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Model for Bathtub-Shaped Hazard Rate: Monte Carlo Study

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MODEL FOR BATHTUB-SHAPED HAZARD RATE:

MONTE CARLO STUDY

by

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ABSTRACT

Model for Bathtub-shaped Hazard Rate:

Monte Carlo Study

by

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Utah State University, 1970

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A new model developed for the entire bathtub-shaped hazard rate curve has been evaluated as to its usefulness as a method of reliability estimation. The model is of the form:

$$F(t) = 1 - \exp - (\theta_1 t^L + \theta_2 t + \theta_3 t^M)$$

where "L" and "M" were assumed known.

The estimate of reliability obtained from the new model was compared with the traditional restricted sample estimate for four different time intervals and was found to have less bias and variance for all time points.

This was a monte carlo study and the data generated showed that the new model has much potential as a method for estimating reliability.

(51 pages)

INTRODUCTION

Reliability has become a very commonly used term today, especially with the advent of space travel. As is common with all new sciences, the state of the art is rapidly changing and improving. This thesis is a combination programming and evaluation of a new model for reliability estimation. For this evaluation the estimate of reliability will be an important criterion.

It is known that the plot of the hazard rate (i.e., the rate at which the component population still in test at time "t" is failing (Bazovsky, 1961)) as function of time has the shape like a bathtub for the entire life of the component. It is the bottom, flat section that is traditionally used for the estimation of reliability. This is called a restricted sample estimate because only data for the middle portion of the component's life are used. The new model proposed will utilize the data for the entire component's life. The distribution function for "time to failure" for this model is:

$$F(t) = 1 - \exp - (\Theta_1 t^L + \Theta_2 t + \Theta_3 t^M)$$

For this study the parameters "L" and "M" will be assumed known.

A good deal of the effort will be spent in the development of computer programs, to generate data to enable certain checks and evaluations to be made on the model. Some of the checks to be made will be to vary the parameters "L", "M" and thetas to see if any irregularities are apparent. Estimate of reliability will be calculated and examined for bias and patterns.

The thesis is divided into several sections. The first will consist of definitions and derivations to be used in the development of the model. A description of the computer programs developed for the obtaining and checking of the data is given in the next section. The third section gives results and findings for all the various checks, tests and evaluations performed on the model. Also included in this section is the comparison of the reliability estimates for the model, restricted sample and true value.

DEFINITIONS AND DERIVATIONS

Reliability

Reliability is defined by Bazovsky (1961, p. 14) as "the probability that a component performing its purpose adequately for the period of time intended under the operating conditions encountered." The term component may be used interchangeably with other terms such as item, part, vehicle, or complete system and still maintain the same meaning. Thus the component user is interested in the length of time that he can expect the component to operate without failure or breakdown. For the non-repairable components this means that the "time to failure" is the critical characteristic. For the astronaut the "time to failure" must exceed the mission time. Thus, for these reasons, it is worthwhile to define reliability in terms of the distribution of the "time to failure." The probability density of "time to failure" is:

$$f(t) \qquad t > 0$$

The distribution of "time to failure" or cumulative probability is given as:

$$F(t) = \int_{o}^{t} f(t)\, dt$$

which is the probability of a failure by time "t." Reliability will now be defined as the probability of no failure by time "t."

$$R(t) = 1 - F(t) = \int_{t}^{\infty} f(t)\, dt$$

Hazard rate

With the definition of reliability given, the hazard rate can now be defined as the conditional probability that a component will fail in a unit time interval after "t," given it has not failed before time "t" (Lloyd and Lipow, 1962). Sometimes hazard rate is called instantaneous failure rate or force of mortality and is given as:

$$h(t)\ dt = f(t)\ dt\ /\ R(t)$$

or

$$h(t) = f(t)\ /\ R(t)$$

or the hazard rate can be defined as (Lindgren, 1968)

$$h(t) = \frac{-d\mathrm{Ln}\ R(t)}{dt}$$

Failure periods

The term failure has been used in the previous definitions and we shall now discuss its role in reliability. A perfectly reliable component is one which never fails. A high reliable component would have a low frequency of failures. Therefore, the goal would be to have components failure free, but experience has shown that even the best designed, engineered, tested, and maintained components do fail. Reliability distinguishes between types of failures. These failures are called burn-in, random, and wearout. Each one of these categories defines a distinct operating period in the lifetime of many components. These periods are of varying length in time and experiences. They are definitely related to each item's hazard rate. The three categories or periods are shown in Figure 1.

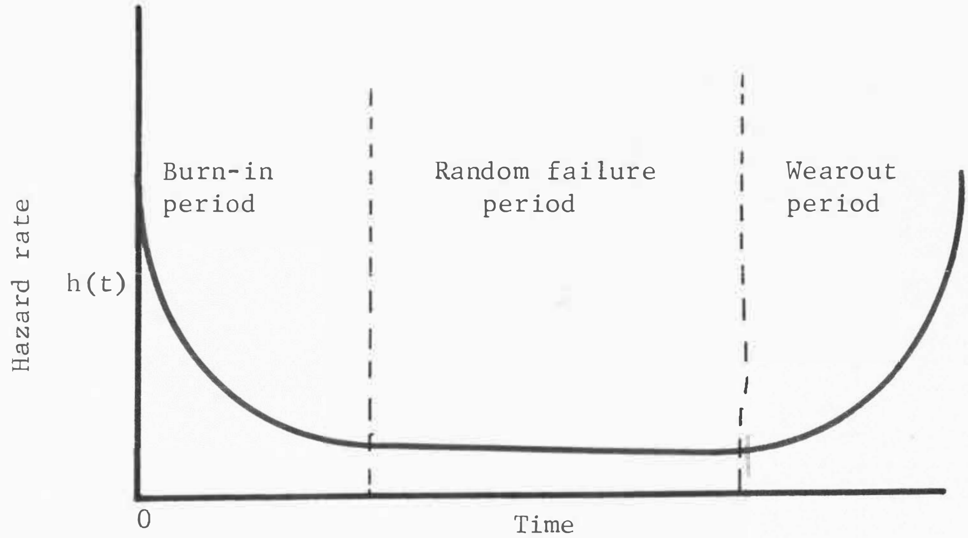

Figure 1. Bathtub-shaped hazard rate curve.

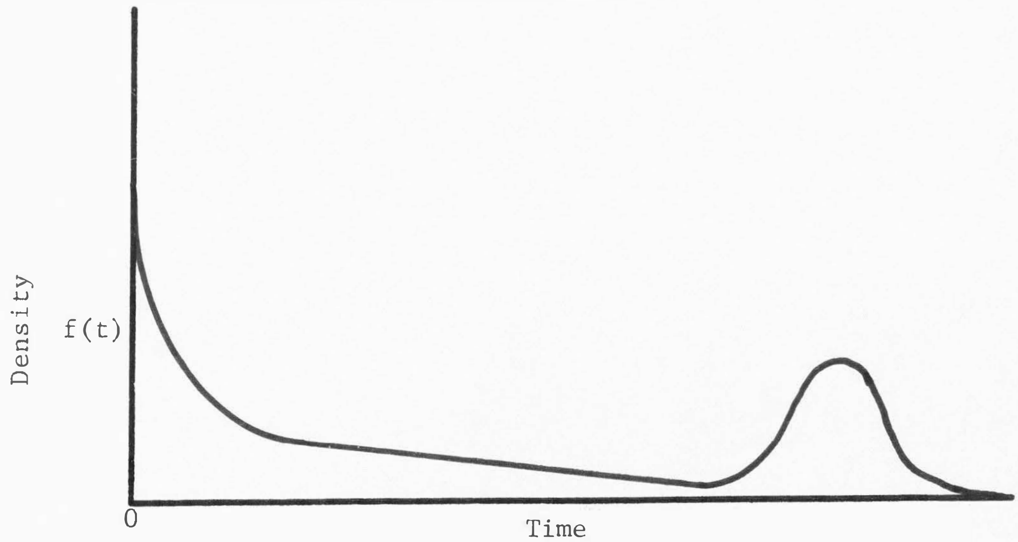

Figure 2. Probability density for bathtub-shaped hazard rate curve.

The model

In order to obtain a mathematical description of a bathtub-shape hazard curve, it must be developed. One such model was proposed by Krohn (1969). He selected an appropriate density for each of the three periods of decreasing, constant and increasing hazard rated as shown in Figure 2. He labeled them $p_1(t)$, $p_2(t)$ and $p_3(t)$ respectively, and used a Wiebull with different shape parameters to represent the periods. With the assumption that only one of the failure causes will occur for each item, each cause will receive a given probability of occurrence: P_1, the probability of failure due to the burn-in period; P_2, probability of failure due to the random period; and P_3, the probability of failure due to the wearout period. Such that:

$$P_1 + P_2 + P_3 = 1$$

A distribution for such a model would be of the form:

$$p(t) = P_1 p_1(t) + P_2 p_2(t) + P_3 p_3(t)$$

and the density function for the above developed model would be of the form:

$$p(t) = P_1 a_1/b_1 t^{a-1} e^{-(t^{a}1/b_1)} + P_2 1/b_2 e^{-(t/b_2)} + P_3 a_3/b_3 t^{a-1} e^{-(t^{a}3/b_3)}$$

where $a_1 < 1$ $a_2 = 1$ and $a_3 > 1$

giving the decreasing, constant and increasing hazard rate respectively. The reliability and hazard rate function can be developed but would be messy.

The above model has too many parameters to be estimated (nine) and is messy and complicated. Thus another model would be more useful.

The proposed model is developed as follows. Investigations in the physics of failure have shown that failures of components may often be attributed to the three failure periods. Within each cause there are potentially many possible failures (Shooman, 1968; Wright, 1968). If we associate with each potential failure a random variable, "time to failure," then the actual failure may be viewed as the minimum value of all those random variables which describe the component. If it is further assumed that the number of potential failures in a component attributable to cause 1 is a poisson random variable with parameter λp_1; similarly if the number of failures caused by 2 and 3 are poisson with parameters λp_2 and λp_3.

Where $p_1 + p_2 + p_3 = 1$ and $p_i > 0$

Then the total number of potential failures is poisson with parameter λ. It has been shown (Canfield, 1970) that for large λ, the distribution function for the components with this failure model may be approximated by:

$$F(t) = 1 - \exp - (\Theta_1 t^L + \Theta_2 t + \Theta_3 t^M)$$

where $L < 1$ to represent the decreasing burn-in period

and $M > 1$ to represent the increasing wearout period

This distribution is the product of three separate distributions as shown in Figure 3. And the product of the three distributions has the shape as shown in Figure 4.

The hazard function is found by using the form:

$$h(t) = \frac{-d \ln R(t)}{dt}$$

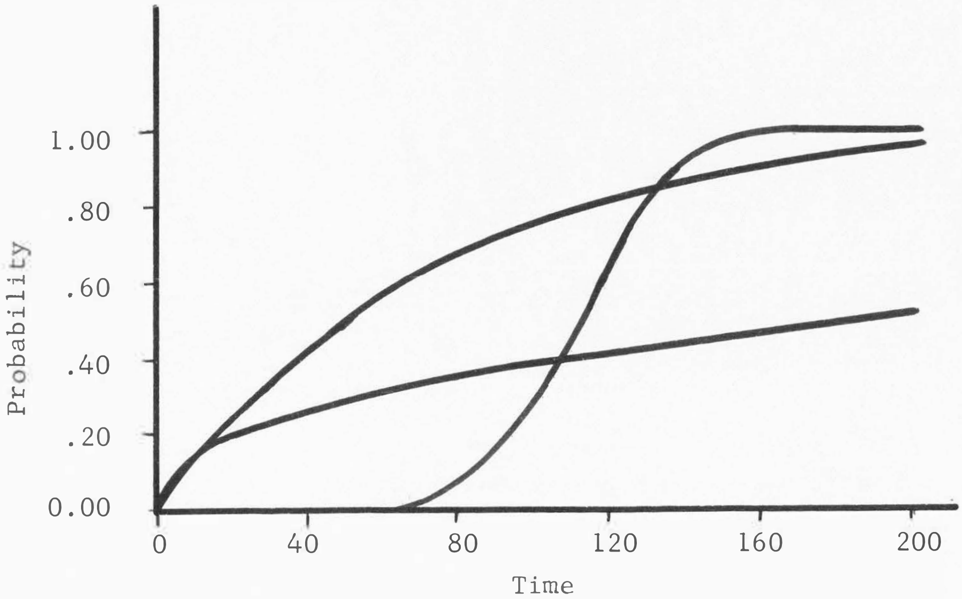

Figure 3. The separate distributions that make up F(t).

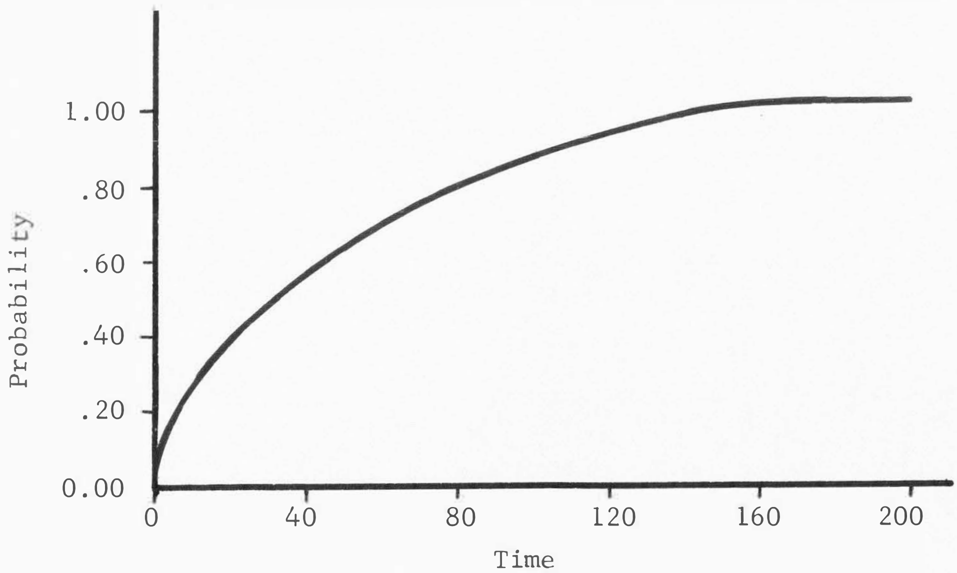

Figure 4. The distribution for the model F(t).

where $R(t) = 1 - F(t) = 1 - [1 - \exp -(\theta_1 t^L + \theta_2 t + \theta_3 t^M)]$

and $\ln R(t) = - [\theta_1 t^L + \theta_2 t + \theta_3 t^M]$

thus $h(t) = L\theta_1 t^{L-1} + \theta_2 + M\theta_3 t^{M-1}$

This hazard rate function has only six parameters to be estimated, and is much neater and easier to work with. For the purpose of this study, "L" and "M" are assumed known.

Interpretation of thetas

A point should be made here concerning the interpretation of the thetas in the above model. These thetas are not the same as commonly seen and used in the Wiebull distribution, the reciprocal of the mean "time to failure." They are a transformation of the form:

$$\theta_1 \sim \theta_1^* P_1$$

$$\theta_2 \sim \theta_2^* P_2$$

$$\theta_3 \sim \theta_3^* P_3$$

where θ_i^* is considered as the reciprocal of the "mean time to failure" as is commonly used in the Wiebull distribution and the P_i are the probability associated with each cause of failure. Therefore, the thetas used in the model are approximately the product of the reciprocal of the "time to failure" and the probability associated with each failure period.

COMPUTER PROGRAMS

This is a study to evaluate the feasibility of the new model for the bathtub-shaped hazard rate function. This means considerable effort required in the development of computer programs to generate data and to check the adaptability of the model. The checks to be made consist of: varying "L" and "M" thetas of the model, and comparing the estimate of reliability. Because the estimate of reliability is the goal of the new model, it was decided that it would be a criterion for its evaluation.

The method commonly used in industry is to place a given number of components on test and record the times at which they fail. With the data from the tests, the three periods are then determined and the mean time to failure is calculated for the random failure period. The computer will be used to simulate the same procedure using the monte carlo techniques. One hundred components will be placed on test and operated till all have failed. The times of their failures will be referred to as the test data set. From the test data sets obtained from this simulation, estimate for the thetas and reliability will be calculated. Along with these two programs--generation of test data and estimation of thetas and reliability--another program will be used to estimate the reliability from the restricted sample.

Obtaining test data sets

The method of simulation described above makes it necessary to generate random times for the failures of the components according to the parameters of the model. To accomplish this, uniform random numbers must first be generated on the interval (0,1) and then equated with the distribution function and then solving for "t." This is accomplished by using the function subroutine RAN (IBM 1965) on the U.S.U. IBM 360 computer library and then proceeding as follows:

let S = uniform random number

and $$G(t) = \exp - (\theta_1 t^L + \theta_2 t + \theta_3 t^M)$$

then $$S = G(t) = \exp - (\theta_1 t^L + \theta_2 t + \theta_3 t^M)$$

and now by taking the logarithm of both sides gives

$$\text{Ln } S = - (\theta_1 t^L + \theta_2 t + \theta_3 t^M)$$

and $$\text{Ln } S + \theta_1 t^L + \theta_2 t + \theta_3 t^M = 0$$

The application of numerical techniques was needed to solve the above equation for "t." The Newton-Raphson method (Duris and Moursund, 1967, p. 29) is used because of its speed and ease to program. Repeated iterations of the following equation give the solution.

$$g(t) = t - f(t) / f'(t)$$

where $$f(t) = \text{Ln } S + \theta_1 t^L + \theta_2 t + \theta_3 t^M$$

and $$f'(t) = l\theta_1 t^{L-1} + \theta_2 + m\theta_3 t^{M-1}$$

In this method an initial guess for "t" is needed, and then on each successive iteration the "t" is replaced by the new value g(t). Most solutions were obtained in less than six iterations with five place accuracy.

This program requires only one data card containing the values of "L," "M," and thetas. The test data sets, with the one hundred random times of failure, are next sorted in ascending order by a subroutine SORT[1] and then written on tape to be used by the other programs.

Before the test data sets can be used for any calculations they must be checked to see if they do indeed follow the desired distribution. The Kolomogrov-Smirnov goodness of fit statistic (Siegel, 1956) is used and is calculated as follows:

$$D = \text{maximum} \left| S(t) - F(t) \right|$$

where S(t) is the theoretical distribution under the null hypothesis which is the empirical distribution (l-i/n+l). F(t) is the observed distribution. The statistic "D" is then compared against the tabular value with appropriate degrees of freedom and selected α level. If the statistic "D" exceeds the tabular value, the null hypothesis will be rejected. The null hypothesis is that the sample has been drawn from the specified distribution.

A listing of the program and output is listed in Appendix A.

Estimation of thetas and conditional reliability

The method used to obtain estimates for each theta of the distribution is a least squares approach proposed by Bain and Antle (1967). The problem is to obtain estimates of the thetas which minimize the squared deviations between the theoretical and observed distributions

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[1] This subroutine was written by Dr. Rex L. Hurst, Department of Applied Statistics and Computer Science, Utah State University, Logan, Utah.

as shown below.

$$Z = \Sigma[S(t) - F(t)]^2$$

where S(t) and F(t) are given above.

$$Z = \sum_{i}^{n}[(1-i/n+1) - (\exp - \{\Theta_1 t^L + \Theta_3 t + \quad t^M\})]^2$$

In this form it is not easily solved so the logarithm will be taken giving:

$$Z = \sum_{i}^{n}[(1-i/n+1) - (\Theta_1 t^L + \Theta_2 t + \Theta_3 t^M)]^2$$

and then the partials will be taken with respect to each theta and set equal to zero giving:

$$\frac{\partial Z}{\partial \Theta_1} = \sum_{i}^{n}[Ln(1-i/n+1)t^L + \Theta_1 t^{2L} + \Theta_2 t^{L+1} + \Theta_3 t^{L+M}]$$

$$\frac{\partial Z}{\partial \Theta_2} = \sum_{i}^{n}[Ln(1-i/n+1)t + \Theta_1 t^{L+1} + \Theta_2 t^2 + \Theta_3 t^{M+1}]$$

$$\frac{\partial Z}{\partial \Theta_3} = \sum_{i}^{n}[Ln(1-i/n+1)t^M + \Theta_1 t^{M+L} + \Theta_2 t^{M+1} + \Theta_3 t^{2M}]$$

Each of the above partials is a linear equation in three unknowns thus giving three equations in three unknowns. The solution for each theta is obtained by using matrix algebra and a method known as Cramer's rule (Stien, 1967). The estimated for each theta test data set is then written on disk for use by the program CORR (Hurst, 1968). This program CORR calculates the mean and standard deviation for the estimates of each theta.

The reliability estimate for each test set is calculated using each of the above estimates. Because reliability is always calculated for the random failure period and the estimates using the model method use the complete data on the entire life of the component, the formula for reliability is given as

$$R(t) = R(t + \triangle t) / R(t)$$

where $\triangle$ is the time interval of interest. This estimate of reliability will be referred to as the conditional reliability. For example, the conditional reliability for t = 1 is found as follows:

$$R(1) = R(21) / R(20)$$

where t = 20 is the end of the burn-in period. This gives the reliability for a component which is to be operated for a unit time period.

These estimates are also written on disk for use by the CORR program.

A listing of this program and output is in Appendix B.

Estimation from restricted sample

The third program is used to calculate the estimate of reliability from the restricted sample assuming a constant hazard rate. The formula is given as:

$$R(t) = \exp - (\lambda t)$$

where "λ" is the failure rate and "t" is the time of operation for the component. The failure rate "λ" is the reciprocal of the mean time to failure, "MTTF," which is calculated using only the failures occurring in the random failure period. Due to the difficulty in writing a program that would evaluate the appropriate times for the beginning and ending times of the random failure period for each test data set, it was decided to use two set times, "T_b" and "T_w." This means that for every test data set the "MTTF" would be calculated using only the failures between "T_b" and "T_w." The values of "T_b" and "T_w" were determined from the theoretical hazard rate curve. The formula now looks like this:

$$R(t) = \exp - (t/MTTF)$$

These estimates are also written on disk for use by CORR.

A listing of this program is in Appendix C.

RESULTS OF CHECKS

Kolmogorov-Smirnov statistic

The first task with any monte carlo study is to test the random number generator and this was done using the Kolmogorov-Smirnov "goodness of fit" statistic as described in the previous section. To make sure the generator for the test data would be valid for small as well as large sample sizes, two different sample sizes were tested--one of size 50 and the other of size 100. Each size has 20 test data sets. The maximum absolute differences "D" between theoretical and generated for each set are listed in ascending order in Table 1. As can be noticed, the null hypothesis, that the random

Table 1. The absolute maximum difference "D" for the Kolmogorov-Smirnov "goodness of fit" statistic

Test data sets of size			
50		100	
.04372	.07923	.05791	.05888
.08039	.08930	.06038	.06121
.09373	.09589	.06953	.07184
.09898	.09908	.07613	.07648
.10465	.10666	.07965	.08061
.10674	.10898	.08241	.08433
.11516	.11810	.08702	.08726
.12153	.13947	.08971	.09826
.14163	.14789	.11520	.11722
.15798	.16137	.12114	.12355
Tabular values			
$\alpha = .05$	.23	.163	
$\alpha = .01$	.19	.136	

times follow the model distribution, was not rejected at either level or for either sample size. Another observation to be made is that the larger the test data sample size the smaller the "D" values indicating that the more items placed on test the better the fit for the generated times. Thus the conclusion that the random times were generated according to the model distribution function for the "times to failure."

Varying "L" and "M"

Because "L" and "M" were assumed to be known for this study, a check was made to see just what effect varying "L" and "M" would have on the estimates of thetas and reliability. The reason was that if "L" and "M" were to be estimated, which they must in practice, it would be helpful to discover exactly what effect, if any, a poor estimate of these values would have on the estimation of reliability. It was decided to vary "L" by $\pm$ .1 and "M" by $\pm$ 1 giving four combinations. All four combinations plus the constant values for "L" and "M" which are .5 and 6 respectively were evaluated using 100 sets of test data and calculating the mean and standard deviation for each theta and conditional reliability for four times. Table 2 contains the deviations from expected theta values for each combination of test data sets. Deviation is defined as follows:

$$\text{Deviation} = \text{observed} - \text{expected}$$

for the thetas and for reliability the term expected is replaced by true value. Table 3 contains the deviations from true values for the conditional reliability estimates for each combination.

Table 2. Deviations from expected theta values for various combinations of "L" and "M"

Combinations of		Deviations from expected		
"L"	"M"	Theta 1	Theta 2	Theta 3
.5	6	-.0034	-.0008	.0468 E-12
.4	5	.1794	.0010	-.3135 E-12
.6	5	.0355	-.0416	-.3134 E-12
.4	7	-.0687	.0355	.2598 E-14
.6	7	-.1788	.0135	.2641 E-14

E-12 mean the number x 10^{-12}

Table 3. Deviations from true reliability for various combinations of "L" and "M"

Combinations of		Deviations from true reliability for times			
"L"	"M"	1 unit	3 units	5 units	30 units
.5	6	.0007	.0382	.0024	-.0005
.4	5	.0019	.0054	.0083	.0156
.6	5	.0015	.0044	-.0067	.0163
.4	7	.0001	-.0001	-.0007	-.0140
.6	7	-.0003	-.0010	-.0020	-.0157

These tables bring out one of the more interesting aspects of this method of reliability estimation and that is that the estimates of reliability are relatively good, while the estimates of thetas bounce all around. This is in part due to the difference of sign on the exponents "L" and "M" which tend to offset each other's errors when used to estimate reliability. The exponent "M" is the more dominate factor for this model distribution, having the most influence in the estimation of thetas which is to be expected due to its magnitude. It would appear that an error is to be made in estimation of "L" and "M" that it is better to over estimate "M" and under estimate "L" yielding less error in reliability estimation. The offsetting tendencies of "L" and "M" would merit further investigation.

The standard deviations for reliability were very small and constant, bearing out the fact that the estimates are constant. From this set of calculations it is concluded that relatively small errors in the exponents "L" and "M" of the model distribution do not appreciably alter the estimates of reliability for short times.

Varying thetas

One of the inherent problems with this type of study is the obtaining of good realistic numbers, because sometimes just any old number may work but not be realistic. Real test data are hard to find. With this in mind, a check was done to see if there were any readily apparent problems or restrictions to be placed on the values selected for thetas. A total of five combinations of various thetas was tried using 20 sets of test data. The deviations of the estimates of thetas and reliability were calculated and tabulated to see if any irregularities appeared. The values for thetas were selected to give different percentages of observations in each of the three failure periods. The results are given in Tables 4 and 5.

These tables show that the estimates for thetas were generally very close, being low for theta 1 and high for the other two. The magnitudes were very consistent for each combination. Again the estimates for reliability were very close with extremely consistent standard deviations, all approximately .045. No problems for different magnitudes for thetas were discovered so with the lack of real test data the above results indicated that any one of the combinations could and would be a feasible choice for the production runs.

The values of:

$$\theta_1 = .0474$$

$$\theta_2 = .0139$$

$$\theta_3 = .3186 \times 10^{-12}$$

were selected to use for production and evaluation of the model.

Table 4. Deviations from expected theta values for various combinations of thetas

Combinations of			Deviations from expected		
Θ_1	Θ_2	Θ_3	Theta 1	Theta 2	Theta 3
.0791	.0100	.5645 E-12	-.0155	.0040	.0199 E-12
.0633	.0120	.4321 E-12	-.0100	.0019	.0251 E-12
.0633	.0139	.6302 E-12	-.0103	.0020	.0402 E-12
.0470	.0139	.3186 E-12	-.0098	.0018	.0221 E-12
.0470	.0035	.3237 E-14	-.0056	.0065	.0178 E-14

E-12 mean the number x 10^{-12}

Table 5. Deviations from true reliability for various combinations of thetas

Combinations of			Deviations from true reliability for time		
Θ_1	Θ_2	Θ_3	15	50	85
.0791	.0100	.5645 E-12	-.0092	-.0056	-.0085
.0633	.0120	.4321 E-12	.0082	-.0052	-.0079
.0633	.0139	.6302 E-12	.0073	-.0060	-.0074
.0470	.0139	.3186 E-12	.0055	-.0049	-.0076
.0470	.0035	.3237 E-14	.0105	.0057	.0001

E-12 means the number x 10^{-12}

FINDINGS

Estimation of thetas

Now that the preliminary checks concerning the random generator, varying exponent "L" and "M" and various combinations of thetas have been described and presented with no apparent handicaps having been discovered, three production runs which consist of the three previously described programs will be used for the evaluation. Each production run consists of 500 test data sets. The only difference between each run will be the argument for the function subroutine RAN, thus giving a completely new set of random times. The distribution for the model now has the form:

$$F(t) = 1 - \exp - (\Theta_1 t^L + \Theta_2 t + \Theta_3 t^M)$$

where $L = .5$ $\quad \Theta_1 = .0474$

$M = 6$ $\quad \Theta_2 = .0139$

$\Theta_3 = .3186 \times 10^{-12}$

The theta values were chosen arbitrarily from the five sets of 20 test data combinations described in the last section. And they will remain constant for the three production runs. For each production run the means and standard deviations were calculated for the estimates of thetas and reliability for time intervals of 1, 3, 5, and 30 units. Table 6 contains the deviations and standard deviations for the estimates of thetas for all three runs.

The estimate of thetas was generally fairly consistent and had standard deviations that were very close to one another for each of

Table 6. Deviations from expected theta values and standard deviations for thetas for each of the three runs

Production run	Theta 1 Dev.	Theta 1 St. dev.	Theta 2 Dev.	Theta 2 St. dev.	Theta 3 Dev.	Theta 3 St. dev.
1	-.0046	.0342	.0010	.0052	-.0473 E-12	.1420 E-12
2	-.0076	.0327	.0012	.0049	-.0500 E-12	.1307 E-12
3	-.0017	.0336	.0003	.0053	-.0278 E-12	.1619 E-12
Average	-.0046	.0335	.0008	.0052	-.0232 E-12	.1449 E-12

E-12 means the number x 10^{-12}

the production runs. The average, taken for the 1500 test data sets, showed small deviations for the true expected values leading to the conclusion that the model and least squares procedure for estimation of thetas is satisfactory, having some small bias.

Estimation of conditional reliability

As mentioned above, the means and standard deviations for the conditional reliability estimate of the three runs of 500 test data sets were obtained and are shown in Table 7. The reliability estimates were based on the constant failure period of the hazard curve for the time between t = 15 and t = 80 units based on the theoretical curve. The actual formula for reliability will be given again:

$$R(t) = R(15 + \Delta t) \,/\, R(15) \qquad t = 1,2,\ldots..N$$

This has been referred to as conditional reliability.

Table 7. Deviations from true reliability and standard deviations for the conditional reliability estimates for the three production runs

Production run	Conditional reliability for time -							
	1 unit		2 units		5 units		30 units	
	Dev.	St. dev.	Dev.	St. dev.	Dev.	St. dev.	Dev.	St. dev.
1	-.0004	.0084	-.0011	.0100	-.0019	.0132	-.0069	.0463
2	-.0003	.0081	-.0007	.0101	-.0013	.0132	-.0061	.0449
3	-.0003	.0067	-.0005	.0108	-.0017	.0146	-.0060	.0530
Average	-.0003	.0082	-.0008	.0103	-.0019	.0137	-.0063	.0480

The table shows that the estimates obtained from the model are very close to the true reliability at the times calculated. In all cases the estimates are slightly lower or conservative. The standard deviations being constant as well as small indicating the estimates are doing a good job, being only slightly biased low. The one pattern that developed and is what would be expected is that as "t" increases the bias and standard deviations also increase.

Consistency and accuracy of these estimates of conditional reliability give promise for this model and method of estimation for reliability. One big factor in its favor is that the estimates are based on the complete life of the components and are easy to calculate.

Comparisons

A criterion established for the evaluation of the proposed model of the bathtub-shaped hazard rate function would be its estimate of reliability as compared to the theoretical value and restricted sample estimates. The comparison at four time intervals is given in Table 7 and proved to be slightly low with a small variance. The comparison with the restricted sample estimates is given in Table 8 below.

This table shows that the model in all four time intervals had much more accurate estimates of reliability. In all but the time interval t = 30 the variance was smaller. This tends to indicate that the new model has potential as possibly a better method of estimating reliability.

One additional supposition is the belief that the restricted sample estimates given are minimum variance. The reason for this belief is that the times for the random failure period were established from the expected empirical hazard rate curve and not for

Table 8. Average deviations from true reliability and average standard deviations of the three production runs for the estimates for reliability computed by the conditional reliability and restricted sample methods

Reliability times	Average deviations		Average st. dev.	
	Cond.	Rest.	Cond.	Rest.
1 unit	-.0003	-.0177	.0082	.0086
3 units	-.0008	-.0505	.0103	.0129
5 units	-.0019	-.0801	.0137	.0178
30 units	-.0063	-.2537	.0480	.0369

each individual test data set. Thus if the test data set was biased either up or low, our method did not take it into account; therefore, more variance and bias would have been introduced. Therefore, it is felt that if it were possible to have treated each set of test data individually, as would have been done in industry, the estimates would have been more biased and have larger variances than was obtained.

The conclusions of the comparisons are that the new model has a great deal of potential and promise. The fact that the estimates of reliability were much closer to the true values for the model as compared to the restricted sample indicates that the additional data are of great value; therefore, in practice, if an estimate of reliability is bias, it is never known because there is no theoretical value with which to compare.

SUMMARY

The purpose for the development of the new model was to facilitate the use of all the available test data compiled on the complete life of the component that is useful over the entire bathtub-shaped hazard rate function. All indications and preliminary checks failed to show any apparent restrictions or limitations for the new procedure. The reliability estimates obtained using the new model showed that they were extremely close to true reliability with smaller variances than the restricted sample estimates currently being used.

The estimates of thetas were not always as close as desired, especially when the parameters "L" and "M" were varied, but the deviations from true reliability for the estimate of reliability were never very large. This points up an interesting facet of the model and that is it appears to be insensitive to moderate errors in its parameters, which is a good trait when they must be estimated.

Another good characteristic is that this model provides a method for determining the point in time at which the conditional reliability for the component reaches the peak. This is convenient because this is the most efficient estimate for the beginning of the random failure period.

Over-all, this new model has very high potential from all preliminary indications and checks completed in this pilot study. Part of the purpose for this study was to do preliminary evaluation and

find areas that would require further study and applications. One such area would be to try and find a feasible method of estimating the parameters "L" and "M" as well as seeing exactly what the limitations on their deviations from expected could be tolerated.

The exact relationship between the P_i and θ_i^* for the thetas in the model would be worthwhile investigating to attach the proper interpretation to them.

There appears to be an invariant property between the estimates of reliability and time. If this property holds there would be the possibility of placing confidence limits on the estimates of reliability.

The last thing to add is that it would be worthwhile to try and locate some real data, not computer generated, to check the real potential of the model.

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APPENDIXES

Appendix A

Listing of the Program to Obtain the Test Data Sets

```
C    THIS PROGRAM IS USED TO GENERATE THE RANDOM TIMES
C    FUNCTION SUBROUTINE RAN GENERATES UNIFORM RANDOM NOS
C    RAN REQUIRES A LARGE ODD INTEGER FOR THE ARGUMENT
C    SUBROUTINE KOL FOR THE KOLMOGUROV-SMIRNOV STATISTIC
C    SUBROUTINE SORT ARRANGES THE TIMES IN ASENDING ORDER
C    THE RANDOM TIMES ARE WRITEN ON TAPE, DRIVE 180 L.U.8
C
C       X IS THE POWER OF THETA1 L
C       Y IS THE POWER OF THETA3 M
C       RN IS THE ARRAY OF RANDOM TIMES
C
        DIMENSION RN(100),IA(100),C(100)
100     FORMAT(F5.5,4F5.0)
200     FORMAT(10X,10F10.5)
201     FORMAT(10X,5F10.5)
203     FORMAT(1H0)
204     FORMAT (1H0,5X,'THE ESTIMATES ARE =',3(2X,E15.8))
300     FORMAT(25F10.5)
301     FORMAT(25F10.5)
400     FORMAT(6E15.8)
500     FORMAT(10X,'THE RANDOM NUMBERS FROM TAPE ')
        READ (5,100) X,Y,THETA1,THETA2,THETA3
        WRITE(6,201)X,Y,THETA1,THETA2,THETA3
        P = X - 1.
        PP = Y - 1.
        DO 1 LI = 1,100
        DO 2 J = 1,100
        S = RAN(5461)
        SS = ALOG(S)
        A = 18.
        N = 0
10      BB = A/THETA1
        CC = A/THETA2
        DD = A/THETA3
        FX = SS + BB**X + CC + DD**Y
        DFX=(X/THETA1)*BB**P+1./THETA2+(Y/THETA3)*DD**PP
        AA = A-FX/DFX
        ER = ABS(A-AA)
        IF (ER .LT. .00005  .OR.N .EQ.  6) GO TO 11
        N = N + 1
        A = ABS(AA)
        GO TO 10
   11   RN(J) = ABS(AA)
2       CONTINUE
        CALL SORT (RN,IA,100)
```

```
      WRITE (6,203)
      WRITE (6,200) (RN(I),I=1,100)
      CALL KOL (THETA1,THETA2,THETA3,X,Y,RN,C)
      WRITE (8,300) (RN(II),II=1,25)
      WRITE (8,300) (RN(II),II=26,50)
      WRITE (8,300) (RN(II),II=51,75)
      WRITE (8,300) (RN(II),II=76,100)
1     CONTINUE
      END FILE 8
      REWIND 8
      WRITE (6,500)
      DO 5 IKI= 1,10
      READ(8,301) (RN(IK),IK=1,25)
      READ(8,301) (RN(IK),IK=26,50)
      READ(8,301) (RN(IK),IK=51,75)
      READ(8,301) (RN(IK),IK=76,100)
      WRITE (6,203)
      WRITE(6,200) (RN(I),I=1,50)
5     CONTINUE
      STOP
      END
```

```
      SUBROUTINE SORT(A,IA,N)
      DIMENSION IA(1),A(1)
      DO 50 I = 1,N
   50 IA(I) = I
      M = N
   51 M = M/2
      IF(M.EQ.0) GO TO 57
      K = N-M
      J = 1
   53 I = J
   54 L = I+M
      IF(A(I).LE.A(L)) GO TO 56
      T = A(I)
      IT = IA(I)
      A(I) = A(L)
      IA(I) = IA(L)
      A(L) = T
      IA(L) = IT
      I = I-M
      IF(I.GE.1) GO TO 54
   56 J = J+1
      IF(J-K) 53,53,51
   57 RETURN
      END
```

```
      SUBROUTINE KOL (THETA1,THETA2,THETA3,X,Y,RN,C)
C
C   THIS SUBROUTINE IS USED TO CALCULATE THE KOLMOGOROV
C   SMIRNOV STATISTIC FOR THE RANDOM TIMES
C
C      RN IS THE RANDCM TIMES ARRAY
C      X IS THE POWER OF THETA1
C      Y IS THE POWER OF THETA3
C      C IS THE ARRAY FOR THE DIFFERENCES BETWEEN THE
C      EXPECTED AND OBSERVED
C      AB IS THE MAXIMUN DIFFERENCE
C
      DIMENSION RN(100), C(100)
202   FORMAT(1H0,10X,'KOLMOGOROV-SMIRNOV STAT =',F8.5)
203   FORMAT (1H0)
      AB = 0.0
      XI = 100
      DO 3 K = 1,100
      U = (RN(K)/THETA1)**X
      V = (RN(K)/THETA2)
      W = (RN(K)/THETA3)**Y
      B = (U+V+W)*(-1.)
      C(K) = 1. - EXP(B)
      D = K/XI
      E = ABS(C(K)-D)
      AB = AMAX1(E,AB)
    3 CONTINUE
      WRITE (6,203)
      WRITE (6,202) AB
      RETURN
      END
```

THIS IS THE OUTPUT FOR THIS PROGRAM

0.0657	0.1216	0.1436	0.1988	0.2081	0.2225
0.2531	0.2555	0.7274	0.7617	1.0774	1.2813
1.4565	1.5559	1.7165	1.8729	1.9758	3.9280
5.0184	5.5171	5.7733	6.7628	7.3718	7.7522
8.7432	10.9153	11.2914	11.4175	12.9592	14.6266
15.8644	15.9137	16.4747	17.1187	18.0268	18.1342
19.0042	20.9290	21.3413	22.2340	24.5336	25.4758
27.2652	29.6529	30.9332	31.2968	31.9929	32.6432
32.6906	33.0432	33.8020	34.2533	35.0415	36.2956
37.9075	39.6758	39.7300	39.9562	40.2555	40.4542
40.6365	40.6394	42.3038	45.3097	45.6750	47.2967
47.7093	49.5451	49.6939	51.8215	51.9766	56.0186
56.4420	61.5444	64.0395	66.8932	69.0001	69.3763
69.7221	73.1965	74.3293	74.5474	76.8851	78.1469
87.1198	87.3239	91.0996	91.4977	91.6901	92.8273
100.9079	101.3193	101.5077	102.9843	104.1672	113.9024
115.8090	116.1287	123.9004	124.1622		

KOLMOGOROV-SMIRNOV STAT = 0.08022

0.0549	0.1253	0.2775	1.5416	2.1097	2.5319
3.0051	3.1137	3.7858	3.7999	5.1295	5.4806
6.4885	6.8682	7.2208	7.2550	8.3043	9.0151
9.7254	10.7087	12.3824	12.6407	12.7215	12.9199
12.9874	13.4392	14.1966	14.5881	14.7780	15.1713
15.4000	15.9756	16.0901	16.3653	16.7613	16.9420
17.5467	17.7065	20.4080	20.9445	22.2323	24.1232
24.1725	24.2560	24.9539	25.4191	28.5977	28.7756
29.2743	29.9482	31.4294	34.0418	34.5379	35.4060
36.8134	37.9755	38.4537	39.6454	41.1039	41.6396
44.1508	44.4324	45.0695	47.3420	49.3668	51.3751
51.7797	53.2613	54.2734	56.8091	60.9427	61.8179
63.0562	63.7952	64.4556	65.5009	67.3540	68.6629
71.3562	72.7712	81.3914	81.4247	83.3227	85.0796
85.4066	86.5196	87.2591	88.4148	93.8603	99.6593
101.3868	101.7317	104.4485	105.3436	105.7602	107.7071
108.5598	133.3321	146.2660	154.2896		

KOLMOGOROV-SMIRNOV STAT = 0.07737

Appendix B

Listing of the Program to Estimate the Thetas and Conditional Reliability

```
C
C    THIS PROGRAM CALCULATED THE ESTIMATES OF THETA'S
C    AND THE ESTIMATES FOR RELIABILITY
C    USES CRAMERS METHCD FOR SCLVING THREE EQUATICNS
C    THIS IS PROGRAMED FOR THE TIME INTERVAL 15 - 80
C
C       X IS THE POWER OF THETA1 L
C       Y IS THE POWER OF THETA3 M
C       RN IS THE ARRAY OF RANDCM TIMES
C       DET IS THE DETERMINANT FOR THE MATRIX A
C       DET1 IS THE DETERMINANT OF THE MATRIX A AND THETA1
C       DET2 IS THE DETERMINANT OF THE MATRIX A ANC THETA2
C       DET3 IS THE DETERMINANT OF THE MATRIX A ANC THETA3
C       REL1 IS THE RELIABILITY FCR TIME = 1 UNIT
C       REL2 IS THE RELIABILITY FOR TIME = 2 UNIT
C       REL4 IS THE RELIABILITY FOR TIME = 30 UNITS
C       OX IS THE ESTIMATE FOR THETA1
C       OY IS THE ESTIMATE FOR THETA2
C       OZ IS THE ESTIMATE FOR THETA3
C
C    RN ARRAY IS STORED ON TAPE, DRIVE 180  L.U.8
C    OX,OY,CZ ARE STORED ON DISK L.U.14
C
C    THE PROGRAM CORR IS USED TO CALCULATE THE MEANS
C
       DIMENSION RN(1CO),IA(100)
       DOUBLE PRECISICN PA1,PA2,PA3,PA4,PA5,PB1,PB2,PB3
       DOUBLE PRECISICN PB4,PB5,PC1,PC2,PC3,PC4,PC5
       DOUBLE PRECISICN OX(500),OY(500),OZ(500),DET,DET1
       DOUBLE PRECISICN DET2,DET3,CAA,OA,CB,OC,OD,OE,OF
       DOUBLE PRECISICN OG,OJ,OI,OH
100    FORMAT (F2.2,F1.0)
200    FORMAT(10X,'THE ESTIMATES ARE GIVEN FOR L= ',F5.2,
     1 '   M  .= ', F5.2/)
204    FCRMAT (12X,'THETA1',12X,'THETA2',12X,'THETA3',9X,
     1 'REL.-=1',9X,'REL.T=3',7X,'REL.T=5',7X,'REL.T=30')
205    FCRMAT(5X,7(E15.8,3X))
300    FORMAT(25F10.5)
400    FORMAT(7E15.8)
       REWIND 8
       REAC(5,10C)X,Y
       WRITE(6,2C0) X,Y
      NN = 0.
       WRITE (6,204)
       DO 1 IL = 1,10C
```

```
      READ(8,300) (RN(II),II=1,25)
      READ(8,300) (RN(II),II=26,50)
      READ(8,300) (RN(II),II=51,75)
      READ(8,300) (RN(II),II=76,100)
      OA = 0.
      OB = 0.0
      OC = 0.0
      OD = 0.0
      OE = 0.0
      OF = 0.0
      OG = 0.0
      OH = 0.0
      OI = 0.0
      OJ = 0.0
      DO 5 L = 1,100
      A = L
      OAA = 1. - (A/101.)
      OB =OB+ (RN(L)*DLOG(OAA))
      OC =OC + RN(L)*RN(L)
      OD =OD + ((RN(L)**X)*DLOG(OAA))
      OE =OE + RN(L)**(X+1.)
      OF =OF + RN(L)**(2*X)
      OG =OG + RN(L)** (X+Y)
      OH =OH + ((RN(L)**Y )*DLOG(OAA))
      OI =OI + RN(L)**(Y +1.)
      OJ =OJ + RN(L)**(Y *2)
 5    CONTINUE
      DET = OF*OC*OJ+OE*OI* OG+OE*OI*OG-OG*OG*OC-OE*OE*
     1 OJ-OI*OI*OF
      DET1=OG*OC*OH+OI*OI*OD+OB*OE*OJ-OD*OC*OJ-OE*OI*
     2 OH-OB*OI*OG
      DET2=OG*OG*OB+OI*OH*OF+OE*OD*OJ-OF*OB*OJ-OD*OI*
     1 OG-OE*OH*OG
      DET3=OF*OI*OB+OE*OE*OH+OD*OG*OC-OF*OC*OH-OE*OB*
     1 OG-OE*OI*OD
      IF (DET .EQ. 0.0) GO TO 6
      NN = NN + 1
      OX(NN) = DET1/DET
      OY(NN) = DET2/DET
      OZ(NN) = DET3/DET
 6    CONTINUE
      PA1 = (15**X)*OX(NN)
      PA2 = (16**X)*OX(NN)
      PA3 = (18**X)*OX(NN)
      PA4 = (20**X)*OX(NN)
```

```
      PA5 = (45**X)*CX(NN)
      PB1 = 15*CY(NN)
      PB2 = 16*CY(NN)
      PB3 = 18*CY(NN)
      PB4 = 20*CY(NN)
      PB5 = 45*CY(NN)
      PC1 = (15**Y)*CZ(NN)
      PC2 = (16**Y)*CZ(NN)
      PC3 = (18**Y)*CZ(NN)
      PC4 = (20**Y)*CZ(NN)
      PC5 = (45**Y)*CZ(NN)
      PO1 = (-1.)*(PA1+PB1+PC1)
      PO2 = (-1.)*(PA2+PB2+PC2)
      PO3 = (-1.)*(PA3+PB3+PC3)
      PO4 = (-1.)*(PA4+PB4+PC4)
      PO5 = (-1.)*(PA5+PB5+PC5)
      PREL1 = EXP(PO1)
      PREL2 = EXP(PO2)
      PREL3 = EXP(PO3)
      PREL4 = EXP(PO4)
      PREL5 = EXP(PO5)
      REL1 = PREL2/PREL1
      REL2 = PREL3/PREL1
      REL3 = PREL4/PREL1
      REL4 = PREL5/PREL1
      WRITE( 6,400) CX(NN),CY(NN),CZ(NN),REL1,REL2
     1 REL3,REL4
      WRITE(14,400) CX(NN),CY(NN),CZ(NN),REL1,REL2
     1 REL3,REL4
    1 CONTINUE
      END FILE 14
      REWIND 14
      STOP
      END
```

THIS IS THE OUTPUT FOR THIS PROGRAM

THE ESTIMATES ARE GIVEN FOR L = 0.50 M = 6.00

THETA1	THETA2	THETA3	REL.T=1	REL.T=3	REL.T=5	REL.T=30
0.0395	0.0156	-.0004	0.9796	0.9405	0.9034	0.5579
-.0107	0.0236	-.0001	0.9781	0.9355	0.8946	0.5082
0.0480	0.0120	-.0002	0.9821	0.9478	0.9152	0.6081
0.0489	0.0111	-.0002	0.9828	0.9498	0.9185	0.6218
0.0695	0.0136	-.0001	0.9778	0.9356	0.8961	0.5449
-.0402	0.0286	-.0001	0.9767	0.9314	0.8876	0.4738
0.0289	0.0165	-.0001	0.9801	0.9417	0.9051	0.5613
0.0666	0.0109	-.0003	0.9808	0.9441	0.9097	0.5938
0.0841	0.0096	-.0002	0.9799	0.9419	0.9064	0.5837
-.0214	0.0243	-.0002	0.9786	0.9369	0.8968	0.5108
0.0410	0.0171	-.0001	0.9780	0.9358	0.8959	0.5326
0.0744	0.0062	-.0003	0.9844	0.9549	0.9271	0.6698
0.0617	0.0132	-.0005	0.9792	0.9396	0.9024	0.5629
0.0389	0.0153	-.0002	0.9800	0.9415	0.9050	0.5647
0.1164	0.0108	-.0004	0.9747	0.9274	0.8836	0.5179
0.0547	0.0125	-.0002	0.9807	0.9438	0.9089	0.5866
0.0258	0.0140	-.0002	0.9828	0.9496	0.9179	0.6094
-.0014	0.0194	-.0001	0.9810	0.9439	0.9083	0.5604
0.0476	0.0134	-.0001	0.9807	0.9437	0.9087	0.5834
0.0107	0.0174	-.0002	0.9814	0.9452	0.9106	0.5735
0.0521	0.0172	-.0001	0.9765	0.9317	0.8896	0.5149
-.0015	0.0170	-.0003	0.9833	0.9507	0.9192	0.6005

Appendix C

Listing of the Program to Estimate the Reliability from the Restricted Sample

```
C
C    THIS PROGRAM CALCULATES THE ESTIMATES OF RELIABILITY
C    USING THE TRADITION MENTHODS AND TECHNIQUES
C    THIS IS PROGRAMED FOR THE TIMES INTERVAL 15 - 80
C
C       RN IS THE ARRAY CONTAINING THE RANDOM TIMES
C       AVE IS THE MEAN TIME TO FAILURE MTTF
C       REL1 IS THE RELIABILITY FOR TIME = 1 UNIT
C       REL2 IS THE RELIABILITY FOR TIME = 3 UNITS
C       REL3 IS THE RELIABILITY FOR TIME = 5 UNITS
C       REL4 IS THE RELIABILITY FOR TIME = 30 UNITS
C
C    THE RN ARRAY IS STORED ON TAPE, DRIVE 180 L.U. 8
C    REL'S ARE STORED ON DISK L.U.14
C
C    THE PROGRAM CORR IS USED TO CALCULATE THE MEANS
C
      DIMENSION RN(100)
102   FORMAT(10X,'AVERAGE',10X,'RELIABILITY T=1',10X,
     1 'RELIABILITY T=3',10X,'RELIABILITY T=5',10X,
     2 'RELIABILITY T=30'/)
100   FORMAT(5X,5(E15.8,5X))
101   FORMAT(5E15.8)
300   FORMAT(25F10.5)
      REWIND 8
      WRITE(6,102)
      DO 1 M = 1,100
      READ (8,300) (RN(I),I=1,25)
      READ (8,300) (RN(I),I=26,50)
      READ (8,300) (RN(I),I=51,75)
      READ (8,300) (RN(I),I=76,100)
      N = 0.0
      XY = 0.0
      DO 2 MM=1,100
      IF (RN(MM) .GE. 15. .AND. RN(MM) .LE. 80.) GO TO 4
      GO TO 2
    4 N = N+1
      XY = XY + (15. - RN(MM))
    2 CONTINUE
      AVE = XY /N
      A = (1./AVE) * 1.
      B = (1./AVE) * 3.
      C = (1./AVE) * 5.
      D = (1./AVE) * 30.
      REL1 = EXP(A)
```

```
      REL2 = EXP(B)
      REL3 = EXP(C)
      REL4 = EXP(D)
      WRITE (6,100) AVE,REL1,REL2,REL3,REL4
      WRITE(14,101) AVE,REL1,REL2,REL3,REL4
    1 CONTINUE
      END FILE 14
      REWIND 14
      STOP
      END
```

THIS IS THE OUTPUT FOR THIS PROGRAM

AVERAGE	REL = 1	REL = 3	REL = 5	REL = 30
-26.94	0.9636	0.8946	0.8306	0.3284
-23.92	0.9599	0.8817	0.8106	0.2838
-26.63	0.9631	0.8935	0.8288	0.3241
-32.10	0.9693	0.9108	0.8558	0.3928
-27.08	0.9637	0.8951	0.8314	0.3302
-29.78	0.9670	0.9042	0.8454	0.3651
-23.67	0.9586	0.8810	0.8096	0.2815
-24.89	0.9606	0.8864	0.8180	0.2996
-25.25	0.9612	0.8880	0.8204	0.3048
-27.59	0.9644	0.8970	0.8342	0.3371
-29.31	0.9665	0.9027	0.8432	0.3593
-24.19	0.9595	0.8834	0.8132	0.2893
-30.29	0.9675	0.9057	0.8478	0.3714
-30.15	0.9674	0.9053	0.8472	0.3697
-19.61	0.9503	0.8581	0.7749	0.2166
-23.79	0.9588	0.8815	0.8104	0.2834
-27.51	0.9643	0.8967	0.8338	0.3361
-25.18	0.9611	0.8877	0.8199	0.3038
-23.87	0.9590	0.8819	0.8110	0.2845
-30.49	0.9677	0.9063	0.8488	0.3739
-27.69	0.9645	0.8973	0.8348	0.3384
-34.79	0.9717	0.9174	0.8661	0.4222
-25.24	0.9612	0.8879	0.8203	0.3046

VITA

Glen S. Leithead

Candidate for the Degree of

Master of Science

Thesis: Model for Bathtub-shaped Hazard Rate: Monte Carlo Study

Major Field: Applied Statistics

Biographical Information:

Personal Data: Born at El Paso, Texas, November 30, 1941, son of Horace L. and Louise Leithead; married JoAnn Goodworth January 21, 1966; two children--Joel and Tracy.

Education: Attended elementary school in Marfa and Hereford, Texas, and also in Lakeview and Bend, Oregon; graduated from Arvada, Colorado, high school in 1960; received the Bachelor of Science degree from Brigham Young University in statistics; completed requirements for the Master of Science degree, in applied statistics, at Utah State University in 1970.

Professional Experience: September 1968 to present, graduate assistant in Applied Statistics Department, Utah State University; September 1966-1968, The Boeing Company, Huntsville, Alabama, statistician on the Saturn V project.