A Logistic System Simulation Model Encompassing Poisson Processes and Normal or Weibull Life

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A LOGISTIC SYSTEM SIMULATION MODEL
ENCOMPASSING POISSON PROCESSES
AND NORMAL OR WEIBULL LIFE

by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Applied Statistics

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1966

ACKNOWLEDGEMENTS

I wish to thank Mr. Tad Norman of the IBM Corporation for his technical assistance on the computer runs of this Simulation Model Program and Miss Anita Flippen who typed seemingly endless drafts of this thesis.

I am indebted to the faculty advisors from the Department of Applied Statistics, Utah State University, who gave their advice and guidance and spent their time reading and commenting on the drafts. Dr. David White furnished expert advice and suggestions concerning the statistical sampling models.

Utah State University — Willard A. Hansen

May, 1966

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INTRODUCTION

This thesis describes a computer simulation model for determining effective spares stock levels for recoverable items at Air Force bases and depots. The simulation model is based on the following fundamental inventory theory; whenever a demand arises, it is satisfied from stock on hand, and the quantity equal to that demand is recorded immediately; when a demand exceeds stock on hand, the excess demand is backordered immediately and when item life expires procurement action is initiated at depot level. The resulting product of the model can be used as a guide for the optimum distribution of available spares or as a computation of the necessary spares which will meet a desired percent fill rate. Outputs from the simulation model will also enable evaluation of the spares level effects as a result of change in other logistic parameters.

The purpose of this thesis is two-fold to the extent that it presents:

(a) A computer simulation model of an Air Force logistic system; and

(b) A discussion of compound Monte-Carlo demand generation involving various analytic failure distributions.

The specific nature of the problem to which the simulation model is applied is described and the model construction and output are discussed in detail.

BACKGROUND AND GENERAL NATURE OF THE PROBLEM

A brief background concerning the techniques used in this paper is appropriate. Many authors using statistical techniques have considered logistic operations, including failure, repair, replacement, etc, on systems consisting largely of identical, independent operating units. Historically, C. Palm (6), following the methods of A. K. Erlang, (2), was one of the first to use properties of the exponential distribution in solving various inventory type problems. Feller (3), popularized and extended the analysis technique. Several other researchers investigating failure and fatigue analysis have since provided empirical mathematical formulation which has proven to be of much practical analysis worth. Probability density functions identified to these type failure analyses are the "normal" gaussian, the gamma, weibull, and the exponential among others. More recently, renewal theory techniques have been used by many authors in solving related inventory theory problems.

Since the advent of the large scale high capacity computers, simulation techniques have become a desirable tool for simultaneous consideration of many time dependent variables. Indeed, simulation represents an excellent experimental medium to examine the time patterns of operational events and the consequences of various policies or decisions. Large scale monte-carlo models, and man-machine simulations, are being developed and utilized by many research organizations including the military.

In this paper, the general interest is in simulating an Air Force multi-echelon logistic system wherein effective spares levels are the major concern, while the interacting effect of the other logistic parameters are secondary. To accomplish this, an appropriate probability or statistical model representing failure or life data must be an integral part of the overall simulation model. To succeed, the simulation model must be able to cope with the following:

a. Components of large logistic systems rarely operate independently.

b. The source of spares supply in a depot - base complex is variable.

c. Item failure characteristics change with respect to life cycle period.

d. Statistical distributions overlay in a given simulation requiring compound monte-carlo generation.

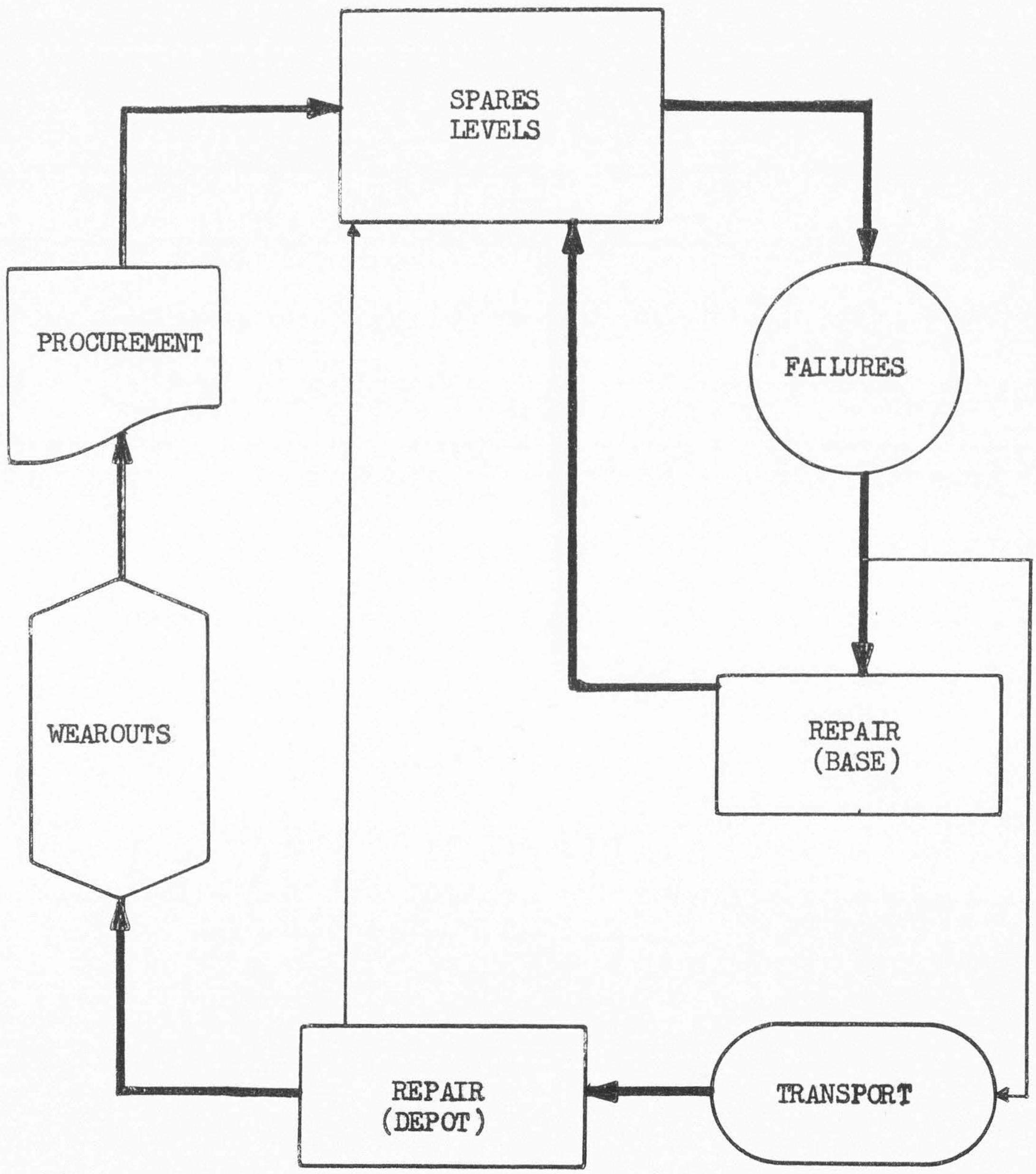
The simulation model must be sufficiently complex to account for these first two problems and the probability models must be precise in order to handle the remaining two problems. In particular, the simulation model must be so designed as to allow detailed study of the effects of varying the input parameters over a wide range with a combination of spares failure and life probability distributions. As in all simulations the degree of success is measured by the degree to which results provided by the model are actually true.

PROBLEM DESCRIPTION

The Air Force depot-base supply process for a recoverable item operates in the following manner. When an item fails in its course at an operational base it is removed from its application and a serviceable item replaced. The source of the replacement item is either from the local base supply shelf or requisitioned directly from an appropriate depot somewhere in the continental United States. The failed item is examined to determine whether repair is possible at base level. If so, the item is scheduled into the base repair shop and following a variable repair cycle time, it is returned to a serviceable condition. If base repair is not indicated the item is either condemned or forwarded to the depot for repair. In the case of a direct replacement from the depot a variable resupply time is experienced. Refer to Figure 1, page 5.

Present Air Force policy (1 authorizes a base to establish a 30-day stockage objective for most recoverable items. The distribution system operates on the basis of stock control levels which includes the base stockage objective plus the number of days of stock required for normal resupply action.

The objective of this model is to simulate the depot-base activity on a given quantity of a single type recoverable item thereby observing the interacting effects of the many variable factors such as increasing, decreasing, or constant failure rates, mortality rates, repair and resupply times, repair capacities, and alternative levels of available spares. Refer to Figure 2, the Model Concept, page 6.



SIMULATES OPERATION OF DEPOT AND BASES ON AN INDIVIDUAL SPARE ITEM.

Figure 1. Model flow

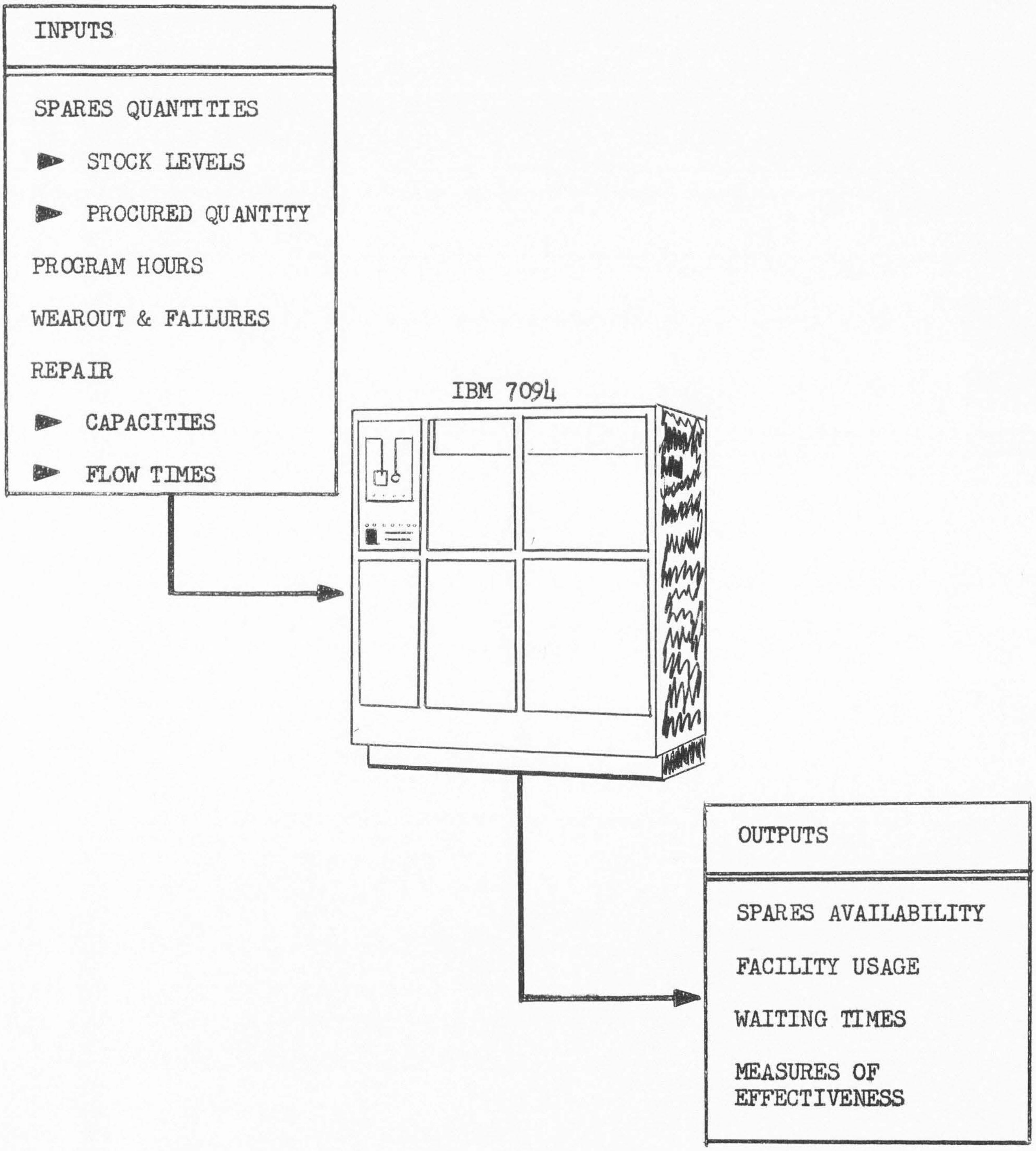


Figure 2. Model concept

SIMULATION MODEL

Description

The simulation model is constructed to utilize the IBM 7090/94 Computer with the IBSYS/FORTRAN (4), system language. The operation to be simulated is described in terms of coded block diagrams. A complete set of rules govern the use of each block in the simulation model program. This methodology in addition to driving the computer provides for study of the logical structure of the operation being simulated.

The computer output is so arranged as to furnish information of the following:

a. The total volume of transactions flowing through all elements of the operation.

b. The distribution of flow times for transactions flowing through each base repair and depot repair element. The resupply of serviceable item flow times are also available.

c. The repair facility utilization.

d. The maximum, minimum, and average queue lengths at desired points in the operation.

Statistical sampling techniques describing failure characteristics are introduced into the simulation model. Levels of priority are assigned to each transaction to provide for various dependent events such as sources of supply points in the logistic simulation. The interdependence of certain variables in the operation, such as repair decisions in view of queue lengths and facility availability or utilization is taken into account and is simulated in the model. A copy of the computer program is included in the Appendix, page 49.

Operation

The program operates by moving transactions from block to block of the simulation model in a manner analogous to the flow of spares through the real operational logistic system. Every movement is an event that is simulating a real event at a particular point in time. The program maintains a record of the times at which the events are due to occur, and it operates by executing the events in their correct time sequence.

Input time parameters are all converted to a single standard time unit and must be consistent throughout a simulation. Typical input parameters are shown in Table I, page 25. All input parameters are entered in the program by a set of machine control cards. The duration of the simulation is controlled either by total number of transactions at a pre-designated point in the simulation model or by a pre-designated number of clock units within the simulation model.

The output product of the simulation run contains the summary statistics which includes number of transactions for most blocks, average utilization of facilities, average time per transaction, average contents, maximum contents, queue lengths, means, variances, and frequency distributions. A typical output product is included in the Appendix and a summary of typical simulation results are reflected in Table II, page 26.

A logistics manager can, with this output, assess the interacting effects of changes in parameters enabling him finally to make effective time oriented decisions relative to procurement and positioning of spares.

METHODS AND PROCEDURES

Failure Phenomenon

Many items that have been analyzed for simulation demonstrate what has come to be regarded in the literature as a classical failure pattern: initially decreasing mortality, followed by a period of essentially constant failure, and ending in a sharp rise in the incidence of failure. Refer to Figure 3.

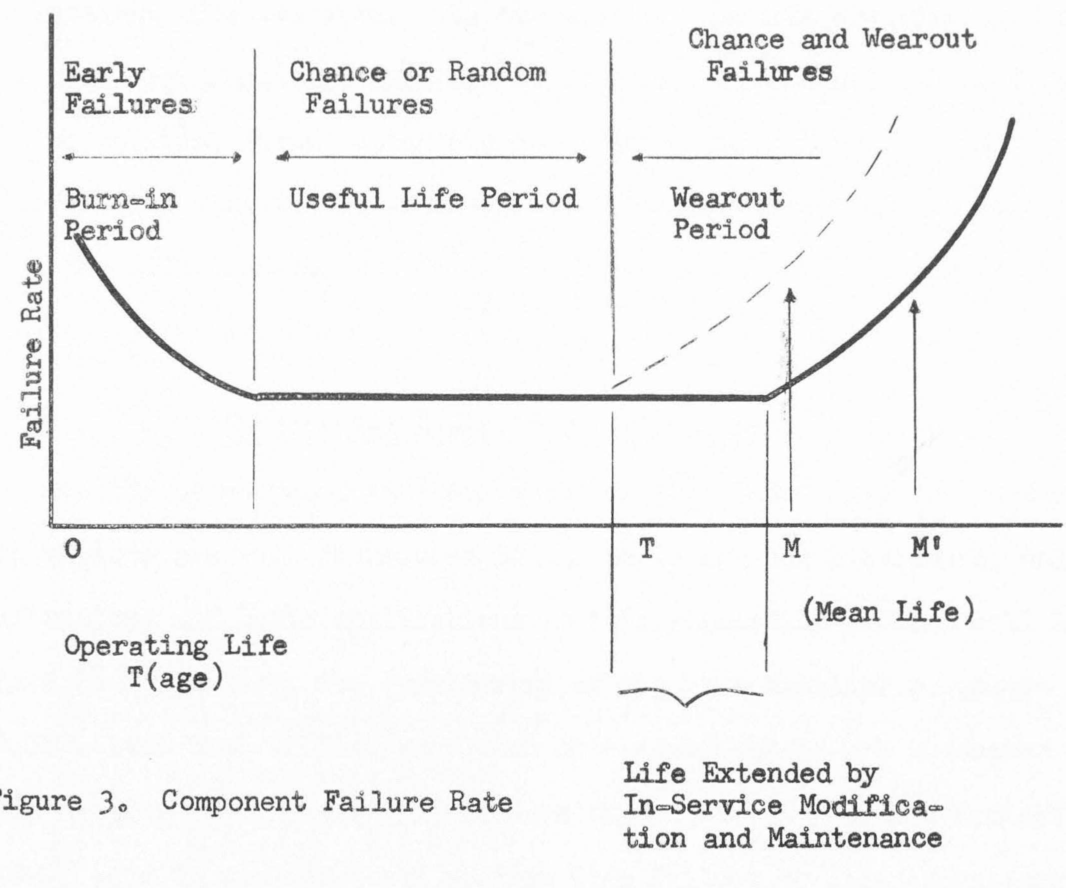


Figure 3. Component Failure Rate

This well known "bath tub curve" modified to consider life extension by an in-service maintenance or modification describes the general component time failure phenomenon. Relatively little direct attention has been given to the high initial failure rate in this model because of observed low incidence of clear infant mortality.

During the chance or random failure period of an item's life, a poisson process is assumed and has been verified on many items. The items are assumed to fail, be repaired, and made available for another application. The intervals between occurrences in this poisson process are generated with an exponential distribution function. During the rapidly increasing failure or wearout period of an item's life the normak, weibull, gamma, or other probability distribution function is utilized as a statistical model to describe life length. During this period items are assumed to wearout and be replaced by procurement of a new item. The two distinct periods are simulated in the model by compound generation of failure. Random selections from the exponential distribution are made sequentially throughout an item's total life to assign the precise failure time. Random selections from the life distributions determine the time at which items will expire. Refer to Figure 5, page 41.

Fundamental Analytic Distributions

Since the development and derivation of the fundamental mathematical expressions are well documented in the texts and the literature, only definitions and their applications to this simulation problem will be discussed. However, the development of the less familiar or unique applications such as the generation of random numbers are presented in more detail. The formulation depicts the probability or statistical models used to represent and analyze item failure or life characteristics within the simulation model.

Poisson

The Poisson probability density function is given by,

$$f(x) = \frac{\lambda^{x} e^{-\lambda}}{x!} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

for $x = 0, 1, 2 \ldots \infty$

where the range of x is infinite, with λ the expected number of failures and x is the random variable, the number of failures.

Exponential

The exponential density function is given by,

$$f(t) = \frac{e^{-\frac{t}{\theta}}}{\theta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

$t \geq 0$

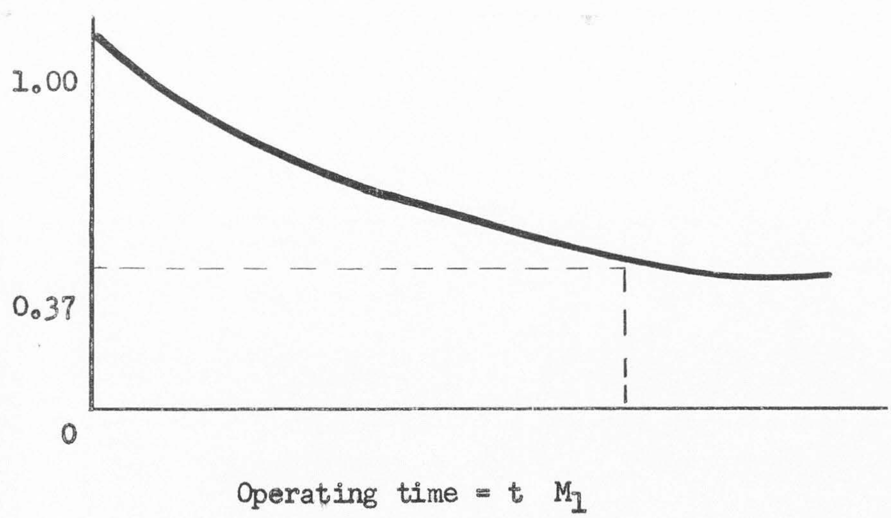


Figure 4. Exponential inter-arrival times

The exponential probability density function is the function of failure versus time (t) with (Θ) equal to the mean-time-between-failures. The probability of failure in the interval (o, t) is given by,

$$\int_{o}^{t} \frac{e^{-\frac{t}{\Theta}}}{\Theta}\,dt = 1 - e^{-\frac{t}{\Theta}} \quad \ldots\ldots\ldots \quad (3)$$

Normal

The normal probability density function is given by,

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}}\; e^{-(t-\Theta)^2/2\sigma^2} \quad \ldots\ldots \quad (4)$$

$0 < t < \infty$

Where;

t = random time for mortality

Θ = mean life

σ = standard deviation

Notice that the regular restriction on the random variable (t) would be $-\infty < t < +\infty$, but in this application less than zero time to life expiration is not appropriate.

The corresponding cumulative distribution function for the normal is;

$$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{t}^{\infty} e^{-(t-\Theta)^2/2\sigma^2}\,dt \quad \ldots\ldots \quad (5)$$

Weibull

The weibull probability density function is given by,

$$f(t) = \left(\frac{\beta}{N}\right)\left(\frac{t}{N}\right)^{\beta-1} e^{-\left(\frac{t}{N}\right)^{\beta}} \quad \ldots\ldots\ldots \quad (6)$$

for $t,\ N,\ \beta > 0$

= 0 otherwise

where;

β = the shape parameter

N = a scale parameter (also known as the characteristic life)

t = random time for mortality

Some authors reflect $(t - \gamma)$ in place of t above, where γ is a location parameter corresponding to an assumed time, prior to which no failures will occur. All application in this model assumes $\gamma = 0$, resulting in no restriction for first mortality. Further, other authors show a scale parameter α related to N by, $N = \alpha^{1/\beta}$, α^{b}. This form is slightly more cumbersome to work with thus the choice of N as the scale parameter with no loss of precision.

The moments of the weibull distribution being,

$$\text{mean} = \gamma + N\,\Gamma\left(\frac{1}{\beta} + 1\right)$$

$$\text{standard deviation} = N\left[\Gamma\left(\frac{2}{\beta} + 1\right) - \Gamma^{2}\left(\frac{1}{\beta} + 1\right)\right]^{\frac{1}{2}}$$

The corresponding cumulative distribution function for the weibull is,

$$F(t) = 1 - e^{-\frac{t}{N}^{\beta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

for $t, N, \beta > 0$

= 0 otherwise

Gamma

The Gamma probability density function is given by,

$$f(t) = \frac{1}{\Gamma(\beta - 1)\,N^{\beta}}\, t^{\beta-1}\, e^{-\frac{t}{N}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

$t > 0$

Where;

β = shape parameter

N = scale parameter

t = random time for mortality

The moments being;

mean = $N(\beta)$

variance $\sigma^2 = N^2(\beta)$

The corresponding cumulative distribution function is given by,

$$F(t) = \int_0^t \frac{1}{\Gamma(\beta-1)\, N^{\beta}}\, t^{\beta-1}\, e^{-\frac{t}{N}}\, dt \quad . \ . \ . \ . \ . \quad (9)$$

$t > 0$

= 0 otherwise

Log Normal

The Log Normal probability density function is;

$$f(t) = \frac{1}{\sigma \sqrt{2\pi}\; t}\, e^{-(\ln \theta\, t)^2/2\sigma^2} \quad . \ . \ . \ . \ . \quad (10)$$

$t > 0$

= 0 otherwise

Where;

t = random time for mortality

θ = mean of the logarithm of (t)

σ = standard deviation of the logarithm of (t)

The corresponding cumulative distribution function is given by,

$$F(t) = 1 - \frac{1}{\sigma \sqrt{2\pi}} \int_t^{\infty} \frac{1}{t}\, e^{-(\ln \theta\, t)^2/2\sigma^2} \quad . \ . \quad (11)$$

Generation of Random Variables

Approach

Within the existing capability of the General Purpose Systems Simulator II (4), are procedures to generate uniform, exponential and normal random variables. This paper extends this capability to the generation of weibull, gamma, and log normal random variables. This additional coverage enables more precision and a greater range of application to hardware items.

In general, there are in existence many possible methods of generating functional variables. The methodology as provided in this thesis is designed to operate specifically within the General Purpose System Simulator II capability.

The computer program contains a uniform random number generator used as the independent variable of any defined function. The value of the generated uniform random number is a fraction greater than 0 but less than 1. For practical purposes, these quantities are equally probable. The methodology for producing functional variables consists of generally defining a specific probability density function $f(t)$ versus t where $f(t)$ is the probability of condition t. In order to generate a specific random variable, the cumulative distribution function $F(t) = \int_0^t f(t)dt$ is evaluated for t in a suitable range. Given the uniform random number, the corresponding value of the function $F(t)$ is selected and the corresponding value of t provided as the desired random variable. Specific examples of this generation are reflected in the discussion on weibull and gamma variables. The appearance of an uniform random number between defined value of t will result in interpolation within the program.

Weibull

A weibull random variable (W) may be generated perhaps in several ways depending on the available computer equipment and the desired precision. One particular advantage to the methodology employed below is the generality of the function. At any point in the simulation model a weibull variable from a parent population with a different mean value may be generated. The generation of weibull variables in this model is accomplished as follows:

Beginning with the weibull cumulative distribution function equation (7);

$$F(t) = 1 - e^{-\left(\frac{t}{N}\right)^{\beta}}$$

an appropriate shape parameter (β) is selected.
In this model values of $\beta = 2$ and $\beta = 3$ were selected as most representative of actual item failure experience. Notice that a value of $\beta = 1$ reduces the expression to the exponential.

If N, the characteristic life, is set equal to unity, and an all inclusive range of F(t) defined, t can then be evaluated. An inclusive range refers to the desired precision which is increased by providing additional values of F(t) in intervals where the rate of change in the value of the function is greatest.

An example of the unitized function is shown below with N = 1, and $\beta = 2$.

$$F(t) = 1 - e^{-t^2}$$

Let F(t), which now has corresponding values of the uniform random numbers between 0 and 1, assume a value equal to 0.39346; now $t^2 = 0.50$, and t is evaluated equal to 0.7071. This result is a particular random value of the weibull unitized function.

At the point of application in the simulation model the random value of t (0.7071) is multiplied by the desired mean life, divided by the quantity $\Gamma(\frac{1}{\beta} + 1)$. Refer to equation (6) and (7), pages 12 and 13.

Given that mean life is,

$$\theta = \gamma + \Gamma(\frac{1}{\beta} + 1)N \quad \text{then,}$$

$$\frac{\theta}{\gamma + \Gamma(\frac{1}{\beta} + 1)} = N \quad \text{(the characteristic life)}$$

Assume that random values are desired from a population with a mean life equal to 130 hours. Recall that N was unitized in the generation function and now at point of use must be operated upon. Using the example uniform random number 0.39346 as above a weibull random variable is generated as follows:

The corresponding F(t) value to 0.39346 yields a value of t = 0.7071;

$$\text{therefore, } W = (t)\frac{\theta}{\Gamma(\frac{1}{\beta} + 1)} = 0.7071\ \frac{130}{0.8862} = 103.9$$

This quantity 103.9 is a random weibull variable from a population with a mean value equal to 130 and shape parameter $\beta = 2$.

Fifty-nine values of the weibull generating function are given in Table 7 in the Appendix, page 42. This data is presented in the same format as provided as input to the simulation program. The first value given is function X followed by its corresponding value function Y . Consequently, the first column is function X values followed by function Y repeating in this manner throughout the display.

Also in the Appendix, pages 43 and 44, Tables 8 and 9, are two example outputs of the weibull generating function. A distribution table of 10,135 (W) variables with a theoretical population mean of 130, $\beta = 2$, and standard deviation equal to 68, and a second table of 10,002 values with a mean equal to 130, and $\beta = 3$ with a standard deviation equal to 48.

Poisson Process

If during the operational period of an item's life the probability of failure is poisson and the failure rate is essentially constant, it is known as a poisson process. The inter-arrival times of failures during this period are distributed as an exponential distribution; see Figure 4, page 11. In most cases of a poisson process simulation can be accomplished by selecting random values from the exponential distribution to decide when an item will fail. On some small recoverable items it is operationally desirable to wait until two or more failures have occured before initiating maintenance action. In this case the exponential distribution fails to adequately describe the times between two or more failures. The gamma distribution is used in this simulation model in those cases where two or more items are held until maintenance action is taken.

It can be shown that the distribution of inter-arrival times between two or more items in a poisson process results in a gamma distribution. Beginning with a summation of the poisson, Wadsworth (7), has shown that the resulting distribution function of time is a gamma distribution.

Let T be the time that is observed until exactly c failures have occurred, where c is a fixed positive interger. Now T is a random variable with a distribution function:

$$G(t_0) = \Pr(T \leq t_0) = 1 - \Pr(T > t_0)$$

$$\Pr(T > t_0) = \sum_{X=0}^{c-1} \frac{(\lambda t_0)^X e^{-\lambda t_0}}{X!}$$

which is a summation of the poisson.

It can be shown by mathematical induction that,

$$\sum_{X=0}^{c-1} \frac{(\lambda t_0)^X e^{-\lambda t_0}}{X!} = \int_{t_0}^{\infty} \frac{Z^{c-1} e^{-Z}}{(c-1)!} dz$$

Now for a $t_0 > 0$

$$G(t_0) = 1 - \int_{t_0}^{\infty} \frac{Z^{c-1} e^{-Z}}{(c-1)!} dz = \int_0^{\lambda t_0} \frac{Z^{c-1} e^{-Z}}{\Gamma c} dz$$

substituting $Z = \lambda y$, for the variable of integration in the integral,

now,

$$G(t_0) = \int_0^{t_0} \frac{\lambda^c y^{c-1} e^{-\lambda y}}{\Gamma(c)} dy \qquad t_0 > 0$$

and the probability distribution function of T is,

$$g(t_0) = \frac{dG(t_0)}{dy} = \frac{\lambda^c t_0^{c-1} e^{-\lambda t_0}}{\Gamma(c)} \qquad 0 < t_0 < \infty$$

$= 0$ otherwise.

It is seen that this is a gamma distribution with shape parameter, $\beta = c$, and N, the characteristic life, equal to $1/\lambda$.

Letting the shape parameter $\beta = 1$ which is equivalent to $c=1$, the function reduces to the exponential in the case of an interarrival time for one item in a poisson process.

Gamma Variables

Using a similar approach to the gamma distribution as was used with the weibull, a unitized gamma function can be derived.

Starting with the cumulative distribution function, equation (9),

$$F(t) = \int_0^t \frac{1}{\Gamma(\beta-1)N^{\beta}}\, t^{\beta-1}\, e^{-\frac{t}{N}}\, dt$$

$t > 0$

This function can only be evaulated by numerical methods unless β is a positive whole number; in the application in this model β is limited to the integral values 2 and 3.

By successive integration by parts the function can be shown to be, (5),

$$F(t) = 1 - \left[1 + \frac{t}{N} + \frac{1}{2!}\left(\frac{t}{N}\right)^2 + \frac{1}{3!}\left(\frac{t}{N}\right)^3 + \ldots + \frac{1}{(\beta-1)!}\left(\frac{t}{N}\right)^{\beta-1}\right] e^{-t}$$

Setting N equal to 1 and letting the shape parameter $\beta = 2$ we obtain;

$$F(t) = 1 - \left[1+t\right] e^{-t}$$

and for $\beta = 3$,

$$F(t) = 1 - \left[1 + t + \frac{1}{2!}(t)^2\right] e^{-t}$$

Now defining a suitable range of F(t) between 0 and 1, and evaluating for t as was done with the weibull distribution, a unitized function which will provide gamma random variables of any desired mean is provided.

As an example, let,

F(t) assume the uniform random number 0.30097.

$F(t) = 1 - \left[1+t\right] e^{-t} = 0.30097.$

Evaluating for t,

$t = 1.1$

Similar to the weibull case, at the point of use in the model, t is multiplied by the desired mean and divided by β. Refer to equation (8) and (9), pages 13 and 14.

Given that the desired mean life is equal to 130, a gamma random variable is generated as follows:

The mean,

$\theta = N\beta$; therefore, $N = \frac{\theta}{\beta}$

Using the example value of F(t) above,

assume a uniform random variable was selected equal to 0.30097, equated to the unitized (G) F(t) yields, t = 1.1. Therefore,

$$(G) = T\left(\frac{\theta}{\beta}\right) = 1.1\left(\frac{130}{2}\right) = 71.5.$$

The value 71.5 is a random gamma variable from a population with a mean equal to 130 and a shape parameter equal to 2. Seventy-seven values of the unitized gamma function were computed for shape parameters 2 as shown in the Appendix, Table 7, page 42. Also shown in the Appendix, page 44 is an example distribution with 10,000(G) variables with a mean of 130 and a shape $\beta = 2$.

Compound Generation

A frequently encountered condition on Air Force recoverable items is a situation where a portion of a set of items is distinctly in the operational life period where chance failures alone occur, while the remaining items are approaching mortality and are subject not only to chance failures but also to wearout. In any given simulation of an item of this type the model must provide for exponential inter-arrival times for chance failure which is independent of item age and wearout failures using one of the mortality distribution functions, i.e., the normal, gamma, or weibull.

The combined effects of random and mortality failures are simulated in the model by dual Monte-Carlo generation of failure. Random selection of exponential times till chance failure occurs is assigned to all items continually for failure after failure throughout any simulation period. For that subset of items that will enter the wearout period random selection from one of the mortality distribution is assigned and as the mortality occurs in the simulation the item is removed from the system and a new item takes its place.

A flow diagram using the coded blocks of the simulation language is contained in the Appendix, page 45. This diagram portrays the flow of transaction through the simulation model. The diagram reflects only that element of the program where compound generation is accomplished. It can be seen that in general as transactions approach this element of the program they are all assigned a time to fail, then are tested to determine to which subset they belong. If they belong to the wearout subset they are assigned a time to wearout. The transactions are then held simulating application or use and remain in this state until their respective times till failure or wearout occurs in the model. They then depart from their held position which simulates the occurrence of failure and are replaced by a transaction obtained from storage which is simulating a stock level.

RESULTS

Specific inventory problems have been defined and examined to demonstrate the use of the simulation model as a technique for decision making. It is not the intent of this section to provide complete and conclusive answers to specific inventory problems, but to establish the fact that simulation runs can be used by operation logistics personnel to evaluate stock levels and other inventory decision rules. For this reason, only a small number of items are selected representing perhaps a narrow range of the total system stock.

Three particular hardware items have been selected as typical examples of operation of the simulation model. The detailed input parameters are described and the resulting outputs from the simulation model are discussed. The items will be referred to as items A, B, and C to prevent violation of security regulations.

Item A is a relatively high cost, recoverable item, with a high demand rate. The question to be investigated by the model is: Given a set quantity of spare items, what is the best distribution of these spares among four operational bases and a support depot? Refer to Table 1, page 25.

Item B is an extremely high cost, low demand, recoverable item. This item is typical when compared to the life cycle pattern. It is subject to random failures, which are capable of being repaired, for a relatively long period of its life but finally criteria such as metal fatigue, wear, and other life expectation elements take over to cause mortality. The objective of the simulation run is to define a set of spare stock levels at base which will meet a desired percent fill rate. Refer to Table 3, page 28.

Item C is a medium cost item, with infrequent demands for repair but is subject to wearout after about two thousand hours of application. The objective of the simulation run is to evaluate the effect on spares levels if the repair is accomplished at the depot or at the bases. Refer to Table 5, page 31.

TABLE 1

MODEL PARAMETERS FOR RUN NUMBER 1

Spare Item	A
Unit Cost	$17,326.00
Total Items	174
Operating	(144)
Spares	(30)
Subject to Wearout	(10)
Duration of Simulation Period	2000 Days
Number of Bases	4
Transportation Time (Depot to Base)	
#1	240 Hours
#2	192 Hours
#3	192 Hours
#4	192 Hours
Repair Facility Flow Time	
Depot	240 ± 20
Base #1	120 ± 20
#2	120 ± 20
#3	120 ± 20
#4	120 ± 20
Stock Levels	(Initial Run)
Depot	4
Base #1	6
#2	4
#3	4
#4	4
Percent Repair (Base-Depot)	90-10
Repair Capacity	
Depot	4 Units
Base #1	2 Units
#2	2 Units
#3	2 Units
#4	2 Units
Failure Rates (MTBF)	Exponential
Base #1	75 Hours
#2	75 Hours
#3	100 Hours
#4	125 Hours

Wearout Rates (MTTW) "Normal" Mean = 5670 Hours = Std Deviation = 480 Hrs
Procurement Lead Time - Normal - Mean = 1420 Hours Std Deviation = 240 Hrs
Quantity (Condemnation Before Procurement) 6

TABLE 2

SIMULATION RESULTS
2,000 Days Simulation

ITEM A

RUN #1

1st Result

	Stock Level	% Time Spares Available	Nr of Failures	MTBF Hours
DEPOT	4	100.0	0	
BASE #1	6	98.5	653	70.5
#2	4	95.0	572	77.4
#3	4	94.0	444	98.8
#4	4	90.5	347	122.0

	Spares Available			Total Repairs	Avg Repair Time Hr	Max Awaiting Repair
	Min	Avg	Max			
DEPOT	2	3.77	8	188	240.5	1
BASE #1	0	5.92	12	589	119.7	10
#2	0	3.45	7	514	118.8	5
#3	0	3.19	6	403	114.9	3
#4	0	2.30	5	314	119.6	3

TABLE 2 (continued)

SIMULATION RESULTS
2,000 Days Simulation

ITEM A

RUN #1

2nd Result

	Stock Level	% Time Spares Available	Nr of Failures	MTBF Hours
DEPOT	4	100.0	0	
BASE #1	5	96.1	642	73.2
#2	4	95.1	580	76.1
#3	4	94.5	438	101.2
#4	5	94.9	342	124.9

	Spares Available			Total Repairs	Avg Repair Time Hr	Max Awaiting Repair
	Min	Avg	Max			
DEPOT	1	2.99	8	193	243.1	2
BASE #1	0	4.31	8	590	118.6	8
#2	0	3.53	6	505	120.2	4
#3	0	3.61	6	421	122.1	4
#4	0	3.30	7	331	119.9	5

TABLE 3

MODEL PARAMETERS FOR RUN NUMBER 2

Spare Item	B
Unit Cost	$43,260.00
Total Items	310 and 388
Operating	288
Spares	Variable (22 and 100)
Subject to Wearout	
Duration of Simulation Period	5 Years
Number of Bases	4
Transportation Time (Depot to Base)	
#1	288 Hours
#2	264 Hours
#3	264 Hours
#4	264 Hours
Repair Facility Flow Time	
Depot	480 ± 48
Base #1	600 ± 120
#2	600 ± 120
#3	600 ± 120
#4	600 ± 120
Stock Levels	1 2 3
Depot	4 1
Base #1	3 5
#2	3 5
#3	3 5
#4	3 5
Percent Repair (Base-Depot)	80-20
Repair Capacity	
Depot	6 Units
Base #1	2 Units
#2	2 Units
#3	2 Units
#4	2 Units
Failure Rates (MTBF)	Exponential
Base #1	225
#2	225
#3	225
#4	225
Wearout Rates (MTTW) "Gamma" Mean =	2525 Hours, Shape Parameter $\beta = 2$
Procurement Lead Time	2160 ± 250
Quantity (Condemnation Before Procurement)	1

TABLE 4

SIMULATION RESULTS
2,000 Days Simulation

ITEM B

RUN #2

1st Result

	Stock Level	% Time Spares Available	Nr of Failures	MTBF Hours
DEPOT	4	100.0	0	
BASE #1	3	66.7	2486	221
#2	3	6.1	290	218
#3	3	2.1	113	227
#4	3	2.1	117	229

	Spares Available			Total Repairs	Avg Repair Time Hr	Max Awaiting Repair
	Min	Avg	Max			
DEPOT	0	1.10	4	45	483	24
BASE #1	0	0	3	50	592	34
#2	0	0	3	38	591	29
#3	0	0	3	36	604	28
#4	0	0	3	32	589	27

TABLE 4 (continued)

SIMULATION RESULTS
2,000 Days Simulation

ITEM B

RUN #2

2nd Result

	Stock Level	% Time Spares Available	Nr of Failures	MTBF Hours
DEPOT	1	85.2	0	
BASE #1	5	91.2	2675	226
#2	5	90.3	2542	228
#3	5	88.3	2371	221
#4	5	90.5	2531	218

	Spares Available			Total Repairs	Avg Repair Time Hr	Max Awaiting Repair
	Min	Avg	Max			
DEPOT	0	0.7	2	38	486.1	23
BASE #1	0	2.3	6	58	593.9	25
#2	0	2.1	5	59	602.4	24
#3	0	2.6	7	62	598.3	22
#4	0	2.0	5	63	600.1	24

TABLE 5

MODEL PARAMETERS FOR RUN NUMBER 3

Spare Item	C	
Unit Cost	$4500.00	
Total Items	775	675
Operating	550	
Spares	225	
Subject to Wearout	675	
Duration of Simulation Period	2 Years	
Number of Bases	4	
Transportation Time (Depot to Base)		
#1	240 Hours	
#2	192 Hours	
#3	192 Hours	
#4	192 Hours	
Repair Facility Flow Time		
Depot	480 ± 100	480 ± 100
Base #1	520 ± 80	0
#2	520 ± 80	0
#3	520 ± 80	0
#4	520 ± 80	0
Stock Levels		
Depot	10	
Base #1	2	
#2	2	
#3	2	
#4	2	
Percent Repair (Base-Depot)		
Repair Capacity	89 - 20	0 - 100
Depot	5 Units	10 Units
Base #1	2 Units	0 Units
#2	2 Units	0 Units
#3	2 Units	0 Units
#4	2 Units	0 Units
Failure Rates (MTBF)	Exponential	
Base #1	596	
#2	596	
#3	596	
#4	596	
Wearout Rates (MTTW) "Weibull" Mean	= 2000 = β = 3	
Procurement Lead Time	1000 ± 150	
Quantity (Condemnation Before Procurement)	1	

TABLE 6

SIMULATION RESULTS
280 Days Simulation

ITEM C

RUN #3

1st Result

	Stock Level	% Time Spares Available	Nr of Failures	MTBF Hours
DEPOT	10	4.0		
BASE #1	2	88.0	1008	420
#2	2	55.0	621	515
#3	2	6.0	232	582
#4	2	5.0	240	578

	Spares Available			Total Repairs	Avg Repair Time Hr	Max Awaiting Repair
	Min	Avg	Max			
DEPOT	0	1.2	0	229	403	29
BASE #1	0	0.5	2	226	507	22
#2	0	0.1	2	227	492	15
#3	0	0.2	2	228	475	5
#4	0	0.3	2	227	493	5

SIMULATION RESULTS
280 Days Simulation

ITEM C

RUN #3

2nd Result

	Stock Level	% Time Spares Available	Nr of Failures	MTBF Hours
DEPOT	10	93	----	---
BASE #1	6	95	1121	415
#2	6	94	1085	426
#3	6	92	1112	412
#4	6	96	1093	426

	Spares Available			Total Repairs	Avg Repair Time Hr	Max Awaiting Repair
	Min	Avg	Max			
DEPOT	0		10	3240	482	231
BASE #1	0		8			
#2	0		7			
#3	0		9			
#4	0		8			

The results of two separate operations of the model are reflected for each of the items A, B, and C. Refer to Tables 2, 3, and 4 for a summary of the simulation results.

It can be seen that for item A, spare stock should be distributed as finally shown in the second result to achieve approximately a 95% availability of spares over all bases. Outstanding in the first result is the fact that only 90% availability of spares occurred at Base #4. Further, it is noteworthy at Base #1 that spares reached a zero level, and also fluctuated to a maximum of 12 units at one point in time in the simulation. This fact demonstrates the occurrence of a wide variability of available spares. However, the system still requires a spares stock position of six units to maintain a high level of fill rate over a long period of time.

The results for item B shows in the second operation of the model that an additional 78 spare items are required and that a spares level increases from three to five units provides for an approximate overall fill rate of 90%. Notice the first results which included 22 total spares and stock level of three spares for each base, indicated extremely low fill rates, 66% for Base #1, 6.1% for Base #2, 2.1% for Base #3, and 2.1% for Base #4. The large quantity of units awaiting repair suggest that improvement in repair times could result in less total spare requirements. The decision to increase capability of repair would require further analysis comparing the cost of increased spares with the cost of increased repair facilities.

The results for item C indicate that with a base repair capability and a total of 125 spares in the system a low unsatisfactory percent of spares availability occurs. The second result reflects that by increasing to a large quantity of 225 spares and further utilizing only depot repair capability near 95% availability of years is possible. However, the large queue of 231 units awaiting repair suggest that the total spares requirement could be reduced significantly if improvement in depot repair time could be achieved. Subsequent runs of the model would further evaluate this condition and establish the proper ratio for a balanced condition.

DISCUSSION

The ever increasing complexity of military operations due to rapidly changing technology under the constraints of limited resources has forced managers to look for new methods by which more alternatives can be considered in making logistics decisions. Some of the recent techniques of logistics evaluation has been the construction of mathematical models to represent some elements of a real operating situation. A computer manipulation of these models is known as simulation. It enables the user to compress time and examine the effects of changes in variables incorporated in the model over simulated time. In this thesis one type of simulation was examined in an attempt to establish this method of evaluation as a useful tool for operating managers.

Items selected as samples to test the simulation model are high cost, high activity items, wherein considerable dollar investments are necessarily involved in any policy decision. Precision in simulating the true environmental effects of these items becomes absolutely necessary and the precise statistical models utilized to simulate failure and wearout are obviously an integral part of the model. As a result of analyzing the output from this simulation model it is concluded that an inventory manager could see the impact in terms of percent of time spares are available, percent of time stockout occurs, the minimum and maximum quantities available, the effects of repair times and capacities, when he applies the various decision rules.

The expanded use of this modelling technique has the potential to give managers a clearer understanding of the alternative courses of action or policies in complex problem situations involving risk and uncertainity where analytical methods cannot be used.

SUMMARY

This paper has presented a simulation model with capability to consider both random and mortality failures for application to the decision making process by managers of logistics functions. The results suggest that computer simulation is a tool for managers that is available for perhaps a wide range of logistics problems where analytical solutions cannot be formed in a reasonable time, and where it is desirable to test decision rules prior to deployment. Since this paper only covered an inventory use of computer simulation in logistics management an examination of other forms of simulation and other problems may establish a basis where these techniques would prove profitable to many types logistics management problems.

An obstacle that is limiting the use of simulation is now perhaps being overcome. This obstacle is the substantial time, cost, and effort needed to develop the statistical models and prepare the computer programs. This paper presented a model programmed in General Purpose Systems Simulation II (GPSS II) language. There are now new programming languages being developed to aid this process, i.e., SIMSCRIPT, SIMPACT, and DYNAMO.

Development of simulation should proceed within an organized framework of construction or within a master plan to permit later integration of subsystems into a higher order system simulation. The system must be simulated with as many interrelationships as exists in the real operational world. If these techniques can reduce the quantity of an item stocked or eliminate stocking of an item, then the dollar savings realized would more than offset the increased cost of applying more intensive and improved inventory decision techniques.

It is recommended that before a decision is made to adjust the stock levels certain costs must also be considered. For example, consideration must be given to the possibilities of deferring procurement action, calculation of the risk of a stockout versus holding costs, expected procurement lead time, expected life duration, and equipment cost versus cost of repair. As a result of these additional cost considerations a more economic inventory level of high cost items will result.

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APPENDIX

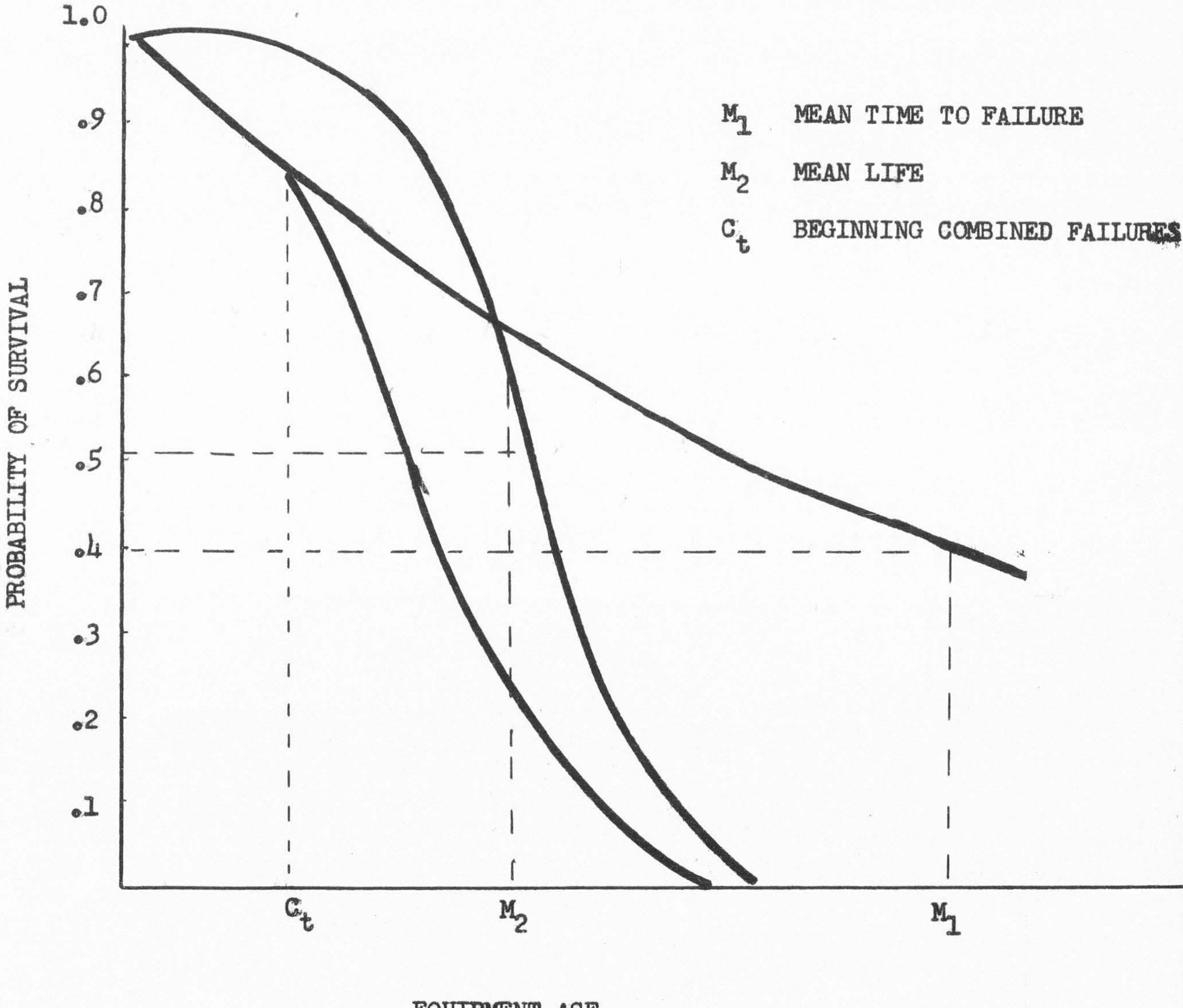

Figure 5. Combined effects of random failures and wearouts

TABLE 7

GENERATING FUNCTIONS

```
 12      FUNCTION          RN1     C59          WEIBULL       B 3
0       0    .00099.10000.00199.12599.00399.15874.00598.18171.00797.20000
.00896.20800.00994.21544.04877.36840.09516.46415.13929.53132.18126.58480
.22119.62996.25918.66943.29531.70472.32967.76380.36237.76630.39346.79370
.42304.81932.45118.84343.47795.86623.50341.88790.52763.90856.55067.92831
.57258.94726.59342.96548.61325.98304.632121.0000.650061.0163.667121.0322
.683361.0476.698811.0626.713491.0772.727461.0913.740751.1052.753401.1186
.765421.1318.776861.1447.798101.1696.817311.1934.834701.2164.850431.2385
.864661.2599.877541.2805.889191.3005.899741.3200.917911.3572.932791.3924
.944971.4260.950211.4422.969801.5182.981681.5874.988891.6509.993261.7099
.997521.8171.999081.9129.999662.0000.999872.0800.999952.1544
 13      FUNCTION          RN1   C59          WEIBULL        B 2
0       0    .00099.03162.00199.04472.00399.06324.00598.07745.00797.08944
.00896.09487.00994.10000.04877.22360.09516.31622.13929.38729.18126.44721
.22119.50000.25918.54772.29531.59160.32967.63245.36237.67082.39346.70710
.42304.76161.45118.77459.47795.80622.50341.83666.52763.86602.55067.89442
.57258.92195.59342.94868.61325.97467.632121.0000.650061.0247.667121.0488
.683361.0724.698811.0954.713491.1130.727461.1402.740751.1619.753401.1832
.765421.2042.776861.2247.798101.2649.817311.3038.834701.3416.850431.3784
.864661.4142.877541.4491.889191.4832.899741.5166.917911.5811.932791.6432
.944971.7029.950211.7321.969801.8708.981682.0000.988892.1213.993262.2361
.997522.4495.999082.6457.999662.8284.999873.0000.999953.1623
 JOB             GAMMA ANALYSIS
 22    FUNCTION     RN1, C77        GAMMA B 2 X 100
0      0       .00005 1.0  .00121 5.0  .0046810.0  .0175220.0  .0369430.0
.0615540.0  .0902150.0  .1057355.0  .1219060.0  .1386265.0  .1557970.0
.1733575.0  .1912180.0  .2092985.0  .2275290.0  .26424100.0 .30097110.0
.33738120.0 .37318130.0 .40816140.0 .44217150.0 .47506160.0 .50676170.0
.53716180.0 .56625190.0 .59398200.0 .62037210.0 .64544220.0 .66914230.0
.69155240.0 .71272250.0 .73263260.0 .75132270.0 .76892280.0 .78542290.0
.80084300.0 .81529310.0 .82881320.0 .84142330.0 .85317340.0 .86410350.0
.87433360.0 .88382370.0 .89262380.0 .90082390.0 .90840400.0 .91549410.0
.92200420.0 .92808430.0 .93374440.0 .93889450.0 .94372460.0 .94813470.0
.95227480.0 .95605490.0 .95956500.0 .96279510.0 .96578520.0 .96856530.0
.97107540.0 .97342550.0 .97558560.0 .97756570.0 .97939580.0 .98109590.0
.98264600.0 .98601625.0 .98875650.0 .99093675.0 .99272700.0 .99532750.0
.99694800.0 .99810850.0 .99880900.0 .99926950.0 .999451000.0
```

TABLE 8

OUTPUT

WEIBULL GENERATING FUNCTION

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION
10135	129.958	68.938

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE
0	0	.00	.0
10	65	.64	.6
20	183	1.81	2.4
30	226	2.23	4.7
40	333	3.29	8.0
50	369	3.64	11.6
60	464	4.58	16.2
70	496	4.89	21.1
80	509	5.02	26.1
90	597	5.89	32.0
100	589	5.81	37.8
110	588	5.80	43.6
120	569	5.61	49.2
130	563	5.56	54.8
140	528	5.21	60.0
150	520	4.63	65.1
160	469	4.63	69.7
170	410	4.05	73.8
180	407	4.02	77.8
190	358	3.53	81.3
200	300	2.96	84.3
210	254	2.51	86.8
220	235	2.32	89.1
230	209	2.06	91.2
240	187	1.85	93.0
250	154	1.52	94.5
260	101	1.00	95.5
270	96	.95	96.5
280	77	.76	97.2
290	64	.63	97.9
300	51	.50	98.4
OVERFLOW	164	1.62	100.0

TABLE 9

OUTPUT

WEIBULL GENERATING FUNCTION

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION
10002	131.680	48.148

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE
0	0	.00	.0
10	11	.11	.1
20	24	.24	.3
30	62	.62	1.0
40	163	1.63	2.6
50	165	1.65	4.2
60	269	2.69	6.9
70	366	3.66	10.6
80	462	4.62	15.2
90	564	5.64	20.9
100	649	6.49	27.3
110	437	4.37	31.7
120	1019	10.19	41.9
130	733	7.33	49.2
140	800	8.00	57.2
150	780	7.80	65.0
160	765	7.65	72.7
170	650	6.50	79.2
180	507	5.07	84.2
190	460	4.60	88.8
200	327	3.27	92.1
210	256	2.56	94.7
220	178	1.78	96.5
230	119	1.19	97.6
240	96	.96	98.6
250	54	.54	99.1
260	27	.27	99.4
270	31	.31	99.7
280	14	.14	99.9
290	10	.10	100.0
300	4	.04	100.0

REMAINING FREQUENCIES ARE ALL ZERO

TABLE 10

COMPOUND GENERATION
SIMULATION FLOW DIAGRAM

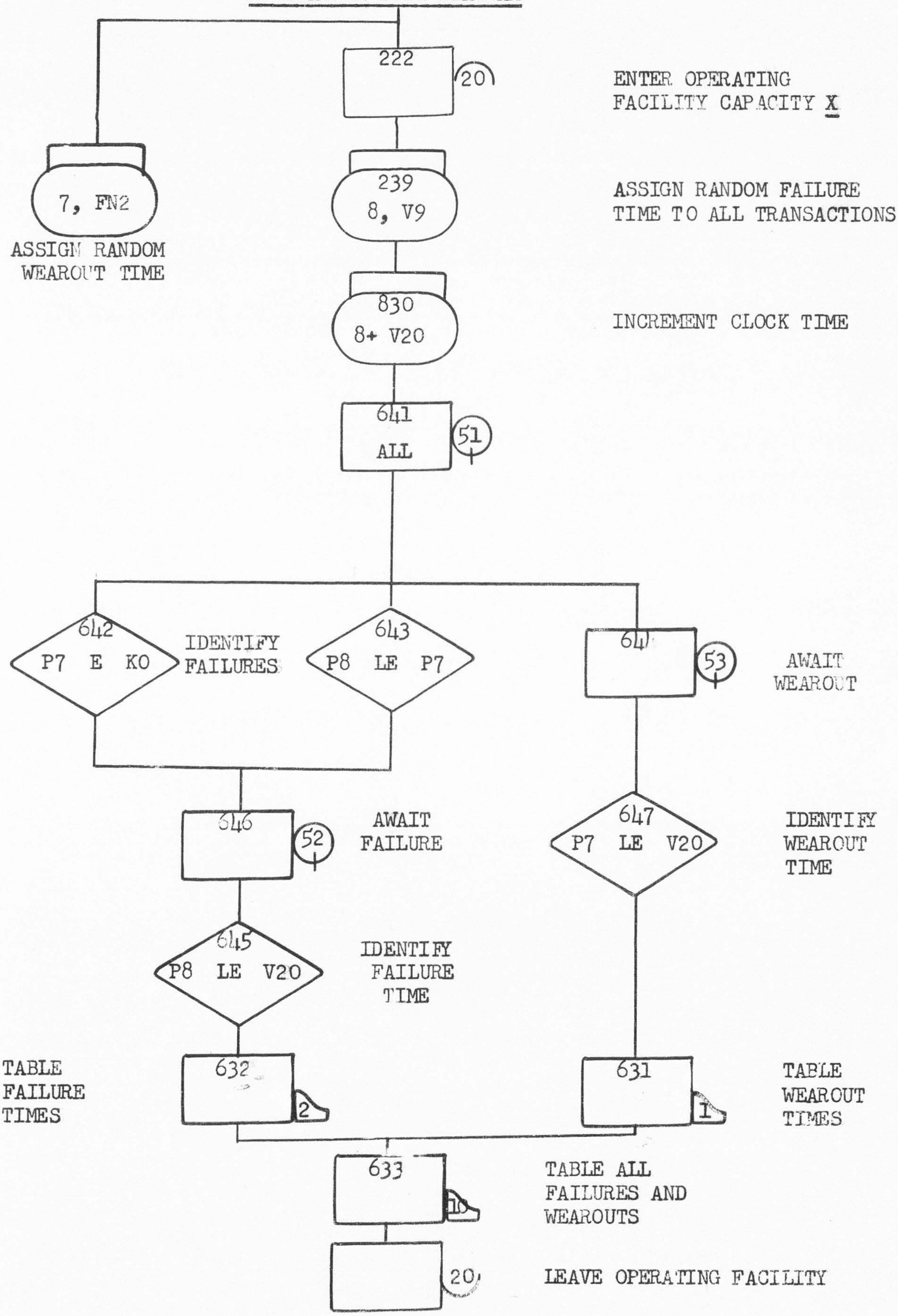

TABLE 11

CARD CHANGES OF MODEL PARAMETERS

Run Number 2 Item B

JOB NR 2 GAMMA WEAROUT

22 FUNCTION RN1 C77 GAMMA B2 x 100

	.00005 1.0	.00121 5.0	.0046810.0	.0175220.0	.0369430.0
.0615540.0	.0902150.0	.1057355.0	.1219060.0	.1386265.0	.1557970.0
.1733575.0	.1912180.0	.2092985.0	.2275290.0	.26424100.0	.30097110.0
.33738120.0	.37318130.0	.40816140.0	.44217150.0	.47506160.0	.50676170.0
.53716180.0	.56625190.0	.59398200.0	.62037210.0	.64544220.0	.66914230.0
.69155240.0	.71272250.0	.73263260.0	.75132270.0	.76892280.0	.78542290.0
.80084300.0	.81529310.0	.82881320.0	.84142330.0	.85317340.0	.86410350.0
.87433360.0	.88382370.0	.89262380.0	.90082390.0	.90840400.0	.91549410.0
.92200420.0	.92808430.0	.93374440.0	.93889450.0	.94372460.0	.94813470.0
.95227480.0	.95605490.0	.95956500.0	.96279510.0	.96578520.0	.96856530.0
.97107540.0	.97342550.0	.97558560.0	.97756570.0	.97939580.0	.98109590.0
.98264600.0	.98601625.0	.98875650.0	.99093675.0	.99272700.0	.995327
.99694800.0	.99810850.0	.99880900.0	.99926950.0	.999451000.0	

9 VARIABLE FN5*K225&1000

22 VARIABLE FN22*1263&K100

1 CAPACITY 4

2 CAPACITY 3

3 CAPACITY 3

4 CAPACITY 3

5 CAPACITY 3

TABLE 11 (continued)

16	CAPACITY	6						
20	CAPACITY	72						
30	CAPACITY	72						
40	CAPACITY	72						
50	CAPACITY	72						
2	QUEUE	99			3			
5	GENERATE	155	1		6			
6	ASSIGN	7	V22		15			
10	GENERATE	155	1		15			
11	ASSIGN	1	K288		12			
12	ASSIGN	2	K264		13			
13	ASSIGN	3	K264		14			
14	ASSIGN	4	K264		20			
170	ENTER	16			180		480	48
46	SPLIT				711	790		
790	ADVANCE				791		2160	250
791	PRIORITY	1			610			
634	ASSIGN	4	KO		15			
250	ADVANCE			.200	260	160		
270	ENTER	9			280		600	120
350	ADVANCE			.200	360	160		
370	ENTER	10			380		600	120
439	ASSIGN	8	V9		130			

TABLE 12

CARD CHANGES OF MODEL PARAMETERS

Run Number 3 Item C

NR 3 WEIBULL WEAROUT

JOB							
9	VARIABLE	FN5*K596&1000					
23	VARIABLE	FN12*2257					
1	CAPACITY	10					
2	CAPACITY	2					
3	CAPACITY	2					
4	CAPACITY	2					
5	CAPACITY	2					
20	CAPACITY	137					
30	CAPACITY	137					
40	CAPACITY	138					
50	CAPACITY	138					
2	QUEUE	99		3			
5	GENERATE	675	1	6			
6	ASSIGN	7	V23	15			
17	ENTER	16		180		480	100
46	SPLIT			711	790		
634	ASSIGN	4	KO	15			
790	ADVANCE			791		1000	150
791	PRIORITY	1		610			
270	ENTER	9		280		520	80
370	ENTER	10		380		520	80
470	ENTER	11		480		520	80
570	ENTER	12		580		520	80

COMPUTER PROGRAM

```
 LOC  NAME        X     Y     Z     SEL   NBA     NBB   MEAN  MOD   REMARKS
      JOB                           BILL  HANSEN  RUN   NR 1  OPT ST
 1    FUNCTION    RN1   C25         NORMAL
0     -50000.00003-40000.00135-30000.00621-25000.02275-20000.06681-15000
.11507-12000.15866-10000.21186-8000 .27425-6000 .34458-4000 .42074-2000
.500000     .579262000  .655424000  .725756000  .788148000  .8413410000
.8849312000 .9331915000 .9772520000 .9937925000 .9986530000 .9999740000
1.0000 50000
 2    FUNCTION    V8    C2
 0    0     99999 99999
 5    FUNCTION    RN1   C24    EXPONENTIAL X 1000
0     0     .1    104   .2    222   .3    355   .4    509   .5    690
.6    915   .7    1200  .75   1380  .8    1600  .84   1830  .88   2120
.9    2300  .92   2520  .94   2810  .95   2990  .96   3200  .97   3500
.98   3900  .99   4600  .995  5300  .998  6200  .999  7000  .9997 8000
 1    CAPACITY    4
 2    CAPACITY    6
 3    CAPACITY    4
 4    CAPACITY    4
 5    CAPACITY    4
 9    CAPACITY    2
 10   CAPACITY    2
 11   CAPACITY    2
 12   CAPACITY    2
 16   CAPACITY    4
 20   CAPACITY    36
 30   CAPACITY    36
 40   CAPACITY    36
 50   CAPACITY    36
     1VARIABLE    P1/K240
     2VARIABLE    P2/K192
     3VARIABLE    P3/K192
     4VARIABLE    P4/K192
     6VARIABLE    FN1*K48000
     7VARIABLE    K10000*K100
     8VARIABLE    V6/V7+K05760
     9VARIABLE    FN5*K75/K1000
    10VARIABLE    FN5*K100/K1000
    11VARIABLE    FN5*K125/K1000
 20   VARIABLE    C1
 1    GENERATE          3                 2
 2    QUEUE       99                      3
 3    ADVANCE                             2                 1
 5    GENERATE          10    1           6
 6    ASSIGN      7     FN2               15
 10   GENERATE          164   1           15
 15   QUEUE       1                 ALL   21      25
 21   GATE        SNF1                    190
 190  PRIORITY    0                       11
 11   ASSIGN      1     K240              12
 12   ASSIGN      2     K192              13
 13   ASSIGN      3     K192              14
```

LOC	NAME	X	Y	Z	SEL	NBA	NBB	MEAN	MOD	REMARKS
14	ASSIGN	4	K192			20				
20	STORE	1			ALL	222	225			
160	QUEUE	10				170				
170	ENTER	16				180		240	20	
180	LEAVE	16				185				
185	SPLIT					191	192			
191	PRIORITY	1				15				
192	TERMINATE	R								
291	PRIORITY	2				200				
200	QUEUE	2				22				
22	STORE	2				222				
41	COMPARE	P7	L	P8		46				
46	SPLIT					710	711			
710	SAVEX	10+	K1		BOTH	701	703			
711	TERMINATE	R								
701	COMPARE	X10	E	K6		702				
702	SAVEX	10	K0			610				
703	TERMINATE									
610	ASSIGN	7	K0			614				
614	ASSIGN	8	K0			620				
620	ASSIGN	1	K0			624				
624	ASSIGN	2	K0			629				
629	ASSIGN	3	K0			634				
634	ASSIGN	4	K0			715				
715	SPLIT					720	725			
720	SPLIT					740	745			
725	SPLIT					750	755			
740	SPLIT					790	790			
745	SPLIT					790	790			
750	SPLIT					790	790			
755	TERMINATE									
790	ADVANCE					791		1420	240	
791	PRIORITY	1				15				
222	ENTER	20				239		*1		
239	ASSIGN	8	V9			630				
630	ASSIGN	8+	V20			641				
641	QUEUE	51				642	644			
642	COMPARE	P7	E	K0		646				
643	COMPARE	P8	LE	P7		646				
644	QUEUE	53				647				
646	QUEUE	52				645				
645	COMPARE	P8	LE	V20		632				
647	COMPARE	P7	LE	V20		631				
631	TABULATE	1				633				
632	TABULATE	2				231				
633	TABULATE	10				240				
231	SAVEX	1+	V1			233				
233	ASSIGN	1	K0			234				
234	ASSIGN	2	K0			235				
235	ASSIGN	3	K0			236				
236	ASSIGN	4	K0			633				
240	LEAVE	20			BOTH	41	661			
661	ASSIGN	8	K0			250				

LOC	NAME	X	Y	Z	SEL	NBA	NBB	MEAN	MOD	REMARKS
250	ADVANCE				.100	260	160			
260	QUEUE	20				270				
270	ENTER	9				280		120	20	
280	LEAVE	9				290				
290	SPLIT					291	292			
292	TERMINATE	R								
391	PRIORITY	2				300				
300	QUEUE	3				23				
23	STORE	3				223				
223	ENTER	30				339		*2		
339	ASSIGN	8	V9			730				
730	ASSIGN	8+	V20			741				
741	QUEUE	54			ALL	742	744			
742	COMPARE	P7	E	KO		747				
743	COMPARE	P8	LE	P7		747				
744	QUEUE	56				748				
747	QUEUE	55				746				
746	COMPARE	P8	LE	V20		732				
748	COMPARE	P7	LE	V20		731				
232	SAVEX	2+	V2			333				
731	TABULATE	3				733				
732	TABULATE	4				232				
733	TABULATE	11				340				
333	ASSIGN	1	KO			334				
334	ASSIGN	2	KO			335				
335	ASSIGN	3	KO			336				
336	ASSIGN	4	KO			733				
340	LEAVE	30			BOTH	41	662			
662	ASSIGN	8	LP			350				
350	ADVANCE				.100	360	160			
360	QUEUE	30				370				
370	ENTER	10				380		120	20	
380	LEAVE	10				390				
390	SPLIT					391	392			
392	TERMINATE	R								
491	PRIORITY	2				400				
400	QUEUE	4				24				
24	STORE	4				224				
224	ENTER	40				439		*3		
439	ASSIGN	8	V10			130				
130	ASSIGN	8+	V20			770				
770	QUEUE	57			ALL	771	773			
771	COMPARE	P7	E	KO		775				
772	COMPARE	P8	LE	P7		775				
775	QUEUE	58				774				
774	COMPARE	P8	LE	V20		132				
773	QUEUE	59				776				
776	COMPARE	P7	LE	V20		131				
50	SAVEX	3+	V3			433				
131	TABULATE	5				133				
132	TABULATE	6				50				
133	TABULATE	12				440				
433	ASSIGN	1	KO			434				

LOC	NAME	X	Y	Z	SEL	NBA	NBB	MEAN	MOD	REMARKS
	434ASSIGN	2	KO			435				
	435ASSIGN	3	KO			436				
	436ASSIGN	4	KO			133				
	440LEAVE	40			BOTH	41	663			
	663ASSIGN	8	KO			450				
450	ADVANCE				.100	460	160			
460	QUEUE	40				470				
470	ENTER	11				480		120	20	
480	LEAVE	11				490				
490	SPLIT					491	492			
492	TERMINATE	R								
591	PRIORITY	2				500				
500	QUEUE	5				25				
25	STORE	5				225				
225	ENTER	50				539		*4		
539	ASSIGN	8	V11			530				
530	ASSIGN	8+	V20			780				
780	QUEUE	61			ALL	781	783			
781	COMPARE	P7	E	KO		785				
782	COMPARE	P8	LE	P7		785				
785	QUEUE	62				784				
784	COMPARE	P8	LE	V20		152				
783	QUEUE	63				786				
54	SAVEX	4+	V4			533				
786	COMPARE	P7	LE	V20		151				
	151TABULATE	7				153				
	152TABULATE	8				54				
	153TABULATE	13				540				
	533ASSIGN	1	KO			534				
	534ASSIGN	2	KO			535				
	535ASSIGN	3	KO			536				
	536ASSIGN	4	KO			153				
	540ASSIGN	50			BOTH	41	664			
	664ASSIGN	8	KO			550				
550	ADVANCE				.100	560	160			
560	QUEUE	50				570				
570	ENTER	12				580		120	20	
580	LEAVE	12				590				
590	SPLIT					591	592			
592	TERMINATE	R								
1	TABLE	IA	0	50	100					
2	TABLE	IA	0	10	100					
3	TABLE	IA	0	50	100					
4	TABLE	IA	0	10	100					
5	TABLE	IA	0	50	100					
6	TABLE	IA	0	10	100					
7	TABLE	IA	0	50	100					
8	TABLE	IA	0	10	100					
10	TABLE	IA	0	50	100					
11	TABLE	IA	0	50	100					
12	TABLE	IA	0	50	100					
13	TABLE	IA	0	50	100					
	START	20								