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The Use of Contingency Table Analysis as a Robust Technique for Analysis of Variance

Mei-Eing Chiu
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THE USE OF CONTINGENCY TABLE ANALYSIS AS A
ROBUST TECHNIQUE FOR ANALYSIS OF VARIANCE

by

Mei-Eing Chiu

A thesis submitted in partial fulfillment
of the requirements for the degree
of
MASTER OF SCIENCE
in
Applied Statistics

UTAH STATE UNIVERSITY
Logan, Utah
1982
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Mei-Eing Chiu
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</tr>
<tr>
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<td>33.</td>
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</tbody>
</table>
ABSTRACT

The Use of Contingency Table Analysis as a Robust Technique for Analysis of Variance

by

Mei-Eing Chiu, Master of Science
Utah State University, 1982

Major Professor: Dr. David White
Department: Applied Statistics

The purpose of this paper is to compare Analysis of Variance with Contingency Table Analysis when the data being analyzed do not satisfy Analysis of Variance assumptions. The criteria for comparison are the powers of the Standard variance-ratio and the Chi-square test.

The test statistic and powers were obtained by Monte Carlo.

1. Calculate test statistic for each of 100 trials, this process was repeated 12 times. Each time different combination of means and variances were used.

2. Powers were obtained for each of 12 combinations of means and variances.

Whether Analysis of Variance or Contingency Table Analysis is a better alternative depends on if we are interested in equality of population means or differences of population variances.

(36 pages)
The One-Way Analysis of Variance (AOV)

The statistical testing procedure to decide whether the differences among sample means are large enough to imply that the corresponding population means are different uses an analysis of variance table. For example, if we wish to test the equality of K population means, we assume that the K sets of measurements are normally distributed, with means given by \( \mu_1, \mu_2, \ldots, \mu_K \) and with a common variance \( \sigma^2 \). The single test of the hypothesis "all K population means are equal" is the F-ratio, obtained from an analysis of variance table.

The Test Statistic

The decision procedure used to test equality of the population means uses the ratio of two estimates of \( \sigma^2 \), namely, \( S^2_W \) and \( S^2_B \). The ratio is denoted as \( F = \frac{S^2_B}{S^2_W} \), where \( S^2_W \) is the variability of the observations within the K populations and \( S^2_B \) is a measure of the variability among the sample means for the K populations. Thus,

\[
S^2_B = \frac{1}{K} \sum_{i=1}^{K} n_i (\bar{Y}_i - \bar{Y})^2 / (K-1)
\]

\[
S^2_W = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_i)^2 / (N-K)
\]
Notation Needed for the AOV of a One-Way Classification

- $Y_{ij}$: The $j$th sample observation obtained from population $i$
- $n_i$: The number of sample observations drawn from population $i$
- $N$: The total sample size drawn from $K$ populations.
- $\bar{Y}_i$: The average of the $n_i$ sample observations obtained from population $i$
- $\bar{Y}$: The average of all sample observations

Contingency Tables: Chi-Square Test of Independence

To compare $K$ different populations, we arrange data in a two-way table such that each observation is classified according to:

1. The population from which it was drawn
2. Each population has $r$ possible responses, i.e., each of $K$ populations with $r$ categories of response. The two-way tables sometimes are called contingency tables.

A test of the independence of two variables arranged in a two-way table, use the test statistic

$$
X^2 = \sum_{i=1}^{r} \sum_{j=1}^{K} \frac{(n_{ij} - E_{ij})^2}{E_{ij}}
$$

where $n_{ij}$ and $E_{ij}$ are, respectively, the observed and expected number of measurements falling in the cell for the $i$th row and the $j$th column.

$$
E_{ij} = \frac{(\text{row i total})(\text{column j total})}{n}
$$
We can use this technique for a one-way analysis of variance table by classifying each of K population into different r categories. This will give us a two-way K x r table as follows:

Table 1
Classification of K Populations in a Two-way K x r Table

<table>
<thead>
<tr>
<th>Class</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>K</th>
<th>Row Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>011</td>
<td>012</td>
<td>01K</td>
<td></td>
<td>R1</td>
</tr>
<tr>
<td>2</td>
<td>021</td>
<td>022</td>
<td>02K</td>
<td></td>
<td>R2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rr</td>
</tr>
<tr>
<td>r</td>
<td>0r1</td>
<td>0r2</td>
<td>0rK</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where

\[ X^2 = \sum_{i=1}^{K} \sum_{j=1}^{r} \left( \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \right) \]

where

- \( O_{ij} \) = the observed number in the ith class in the sample from the jth population
- \( E_{ij} \) = the expected number in the ith class in the sample from the jth population
Robustness and the Inequality of Variances

It has been proven by Fisher that the variances ratio follows a non-central F distribution if the parent distributions are normal, and if the variances of samples are equal. Among the underlying assumptions made in deriving statistical methods are usually some that are apt to be violated in applications and are introduced only to ease the mathematics of the derivation. For instance, the assumption of normality. Statistical methods have been called "robust" if the inferences are not seriously invalidated by the violation of such assumptions. In this paper we are going to see if Chi-square test will work better than the F test under the assumptions of normality and under the violation of common variance assumption, i.e., in the case of inequality of variance. Scheffé reports work (1959) which shows that the F test is not valid in the case of inequality of variance. These results will be described in Chapter II. In Chapter III the power of the test using the Variance ratio is explained. In doing tests, we wish that data are randomly obtained, and will be normally distributed with mean AVE and standard deviation SD. The procedure for generating data which are needed is described in Chapter V.

Finally, data will be generated for one-way analysis using the F test and Chi-square test for the same data, and the results will be compared. In Chapter VII, conclusions are given concerning these results.
Scheffé reports results (1959) which have shown that in the case of two equal size groups, the F-ratio is exceedingly well behaved with regard to violation of the equality-of-variance assumption, since it shows no effect at all for large \( n \). However, when we now consider the case of \( K \) groups in the one-way layout under the fixed-effects model, we shall find that violating the equality-of-variance assumption when \( K > 2 \) has some effect even when group sizes are equal, although it then appears to be slight. For the one-way layout, the probabilities of a Type-I Error with F-test for equality of means at the nominal 5% level are shown in Table 1. Let \( \theta \) denote the ratio of the maximum to minimum of the \( \{ \sigma_i^2 \}^* \), the last line of the table in the case of equal group sizes, where \( \theta = 7 \), was included to show how bad the deviation might be approached ordinarily in practice. For two of the other three lines, if not for all three, where the group sizes are equal, the deviations can be considered bearable. However, this cannot be said for six of the eight lines where the group sizes are unequal.

---

*The brace notation denotes the set of quantities indicated: In this case \( \{ \sigma_i^2 \} \) means the set consisting of \( K \) quantities \( \sigma_i^2 \) with \( i = 1, 2, \ldots, K \).
Table 2

Effect of Inequality of Variances on Probability of Type-I Error with F Test for Equality of Means in One-way Layout at Nominal 5% Level

<table>
<thead>
<tr>
<th>No. of Groups, K</th>
<th>Ratio of Group Variances ( { \sigma^2_i } )</th>
<th>Group Sizes ( { n_i } )</th>
<th>n*</th>
<th>Probability of Type-I Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1:2:3</td>
<td>5,5,5</td>
<td>15</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,9,3</td>
<td>15</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7,5,3</td>
<td>15</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,5,7</td>
<td>15</td>
<td>0.040</td>
</tr>
<tr>
<td>3</td>
<td>1:1:3</td>
<td>5,5,5</td>
<td>15</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7,5,3</td>
<td>15</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9,5,1</td>
<td>15</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,5,9</td>
<td>15</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>1:1:1:1:3</td>
<td>5,5,5,5,5</td>
<td>25</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9,5,5,5,1</td>
<td>25</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,5,5,5,9</td>
<td>25</td>
<td>0.025</td>
</tr>
<tr>
<td>7</td>
<td>1:1:...:7</td>
<td>3,3,...,3</td>
<td>21</td>
<td>0.12</td>
</tr>
</tbody>
</table>

*n = total number of observations.
CHAPTER III
PROBABILITY OF ACCEPTING A FALSE HYPOTHESIS

Alternatives and Two Types of Error

A test of statistical hypothesis consists of choosing a test statistic and selecting a critical region. When the hypothesis is true, the test specifies that the chance of rejecting the hypothesis is some preassigned level of significance $\alpha$. The Type-I error would be an error to reject the hypothesis when it is true, and this type of error is called an $\alpha$ error. If the hypothesis is not true, accepting it is also a mistake, and this Type-II error is called a $\beta$ error. Only one of these two errors is possible in a single problem. For a test of hypothesis $H_0: \mu = \mu_0$ and small if is very different from $\mu_0$. The probability of rejecting a false hypothesis is called the power of the test.

Power of the Analysis-of-Variance Tests

The analysis-of-variance technique is used to compare means of several populations. It is assumed that each of the populations has a normal distribution with a common value $\sigma^2$ for the variance. The means of the populations are $\mu_1, \mu_2, \ldots, \mu_K$, and the usual hypothesis to be tested is $\mu_1 = \mu_2 = \ldots = \mu_K$. Alternatives to this hypothesis would specify that values for the $\mu_1, \mu_2, \ldots, \mu_K$ are not all the same. We can measure the dispersion of the $\mu$'s by using the variance of these quantities.
\[
\frac{K}{\sum_{i=1}^{K} (\mu_i - \bar{\mu})^2 / K}
\]

\[
\bar{\mu} = \frac{K}{\sum_{i=1}^{K} \mu_i / K}
\]

where K is the number of populations.

It is convenient to divide this quantity by \(\sigma^2 / n\), and call it

\[
\phi^2 = \frac{\sum_{i=1}^{K} (\mu_i - \bar{\mu})^2 / K}{\sigma^2 / n}
\]

where n is the number of observations from each population.

In this paper, when dealing with the case of unequal variances of the populations, we assume that

\[
\phi^2 = \frac{1}{K} \left( \sum_{i=1}^{K} \frac{(\mu_i - \bar{\mu})^2}{\sigma_i^2 / n} \right)
\]

is used. The number \(\phi^2\) can be used to measure an alternative. The probability of rejecting the hypothesis \(\mu_1 = \mu_2 = \ldots = \mu_K\) when actually unequal values are specified can be obtained in terms of \(\phi^2\).

In Table 2 is a graph giving the value of \(1-\beta\) on the vertical scale related to \(\phi\) on the horizontal. The graph is for two levels of significance, \(\alpha = 0.01\) and \(\alpha = 0.05\), for the given value of \(v_1\), the number of degrees of freedom in the denominator of the F ratio. Note that there is a different curve for each set of values \(v_1, v_2,\) and \(\alpha\).

Consider the curve of the table for \(\alpha = 0.05, v_1 = 3,\) and \(v_2 = 12\). This would be used, for example, in testing the hypothesis that four
Power Function for Analysis of Variance (fixed effects model)

Figure 1. Power and $\phi$. 
(v_1 = 4 - 1 = 3) populations have equal means with samples of size 
n = 4 from each population (v_2 = 16 - 4 = 12) reading above \( \phi = 2 \).

We see that the chance of recognizing that the four populations do
not have equal means when actually \( \phi^2 = 4 \) is \( 1 - \beta = 0.82 \). Thus,
the power of the test against any set of values \( \mu_1, \mu_2, \mu_3, \mu_4 \)
giving \( \phi^2 = 4 \) is 0.82. The alternatives are in terms of \( \phi^2 \) alone,
not distinguishing among different sets of means which give the same
value to \( \phi^2 \). For example, with \( \sigma^2 = 1 \) the four population means could
be 50, 50, 50, 52 or 50, 50, 50, 52, 31 and give \( \phi^2 = 4 \), where \( \phi \) is
a function of \( \sigma^2 \).
CHAPTER IV
DESIGN OF EXPERIMENTS

The hypothesis to be tested is that $H_0: \mu_1 = \mu_2 \ldots = \mu_k$ with controlled $\alpha$, the Type-I error, and normality assumption. We wish to conduct an empirical study (so called Monte Carlo method) to determine the effect of different combination of means and variances on Type-I and Type-II errors, using

1. Standard variance-ratio test
2. Chi-square analysis on the contingency table

Given the raw data for an analysis of variance, we can always construct a contingency table (Table 3) with the same set of data such that each observation is classified according to the population from which it was drawn, where each population has $r$ categories response. For example, if a set of data which consists of three groups (populations) is used to conduct the hypothesis test, and each of three populations with three possible responses, the set of data is as shown in Table 3. This will give us a $3 \times 3$ of two-way table as shown in Table 4.

The study will involve the following different combinations of means and variances:

1. Equal means with
   a. Equal variances
   b. Slightly unequal variances
c. Very unequal variances
d. Extremely different variances

2. Slightly unequal means with
   a. Equal variances
   b. Slightly unequal variances
   c. Very unequal variances
   d. Extremely different variances

3. Very unequal means with
   a. Equal variances
   b. Slightly unequal variances
   c. Very unequal variances
   d. Extremely different variances

Those above will give 12 combinations; the details about means and variances will come in the following chapter.

Table 3
Example Raw Data for a 3 x 3 Layout

<table>
<thead>
<tr>
<th></th>
<th>Population #1</th>
<th>Population #2</th>
<th>Population #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>X &lt; 12.08</td>
<td>6.34 9.16</td>
<td>7.68</td>
<td>5.42</td>
</tr>
<tr>
<td></td>
<td>7.60 8.23</td>
<td></td>
<td>6.51</td>
</tr>
<tr>
<td></td>
<td>10.76 11.75</td>
<td></td>
<td>-8.99</td>
</tr>
<tr>
<td>12.08 &lt; X &lt; 17.49</td>
<td>14.72</td>
<td>14.80 13.38</td>
<td>14.20</td>
</tr>
<tr>
<td></td>
<td>12.42</td>
<td>15.42 16.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.61</td>
<td>14.29 15.16</td>
<td></td>
</tr>
<tr>
<td>17.49 &lt; X</td>
<td>19.39</td>
<td>20.21</td>
<td>45.91 28.97</td>
</tr>
<tr>
<td></td>
<td>18.41</td>
<td></td>
<td>23.42 34.94</td>
</tr>
<tr>
<td></td>
<td>18.35</td>
<td></td>
<td>29.34 18.34</td>
</tr>
</tbody>
</table>
Table 4
Contingency Table for the 3 x 3 Layout Example

<table>
<thead>
<tr>
<th></th>
<th>Population #1</th>
<th>Population #2</th>
<th>Population #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>X &lt; 12.08</td>
<td>6</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>12.08 &lt; X &lt; 17.49</td>
<td>3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>17.49 &lt; X</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
CHAPTER V
PROCEDURE OF GENERATING DATA AND PROGRAMS USED

The Algorithms for Generating Data

The procedure for comparing F and chi-square tests is by generating 100 random data sets, for analysis of variance with three groups in each set and ten observations in each group (n = 10, i = 1, 2, 3). Each set will consist of certain mean and standard deviation analyzed in each of three mean situations, which run with four different combinations of variances:

1. Equal means, \( \mu_1 = \mu_2 = \mu_3 = 15 \), with four combinations of variances.
2. Slightly unequal means, \( \mu_1 = 13, \mu_2 = 15, \mu_3 = 17 \), with four combinations of variances.
3. Very unequal means, \( \mu_1 = 11, \mu_2 = 15, \mu_3 = 19 \), with four combinations of variances.

The four combinations of variances are:

1. Equal variances:
   The ratio of the variances of the three populations is
   \[ \sigma_1: \sigma_2: \sigma_3 = 1:1:1 \]

2. Slightly unequal variances:
   The ratio of the variances of the three populations is
   \[ \sigma_1: \sigma_2: \sigma_3 = 1:1:2 \]
3. Very unequal variances:
   The ratio of the variances of the three populations is
   \[ \sigma_1 : \sigma_2 : \sigma_3 = 1:1:4 \]

4. Extremely unequal variances:
   The ratio of the variances of the three populations is
   \[ \sigma_1 : \sigma_2 : \sigma_3 = 1:1:7 \]

These give 12 combinations, one combination for each 100 data sets
and both F-test and chi-square test will be run with each data set.
The proportions of times that the hypothesis is rejected will be cal-
culated for each combination. The proportions of times the hypothesis
is rejected will be the Type-I error if the hypothesis is true, and
will be the power of the test when the hypothesis is false. The power
of the test is expressed as 1-\( \beta \), where \( \beta \) is the probability of accepting
the hypothesis when it is false.

In deciding the value of the variances to be used in each of the
four combinations of variances, we assume that each set of data will
consist of certain standard deviation such that the value of \( \phi \) will not
exceed 3.5, the maximum value of \( \phi \) in Figure 2 for the given value \( \nu_1 = 2 \)
(three populations) in the case of the very unequal means, \( \mu_1 = 11 \),
\( \mu_2 = 15 \), and \( \mu_3 = 19 \). For instance, in the case of the combination of
variances II, \( \sigma_1 : \sigma_2 : \sigma_3 = 1:1:2 \), give \( \sigma_1 = \sigma_2 = 4.5 \), \( \sigma_3 = 9 \) the \( \phi^2 = 9.8 \),
\( \phi = 3.14 \).
Power Function for Analysis of Variance (fixed effects model)

Figure 2. Power and $\phi$. 

\[ \text{Power} = 1 - \beta \]
Programs for Generating Data Randomly, and for F-Test and Chi-Square Test

There are three major parts to the programming done for this thesis:

1. A random number generator.
2. Generation of the analysis of variance data and computation of the F-ratio.
3. Construction of the contingency table and computation of the Chi-Square.

A Random Number Generator

FUNCTION RNOR (IR) generates numbers which are normally distributed with mean = 1 and standard deviation = 1 (Figure 3).

```
*FUNCTION FOR GENERATING RANDOM NORMAL NUMBERS, MEAN = 0
*STANDARD DEVIATION = 1.
*A PAIR OF VARIABLES ARE GENERATED AND RETURNED ONE AT A TIME ON ALTERNATE CALLS.
+SET OUR!
+DATA I/O/
+RESET OUR!

IF (I,GT,0) GO TO 30
10 X=2*RANDOM(IR)-1.0
  Y=2*RANDOM(IR)-1.0
  S=XX+YY
  IF (S,GE,(1.0)) GO TO 10
  S=SORT(-2.0%ALOG(S)/S)
  RNOR=X*S
+SET OUR!
  RNORT=Y*S
+RESET OUR!
  I=1
  GO TO 40
30 RNOR=RNORT
  I=0
40 RETURN
END
```

Figure 3. Random number generator program.
Generation of the Analysis of Variance Data

The following program segment (Figure 4) generates data normally distributed with mean AVE and standard deviation SD.

```fortran
DIMENSION X(3,30),NOBS(3),AVE(3),SD(3),NGROUP(100),Y(100)
1 FORMAT(5011)
2 FORMAT(3I2)
3 FORMAT(3(I2,F3,1))
NX=TIME(11)
READ(5,/)NSET
READ(5,1)(NGROUP(N),N=1,NSET)
DO 55 N=1,NSET
   DO 25 K=1,NGROUP(N)
      NOBS(K)=0
      AVE(K)=0
      SD(K)=0
25   CONTINUE
   DO 45 I=1,NGROUP(N)
      DO 35 J=1,NOBS(I)
         X(I,J)=RNOR(NX)*SD(I)+AVE(I)
      35 CONTINUE
   WRITE(8,100)(X(I,J),J=1,NOBS(I))
100 FORMAT(5(2X,F12.8))
45 CONTINUE
   CNT=0
   CALL BOUND(NGROUP,NOBS,X,Y,N,UBOUND,LBOUND,N)
55 CONTINUE
STOP
END
```

Figure 4. Program which generates normally distributed data.

SUBROUTINE BOUND sorts data in each set into descending order and classifies those data into three groups which are separated by two critical points, UBOUND and LBOUND.
SUBROUTINE BOUND(NGROUP, NOBS, X, Y, N, UBOUND, LBOUND, M)
REAL LBOUND
DIMENSION X(3, 30), NGROUP(100), NOBS(30), Y(100)
CNT=1
DO 70 I=1, NGROUP(N)
  DO 60 J=1, NOBS(I)
    Y(CNT)=X(I, J)
    CNT=CNT+1
  60 CONTINUE
70 CONTINUE
M=CNT-1
DO 30 I=1, M-2
  DO J=1, M-1
    IF(Y(J).GT.Y(J+1)) GO TO 20
    TEMP=Y(J)
    Y(J)=Y(J+1)
    Y(J+1)=TEMP
  20 CONTINUE
30 CONTINUE
UBOUND=(Y(M/3)+Y(M/3+1))/2
LBOUND=(Y(M/3*2)+Y(M/3*2+1))/2
WRITE(8, 200) UBOUND, LBOUND
200 FORMAT(2(2X, F12.8))
RETURN
END

Figure 5. Subprogram which sorts data and separates into three groups.
The following program (Figure 6) will compute the F-ratio for each of the data sets.

```fortran
FILE 15(KIND=DISK,TITLE="DATA",FILETYPE=7)
DIMENSION SUM(100),TOTSUM(100),TNB15(100),AOBS2(100),
*AOBS(200),RSSTST(100),SSTD15(100),SSTRT(100),SSER15(100),
*AMSTRT(100),CT(100),MSER15(100),RSSTOT(100),FRATIO(100),
*SUMGRP(60),GRPS2(60),NGROUP(100),NOBS(3)

REAL LBOUND
1 FORMAT(50I1)
2 FORMAT(3I2)
READ(5*1)(NGROUP(N),N=1,NSET)
DO 60 N=1,NSET
   DO 55 K=1,NGROUP(N)
      NOBS(K)=0
   55 CONTINUE
READ(5*2)(NOBS(I),I=1,NGROUP(N))
C**************************************************************************
C* SUMGRP  =  Yij  =  SUM OF GROUP I
C* GRPS2   =  Yij*Yij
C* AOBS    =  Yij  =  jth observation in ith group z
C* SUM     =  Yij  =  SUM OF ALL OBSERVATIONS IN ONE SET
C* TNB15   =  N = TOTAL NUMBER OF OBSERVATIONS IN ONE SET
C* AOBS2   =  Yij*Yij
C* RSSTOT  =  SUM( Yij^2 )
C* RSSTRT  =  SUM( Yij^2 / ni )
C* TOTSUM  =  Yij*Yij
C* CT      =  Yij / N = CORRECTION TERM
C* SSTD15  =  RSSTOT - CT
C* SSTRT   =  RSSTRT - CT
C* SSER15  =  RSSTOT - RSSTRT
C* AMSTRT  =  SSTRT/(K-1)
C* K       =  NUMBER OF GROUPS
C* AMSER15 =  SSER15/(N-K)
C* N       =  TOTAL NUMBER OF OBSERVATIONS IN ONE SET
C**************************************************************************
DO 50 I=1,NGROUP(N)
   GRPS2(I)=0
   SUMGRP(I)=0
READ(15*100)(AOBS(M),M=1,NOBS(I))
100 FORMAT(5(2X,F12.8))
DO 40 J=1,NOBS(I)
   SUM(N)=SUM(N)+AOBS(J)
   TNB15(N)=TNB15(N)+1
   AOBS2(N)=AOBS(J)**2
   SUMGRP(I)=SUMGRP(I)+AOBS(J)
   RSSTOT(N)=RSSTOT(N)+AOBS2(N)
40 CONTINUE

Figure 6. Program which computes the F-ratio.
Construction of the Contingency Table and Computation of the Chi-Square Algorithm. For the Chi-square test, we put all observations in a set in descending order first, then divided them into three groups with 10 observations in each group giving C1, C2 (we called UBOUND and LBOUND in the SUBROUTINE BOUND) such that

\[ X_1, X_2, \ldots, X_{10} < C1 \]

\[ C1 < X_{11}, X_{12}, \ldots, X_{20} < C2 \]

\[ C2 < X_{21}, X_{22}, \ldots, X_{30} \]

where \( X_1, X_2, \ldots, X_{30} \) are in descending order.
FILE 15(KIND=DISK, TITLE="DATA", FILETYPE=7)
DIMENSION X(3,30), F(3,3), VE(3,3), ROWSUM(3), COLSUM(3),
     CHISQR(100), NGROUP(100), NOBS(30)
1  FORMAT(50I1)
2  FORMAT(3I2)
100  FORMAT(5(2X,F12.8))
200  FORMAT(5(5X,F3,2))
READ(5,/)NSET
READ(5,1)(NGROUP(N), N=1,NSET)
DO 70  N=1,NSET
     TOTSUM=0
     DO 44  I=1,3
            ROWSUM(I)=0
            COLSUM(I)=0
     DO 33  J=1,3
            F(I,J)=0
            VE(I,J)=0
     CONTINUE
33    CONTINUE
44    CONTINUE
     READ(5,2)(NOBS(I), I=1,NGROUP(N))
     READ(15,100)((X(I,J), J=1,NOBS(I)), I=1,NGROUP(N))

CALL FREQ(NGROUP, NOBS, X, F, N)
CALL SUMROW(ROWSUM, F)
CALL SUMCOL(COLSUM, F)
CALL SUMTOT(TOTSUM, COLSUM)
CALL EXPFRE(ROWSUM, COLSUM, TOTSUM, CHISQR, VE, F, N)
70    CONTINUE
WRITE(6,200)(CHISQR(I), I=1,NSET)
STOP
END

Figure 7. Main program.
Figure 8. Subprogram for computing the frequencies of a two-way table.

```fortran
C*  SUBPROGRAM FOR COMPUTING THE FREQUENCIES OF  *
C*  A TWO-WAY TABLE                                 *
C*****************************************************************************
SUBROUTINE FREQ(NGROUP,NOBS,X,F,N)
DIMENSION X(3,30),F(3,3),NOBS(30),NGROUP(100)
REAL LBOUND
READ(5,300)UBOUND,LEBOUND
300 FORMAT(2(2X,F12.8))
DO 50 I=1,NGROUP(N)
   DO 40 J=1,NOBS(I)
      IF(X(I,J),GT,UBOUND) GO TO 11
      IF(X(I,J),GE,LEBOUND) GO TO 22
      F(3,I)=F(3,I)+1
      GO TO 40
11     F(1,I)=F(1,I)+1
      GO TO 40
22     F(2,I)=F(2,I)+1
   40 CONTINUE
50 CONTINUE
RETURN
END

C*  SUBPROGRAM FOR COMPUTING ROW'S TOTAL                        *
C*****************************************************************************
SUBROUTINE SUMROW(ROWSUM,F)
DIMENSION ROWSUM(3),F(3,3)
DO 122 I=1,3
   DO 122 J=1,3
      ROWSUM(I)=ROWSUM(I)+F(I,J)
   122 CONTINUE
RETURN
END
```

Figure 8. Subprogram for computing the frequencies of a two-way table.
* SUBROUTINE SUMCOL(COLSUM,F)  
DIMENSION COLSUM(3),F(3,3)  
DO 124 J=1,3  
   DO 114 I=1,3  
      COLSUM(J)=COLSUM(J)+F(I,J)  
114    CONTINUE  
124   CONTINUE  
RETURN  
END

* SUBROUTINE SUMTOT(TOTSUM,COLSUM)  
DIMENSION COLSUM(3)  
DO 134 J=1,3  
   TOTSUM=TOTSUM+COLSUM(J)  
134   CONTINUE  
RETURN  
END

Figure 9. Subprogram for computing column's total.
CHAPTER VI
RESULTS

In testing the equality of three population means, we hypothesized that $\mu_1 = \mu_2 = \mu_3$ and the alternative to this hypothesis would be the specified value for $\mu_1$, $\mu_2$, $\mu_3$ are not all the same, where $\mu_1$, $\mu_2$, $\mu_3$ are the means of populations.

In order for the Chi-square test to give better results than the F-ratio test, it should give the following quantities:

1. The probability of rejecting the hypothesis is closer to the controlled $\alpha$, the Type-I error, than the F-ratio test when the hypothesis is true.

2. Has a higher power, the probability of rejecting the hypothesis than the F-ratio test when the hypothesis is false.

In Table 4 the probability of rejecting the hypothesis is the significance level, the Type-I error, when the hypothesis is true. When the hypothesis is false, it is the power of the test which is denoted as $1-\beta$ in Figure 1 and Figure 2.

From the experiments that we conducted, the following results were obtained and are also shown in Table 5.

**Common Variance**

1. The probability of rejecting the hypothesis using the F-ratio test is equal to the controlled $\alpha(= 0.05)$, and using the Chi-square test is close to $\alpha$, when the hypothesis is true.
Table 5
Probabilities of Rejecting Hypothesis for Various Ratios of Variances

<table>
<thead>
<tr>
<th>Ratio of Variances</th>
<th>Population Means</th>
<th>Probabilities of Rejecting Hypothesis P</th>
<th>$\phi$</th>
<th>$P^*$</th>
<th>95% Confidence Interval for $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>F</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1:1</td>
<td></td>
<td>0.04</td>
<td>0.07</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>15 15 15</td>
<td>0.19</td>
<td>0.14</td>
<td>1.63</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>13 15 17</td>
<td>0.77</td>
<td>0.52</td>
<td>3.25</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>11 15 19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1:2</td>
<td>15 15 15</td>
<td>0.09</td>
<td>0.15</td>
<td>0.05</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>13 15 17</td>
<td>0.17</td>
<td>0.31</td>
<td>1.57</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>11 15 19</td>
<td>0.65</td>
<td>0.59</td>
<td>3.14</td>
<td>1.00</td>
</tr>
<tr>
<td>1:1:4</td>
<td>15 15 15</td>
<td>0.09</td>
<td>0.33</td>
<td>0.05</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>13 15 17</td>
<td>0.18</td>
<td>0.52</td>
<td>1.63</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>11 15 19</td>
<td>0.38</td>
<td>0.65</td>
<td>3.26</td>
<td>1.00</td>
</tr>
<tr>
<td>1:1:7</td>
<td>15 15 15</td>
<td>0.09</td>
<td>0.47</td>
<td>0.05</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>13 15 17</td>
<td>0.13</td>
<td>0.63</td>
<td>1.60</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>11 15 19</td>
<td>0.28</td>
<td>0.73</td>
<td>3.19</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*: $P$ is the theoretical power of the analysis-of-variance given $\phi$ value, which is according to Figure 2, which assumes equal variances.
2. The power of the F test is higher than the Chi-square test $0.19 > 0.14$ and $0.77 > 0.52$, in the case of equal population variances. However, both the powers of the test of F and Chi-square tests are less than the theoretical probability of rejecting the hypothesis when it is false.

**Slightly Unequal Variances**

1. The Type-I error from Chi-square test is about 10% higher than the controlled $\alpha$, while F test has about 4% higher only in the case of equal means.

2. In the case of unequal means, the F test has a higher power of the test than the Chi-square test, $0.65 > 0.59$, when the difference between means is larger.

However, both F and Chi-square tests gave a lower power of the test than the theoretical power of the analysis-of-variance given a $\phi$ value. For instance, the power of the analysis-of-variance should be 0.57 according to Table 3 when $\phi$ is 1.57 and should be 1.00 when $\phi$ is 3.14.

**Very Unequal Variances**

1. The probability of rejecting the hypothesis using the F test is still only 4% higher than the controlled $\alpha$, but the Type-I error from the Chi-square test increased to 28% higher in the case of equal means when the variances are very unequal.
2. In the case of unequal means, the power of the test from Chi-square test is larger than which is from the F test. Both of the power from F and Chi-square tests are lower than the theoretical power of the analysis-of-variance according to Figure 2. The differences get larger when the differences between means are larger. For instance, in Figure 3, in the case of the $\mu_1$, $\mu_2$, $\mu_3$ are 13, 15, 17, it showed that $P$ is 0.65 while F and Chi-square have only 0.18 and 0.52 of the power of the test. When $\mu_1$, $\mu_2$, $\mu_3$ became 11, 15, 19, the powers of the test of F and Chi-square tests are 0.38 and 0.65, but $P$ is 1.00.

**Extremely Unequal Variances**

1. In the case of equal means, the Type-I error of F test is still 4% higher than the controlled $\alpha$, but the Chi-square test has 42% higher Type-I error.

2. The powers of the Chi-square test are higher than the power of the F test in the cases of unequal means, but both tests still have lower power than the actual one, $P$.

**95% Confidence Interval**

The 95% confidence interval for $P$, the probability of rejecting hypothesis, showed that whenever the $P$ is equal to (or very close to $\alpha$), the confidence interval includes the $\alpha$, otherwise $\alpha$ is not within the interval.
Figures 10, 11, 12, and 13 are the set of the power curves of the F and Chi-square tests given \( \phi \) values in the cases of four different combinations of variances with equal means, slightly unequal means, and very unequal means, respectively. Points A are the powers of the F test, points B are the powers of the Chi-square test, and points C are the theoretical powers of the analysis-of variance at different given \( \phi \) values according to Figure 2.
Figure 10. The power curve of tests in the case of combination of common variances.
Figure 11. The power curve of tests in the case of combination of slightly unequal variances.
Figure 12. The power curve of tests in the case of combination of very unequal variances.
Figure 13. The power curve of tests in the case of combination of extremely unequal variances.
CHAPTER VII
DISCUSSION AND CONCLUSION

In the case of equal means, the Type-I error of F test is equal to the controlled $\alpha$ when populations have common variance and is a little higher than the controlled $\alpha$ even when the variances are very different. But the Chi-square test has the Type-I error close to the controlled $\alpha$, only when populations have common variance in the case of equal means. The Type-I error of the Chi-square test increased as the ratio of the maximum to the minimum ($\sigma_1$) increased.

When the hypothesis is false, the power of the F test increased as the difference between means increased in either case of common variance or unequal variances, so did the Chi-square test. For instance, in Table 4, the power of the F test went up from 0.19 to 0.77 and from 0.18 to 0.38 when the means change from 13, 15, 17 to 11, 15, 19 in the case of common variance and the case of $\sigma_1: \sigma_2: \sigma_3 = 1:1:4$.

In the case of unequal means, the hypothesis is false, the power of the F test decreased as $\theta$, the ratio of the maximum to the minimum of ($\sigma_1$), goes up. The Chi-square test did contrarily. For instance, in Table 4, in the case of means are 13, 15, 17, the power of the F test went down from 0.19 to 0.13 as $\theta$ went up from 1 to 7, but the power of the Chi-square went up from 0.14 to 0.63.

In view of the experiments, the F test is better when we are interested in the case of unequal means with the assumption of common
variance. Otherwise, if we're interested in the case of unequal means with unequal variances, the Chi-square test which has higher power is better than the F test. And the F test is better than the Chi-square test whenever the populations have equal means.

Thus, the F test, used as a non-parametric procedure, is superior to the Chi-square test if we are concerned only with equality of means. Since the Chi-square test is sensitive to variance differences, it may be a good alternative if we wish to detect these differences.
REFERENCES