A Two Sample Test of the Reliability Performance of Equipment Components

Miki Lynne Coleman

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A TWO SAMPLE TEST OF THE RELIABILITY PERFORMANCE
OF EQUIPMENT COMPONENTS

by

Miki Lynne Coleman

A thesis submitted in partial fulfillment
of the requirements for the degree
of
MASTER OF SCIENCE
in
Applied Statistics

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UTAH STATE UNIVERSITY
Logan, Utah
1972
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ABSTRACT

A Two Sample Test of the Reliability Performance of Equipment Components

by

Miki Lynne Coleman, Master of Science

Utah State University, 1972

Major Professor: Dr. Ronald V. Canfield
Department: Applied Statistics

The purpose of this study was to develop a test which can be used to compare the reliability performances of two types of equipment components to determine whether or not the new component satisfies a given feasibility criterion.

Two types of tests were presented and compared: the fixed sample size test and the truncated sequential probability ratio test. Both of these tests involve use of a statistic which is approximately distributed as F.

This study showed that the truncated sequential probability ratio test has good potential as a means of comparing two component types to see whether or not the reliability of the new component is at least a certain number of times greater than the reliability of the old component.

(58 pages)
CHAPTER I

INTRODUCTION

As technology develops, new and more reliable equipment components become available for industrial use. An increase in cost usually accompanies the new type of equipment. Thus, users of the equipment components must be able to determine whether or not it would be economical for them to use the new components. This decision could be based on a comparison of the reliability performances of the old and new components. In this study, the reliability of each component will be measured by the "mean time to failure" for the component.

Suppose that replacement of an old equipment component by a new type of component which performs the same function is being considered. Because there is generally an increase in cost for the new component, the users of the equipment components must decide on the amount by which the new component must outperform the old component in order to make use of the new component feasible. Based on economic considerations, they must determine by some measure the degree to which the new component must outperform the old component. Determination of this amount is not a part of this study but rather the task of the users of the equipment.

The purpose of this study is to present and compare two types of statistical tests which can be used to compare the reliability performances of the two components. The two tests to be discussed are: (1) the fixed sample size test and (2) the truncated sequential probability ratio test.
This study will be restricted to the exponential reliability function

\[ R(t; \theta) = e^{-\frac{t}{\theta}} \quad \theta > 0, \ t \geq 0. \]

This function is used to describe the life characteristics of many types of equipment components and, thus, has great applicability.

Both the fixed sample size test and the truncated sequential probability ratio test will be conducted assuming replacement. This means that it is assumed that as soon as a component fails, it will be immediately replaced by a component of the corresponding type. Thus, there will be no loss in test time due to a component failure.

This study is divided into several sections. The first part of this study gives the mathematical formulation of hypothesis testing, of the type I and type II errors, and of the power of a test. Also included in this chapter is the development of the test statistic to be used in this study, along with the formulation of the fixed sample size and the truncated sequential probability ratio tests. Chapters III and IV deal with the application of the fixed sample size and the truncated sequential probability ratio tests to life testing of equipment components. These chapters explain the procedures for carrying out the tests. Chapter V presents the results of the simulation of the two tests on the computer. The power curves and sample sizes of the stated tests are presented and compared. Also, effects on the power curve and the average sample size caused by altering test parameters of the truncated sequential test are investigated. The final chapter presents a summary and conclusion of this study.
CHAPTER II
MATHEMATICAL FORMULATION

Testing Hypotheses

Many of the problems of practical statistics are concerned with testing whether or not some statistical hypothesis is true. This hypothesis is generally a statement about the values of one or more parameters of the population.

In order to test the validity of a given hypothesis, an experiment is conducted. The hypothesis which is formulated and tested by experiment is called the null hypothesis, designated as \( H_0 \). On the basis of this experiment, one can: (1) reject the null hypothesis if the results obtained from the experiment are not likely under the hypothesis, (2) accept the null hypothesis if the results are not improbable under \( H_0 \), or (3) decide that further sampling is necessary in order to make a decision. In the case of a fixed sample size test, the third alternative is not available; however, in a sequential test, the experimentation may be continued until one is convinced of taking either alternative (1) or (2).

Definition of Type I and Type II Errors

In taking such an action on the basis of a sample, there are two types of errors which we can make:

(1) We may reject a null hypothesis which is really true; this is called a type I error.
(2) We may accept a hypothesis which is really false; this is called a type II error.

The probability of making a type I error is designated as $\alpha$, while the probability of making a type II error is designated as $\beta$. These relationships are demonstrated in Table 1.

Table 1. Relation of type I and type II errors

<table>
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<tr>
<th>Decision</th>
<th>Actual situation (unknown)</th>
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<td></td>
<td>Hypothesis is true</td>
<td>Hypothesis is false</td>
</tr>
<tr>
<td>Accept the hypothesis</td>
<td>No error</td>
<td>Type II error</td>
</tr>
<tr>
<td>Reject the hypothesis</td>
<td>Type I error</td>
<td>No error</td>
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</table>

When setting up an experiment to test a hypothesis, it is desirable to minimize the risk of committing these errors.

In order to test a hypothesis, the possible outcomes of an experiment are divided into two regions. One of these regions consists of outcomes that are likely if the null hypothesis is true. This region is known as the acceptance region. The other region consists of results that are not probable if the null hypothesis is true. This region is known as the rejection or critical region. The sizes of these regions are fixed by $\alpha$ and $\beta$, the probabilities of making type I and type II errors, respectively.

Then some statistic from the data of the experiment is computed and tested to see whether it falls in the acceptance region or in the rejection region. The hypothesis $H_0$ is rejected if the computed value
lies in the rejection region; otherwise, $H_0$ is accepted. In order to aid in the determination of the rejection region, alternative(s) to the null hypothesis are stated. An alternative hypothesis is often denoted by $H_1$.

Simple and Composite Hypotheses

Suppose that $\theta_1, \ldots, \theta_k$ are the unknown parameters of the distribution of some random variable. A simple hypothesis is a statement about the parameters $\theta_1, \ldots, \theta_k$ that uniquely determines the values of all $k$ parameters. For example, consider the exponential distribution, with the single parameter $\theta$, described by the density function

$$f(t; \theta) = \frac{1}{\theta} e^{-\frac{t}{\theta}}, \quad \theta > 0, \ t \geq 0.$$ 

The hypothesis that $\theta$ is equal to some specified $\theta_0$ ($H_0 : \theta = \theta_0$) is a simple hypothesis, since it uniquely specifies the value of the single parameter $\theta$ of the exponential distribution. It is possible that the alternate hypothesis could also be simple, as is the case with a hypothesis that $\theta = \theta_1$, where $\theta_1$ is some specified constant ($H_1 : \theta = \theta_1$). It might be the case, however, that the alternate hypothesis could be composite. A composite hypothesis does not specifically define the values of the unknown parameters of a given distribution. It could be a two-sided alternative ($H_1 : \theta \neq \theta_0$, i.e. that $\theta$ is either less than or greater than $\theta_0$), or it could be a one-sided alternative ($H_1 : \theta > \theta_0$, supposing one is justified in believing that $\theta$ cannot be less than $\theta_0$; or $H_1 : \theta < \theta_0$, if it is believed
that \( \theta \) cannot be greater than \( \theta_0 \). A null hypothesis could also be a composite hypothesis, such as \( H_0 : \theta < \theta_0 \).

**Power of a Test**

Suppose it is desired to test the simple null hypothesis \( H_0 : \theta = \theta_0 \) against the simple alternative \( H_1 : \theta = \theta_1 \), using some test statistic \( \tau \). In order to determine the rejection region, it is necessary to know the density function of the test statistic when \( \theta = \theta_0 \), call it \( f(t; \theta_0) \).

Denote the rejection region by \( R \), where \( R \) is some interval \( t > t_\alpha \) on the \( t \) axis, and denote the acceptance region by \( A \), where the acceptance region is all possible values of \( t \) outside of \( R \). The probability of committing a type I error (rejecting \( H_0 \) when it is true) is given by

\[
\alpha = \int_{R} f(t; \theta_0) \, dt.
\]

If the alternate hypothesis \( H_1 \) is true, \( \tau \) will have a different density given by \( f(t; \theta_1) \). The probability of committing a type II error (accepting \( H_0 \) when it is false) is given by

\[
\beta = \int_{A} f(t; \theta_1) \, dt.
\]

In Figure 1, \( \alpha \) is represented by the shaded region, while \( \beta \) is represented by the striped region.
The power of a test depends on the $\theta_1$ specified in the alternate hypothesis. More specifically, it depends on the difference between the value of the parameter specified by the null hypothesis and the actual (unknown) value. If $\theta$ is any value of $\theta_1$, then

$$\pi(\theta) = 1 - \beta(\theta)$$

is called the power function for the test of $\theta_0$ against $\theta$. At $\theta = \theta_0$, the power is equal to $\alpha$. For $\theta$ close to $\theta_0$, $\beta$ will usually be quite large since it might be fairly difficult to distinguish between the two hypothesized values. This, in turn, implies that the power for $\theta$ close to $\theta_0$ will usually be small. For the case of $\theta$ far removed from $\theta_0$, $\beta$ should be reasonably small since it should be fairly easy to distinguish between two very different hypothesized values. Thus, for $\theta$ very different from $\theta_0$, the power will usually be close to one.
Statement of Statistical Hypotheses for Life Testing

In quality control engineering, it is often assumed that the various components of a piece of equipment have a lifetime described by the exponential distribution. This distribution is defined by the probability density function \( f(t; \theta) \) of the form

\[
f(t; \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{t}{\theta}} & t \geq 0 \\ 0 & t < 0 \end{cases}
\]

where the parameter \( \theta > 0 \).

The exponential reliability function, given by \( 1 - \int_{0}^{t} f(t; \theta) \, dt \) is

\[
R(t; \theta) = 1 - \int_{0}^{t} \frac{1}{\theta} e^{-\frac{t}{\theta}} \, dt = 1 - (1 - e^{-\frac{t}{\theta}}) = e^{-\frac{t}{\theta}}.
\]

Since

\[
E(T) = \int_{0}^{\infty} t \frac{1}{\theta} e^{-\frac{t}{\theta}} \, dt = \theta,
\]

\( \theta \) is referred to as the "mean time to failure," or MTTF. Thus, the reliability performance of the exponential distribution may be measured by the parameter \( \theta \), or MTTF.

Let \( \theta_1 \) by the MTTF of the new component and \( \theta_0 \) be the MTTF of the old component. The problem can be stated statistically by the hypothesis, \( H_0 : \frac{\theta_1}{\theta_0} < c \), where \( c \) is a constant which must be determined by the persons conducting the test. The alternate hypothesis is given by
For example, suppose that users of a certain equipment component have decided that in order to make the use of a new component feasible, the MTTF of the new component must be more than three times greater than that of the old. The hypotheses could be stated as

$$H_0 : \frac{\theta_1}{\theta_0} \leq 3, \quad H_1 : \frac{\theta_1}{\theta_0} > 3.$$  

It might be noted that another set of hypotheses could be used for this same problem. These hypotheses would be

$$H_0 : \theta_1 - \theta_0 < k, \quad H_1 : \theta_1 - \theta_0 > k,$$

where $k$ is some constant determined by the persons conducting the test. However, the distribution of the test statistic for a specified difference between the parameters is not independent of the parameters themselves. On the other hand, the test statistic for a specified ratio $\frac{\theta_1}{\theta_0}$ is independent of the parameters. Therefore, the hypotheses involving the ratio $\frac{\theta_1}{\theta_0}$ have been chosen for this study.

Suppose there are $n_0$ units on test of the old component and $n_1$ units on test of the new component. Assume that the lifetime of each of the $n_0$ units of the old component is described by the density function

$$f_0 (t; \theta_0) = \frac{1}{\theta_0} e^{-\frac{t}{\theta_0}}, \quad t \geq 0,$$

and the lifetime of each of the $n_1$ units of the new component is described by the density function
where \( \theta_0 > 0 \) is the MTTF of the old component and

\( \theta_1 > 0 \) is the MTTF of the new component.

The problem is to test the null hypothesis

\[
H_0 : \frac{\theta_1}{\theta_0} < c
\]

against the alternate hypothesis

\[
H_1 : \frac{\theta_1}{\theta_0} > c,
\]

where \( c \) is some predetermined constant.

**Development of the Test Statistic**

Because of practical limitations, it is often necessary to terminate a life test after a preassigned amount of time \( t^* \) has elapsed. Therefore, suppose that \( t^* \) is the termination time of the life test.

Since it has been assumed that components which fail are immediately replaced, there is no loss in test time due to a failure. Define

\[
T_i = \sum_{t^*} t^* \text{, } i = 0, 1.
\]

Then \( T_i \) is the total accumulated test time (for the old component if \( i = 0 \) or for the new component if \( i = 1 \)). For the special case \( n_0 = n_1 \), then

\[
T_0 = n_0 t^* = n_1 t^* = T_1.
\]

Suppose that \( k_i \) is the total number of units (of the old component
if $i = 0$ or of the new component if $i = 1$) which fail in the time

$[0, \tau^*]$. It is known that $2T_i / \theta_i$ is approximately distributed as a

chi-square random variable with $(2k_i + 2)$ degrees of freedom (Epstein,

1960).

Now define $\psi$ as follows:

$$\psi = \frac{(2T_0 / \theta_0) / (2k_0 + 2)}{(2T_1 / \theta_1) / (2k_1 + 2)}.$$

The statistic $\psi$ is the ratio of two chi-square random variables

divided by their respective degrees of freedom. Therefore, it is

approximately distributed as $F$ with $(2k_0 + 2)$ degrees of freedom in

the numerator and $(2k_1 + 2)$ degrees of freedom in the denominator


The test of a composite null hypothesis against a composite alternate

hypothesis is an extension of the test of a simple hypothesis

against a simple alternative. Therefore, suppose the problem is

stated as a simple hypothesis, $H_0 : \frac{\theta_1}{\theta_0} = c_0$ against a simple alternative, $H_1 : \frac{\theta_1}{\theta_0} = c_1$. Then under the null hypothesis, $H_0 : \frac{\theta_1}{\theta_0} = c_0$,

$$\psi = \frac{c_0 T_0 / (2k_0 + 2)}{T_1 / (2k_1 + 2)},$$

and $\psi$ is $F$ distributed with density

$$f_0(\psi) = \frac{\Gamma \left( \frac{(2k_0 + 2) + (2k_1 + 2)}{2} \right) \Gamma \left( \frac{2k_0 + 2}{2} \right) \Gamma \left( \frac{2k_1 + 2}{2} \right)}{\Gamma \left( \frac{2k_0 + 2}{2} \right) \Gamma \left( \frac{2k_1 + 2}{2} \right) \Gamma \left( \frac{(2k_0 + 2)(2k_1 + 2)}{2} \right)} \left( \frac{(2k_0 + 2) - 2}{2} \right)^{\frac{(2k_0 + 2) - 2}{2}} \psi^{\frac{(2k_0 + 2)(2k_1 + 2) - (2k_0 + 2)(2k_1 + 2)}{2}}.$$
Under the alternate hypothesis, $H_1 : \frac{\Theta_1}{\Theta_0} = c_1$, the cumulative distribution function for $\psi$ is

$$F(y) = P(\psi \leq y) = P\left(\frac{c_1}{c_0} \psi \leq \frac{c_1}{c_0} y\right) = F\left(\frac{c_1}{c_0} y\right).$$

Therefore, the density for $\psi$ under $H_1$ is

$$f_1(y) = \frac{d}{dy} F\left(\frac{c_1}{c_0} y\right) = f_0\left(\frac{c_1}{c_0} y\right) \frac{d}{dy} \left(\frac{c_1}{c_0} y\right) = \frac{c_1}{c_0} f_0\left(\frac{c_1}{c_0} y\right).$$

Therefore, under the alternate hypothesis, the density function for $\psi$ is given by

$$f_1(\psi) = \frac{c_1}{c_0} f_0\left(\frac{c_1}{c_0} \psi\right) = \frac{c_1}{c_0} \frac{\Gamma\left(\frac{2 k_0 + 2}{2}\right)\Gamma\left(\frac{2 k_1 + 2}{2}\right)}{\Gamma\left(\frac{2 k_0 + 2}{2}\right)\Gamma\left(\frac{2 k_1 + 2}{2}\right)} \left(1 + \frac{2 k_0 + 2}{2 k_1 + 2}\right)\left(\frac{c_1}{c_0} \psi\right)^{(2 k_0 + 2) + (2 k_1 + 2)} - 1 - \frac{c_1}{c_0} \psi\left(\frac{c_1}{c_0} \psi\right)^{(2 k_0 + 2) + (2 k_1 + 2)}.$$ 

The test is to reject the null hypothesis if $\psi < F(2 k_0 + 2, 2 k_1 + 2)$ at some appropriate $\alpha$ value.

One might wonder why this particular test statistic $\psi$ is being used, since it is known that the number of failures of certain equipment components is Poisson distributed (Lindgren, 1969, p. 162). Although the Poisson model is useful in describing the number of failures of a component, cumulative distribution tables of the ratio of two Poisson random variables are not available. On the other hand,
the statistic $\psi$ is approximately $F$ distributed, and tables of the $F$

distribution are readily available. Thus, use of the $\psi$ statistic
creates a simple test of the stated hypotheses.

The Fixed Sample Size Test

In a fixed sample size test of a simple hypothesis against a
simple alternative, the most powerful test is given by a critical
region of the form

$$\Lambda_n = \frac{f_0 (X_1, X_2, \ldots, X_n)}{f_1 (X_1, X_2, \ldots, X_n)} < k,$$

where $f_0 (X_1, \ldots, X_n)$ is the joint density function of the observations

corresponding to $H_0$ and $f_1 (X_1, \ldots, X_n)$ is the joint density function

of the observations corresponding to $H_1$ (Lindgren, 1969, p. 338). The four quantities $\alpha$, $\beta$, $n$, and $k$ determine the test, and given any
two of them, the other two are determined by the relations

$$\alpha = P_{H_0} (\Lambda_n < k), \quad \beta = P_{H_1} (\Lambda_n \geq k).$$

For this test, one may pick values of $\alpha$ and $\beta$ and have the values of

$n$ and $k$ determined from the above relations, or one may choose values

of $\alpha$ and $n$ and have the other two determined, and so forth.

For the fixed sample size test, a sample of size $n$ is drawn, and

$\Lambda_n$ is computed. The null hypothesis $H_0$ is accepted if $\Lambda_n \geq k$ and

rejected if $\Lambda_n < k$.

This technique of using a fixed sample size does not give one the

opportunity to cut the sampling short if a conclusion becomes evident

in the early stages of sampling, nor does it allow one to take a larger
sample if the nature of the data does not convincingly suggest acceptance or rejection of $H_0$. This perhaps suggests that an improvement of such a test might be possible if the sample size is not determined in advance but is allowed to depend upon the nature of the observations.

The sequential probability ratio test is such a test in which the sample size is dependent upon the observations. It is known that when a sequential test is available, such a test, on the average, requires a fewer number of observations than the fixed sample size test with corresponding $\alpha$ and $\beta$ values (Lehmann, 1959, p. 98).

The Sequential Probability Ratio Test

The second test of a simple hypothesis against a simple alternative is the sequential probability ratio test.

Suppose $H_0$ is the hypothesis that the population density function is $f_0(x)$, and $H_1$ is the hypothesis that it is $f_1(x)$. Two boundaries $A$ and $B$ are chosen, where $A < B$. A single observation at a time is taken, and after each observation one of three decisions is made: (1) accept $H_0$, (2) reject $H_0$, or (3) continue sampling.

The sequential probability ratio test presented in this study is a modification of the ordinary sequential probability ratio test. In the usual sequential probability ratio test, the decision to accept or reject $H_0$ or to continue sampling is based upon the likelihood ratio of the observations,

$$A_n = \frac{f_0(X_1, \ldots, X_n)}{f_1(X_1, \ldots, X_n)}.$$
where \( f_0 \) and \( f_1 \) are the joint density functions of \( X_1, \ldots, X_n \) under \( H_0 \) and \( H_1 \), respectively. In the case of \( n \) independent observations \( X_1, \ldots, X_n \), the joint density function of \( X_1, \ldots, X_n \) is the product of the individual density functions of the \( X_i \), \( i = 1, \ldots, n \), i.e.

\[
f(X_1, \ldots, X_n) = f(X_1) f(X_2) \cdots f(X_n).
\]

For the probability ratio test considered in this study, the density functions \( f_0(X_1, \ldots, X_n) \) and \( f_1(X_1, \ldots, X_n) \) are functions of the \( n \) observations, but they are not the product of the individual density functions for the \( n \) independent observations.

As in the usual sequential probability ratio test, the decision is based upon the ratio of the two functions,

\[
\Lambda_n = \frac{f_0(X_1, \ldots, X_n)}{f_1(X_1, \ldots, X_n)},
\]

where \( f_0(X_1, \ldots, X_n) \) is the density function under \( H_0 \) and \( f_1(X_1, \ldots, X_n) \) is the density function under \( H_1 \).

The null hypothesis \( H_0 \) is accepted if \( \Lambda_n \geq B \) and rejected if \( \Lambda_n \leq A \). If \( A < \Lambda_n < B \), another observation is drawn.

Another modification of the sequential test used in this study is the incorporation into the test of truncation. It is decided in advance that the test will be discontinued if the sample size reaches a certain number \( n' \). If no decision has been reached by the time \( n' \) observations have been taken, the test is truncated, and some new criterion is used to make a decision concerning the null hypothesis.

Wald (1947, p. 61) gives a simple and reasonable rule for acceptance or rejection of \( H_0 \) when the test is truncated at the \( n' \)th
trial: If no decision has been reached for \( n \leq n' \), accept \( H_0 \) on the \( n' \)th trial if \( 1 < \Lambda_{n'} < B \), and reject \( H_0 \) when \( A < \Lambda_{n'} < 1 \). This rule seems reasonable because when \( \Lambda_{n'} = \frac{f_0(x_1, \ldots, x_n)}{f_1(x_1, \ldots, x_n)} = 1 \), then the densities \( f_0 \) and \( f_1 \) of the \( n' \) observations are the same, and one would not expect to be able to distinguish between the null and alternate hypotheses.

This method of truncation does have an effect on the probabilities of the type I and type II errors, as pointed out by Wald (1947, p. 61). The effect on the \( \alpha \) and \( \beta \) depends upon the truncation value \( n' \). The larger the \( n' \), the smaller the effect due to truncation on \( \alpha \) and \( \beta \).

**Relations Among \( \alpha, \beta, A, \) and \( B \)**

The \( \alpha \) and \( \beta \) of the sequential ratio test can be expressed in terms of the numbers \( A \) and \( B \) which define the boundaries of the test:

\[
\alpha = P_{H_0} (\Lambda_1 \leq A) + P_{H_0} (A < \Lambda_1 < B \text{ and } \Lambda_2 \leq A) + \ldots
\]

\[
\beta = P_{H_1} (\Lambda_1 \geq B) + P_{H_1} (A < \Lambda_1 < B \text{ and } \Lambda_2 \geq B) + \ldots
\]

Since these equations cannot be easily solved for \( A \) and \( B \) in terms of \( \alpha \) and \( \beta \), the following approximations are used:

\[
A \approx \frac{\alpha}{1 - \beta} \quad \text{and} \quad B \approx \frac{1 - \alpha}{\beta} \quad \text{(Lindgren, 1969, p. 341)}.
\]

Since the values of \( A \) and \( B \) are only approximations, the actual values of \( \alpha \) and \( \beta \) vary somewhat from those specified in the test. However, it has been shown that when using these approximation formulas for \( A \)
and B, at most one of the error sizes will be larger than specified (Lindgren, 1969, p. 342). The actual $\alpha$, call it $\alpha'$, and the actual $\beta$, call it $\beta'$, satisfy the relations

$$\alpha' \leq \alpha (1 + \beta) \quad \text{and} \quad \beta' \leq \beta (1 + \alpha).$$

It is also possible that the true error sizes $\alpha'$ and $\beta'$ are smaller than the $\alpha$ and $\beta$ sizes specified for the test. This would allow a smaller sample size $n$ to be taken, with the results still being held within the $\alpha$ and $\beta$ limits. However, the effect caused by the error sizes being smaller than those specified by the test should not be any more significant than that caused by the increase in size of the specified $\alpha$ and $\beta$ (Lindgren, 1969, p. 343).

**Logarithm Form of the Sequential Test**

There are several cases in which the logarithm of the ratio is easier to work with than the ratio of the density functions itself. For the logarithm form,

$$\log \Lambda_n = \log \frac{f_0 (X_1, \ldots, X_n)}{f_1 (X_1, \ldots, X_n)}.$$ 

If the logarithm form of the ratio is used, the inequality for continuing sampling can be written as

$$\log A < \log \Lambda_n < \log B.$$ 

In this case, usually $\log A < 0$ and $\log B > 0$. Figure 2 shows a typical plot of $n$ against $\log \Lambda_n$. If the boundary determined by $\log B$ is crossed first, $H_0$ is accepted. If the lower boundary determined by $\log A$ is crossed first, $H_0$ is rejected. In the case of either accept-
tance or rejection of $H_0$, sampling is discontinued. In the sample plot shown in Figure 2, sampling is discontinued after the eighth observation is taken.

In the case of a truncated test, where truncation occurs if no decision has been reached by the n'th trial, the following rule given by Wald (1947, p. 61) for acceptance or rejection of the null hypothesis may be used: Accept $H_0$ at the n'th trial if $0 < \log A_n < \log B$, and reject $H_0$ at the n'th trial if $\log A < \log A_n < 0$.

Figure 2. Sample plot of $(n, \log A_n)$.
CHAPTER III
APPLICATION OF THE FIXED SAMPLE SIZE TEST
TO LIFE TESTING OF EQUIPMENT COMPONENTS

Suppose the problem is to test the hypothesis

\[
H_0 : \frac{\theta_1}{\theta_0} \leq c_0
\]

against the alternate hypothesis

\[
H_1 : \frac{\theta_1}{\theta_0} > c_0,
\]

where \( \theta_0 \) is the MTTF of the old component, \( \theta_1 \) is the MTTF of the new component, and \( c_0 \) is a constant determined from economic considerations.

Suppose there are \( n_0 \) units of the old component and \( n_1 \) units of the new component on test for time \( t^* \), where \( t^* \) is the sample size in some appropriate units of time. Then \( T_i = n_i t^* \) is the total accumulated test time for a particular component type. For example, suppose a test on \( n_0 \) units is to be run for 30 days. Then \( (30 \text{ days}) \times (24 \text{ hours/day}) \times (n_0) = (720) \times (n_0 \text{ hours}) \) is the total accumulated test time in hours for the old component.

In testing the lifetimes of the given components, it is assumed that when a component fails, it is immediately replaced by a new component of the same type. Thus, there is assumed to be no loss in test time due to a failure.

The test is then carried out as follows:
The test begins with $n_0$ units of the old component and $n_1$ units of the new component on test. As a component fails, one is added to the count $k_i$ of the number of failures (of the old component if $i = 0$ or the new component if $i = 1$), and the failure is immediately replaced by a new component of the corresponding type. This procedure is continued until the time $t^*$ has been reached.

The test statistic, given by

$$\psi = \frac{2T_0}{(\theta_0 (2k_0 + 2))} \frac{2T_1}{(\theta_1 (2k_1 + 2))}$$

is approximately distributed as $F$ with $(2k_0 + 2)$ degrees of freedom in the numerator and $(2k_1 + 2)$ degrees of freedom in the denominator.

If both types of components have the same number of units on test, then $T_0 = n_0 t^* = n_1 t^* = T_1$, and the above statistic reduces to

$$\psi = \frac{\theta_1 (2k_1 + 2)}{\theta_0 (2k_0 + 2)}.$$ 

Under the null hypothesis, the statistic becomes

$$\psi = \frac{c_0 (2k_1 + 2)}{(2k_0 + 2)},$$

and it is distributed as $F(2k_0 + 2, 2k_1 + 2)$. 

This statistic $\psi$ is compared with the value of $F$ with $(2k_0 + 2, 2k_1 + 2)$ degrees of freedom at the specified $\alpha$ value in a table of $F$ distribution values. If

$$\frac{c_0 (2k_1 + 2)}{(2k_0 + 2)} < F(2k_0 + 2, 2k_1 + 2; \alpha),$$

the null hypothesis is rejected; otherwise, the null hypothesis is accepted.

**Example:** Suppose it is desired to see if the MTTF of a new component is more than three times greater than the MTTF of an old component, i.e., to test

\[ H_0 : \frac{1}{\theta_0} \leq 3, \quad H_1 : \frac{1}{\theta_0} > 3. \]

A fixed sample size test with a sample size of 30 days and \( \alpha = .05 \) is to be run.

Suppose there are \( n_0 = 3 \) and \( n_1 = 3 \) units on test of the old and new components, respectively. If the length of the test time in hours is \( t^* = (30 \text{ days}) (24 \text{ hours/day}) = 720 \text{ hours} \), then \( T_0 = n_0 t^* = 3 \) (720 hours) = \( n_1 t^* = T_1 \), i.e., the old and new components have the same total accumulated test time. Then, under \( H_0 \),

\[
\psi = \frac{\frac{2}{T_0}}{\frac{2}{T_1}} = \frac{\frac{2}{T_0}}{\frac{2}{T_1}} = \frac{3}{2} \frac{T_0 t^*}{T_1 t^*} = \frac{3}{2} \left( \frac{n_0}{n_1} \right) \frac{T_0}{T_1}
\]

is \( F \) distributed with \( (2k_0 + 2, 2k_1 + 2) \) degrees of freedom.

Suppose at the end of 30 days, the following test data is available:

- number of failures of the old component = \( k_0 = 48 \)
- number of failures of the new component = \( k_1 = 8 \).

Then \( \psi = \frac{3}{2} \frac{(2(8) + 2)}{(2(48) + 2)} = \frac{3}{2} \frac{18}{98} = \frac{54}{98} = .55. \)

From a table of \( F \) distribution values, \( F(98, 18; \alpha = .05) = .59. \)

Since \(.55 < .59\), \( H_0 \) is rejected.
CHAPTER IV
APPLICATION OF THE TRUNCATED SEQUENTIAL PROBABILITY RATIO TEST TO LIFE TESTING OF EQUIPMENT COMPONENTS

Now consider application of a truncated sequential probability ratio test to test the hypothesis $H_0 : \frac{\theta_1}{\theta_0} \leq c$ against $H_1 : \frac{\theta_1}{\theta_0} > c$.

In order to apply the sequential test, the problem is usually reduced to a test of a simple hypothesis against a simple alternative (Wald, 1947, pp. 78-79). Values $c_0$ and $c_1$ for the null and alternate hypothesis, respectively, are chosen so that the probability of a type I error is less than or equal to a preassigned $\alpha$ whenever $\frac{\theta_1}{\theta_0} \leq c_0$ and the probability of a type II error is less than or equal to a preassigned $\beta$ whenever $\frac{\theta_1}{\theta_0} \geq c_1$. Therefore, suppose the hypotheses can be stated as $H_0 : \frac{\theta_1}{\theta_0} = c_0$ against $H_1 : \frac{\theta_1}{\theta_0} = c_1$, where $c_1 > c_0$ is some particular value from the original composite alternate hypothesis.

The test procedure for the truncated sequential probability ratio test is as follows:

First, values for the probabilities of the type I and type II errors must be chosen. As explained in Chapter II, the numbers $A$ and $B$, which define the boundaries of the test, are determined from the relations

$$A = \frac{\alpha}{1 - \beta} \quad \text{and} \quad B = \frac{1 - \alpha}{\beta}.$$
Suppose the test is to be observed every $t'$ units of time. For instance, observations may be taken at the end of a day, i.e., every 24 hours. In order to determine the sample size, a record is to be kept of the number of $t'$ periods required to reach a decision to either accept or reject the null hypothesis. The count of the number of $t'$ periods is set equal to one at the beginning of the first period.

For each $t'$ period, counts $k_0$ and $k_1$ of the number of failures of the old component and the new component, respectively, are to be recorded.

As in the fixed sample size test, suppose there are $n_0$ units of the old component and $n_1$ units of the new component on test. In any $j$th $t'$ period, as a component fails, one is added to the count $k_j$ of the number of failures of the appropriate component in that $j$th period, and the failure is replaced by another component of its type.

At the end of the first $t'$ time period, the test statistic $\psi$, under the null hypothesis, is computed as

$$\psi = \frac{\frac{2}{2} \frac{T_0}{T_1} / \left(\theta_0 (2 k_0 + 2)\right)}{\frac{2}{2} \frac{T_1}{T_0} / \left(\theta_1 (2 k_1 + 2)\right)} = \frac{(c_0) 2 T_0}{(2 k_0 + 2)}$$

where $k_i = \sum_{j=1}^{1} k_{j i}$, $i = 1, 2$.

For the special case $n_0 = n_1$, then $T_0 = n_0 t^* = n_1 t^* = T_1$, and the test statistic $\psi$ can be simplified to

$$\psi = \frac{c_0 (2 k_1 + 2)}{(2 k_0 + 2)}.$$

Now $\psi$ is approximately F distributed with $(2 k_0 + 2)$ degrees of freedom in the numerator and $(2 k_1 + 2)$ degrees of freedom in the denominator.
As stated previously, the density function associated with the null hypothesis is given by

\[
\begin{align*}
  f_0(y) &= \frac{\Gamma\left(\frac{2k_0 + 2}{2}\right) + \frac{2k_0 + 2}{2k_1 + 2}}{\Gamma\left(\frac{2k_0 + 2}{2}\right) \Gamma\left(\frac{2k_1 + 2}{2}\right) (1 + \frac{(2k_0 + 2)}{2k_1 + 2} y)} \\
  f_1(y) &= \frac{\Gamma\left(\frac{2k_0 + 2}{2}\right) + \frac{2k_0 + 2}{2k_1 + 2}}{\Gamma\left(\frac{2k_0 + 2}{2}\right) \Gamma\left(\frac{2k_1 + 2}{2}\right) (1 + \frac{(2k_0 + 2)}{2k_1 + 2} c_1 c_0 y)}
\end{align*}
\]

and the density function associated with the alternate hypothesis is given by

\[
\begin{align*}
  f_0(y) &= \frac{\Gamma\left(\frac{2k_0 + 2}{2}\right) + \frac{2k_0 + 2}{2k_1 + 2}}{\Gamma\left(\frac{2k_0 + 2}{2}\right) \Gamma\left(\frac{2k_1 + 2}{2}\right) (1 + \frac{(2k_0 + 2)}{2k_1 + 2} c_1 c_0 y)} \\
  f_1(y) &= \frac{\Gamma\left(\frac{2k_0 + 2}{2}\right) + \frac{2k_0 + 2}{2k_1 + 2}}{\Gamma\left(\frac{2k_0 + 2}{2}\right) \Gamma\left(\frac{2k_1 + 2}{2}\right) (1 + \frac{(2k_0 + 2)}{2k_1 + 2} c_1 c_0 y)}
\end{align*}
\]

Forming the ratio of these two density functions gives

\[
\frac{f_0(y)}{f_1(y)} = \frac{\Gamma\left(\frac{2k_0 + 2}{2}\right) + \frac{2k_0 + 2}{2k_1 + 2}}{\Gamma\left(\frac{2k_0 + 2}{2}\right) \Gamma\left(\frac{2k_1 + 2}{2}\right) (1 + \frac{(2k_0 + 2)}{2k_1 + 2} c_1 c_0 y)}
\]

Some simplification of the above yields
Now, evaluation of $f_0$ and $f_1$ at $\psi = \frac{c_0(2 k_1 + 2)}{(2 k_0 + 2)}$ yields

$$\frac{f_0(\psi)}{f_1(\psi)} = \frac{\frac{c_0}{c_1} \left( \frac{c_0}{c_0} \right)^{k_0} \left( \frac{c_0}{c_0} \right)^{(2 k_1 + 2)}}{\left(1 + \frac{(2 k_0 + 2)}{(2 k_1 + 2)} \right) \left( \frac{c_0}{c_0} \right)^{(2 k_1 + 2)}}$$

This reduces finally to

$$\frac{f_0(\psi)}{f_1(\psi)} = \left( \frac{c_0}{c_1} \right)^{k_0 + 1} \left( \frac{1 + \frac{c_1}{c_0}}{1 + c_0} \right)^{k_0 + k_1 + 2}$$

After the first $t'$ period, the ratio $\Lambda_1 = \frac{f_0(\psi)}{f_1(\psi)}$, based on the first $t'$ period, is compared with the $A$ and $B$ values to see if a final decision can be reached. If $\Lambda_1 < A$, $H_0$ is rejected; if $\Lambda_1 > B$, $H_0$ is accepted.

Either of these two cases causes termination of the sequential test. If, however, $A < \Lambda_1 < B$, no decision concerning $H_0$ can be
made. Instead, sampling must be continued by testing for another t' time period.

If sampling must be continued, one is added to the count of the number of t' periods needed to reach a decision, and the test is run for another period. At the end of the second t' period, the number of failures $k_{2i}$ is added to the total accumulated number of failures for the appropriate component. Thus, the total number of failures for a component at the end of the second period becomes

$$k_i = \sum_{j=1}^{2} k_{ji}, i = 0, 1.$$  

The statistic $\psi$ is recalculated with the new values of $k_0$ and $k_1$. Then $\Lambda_2 = \frac{f_0(\psi)}{f_1(\psi)}$ is compared with the A and B values. If $A < \Lambda_2 < B$, one is added to the count of t' periods, and the test is continued for another t' period, and so forth. Otherwise, a final decision to either accept or reject the null hypothesis is made, and the sequential test terminates.

If it should be the case that no decision has been reached by the truncation time (the n'th t' test period), the test will automatically be truncated after the n'th period. Then $H_0$ will be accepted if $1 \leq \Lambda_n < B$ and rejected if $A < \Lambda_n < 1$.

**Example:** Suppose it is desired to test the hypothesis $H_0 : \frac{\theta_1}{\theta_0} < 3$ against $H_1 : \frac{\theta_1}{\theta_0} > 3$. In order to run the truncated sequential test, the hypotheses are reduced to simple ones. Therefore, suppose it is desired to test $H_0 : \frac{\theta_1}{\theta_0} = 3$ against $H_1 : \frac{\theta_1}{\theta_0} = 6$. A sequential test with parameters $\alpha = .05$ and $\beta = .15$ is to be run.
Suppose that there are an equal number of components of each type on test, i.e., \( n_0 = n_1 = 3 \). Also, suppose that the test is to be run at the end of each day, using the total accumulated failures for each component type up to that time as the \( k_0 \) and \( k_1 \) values. Then \( t^* = 24 \) hours, and \( \frac{n_0}{T_0} = \frac{n_1}{t^*} = \frac{3}{3(2k_1+2)} \) is \( F \) distributed with \( (2k_0 + 2, 2k_1 + 2) \) degrees of freedom.

The ratio of the two densities is

\[
\frac{f_0(\psi)}{f_1(\psi)} = \left( \frac{1}{2} \right)^{3k_0 + 1} \frac{1 + 6}{1 + 3} \left( \frac{1}{2} \right)^{k_0 + k_1 + 2} = \left( \frac{1}{2} \right)^{k_0 + 1} \frac{1}{(1.75)^{k_0 + k_1 + 2}}
\]

The test boundaries \( A \) and \( B \) are

\[
A = \frac{\alpha}{1-\beta} = \frac{.05}{.85} = .059, \quad B = \frac{1-\alpha}{\beta} = \frac{.95}{.15} = 6.33.
\]

Suppose the truncated sequential test was run and that the observations and the corresponding ratios \( \Lambda_n \) are as given in Table 2.
Table 2. Data for truncated sequential test example

<table>
<thead>
<tr>
<th>Day</th>
<th>$k_0$</th>
<th>$k_1$</th>
<th>$\Lambda_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3.59</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2.41</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2</td>
<td>2.10</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>2</td>
<td>1.84</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>2</td>
<td>1.08</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>2</td>
<td>0.83</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>2</td>
<td>0.72</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>2</td>
<td>0.63</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
<td>3</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>4</td>
<td>1.70</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>4</td>
<td>1.48</td>
</tr>
<tr>
<td>12</td>
<td>19</td>
<td>4</td>
<td>1.14</td>
</tr>
<tr>
<td>13</td>
<td>21</td>
<td>4</td>
<td>0.87</td>
</tr>
<tr>
<td>14</td>
<td>25</td>
<td>4</td>
<td>0.51</td>
</tr>
<tr>
<td>15</td>
<td>26</td>
<td>4</td>
<td>0.45</td>
</tr>
<tr>
<td>16</td>
<td>26</td>
<td>6</td>
<td>1.37</td>
</tr>
<tr>
<td>17</td>
<td>27</td>
<td>8</td>
<td>3.66</td>
</tr>
<tr>
<td>18</td>
<td>27</td>
<td>9</td>
<td>6.41</td>
</tr>
</tbody>
</table>

After the 18th day, $\Lambda_{18} = 6.41 > B = 6.33$.

Therefore $H_0 : \frac{0.1}{\theta_0} \leq 3$ is accepted.

This example can be pictured graphically in Figure 3.

For simplicity in this study, one might want to consider the logarithm of the ratio. Then the criterion for continued sampling could be transformed as follows:

$$A < \Lambda_n < B$$

Taking the log of each quantity,

$$\log A < \log \Lambda_n < \log B.$$
Figure 3. Plot of $(n, \Lambda_n)$ for truncated sequential test example.
= (k_0 + 1)(\log c_0 - \log c_1) + (k_0 + k_1 + 2)(\log (1 + c_1) - \log (1 + c_0)) \\
= k_0(\log c_0 - \log c_1 + \log (1 + c_1) - \log (1 + c_0)) \\
+ k_1(\log (1 + c_1) - \log (1 + c_0)) + \log c_0 - \log c_1 \\
+ 2(\log (1 + c_1) - \log (1 + c_0)).

Therefore,

\[
\log A < k_0(\log c_0 - \log c_1 + \log (1 + c_1) - \log (1 + c_0)) + \\
k_1(\log (1 + c_1) - \log (1 + c_0)) + \log c_0 - \log c_1 \\
+ 2(\log (1 + c_1) - \log (1 + c_0)) < \log B.
\]

Since the quantity \(D = -\log c_0 + \log c_1 - 2(\log (1 + c_1) - \log (1 + c_0))\)

is just a constant, it can be added to each term giving

\[
\log A + (-\log c_0 + \log c_1 - 2(\log (1 + c_1) - \log (1 + c_0))) < \\
k_0(\log c_0 - \log c_1 + \log (1 + c_1) - \log (1 + c_0)) + k_1(\log \\
(1 + c_1) - \log (1 + c_0)) \\
< \log B + (-\log c_0 + \log c_1 - 2(\log (1 + c_1) - \log (1 + c_0))).
\]

This set of inequalities now has the form \(\log A + D < T < \log B + D\),

where \(D\) is a constant and \(T\) is a function of \(k_0\) and \(k_1\). The decision

for continued sampling would then be based upon the above inequalities. 

For the special case of a truncated test, where truncation occurs at

the \(n\)'th trial if no decision can be reached for \(n \leq n'\), the following

decision rule could be used: Accept \(H_0\) if

\[
\log B + D > k_0(\log c_0 - \log c_1 + \log (1 + c_1) - \log (1 + c_0)) \\
+ k_1(\log (1 + c_1) - \log (1 + c_0)) \geq 0
\]
and reject $H_0$ if

$$0 > k_0 (\log c_0 - \log c_1 + \log (1 + c_1) - \log (1 + c_0))$$

$$+ k_1 (\log (1 + c_1) - \log (1 + c_0)) > \log A + D.$$
CHAPTER V  
RESULTS OF THE MONTE CARLO STUDY

Procedures for Simulation of the Number of Component Failures

The method commonly used in industry to carry out either of the stated tests is to place a given number of components of each type on test and record the number of failures that occur in a certain time $t$. The hypotheses can then be tested using the test data. In order to simulate an actual testing situation, it was necessary to generate one Poisson observation to represent the number of failures of the old component and another to represent the number of failures of the new component. This process was used in both the fixed sample size test and the truncated sequential probability ratio test.

These Poisson observations were generated according to the Poisson distribution with parameter $\frac{1}{\theta} t$, where $\theta$ was the hypothesized MTTF of the appropriate component, and $t$ was the total test time. For example, suppose a fixed sample size test of 30 days (720 hours) with three components of each type on test was to be run. Suppose also that the hypothesized values of $\theta_0$ and $\theta_1$ were 50 hours and 100 hours, respectively. Then the number of failures for the old component was generated according to the Poisson distribution with parameter $\frac{1}{\theta_0} t = \frac{1}{\frac{50}{(720 \text{ hours})}} (720 \text{ hours}) = 14.4$.

Similarly, the number of failures for the new component was generated according to the Poisson distribution with parameter $\frac{1}{\theta_1} t = 7.2$. 

The method used for generating the Poisson observations representing the number of failures for each component is given in Appendix A.

**Procedure for Simulation of the Fixed Sample Size Test and Its Power Curve**

For the fixed sample size test simulation, hypothesized values of $\theta_0$ and $\theta_1$ (and $\frac{\theta_1}{\theta_0}$) were decided upon. Then a sample size of $n$ days was chosen, along with the number $u$ representing the number of units of each type of component on test (assuming both components had the same number of units on test). Poisson observations, representing the number of failures $k_0$ and $k_1$ of the old and new components, respectively, were generated. The test statistic $\psi$ was computed using the hypothesized value of $\frac{\theta_1}{\theta_0}$. The statistic $\psi$ was then compared with the appropriate rejection region for $\alpha = .05$ (tabular $F(2k_0 + 2, 2k_1 + 2; \alpha = .05)$).

In order to simulate the power curve for the fixed sample size test, five-hundred fixed sample size tests were run for each true value of $\frac{\theta_1}{\theta_0}$, and the power of the test for each actual $\frac{\theta_1}{\theta_0}$ was determined from the percentage of rejections of $H_0$ for that value. The computer program used to simulate the fixed sample size test and determine the power curve associated with the test is given in Appendix B.
Procedure for Simulation of the Truncated Sequential Probability Ratio Test, Its Power Curve and Expected Sample Size

The first step in the simulation of the truncated sequential probability ratio test was the determination of various values. Two sets of values of $\theta_0$ and $\theta_1$ (and $\frac{\theta_1}{\theta_0}$) were chosen in order to determine the densities of $\psi$ under the null and alternate hypotheses. The test time unit $t'$ and the number of units $u$ of each component on test were selected. Then the following test parameters were chosen: $\alpha$ and $\beta$, the probabilities of the type I and type II errors, respectively, and $n'$, the truncation point.

The boundaries $A$ and $B$ which determined the sequential test were computed using the chosen $\alpha$ and $\beta$ values. Then Poisson observations representing the number of failures of each of the two types of components were generated.

Using the hypothesized value of $\frac{\theta_1}{\theta_0}$ and the accumulated number of failures $k_0$ and $k_1$, the statistic $\psi = \frac{\theta_1 (2k_1 + 2)}{\theta_0 (2k_0 + 2)}$ was computed.

Then $\Lambda_n$, the ratio of the densities for $\psi$ under $H_0$ and $H_1$, was evaluated. If $\Lambda_n \leq A$, $H_0$ was rejected; if $\Lambda_n \geq B$, $H_0$ was accepted. If $A < \Lambda_n < B$, another set of Poisson observations was generated, and the total number of failures for each component up to that time (i.e., the sum of the Poisson observations) was used in computing a new ratio $\Lambda_n$. This ratio was tested against $A$ and $B$, and $H_0$ was accepted or rejected, or the process of generating more Poisson observations was repeated, and so forth.
As in the case of the fixed sample size test, five-hundred sequential tests were run for each actual value of \( \frac{\theta_1}{\theta_0} \) tested. Then the power of the sequential test of \( H_0 \) against \( \frac{\theta_1}{\theta_0} \) was determined from the percentage of rejections of \( H_0 \) for the corresponding true value of \( \frac{\theta_1}{\theta_0} \). In order to get an estimate of the expected sample size when \( \frac{\theta_1}{\theta_0} \) was the true value of the parameter, the program computed the average sample size required to reach a decision for each \( \frac{\theta_1}{\theta_0} \) value used to generate the data.

The program also tabulated a frequency distribution of the sizes of the samples and computed the percentage of sample sizes which were less than or equal to the fixed sample size of 70 days. The computer program for simulation of the truncated sequential probability ratio test and estimation of its power curve and expected sample size is found in Appendix C.

If no decision had been reached after \( n' \) test periods, the pre-assigned truncation time, the test was truncated. In the case of truncation, the following criteria were used to make a decision:

1. If \( \Lambda_n' \geq 1 \), \( H_0 \) was accepted.
2. If \( \Lambda_n' < 1 \), \( H_0 \) was rejected.

**Comparison of Power Curves for the Two Tests**

First of all, fixed sample size tests with various sample sizes were run in order to determine a sample size which created a reasonably powerful test. The fixed sample size test with \( n = 70 \) and \( \alpha = .05 \) was chosen for the comparison with a sequential test with approximately the same power curve.
Then a truncated sequential test whose power curve matched that of the chosen fixed sample size test was devised. The $\beta$ value and the value of $\frac{\theta_1}{\theta_0}$ used to determine the density of $\psi$ under the alternate hypothesis were varied until the curve produced by the truncated sequential test closely approximated that of the fixed sample size test. The resulting power curves for the fixed sample size test (-----) of $n = 70$ days, $\alpha = .05$ and the truncated sequential test (-----) with $\alpha = .05$, $\beta = .10$, $\frac{\theta_1}{\theta_0} = 3$ for the density of $\psi$ under $H_0$, and $\frac{\theta_1}{\theta_0} = 6$ for the density of $\psi$ under $H_1$, are shown in Figure 4.

As illustrated by Figure 4, the power of the truncated sequential test is very close to that of the fixed sample size test for the values of $\frac{\theta_1}{\theta_0}$ in the region close to the hypothesized value. It is slightly less powerful in the region of $\frac{\theta_1}{\theta_0}$ values far removed from the hypothesized value, but this region is of less importance, because it would be fairly obvious that values in this region were greater than the hypothesized value of $\frac{\theta_1}{\theta_0}$.

**Comparison of Sample Size for the Two Tests**

Consider again the same fixed sample size test and the same truncated sequential test whose power curves were compared in the previous section. It is known that the sequential probability ratio test requires, on the average, fewer observations than the fixed sample size test with the corresponding $\alpha$ and $\beta$ error levels (Lehmann, 1959, p. 98). In order to compare the sample sizes of the two tests, the average sample size required to reach a decision was computed for each
Figure 4. Power curves for the fixed sample size test (-----) of \( n = 70 \) days, \( \alpha = .05 \) and the truncated sequential test (----) with \( \alpha = .05, \beta = .10 \).

\[
\frac{\theta_1}{\theta_0} = 3 \text{ under } H_0, \quad \frac{\theta_1}{\theta_0} = 6 \text{ under } H_1.
\]
used to generate the data in the truncated sequential test. Figure 5 shows a plot of the average sample size for the truncated sequential test (-----) against a fixed sample size (-----) of n = 70 days.

Figure 5. Plot of $\frac{\theta_1}{\theta_0}$ against sample size for the truncated sequential test (-----) with $\alpha = .05$, $\beta = .10$, $\frac{\theta_1}{\theta_0} = 3$ under $H_0$, $\frac{\theta_1}{\theta_0} = 6$ under $H_1$ and the fixed sample size test (-----) with n = 70, $\alpha = .05$. 
As illustrated in Figure 5, there was a substantial reduction in sample size when the truncated sequential probability ratio test was applied. The sample size for the truncated sequential test most closely approximates the fixed sample size at a point about midway between the hypothesized \( \frac{\theta_1}{\theta_0} \) and the true \( \frac{\theta_1}{\theta_0} \) (the \( \frac{\theta_1}{\theta_0} \) used to generate the data).

Also shown in Figure 5 are the approximate .05 and .95 percentile points (dots connected by lines) for the frequency distribution of the sample sizes. The .05 point is the sample size which is exceeded by 95 percent of the samples. Similarly, the sample size which was an upper limit for 95 percent of the samples is given by the .95 point.

Table 3 gives for each actual \( \frac{\theta_1}{\theta_0} \) value the percentage of sequential tests (based on the 500 tests given for each \( \frac{\theta_1}{\theta_0} \) ) whose sample sizes were less than or equal to the fixed sample size of 70 days.

<table>
<thead>
<tr>
<th>True ( \frac{\theta_1}{\theta_0} ) value</th>
<th>Percent of sample sizes ≤ 70 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2.5</td>
<td>99.8</td>
</tr>
<tr>
<td>3.0</td>
<td>93.2</td>
</tr>
<tr>
<td>3.2</td>
<td>89.0</td>
</tr>
<tr>
<td>3.6</td>
<td>80.6</td>
</tr>
<tr>
<td>4.0</td>
<td>74.4</td>
</tr>
<tr>
<td>5.0</td>
<td>71.8</td>
</tr>
<tr>
<td>6.0</td>
<td>81.4</td>
</tr>
<tr>
<td>8.0</td>
<td>94.0</td>
</tr>
<tr>
<td>10.0</td>
<td>98.2</td>
</tr>
</tbody>
</table>
Table 3 demonstrates that at worst at least 70 percent of the truncated sequential tests have sample sizes less than or equal to the fixed sample size of 70 days.

**Effects on Power Curve of Altering the Sequential Probability Ratio Test Parameters**

Two of the parameters of the sequential ratio test are the $\beta$ and the $\frac{\theta_1}{\theta_0}$ used to determine the density of $\psi$ under the alternate hypothesis. These two parameters were altered in order to see the effects on the power curve of the truncated sequential probability ratio test. First of all, the $\alpha$ and $\beta$ values were fixed at .05 and .10, respectively, and the $c_1$ in the alternate hypothesis $H_1 : \frac{\theta_1}{\theta_0} = c_1$ was given the values five, six and seven. The resulting power curves are shown in Figure 6. The center curve in this figure is the same as the curve given for the truncated sequential test (-----) in Figure 4.

The second parameter to be altered was $\beta$. In this case, a sequential test with $\alpha$ fixed at .05 and the $c_1$ of the alternate hypothesis set at six was simulated. The parameter $\beta$ was allowed to assume the values .08, .10, and .15. The power curves which resulted for these various tests with differing values of the parameter $\beta$ are shown in Figure 7. The curve for $\beta = .10$ in this figure is the same as the curve given for the truncated sequential test (-----) in Figure 4.
Figure 6. Power curves for the truncated sequential test with $\alpha = .05$, $\beta = .10$, $\frac{\theta_1}{\theta_0} = 3$ under $H_0$, and various values of $c_1$ for $H_1: \frac{\theta_1}{\theta_0} = c_1$. 
Figure 7. Power curves for the truncated sequential test with $\alpha = .05$, $\frac{\theta_1}{\theta_0} = 3$ under $H_0$, $\frac{\theta_1}{\theta_0} = 6$ under $H_1$, and various values of $\beta$. 

$1 - \beta\left(\frac{\theta_1}{\theta_0}\right)$
Consider again the truncated sequential test with $\alpha$ and $\beta$ fixed at .05 and .10, respectively, $H_0 : \frac{\theta_1}{\theta_0} = 3$, and $c_1$ varied over the values five, six and seven, for $H_1 : \frac{\theta_1}{\theta_0} = c_1$. The average sample size required to reach a decision in the sequential test is plotted for the various alternate $\frac{\theta_1}{\theta_0}$ values in Figure 8. The center curve in this figure is the same curve as the one for the truncated sequential test (-----) in Figure 5.

Figure 8. Plot of $\frac{\theta_1}{\theta_0}$ against sample size for the truncated sequential test with $\alpha = .05$, $\beta = .10$, $\frac{\theta_1}{\theta_0} = 3$ under $H_0$, and various values of $c_1$ for $H_1 : \frac{\theta_1}{\theta_0} = c_1$.
Figure 9 shows a plot of the average sample size required to reach a decision in the truncated sequential test with $\alpha = .05$, $H_0: \frac{\theta_1}{\theta_0} = 3$, $H_1: \frac{\theta_1}{\theta_0} = 6$, and various values of the parameter $\beta$. Again, the middle curve in Figure 9 is the same curve given for the truncated sequential test (----) in Figure 5.

![Figure 9. Plot of $\frac{\theta_1}{\theta_0}$ against sample size for the truncated sequential test with $\alpha = .05$, $\frac{\theta_1}{\theta_0} = 3$ under $H_0$, $\frac{\theta_1}{\theta_0} = 6$ under $H_1$, and various values of $\beta$.](image)
As can be seen in Figure 8, as \( \frac{\theta_1}{\theta_0} = c_1 \) gets closer to the hypothesized value, the average sample size for the stated sequential test increases. This is reasonable as it seems that it would become more difficult (and thus require a larger sample size) to distinguish between the null and alternate hypotheses as the alternate value moved closer to the hypothesized value.

Figure 9 shows that as \( \beta \) decreases, with the other truncated sequential test parameters held constant, the average sample size increases.

It should be noted that the sample sizes required by these tests are reasonable ones; that is, they are sample sizes which could very well be used in actual test situations.
CHAPTER VI
SUMMARY AND CONCLUSION

Many users of equipment today are faced with the problem of deciding whether or not old equipment should be replaced by a new type of equipment which performs the same function. In order to make this decision, they often need to test the reliability performance of the new equipment components relative to that of the old components. Then, based on a comparison of the reliability performances of the new and old components, they can decide whether or not it is economical to use the new equipment.

The purpose of this study has been to develop a test which can be used to compare two different component types to determine whether or not the "mean time to failure" of the new component is less than or equal to a certain number times the "mean time to failure" of the old component. A statistic with an F distribution has been presented, and because tables of this distribution are readily available, the test is easily and conveniently run.

Two types of tests have been considered in this paper: the fixed sample size test and the truncated sequential probability ratio test. A comparison of the two tests has been based on a Monte Carlo study. The results of the simulated power curves and required sample sizes have demonstrated the potential of the truncated sequential test for use in two sample life tests. A truncated sequential test with power almost equal to that of the corresponding fixed sample size test in the region of foremost consideration was developed. It was found that this
truncated sequential test required a significantly smaller sample size than the fixed sample size test with corresponding $\alpha$ and $\beta$ error levels. Thus, a truncated sequential probability ratio test based on the $F$ distributed statistic $\psi$ seems to be an efficient and effective means of comparing two component types to see if the new one meets a certain feasibility criterion.
LITERATURE CITED


Appendix A

Method of Generating Poisson Random Variables

Consider the following process. A point $v_1$ is picked at random from the unit interval $[0, 1]$. Then a second point $v_2$ is picked at random from $[0, v_1]$, a third point is picked at random from $[0, v_2]$, and so forth. An approximate model for the distribution of $v_1, v_2, \ldots, v_n$ is $v_1 = x_1, v_2 = x_1 x_2, \ldots, v_n = x_1 x_2 \ldots x_n$, and so on. Now $x_1, x_2, \ldots, x_n$ are mutually independent random variables, and each is uniformly distributed on the interval $[0, 1]$. Suppose that $Z$ is the number of points that fall in some interval $[c, 1]$ where $0 < c < 1$. Then $Z$ is distributed as a Poisson random variable with parameter $\lambda = -\log c$ (Dwass, 1970, pp. 307-308).

The following algorithm for generating a single random variable from the Poisson distribution with parameter $\lambda$ is found in Knuth (1971, p. 117).

1. Calculate the expression $p = e^{-\lambda}$.
   
   Set $N = 0$ and $q = 1$.

2. Pick a random variable $U$ from the uniform distribution on $[0, 1]$.

3. Multiply $q$ by $U$ to obtain a new variable $q$. ($q = q(U)$).

4. Test to see if $q \geq p$. If it is, add one to $N$ ($N = N + 1$) and return to step 2. Otherwise, stop and output $N$ as the Poisson observation.

The method of simulation used in the computer programs of Appendices B and C incorporates the above algorithm to generate random
Poisson numbers to represent the number of failures of the two components. The Poisson observations are generated according to the Poisson distribution with the parameter $\frac{1}{\theta} t$, where $\theta$ is the MTTF of the specified component, and $t$ is the test time. A random number generator is used to generate the numbers from the uniform distribution on $[0, 1]$. 
Appendix B

Computer Program for Simulation of Fixed Sample Size Test and Estimation of Power Curve

To simulate the number of failures of each component, this program incorporates the method of generating Poisson observations given in Appendix A. The number of failures of each component type and the hypothesized value are used to generate the statistic. The function PRBF is then called to compute the probability of the resulting F value. This probability is then compared with \( \alpha \), the specified probability of a type I error, to see if it falls in the critical region. If it does, \( H_0 \) is rejected; if not, \( H_0 \) is accepted. The power of the test for each true value of \( \frac{\theta_1}{\theta_0} \) is computed based on the percentage of rejections of \( H_0 \).
IMPLICIT REAL*8(A-H,O-Z)
READ(5,2) N,ALPHA,D,U
2 FORMAT(I4,F10.0,2F2.0)
40 READ(5,1,END=100)TO,T1,IARG
1 FORMAT(2F3.0,I10)
WRITE(6,3) TO,T1
3 FORMAT('O', 'TO = ',F7.2,' T1 = ',F7.21
K=0
KR=0
C=3.0
TIME=D*24.0*U
PO=DEXP(-TIME/TO)
Pl=DEXP(-TIME/T1)
DO 31 I=1,N
NO=0
N1=0
Q0=1
Q1=1
K=K+1
10 Q0=Q0*RN(IARG)
   IF(Q0.LT.PO) GO TO 20
   NO=NO+1
   GO TO 10
20 Q1=Q1*RN(IARG)
   IF(Q1.LT.Pl) GO TO 30
   N1=N1+1
   GO TO 20
30 DFO=2.0*NO+2.0
   DF1=2.0*N1+2.0
   FR=(C*DF1)/DFO
   Z=1.0-PRBF(DFO,DF1,FR)
   IF(MOD(K,100).NE.0) GO TO 33
   WRITE(6,34) DFO,DF1,FR,Z
34 FORMAT(' ',3F9.2,F9.5)
33 IF(Z.LT.ALPHA) KR=KR+1
31 CONTINUE
   REJ=DFLOAT(KR)/DFLOAT(N)
   WRITE(6,32) KR,REJ
32 FORMAT('O','NO. REJECTIONS = ',I4,' PERCENT REJECTION = ',F10.6)
GO TO 40
100 STOP
END
DOUBLE PRECISION FUNCTION PRBF(DA, DB, FR)
IMPLICIT REAL*8(A-H,O-Z)
PRBF=1.0
IF(DA*DB*FR .EQ. 0.0) RETURN
IF(FR .LT. 1.0) GO TO 5
A=DA
B=DB
F=FR
GO TO 10
5 A=DB
B=DA
F=1.0/FR
10 AA=2.0/(9.0*A)
BB=2.0/(9.0*B)
Z1=((1.0-BB)*F**((1.0/3.0)-1.0+AA)/DSQRT(BB*F**((2.0/3.0)+AA))
Z=ZABS(Z1)
IF(B .LT. 4.0) Z=Z*(1.0+0.08*Z**4/B**3)
PRBF=0.5/(1.0+Z*(0.196854 + Z*(0.115194+ Z*
1(0.000344+Z*0.019527))))**4
IF(FR .LT. 1.0) PRBF=1.0-PRBF
IF(Z1 .LT. 0.0) PRBF=1.0-PRBF
RETURN
END
Appendix C

Computer Program for Simulation of Truncated Sequential Probability Ratio Test and Estimation of the Power Curve and Expected Sample Size

To simulate the number of failures of the two component types, this program uses the Poisson generation method described in Appendix A. The power of the test for each true value of $\frac{\theta_1}{\theta_0}$ is evaluated based on the percentage of rejections of $H_0$. The average sample size required to reach a decision is computed as the simple average of the sample sizes required for all five-hundred tests that were run for each $\frac{\theta_1}{\theta_0}$ value. This program also tabulates a frequency distribution for the sample sizes and counts the number of tests whose sample sizes are less than or equal to the fixed sample size of 70 days.
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION K(25),PC(25)
IRD=5
IPR=6
C0=3.0
Cl=6.0
WRITE(IPR,202) CO
202 FORMAT('O', 'HO - THETA 1/THETA 0 = ',F4.1)
READ(IRD,2) ALPHA,BETA,IARG,D,U,N
2 FORMAT(2F10.0,1I0,2F2.0,I5)
WRITE(IPR,203) ALPHA,BETA,D,U
203 FORMAT(' ', 'ALPHA = ', F6.3, ' BETA = ', F6.3/1X, 'TRIAL SIZE = ', IF5.1,
'NUMBER OF UNITS OF EACH COMPONENT ON TEST = ', F4.0)
A=ALPHA/(1.0-BETA)
B=(1.0-ALPHA)/BETA
WRITE(IPR,201) A,B
201 FORMAT(' ', 'TEST BOUNDS - ', 'A = ', F7.3, ' B = ', F7.3/1/
TIME=24.0*U*D
40 READ(IRD,4,END=100) TO,T1
4 FORMAT(2F3.0)
NR=0
N100=0
NX=0
DO 11 L=1,25
11 K(L)=0
C=T1/TO
WRITE(IPR,3) TO,T1,C
3 FORMAT('O', 'DATA IS GENERATED ACCORDING TO - ', 'THETA 0 = F5.1,
THETA 1 = F5.1, THETA 1/THETA 0 = F4.1)
DA=(-1.0*TIME)/TO
PO=DEXP(DA)
DB=(-1.0*TIME)/T1
P1=DEXP(DB)
KR=0
TOTD=0.0
DO 31 II=1,N
NTOTO=0
NTOT1=0
ND=0
R=(.5)*(1.75)**2
31 ND=ND+1
NO=0
N1=0
QO=1
Q1=1
10 Q0=Q0*RN(IARG)
IF(QO.LT.PO) GO TO 20
NO=NO+1
GO TO 10
20 QL=Q1*RN(IARG)
    IF(QL.LT.P1) GO TO 30
    N1=N1+1
    GO TO 20
30 R=R*(.5)**NO*(1.75)**(NO+N1)
34 IF(R.LE.A) GO TO 22
    IF(R.GE.B) GO TO 23
    IF(ND.GE.100) GO TO 43
    GO TO 21
43 N100=N100+1
    IF(R.GE.1.0) GO TO 23
    NR=NR+1
22 KR=KR+1
23 TOTD=TOTD+DFLOAT(ND)
    IF(ND.LE.70) NX=NX+1
    L=ND/4.001+1
    K(L)=K(L)+1
31 CONTINUE
    SS=TOTD/DFLOAT(N)
    REJ=DFLOAT(KR)/DFLOAT(N)
    WRITE(IPR,32) KR,REJ
32 FORMAT('0','NO. REJECTIONS = ',I4,' PERCENT REJECTION = ',F7.3)
    WRITE(IPR,33) SS
33 FORMAT(' ','AVE SAMPLE SIZE IN DAYS = ',F8.2)
    WRITE(IPR,206) N100,NR
206 FORMAT('NO, TRUNCATED TESTS = ',I5,' WITH ',I5,' REJECTS'//)
    WRITE(IPR,210)
210 FORMAT('0',5X,'INTERVAL',9X,'FREQ',4X,'PERCENT'//)
    JJL=1
    DO 208 JJ=1,25
    JJU=JJL+3
    PC(JJ)=DFLOAT(K(JJ))/DFLOAT(N)
    WRITE(IPR,207) JJL, JJU, K(JJ), PC(JJ)
207 FORMAT(' ',I5, ' GE K LE ',I5,3X,I5,3X,F7.3)
208 JJL=JJU+1
    PP=DFLOAT(NX)/DFLOAT(N)
    WRITE(IPR,209) PP
209 FORMAT('0','PERCENT OF SAMPLE SIZES LE FIXED SAMPLE SIZE OF 70',
    '1' DAYS = ',F7.3)
    GO TO 40
100 STOP
END
VITA

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