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Distribution of Particle Image Velocimetry (PIV) Errors in a Planar Jet

Jaron A. Howell
Utah State University

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DISTRIBUTION OF PARTICLE IMAGE VELOCIMETRY (PIV) ERRORS IN A
PLANAR JET

by

Jaron A. Howell

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Mechanical Engineering

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UTAH STATE UNIVERSITY
Logan, Utah

2018
ABSTRACT

Distribution of Particle Image Velocimetry (PIV) errors in a planar jet

by

Jaron A. Howell, Master of Science
Utah State University, 2018

Major Professor: Barton Smith, Ph.D.
Department: Mechanical and Aerospace Engineering

Particle Image Velocimetry (PIV) is an unobtrusive fluid measurement technique used to capture full-field measurements of a flow. A comparison of the measurement error and uncertainty produced by the CS PIV-UQ method is performed in this study. Data was obtained for a rectangular jet and is used to calculate measurement error in PIV. The purpose of this study is to examine the distribution of measurement errors, examine failure of the CS PIV-UQ method, investigate convergence of PIV correlation peaks for reliable CS results, examine correlation of error distribution in PIV, and examine the correlation of random error in space.

Error distribution is investigated by examining histograms of the error distribution at locations of interest and by computing the skewness and kurtosis of error distributions along a vertical cut through the jet. It was found that error distributions are generally Gaussian except for in regions where large shear or through-plane motion are present. It is also in regions of large shear that discrepancies between $S_e$ and $RMS(U)$ were observed through a vertical cut of the jet.

The CS method requires that PIV correlation peaks are "sufficiently converged" [1] in order for reliable uncertainty estimations to be obtained. This was investigated by performing multi-pass PIV processing on MS data for 2-9 passes with 50% and 75% IW
overlap. Results of the PIV processing were compared to CS uncertainty predictions and evaluated to determine when correlation peak convergence occurs.

Error distribution was examined and found to be correlated with particle seeding density of the flow as well as flow shear. This was used to develop a new error distribution prediction model for PIV systems and is given as Eq. 4.1. Correlation of random error was investigated and determined to be significantly larger in shear regions than in the jet core for all cases.
PUBLIC ABSTRACT

Distribution of Particle Image Velocimetry (PIV) errors in a planar jet

Jaron A. Howell

Particle Image Velocimetry (PIV) is an optical fluid measurement technique used to obtain velocity measurements. Two PIV systems were used to capture data simultaneously and measurement error for the MS PIV system is calculated. An investigation of error distribution is performed to determine when uncertainty estimations fail for the CS PIV-UQ method. Investigation of when results from multi-pass PIV processing are achieve were performed so that reliable uncertainty estimations are produced with the CS method. An investigation was also performed which determined that error distributions in PIV systems are correlated with flow shear and particle seeding density. Correlation of random errors in space was also performed at the jet core and shear regions of the flow.

It was found that in flow regions with large shear that error distributions were non-Gaussian. It was also found in regions of large shear that CS uncertainty results did not match the error. For multi-pass PIV processing with 50% and 75% IW overlap it was found that 4 and 6 passes should be used, respectively, in order for CS uncertainty estimations to be reliable. It was also found that the correlation of random errors in space is much larger in shear regions of the jet flow than in the jet core.
ACKNOWLEDGMENTS

I would like to thank Steven Beresh, of Sandia National Laboratories, for funding this research and providing me with this opportunity. I would also like to thank both Andrea Sciacchitano and Doug Neal for providing assistance and direction in my research. Finally, thanks to Dr. Barton Smith, my adviser, for the constant encouragement and assistance through all stages of my thesis work.

Jaron Howell
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CHAPTER 1

Introduction

Particle Image Velocimetry (PIV) is an unobtrusive full-field fluid measurement technique. PIV offers the ability to perform in detail velocity measurements unobtrusively within a fluid through the application of lasers, high speed cameras, and PIV processing software. PIV allows for high accuracy measurements to be performed based on image correlation of particle movements in a flow.

A standard PIV setup, involving a single camera, is capable of determining the two velocity components that are in-plane with the laser sheet. However, a third component of velocity may be obtained in the through-plane direction when a two camera setup is used. This is known as stereo PIV. Additional useful quantities that are a function of velocity can be computed, such as vorticity and shear strength.

As with any measurement system, it is important to understand the measurements that one is taking, in addition to the limitations of the tools that one is using. A very important part of understanding measurement limitations is the error in your measurement. Uncertainty is a range where the actual, or true, value of a measurement lies. A priori uncertainty analysis is very useful when designing an experiment and is used to predict the accuracy of measurements associated with an experiment setup [2]. A posteriori uncertainty analysis determines the uncertainty of a data set after it has been obtained. Without a thorough understanding of the error, uncertainty, and how they are related, one can misinterpret or obtain inconsequential results.

Measurement error is defined as the difference between the true and measured values. Uncertainty describes the width of the distribution of error.

This work discusses error and uncertainty of PIV measurements with particular interest in the distribution of error through a planar jet. Different areas of the jet are investigated, which include: the potential core near the jet exit, mixed flow, and downstream turbulent
flow. Particular attention is given to the distribution of PIV error in regions of high shear by examining the skewness and kurtosis of the error distributions.

This work uses the PIV data obtained by Neal et al [3], which used two PIV measurement systems to analyze the same rectangular jet flow. The first PIV system was a basic two-component (2C) PIV measurement system. This system was called the measurement system (MS). The other PIV setup was a highly accurate stereo PIV system. This system has a high dynamic range (HDR), compared to the MS system. Because the HDR system is significantly more accurate than the MS system, it is considered to measure the true value of the jet flow. By using both the MS and HDR systems, the error associated with the measurement PIV system can be directly computed (cf. Eq. 2.1). Because measurement error for the MS system can be calculated, the distribution of errors for several flow cases are examined.

It has only been in recent years that a posteriori uncertainty quantification of PIV measurements has been possible. In the work of Sciacchitano et al [4] four PIV uncertainty quantification (PIV-UQ) methods were compared: Uncertainty Surface, Peak Ratio, Particle Disparity, and Correlation Statics. It was found that the correlation statistics method was the most accurate, and is currently employed for uncertainty estimations in the LaVision DaVis PIV software.

This work investigates cases when the correlation statistics (CS) method does not accurately predict the actual error distribution. It was determined that in regions of high shear the error distribution is non-Gaussian in shape, which the CS uncertainty method is unable to accurately predict. The CS method assumes that the particle displacement, found through PIV processing, is sufficiently converged. An investigation of how many passes should be used in a PIV processing scheme in order to be confident that the solution is converged and the results of the CS method are reliable is performed. This was investigated for all data cases using both 50% and 75% interrogation region overlap. For the B009 and I013 data cases 87% interrogation region overlap is also investigated.

Because this work provides information about the measurement errors of PIV, the
accuracy of the error prediction model presented by McClure et al [5] was investigated. In the work of McClure et al [5] the error of PIV/PTV based pressure calculations are investigated. As part of their work, a PIV error model was used to predict the standard deviation of the random error. This distribution model is described by two terms: the first term relates random error to the normalized velocity gradient of the flow, and the second term relates random error that stem from correlation peak identification. Because the measurement error for a planar jet can be obtained due to the work of Neal et al [3], a comparison of the actual error distribution of a jet and the error prediction model used by McClure et al [5] was performed. From this work it was determined that error distribution is not only related to the flow shear, but also is dependent upon the seeding density used in PIV measurements. It is recommended that future PIV error distribution models consider both flow shear and seeding density to predict error distributions.

Correlated random error in space is also investigated for multiple flow regions. Correlated random error exists due to interrogation region overlap and image distortion. As discussed by Cressel and Smith [6], when the same particles are used for multiple calculations there is potential for random error correlation in space. When interrogation window overlap is applied, the same particles in an image are used for multiple vector calculations. This work investigates the existence of correlated random error in all four spatial directions: upstream (left), downstream (right), up, and down. It was found that the correlation of random error was significantly larger in shear regions of the jet than in the jet core.
CHAPTER 2
Literature Review

This chapter discusses the relevance of this work by reviewing pertinent topics and literature. The topics of discussion begin with the basics of error and uncertainty. Next, Particle Image Velocimetry is discussed along with the current work of Particle Image Velocimetry Uncertainty Quantification. Finally, discussion of the random error prediction model for PIV data used by McClure et al [5] will be performed.

2.1 Error and Uncertainty

Although error and uncertainty are related, they are fundamentally different and are commonly misunderstood. The purpose of this section is to clarify what error and uncertainty are, in order to have a solid basis for discussion of PIV uncertainty. Much of the discussion of error and uncertainty stem from Coleman and Steele [2].

2.1.1 Error

Error is present in all measured values and is defined as the difference between the true and measured value of a measurement,

\[ x_{\text{error}} = x_{\text{true}} - x_{\text{measured}}. \] (2.1)

In most experiments the true value is unknown. Consequently, this also makes the error of the measurement unknown. There is always an error associated with measured data, although often varying in size. Measurements are not completely accurate and therefore must contain a certain amount of error. The error of a measurement is described by both a sign and magnitude.

There are two main types of error, bias and random errors. Random errors are statistical variations in measured data that vary in the positive and negative directions. Bias
errors result in measurement data being pulled in a single direction because of a persistent
error in the system. Bias error is reproducible in data sets and is considered to be all error
not detected by statistical variation.

Bias error can be difficult to quantify because the entire data set is affected, and its
value is unknown. The simplest bias error affects the entire data set by shifting each data
point a certain amount in the same direction. Bias errors tend to affect all points in a data
set and can cause the mean of the data to shift or drift. Bias may stem from many different
sources, which may include the manufacturing processes of instruments, the instrument
calibration process, etc.

Random errors are unpredictable in nature. In most cases random errors can be de-
scribed as a Gaussian statistical distribution about the mean of the obtained data.

2.1.2 Uncertainty

Whenever data is determined, whether by simulation or experiment, it is always pru-
dent to consider to what degree that data should be trusted. Uncertainty allows for esti-
mation of accuracy to be determined. This is done by estimating an upper and lower value
that the actual error is likely to be contained within. While errors are described by both a
sign and magnitude, uncertainty is generally described only by a value of magnitude and is
applied in both the positive and negative direction from the measured value.

Combined uncertainty consists of both random and bias uncertainty. Combined un-
certainty strives to take into account the error caused from both bias and random sources.
Because uncertainty describes the spread of error of a system, uncertainty estimations are
described to a certain degree of confidence. A common degree of confidence in Mechanical
Engineering is 95%. What this means is 95 out of 100 times the error of the system will
fall within the defined uncertainty bands.

2.1.3 Statistics

In order to understand how uncertainty is quantified, some basic statistics need to be
understood. There are two types of populations considered in statistics, parent and sample
populations. Parent populations are of an infinite number of data points, or enough data points that the behavior of the data is completely known. A sample population is a data set of a finite number of samples. The sample population is representative of only a part of the parent population. Significant quantities of interest for each population type are the mean, standard deviation, and standard deviation of the mean.

The mean of the parent population is

$$\mu = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i.$$  \hspace{1cm} (2.2)

Where \( \mu \) is the mean, \( N \) is the number of data points, and \( x_i \) are the data point values. The standard deviation of a parent population is defined as

$$\sigma = \lim_{N \to \infty} \left[ \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 \right]^{1/2}. \hspace{1cm} (2.3)$$

The mean and standard deviation of a sample population are calculated similarly to how the mean and standard deviation of a parent population are calculated. But, are calculated with only a finite number of terms. The sample population mean is

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i,$$  \hspace{1cm} (2.4)

and the standard deviation of a sample population is

$$s_x = \left[ \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \right]^{1/2}. \hspace{1cm} (2.5)$$

The mean and standard deviation are useful in understanding Gaussian statistics. The standard deviation of the mean allows one to assess the variance of the mean across multiple data sets. This is important because it is likely that from one data set to another the mean will not be exactly the same. Assuming independent samples, Eq. 2.7 demonstrates how the standard deviation of the mean decreases as the number of sample points increases.
The standard deviation of the mean for a parent population is calculated as

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}.$$ \hspace{1cm} (2.6)

The standard deviation of the sample mean is calculated as

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}.$$ \hspace{1cm} (2.7)

### 2.1.4 Statistics Applied to Uncertainty

The sample standard deviation is the random uncertainty of a data set. Unfortunately, the bias error is not detectable by the simple scatter of data. If the measurement error is known, bias can be calculated by computing the mean of the error. The goal of performing uncertainty analysis is to determine the range associated with a measured value in order to contain the true value of a data point to a certain degree of confidence [2].

In order to perform a full uncertainty analysis, both the random and bias errors must be taken into account. The total uncertainty associated with a measurement is equal to the root mean square of the random and bias uncertainties,

$$u_c = \sqrt{s^2_x + b^2_k}$$ \hspace{1cm} (2.8)

[2]. Where \(s_x\) represents the uncertainty component from random error and \(b_k\) represents uncertainty due to the bias error of the system.

Coleman and Steele [2] show that for a measurement, \(r\), that is a function of multiple variables, \(x\) and \(y\), there exists a possibility of correlated uncertainties. The random uncertainty associated with \(r\) is

$$s_r = \sqrt{\theta_x^2 s_x^2 + \theta_y^2 s_y^2 + 2 \theta_x \theta_y s_{xy}}.$$ \hspace{1cm} (2.9)
Where $s_{xy}$ represents the covariance term that accounts for the correlated random uncertainty between $x$ and $y$, and $\theta$ terms are defined as

$$\theta_x = \frac{\partial r}{\partial x}. \tag{2.10}$$

The random covariance, $s_{xy}$, accounts for random error sources that affect both $x$ and $y$ at the same time. It is defined as

$$s_{xy} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \epsilon_{xk} \epsilon_{yk}, \tag{2.11}$$

where $\epsilon_{xk}$ is the random error in $x$ at time $k$. The total random uncertainty can be rewritten for $J$ number of variables as

$$s_r^2 = \sum_{i=1}^{J} \theta_i^2 s_i^2 + 2 \sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \theta_i \theta_k s_{ik}. \tag{2.12}$$

Although it is more common to consider correlated bias error, random error is dominant in PIV systems. Similar equations as those shown above can be written to describe bias uncertainty, bias covariance, and total bias uncertainty but these are insignificant in PIV.

### 2.1.5 Comparing Error and Uncertainty of PIV systems

PIV uncertainty quantification methods predict uncertainty estimates of the error that exists in PIV systems. This section will discuss how error and uncertainty are compared. This discussion is based off the appendix of Sciacchitano et al [4].

Let $\epsilon_i$ be error and $U_i$ be the uncertainty prediction determined by PIV-UQ methods, evaluated to a 68% confidence interval. Where samples range from $i = 1, 2, \ldots, n$.

For simplicity, the error is assumed to be described by a Gaussian distribution with a zero mean and the uncertainty prediction, $U$, is determined perfectly. Under these assumptions $U = \sigma$, where $\sigma$ is the standard deviation of the error distribution. The relationship
between error and uncertainty is shown as

\[ U \approx \sqrt{\frac{\sum_{i=1}^{n} \epsilon_i^2}{n}}. \tag{2.13} \]

Equation 2.13 can be rearranged and written as

\[ nU^2 = \sum_{i=1}^{n} U^2 \approx \sum_{i=1}^{n} \epsilon_i^2. \tag{2.14} \]

Where the approximately equal sign (\(\approx\)) can be substituted for exactly equal (=) as \(n\) approaches infinity. It is worth noting that for cases without bias error that the root mean square of error and the standard deviation of error are equal. This is due to the error distribution having a zero mean.

Considering the case that the error distribution is described not by one, but multiple, Gaussian distributions with unique variances, Eq. 2.14 can be represented as a superposition of the multiple distributions. If there are \(K\) distributions, then Eq. 2.14 can be written to describe multiple distributions as

\[ \sum_{i=1}^{n_1} U_{1i}^2 + \ldots + \sum_{i=1}^{n_K} U_{Ki}^2 \approx \sum_{i=1}^{n_1} \epsilon_{1i}^2 + \ldots + \sum_{i=1}^{n_K} \epsilon_{Ki}^2. \tag{2.15} \]

The number of terms in Eq. 2.15 can be simplified to a single summation by applying

\[ N = \sum_{i=1}^{K} n_i. \tag{2.16} \]

Where \(N\) is the combined number of samples in all considered distributions. After applying Eq. 2.16, dividing by \(N\), and taking the square root, the simplification of Eq. 2.15 results in

\[ RMS(U) = \sqrt{\frac{\sum_{i=1}^{N} U_i^2}{N}} \approx \sqrt{\frac{\sum_{i=1}^{N} \epsilon_i^2}{N}} = RMS(\epsilon). \tag{2.17} \]
By continuing the assumption that the error distributions being considered have a zero mean and that the number of samples approaches infinity, Eq. 2.17 can be restated as

$$RMS(U) = s_\epsilon.$$  \hspace{1cm} (2.18)

This equation allows for a direct comparison between the measurement error and uncertainty quantification of PIV systems. In the proposed work, this relationship will be used to determine the effectiveness of uncertainty estimations. Ideally, these quantities should equal each other. Cases for which this does not occur will be investigated for possible reasons.

### 2.2 Particle Image Velocimetry

Particle Image Velocimetry (PIV) is an unobtrusive full-field fluid measurement technique. Seed particles are deposited into a flow at a particular seeding density, a laser sheet is fired through the flow, and cameras capture the illuminated seed particles. Images of the particles are compared using PIV software to determine the distance and direction it is most likely that the fluid moved. This is done by image cross-correlation [7]. With the known timing of the laser shots a velocity can be determined by computing the distance traveled over time. The following subsections will outline PIV in more detail.

#### 2.2.1 PIV Setup

Although PIV is an unobtrusive measurement technique, in order for PIV data to be obtained the flow must be visually transparent.

**Seeding**

A key part of PIV is effectively seeding the fluid flow of interest with tracer particles. Tracer particles ideally should be continuously spaced, are neutrally buoyant [7], and are of a controlled seeding density. With too few or too many particles in the flow good PIV measurements become difficult to obtain. The light that is reflected off of seed particles is captured in PIV images. The corresponding pixel values are used in computing image
correlations. Tracer particle possibilities vary from fluid to fluid. For instance, when performing PIV in a liquid, small glass spheres may be used, but in air, creating small particles from olive oil is commonly accepted. For many environments a device designed to disperse seed is used to control seeding density of a flow. This is commonly referred to as a seeder.

Illuminating the Seed and Capturing Images

High speed cameras are used to capture images of the illuminated seed in a flow. A cylindrical lens is attached to a laser in order to change the laser beam into a laser sheet. The seed particles in the fluid reflect the light from the laser sheet and are captured by the high-speed cameras. Because the cameras are digital, the image is broken down into pixels, at the resolution of the camera. The intensity of light reflected off of the seed particles are quantified by an analog to digital converter multiplexed to each pixel. The number of brightness levels that a camera is able to detect depends upon the bit depth of the camera.

Adjustments should be made to camera aperture in order to capture an optimal particle diameter size of tracer particles. Particle images captured too small, less than one pixel in diameter, may contribute to a bias error known as peak locking [7]. While particle images too large increase the magnitude of random error [8]. It is ideal to capture particles with a diameter of approximately three pixels [9]. After the images are captured, they are stored and later processed using PIV processing software.

2C and Stereo PIV

Depending on how much information one is trying to obtain with a PIV system there are two types of PIV one can perform: two-component (2C) PIV and stereo PIV. Two or three components of velocity can be determined respectively. The trade off between 2C and stereo PIV is the amount of information obtained vs. setup complexity. Because stereo PIV obtains three components of velocity, error sources such as through-plane motion are eliminated.

2C PIV requires one camera and a laser for its setup, whereas stereo PIV requires two cameras and a laser. Both 2C and stereo PIV also require a timing unit. The two
components of velocity that are obtained through the use of 2C PIV are the two components that are in-plane with the laser sheet, the \( u \) and \( v \) components. By using stereo PIV, it is possible to capture both of the in-plane velocities as well as the through-plane velocity. Effectively calculating the \( u \), \( v \), and \( w \) components of velocity. Because stereo PIV calculates the through-plane velocity, it is necessary to adjust the laser sheet thickness to allow for a recognizable depth displacement of particles in the \( z \) direction.

**Calibration**

From PIV calibration, a relationship between pixels of the camera’s field of view and physical space is established. This relationship converts from units of pixels to millimeters and is known as a scaling factor (SF). This is important for the processing of PIV images since all of the initial data from digital cameras are stored in units of pixels.

Calibration for 2C PIV requires a scale that is in-plane with the laser sheet. Because only one camera is used for 2C PIV, it is necessary to make sure that the camera is square to both the flow region of interest and the laser sheet. The calibration needed for stereo PIV is more complicated since it requires properly calibrating two cameras to the laser sheet. For stereo PIV, it is necessary to use a three dimensional calibration plate because a 3-dimensional space is to be mapped to two 2-dimensional sensors. A stereo calibration plate is accurately machined and has dots of known location. It is used to map the \( x \), \( y \), and \( z \) directions between the two camera’s fields of view.

Before calibration can be performed, the PIV system must be setup with cameras oriented as they would be used when obtaining PIV data. Images of a calibration plate are then obtained. The calibration plate must be located and oriented similar to the laser sheet that will illuminate particles when obtaining PIV data. For 2C PIV only a single image of the calibration plate is necessary. However, for stereo PIV two images must be obtained, one image from each camera. Once calibration images are obtained, physical space is related to pixel location of images through a mapping function. Because stereo PIV cameras are angled toward the calibration plate the images appear warped. During calibration warped images are dewarped by using a mapping function. Calibration is commonly performed
using either a pinhole model or polynomial fit model [7].

The pinhole model assumes that all light rays between the field of view and the camera sensor pass through a single point [7]. The polynomial model uses a polynomial fit to map the object and image locations together [7]. The polynomial model is more versatile than the pinhole model because the pinhole model can not be used when a change in index of refraction occurs between the camera and the target. This limits the pinhole model to PIV measurements that are performed in air. However, an advantage of the pinhole model is that the calibration plate does not have to fill the entire measurement field of view. This is because the pinhole model is capable of extrapolating the mapping function beyond the area of the calibration plate.

It is possible that errors are introduced during image mapping. Even more errors are potentially introduced when the calibration plate and the laser plane are not exactly aligned. An attempt is made to minimize the effects of laser mis-alignment for stereo PIV during calibration by performing stereo self-calibration [7]. If the calibration plate is perfectly aligned with the laser sheet, mapped images from both cameras in a stereo PIV setup will exactly match. When a mis-alignment is present between the calibration plate and the laser sheet a disparity occurs between the mapped stereo images. Self-calibration performs image cross correlation between PIV images collected by camera 1 and camera 2 of a stereo setup to determine the disparity between cameras. Once the disparity between cameras is known, the calculated disparity can be applied to PIV mapping in order to correct for laser plane mis-alignment.

2.2.2 PIV Processing

Once PIV images have been obtained they are processed using PIV software. There are three stages of processing PIV data: pre-processing, processing, and post-processing. During pre-processing, excess noise and stationary objects in PIV images are removed in order to allow for a more confident estimation of particle displacement between images. During the actual processing of the data, a cross correlation algorithm is used to estimate the actual displacement of particles. This is directly used to produce PIV velocity fields. Post-
processing of PIV data examines the processed data and determines if outliers are present. An example of an outlier is a vector with direction or magnitude that is considerably different from the expected flow or its neighboring values. If outliers are present it determines how to replace the bad vectors.

**Pre-Processing**

When obtaining images, it is likely that PIV cameras picked up bias and random errors. Random error is commonly known as noise. Possible contributions to bias error in PIV images may be from ambient light sources when the camera shutters are open or reflected laser light from the setup. The pre-processing stage is used to filter out noise and mask bias errors from PIV images.

**Processing: Interrogation Regions**

As a PIV image is processed, it is broken up into small regions called interrogation regions or interrogation windows. In these regions, a cross correlation calculation is performed to determine the movement of the particles. Appropriate interrogation region size is determined by the displacement of particles between images. By the quarter rule [7], it is best for the interrogation region size of the initial pass to have the displacement of particles be no more than one quarter of the interrogation region. Often multiple passes are made with the interrogation regions being refined to smaller regions in later passes. This is done to improve spacial resolution.

**Processing: Cross Correlation**

Once the interrogation region size is determined, the cross correlation algorithm is applied to each interrogation region. The cross correlation equation is

\[
C(r, s) = \sum_{i=0}^{D_1/2} \sum_{j=0}^{D_2/2} IA_1(i, j) IA_2(i + r, j + s)
\]  

(2.19) [7]. The cross correlation algorithm examines one interrogation region of an image at time,
t, and compares it to the image at time $t + dt$. $IA_1$ and $IA_2$ represent the intensity values in the first and second images respectively. $IA_2$ is shifted by $r$ and $s$, pixels, and the image values of $IA_1$ and $IA_2$ are multiplied together. The algorithm sweeps through $\pm r, \pm s$ of size $D_I/2$, where $D_I$ is the size of the interrogation region. Ideally, when the sweep is complete a clear cross correlation peak is apparent in the correlation. This peak will correspond with a position, $(r, s)$. The location of the peak is the best estimated displacement that the seed particles moved between $dt$. A good PIV image set will produce a sharp and unequivocal peak in the cross correlation. An example of this is shown as Figure 2.1. The peak of the correlation map at $(r, s)$, describes the direction and distance from the center of the interrogation region that the particles traveled during the time $dt$. The displacement of this shift is then stored as a velocity vector in units of pixels/sec. Note that the actual magnitude of the cross correlation peak is not significant. Only the corresponding location of the largest correlation peak is notable.

Fig. 2.1: Cross correlation map, peak location from center indicates the most likely displacement of particles.
For each interrogation region, a single velocity vector is computed. As the PIV algorithm sweeps through all of the interrogation regions, a vector field is developed. In order to create a more dense vector field, interrogation regions may be overlapped and size of interrogation regions may be decreased after completion of the first interrogation region pass. The increase of interrogation region overlap results in a more dense vector field to describe the fluid motion under consideration.

**Processing: Multi-Pass and Image Distortion**

Multi-pass processing is used to more accurately calculate the displacement of particles by using image distortion [10]. Image distortion is used to compare the first and second images used in image correlation to each other. This is done to increase displacement prediction accuracy. The displacement, $\vec{u}$, is found from the first pass of PIV processing. Both the first and second images are distorted by $\vec{u}/2$ and then are compared. If the actual displacement of particles were determined from the image correlation, the images would overlay perfectly and look the same. After the images are distorted, a new corrector displacement is computed by performing image cross correlation of the distorted images. This displacement is then added to the initial displacement. This is performed iteratively for multi-pass processing to obtain convergence of displacement solutions.

**Post Processing**

Although measures have been taken to filter noise and to process data as accurately as possible, it is still possible for invalid calculations, or bad vectors, to occur as a result of PIV processing. PIV post-processing examines each vector in the velocity vector field for similarity to the vectors surrounding it. There are multiple methods of removing bad vectors as well as multiple options of how to replace the removed vectors. Two option are to replace the bad vector with a zero or replace it with an average of the surrounding vector values.
2.3 Particle Image Velocimetry Uncertainty Quantification Methods

Until 2012, uncertainty estimates of PIV calculations had not been assessed a posteriori. In the work of Sciacchitano et al [4] the effectiveness of four Particle Image Velocimetry Uncertainty Quantification (PIV-UQ) methods were examined to determine the accuracy of uncertainty predictions in PIV measurements. The four PIV-UQ methods examined in this comparison were: Uncertainty Surface [9], Particle Disparity [11], Peak Ratio [12], and Correlation statistics [1]. Because each method determines the uncertainty in different ways, they behave differently depending on the flow type and error sources that are present. In the comparison performed by Sciacchitano et al [4], several flow and error sources were examined. Specifically: near the jet inside the potential core, measurements with out of plane motion, measurements considering the effects of small particle images, and measurements considering the effects of low seeding density. A brief explanation of how each of the compared PIV-UQ methods work is next. This will be followed by the results of the comparison performed by Sciacchitano et al [4].

2.3.1 Uncertainty Surface Method

The Uncertainty Surface (US) method was developed and by Timmons et al [9]. This method processes randomly generated synthetic images which contain known error sources with the PIV processing scheme being considered in order to produce a distribution of errors. This error distribution is used in determining an uncertainty prediction. The uncertainty prediction is performed by using Monte Carlo analysis on the error distributions. The error sources considered by the US method are particle image size, particle density, particle displacement, and flow shear. This method forms an uncertainty surface map which is referenced at each vector location in time to determine the uncertainty. Once the uncertainty surface map is produced, the actual PIV data being considered are analyzed to determine which error sources are present. The uncertainty surface map is then searched for the corresponding uncertainty due to the measured values of the error sources. Because the US method uses a Monte Carlo analysis, a unique upper and lower bound of uncertainty are obtained in both the $x$ and $y$ directions.
2.3.2 Peak Ratio Method

The Peak Ratio (PR) method produces a magnitude of uncertainty based on the ratio of the first and second highest peaks in the correlation. In PIV processing the largest peak of a correlation map is used to determine the displacement of particles. However, it is common that a correlation map contains many peaks due to noise in the PIV system. The PR method postulates a relationship of the ratio of the highest and next highest peaks in a PIV correlation map to the uncertainty associated with PIV measurements. The equation used to describe the relationship between uncertainty, $U$, and the peak ratio, $PR$ is presented by Charonko and Vlachos [12] as

$$U = \alpha (PR)^{-\beta}.$$  \hspace{1cm} (2.20)

As with the US method, the PR method requires a calibration process with synthetically generated images and the PIV processing scheme being used. The PR method uses that calibration and Monte Carlo analysis to determine the values of the scaling coefficient, $\alpha$, and exponential decay rate, $\beta$.

2.3.3 Particle Disparity Method

The Particle Disparity (PD) method, developed by Sciacchitano et al [11], builds from the principles of image distortion. Inside interrogation regions, particle image pairs are identified between images at times $t$ and $t+dt$. The particle image pairs are compared with sub-pixel accuracy and the particle disparities, difference in position of the center of the particles, are calculated. The uncertainty is determined by examining each interrogation region and computing the average particle disparity between all particles in the region and examining the statistical distribution of disparity.

2.3.4 Correlation Statistics Method

The Correlation Statistics (CS) method (Wieneke [1]), similar to the PD method, is based off image distortion. Image distortion is performed and the disparity between image pairs is calculated. The difference between the PD and CS methods is that while the PD
method examines the disparities of particles, the CS method examines the disparity for each pixel location between images.

The CS method takes into account each pixel in PIV images and how they effect the shape of the image correlation peak. If particles are not perfectly overlayed the PIV correlation peak will not be symmetric. When using the CS method, it is assumed that the displacement correlation function, \( C(u) \), is at a maximum with zero slope. This demonstrates a sufficiently converged solution. If the achieved solution is converged the correlation peak will be symmetric and the correlation values evaluated a small distance, \( \pm \Delta x \), from the peak should be equal. The difference between the two values, \( C(-\Delta x) \), and \( C(+\Delta x) \), should be approximately zero. If this is not the case, the residual displacement, \( \delta u \), can be found by fitting a Gaussian curve through the points \( C(u) \), \( C(-\Delta x) \), and \( C(+\Delta x) \). After performing the predictor-corrector scheme as discussed by Wieneke [1] the difference between \( C(-\Delta x) \) and \( C(+\Delta x) \) should be zero. An uncertainty estimation is then computed using calculations that include the variance of \( \Delta C \), where \( \Delta C \) is the difference between \( C(+\Delta x) \) and \( C(-\Delta x) \).

By determining uncertainty through this method factors such as particle image size, disparity, background image noise, and out of plane motion may be taken into account [4]. Because the CS method assumes the PIV algorithm is converged, it primarily estimates random uncertainty. The result of a fully converged PIV algorithm produces a correlation peak that is symmetric, this eliminates most bias error of the system. As part of the proposed work, we will investigate the effect of the number of PIV algorithm final passes and interrogation window overlap on the CS method uncertainty results to determine when the PIV processing algorithm becomes sufficiently converged and the CS result are reliable.

### 2.3.5 Comparison of Uncertainty Quantification Methods

The PIV-UQ methods discussed above were used to analyze data obtained by Neal et al [3], for which the error of the measurement system is known. A comparison of the effectiveness of the PIV-UQ methods was performed by Sciacchitano et al [4] and a summary of their results are presented in this section. The PIV-UQ methods were performed on the
MS PIV system measurements [3] and the uncertainty was estimated to a confidence interval of one sigma, 68%. The methods were compared using data obtained from the flow of a rectangular jet in air with a velocity of 5 m/s. The cases considered are as follows: Unsteady inviscid core just beyond the jet exit, measurement with out of plane motion, measurements considering the effects of small particle images, and measurements considering the effects of low seeding density.

For the unsteady inviscid core case, data were taken inside the potential core near the exit of the jet. It was found that the random error was much larger than the systematic error. The root mean square (RMS) of error, the difference between the MS and HDR system measurements, ranged from 0.04 - 0.05 pixels. The uncertainty resulting from the CS method closely matches the profile of the RMS of error, which is to be expected from Eq. 2.18. The actual coverage of error being 58%. The US method underestimated the error and had an actual error coverage of 39%. Both the PD and PR methods overestimated uncertainty significantly and had actual error coverage of 76% and 82% respectively. The CS method was able to be within 0.005 pixels of the actual one sigma error distribution bounds.

The case considering out of plane motion was measured by rotating the laser sheet 16 degrees from the normal direction of the flow and the laser sheet thickness was adjusted to 1.7 mm. This case was performed to investigate the effectiveness of the PIV-UQ methods in addressing error sources due to out of plane motion, while the potential core of the jet is still the location of interest. The largest errors occur for this case in the center of the jet, which is where the out of plane motion is largest. The errors obtained for this case range from 0.04 - 0.23 pixels, which is significantly higher than the previous case. The existence of bias as well as random errors are present. The CS and PD methods both slightly underestimate the uncertainty, with the actual error coverage being 65% and 64% respectively. The PR method underestimates the uncertainty while still having an actual error coverage of 51%. The US method was ineffective for this case. The actual error coverage for the US method was only 5%, this is a due to fact that the US method does not take into account out of
plane motion.

Examining the effects of small particle images was considered by obtaining data in the potential core with small particle image diameters. The seeding density of the flow was approximately 0.05 particles per pixel (ppp). The size of particle images used for this case was approximately 1 pixel in diameter. This small of particle diameter results in peak locking. For this case a cross correlation coefficient was calculated between error magnitude and estimated uncertainty in order to gauge the correlation of PIV-UQ method results with error. The PD and CS methods result in correlation coefficients of 0.94 and 0.93 respectively, while the US and PR methods result in correlation coefficients of 0.87 and 0.85 respectively. This shows that the PD and CS methods better estimated uncertainty for cases with small particle images. The CS method was able to estimate the random error closely. The US and PD methods were also able to detect the trend of random error but the US method underestimated the error by 20% and the PD method overestimated the error by 50%. Unfortunately, the PR method exhibited lower sensitivity to the effects of small particle images.

In order to investigate the effects of low seeding density the small particle image case was repeated, but with a seeding density of 0.02 ppp and the particle image diameter size was increased to 1.5 pixels. As a result of lower seeding density, the PIV-UQ methods relying upon statistical calculations become disadvantaged since there are simply less particles to analyze. The PR method exhibited uncertainty predictions similar in size to the previous case while the actual error of this case significantly increased. The PR method resulted in an actual error coverage of 39%. The PD method also only increased its uncertainty prediction minimally which resulted in an actual error coverage of 38%. The US and CS method uncertainty results produce an actual error coverage of only 39% and 47% respectively. The CS method resulted in the best error prediction, although all of the PIV-UQ methods produced uncertainty results that significantly underestimated error.

The investigation of Sciacchitano et al [4] concludes that the CS and PD methods produce the best results. The CS method provides the best predictions of uncertainty, with
85% accuracy for the cases of unsteady inviscid core and out of plane motion. The CS method still produced an accuracy of 75% for the cases of small particle image diameter and low seeding density. The PD method performed well with an accuracy of 70% when the case of small particle images is ignored. Unfortunately the PR and US methods did not perform as well.

2.4 Error Distribution Prediction Models

Calculations such as pressure estimations propagate velocity error forward. To help predict the impact of velocity error on pressure calculations, error models have been developed. In the work of McClure et al [5] the error of PIV/PTV based pressure calculations are investigated. As part of their work, a PIV error model was used to describe the expected distribution of PIV/PTV velocity measurement errors.

In their work, the distribution of random error is described as

$$\sigma_i = \alpha \bar{u}_{peak} \frac{|\nabla \bar{u}_{ex}|}{\max(|\nabla u_{ex}|)} + \beta \frac{SF}{\delta t}.$$  (2.21)

Where $\sigma_i$ represents the standard deviation of random error, $\nabla \bar{u}_{ex}$ is the exact velocity gradient, $\bar{u}_{peak}$ is the peak velocity and $SF$ is the scaling factor. The first term is flow dependent, scales according to shear, and results in a term that is a percent of the maximum, or peak, velocity. The second term is not dependent on flow characteristics, but rather contributes to the distribution of random error by taking into account the random errors associated with identifying correlation peaks of interrogation regions over a time separation $\delta t$. Both $\alpha$ and $\beta$ are used to scale the first and second terms of Eq. 2.21 respectively.
CHAPTER 3
Approach

3.1 PIV Processing

The MS and HDR image sets were processed using the DaVis 8.3.1 PIV processing software. The same processing schemes are used as discussed by Neal et al [3]. The pre-processing used on both MS and HDR data was a subtraction of the minimum value over time for all images in each data set. Masks were applied as necessary for both the MS and HDR systems in order to remove laser reflections off of the experimental setup. The MS images were processed using a standard single frame time-series vector calculation. The MS processing was performed with a final interrogation region size of 16x16 pixels with a 75% overlap. Post-processing was performed using the universal outlier detection filter. Uncertainty of the MS data was computed using the built in DaVis uncertainty calculation function, which uses the Correlation Statistics (CS) uncertainty quantification method [1]. The HDR data was processed using the sliding sum-of-correlation algorithm over a kernel of 5 image pairs. The stereo cross correlation was performed with a final interrogation window size of 32x32 pixels with a 75% overlap.

3.2 Scaling of HDR Velocity Fields and Computing Error

All PIV data used in this work were obtained by Neal et al [3]. Two PIV systems were used to simultaneously measure regions of interest of flow for a rectangular jet. Regions of interest in the jet, visually represented in Fig. 3.1, were the inviscid steady flow, inviscid unsteady flow, and developed turbulence flow. The effects of through-plane motion were also investigated in the inviscid steady flow region.

Each experimental setup included two PIV systems, a measurement system (MS) and high dynamic range (HDR) system. The measurement system used a single camera to
perform 2C PIV. The HDR system used two cameras to perform stereo PIV. Because the HDR system had a significantly higher dynamic range than the MS system, the HDR system is considered the true measurement. Error for the MS system can be obtained by using Eq. 2.1. The experimental setup used to obtain data for all cases except through-plane motion had the laser sheet parallel with the direction of the jet flow. The experimental setup used to consider the effects of through-plane motion had the laser rotated 16 degrees from the flow direction. Both PIV setups are shown in Fig. 3.2.

The error associated with the MS system can be easily calculated by taking the difference between the MS and HDR velocity fields once the two PIV systems have been properly aligned. Because the MS and HDR systems each had a different size field of view, the HDR system had to be scaled and mapped onto the MS coordinate system. A series of matlab codes were composed by Sciacchitano [13] to allow the HDR velocity fields to be accurately mapped onto the MS coordinate system. Three codes were used to map the HDR velocity fields and ensure that the mapping was accurate. These codes map PIV images using image correlation, scale HDR velocity fields, and evaluate how well the MS and HDR velocity
Fig. 3.2: Schematic showing setup of MS and HDR systems used to obtain jet data. (Used with Permission [3]) (a) Setup used to obtain data for B, F, and H data sets. (b) Setup used to study through-plane motion. Laser is rotated 16 degrees from the jet flow.

The first step in mapping the HDR velocity field onto the MS coordinate system uses cross correlation of PIV images to determine a scaling factor and necessary shifting values in the $x$ and $y$ directions. The scaling factor and shifting values are obtained by using the sum of image correlations between raw MS images and dewarped HDR images. The scaling factor, $scal$, will later be used in scaling the HDR velocity fields to the same size as the MS velocity fields. Once the scaling factor was determined, the location of the largest image correlation peak was determined in order to obtain the necessary shifting values, $x_0$ and $y_0$. The shifting values are used to adjust the scaled HDR coordinates so that the HDR velocity fields overlay with the MS velocity fields correctly. To show the relative size of the HDR and MS fields of view, a scaled and mapped HDR image is overlayed onto an MS image in Fig. 3.3 for the B009 data case.
The next step applies the scaling factor and shifting values to the HDR velocity fields. The HDR coordinate system is scaled and shifted using

$$x' = x_0 \frac{x}{scal}$$
$$y' = y_0 \frac{y}{scal}.$$  \hspace{1cm} (3.1)

Where $x'$ and $y'$ are the new scaled and shifted HDR coordinates that match the MS coordinate system. The HDR velocities are scaled by

$$u' = \frac{u}{scal}$$
$$v' = \frac{v}{scal}.$$  \hspace{1cm} (3.2)

where $u'$ and $v'$ are the new scaled HDR velocity components.

The final step in scaling the HDR velocity was ensuring that the MS and HDR velocity fields were properly aligned. Alignment of the MS and HDR velocity fields was verified by computing the RMS of error for a selected region of interest of the MS and HDR velocities.
Once the region of interest has been selected, the HDR region was shifted in the $x$ and $y$ directions in small increments. At every shifted, $x$ and $y$, location the RMS of error is computed and a map of the RMS values is produced. Once the RMS map is produced the location of the minimum RMS value is found. If an optimal scaling factor and shifting values were applied to the HDR velocity fields, the MS and HDR velocity fields would be perfectly aligned and the minimum $RMS(\text{Error})$ value would occur at the shifting location (0,0). Because perfect scaling factor and shifting values are not actually applied, the minimum likely will not occur at the (0,0) location. The HDR velocity is considered adequately mapped if the minimum of the RMS map occurs within $\pm 0.2$ pixels in the $x$ and $y$ directions. Once this was accomplished verification that the HDR and MS velocity fields were sufficiently aligned was complete. Once the alignment was verified the error of the MS system was calculated using Eq. 2.1.

3.3 Investigating the Distribution of Error in the MS System

Upon completion of performing PIV processing, velocity fields were exported to allow for additional analysis of the data. Some of the calculations that were performed are the standard deviation of the error and the root mean square of uncertainty. The effects of flow shear on error distributions were examined by computing histograms of error distributions and examining the skewness and kurtosis of the distributions. It was found that in regions where high flow shear was present that the error distributions were skewed and had high levels of kurtosis.

3.4 Investigation of Reasons for Cases with Uncertainty from CS that Does Not Represent Error

Of the cases obtained by Neal el al [3], there are cases for which the Correlation Statistics uncertainty quantification method [1] does not predict error well. It was found that this occurred consistently in regions of high shear, reasons for this were investigated. In order to determine which cases struggled to predict error through the CS method, the standard deviation of error and root mean square of uncertainty were compared. For cases
that did not satisfy Eq. 2.18, the distribution of error was examined by calculating skewness and kurtosis to determine if the error distribution was Gaussian. The CS method was also investigated for other reasons of uncertainty estimation failure.

3.5 Investigation of Effect of Multipass Convergence on CS Method

The CS method assumes that the correlation peak solution from PIV processing is "sufficiently converged" [1]. An investigation of when this actually occurs was performed. The PIV processing scheme used to process MS data varied from 2 to 9 passes for interrogation region overlap of both 50% and 75%. For select cases 87% interrogation region overlap was also performed. The results of the measurement error and CS uncertainty predictions were compared using Eq. 2.18. It was determined from this comparison when the PIV processing solution is sufficiently converged and results in reliable uncertainty calculations.

3.6 Performance of Error Model

The model presented by McClure et al [5] to predict the distribution of PIV random errors is shown as Eq. 2.21. The first term of this model assumes that the contribution to the random error distribution scales with the shear of the flow. The second term is not flow dependent, and is assumed to have a constant contribution to the random error distribution by taking into account the error associated with correlation peak identification. This contribution is dependent on the PIV processing scheme used and is associated with scaling factor, \( SF \), and time separation between images, \( \delta t \).

To determine the accuracy of this model, the standard deviation of error was calculated and compared with shear values of data sets. It was determined that distribution of random error for PIV systems are correlated with both flow shear and particle seeding density of the flow. Linear regression was performed to confirm a positive correlation between error distribution and flow shear. Recommendations for future error models were presented based on the correlation of error distribution, flow shear, and seeding density.
3.7 Investigation of Random Error Correlation in Space

In order to increase spatial resolution and accuracy of PIV measurements, interrogation region overlap is often applied. One of the potential problems with using interrogation region overlap is that the same particle images are used for calculating multiple vectors. When this occurs, it is possible for correlated error to be present in neighboring vectors. Raffel et al [14] discusses an increased correlated random uncertainty in the vorticity of a laminar vortex ring due to increased interrogation window overlap and differentiation scheme. As interrogation window overlap increased from 50% to 75%, the vorticity contour became less smooth. This was attributed to an increase in random error. The causes for this were identified as: a decrease in grid spacing while the measurement uncertainty remained unchanged, and an increased overlap in data used.

In the work of Cressel and Smith [6], correlated random error due to interrogation window overlap began to be investigated. This work continues the investigation of correlated random error by applying Eq. 2.11 to determine if correlated random error exists. The investigation of correlated random error was performed in the jet core and shear regions in all four spatial directions: left, right, up, and down.
CHAPTER 4

Results

A discussion of this work is performed by grouping data sets together by locations of interest and a special case of flow from a rectangular jet. These groupings are: inviscid steady flow, inviscid unsteady flow, developed turbulence, and through-plane motion. The distribution of errors for each case will be discussed first. Next, failure of the CS PIV-UQ Method to predict error are considered. Specific interest is given to how non-Gaussian error distributions influence the failure of the CS method. This will be followed by presenting when convergence of correlation peaks in PIV processing occur in order to produce reliable CS method uncertainty results. This is done by comparing the results of PIV processing that used 2-9 passes for both 50% and 75% IW overlap. Convergence for 87% IW overlap is also investigated for select data cases. A discussion of current error models is presented and discussed by using actual PIV error data. Finally, the results of random error correlation in space are discussed.

It is necessary to understand the seeding characteristics in PIV measurements in order to present accurate results. Flow characteristics of interest are particle image diameter size and particle seeding density. Particle seeding density is of particular interest in determining if particle seeding density and flow shear are correlated with random errors in PIV systems. Seeding characteristics for all cases investigated in this work are obtained through the use of an auto-correlation based method that uses PIV images to determine particle diameter and seeding density. This method was developed and presented by Warner et al [15].

Distribution of Errors

The distribution of errors for each case examined in this work are examined along a vertical slice of data. The cut location was determined by selecting the location for which Eq. 2.18 was best satisfied. The selected locations for each data case are discussed and visually
shown in their associated data case discussions below. Along the cut, averaged values of MS, HDR and MS error are shown to demonstrate how well the MS and HDR systems align. This is also used in the discussion of bias error and to show where maximal bias error occurs. The averaged MS velocity errors describe bias while the standard deviation of error quantifies random error.

As it is of specific interest to investigate the distribution of errors, three points are selected for each data case at locations of interest. The locations for each point of interest are shown along a velocity profile of the vertical cut for each case. Associated histograms are generated to allow for a visual investigation of error distributions. A discussion for each data case is performed in the associated case sections below.

Investigation of CS PIV-UQ Method Failures

Regions where the CS PIV-UQ Method fails are identified using Eq. 2.18. $S_\epsilon$ and $RMS(U)$ are calculated along vertical cuts and the difference between the two values is plotted. In regions that the CS method fails to accurately predict the error distribution, it is hypothesized that the error distribution may be non-Gaussian. The degree to which an error distribution is non-Gaussian is investigated by computing the skewness and kurtosis of the error distributions. The extent to which skewness and kurtosis values deviate from 0 and 3 respectively the more non-Gaussian the distribution is. Skewness is a measure of the distributions symmetry about its center while kurtosis is a measure of how many outliers are present in the data. If there are a large number of outliers present in the data, the distribution will look wider and the tails of the distribution will be thicker. A distribution that has a large amount of kurtosis is commonly referred to as a heavy-tailed distribution.

The assumption used to evaluate CS method results, Eq. 2.18, assumes that the error distribution is Gaussian and has zero mean [4]. Upon further investigation, it was determined that error distributions need to be non-symmetrical, or skewed, in order for the metric that compares error and uncertainty, Eq. 2.18, to fail [16].

Figures are presented for each data case to show how $S_\epsilon$, $RMS(U)$, skewness, and kurtosis relate. Because the difference between $S_\epsilon$ and $RMS(U)$ is of specific interest, a
plot of this value along a cut is shown for each data case. This calculation shows locations where the CS method or the metric used to compare error and uncertainty, Eq. 2.18, has failed. It is possible that when discrepancies between $S_\epsilon$ and $RMS(U)$ occur that the error distributions have either a bias or skewed distribution, causing the metric to fail.

**Correlation Peak Convergence for Reliable CS Method Uncertainty Results**

The CS PIV-UQ Method relies upon a "sufficiently converged" correlation peak to be produced from PIV processing in order to provide reliable CS uncertainty estimations [1]. Because of this requirement it becomes necessary to know how many passes in PIV processing must be used for a sufficiently converged correlation peak to occur. This analysis was performed by processing MS PIV data with an increasing number of passes and comparing the results of the uncertainty produced by the CS PIV-UQ method with the MS error. The PIV processing schemes varied from 2 to 9 passes for both 50% and 75% IW overlap. For the B009 and I013 cases, an IW of 87% overlap was also performed.

After the necessary processing was performed, the mapped HDR velocity fields were compared with each of the varied multi-pass processing scheme velocity fields. Both terms of Eq. 2.18 were computed for each number of passes used. The absolute value of the difference between $S_\epsilon$ and $RMS(U)$ was computed for areas of MS and HDR overlay and averaged over regions of interest.

**Investigation of PIV System Error Distributions**

In the work of McClure et al [5], how velocity uncertainty impacts pressure calculations was investigated. Because an existing model to describe the distribution of random error for PIV/PTV based measurements did not exist, they presented a model to describe error distribution according to flow conditions. This model scales with shear and the dynamic range of the setup and is shown by Eq. 2.21. Because the error can be computed for the data obtained by Neal et al [3] a comparison of Eq. 2.21 and actual error distributions was performed. For each data case, $S_\epsilon$ and flow shear are compared to determine if a correlation exists between the distribution of error and flow shear.
Correlation of Random Error in Space

Correlation of random errors in space is determined using Eq. 2.11. A single vector location was selected and set as the anchor vector location. The error at that location is used for, \( \epsilon_xk \), while \( \epsilon_yk \) is the error a distance \( \Delta i \) away from the anchor vector location. Where \( \Delta i \) is the spacing between vectors. The correlation of random error was performed in each spatial direction; right, left, up, and down. Because an IW overlap of 75% was used for processing both the MS and HDR systems, the same particles in a PIV image are used in up to four neighboring vectors, \( \Delta i = 3 \). Random errors in PIV measurements may be correlated because the same particles are used in multiple calculations. In the absence of image deformation, correlated random error beyond \( \Delta i = 3 \) should not exist. The correlation of random errors is computed at locations of interest for each data case. For cases that contain a jet core and regions of high shear, correlation of random error is performed in both locations. The correlation of random error at the anchor vector location and itself, \( \Delta i = 0 \), is equal to the square of \( S_\epsilon \) at that location [6].
4.1 B Data Cases: Inviscid Steady Flow

This section will examine the inviscid steady flow near the exit of a rectangular jet. This region of interest of the jet flow is shown visually in Fig. 3.1. The three data cases obtained at this region of interest are B009, B011, and B013. Each data set examines the same area of the jet flow, but have different seeding characteristics. The actual seeding densities for these cases are 0.093, 0.073, and 0.083 ppp respectively and the particle image diameter for all B cases was found to be approximately 2 pixels. This region contains a potential core as well as strong shear regions where the moving jet and stationary ambient air interact.

Examining the Distribution of Errors

Scaled and shifted HDR velocity fields for each data case are shown with the location of the selected vertical cut for each B case in Fig. 4.1. The indices in Fig. 4.1 are arbitrary vector locations in the $x$ and $y$ directions for the mapped HDR velocity fields. The cut locations for B009, B011, and B013 data cases are 45, 43, and 45 respectively, in the $x$ direction.
After mapping the HDR system onto the MS coordinate system, it was necessary to validate that the velocity vectors of the HDR system were in the correct locations. This was performed in the mapping process, but is again demonstrated for each B case in Figure 4.2 by showing how well the MS and HDR velocity profiles align. This figure also shows the relative size of the average MS error. The average error reveals the existence of bias error in the system. The B cases show bias error at least in part because the particle diameters are small, this causes bias due to peak locking. Figure 4.2 shows that the largest average errors for all B data cases exist at or near the shear regions.
Fig. 4.2: Average MS, HDR, and MS Error Velocities along vertical cut for: (a) Data Case: B009; (b) Data Case: B011; (c) Data Case: B013.

Three locations along the vertical cut are selected as points of interest for each data case. The points of interest for each of the B data cases are: 1) just outside the jet, 2) in the jet core, 3) in the shear region. Figures 4.3 - 4.5 show the location of the points along the velocity profile and displays a histogram of the MS error for each point of interest. It is
easily observed that the error distributions at the locations of interest widely vary for each data case. The error distributions for data cases B009, B011, and B013 are examined and discussed with skewness and kurtosis in mind.

Figure 4.3 shows that the error distributions for points 1, 2 and 3 of the B009 data case grow consecutively wider. This indicates an increasing number of outliers in the MS error and causes the distribution to become heavy-tailed. The width of the error distribution in the shear layer, point 3, is significantly larger than the distribution widths of either in the jet core or immediately outside of the jet. Skewness isn’t largely prevalent for this data case, however slight positive skewness can be seen in the B009 case histograms. The error distribution at the center of the jet core, point 2, appears to be the most Gaussian distribution for this data set.

Data case B011, shown in Fig. 4.4, displays a more noticeable amount of both skewness and kurtosis in error distributions than was observed in the B009 data case. For this case, all three points of interest have error distributions that look skewed, although point 1 is the least noticeable. The error distributions of both point 1 and point 2 are similar and are much more Gaussian in shape than point 3. The error distribution for point 3 is very heavy-tailed and is the most noticeably skewed of all error distributions for the B data cases.

Data case B013, shown in Fig. 4.5, shows a steady increase in kurtosis from point 1 to point 3. The point 1 error distribution appears Gaussian, having low amounts of both skewness and kurtosis. The second point is skewed in the positive direction and is heavy-tailed. The third point, located in the shear region, is the most heavy-tailed error distribution for this data case, but is surprisingly symmetric.

For all of the B cases, significantly higher kurtosis levels were evident in shear regions. Data case B011 exhibits the highest amounts of kurtosis compared to both B009 and B013. It is interesting to note that higher kurtosis levels were exhibited in the shear region for cases of lower seeding density. It was observed that the seeding density of a flow and kurtosis of the error distribution in the shear region are inversely related. Higher levels of skewness
were also noticed in cases B011 and B013 than in B009. Data case B009 behaves the most ideally of all B cases examined. This is hypothesized to be due to the fact that B009 has the largest seeding density of all of the B data cases.
Fig. 4.3: Locations of points of interest along average HDR velocity profile and histograms of MS velocity error distributions at points of interest for the B009 data case. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
Fig. 4.4: Locations of points of interest along average HDR velocity profile and histograms of MS velocity error distributions at points of interest for the B011 data case. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
Fig. 4.5: Locations of points of interest along average HDR velocity profile and histograms of MS velocity error distributions at points of interest for the B013 data case. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.

Because it was noticed that the mean of the error distributions varies from one point of
interest to another and from one data case to another, an investigation of what causes the changing bias of error distributions was investigated. In the process of mapping the HDR velocity fields onto the MS coordinate system it is possible that a misalignment still exists, even after verifying that the velocity fields match well. Additional manual shift was applied in the vertical direction to the mapping of the HDR velocity fields to investigate if the bias of the error distributions varied. Manual shifts of 0.1, 0.3, 0.5, 0.75, 1, and -0.5 were applied in the vertical direction for the data case B009. It was noticed that for positive manual shift that the point 3 error distribution moved further negative, while a negative manual shift results in the error distribution being moved in the positive direction. The manual shifts of 0.3, 0.5, and -0.5 are shown in Fig. 4.6 - 4.8. By comparing Fig. 4.3 with the shifted HDR error distributions it is evident that bias of the error distribution is likely due to misalignment of the MS and HDR systems. Only the point 3 distributions are effected because it is located in the shear region where the velocity gradient is high and therefore is highly sensitive to misalignment.

Once it was determined that the bias of the error distribution corresponds with the vertical misalignment of the MS and HDR systems in the B009 case, an investigation of the error distributions in both shear regions was performed. Vertical shifts of 0, 0.3, 0.5 and -0.5 were applied to the HDR system during the mapping process and the error distributions of both shear regions were investigated. A Gaussian curve of the error distributions in the upper and lower shear regions are shown in Fig. 4.9. It was found that as a vertical shift is applied that the error distributions moved in opposite directions. Understanding that the bias of the error distribution can be used to determine misalignment between the MS and HDR system allows for the use of both error distributions to more accurately scale the HDR system and result in better alignment.
Fig. 4.6: Locations of points of interest along average HDR velocity profile and histograms of MS velocity error distributions at points of interest for the B009 data case with a vertical shift of +0.3 pixels. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
Fig. 4.7: Locations of points of interest along average HDR velocity profile and histograms of MS velocity error distributions at points of interest for the B009 data case with a vertical shift of +0.5 pixels. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
Fig. 4.8: Locations of points of interest along average HDR velocity profile and histograms of MS velocity error distributions at points of interest for the B009 data case with a vertical shift of -0.5 pixels. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
Fig. 4.9: Various Vertical Shifts applied to HDR mapping for data case B009, looking at shear region histograms of error (a) Profile of average HDR velocity along vertical cut and location of interest in shear regions. (b) Vertical Shift = 0 (c) Vertical Shift = 0.3 (d) Vertical Shift = 0.5 (e) Vertical Shift = -0.5
Investigation of CS PIV-UQ Method Failures

Figure 4.10 shows the values of the difference between $S_{\epsilon}$ and $RMS(U)$ for each B case. This value is important because regions where $S_{\epsilon} - RMS(U)$ is large indicates that either the CS method or the metric used to compare error and CS uncertainty has failed. This plot shows where either the CS uncertainty prediction or comparison metric fails to accurately predict the actual error of the MS system. The largest discrepancy occurs in regions of high shear for all B data cases. It is also interesting to note that the difference between $S_{\epsilon}$ and $RMS(U)$ increases inversely proportional to particle seeding density for B cases.

By examining Fig. 4.11 - 4.13 it can be seen that when $S_{\epsilon}$ does not match $RMS(U)$ that skewness and kurtosis of error distributions are generally large. This especially occurs in all shear regions of the B cases. This shows that for regions of high shear that the error distribution is non-Gaussian. When the error distribution is non-Gaussian due to skewness the error and uncertainty comparison metric, Eq. 2.18, fails.

Figures 4.11 - 4.13 show the skewness and kurtosis values along the vertical cuts for each data case. These figures clearly show that data case B009 has the lowest values of both skewness and kurtosis. Case B011 contains the highest kurtosis values and case B013 contains the most skewed error distributions. It is interesting to note that for all cases, the levels of both skewness and kurtosis increased in regions of shear and decreased in value at the jet core. Kurtosis has a value of three at the jet core for both the B009 and B013 cases, while the kurtosis value for B011 at the jet core was only slightly greater than three. Although skewness decreased in the jet core for all B cases, only a zero skewness value was obtained in the core for case B013. The lowest values of skewness at the jet core for cases B009 and B011 are approximately 0.25 and -0.5 respectively.

For two of the three B data cases, B009 and B013, Eq. 2.18 is not satisfied in the potential core region of the jet. This is interesting because for both cases, especially in B013, the skewness and kurtosis are at low values, which indicates a Gaussian distribution of errors. However, in the case B011, where Eq. 2.18 is satisfied in the jet core, there is a
higher amount of skewness in the error distributions than in the other B data cases. When the \( S_\epsilon \) and \( RMS(U) \) don’t agree in the jet core it was noticed that a bias error is present. This indicates that the comparison method of error and uncertainty, Eq. 2.18, may be what is failing in the jet core rather than the CS results.

Fig. 4.10: Difference between \( S_\epsilon \) and \( RMS(U) \) for all B cases along a vertical cut.
Fig. 4.11: Comparison of $S_\epsilon$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for Data Case B009. (a) Calculated $S_\epsilon$ and $RMS(U)$ along cut; (b) Skewness of Error Distribution; (c) Kurtosis of Error Distribution.
Fig. 4.12: Comparison of $S_\epsilon$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for Data Case B011. (a) Calculated $S_\epsilon$ and $RMS(U)$ along cut; (b) Skewness of Error Distribution; (c) Kurtosis of Error Distribution.
Fig. 4.13: Comparison of $S_\epsilon$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for Data Case B013. (a) Calculated $S_\epsilon$ and $RMS(U)$ along cut; (b) Skewness of Error Distribution; (c) Kurtosis of Error Distribution.

**Correlation Peak Convergence for CS Method with Increasing PIV Passes**

The PIV processing schemes used to process MS data varied from 2 to 9 passes for both 50% and 75% IW overlap for all B data cases. The processed data was used in determining when PIV processing results in a converged correlation peak. This is important
because a converged correlation peak is a prerequisite for the CS method to result in reliable uncertainty predictions. PIV processing with an 87% IW overlap was also performed for data case B009.

Figures 4.14 and 4.15 show the value of $S_{\epsilon} - RMS(U)$ along a vertical cut for each number of passes used in PIV processing. It is observed that the value $S_{\epsilon} - RMS(U)$ decrease as the number of passes used on the MS PIV data increases. When $S_{\epsilon} - RMS(U)$ stops decreasing as additional passes are performed, convergence of the correlation peak has been achieved. Figure 4.14 shows $S_{\epsilon} - RMS(U)$ for the B009 data case processed with 75% and 87% IW overlap. Figure 4.15 shows $S_{\epsilon} - RMS(U)$ for the B011 and B013 data cases where 75% IW overlap is used in the PIV processing scheme. From both of these figures it is easy to see that as the number of passes used in PIV processing increases, that the value $S_{\epsilon} - RMS(U)$ decreases until convergence is achieved. For data case B009, by Fig. 4.14, convergence is achieved at approximately 6 passes for 75% IW overlap. But for 87% IW overlap, convergence is never fully achieved although changes in $S_{\epsilon} - RMS(U)$ between passes significantly decrease by 9 passes. By Fig. 4.15, cases B011 and B013 achieve convergence at approximately 8 passes for 75% IW overlap.

The absolute value of $S_{\epsilon} - RMS(U)$ was performed for the area of MS and HDR overlay and averaged over two different regions of interest. The first region of interest was the entire area of overlay between the MS and HDR systems. The second region considered was the overlay of only the shear regions of the jet. The averaged values are plotted against the number of PIV passes used and shown in Fig. 4.16 for all B cases. The region of interest that only considering the shear region overlay resulted in slightly higher averaged values than the regions that examined the entire overlay of the MS and HDR velocity fields.

Figure 4.16 shows that the larger the IW overlap that is used the more passes are required for convergence of the correlation peak to be achieved. It is apparent from this figure that reliable uncertainty results are produced for PIV processing that used 50% IW overlap after only three passes for data case B011, and four passes for data cases B009 and B013. For processing schemes that used either 75% or 87% IW overlap, it took longer
for the correlation peak to become sufficiently converged and for reliable CS uncertainty estimations to be produced. When 75% IW overlap is used in PIV processing, sufficient convergence of the correlation peak is achieved after 5 passes for B009, 8 passes for B011, and 6 passes for B013. By examining Fig. 4.14 and Fig. 4.15 it can be seen that the value of $S_e - RMS(U)$ stops decreasing at the same number of passes as convergence is reached in Fig. 4.16. For the case of B009, which produced a lower uncertainty than both B011 and B013, it is interesting to note that the 75% IW overlay took 6 passes to be become converged. For case B009 an IW overlap of 87% was also investigated and found that 9 passes in the PIV processing still had not reached a sufficiently converged correlation peak. Although convergence still had not been reached by 9 passes, it is evident that a lower value is reached for the average absolute value of $S_e - RMS(U)$ than for either the 50% or 75% IW overlap. This indicates that eventually a slightly more accurate uncertainty estimation would be achieved once convergence occurs.
Fig. 4.14: Difference between StdDev(Error) and RMS(Uncertainty) for the B009 Data Case with PIV processing that used: (a) 75% IW Overlap; (b) 87% IW Overlap.
Fig. 4.15: Difference between StdDev(Error) and RMS(Uncertainty) for 2-9 passes of PIV processing with 75% IW overlap for: (a) Data Case B011; (b) Data Case B013.
Fig. 4.16: Average absolute value of $S_\epsilon - R M S(U)$ vs. number of passes used in PIV processing for B Data cases. Where 50%, 75%, and 87% IW overlap of PIV processing is used to determine when Correlation Peak convergence occurs for: (a) Data Case B009; (b) Data Case B011; (c) Data Case B013.

Correlation of Error Distribution and Flow Shear

In order to investigate if the distribution of error, $S_\epsilon$, and flow shear are correlated, they are compared to each other for all B data cases. $S_\epsilon$ and flow shear are compared by
plotting them against each other for each point through the shear regions of a vertical cut of the jet. Once this was complete, a linear fit of each case was performed. The results are shown by Fig. 4.17. It is readily apparent from this figure that the distribution of error increases as the shear in the flow increases. It is also interesting that $S_\varepsilon$ increases inversely proportional to the seeding density of the associated data cases. This shows that the particle seeding density as well as flow shear should be considered when estimating the distribution of random errors for PIV systems.

Figure 4.18 shows the slopes of the fits in fig. 4.17 as a function of particle seeding density. It can be seen that a near linear relationship exists between the B case slopes and particle seeding density. A model to predict error distribution based off correlation of slope and seeding density is written as

$$S_{P\varepsilon} = mJ + b.$$  (4.1)

Where $S_{P\varepsilon}$ is the predicted error distribution based on shear and seeding density. The fit values of slope vs. seeding density, $m$, is the value of the linear fit of Fig. 4.18 for each case seeding density. The flow shear for each case is $J$, and $b$ is the intercept of Fig. 4.17. The $m$ term can also be written as

$$m = ap + c,$$  (4.2)

where $a$ and $c$ are the slope and intercept of Fig. 4.18, and $p$ is the seeding density of the cases.

The model’s prediction of error distribution, $S_{P\varepsilon}$, is determined by Eq. 4.1 and is compared to the actual error distributions, $S_\varepsilon$. This comparison is shown in Fig. 4.19. This shows a strong correlation exists between error distribution, seeding density, and flow shear. As a result of this comparison, it is recommended that future models that are made to predict the random error distribution of PIV systems should consider both the flow shear and seeding density of the flow.
Fig. 4.17: Comparison of $S_{\epsilon}$ and shear in the flow with linear fits for all B data cases of varying particle density.

Fig. 4.18: Plot of Seeding Density vs Slope of $S_{\epsilon}$ vs du/dy plot, Fig. 4.17, for B data cases.
Correlation of Random Errors in Space

For the B data cases, two locations are of specific interest when performing correlation of random error, the jet core and the shear layer. Correlation of random error is performed at both of these locations in all directions. Figures 4.20 - 4.25 show the correlation of random error in space at both the jet core and shear layer for all three B cases. It is interesting that when correlation of random error in space is computed in the shear region that $S_{xy}$ is an order of magnitude higher for cases B009 and B013 and two orders of magnitude larger for case B011 than when correlation of random error is computed in the jet core. This shows significantly larger correlation of random error exist in the shear region than in the jet core. Correlated random error, $S_{xy}$ should be absent beyond $\Delta i = 3$ when no residual random error correlation is present.

Two trends are observed for values of $S_{xy}$ in the jet core. The first trend is that values of $S_{xy}$ decrease linearly from $\Delta i = 0$ to $\Delta i = 6$, where $S_{xy}$ reaches a near zero value. The second trend observed is a linear decrease in $S_{xy}$ until $\Delta i = 4$ and then the $S_{xy}$ value flattens off at a non-zero value. Both of these trends are odd. The first trend is present for all B
cases when considering the correlation of random error in space in the vertical directions at the jet core. While the second trend of correlated random error occurs for all B cases in the horizontal directions at the jet core. The correlation values that the B cases level out at for the horizontal directions are approximately 0.003 for B009, 0.002 for B011, and 0.005 for B013. This shows that there are minimal residual random error correlations in the horizontal directions at the jet core for all B cases. When considering the correlation of random error in the vertical direction at the jet core, it is interesting to note that for the cases B009, B013, and B011 in the negative direction that the correlation steadily decreases to roughly a zero value by $\Delta i = 6$. Data case B011, in the positive vertical direction, never reaches a zero correlation value but instead settles at roughly 0.001. These results show that at the jet core all errors at surrounding vector locations to the anchor vector location experience at least minimal correlated random error until at least $\Delta i = 6$.

When considering correlation of random error in the shear region for all B cases, the values of $S_{xy}$ are much larger than observed in the jet core. The values of $S_{xy}$ for data cases B009 and B013 are on the order of magnitude of $10^{-2}$ and values of $S_{xy}$ for data case B011 are on the order of magnitude of $10^{-1}$. It is interesting that the lowest seeding density case, B011, has the highest correlated random errors in space in the shear region. In the horizontal directions of the shear layer, it is observed that beyond $\Delta i = 4$ that correlation of random error in space is absent for all cases and directions, except for case B011 in the horizontal directions. For case B011 in the horizontal directions it is observed that $S_{xy}$ values decrease linearly until $\Delta i = 6$ where zero values are reached. These results show that although correlation of random error is larger in the shear region than in the jet core, that correlation of random error is generally absent beyond $\Delta i = 3$ in the shear region.
Fig. 4.20: Correlated Random Error for Data Case B009 in the jet core: (a) Location (point); (b) Correlation of Random Err in both the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.21: Correlated Random Error for Data Case B009 in the shear region: (a) Location (point); (b) Correlation of Random Error in the $x$ and $y$ directions. (c) Correlation of Random Error in the $x$-directions. (d) Correlation of Random Error in the $y$-directions.
Fig. 4.22: Correlated Random Error for Data Case B011 in the jet core: (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.23: Correlated Random Error for Data Case B011 in the shear region: (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.24: Correlated Random Error for Data Case B013 in the jet core: (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.25: Correlated Random Error for Data Case B013 in the shear region: (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
4.2 F Data Cases: Inviscid Unsteady Flow

This section examines the region of inviscid unsteady flow of a rectangular jet. The data sets for this region were obtained downstream of the inviscid steady flow cases and the location is shown in Fig. 3.1. This location allowed for vortices to develop in the shear regions. The inviscid unsteady region still contains a potential core but has wider and less intense shear regions where the jet and stationary air interact than in the inviscid steady cases. The two data cases used for this case are F001 and F005. The first has a low particle seeding density, 0.058 ppp, while the latter has a larger seeding density of 0.068 ppp. Both of these data cases have similar particle diameter sizes since the f-stop of the cameras used was the same for both cases, the size of particle diameters are approximately 2.5 pixels.

Examining the Distribution of Errors

Scaled and shifted HDR velocity fields are shown with the selected cut locations for both of the F cases in Fig. 4.26. The indices used in this figure are arbitrary vector locations in the \( x \) and \( y \) directions. The cut location for data case F001 is 99 and the cut location for the F005 case is 90 in the \( x \)-direction.

Fig. 4.26: Mapped HDR \( u \) velocity field with associated cut locations: (a) Data Case: F001; (b) Data Case: F005.
Figure 4.27 demonstrates how well the MS and HDR velocities align by examining the averaged MS velocity, HDR velocity, and MS velocity error along the associated cuts for each data case. This figure also shows the relative size of the bias error along the cut. It is interesting to see that the maximum bias error for both cases is similar but for the case of higher seeding density, F005, the bias error is a positive value during both regions of shear. The positive bias error simply means that the HDR velocity is larger than the MS velocity since $u_{\text{error}} = u_{\text{HDR}} - u_{\text{MS}}$. It is also interesting that the higher seeding density case, F005, has slightly higher bias errors than the lower seeding density case.
Figure 4.27 shows that the largest average errors for both F data cases exist at or near the shear regions. By selecting a few points along the cuts for each data case, the distribution of MS error may be examined. The error distribution for each location of interest will indicate the amount of random error in the system. Three points are selected for each of the F cases. These locations are outside the flow of the jet, in the jet core, and in the shear region. Figures 4.28 and 4.29 show the location of the three points along
the velocity profile cut and shows histograms of the errors for the F001 and F005 cases respectively.

Figure 4.28 shows the distribution of errors for all three locations of interest for data case F001. It can be seen that there are only minor variations in the distribution of errors for each location of interest. All of the error distributions look similar and have low levels of skewness and kurtosis. The first point of interest is just outside the jet’s shear region. This error distribution has a slightly heavier tail compared to point 2 and is only very slightly skewed. The second point examines the error distribution at the jet core. This distribution of errors has a mean that is the closest to zero of all three points considered and looks less skewed and lighter-tailed than the other distributions. This shows that the most Gaussian error distribution occurs in the jet core. The third point examines the distribution of errors in the shear region. This distribution doesn’t look skewed but is the most heavy-tailed distribution of all points considered for case F001.

Figure 4.29 examines the distribution of error for data case F005. The distribution of errors for the first and second points look very similar. Both points are not noticeably skewed or have significantly heavy-tails. Point 3 is the most interesting because it exhibits both skewness and kurtosis. Point 3 is negatively skewed and is more heavy-tailed than either points 1 or 2.

It is interesting that the F cases behave more ideal and don’t have nearly as non-Gaussian error distributions as the B cases. It is also interesting that case F005 has slightly more variation in error distributions than case F001, even though F005 has the higher seeding density.
Fig. 4.28: Locations of points of interest along average HDR velocity profile and histograms of MS error distributions at points of interest for the F001 data case. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histogram of MS error distributions at points of interest.
Fig. 4.29: Locations of points of interest along average HDR velocity profile and histograms of MS error distributions at points of interest for the F005 data case. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
Investigation of CS PIV-UQ Method Failures

By examining Fig. 4.30 it can be seen that largest values of $S_\epsilon - RMS(U)$ primarily occur in the shear regions of the jet. By looking at Fig. 4.31 - 4.32 it can be seen that when $S_\epsilon$ and $RMS(U)$ do not match that the skewness and kurtosis are generally larger. This shows that for regions of shear it is more likely that the error distribution is not Gaussian and that either the CS method does not accurately predict the error in those regions or the metric used to compare error and uncertainty fails. This was also demonstrated by examining the histograms of error distribution above. Although the results displayed by the error histograms for the F cases are not as extreme as the B data cases, the kurtosis of error distributions in the shear region were more heavy-tailed.

As mentioned in the discussion of error distribution at the points of interest, there isn’t a high amount of either skewness or kurtosis present when visually investigating the error distributions. This is confirmed by Fig. 4.31 and Fig. 4.32. The highest magnitude of skewness that is calculated for the distribution of error for case F001 is 0.4 and only 0.2 for case F005. Kurtosis values are also low for both of the F cases. Data case F001 exhibited a maximum kurtosis value of 4.38 while the F005 case only reached a value of 3.58. It is also noteworthy that the higher seeding density case, F005, had skewness and kurtosis values that were roughly half of the values associated with the lower seeding density case, F001.
Fig. 4.30: Difference between $S_\epsilon$ and $RMS(U)$ for data cases F001 and F005.
Fig. 4.31: Comparison of $S_\epsilon$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for Data Case F001. (a) Calculated $S_\epsilon$ and $RMS(U)$ along cut. (b) Skewness of Error Distribution. (c) Kurtosis of Error Distribution.
Fig. 4.32: Comparison of $S_e$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for Data Case F005. (a) Calculated $S_e$ and $RMS(U)$ along cut. (b) Skewness of Error Distribution. (c) Kurtosis of Error Distribution.

Correlation Peak Convergence for CS Method with Increasing PIV Passes

When determining the convergence of the PIV processing correlation peak is investigated, there are two regions are of interest for the F data cases. The regions of interest for cases F001 and F005 are the entire jet overlay and the overlay of only the shear regions.
Figure 4.35 shows the results of the averaged regions while Fig. 4.33 and 4.34 show the value of $S_{\epsilon - RMS(U)}$ along a single cut with the number of passes used for both 50% and 75% IW overlap.

Figures 4.33 and 4.34 show that significant decrease of $S_{\epsilon - RMS(U)}$ ceases after 3 and 4 passes for F001 and F005 respectively when an IW overlap of 50% is used. From Fig. 4.35 it is also clearly evident that convergence of uncertainty results occurs after 3 passes for 50% IW overlap. When PIV processing is performed with 75% IW overlap it takes more passes before convergence of the CS uncertainty results are achieved. Figures 4.33 and 4.34 show that a steady decrease in $S_{\epsilon - RMS(U)}$ occurs until 6 and 5 PIV processing passes are performed for F001 and F005 cases respectively. Figure 4.35 shows convergence has been achieved by roughly 5 passes for both F cases with 75% IW overlap. Any passes performed beyond the number of passes when convergence is achieved does not result in a more reliable CS uncertainty result and is therefore unnecessary additional processing.
Fig. 4.33: Difference between StdDev(Error) and RMS(Uncertainty) for F001 Data Case with PIV processing that used: (a) 50% Overlap; (b) 75% Overlap.
Fig. 4.34: Difference between StdDev(Error) and RMS(Uncertainty) for F005 Data Case with PIV processing that used: (a) 50% Overlap; (b) 75% Overlap.
Fig. 4.35: Average absolute value of $S_e - RMS(U)$ vs. number of passes in PIV processing for F001 and F005 data cases. Where 50% and 75% IW overlap of PIV processing is used to determine when Correlation Peak convergence occurs for: (a) Data Case F001; (b) Data Case F005.

Correlation of Error Distribution and Flow Shear

The investigation of correlation between error distribution and flow shear began by plotting $S_e$ and flow shear against each other. Points along a vertical cut in the shear regions for data cases F001 and F005 are used. This plot is shown as Fig. 4.36. A linear fit for each data case was performed. The slopes of these fits were very similar but the lower
seeding density case, F001, has a slightly steeper slope. The trend that $S_\epsilon$ and fit slope are inversely related hold for the F cases, although not as strong as the B cases. The values of seeding density and fit slope for each F case were plotted. A linear fit was performed between the two points and is shown in Fig. 4.37.

A predicted error distribution was calculated based off the correlation of slope and seeding density of the F cases. The equation used is Eq. 4.1, where $m$ for the F cases is the value of the fit at each seeding density from Fig. 4.37, $J$ is the flow shear of each case, and $b$ is the intercept value of the fits from Fig. 4.36. This predicted error distribution, $S_{P\epsilon}$, is plotted against $S_\epsilon$ and shown as Fig. 4.38. From this plot it is again seen that $S_\epsilon$ and $S_{P\epsilon}$ are correlated. This shows, similar to the B cases, that error distribution is correlated with both seeding density and flow shear for the F data cases.

![Fig. 4.36: Comparison of $S_\epsilon$ and shear in the flow with linear fits for F001 and F005 data cases of different particle seeding densities.](image-url)
Seeding Density (ppp)

Slope of Linear Fit through $S_\epsilon$ vs. $du/dy$

- $F_{001}$ (.058 ppp)
- $F_{005}$ (.069 ppp)
- Linear Fit

Fig. 4.37: Plot of Seeding Density vs. slopes of Fig. 4.36, for F data cases.

du/dy of HDR System

StdDev(Err) (pixels)

- $F_{001}$ (.058 ppp)
- $F_{005}$ (.069 ppp)

Fig. 4.38: Plot showing Predicted error distributions, $S_{Pr}$, vs. actual error distributions, $S_\epsilon$, for the F001 and F005 data cases.
Correlation of Random Errors in Space

Figures 4.39 - 4.42 show the correlation of random error in space for the F cases in both the jet core and shear region. The correlation of random error was computed for both cases at the center of the jet and in the shear region using Eq. 2.11. It was found that values of \( S_{xy} \) in the shear region were approximately twice as large as the \( S_{xy} \) values calculated in the jet core. The \( S_{xy} \) values at \( \Delta i = 0 \) are approximately 0.002 and 0.004 for the jet core and shear region respectively. All F data cases follow the trend of linearly decreasing \( S_{xy} \) values until a near zero value is reached and the \( S_{xy} \) values level off.

When examining the correlation of random error in the jet core, the horizontal directions behave slightly better than in the vertical directions. Values of \( S_{xy} \) for the horizontal direction consistently reach a near zero value at \( \Delta i = 4 \) and remain near zero. This shows that correlation of random error does not exists beyond where particles are shared when performing PIV processing in the horizontal directions. The vertical directions also reach \( S_{xy} \) values of near zero at approximately \( \Delta i = 4 \), but don’t remain at zero as \( \Delta i \) continues to increase. Data case F001 in the positive vertical direction continues the downward trend to a negative correlation value at \( \Delta i = 6 \). Interestingly, at \( \Delta i = 6 \), the positive and negative directions vary slightly from zero in opposite directions for both F001 and F005 cases. For case F001, at \( \Delta i = 6 \), the positive vertical direction has a \( S_{xy} \) value that is slightly negative, while the negative vertical direction remains at a zero value for \( S_{xy} \). For the case F005, the positive and negative vertical directions deviate from a zero \( S_{xy} \) value and reach a slightly positive and negative \( S_{xy} \) values in the positive and negative vertical directions respectively. Although the deviation of \( S_{xy} \) is small for F005 at \( \Delta i = 6 \), it is interesting that \( S_{xy} \) moved in opposite directions.

When examining the correlation of random error in the shear region of both the F001 and F005 cases, minimal correlation of random error is observed beyond \( \Delta i = 3 \). The correlation of random error results in the horizontal directions for F001 case, beyond \( \Delta i = 3 \), agree very well in both the positive and negative directions. For the case of F005, considering the horizontal directions, zero values of correlation are obtained at \( \Delta i = 5 \). It is interesting
that the higher seeding density had slight correlation of random error at $\Delta i = 4$, while the lower seeding density performed slightly better and reached a near zero correlation value at $\Delta i = 4$. It is only in the vertical directions for both F001 and F005 that it is observed that zero correlation value is obtained at $\Delta i = 3$. Case F001 in the negative vertical direction reaches a zero value for correlation and remains there from $\Delta i = 3$ to $\Delta i = 6$. For case F005 in the positive vertical direction, near zero correlation values are achieved at and beyond $\Delta i = 3$. In the negative vertical direction for case F005, the correlation is the furthest negative at $\Delta i = 3$ and increases back to stable zero values of $S_{xy}$ at $\Delta i = 5$.

Fig. 4.39: Correlated Random Error for Data Case F001 in the jet core. (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.40: Correlated Random Error for Data Case F001 in the shear region. (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.41: Correlated Random Error for Data Case F005 in the jet core. (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
Fig. 4.42: Correlated Random Error for Data Case F005 in the shear region. (a) Location (point); (b) Correlation of Random Err in the x and y directions. (c) Correlation of Random Err in the x-directions. (d) Correlation of Random Err in the y-directions.
4.3 H015 Data Case: Developed Turbulence

This section examines the downstream developed turbulent flow of a rectangular jet. For this area of interest there is only a single data case, H015. Unlike the inviscid steady or inviscid unsteady flow cases, this region does not have any significant shear regions in the flow. This is because we are examining a portion of the flow far enough downstream that it is completely mixed and fully turbulent. The seeding density associated with this case is 0.063 ppp with particle diameters of 2.8 pixels. The location of this data case in the jet flow is shown in Fig. 3.1.

Examining the Distribution of Errors

A scaled and shifted HDR velocity field is shown with the selected cut location for the H015 case in Fig. 4.43. The cut location for the H015 data case is 103 in the $x$-direction. Figure 4.44 demonstrates how well the MS and HDR velocities align for this data case. It is interesting that for the H015 data case that there is a slight negative bias error along the entire cut, although not large in magnitude.

Fig. 4.43: Mapped HDR $u$ velocity field with associated cut location shown for data case H015.
Three points of interest are selected along the vertical cut for case H015. The first and third points are located in regions closer to the edges of the turbulent flow. The second point is located in the region of largest velocity. Figure 4.45 shows the location of the three points as well as the histogram of the errors for the H015 data case.

The distribution of errors for all three points of H015 case are very similar. The first point has a slightly wider error distribution than the other two points, but doesn’t exhibit significant skewness or kurtosis. The second and third points look identical. The histograms of the second and third points are only slightly taller than the histogram for point 1, this indicates that points 2 and 3 have slightly lower kurtosis values. The error distributions for all of the points investigated for case H015 look normal in distribution and have a near zero mean.

Investigation of CS PIV-UQ Method Failures

By examining Fig. 4.47, it can be seen that $S_\epsilon$ and $RMS(U)$ match fairly well and follow the same general trend. This figure also shows that there are only minimal amounts of skewness in the distribution of errors along the vertical cut but that kurtosis is always
Fig. 4.45: Locations of points of interest along average HDR velocity profile and histograms of MS error distributions at points of interest for the H015 Data Case: (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
slightly higher than 3 for all error distributions. Figure 4.46 shows the value $S'_\epsilon - RMS(U)$ along the vertical cut for data case H015. The largest amounts of skewness and kurtosis that are seen for H015 are approximately 0.21 and 4.5 respectively. This demonstrates that with the absence of large amounts of shear in the flow that the distribution of errors remain Gaussian. The difference between $S'_\epsilon$ and $RMS(U)$ is much smaller for this case than any of the previous cases that have been examined. This shows that the CS uncertainty method predicts the error distribution very well for the case of developed turbulence.

Fig. 4.46: Difference between $S'_\epsilon$ and $RMS(U)$ along a vertical cut for Data Case H015.
Fig. 4.47: Comparison of $S_\epsilon$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for Data Case H015.  
(a) Calculated $S_\epsilon$ and $RMS(U)$ along cut.  
(b) Skewness of Error Distributions.  
(c) Kurtosis of Error Distributions.

**Correlation Peak Convergence for CS Method with Increasing PIV Passes**

The PIV processing schemes used to process MS data varied from 2 to 9 passes for both 50% and 75% IW overlap for data case H015. The processed MS velocity fields were compared with the overlayed HDR velocity fields and the $S_\epsilon$ and $RMS(U)$ were computed.
The value of $S_\epsilon - RMS(U)$ was computed for each number of passes used in PIV processing and plotted along a vertical cut in Fig. 4.48 for both 50% and 75% IW overlap. From this figure it can be seen that changes in the value $S_\epsilon - RMS(U)$ cease to occur at 3 passes for 50% IW overlap and 6 passes for 75% IW overlap.

In order to further investigate when convergence of the correlation peak in PIV processing occurs, the average of the absolute value of $S_\epsilon - RMS(U)$ was computed. Because there is no significant shear layer present for the developed turbulence flow, this value is only averaged over the entire MS and HDR velocity overlay. The results of this calculation are plotted against the number of passes used in PIV processing to assess when the convergence of the correlation peak is achieved and shown in Fig. 4.49. For 50% IW overlap, sufficient correlation peak convergence is achieved by 3 passes. For 75% IW overlap convergence is achieved at approximately 7 passes.
Fig. 4.48: Difference between StdDev(Error) and RMS(Uncertainty) for H015 Data Case with PIV processing that used: (a) 50% Overlap; (b) 75% Overlap.
Fig. 4.49: Average absolute value of $S - RMS(U)$ vs. number of passes in PIV processing for H015 Data Case. Where 50% and 75% IW overlap of PIV processing is used to determine when Correlation Peak convergence occurs.

**Correlation of Error Distribution and Flow Shear**

A comparison of error distribution and flow shear was performed by plotting all of the points through data case H015 along a vertical cut and performing a linear fit of the data. The results of this are shown in Fig. 4.50. Although the data is noisy, a positive correlation of error distribution and flow shear is present. This shows that even for developed turbulent flows that don’t have large shear regions, that error distribution is still correlated with the amount of shear in the flow.

**Correlation of Random Errors in Space**

For the case of H015, correlation of random error in space is performed at only one location of interest because of the lack of flow shear present in this case. The location of interest is at the center of the velocity field. The results of the correlation of random error show that correlation is not present beyond where shared particles are used to calculate neighboring vectors. It is also noted that the positive and negative directions for both the horizontal and vertical directions agree well with each other and is shown in Fig. 4.51.

When considering the correlation of random error in the horizontal direction, the results
of both the positive and negative directions are nearly identical. Beyond $\Delta i = 3$ the correlations in the horizontal directions reach and stay at near zero values. Considering the vertical directions, the positive vertical direction reaches a near zero correlation value at $\Delta i = 3$ and the negative direction reaches a near zero value at $\Delta i = 4$. Both horizontal and vertical directions show trends that demonstrates correlation of random error does not exist beyond the calculation of vectors that use the same particles for developed turbulent flow.
Fig. 4.51: Correlation of Random Error in Space in the developed turbulence region, data case H015. (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
4.4 I013 Data Case: Through-Plane Motion

This section will examine the inviscid steady flow at the exit of a rectangular jet with the added component of through-plane motion to the PIV measurement. The region of interest for this data case is shown in Fig. 3.1 as the inviscid steady flow region. For this case, the laser was rotated 16 degrees from paralleled with the flow direction so that through-plane motion was present, this setup is shown in Fig. 3.2. The same region of flow is analyzed for this case as for the inviscid steady flow, B, cases. The same potential core and large shear regions that are present in the B cases are present in this case. The seeding density of the flow for this case is 0.062 ppp and the particle image diameter is 2.6 pixels.

Examining the Distribution of Errors

A scaled and shifted HDR velocity field is shown with the selected cut location for the I013 case in Fig. 4.52. The cut location for the I013 data case is at 90 in the $x$-direction. Figure 4.53 shows that average velocity profiles of both the MS and HDR systems. It is seen that the velocity profiles match well and that there is a small bias error through the jet core present for this case. It is noticed here, as with the B and F cases, that the bias error is larger in the shear regions than in the jet core.

By selecting a few points along the vertical cut of the I013 data case, the distribution of MS error may be examined. Three points are selected, outside the flow of the jet, in the jet core, and in the shear region. Figure 4.54 visually shows the location of the selected points as well as the histogram of error distributions that are associated with each point of interest.

From the histograms in Fig. 4.54 the distribution of errors for each point of interest are examined. The first point, just outside of the jet flow, isn’t noticeably skewed or heavy-tailed. The second and third points look similar although the third point is more skewed than the second. Both of these distributions are heavy-tailed, which indicates a large number of outliers and therefore high amounts of kurtosis. It is very interesting in this case that the error distribution in the shear region and at the jet core are so similar. This may indicate that the amount of error that is introduced by through-plane motion
causes a similar amounts of outliers to be calculated as are present in high shear regions during PIV processing. In the B cases, the distribution of errors at the jet core appeared much more Gaussian than the jet core distribution for this case.

Fig. 4.52: Mapped HDR $u$ velocity field with associated cut location for data case I013.

Fig. 4.53: Average MS, HDR, and MS Error Velocities along vertical cut for Data Case I013.

Investigation of CS PIV-UQ Method Failure

The difference between $S_e$ and $RMS(U)$ is shown along the vertical cut for the I013
Fig. 4.54: Locations of points of interest along average HDR velocity profile and histograms of MS error distributions at points of interest for the I013 data case. (a) Profile of average HDR velocity along vertical cut with the location of points of interest. (b) Histograms of MS error distributions at points of interest.
case in Fig. 4.55. This figure shows that the regions with the most significant discrepancy between CS uncertainty predictions and actual error occur in the shear regions. It is interesting that $S_\epsilon - RMS(U)$ is positive through the shear regions, but is negative through the entire jet core. This indicates that the uncertainty is underestimated in regions of shear and is slightly overestimated at regions of through-plane motion, which occurs through the jet core.

Figure 4.56 shows the values of $S_\epsilon$ and $RMS(U)$ along with the skewness and kurtosis of the error distributions along the cut. In the case of through-plane motion, it is interesting that the $S_\epsilon$ and $RMS(U)$ values remain much higher than in any of the B cases in the jet core. This shows that the through-plane motion has introduced significant random error to the PIV measurements in the jet core. Similar to the B cases, the highest skewness and kurtosis values are found in the shear regions and are low for the jet core region. This is interesting because the histograms shown in Fig. 4.54 does not show significant difference between the error distributions of points located in the jet core and shear regions.

![Fig. 4.55: Difference between $S_\epsilon$ and $RMS(U)$ along the vertical cut for the I013 data case.](image_url)
Correlation Peak Convergence for CS Method with Increasing PIV Passes

After PIV processing was performed on the MS data, the mapped HDR velocity results were compared with each of the varied multi-pass processing cases. Multi-pass PIV processing was performed using 50%, 75%, and 87% IW overlap for the data case I013.

Fig. 4.56: Comparison of $S_{\epsilon}$ and $RMS(U)$ with Skewness and Kurtosis of Error Distributions for data case I013. (a) Calculated $S_{\epsilon}$ and $RMS(U)$ along cut. (b) Skewness of Error Distributions. (c) Kurtosis of Error Distributions.
Figure 4.57 shows $S_e - RMS(U)$ for both 75% and 87% IW overlap against the number of passes used in PIV processing. Both terms of Eq. 2.18 were computed for the area of MS and HDR system overlay and the difference between $S_e$ and $RMS(U)$ was calculated. The absolute value of the difference was taken and averaged over two regions of interest. The regions of interest investigated are the total MS and HDR overlay and the overlay of only the shear regions. Results of this are shown in Fig. 4.58.

It is seen that processing schemes that used a 50% IW overlap converged quickly and without much improvement of the CS uncertainty results. Correlation peak convergence for 50% IW overlap occurred after 4 passes. Figure 4.57 shows that all values of $S_e - RMS(U)$ along the cut remain unchanged after 6 passes for 75% overlap, but significant decrease ceases after 5 passes. Figure 4.58 shows that convergence of the correlation peak is achieved for 75% overlap after only 4 passes, the same as with the 50% IW overlap case. Figure 4.58 shows that convergence of the CS uncertainty results occurs at 4 and 5 passes considering the entire overlay and only shear region overlays respectively. For IW overlap of 87%, full convergence is not able to be achieved within the 9 passes used in this investigation. From Fig. 4.57, slow and steady convergence of $S_e - RMS(U)$ is observed, but full convergence is not achieved in this study. It is similarly shown in Fig. 4.58, that full convergence begins to be approached for both regions of interest that are investigated but that full convergence is not achieved. It is interesting that the 75% IW overlap processing converged so quickly, while the B011 and B013 cases required more passes for convergence to be achieved with 75% IW overlap even though through-plane motion is not present for those cases and a lower seeding density was present in the I013 case.
Fig. 4.57: Difference between StdDev(Error) and RMS(Uncertainty) for Data Case I013 with PIV processing that used: (a) 75% Overlap; (b) 87% Overlap.
Fig. 4.58: Average absolute value of $S_\epsilon - RMS(U)$ vs. number of passes used in PIV processing for I013 Data Case. Where 50%, 75%, and 87% IW overlap of PIV processing is used to determine when Correlation Peak convergence occurs for Data Case I013.

**Correlation of Error Distribution and Flow Shear**

Error distribution and flow shear are compared by plotting $S_\epsilon$ and flow shear against each other and performing a line fit through the I013 data. This is performed and shown as Fig. 4.59. A positive correlation is observed for the data case I013, indicating that correlation exists between the distribution of random error for PIV systems and the shear in the flow being considered. As seen with the B data case, the I013 data case also reaches high values of shear that are highly correlated with error distribution. It is interesting that the data shown in Fig. 4.59 appears bi-modal, although an explanation for this is not known.

**Correlation of Random Errors in Space**

For the I013 data case, correlation of random error in space is considered at two regions of interest. The regions considered are in the jet core and in the shear region. The correlation of random error at both locations of interest indicate that random error does not exist
Fig. 4.59: Comparison of $S_x$ and shear in the flow with linear fit for data case I013.

Beyond vectors that share particle images during PIV processing and is shown if Fig. 4.60 and 4.61. This is surprising because the B cases, which is the same area of interest as I013, exhibits correlated random error beyond $\Delta i = 3$. It was observed that correlation of random error values in the shear region are approximately twice the size as the correlated random error values in the jet core. This indicates that shear region has larger presence of correlated random error than the jet core.

In the center of the jet, the horizontal and vertical direction both produce correlation of random error with zero $S_{xy}$ values beyond $\Delta i = 3$. As can be seen from Fig. 4.60, at each $\Delta i$ the values of correlation in the positive and negative directions match each other very well. This occurs in both the horizontal and vertical directions, and shows that correlation of random errors does not exist beyond the vectors that share particles in the computation of the PIV processing.

The correlation of random error in the shear layer also has closely matching correlation values at each $\Delta i$ in the positive and negative horizontal and vertical directions, shown in Fig. 4.61. The horizontal directions have slightly negative values of correlation beyond
Δi = 3. This occurs more in the negative than positive horizontal direction. The vertical directions are interesting because the correlation of random error reached an initial near zero correlation value at Δi = 3 for both the positive and negative directions. It is interesting that zero correlation is obtained at Δi = 3 because this indicates that random error due to shared particles does not exist at Δi = 3. This is surprising because it was expected that correlation of random error would exist up to and including Δi = 3 because a 75% IW overlap is used in PIV processing and shared particles are used to calculate velocity vectors.

Fig. 4.60: Correlated Random Error for Data Case I013 in the jet core. (a) Location (point); (b) Correlation of Random Err in the x and y directions. (c) Correlation of Random Err in the x-directions. (d) Correlation of Random Err in the y-directions.
Fig. 4.61: Correlated Random Error for Data Case I013 in the shear region: (a) Location (point); (b) Correlation of Random Err in the $x$ and $y$ directions. (c) Correlation of Random Err in the $x$-directions. (d) Correlation of Random Err in the $y$-directions.
CHAPTER 5
Discussion of Results

This section will discuss the general observations and results of all cases examined in this work by addressing each topic of interest for all cases. The topics that will be discussed are the same as were examined in each data case: the distribution of errors at points of interest in the flow, failure of the CS PIV-UQ Method, convergence of correlation peaks in PIV processing to obtain reliable CS uncertainty estimations, error distribution correlation in PIV systems, and correlation or random errors in space.

5.1 Distribution of Errors at Locations of Interest in a Flow

An apparent bias stems from misalignment of the data sets. In the section 4.1, a manual shift was applied to the HDR system in order to determine the effect of misalignment between the MS and HDR systems. It was found that a misalignment between the two systems would not cause the skewness or kurtosis of the error distributions to be altered. It was also observed that no changes occurred in the mean position of the error distributions for the points of interest outside of the jet flow or in the jet core. But, the mean of the error distribution in the shear region moved in the negative direction for a positive $y$ shift and in the positive direction for a negative $y$ shift. It was found that the bias of the MS error distribution shifted half of the amount of applied vertical shift of the HDR system. The non-shifted histogram of case B009 is shown in Fig. 4.3 and the shifted HDR system histograms are shown in Fig. 4.6 - 4.8. Only the shear region was observed to have a bias error change due to MS and HDR velocity field misalignment. By understanding how the bias of error distributions react to an applied shift in the mapping of the HDR system, this can be used to more accurately align the MS and HDR systems.

We find that a manual shift of the HDR system effects error distributions of both shear layers. The Gaussian distribution of MS error for the upper and lower shear regions are
shown in Fig. 4.9 with applied vertical shifts of 0, 0.3, 0.5 and -0.5 pixels applied to the HDR system. By using the bias of both error distributions to identify when both shear regions are well aligned, the scaling factor that should be applied to the HDR system can be more accurately determined. Error bias in the shear regions can be explained by MS and HDR system misalignment, but still does not explain the biases in the jet core or stagnant regions. A discussion of the observed bias errors of each case is performed next.

It was observed for all cases with a large laminar shear region, the B, F and I cases, that bias error was present and significantly larger in the shear regions than in the jet core. This is shown in Figures 4.2, 4.27 and 4.53. The largest bias error exhibited for the B cases is -0.468 pixels for B011 (0.073 ppp), -0.399 pixels for B013 (0.083 ppp), and -0.398 pixels for B009 (0.093 ppp). This shows that the lowest seeding density case, B011, had a significantly larger maximum bias error than the other B cases. The I013 case, which has a seeding density of 0.062 ppp, had a maximum bias error in the shear region of -0.277 pixels, which is significantly lower than any of the B cases. The bias error in the core of case I013 was between -0.1 and -0.12 pixels. The bias errors in the jet core of the B cases are approximately -0.1 pixels. Because the calculation for bias error is the MS mean $u$ velocity subtracted from the HDR $u$ mean velocity, the negative values for bias error indicate that the MS system consistently estimates a higher velocity than the HDR system for the B and I data cases. It is possible that the bias error observed in the jet core is due to peak locking since the B cases have particle image diameters of approximately 2 pixels and the I013 case has a particle image diameter of 2.6 pixels.

The F cases showed significantly lower bias error in their shear regions with maximums of 0.062 and 0.068 pixels for cases F001 and F005 respectively. The bias error found in the jet core of these cases ranged from -0.002 to 0.012 pixels for F001 and -0.002 to -0.022 pixels for F005. It is seen with the F cases that slightly larger bias errors are present in the higher seeding density case. It is also seen that the HDR system measures a slightly higher velocity than the MS system for the F cases. Because particle image diameters for the F cases are approximately 2.5 pixels, it is possible that the small bias errors present in the
core are due to peak locking.

The H015 case, being the only data case without the presence of a shear layer, shows that the highest bias error occurs at the center of the flow for developed turbulence. The maximum bias error exhibited in the H015 data case is -0.017 pixels, which is significantly smaller than the maximum error of all other data cases. It is noted that while the H015 case exhibits a significantly smaller bias error than the rest of the cases, that it also has a slightly larger particle image diameter than the other cases, 2.8 pixels.

The error distributions at the different locations of interest have varied distributions for both the B and I cases. The inviscid steady flow cases, B and I, contain the largest values of shear in the shear regions of all flow types considered. For these cases the points of interest that were investigated are outside of the jet flow, in the jet core, and in the shear region. The error distributions for all of the B cases for both the first and second points are similar and appeared to have Gaussian distributions. However, the point of interest located in the large laminar shear region varied greatly for the B cases. The error distribution in the laminar shear region grew increasing non-Gaussian in both skewness and kurtosis as the particle seeding density of the flow decreased.

The error distributions for the F cases, inviscid unsteady region, had Gaussian distributions, even for the distribution in the shear region. The distributions of the point of interest that is located in the shear region is slightly heavier-tailed than the distributions of errors outside the jet or at the jet core. For all locations investigated in the H015 case, the developed turbulence region, the error distributions overlay well and are almost identical with Gaussian distributions.

Data case I013 investigates the effects of through-plane motion. This case has heavy-tailed error distributions in both the jet core and the shear region. It was expected that the I013 case would behave similar to the B cases because it examines the same region of flow. However, it was found that the distribution for the point outside of the jet flow was narrower for the I013 case than for any of the B cases. Even more surprising is that the error distributions for the location in the jet core and in the shear region look identical except
that the shear region error distribution has a larger negative bias. The error distribution at the jet core for the I013 case is significantly more heavy-tailed than any of the B cases. This indicates that the presence of through-plane motion has increased the amount of outliers produced, but that the distribution remained symmetric. Interestingly, the addition of through-plane motion to the MS system results in the same error distribution as the large valued shear region in I013.

It can be seen from the results of all data cases that as the magnitude of shear increases, the error distribution becomes less Gaussian. The flow with the least amount of shear, the developed turbulence case, is the most Gaussian with similar error distributions for all points of interest. As the amount of shear in the flow increases, more varied and non-Gaussian error distributions emerge due to both skewness and kurtosis. This is seen with both the inviscid unsteady flow, F cases, and the inviscid steady flow, B cases. The flow that contains the largest values of shear, the B and I013 data cases, exhibit the most variation in error distributions. The most non-Gaussian error distributions are observed at the point of interest in the shear region and is found that as the particle seeding density of the flow decreases the error distribution becomes significantly less Gaussian by an increase in both skewness and kurtosis. Error distributions are found to be only slightly non-Gaussian and heavy-tailed in the jet core when through-plane motion is present for 2C PIV.

5.2 Investigation of CS PIV-UQ Method Performance

The quantity \( S_\epsilon - RMS(U) \) is used to determine the regions where either the CS method does not accurately predict the actual error of the MS PIV system or the metric used to compare error and uncertainty, Eq. 2.18, fails. Equation 2.18 shows that the \( S_\epsilon \) and \( RMS(U) \) should be equal when uncertainty is accurately predicted and error distributions are not skewed or contain a bias \[16\]. Figures 4.10, 4.30, 4.46, and 4.55 show the values of \( S_\epsilon - RMS(U) \) for the B, F, H, and I data cases, respectively.

It is observed that the discrepancy between \( S_\epsilon \) and \( RMS(U) \) increases with flow shear. The developed turbulence case, H015, which does not contain a high shear region, has the lowest values of \( S_\epsilon - RMS(U) \) along a vertical cut. The largest magnitude value of
$S_ε - RMS(U)$ for case H015 is -0.0046 pixels. The flow cases with the next lowest amount of shear present are the F cases. The values of $S_ε - RMS(U)$ increase in value for the F cases in the laminar shear regions. The largest magnitude values of $S_ε - RMS(U)$ for the F001 and F005 cases are 0.01 and 0.008 pixels respectively. Note that the higher seeding density of the F cases had a slightly smaller $S_ε - RMS(U)$ value.

Considering the flow cases with the largest amounts of laminar shear, B and I013 cases, it again is true that the highest discrepancy between $S_ε$ and $RMS(U)$ occurs in the high shear regions. It is interesting for these cases that the value of $S_ε - RMS(U)$ not only is largest in the shear regions, but also increases as particles seeding density of the flow decreases. It is noted that lower seeding density provides less information for the CS method to use, which means that the CS method will not perform as well in low seeding density circumstances [1]. The largest values of $S_ε - RMS(U)$ experienced for B009 is 0.0259 pixels, B013 is 0.0727 pixels, and B011 is 0.1816. These values correspond with the flow seeding densities of 0.073, 0.083, and 0.093 ppp respectively. The maximum magnitude value of $S_ε - RMS(U)$ for the case I013 is 0.03 pixels, which corresponds to a seeding density of 0.062 ppp. In the data case I013, the value $S_ε - RMS(U)$ is steady through the jet core, with an average value of -0.01 pixels. It is interesting that the uncertainty prediction is overestimated in the jet core for the I013 case, but is underestimated in regions of high shear.

The values of $S_ε - RMS(U)$ were used to recognize the locations of jet flow that either the CS method fails to accurately predict error distributions or the metric, Eq. 2.18, fails. We now discuss reasons for this by examining the distributions or error at those locations. The error distribution is analyzed using the measures of skewness and kurtosis in order to evaluate how Gaussian the error distribution is.

It is seen for all data cases in this study, that skewness and kurtosis of error distributions are largest in shear regions of the flow. This corresponds with the previous observation that discrepancies exist between $S_ε$ and $RMS(U)$ in the shear regions of all inviscid steady and inviscid unsteady cases. The metric used to determine the quality of the results produced
by the CS method assumes that errors stem from Gaussian distributions with zero mean values [4]. Further investigation of the metric used to compare error and uncertainty, Eq. 2.18, revealed that the metric is not valid when error distributions are skewed [16]. This means that the metric used to determine accuracy of the CS method may fail when bias errors are present or error distributions are not symmetric.

In cases that there exists a discrepancy between $S_\epsilon$ and $RMS(U)$ outside of the shear region, or in the case of H015 that does not have a shear region, it is also generally found that larger values of at least skewness or kurtosis exist. Exceptions to this occur for data cases F005, B009, B013, and I013 in the jet core. For these cases, it is observed that there is a discrepancy between $S_\epsilon$ and $RMS(U)$ at the jet core, but values of skewness and kurtosis show that the error distributions are Gaussian. The opposite is observed for data case B011, in that the $S_\epsilon$ and $RMS(U)$ match very well in the jet core but the skewness of the error distribution is fairly large, with a value of -1. Because of these nuances, it is determined that when the CS uncertainty prediction fails in the region of the jet core that it is due to reasons other than that of non-Gaussian error distributions. It is also possible that the reason for the failure at the jet core is not due to the CS method, but rather due to the fact that the comparison of Eq. 2.18 is only valid when a zero bias error and skewed error distributions are not present. All cases that show a failure of the CS method in the jet core also have at least a slight bias in the jet core. This makes it possible that it is not the CS method that fails, but rather the measurement used to evaluate the CS method fails. It is also noted in the case of H015, which does not have a shear region, that although the $S_\epsilon$ and $RMS(U)$ match well the skewness and kurtosis have general averages of -0.1 and 3.4 respectively. But, when large discrepancies appear between $S_\epsilon$ and $RMS(U)$ the skewness and kurtosis still peak at much larger values.

5.3 Impact of Multipass Convergence on CS Method

As more passes are used in multi-pass PIV processing, convergence of the image correlation peak is improved. In order to produce reliable uncertainty predictions using the CS PIV-UQ Method, it is necessary for the correlation peak to be "sufficiently converged" [1].
In this work, we investigate how many passes are required in order for the correlation peak to be sufficiently converged so that reliable CS uncertainty results are rendered. This investigation is performed for all data case using 50% and 75% IW overlap. This investigation is also performed with an 87% IW overlap for the B009 and I013 cases. Multi-pass processing of MS PIV data was performed for 2-9 passes for each IW overlap considered. Convergence of the correlation peak is achieved when $S_\epsilon$ and $RMS(U)$ of the MS data match. The average of the absolute value of $S_\epsilon - RMS(U)$ is performed over the MS and HDR overlay. When this converges to a value, convergence of the correlation peak has also been achieved.

We find that more IW overlap used in PIV processing, requires more passes for convergence to be achieved. Therefore, the PIV processing that used 50% IW overlap converged much more quickly than either the 75% or 87% IW overlap. It is also interesting to note that the average of the absolute value of $S_\epsilon - RMS(U)$ achieved lower values, except for the F001 case, as larger IW overlap was applied. This shows that better uncertainty estimations can be achieved with larger IW overlap, although it is also more computationally expensive to achieve. It was surprising how quickly the 50% IW overlap converged. Convergence of the PIV correlation peak occurred after only 3 passes for cases B011, F001, and H015. All other data cases, B009, B013, F005, and I013 achieved correlation peak convergence after 4 passes with 50% IW overlap.

For multi-pass processing with 75% IW overlap, convergence generally took longer than for 50% IW overlap. The earliest correlation peak convergence for 75% IW overlap occurred for data case B009 at only 3 passes. Four passes were required for data case I013, and 5 passes were required for data cases B013, F001, and F005. The data cases that took the most amount of passes for convergence to be achieved with 75% IW overlap were the B011 and H015 data cases. These cases converged after 7 passes. It is interesting for the B data cases, that for lower seeding density more passes were required for convergence. For instance, case B011, having a seeding density of 0.073 ppp, required 7 passes for convergence while the B013 case required 5 passes and the B009 case required only 3 passes for convergence. Where the B013 and B009 cases have seeding densities of 0.083 and 0.093 ppp respectively. More
passes are necessary to achieve converged CS uncertainty results when low seeding particle
seeding density is present in PIV data. This is due to the fact that the CS uncertainty
method has less information to determine uncertainty and as a result more passes are
required [1]. Convergence of all data cases is shown for PIV processing using 75% IW
overlap in Fig. 5.1.

For the two cases that were processed with 87% IW overlap, cases B009 and I013, full
convergence was not achieved within 9 passes. Although convergence of the correlation peak
was approached for both of these cases, neither achieved a fully converged result during this
study. The incredible computational cost of large overlap values prevent further study of
this issue.

5.4 Proposed Model of PIV Error Accounting for Shear and Particle Density

The distribution of the MS error was used to determine if a correlation exists between
error and other variables of PIV systems. The model presented by McClure et al [5] sug-
gested that error distribution scaled with shear of the flow and dynamic range of the system.
By examining the error distributions for various data case obtained by Neal et al [3], a new
suggested model for predicting error distribution for PIV systems is presented. It was found
that error distribution is a function of both flow shear and particle seeding density.

In all data cases considered, even the case of developed turbulence, it was found that
error distribution was correlated positively with flow shear. By examining the B and F
cases it was also shown that for lower particle seeding densities that the error distribution,
$S_e$, reached larger values. The trend of increased error distribution, $S_e$, with low particle
density is shown for both the B and F cases in Fig. 4.17 and 4.36 respectively. In order to
properly compare the correlation of random error with flow shear for all data cases, all of
the linear fits for each data case are shown together in Fig. 5.2. Although there is variation
in how much the slopes change due to particle seeding density between the B and F cases,
it is shown consistently that a lower seeding density results in a steeper slope for B and F
cases. This is also shown in the work of Timmons et al [9].

For all data cases examined, correlation between error distribution magnitude and flow
Fig. 5.1: Average absolute value of $S_e - RMS(U)$ vs. number of passes used in PIV processing for all data cases. Where 75% IW overlap of PIV processing is used to determine when correlation Peak convergence occurs. (a) B and I Cases. (b) F and H Cases.
shear was observed. It is interesting that the fit slopes shown in Fig. 5.2 were all generally similar, even for the cases of developed turbulence, case I015, and through-plane motion, case I013.

The B and F cases can relate both flow shear and seeding density by Eq. 4.1, $S_{P\epsilon} = mJ + b$, in order to produce a predicted error distribution, $S_{P\epsilon}$. This model produces a predicted error distribution by using $m$ values from fits of Fig. 4.18 for the B cases and Fig. 4.37 for the F cases, that correspond with particle seeding density. These figures relate the slope of the error and shear correlation with particle seeding density. The fits of these figures are very different for the B and F cases. The slopes of the B and F cases are -29.4 and -1.58 respectively, and the intercepts are 2.86 and 0.19 respectively. A possible explanation for the large differences between the B and F cases is that the response of the relationship between correlation slope and particle seeding density is non-linear for the considered jet regions. The results of $S_{P\epsilon}$ from Eq. 4.1 compare well with $S_{\epsilon}$, and are shown for the B and F cases in Fig. 4.19 and 4.38 respectively.
du/dy of HDR System

StdDev(Err) (pixels)

(a)

(b)

Fig. 5.2: Linear fits of all data cases showing the correlation between $S_e$ and flow shear. (a) Showing all Data Case Linear fits. (b) Figure showing Linear fits of all Data Cases, zoomed in to show details of F and H cases.
5.5 Correlation of Random Error in Space

With the use of interrogation window overlap in PIV processing, the same particle images are used in calculating multiple velocity vectors. The PIV processing for both the MS and HDR systems in this study use an interrogation window overlap of 75%. For 75% IW overlap, without image distortion, the same particle images are used for the calculation of four neighboring velocity vectors. Because particle images are shared for those vectors, if a non-ideal particle causes an error in PIV processing it will result in an error in all velocity vectors produced with that particle. This will result in a correlation of random errors in space to be present.

An anchor vector location is selected and the correlation of random error in space is computed between the errors at the anchor location and up to 6 neighboring errors using Eq. 2.11. Correlation of random error in space is computed both at the jet core and in the shear region for cases with a shear region.

The magnitude for correlation of random error varied depending on the location of interest at which flow cases were being examined. The B data cases, with the location of interest in the jet core, had correlation magnitudes of $10^{-3}$, while in the shear region had a magnitude of $10^{-2}$ and $10^{-1}$ depending on the seeding density of the case. The F cases experienced correlation of random error magnitudes of $10^{-3}$ in both the jet core and shear regions. The H015 and I013 cases also experienced a correlation magnitude of $10^{-3}$ in both the jet core and shear regions.

The correlation of random error in space computed at the center of the jet was computed for all data sets and varied results were observed for different data cases. It was observed for all cases, at the jet core location, that correlation of random error leveled off at $\Delta i = 4$. However, different minimal values of correlated random error were found. For the B009, B011, and B013 cases the correlation leveled off at values of 0.003, 0.002, and 0.005 pixels$^2$, while all other data cases leveled off at a near zero correlation value. The variation of where the B cases leveled off seem to be random, as they do not correlate with the seeding density of the cases. When considering the correlation of random error in the vertical directions, the
B cases do not converge, but steadily decrease to a zero correlation values at $\Delta i = 6$. This occurs for B009 in all directions, B013 in all directions, and B011 in the negative vertical direction. The B011 case in the positive vertical direction reaches a minimum correlation value of 0.001 at $\Delta i = 5$ and then increases again. For all other data cases, zero correlation values are reached at $\Delta i = 4$.

For all cases except H015, the correlation of random error in space was also computed in the shear region. Data case H015 was excluded from this because it examines a developed turbulent flow that does not have a significant shear region. It was observed that the magnitude of which correlation of random error existed in the shear region was larger than in the jet core, particularly for the B data cases. For all data cases except for B011 in the horizontal directions, the correlation of random error leveled off once a zero correlation value was reached. Data case B011, in the horizontal directions, reach a near zero value at $\Delta i = 6$. For all other data cases in the horizontal directions, the correlation of random error reached zero values at $\Delta i = 4$ or 5. In the vertical directions, the correlation of random error reached zero values at $\Delta i = 3$ or 4.
CHAPTER 6
Conclusion

6.1 Error Distributions

For each data case used in this study the error distribution of multiple points of interest through the flow were observed. It was observed that for the case of developed turbulence that the error distributions at all points were identical and that Gaussian error distributions are present. For flow cases that have high shear values, the B and I013 cases, large variation in error distributions are observed between the points of interest located outside the jet flow, in the jet core, and in the shear region. For all B data cases the error distributions for locations outside the jet and in the jet core remain mostly Gaussian. However, for the B and I data cases, the error distribution for the location in the shear region are non-Gaussian. For the B cases, the error distribution become increasingly non-Gaussian as particle seeding density of the flow decreases. For the I013 case, in addition to a non-Gaussian error distribution in the region of high shear the error distribution was also non-Gaussian at the jet core, where error due to through-plane motion is present.

It is concluded that error distributions are generally Gaussian except for when high levels of flow shear or through-plane motion are present in the flow. It is also noted that the severity of non-Gaussian error distributions increases as particle seeding density of the flow decreases. It was observed that the error distributions in the shear region grew increasingly non-Gaussian in both skewness and kurtosis as the particle seeding density of the flow decreased.

6.2 Performance of CS PIV-UQ Method

It was found that either the CS method or the metric used to compare error and uncertainty fail to accurately predict error in regions of high laminar flow shear where the
error distributions are non-Gaussian. By the error distribution being non-Gaussian, the predicted uncertainty bands produced by the CS method do not contain the actual error accurately. Because it is observed that the CS PIV-UQ method seems to fail when non-Gaussian error distributions are present, this shows that a correlation between non-Gaussian error distributions and CS PIV-UQ method failure exists, but is not necessarily direct caused by the non-Gaussian error distributions. It is worth noting that the CS method produces more robust uncertainty estimations when larger particle seeding densities are used.

It was also noticed for some data cases that in the region of the jet core that the CS method still does not accurately predict the error distribution, even though error distributions are Gaussian. It is possible that the reason for the failure at the jet core is not due to the CS method, but rather due to the fact that the metric used to compare error and uncertainty, Eq. 2.18, is only valid when a zero bias error and symmetric error distributions are present. All cases that show a failure of Eq. 2.18 in the jet core also have a bias in the jet core. This makes it plausible that it is not the CS method that fails, but rather the metric used to evaluate the CS method fails due to the presence of bias.

6.3 Impact of Multipass Convergence

From the present work, it was determined that PIV correlation peak identification reaches convergence when using 50% IW overlap after 4 passes for the flows considered, which include large shear regions, developed turbulence and through-plane motion. When multi-pass processing was performed with 75% IW overlap, convergence was reached between 3 and 7 passes depending on the data case. When using 75% IW overlap, 6 passes is generally optimal to obtain a sufficiently converged correlation peak for most data cases. For 87% IW overlap, after processing 9 passes for data cases B009 and I013, although convergence was approached full convergence was never achieved. This shows that more than 9 passes are required in order to obtain converged correlation peaks when an IW overlap of 87% is used. It was observed for the inviscid steady flow, B data cases, that the lower the seeding density of the flow, the more passes were necessary for correlation peak convergence to be achieved in PIV processing. This means that additional passes may be required when
processing is performed on low seeding density PIV data.

6.4 Proposed Error Model

The work performed in this study indicates that error distributions for PIV systems scale with both shear of the flow and particle seeding density. The equation that was developed in this work and is suggested to predict error distribution is given as Eq. 4.1. Because the relationship that links particle density is sensitive to the flow type, it is necessary to use this model with appropriate $m$ relationships to accurately predict error distribution with the model presented.

6.5 Correlation of Random Error in Space

Correlation of random error in space was found to exist for all cases examined. It was observed that the correlation of random error in space decreased in a linear fashion until a minimum correlation of random error was reached. Correlation of random error was found to be roughly twice the value in shear regions of a flow than in turbulent or laminar regions. The correlation of random error in the shear region of the inviscid steady flow cases were found to be one to two orders of magnitude larger than at the jet core. This shows that regions of strong flow shear have more correlated random error present. When investigating the correlation of random error at the jet core, it was found that correlated random error existed beyond $\Delta i = 3$ only for the inviscid steady flow cases in the vertical directions. For all other cases and directions at the jet core, residual correlated random error did not exist beyond $\Delta i = 4$. This indicates that extended correlation or random error is only present in the vertical directions for the inviscid steady flow cases. It was also found that in the shear region that correlation values behaved more as expected in both the horizontal and vertical directions by exhibiting near zero correlation values beyond $\Delta i = 3$. 
REFERENCES


APPENDIX
March 2, 2018

Jaron Howell
MAE Graduate Student
Utah State University

Dear Dr. Neal

I am in the process of preparing my Thesis in the Mechanical Engineering department at Utah State University.

I am requesting your permission to include figures from your work, *Collaborative framework for PIV uncertainty quantification: the experimental database*. I will include acknowledgments and appropriate citations to your work and copyright and reprint rights information in a special appendix. The bibliographic citation will appear at the end of my manuscript.

Please indicate your approval of this request by signing in the space provided, attaching any other form or instruction necessary to confirm permission. If you have any questions, please call me at the number below.

I hope you will be able to reply quickly.

Thank you for your cooperation,

Jaron Howell
801-712-2286

I hereby give permission to Jaron Howell to reprint the material in his thesis for the following material.


- **Figure 1.** The experimental setup (left: schematic view)
- **Figure 11.** Magnitude of the x- and y-Reynolds stress and overview of the measurement regions.
- **Figure 12.** Experimental configuration to study the effects of mean out-of-plane motion (left: perspective view; right: top view)

Signed: _____________________________

Douglas Neal, Ph.D.