for the non-pluton-associated system (Figure 11), as is indicated by the area between the heat flux profile and the zero axis in each case. This greater surface heat flux, however, could easily be misinterpreted as resulting from a higher regional heat flux, rather than from a local intrusion.

Transient Cooling of an Igneous Pluton

The steady-state simulations presented in the previous section illustrate the characteristics of convecting hydrothermal systems and the possible impact of surface topography on quasi-steady-state systems. In this section, the impact of surface topography on the evolution of the patterns of fluid flow and the thermal regime associated with a single intrusive event is evaluated by simulating the transient cooling of an igneous pluton. For the transient simulations, the pluton is assumed to be injected at 700 °C plus an additional 280 °C (where ΔH = 2.7 × 10^2 J kg⁻¹, Norton and Cathles (1979)) to account for the latent heat of crystallization. In addition, a higher basal heat flux of 90 mW m⁻² is
used to reflect conditions that are more likely to exist in volcanic terrain. \( k_u \) is also increased to \( 2 \times 10^{-16} \) m\(^2\) to more closely reflect the value thought to exist in most hydrothermal systems. The steady-state simulations discussed previously illustrate how topography induces a component of fluid flow which interacts with buoyancy to produce two separate flow systems. In transient systems, however, the buoyancy component of flow will change through time as the temperatures change. The patterns and rates of fluid flow will, thus, change through time. The transient interaction of the two fluid flow driving mechanisms is evaluated by simulating the cooling history of an igneous pluton beneath sloping surface topography of 1 km relief in a horizontal distance of 3 km. The results of this simulation are compared to the results obtained for a similar system with flat surface topography in order to evaluate the impact of surface topography on the evolving fluid flow and thermal regimes.

**Initial Conditions**

Figure 13A shows the thermal regime of the flat topography case prior to injection of the pluton. In this case, the fluid is assumed to be stagnant, thus heat is transferred purely by conduction. Figure 13B shows the fluid flow and thermal regimes of the sloping topography case prior to pluton injection. In this case, the thermal regime is perturbed due to topographically-driven fluid flow; the isotherms are down-warped below the mountain and correspondingly up-warped beneath the valley floor. Note that the vectors in this figure and in all the figures in this series are scaled to the maximum fluid flux rate \((2.86 \times 10^8 \) m\(^3\)) that occurs in the flat topography case after 1,000 years of cooling (Figure 14A). Note also that the region in which the pluton is to be injected is noticeably cooler than in the flat topography case.

**Cooling History**

Figures 14A and 14B show the fluid flow and thermal regimes obtained for the flat and sloping topography systems after 1,000 years of cooling. Near the pluton little difference can be seen in the thermal regimes. Farther away from the pluton, however, the
Fig. 13. Initial fluid flow and thermal regimes: (A) flat topography case and (B) sloping topography case. Profiles B1 and B2 represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C.
Fig. 14. Fluid flow and thermal regimes after 1,000 years of cooling: (A) flat topography case and (B) sloping topography case. Profiles A1-B1 and A2-B2 represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^{-4} \text{ m s}^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dots represent the centers of convection.
thermal regimes maintain the differences that existed prior to pluton injection (Figure 13). Two flow systems have developed in the sloping topography case of Figure 14B. The interface (dashed line) between the topographically-driven and buoyancy-driven flow systems occurs below the mountain and curves downwards around the pluton to intersect the lower boundary. Note that the topographically-driven flow system in the sloping topography case forces the center of convection to form at a greater depth than in the flat topography case.

In both cases, surface heat and fluid flux have changed little after 1,000 years of cooling, as seen by comparing the histograms of the surface heat and fluid fluxes in Figures 13 and 14. The fluid flux rates within the domains of the two cases are similar because the net buoyancy driving force is located deep within the domain where topographically-induced hydraulic gradients are low. As the thermal plume (defined as the region with temperatures above ambient temperatures) migrates towards the surface with time, however, the influence of topography on the fluid flux rates will become more noticeable.

An additional 4,000 years (total of 5,000 years) of cooling amplifies the difference between the thermal regimes of the two systems in the vicinity of the pluton (Figures 15A and 15B). The average pluton temperature after 5,000 years of cooling is about 50 °C cooler for the sloping topography case. The primary reason for the accelerated cooling rate is that the pluton was injected into a significantly cooler host rock, and thus, the rate of conductive cooling is greater. The thermal plume also advances at a slower rate, due primarily to the greater amount of heat being absorbed by the cooler media. The slight lateral shift of the upper portion of the plume results from the larger lateral component of fluid flow directed along the interface between the two flow systems. An additional reason for the slower advance of the thermal plume is that the fluid flux rates above the pluton are beginning to be significantly influenced by the presence of the surface topography. Differences shown in Figure 16A indicates that fluid flux rates below the mountain are reduced by as much as 3.2 times that of the flat topography case after 5,000 years of
Fig. 15. Fluid flow and thermal regimes after 5,000 years of cooling: (A) flat topography case and (B) sloping topography case. Profiles A1-B1 and A2-B2 represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m $s^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dots represent the centers of convection.
Fig. 16. Fluid flux differences between the flat and sloping topography cases: (A) after 5,000 and (B) 15,000 years of cooling. Contours represent the factor of the fluid flux differences of the sloping topography case relative to the flat topography case.
cooling. Note that fluid fluxes throughout most of the domain, especially between the pluton and the overlying mountain, are lower for the sloping topography case, a phenomenon which was also noted in the previous steady-state simulations.

Note the bimodal pattern of the temperature maxima in Figure 15B. The deeper temperature maximum exists within the less permeable basal unit because heat is transported laterally away from the pluton primarily by conduction within this unit, whereas heat is transported towards the pluton by convection in the overlying more permeable unit.

In the time period between 1,000 and 5,000 years, the interface moves farther away from the pluton as the convection cell expands. The shape of the interface, however, remains unchanged. As the convection cell expands, the center of convection migrates upwards. The presence of the upper topographically-driven flow system, however, slows the rate of upward migration of the center of convection in comparison with the flat topography case (Figure 15A and 15B). After 5,000 years of cooling, the center of convection in the sloping topography case is 700 m below that of the flat topography case.

The recharge/discharge profile of the flat topography case after 5,000 years (Figure 15A) shows that the maximum rate of discharge of fluid across the surface has significantly increased, from 0.0 kg sec\(^{-1}\) m\(^{2}\) initially, since the system was stagnant, to \(2.7 \times 10^{-7}\) kg sec\(^{-1}\) m\(^{2}\). Correspondingly, the maximum surface heat flux has also increased somewhat from about 90 mW m\(^{-1}\) to 130 mW m\(^{-1}\). Figure 15B shows, however, that the maximum surface fluid flux and heat flux have changed little in 5,000 years for the sloping topography case, indicating that the presence of the topographically-driven flow system is damping out the surface effects of the buoyancy-driven flow system.

After 15,000 years of cooling, boiling occurred at a depth of about 1,300 m in the flat topography case (Figure 17A) and the simulation was terminated. Simulation of the sloping topography case, however, was continued beyond 15,000 years. The flat topography simulation was terminated once steam was detected at 6 nodes of the finite element mesh. Boiling at fewer than 6 nodes was allowed because the area of boiling was thought to be too small to cause significant error in the calculations.
Fig. 17. Fluid flow and thermal regimes after 15,000 years of cooling: (A) flat topography case and (B) sloping topography case. Profiles A1-B1 and A2-B2 represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dots represent the centers of convection. Shaded area indicates the region of boiling.
After 15,000 years, the average pluton temperature is 75 °C cooler than in the flat topography case. In addition, the vertical position of the thermal plume for the sloping topography case is significantly deeper and is shifted farther to the right (Figures 17A, 17B). The upward migration rate of the thermal plume, which earlier was reduced by heat adsorption of the cooler host rock near the pluton, is now influenced more by the opposing topographically-induced hydraulic gradients. Figure 16B shows that the fluid flux rates below the mountain are as much as 100 times less than those of the flat topography case. Vertical heat transport is, thus reduced substantially in this region.

In the interval between 5,000 and 15,000 years, the interface has migrated up through the system to intersect the surface near the break-in-slope, similar to the behavior of the interface shown in the steady-state simulations (Figures 10B and 10C). The maximum surface fluid discharge rate after 15,000 years has increased in the flat topography case to \(5.94 \times 10^{-7}\) kg sec\(^{-1}\) m\(^{-2}\), and the maximum surface heat flux has increased to 495 mW m\(^{-2}\). The surface heat and fluid flux for the sloping topography case, however, have undergone less dramatic changes, as the shallow topographically-driven flow system damps out the surface effects of free convection. Note however, the slight increase in discharge near the break-in-slope, while recharge occurs farther out in the valley floor as a result of the interface intersecting the ground surface. Also as a consequence of the interface intersecting the ground surface, the buoyancy-driven fluids containing the signature of the hotter, deeper environment are allowed to discharge at the surface.

The center of convection continues to move upwards until 20,000 years when it is about 2,700 m below valley elevation, 500 m deeper than the center of convection was in the flat topography case at the onset of boiling (15,000 years). At about 22,000 years (Figure 18), minor boiling occurs at a depth of about 1,900 m, but only in a limited area (maximum of 2 nodes). The error associated with neglecting two-phase flow and the phase change is thought to be minimal and the simulation is continued. Note also in Figure 18 that a secondary convection cell has developed in the lower right hand region due to the interaction of the larger convection cell with the right lateral boundary.
Fig. 18. Fluid flow and thermal regimes after 22,000 years: sloping topography case. Profiles (A) and (B) represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dot represents the center of convection. Shaded area indicates the region of boiling.
Although the center of convection begins to recede after 20,000 years, the maximum surface heat flux of 600 mW m\(^{-2}\), corresponding with the maximum fluid discharge rate at the break-in-slope of \(8.5 \times 10^{-7}\) kg sec\(^{-1}\) m\(^{-2}\), does not occur until 40,000 years (Figure 19). In addition, the temperatures at depth have decreased substantially between 20,000 and 40,000 years. This suggests that by the time the surface effect of the pluton is observed, the exploitable thermal anomaly may have diminished substantially. The interface intersects the ground surface until about 45,000 years of cooling at which time it dives down to intersect the right lateral boundary. Between 15,000 years, when the interface first intersects the ground surface and 45,000 years, the pattern of fluid flow throughout the system changes little (Figures 17B, 18 and 19). Note that beneath the mountain the temperatures and the position of the interface remain relatively constant over this period of 30,000 years. The stability of the temperature-pressure environments and the patterns of fluid flow may have important implications for ore deposition, as will be discussed in a subsequent section.

After 50,000 years (Figure 20), the pluton has cooled substantially, and the center of convection has moved downwards to a depth of 3,900 m below the valley floor. Temperatures below the mountain are reduced as cooler topographically-driven fluids move deeper into the system with diminishing resistance from the upward buoyancy forces. Note however, the surface heat and fluid fluxes remain relatively high. At 100,000 years (Figure 21), the center of convection has dropped an additional 600 m, and the temperatures below the mountain continue to fall. The surface heat and fluid fluxes are also beginning to decrease substantially. At 200,000 years the convection cell is obliterated and the entire system is dominated by topographically-driven flow. Temperatures below the mountain continue to fall while the temperatures below the valley correspondingly begin to increase as fluids absorb heat and transport it towards the discharge area in the valley. This process continues until the thermal regime has completely reversed and steady-state conditions, identical to the initial steady-state conditions (Figure 13B), are obtained after 2 million years of cooling.
Fig. 19. Fluid flow and thermal regimes after 40,000 years: sloping topography case. Profiles (A) and (B) represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dot represents the center of convection.
Fig. 20. Fluid flow and thermal regimes after 50,000 years: sloping topography case. Profiles (A) and (B) represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^8$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dot represents the center of convection.
Fig. 21. Fluid flow and thermal regimes after 100,000 years: sloping topography case. Profiles (A) and (B) represent surface heat and fluid flux, respectively. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Dashed line indicates the interface between the two flow systems. Solid dot represents the center of convection.
DISCUSSION OF NUMERICAL RESULTS

The transient simulations presented in the previous section indicate that sloping surface topography complicates the fluid flow and thermal regimes near a cooling pluton by providing an additional fluid flow driving mechanism. In the absence of topographic relief, buoyancy is the only mechanism driving fluid flow and the classic convection cell develops as demonstrated by numerous workers (e.g. Cathles, 1977, 1981; Norton and Knight, 1977). If the permeability of the host rock is sufficiently high, fluid convection will redistribute heat within the system to produce the characteristic mushroom-shaped thermal regimes shown in Figures 14A, 15A, and 17A. High temperature contrasts between the rising thermal plume and the overlying medium generate a substantial density contrast that induces a large buoyancy force. The net buoyancy force, thus, migrates upwards with the thermal plume, causing the center of convection to move upwards. As the center of convection nears the surface, the surface fluid and heat flux increase substantially. The thermal plume eventually begins to dissipate, causing the net buoyancy force to be reduced, and the convective system begins to wane. Eventually a final steady-state condition is attained.

In contrast with the relatively simple evolution of hydrothermal systems under flat surface topography, hydrothermal systems under sloping surface topography evolve in a more complex fashion. To begin with, the thermal regime prior to injection of the pluton differs markedly from the corresponding flat topography case as a result of advective heat transport. The region below the mountain slope is cooled by downward flowing fluids, and the region below the valley is heated as fluids absorb heat and transport it to the discharge area. This initial difference in the thermal regimes of the flat and sloping topography systems has a major impact on the rate of movement of the thermal plume and the cooling rate of the pluton. Sloping topography also induces a hydraulic gradient, and thus a second fluid flow driving mechanism which operates throughout the cooling history of the pluton. The interaction of the two fluid flow driving mechanisms produces two separate flow systems, the spatial evolution of which depends on the relative strength of the buoyancy
force, which changes through time. Early in the cooling history, temperature contrasts near
the pluton are large, leading to large buoyancy forces, much greater than the topographic
driving forces at this depth. Farther away from the pluton, however, the temperatures have
changed little, and thus, the topographic driving force remains dominant. The
buoyancy-driven flow system is, therefore, localized near the pluton, while the
topographically-driven flow system occupies the rest of the domain. This interaction leads
to a complicated pattern of flow, a convection cell near the pluton and
topographically-driven flow that curves down and around the convection cell.

With time, the thermal plume migrates upwards towards the surface through the
combined processes of conductive and convective heat transfer. The buoyancy-driven flow
system expands, displacing the topographically-driven flow system upwards. The
topographically-driven flow system interacts with the buoyancy-driven flow system to
produce a large lateral component of flow near the interface, causing the thermal plume to
migrate laterally. As the buoyancy-driven flow system continues to rise, it may intersect
the ground surface to allow both discharge and recharge at the valley floor, resulting in a
more complicated pattern of fluid flow near the surface. Eventually, the thermal plume
dissipates, and the force of buoyancy diminishes, allowing the topographically-driven flow
system to dominate. The buoyancy-driven flow system is pushed back down, and
eventually, the force of buoyancy is less than that needed to maintain free convection in the
presence of the topographic driving force, and the convection cell is obliterated. At this
point, the topographically-driven flow system extends throughout the entire domain to
produce a thermal regime similar to that which existed initially.

The presence of topographic relief, besides complicating the patterns of fluid flow
and the thermal regime, may also mask the existence of the buoyancy-driven flow system
and in so doing, mask the existence of the heat source (pluton) at depth. As shown in
Figures 14B-19B, topography controls fluid flow at shallow depth during most of the
cooling history of the pluton; direct contact of the buoyancy-driven fluids with the surface
occurs only between 15,000 and about 45,000 years after pluton emplacement. As a
consequence, the chemical character of fluids collected at or near the surface may not be indicative of the temperature conditions that exist at depth. Further evaluation of the rates and patterns of solute transport using a particle-tracking procedure would provide useful insight into the chemistry of fluids discharging at the surface. In addition to the influence of surface topography on the chemistry of near-surface fluids, topographically-driven fluid flow strongly influences the near-surface thermal regimes. Shallow heat flow measurements may therefore not clearly indicate the presence of the hot environment at depth. The lag time between the thermal peak and the maximum surface heat flux further implies that temperatures at depth in regions of high surface heat flow may not be as high as would be calculated from shallow heat flow measurements. This may have implications for geothermal exploration.

Simulation results indicate that the presence of sloping surface topography increases the cooling rate of the igneous pluton. Increased cooling rates are due primarily to the low initial temperature of the host rock prior to pluton injection. Low host rock temperatures result in higher temperature gradients near the pluton which, in turn, cause higher rates of conductive cooling. A number of other factors associated with topographic relief, however, tend to decrease the cooling rate of the pluton, but not enough to counteract the effect of the low host rock temperature. For example, the presence of the topographically-driven flow system above the buoyancy-driven system forces fluids to flow parallel to the interface rather than vertically. If the heated fluids were instead allowed to discharge at the surface, the cooling rate of the entire system would increase substantially, as suggested by the numerical results of Cathles (1977). The interface, however, impedes vertical heat flow because heat is transferred between the two flow systems by conduction and local mixing alone, which are less efficient mechanisms of heat transfer than is convection. Reduced flow rates between the pluton and the mountain, caused by the opposing interaction of topographically-induced hydraulic gradients and buoyancy, also reduce the rate of convective cooling of the pluton. In addition, the added mass of the mountain slope effectively reduces the overall thermal gradient by lengthening the distance between the
surface and the pluton. This factor is probably negligible in comparison to the other factors mentioned above because the overall thermal gradient is reduced only slightly.

If the initial thermal regime in the vicinity of the pluton is warmer, as would be the case when multiple intrusive events occur, simulation results indicate that the cooling rate of the second pluton is reduced by the factors mentioned in the previous paragraph. Cooling of a reinjected pluton is simulated using the initial conditions that result from purely conductive heat transfer in the absence of a pluton (Figure 22). This thermal regime is similar to the thermal regime that exists after an initial intrusion has cooled for approximately 140,000 years. Cooling of the reinjected pluton is simulated up to the onset of extensive boiling (6 nodes) at 21,000 years after reinjection (Figure 23). Note that the difference in the overall character of the patterns of fluid flow and the thermal regimes between this case and the single intrusive case after 22,000 years (Figure 18) results from

Fig. 22. Fluid flow and thermal regimes used to initiate simulation of pluton reinjection. The thermal regime used is that which would exist from purely conductive heat transfer in the absence of a pluton. This thermal regime is approximately the same as the thermal regime that exists after an initial pluton has cooled 140,000 years. Fluid flux is represented schematically by the vectors.
Fig. 23. Fluid flow and thermal regimes at the onset of boiling (21,000 years): reinjected pluton under sloping surface topography. Vectors are logarithmically scaled to a fluid flux rate of $2.86 \times 10^4$ m s$^{-1}$, which corresponds with a length of 0.9 cm. Isotherm contour interval is 100 °C. Shaded area indicates the region of boiling.

the initial conditions used in each case (Figures 22 and 13B). The cooling rates of the pluton under the three different scenarios are plotted in Figure 24; curve A represents the flat topography case, curve B represents the single intrusive event under sloping topography, and curve C represents the secondary intrusive event under sloping topography. This figure illustrates that the conditions that exist prior to pluton injection have a significant impact on the cooling rate of the pluton.

The cooling rate of the reinjected pluton under sloping surface topography, in comparison with that of the simulated flat topography case, is reduced only slightly (Figure 24). If $k_s$ in each case is greater than that used in this study ($2 \times 10^{16}$ m$^2$), however, one might expect that the presence of sloping topography would reduce the cooling rate even more, relative to the flat topography case. Numerical studies of Cathles (1977) indicate
that plutons under flat surface topography cool much faster in highly permeable systems due to large quantities of heat being transferred out of the system by the discharge of convecting fluids at the surface. Results of this study indicate, however, that the presence of surface topography inhibits the discharge of convecting fluids. The cooling rate of the pluton may thus be reduced relative to the flat topography case. Note, however, that in the case of a single intrusive event, higher permeability would result in initially cooler temperatures below the mountain as a result of greater advective cooling from downward-flowing fluids prior to pluton injection. The cool temperature of the host rock near the injected pluton would then cause the conductive cooling rate to increase substantially, and thus the cooling rate of the pluton will exceed that of the flat topography case.
Simulation results indicate that surface topography also delays or even prevents the onset of boiling. Negligible boiling occurred in the simulated single intrusive event under sloping surface topography. Boiling is prevented primarily because the cooler host rock absorbs most of the heat. Thus an insufficient amount of heat remains to raise temperatures at shallow depth to the boiling point. Recall also that the upward movement of the thermal plume is impeded somewhat by the presence of the topographically-driven flow system. In the case of a reinjected pluton, the temperature of the host rock near the pluton is initially similar to that in the flat topography case (Figures 22, 13A). Boiling in this case, however, is still delayed by 6,000 years (boiling occurs at 21,000 years compared with 15,000 years for the flat topography case). Boiling is delayed in part because, as in the previous sloping topography case, the upward migration rate of the thermal plume is reduced compared with the flat topography case due to the presence of the topographically-driven flow system. Boiling is also delayed because the pressure at shallow depth is greater than in the flat topography case, as illustrated by Figure 25. The thermal plume therefore, must migrate closer to the surface before the temperature-pressure conditions required for boiling are attained. The pressure is greater in the sloping topography case, in part, because fluid within the mountain slope exerts additional pressure on the fluids throughout the system. Pressures also differ because the thermal regimes of the two cases differ.

In summary, the simulation results suggest that topography significantly influences the fluid flow and thermal regimes near cooling igneous plutons. The extent of the impact of topography depends on a number of factors, many of which have not been investigated in this study. For example, the scale of the topographic features relative to the size of the pluton will affect the ratio between the strengths of the topographic and buoyancy components of fluid flow which will in turn affect the evolution of the patterns of fluid flow and the thermal regime. The position of topographic features relative to the pluton position will also affect the fluid flow and thermal regimes. For example, if the pluton is positioned below the valley floor rather than below the mountain, upward migration of the
Fig. 25. Fluid pressure after 15,000 years of cooling: flat topography case (dashed contours) and second intrusive event under sloping topography (solid contours). Contour interval is 5 MPa.

thermal plume will be enhanced rather than impeded because the host rock below the valley is initially hotter and thus will not absorb as much heat. Furthermore, topographically-induced hydraulic gradients below the valley floor will complement rather than oppose buoyancy-driven flow; thus fluid velocities will be greater near the pluton. Another potentially important factor is that of bulk permeability of the system. The influence of surface topography on the thermal regimes may be greater for systems with higher permeability since the fluid flow rates would be greater and thus the amount of heat transferred by convection would be greater. Additional numerical investigation, however, is required to ascertain the influence of surface topography on systems with differing permeabilities.
IMPLICATIONS FOR EPITHERMAL ORE DEPOSITION

A number of conceptual models have been proposed to explain the genesis of epithermal ore deposits. Berger and Eiman (1983) proposed three conceptual models, each of which is designed to represent the genesis of ore in a different geologic and hydrogeologic environment. One of the three models, referred to as the Stacked-cell convection model, depicts ore deposition partially as a result of mixing of deep hydrothermal fluids with shallow cooler groundwater. Berger suggests that a low permeability zone is needed to separate the cooler groundwater from the hotter hydrothermal fluids, and that mixing and boiling occur near breaks in the low permeability barrier, resulting in ore deposition.

More recently, Heald et al. (1987) have proposed a similar model, referred to as the Adularia-sericite-type ore deposition model, which they suggest explains the genesis of the majority of epithermal ore deposits. According to Heald et al., Adularia-sericite-type deposits form in extensive lateral-flow systems which occur high above, and perhaps offset from, the heat source. The lateral-flow system is assumed to result from the interaction of topographically-driven fluid flow and the deeper convecting hydrothermal fluids. In their conceptual model, a low permeability zone is not considered necessary to create an interface between cooler groundwater and deeper hydrothermal fluids. The idea that stacked systems can develop in the absence of a low permeability barrier is supported by the numerical studies of Hanoaka (1980) and the results of this study.

Heald et al.'s (1987) conceptual model is consistent with observations of active and fossil geothermal systems. According to Heald et al. (1987), Hedenquist, (1983) has found evidence supporting the presence of multiple fluids in the active geothermal system at Waiotapu, New Zealand. Evidence supporting the presence of multiple fluids is also found in the fossil geothermal system at Creede Colorado (Foley et al., 1982). Isotope studies of fluid inclusions at Creede suggest that ore deposition resulted from mixing of the fluids (Hayba et al., 1985). The mixing of shallow and deep fluids is also suggested to have caused ore deposition in the Kushikino mining area near Kyushu, Japan (Matsuhisa et al.,
The existence of a shallow groundwater flow system in Heald et al.'s (1987) conceptual model implies that surface topography is an important factor in the genesis of ore deposits. The topographic driving force interacts with the buoyancy driving force to produce a large lateral-flow system where deposition of ore is presumed to occur. The transient nature of the interaction of topographically-driven fluid flow and buoyancy-driven fluid flow occurring in pluton-associated hydrothermal systems, however, has not previously been considered. Because pluton associated hydrothermal systems are transient in nature, a reasonable question that could be asked is, how long are the conditions depicted in the conceptual model of Heald et al. likely to last? Do the temperature-pressure conditions and patterns of fluid flow change continuously through time? Or, are they stable for extended periods of time? These questions may be important with respect to the time required to form sizable ore deposits.

Previous studies of cooling plutons assuming flat surface topography indicate that in most cases, the temperature-pressure environments and the pattern of fluid flow change continuously as the pluton cools (Cathles, 1977; Norton and Knight, 1977). Simulations of a cooling pluton in the presence of sloping surface topography presented in this study, however, indicate that there are regions within the system where temperature remains relatively constant for lengthy periods of time. Figure 26 illustrates this by showing the position of the 150 °C isotherm between 15,000 and 65,000 years (most epithermal ore deposits form at temperatures between 150 °C and 350 °C according to Barnes (1979)). Note that the region along the left boundary remains at relatively constant temperature during this 50,000 year period. Not only do temperatures, and thus pressures, remain stable for a lengthy period, the zone of mixing, which occurs at the interface between the topographically-driven flow system and the buoyancy-driven flow system, also remains in a relatively constant position for about the same period of time (Figure 27).
It should be understood that identifying favorable temperature-pressure environments alone are insufficient for delineating where ore deposition will occur with any degree of certainty. The genesis of sizable ore bodies is a complicated process, and the relatively rare occurrence of sizable ore bodies attests to the low frequency in which ore-forming solutions encounter suitable chemical and physical conditions necessary for mineral precipitation (Barnes, 1979). The mechanisms responsible for ore deposition (temperature changes, pressure changes, fluid-rock interactions, and mixing) must be taken into account, in addition to the source and transportation of ore constituents. Although these factors are unaccounted for in this study, speculation regarding the likelihood for formation of sizable ore bodies can be based on the longevity of favorable temperature-pressure environments and the spatial stability of the zone of mixing. Unfortunately, because of the limitation of current dating techniques, little is known about the actual time required to form ore bodies.
Fig. 27. Position of the zone of mixing from 15,000 and 65,000 years. Curves A, B, C, and D represent the positions of the interface after 15,000, 20,000, 30,000, and 65,000 years of cooling, respectively.

(Barnes, 1979). One would expect that the possibility of developing a sizable ore body would be greater in locations where the fluid flow patterns and the temperature-pressure environments remain constant for lengthy periods of time.

In summary, the general features of the transient interaction of topographically-driven fluid flow and buoyancy-driven fluid flow are consistent with the conceptual model of Heald et al. (1987). The two driving mechanisms interact to produce a shallow cooler flow system overlying a deeper hotter flow system. The interface between these flow systems produces an extensive region of lateral fluid flow where ore deposition is presumed to occur as a result of mixing at the interface. The patterns of fluid flow and the temperatures change through time, as discussed previously. If the pluton is located below sloping surface topography similar to the system envisioned in this study, however, there may be regions that remain at a relatively constant temperature and the zone of mixing may remain in a relatively stable position for an extended period of time.
stability of both the temperature-pressure conditions and the zone of mixing in the region below the mountain, may increase the likelihood for depositing sizable ore bodies in this region.
SUMMARY AND CONCLUSIONS

Previous numerical studies on the cooling of igneous plutons have assumed that topographic relief in the vicinity of the pluton is negligible. Significant topographic relief, however, exists in andesitic and silicic volcanic terrain, which raises the question of what the potential impact may be. Field observations at numerous active geothermal sites suggest that topographic relief is driving groundwater flow that interacts with deeper convecting fluids to produce large lateral-flow systems. In this study, a quantitative investigation of this phenomenon is conducted using a finite element numerical model to simulate the fluid flow and thermal regimes associated with a cooling igneous pluton in the vicinity of significant topographic relief. The primary objectives of the study were to assess the need for considering topographic relief when studying pluton-associated hydrothermal systems and to investigate possible implications for the genesis of epithermal ore bodies.

Conclusions

The following conclusions are based on the specific geometry of the system envisioned in this study; a pluton centered at depth below a mountain. It is recognized, however, that the manner in which topography influences convecting hydrothermal systems will change for differing locations of the pluton relative to the topographic features, and that further investigation of different scenarios is warranted.

The primary conclusions of this study are as follows:

1) Hydraulic gradients induced by sloping surface topography interact with buoyancy forces to produce two separate flow systems, a shallow topographically-driven flow system and a deeper buoyancy-driven flow system. The two flow systems interact to produce a large zone of predominantly lateral fluid flow. The rates and patterns of fluid flow within each flow system evolve in a complicated fashion as the pluton cools, and as a result, the thermal regime evolves in a complicated manner, unlike the behavior of flat topography systems.
2) The presence of the shallow topographically-driven flow system may partially mask the convecting hydrothermal system at depth by inhibiting the convecting fluids from reaching the ground surface; thus near-surface fluids may not contain the chemical signature of the deeper hotter environment. The shallow topographically-driven flow system also strongly influences temperatures at shallow depth, which may complicate interpretation of shallow heat flow measurements.

3) Sloping surface topography may increase the cooling rate of the pluton; however, this is caused primarily by the effect of topographically-driven fluid flow on the thermal regime prior to injection of the pluton. The temperatures below the mountain where the pluton is emplaced are reduced substantially by downward-flowing fluids originating at the surface. If, however, a previous intrusive event has increased the temperature of the host rock substantially, the cooling rate of the second pluton may actually be reduced as a result of topographic relief. The shallow topographically-driven flow system inhibits hot convecting fluids from reaching the surface, and thus inhibits heat from leaving the system.

4) The presence of topographic relief inhibits or prevents boiling. If the initial thermal regime has been unaffected by previous intrusive events, the heat released by the pluton may be insufficient to raise near-surface temperatures to the boiling point. If, however, a previous intrusive event has raised the temperatures throughout the regime, boiling will still be inhibited somewhat because the presence of the topographically-driven flow system impedes the upward movement of the thermal plume. Fluid pressures throughout the system are also higher in sloping topography systems than in flat topography systems due to the additional pressure exerted by the weight of the fluid within the mountain slope. The thermal plume must therefore migrate closer to the surface in order for boiling to occur.

5) The fluid flow and thermal regimes below the mountain slope may remain relatively unchanged for an extended period of time; the zone of mixing of topographically-driven fluids and buoyancy-driven fluids remains constant spatially and the temperatures within this zone remain relatively unchanged. The relative stability of the
zone of mixing, coupled with constant temperature-pressure conditions, may increase the likelihood of the formation of sizable ore bodies.

Overall, the results suggest that topographic relief should be considered when studying pluton-associated hydrothermal systems. Hydrothermal systems appear to evolve in a significantly different fashion in the presence of topographic relief than when topographic relief is absent. A better understanding of ore depositing hydrothermal systems may, therefore, be gained by considering the effects of surface topography.

Recommendations for Further Study

This study considers a very simple system in order to gain an understanding of the basic principles of the interaction of topographically- and buoyancy-driven fluid flow. In nature the scale and character of the topography vary considerably as do the size, shape, temperature, and depth of the pluton. These factors can be accounted for in the "minds eye", to a degree; however, other factors, such as differing spatial settings of the pluton relative to the mountain slope, need further numerical investigation. For example, one would expect a far different result if the pluton is situated beneath the valley floor, as opposed to beneath the mountain slope. A spatial setting such as this might be expected in collapsed caldera structures having a pluton located in the center of the caldera, with the rim of the caldera some distance away.

Another factor needing further investigation is the effect of variations in permeability on the interaction of topographically-driven and buoyancy-driven fluid flow. Permeability will not change the relative strengths of the topographic and buoyancy components of flow. Permeability will, however, affect the spatial temperature distribution, and thus the spatial distribution of the buoyancy forces throughout the system.

The assumption was made in this study that the scale of fracturing of the rocks within the system is small enough in comparison to the scale of the overall domain to be represented as an equivalent porous medium. In many active geothermal systems, however, fluid flow appears to be controlled by large-scale fracture networks. In addition, many ore
deposits are found associated with large fracture zones which are presumed to have provided conduits for the ore-depositing hydrothermal fluids. The influence of large fracture zones on the patterns of fluid flow and thermal regimes of pluton associated hydrothermal systems is, therefore, of interest and warrants further investigation.

The most important area needing further consideration is that of substantiating model results with field data. Currently, however, few field data are available due to the lack of deep bore holes in active and inactive geothermal areas. Drilling in volcanic systems such as at Newberry Volcano, Oregon has provided limited data. Future drilling programs at Creede, Colorado, and in the active volcanic system of White Island, New Zealand, and the ongoing drilling program at the Valles Caldera, New Mexico, are expected to yield very useful results. The study of alteration assemblages, such as those associated with porphyry copper deposits, may also provide clues as to whether model results are valid.
REFERENCES


Appendix A.

Transient Formulation and Time-Stepping Method

The numerical model used in this study is based on the Galerkin formulation of the governing equations of heat and fluid flow. The solution is obtained at each time step by solving the transient heat transfer equation (Equation 8) and the steady-state fluid flow equation (Equation 6) in an iterative fashion. The steady-state fluid flow equation is used, rather than its transient equivalent, because the medium is presumed to be relatively incompressible. As a consequence, the hydraulic head throughout the system equilibrates at a much faster rate than does the temperature (Sorey, 1978). At any given time, the fluid flow regime is thus approximately at equilibrium with respect to the existing thermal regime.

Time steps are taken using a finite difference approximation of the time derivative term in Equation 8. Various time weighting schemes can be used by explicitly choosing the value of the time weighting factor ($0 \leq \theta \leq 1$). A fully implicit weighting scheme ($\theta = 1$), however, is used in this study because it is unconditionally stable and the calculated values do not oscillate about the correct values (Segerlind, 1984).

The lumped formulation of the time derivative term in Equation 8 is used in order to allow larger time steps to be taken and more flexibility in the choice of $\theta$ (Segerlind, 1984). This formulation uses a stepped basis function rather than the linear basis function which is used in all other terms of Equations 6 and 8. One disadvantage of the lumped formulation, however, is that it may be slightly less accurate than the consistent formulation. The overall benefits of the lumped formulation, however, outweigh the disadvantages.

Time increments ($\Delta t$) are automatically calculated using a modified version of the method used by England and Freeze (1988). The percentage change in temperature at each time step is calculated at each node within the domain. $\Delta t$ for the succeeding time step is calculated based on the maximum percentage temperature change detected throughout the
domain for the current time step using the following equation:

\[ \Delta T = \Delta T \left(1 - \frac{T_{\text{arg}} - \%T}{T_{\text{arg}}} \right) \]  

(15)

where \( T_{\text{arg}} \) is the desired percentage temperature change, and \( \%T \) is the maximum percentage temperature change detected within the domain. On occasion, unacceptably high values of \( \%T \) occur because the temperature-pressure properties of water (\( \mu, \lambda', \) and \( C_d \)) change sharply. A tolerance percentage temperature change (\( \text{Tol} \)) is therefore specified. If \( \%T \) is greater than \( \text{Tol} \), \( \Delta t \) is reduced by half and the time step is taken over. Values of \( T_{\text{arg}} \) equal to 3.0% and \( \text{Tol} \) equal to 7.0% were found to produce acceptable results.
Appendix B.

Model Verification

The model used in this study is a modified version of the steady-state coupled heat and fluid flow free-surface model developed by Forster and Smith (1988a). The original model employs an iterative procedure in which the finite element mesh is allowed to deform in response to the calculation of the water table configuration. The deforming-mesh iterative procedure requires a rather complicated algorithm, which is not required for the problem envisioned in this study. The subroutines were, therefore, stripped out of the deforming-mesh framework to construct a less complicated non-deforming-mesh steady-state model which provides the foundation of the transient model developed in this study.

Due to the amount of manipulation of the subroutines and the transfers between subroutines, reverification of some of the steady-state capabilities of the original model was required. One-dimensional and two-dimensional steady-state pure conductive heat flow capabilities were, thus, reverified, along with the equivalent isothermal fluid flow capabilities and fluid flux calculations. One-dimensional numerical results were compared with the easily derived analytical solutions, while the two-dimensional capabilities were compared with Tóth's (1962) analytical solution. In all, seven analytical solutions were coded and run in order to verify the steady-state and transient capabilities of the new model. Numerical results were also compared against numerical results of Cathles (1977) and Smith and Chapman (1983).

Transient Pure Conductive Heat Transfer

One-dimensional conductive heat transfer. After the steady-state capabilities of the new model were verified, transient one-dimensional conductive heat flow results were compared against analytical results to insure the accuracy of the time-stepping scheme, to check that the transient form of the finite element equations were properly coded, and to insure that the boundary conditions were being applied correctly. Transient solutions were verified for both constant temperature and constant flux boundary conditions. The
simulated region measured 20 m wide by 6 m high and was discretized into 120 elements. The initial temperature of the media was set at 0 °C, while the left and right boundaries were set at constant temperatures of 10 and 0 °C, respectively, and the upper and lower boundaries were insulated. The parameter values used in this simulation are shown in Table 2.

The equation for one-dimensional transient pure conductive heat flow is:

\[
\lambda \frac{s}{\rho C_p} \frac{\partial^2 T}{\partial x^2} - \rho C_s \frac{\partial T}{\partial t} = 0
\]  

(16)

\[ T(x,t) = T_i + \frac{T_f - T_i}{2} \operatorname{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) \]

where \( \alpha \) is thermal diffusivity \( (\lambda' / \rho C_s) \).

TABLE 2. Model Parameters: Transient Pure Conductive Verification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda' ), Thermal conductivity</td>
<td>2.5 W m(^{-1}) °C(^{-1})</td>
</tr>
<tr>
<td>( \rho_p ), Density</td>
<td>2,700 kg m(^{-3})</td>
</tr>
<tr>
<td>( C_s ), Specific heat</td>
<td>12 J kg(^{-1}) °C(^{-1})</td>
</tr>
<tr>
<td>( \eta ), Porosity</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Temperature profiles were compared at 1.0 \times 10^4 s, 6.0 \times 10^4 s, 1.1 \times 10^5 s, 2.1 \times 10^5 s and 3.0 \times 10^5 s (Figure 28). At all time steps, the results match very closely. Longer simulations were not run for this particular example because of the potential influence of the constant temperature lower boundary which violates the semi-infinite nature of the analytical solution.

The experiment was repeated for a constant flux left boundary, and the results were compared against another semi-infinite analytical solution of Holman (1981) which is
Fig. 28. Transient one-dimensional pure conductive heat flow verification against an analytical solution of Holman (1981): constant temperature boundary conditions. Curves A, B, C, D, and E are at times $1.0 \times 10^4$ sec, $6.0 \times 10^4$ sec, $1.1 \times 10^5$ sec, $2.1 \times 10^5$ sec and $3.0 \times 10^5$ sec, respectively. Solid and dashed curves represent the analytical and numerical results, respectively.
written as:

\[ T - T_i = \frac{2H_b \sqrt{\frac{\alpha t}{\lambda^s A}}}{\lambda^s A} \exp \left( \frac{-x^2}{4\alpha t} \right) - \frac{H_b x}{\lambda^s A} \left( 1 - \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) \right) \]  

(18)

where \( A \) is area.

The simulation was essentially the same as the one above except that a constant flux of 0.5 W m\(^{-2}\) was applied to the lower boundary while the upper boundary was held fixed at a constant temperature of 0 °C. Model results were compared with analytical results at 1.0 \( \times \) 10\(^4\) s, 6.0 \( \times \) 10\(^4\) s, 1.1 \( \times \) 10\(^5\) s, 1.6 \( \times \) 10\(^5\) s and 2.1 \( \times \) 10\(^5\) s (Figure 29). At all time steps except for the earliest, (1.0 \( \times \) 10\(^4\) s, curve A) the results are good. The slight discrepancy at 1.0 \( \times \) 10\(^4\) s is attributed to the coarseness of the mesh.

Two-dimensional conductive heat transfer. Model results for transient two-dimensional pure conductive heat flow were compared with analytical results for a problem having constant temperature boundary conditions. The purpose of this test was to insure that the model calculated transient two-dimensional conductive heat flow correctly. The two-dimensional transient conductive heat flow equation is a simple extension of Equation 16 and is not presented here. The analytical solution (Carslaw and Jeager, 1986) used in this test is:

\[ T = \varphi(x,a)\varphi(z,b) \] 

(19a)

\[ \varphi(x,a) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)} e^{-\lambda^2(2n+1)^2\pi^2a^2/4a^2} \cos \left( \frac{(2n+1)\pi x}{2a} \right) \] 

(19b)

where \( a \) and \( b \) are the length and height of the domain, respectively. The expression for \( \varphi(z,b) \) in 19a is derived by substituting \( z \) and \( b \) for \( x \) and \( a \) in Equation 19b.

The simulated area was 20 m wide by 20 m high and was discretized into 200 elements. Temperatures for all boundaries are given a value of 0 °C while the initial temperature of the interior was set at 1 °C. The parameter values used in this test are
Fig. 29. Transient one-dimensional pure conductive heat flow verification against an analytical solution of Holman (1981): constant flux boundary conditions. Curves A, B, C, D, and E are at times $1.0 \times 10^4$ sec, $6 \times 10^4$ sec, $1.1 \times 10^5$ sec, $1.6 \times 10^5$ sec and $2.1 \times 10^5$ sec, respectively. Solid and dashed curves represent the analytical and numerical results, respectively.
shown in Table 2. Three horizontal profiles and one vertical profile through the region were checked against the analytical results. The horizontal profiles were 4, 10, and 16 m from the bottom of the region, and the vertical profile was in the center. Results were compared with analytical results at \(5.0 \times 10^4\) s, \(1.0 \times 10^5\) s, \(1.5 \times 10^5\) s and \(2.0 \times 10^5\) s. Model results match the analytical solutions well as shown in Figures 30 and 31. Figure 30 shows the results obtained for the center vertical (V1) and horizontal (H1) profiles. Note that the model results at the various times for V1 and H1 are identical (dashed curves), due to the symmetry of the domain. Figure 31 shows the results obtained for the two horizontal profiles (H2 and H3) taken symmetrically about the center (4 m and 16 m from the bottom of the domain). The model results at the various times for H2 and H3, are also identical (dashed curves) due to symmetry. Figures 30 and 31 show that the model results (dashed curves) match the analytical results (solid curves) reasonably well. Note that the match between numerical and analytical results improves with time. The fully implicit weighting scheme, however, is known to be somewhat less accurate during later simulation times than other weighting schemes. The simulation was therefore continued until \(8.0 \times 10^5\) s and the results were again compared with analytical results. The match at \(8.0 \times 10^5\) s, however, is good (not shown), comparable to the match of curve D in Figure 31.

To test the model for pure conductive heat transfer in the context of cooling plutons, numerical results were compared with those of Cathles (1977). The pluton measures 0.75 km wide by 2.25 km high and is 2.75 km below the surface. A constant temperature of 20°C is assigned to the upper boundary, the lateral boundaries are insulated, and the lower boundary is assigned a constant heat flux of 60 mW m\(^{-2}\). Cathles assumed that the pluton was injected instantaneously at 700°C and that 36 calories per gram of magma were liberated over the cooling history of the pluton as a result of exothermic chemical reactions. The model used in this study does not account for heat generation, and therefore, an alternative method was used to account for exothermic reactions. The heat generated by the exothermic reactions was accounted for by adding 180°C to the initial temperature of
Fig. 30. Transient two-dimensional pure conductive heat flow verification against an analytical solution of Carslaw and Jeager (1986): vertical (V1) and horizontal (H1) profiles through the center of the domain. Curves A, B, C, and D are at times $5.0 \times 10^4$ sec, $1.0 \times 10^5$ sec, $1.5 \times 10^5$ sec, and $2.0 \times 10^5$ sec, respectively. Solid and dashed curves represent the analytical and numerical results, respectively.
Fig. 31. Transient two-dimensional pure conductive heat flow verification against an analytical solution of Carslaw and Jeager (1986): horizontal profiles 4 m (H2) and 16 m (H3) from the bottom of the domain. Curves A, B, C, and D are at times $5.0 \times 10^4$ sec, $1.0 \times 10^5$ sec, $1.5 \times 10^5$ sec, and $2.0 \times 10^5$ sec, respectively. Solid and dashed curves represent the analytical and numerical results, respectively.
the pluton, based on the density of the water-saturated media (2700 kg m\(^{-3}\)) and the specific heat of the fluid-media composite (837.2 J kg\(^{-1}\) °C\(^{-1}\)).

Figure 32 shows that the results compare reasonably well considering the differences in the modeling approach. The effect of assigning the additional temperature to the pluton is obvious. The thermal front advances at a faster rate than that of Cathles (1977) due to the greater temperature gradient. Also, differences in the mesh density, differences in the methods of calculating fluid properties, and also, differences in the numerical procedures contributed to the observed disparities. Overall, however, the results are satisfactory.

**Coupled Heat and Fluid Flow**

One-dimensional steady-state coupled heat and fluid flow. After completing the pure conductive heat flow verification, one-dimensional steady-state coupled heat and fluid flow results were compared with an analytical solution to insure that the coupling of the heat flow and fluid flow equations was coded properly. The simulated region is the same as that used for the pure conductive heat flow simulations above and the same finite element mesh is used. The left boundary is assigned a constant temperature of 10 °C and an constant equivalent freshwater head of 1 m; the right boundary is assigned a constant temperature of 0 °C and an equivalent freshwater head of 0 m; and the upper and lower boundaries are insulated and impermeable. Parameters used in this simulation are shown in Table 3. Note that fluid properties ($\mu$, $\lambda'$, $\rho_o$ and $C_o$), also shown in Table 3, are held constant since simple analytical solutions do not exist when the fluid properties are allowed to vary.

One dimensional steady-state conductive and advective heat flow is described as:

\[
\lambda'_{xx} \frac{\partial^2 T}{\partial x^2} - \rho_f C_f \frac{\partial}{\partial x} \frac{\partial T}{\partial x} = 0
\]

where $q_x$ is derived from Darcy's law:

\[
q_x = - \frac{kp_g}{\mu} \frac{\partial h}{\partial x}
\]
Fig. 32. Transient two-dimensional pure conductive cooling of a pluton: comparison against numerical results of Cathles (1977). A, B, and C are the thermal regimes resulting after 5,000, 10,000, and 20,000 years, respectively. Solid and dotted isotherms represent the results of Cathles and the model used in this study, respectively. Isotherm contour interval is 100 °C.
h is obtained by solving the one-dimensional steady-state fluid flow equation:

\[
\frac{\partial^2 h}{\partial x^2} = 0
\]  

(22)

The results were compared with the solution of the steady-state analytical solution of Bredehoeft and Papadopulos (1965) which is written:

\[
T = \frac{(T_1 - T_2) \left[ \exp \left( \frac{\beta x}{L} \right) - 1 \right]}{\exp(\beta) - 1} + T_2
\]  

(23)

where \( \beta = \frac{\rho^e C^e \alpha^e L}{\lambda} \).

Figure 33 shows that the numerical results match exactly with the analytical results.

TABLE 3. Model Parameters: Coupled Heat and Fluid Flow Verification

<table>
<thead>
<tr>
<th>Medium Parameters</th>
<th>Fluid Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k ), Permeability</td>
<td>( \lambda^f ), Thermal conductivity</td>
</tr>
<tr>
<td>( \lambda^s ), Thermal conductivity</td>
<td>( \rho^f ), Density</td>
</tr>
<tr>
<td>( \rho^e ), Density</td>
<td>( C^e ), Specific heat</td>
</tr>
<tr>
<td>( C^s ), Specific heat</td>
<td>( \eta ), Porosity</td>
</tr>
<tr>
<td>( \eta ), Porosity</td>
<td>( \lambda^f ), Thermal conductivity</td>
</tr>
<tr>
<td>( \mu ), Viscosity</td>
<td>( \rho^f ), Density</td>
</tr>
<tr>
<td>( \rho^e ), Density</td>
<td>( C^s ), Specific heat</td>
</tr>
<tr>
<td>( C^f ), Specific heat</td>
<td>( \mu ), Viscosity</td>
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One-dimensional transient coupled heat and fluid flow. Numerical results for transient one-dimensional conductive and advective heat flow were compared with analytical results of Ogata and Banks (1961) to check that the time-stepping method works correctly for coupled heat and fluid flow problems. For this problem, fluid properties are again held constant, and the boundary conditions are the same as those used in the steady-state problem above, with the medium initially at \( 0^\circ C \). It was found, however, that a denser mesh is needed in order to obtain acceptable results. The mesh used in this test is double the density of the mesh used for the steady-state problem.
Fig. 33. Steady state one-dimensional coupled heat and fluid flow verification against an analytical solution of Bredehoeft and Papadopoulos (1965). Solid and dashed curves represent the analytical and numerical results, respectively. Note that solid and dashed lines match exactly.
The transient one-dimensional conductive and advective heat transfer equation is a simple extension of Equation 19, and $q_s$ is obtained in the same manner as in the one-dimensional steady-state coupled problem (i.e. solving Equations 22 and 21). The one-dimensional analytical solution of Ogata and Banks (1961), recast to the context of heat transport, is:

$$T(x,t) = T_i + (T_0 - T_i) \left\{ \frac{1}{2} \text{erfc} \left[ \frac{\rho_i C_p x - q_s t}{2 \sqrt{\lambda \rho_i C_p t}} \right] + \frac{1}{2} \exp \left( \frac{q_s x}{\lambda} \right) \text{erfc} \left[ \frac{\rho_i C_p x + q_s t}{2 \sqrt{\lambda \rho_i C_p t}} \right] \right\}$$

(24)

Model results were compared with analytical results at $2.5 \times 10^6$ s, $5.0 \times 10^6$ s, $7.5 \times 10^6$ s, $1.0 \times 10^7$ s, and $2.0 \times 10^7$ s. Figure 34 shows that the results compare reasonably well. The simulation was also continued until a final steady-state condition was obtained to ensure the model converged properly. Results upon convergence are identical to the results of the steady-state analytical and numerical results shown in Figure 33.

Two-dimensional steady-state coupled heat and fluid flow. Two dimensional steady-state coupled heat and fluid flow for a regional scale groundwater flow system was simulated and compared with published results of Smith and Chapman (1983). The purpose of this test was to check that the model functioned properly using variable fluid properties, and to insure that the model is capable of producing results similar to results of another well-tested model.

The system simulated is a long thin regional groundwater flow system measuring 40 km long by 5 km high. The lateral boundaries are assumed to correspond with natural groundwater divides, and are, therefore, insulated and impermeable. The basal boundary is assumed to be impermeable with constant heat flux of 60 mW m$^{-2}$. The upper boundary, which coincides with the water table, is assigned a constant temperature of 20 °C and an equivalent freshwater head corresponding to the elevation of the upper surface. The upper surface is linear, with a total relief of 0.5 km over the 40 km horizontal distance.
Fig. 34. Transient one-dimensional coupled heat and fluid flow verification against an analytical solution of Ogata and Banks (1961). Curves A, B, C, D, and E are at times $2.5 \times 10^6$ sec, $5.0 \times 10^6$ sec, $7.5 \times 10^6$ sec, $1.0 \times 10^7$ sec, and $2.0 \times 10^7$ sec, respectively. Solid and dashed curves represent the analytical and numerical results, respectively.
Figure 35 shows that the results for three different values of $k$ compare well with those of Smith and Chapman (1983). Slight differences are attributed to the relative coarseness of the mesh, and the use of a slightly different vertical variation in $\eta$ (Smith and Chapman assumed $\eta$ varied step-wise from 0.2 at the surface, to 0.02 at -5 km elevation) which stems from the use of a different mesh discretization.
Fig. 35. Steady state two-dimensional coupled heat and fluid flow in a regional scale groundwater flow system: comparison against numerical results of Smith and Chapman (1983). A, B, and C are the thermal regimes resulting from k equal to $1.0 \times 10^{18}$ m$^2$, $2.0 \times 10^{18}$ m$^2$, and $5.0 \times 10^{16}$ m$^2$, respectively. Solid and dotted isotherms represent the results of Smith and Chapman and the model used in this study, respectively. Isotherm contour interval is 20 °C.