A Comparison of Rank and Bootstrap Procedures for Completely Randomized Designs with Jittering

Feng-ling Lee
Utah State University

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A COMPARISON OF RANK AND BOOTSTRAP PROCEDURES
FOR COMPLETELY RANDOMIZED DESIGNS
WITH JITTERING

by
Feng-ling Lee

A thesis submitted in partial fulfillment
of the requirement for the degree
of
MASTER OF SCIENCE
in
Applied Statistics

UTAH STATE UNIVERSITY
Logan, Utah
1987
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I would like to thank my major professor, Dr. David L. Turner for his encouragement and help above and beyond the call of duty. In addition, my gratitude goes to my committee members, Dr. Ronald V. Canfield and Dr. Robert Campbell.

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I would also like to express my sincere thanks to my parents for their encouragement and support in my graduate studies.

Feng-Ling Lee
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ABSTRACT

A Comparison of Rank and Bootstrap Procedures for Completely Randomized Designs with Jittering

by

Feng-Ling Lee, Master of Science Utah State University, 1987

Major Professor: Dr. David L. Turner Department: Applied Statistics

This paper discusses results of a computer simulation to investigate the effect of jittering to simulate measurement error. In addition, the classical F ratio, the bootstrap F and the F for ranked data are compared. Empirical powers and p-values suggest the bootstrap is a good and robust procedure and the rank procedure seems to be too liberal when compared to the classical F ratio.

(74 Pages)
CHAPTER I
INTRODUCTION

The Purpose for This Thesis

The purpose for this report is to compare several analysis techniques for a completely randomized design: the traditional analysis of variance (ANOVA) as developed by Fisher (1954), the ANOVA on ranks as suggested by Conover and Iman (1981), and the Bootstrap technique developed by Efron (1982). In addition, robustness of these techniques will be investigated, when the data may be inaccurate or when there may be errors in observations as simulated by jittering the data (Chambers et. al. 1983). These problems will be investigated all in the context of a 3 group completely randomized design.

Definition of Analysis Methods

The basic technique used for this study is the analysis of variance for a completely randomized design with \( n \) observations in each of \( k \) groups. This result in an F statistic which is the ratio \( (\text{MSTreatment}/\text{MSError}) \), where

\[
\text{MSTreatment} = \left( \frac{\sum_{i=1}^{k} \left( \sum_{j=1}^{n_i} Y_{ij}^2 \right) - \left( \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} Y_{ij}^2}{nk} \right)^2}{k-1} \right)
\]

and

\[
\text{MSError} = \left( \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} \left( Y_{ij} - \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} Y_{ij}}{nk} \right)^2}{k(n-1)} \right)
\]

One nonparametric variation suggested by Conover and Iman (1981) is to replace each observation by its rank in the aggregate sample. An F statistic is then calculated for the ranks.

The Bootstrap is a recent alternative introduced by Efron (1982).
This is a nonparametric resampling procedure in which a large number of samples with replacement are selected from the original data. The basic assumption of the bootstrap procedure is that the underlying unknown population distribution function is very well approximated by the sample or empirical distribution function. If this is true, then sample statistics computed for samples from this empirical distribution, the empirical distribution of the sample test statistics for the observed samples, should closely approximate the results which would be obtained by sampling from the true distribution function. Each of these samples is then analyzed.

The empirical distribution of the sample statistic under investigation is the traditional ANOVA F statistic in this paper. The F statistic for the original sample is then compared to the distribution of bootstrap sample F ratios to determine a p-value, the probability of a more extreme result. Each bootstrap sample was also subjected to the rank procedure and bootstrap distributions for the rank F's were also found.

To check the original data F ratios, the rank procedure and the bootstrap procedure for robustness under measurement errors, the data was jittered and then analyzed. The jittering was done by adding "noise" to the data to simulate measurement error. Jittering is sometimes useful to help display discrete data sets as in Figures 1 and 2. Figure 2 presents a jittered version of Figure 1, where a small random error has been added(or subtracted) from each observation.
Figure 1
Plot of Nonjittered Data

Figure 2
Plot of Jittered Data
The Basic Data for The Study

The basic data for this study was generated for different 3 group CRD designs which would have a range of powers. The charts in Dixon and Massey (1969) were used to find \( u_i \)'s for the selected powers ranging from 0.1 through almost 1.0. These charts require the parameter

\[
\phi^2 = \frac{\sum_{i=1}^{k} (u_i - \bar{u})^2}{k}/(\sigma^2/n) \tag{3}
\]

where the \( k \) different \( u_i \)'s are the population means involved. If \( \phi^2 \) is equal to zero, then the null hypothesis of equal group means is true. Dixon and Messay's \( \phi^2 \) parameter is related to the noncentrality parameter discussed by Graybill(1976),

\[
\lambda = \frac{\sum_{i=1}^{k} (u_i - \bar{u})^2}{\sigma^2(k-1)/n} \tag{4}
\]

by the function \( \phi^2 = \lambda (k-1)/k \). The noncentrality parameter \( \lambda \), the degrees of freedom for numerator and degrees of freedom for the denominator are required to completely specify the distribution of the F statistic.

Once \( \phi^2 \) is chosen to achieve a particular power, then 3 \( u_i \)'s were selected by trial and error to give the appropriate \( \phi^2 \). The selected powers and the corresponding \( \phi^2, \lambda \) and \( u_i \)'s are listed in Table 1.

Although the population variance certainly influences F values, it was not considered as a factor for this study, i.e. \( \sigma^2 \) was set to 9 for all data generated for this study. Given the selected \( u_i \)'s and \( \sigma^2 = 9 \), 100 data sets were generated for each selected power and then the various F ratios, jittering, etc. were computed for each sample.
Table 1

Selected Values of $\phi^2$, $\lambda$, $u_1$, $u_2$, and $u_3$ to Achieve the Indicated Powers when $\sigma^2 = 9$ and $\alpha = 0.05$

<table>
<thead>
<tr>
<th>Power $(1 - \beta)$</th>
<th>$\phi^2$</th>
<th>$\lambda$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>4.796</td>
<td>7.168</td>
<td>10.54</td>
<td>8</td>
<td>5.46</td>
</tr>
<tr>
<td>0.8</td>
<td>3.610</td>
<td>5.427</td>
<td>12.21</td>
<td>10</td>
<td>7.79</td>
</tr>
<tr>
<td>0.7</td>
<td>2.890</td>
<td>4.356</td>
<td>14.98</td>
<td>13</td>
<td>11.02</td>
</tr>
<tr>
<td>0.6</td>
<td>2.250</td>
<td>3.364</td>
<td>13.74</td>
<td>12</td>
<td>10.26</td>
</tr>
<tr>
<td>0.5</td>
<td>1.932</td>
<td>2.916</td>
<td>12.62</td>
<td>11</td>
<td>9.38</td>
</tr>
<tr>
<td>0.4</td>
<td>1.488</td>
<td>2.240</td>
<td>10.42</td>
<td>9</td>
<td>7.58</td>
</tr>
<tr>
<td>0.3</td>
<td>1.232</td>
<td>1.973</td>
<td>8.29</td>
<td>7</td>
<td>5.71</td>
</tr>
<tr>
<td>0.2</td>
<td>0.723</td>
<td>1.089</td>
<td>11.99</td>
<td>11</td>
<td>10.01</td>
</tr>
<tr>
<td>0.1</td>
<td>0.423</td>
<td>0.642</td>
<td>10.76</td>
<td>10</td>
<td>9.24</td>
</tr>
</tbody>
</table>
CHAPTER II
DATA GENERATION AND VALIDATION

Simulation Program

The simulation program was first written in Turbo Pascal for an IBM PC. Since it took more than 5 hours for the PC to generate the 100 samples and the associated bootstrap samples for a single set of $u_i$'s, the program was converted to Pascal for the VAX 11/780 at USU. This decreased the time per run to about 20 minutes of VAX CPU time.

For a particular run, the three group means necessary to obtain a particular power, $\lambda$ or $\phi^2$ from Table 1 were input. While differences in variances would undoubtedly influence the outcome of tests, this report used $\sigma^2 = 9$ for all runs.

Generate Original Data

To generate the original data, pairs of independent uniform(0,1) random numbers were generated and then transformed to pairs of independent standard normal random numbers using the polar coordinate transformation of Muller(1958). If $W_1$ and $W_2$ are independent uniform random variables, then

$$Z_1 = (-2 \ln W_1)^{0.5} \cos(2 \pi W_2)$$

(5)

and

$$Z_2 = (-2 \ln W_1)^{0.5} \sin(2 \pi W_2)$$

(6)

will be independent standard normal random variables.

These independent standard normal random numbers were then transformed to the CRD data set by multiplying each by 3, the assumed population standard deviation, and then adding $u_i$, the appropriate
population mean.

These original samples were analyzed using the usual F ratio. Since the $u_i$'s are not all equal, the F ratios follow the F' or noncentral F distribution. These original F's were examined to see if the simulation was generating data which really did follow the appropriate noncentral F' distribution.

The expectation of a noncentral F' is

$$E(F') = \frac{\nu_2}{\nu_2 - 2} \left( 1 + \frac{2\lambda}{\nu_1} \right)$$

and the variance of F' is

$$V(F') = \frac{2\nu_2^2}{\nu_1^2(\nu_2 - 2)} \left[ \frac{(\nu_1 + 2\lambda)^2}{(\nu_2 - 2)(\nu_2 - 4)} + \frac{\nu_1 + 4\lambda}{\nu_2 - 4} \right]$$

For this study, the $\lambda$'s were known since they were a function of the $u_i$'s and had been used to generate the data. Hence $E(F')$ and $V(F')$ are listed in Table 2 along with the sample averages and variances obtained.

Figures 3 and 4 plot F' against $E(F')$ and the $S_{F'}^2$ against $V(F')$ for the 100 samples taken for each power. From the plots, the F's are very close to $E(F')$ and $S_{F'}^2$ values are also close to $V(F')$, indicating that the original F ratios do follow F' distribution.

QQ Plots and F' Score for Original Data

Another way to assess the "closeness" of one distribution to another is by using quantile-quantile QQ plots, as suggested in Chambers et. al.(1983). A theoretical QQ plot is obtained by plotting the order statistics or quantiles of one distribution or data set against the theoretical order statistics or quantiles from the
Table 2

Average and Expected F' Values, Sample and Population Variances of F' for the Original Data for 100 Samples from the Simulated CRD Experiments for Indicated Powers

<table>
<thead>
<tr>
<th>Power (1-β)</th>
<th>Average F' values F</th>
<th>Expected F' values E(F')</th>
<th>Sample variances $s^2_F$</th>
<th>Population variances $V(F')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>8.769</td>
<td>8.821</td>
<td>24.266</td>
<td>26.210</td>
</tr>
<tr>
<td>0.8</td>
<td>6.912</td>
<td>6.941</td>
<td>21.326</td>
<td>19.218</td>
</tr>
<tr>
<td>0.7</td>
<td>6.293</td>
<td>5.784</td>
<td>15.008</td>
<td>15.223</td>
</tr>
<tr>
<td>0.6</td>
<td>4.660</td>
<td>4.713</td>
<td>8.626</td>
<td>11.729</td>
</tr>
<tr>
<td>0.5</td>
<td>4.392</td>
<td>4.229</td>
<td>7.596</td>
<td>10.217</td>
</tr>
<tr>
<td>0.4</td>
<td>3.209</td>
<td>3.500</td>
<td>5.369</td>
<td>8.014</td>
</tr>
<tr>
<td>0.3</td>
<td>3.212</td>
<td>3.211</td>
<td>7.779</td>
<td>7.168</td>
</tr>
<tr>
<td>0.2</td>
<td>2.493</td>
<td>2.256</td>
<td>4.080</td>
<td>4.472</td>
</tr>
<tr>
<td>0.1</td>
<td>1.815</td>
<td>1.773</td>
<td>2.752</td>
<td>3.169</td>
</tr>
</tbody>
</table>
Figure 3

Average and Expected F' Values for 100 Original Samples from the Simulated CRD Experiments for Indicated Powers.
Figure 4
Sample and Expected Variances for 100 Original Samples from the Simulated CRD Experiments for Indicated Powers.
theoretical distribution or against the order statistics or quantiles of another data set.

Three steps are required to make QQ plots to compare a data set to a theoretical distribution. First, the order statistics are formed by sorting the original data into ascending order. Next, quantiles or order statistics for the chosen or target theoretical distribution or second sample are computed. Finally, a plot of the sorted data from the original sample against the corresponding quantiles or order statistics of the theoretical distribution or second sample should be a straight line if the data follows the target distribution, or if the two samples come from the same distribution. Departures from a straight line indicate different types of departures of the two distribution. For further discussion and examples, see Chambers et. al.(1983).

To find the quantiles of the order statistics, David(1981) shows that the $i^{th}$ order statistic from large samples is approximately the $100[i/(n+1)]^{th}$ percentile.

For this study, $F'$ score plots were needed, which requires approximating the inverse distribution function for $F'$. Formula (26.6.25) from Abramowitz and Stegun(1965) approximates the noncentral $F$ distribution using a 2 moment approximation with the central $F$ distribution,

$$ P[F'_{v_1},v_2 < x'] \approx P[F_{v_1},v_2 < x] $$

where

$$ v_1 = \frac{(v_1 + \lambda)^2}{v_1 + 2\lambda} $$

$$ x = \left( \frac{v_2}{v_1 + \lambda} \right) x' \implies x' = x \left( \frac{v_1 + \lambda}{v_1} \right) $$

As an example, the Minitab commands to generate the theoretical
percentiles for power \((1 - \beta) = 0.9\), \(\lambda = 7.168\), \(v_1 = 2\) and \(v_2 = 27\) using this approximation, assuming the original F ratios are in column Cl are listed below.

```plaintext
# form the sample order statistics
Sort cl into c1
# generate the theoretical percentiles
Set c31
1:100
Let c31 = c31/101
# 7.168 is the noncentrality parameter
Let k1 = (2+7.168)**2 / (2+2*7.168)
Invcdf c31 into c41;
F k1,27.
# form the theoretical percentiles
Let c41 = c41*(2+7.168)/2
# distribution of F'
Plot cl c41
# straight line indicates good fit
```

Figures 5, 6 and 7 show such F' score plots for powers of 0.1, 0.5 and 0.9 respectively. The relatively straight lines indicate that the F ratios calculated for the original data follow the appropriate noncentral F distribution fairly well. For \((1 - \beta) = 0.9\) and 0.5, there appears to be a slight "hook" at the upper ends of the F' score plots. This may be due to problems with the random number generator or the F' approximation. This slight departure was not deemed serious.

**Empirical Powers for Original Data**

Another way to check on the validity of the simulation is by comparing the empirical power with the hypothetical power. The empirical power is defined to be the proportion of calculated F's which exceed the 95\(^{th}\) percentile of the central F distribution based on 2 and 27 degrees of freedom. From this plot, it appears that the empirical powers are close to but not exactly equal to those for an \(\alpha = 0.05\) test. This is likely due to the graphic method used to read
Figure 5

F' Score Plots for F Ratios from Original Samples for Power (1- \( \beta \)) = 0.9. The Fairly Straight Line Indicates that the F' Values Calculated for the Original Samples are Close to the Target Noncentral F Distribution with \( \lambda = 7.168 \), \( v_1 = 2 \) and \( v_2 = 27 \).
F' Score Plots for F Ratios from Original Samples for Power \((1 - \beta) = 0.5\). The Fairly Straight Line Indicates that the F' Values Calculated for the Original Samples are Close to the Target Noncentral F Distribution with \(\lambda = 2.916\), \(v_1 = 2\) and \(v_2 = 27\).
$F'$ Score Plots for $F$ Ratios from Original Samples for Power (1-\(\beta\)) = 0.1. The Fairly Straight Line Indicates that the $F'$ Values Calculated for the Original Samples are Close to the Target Noncentral $F$ Distribution with $\lambda = 0.642$, $v_1 = 2$ and $v_2 = 27$. 
\[ \phi^2 \] from the charts in Dixon and Massey (1969) for \( \alpha = 0.05 \) tests.

If the observed F ratios are compared to 90\textsuperscript{th} or 99\textsuperscript{th} percentiles, then it appears in Table 3 and Figure 8 that the empirical power curves for \( \alpha = 0.1 \) and \( \alpha = 0.01 \) tests are, as expected, less and more powerful respectively than the \( \alpha = 0.05 \) test.

The empirical powers for \( \alpha = 0.05 \) are listed in Table 4, and plotted in Figure 9. Table 4 and Figure 9 have a column or line for the original F' ratios, the F ratios calculated for the ranked data, the F ratios calculated for the jittered data, the F ratios calculated for the ranks of the jittered data and the F ratios calculated for the bootstrap data. These are plotted in Figure 10 as deviations from the theoretical power values. The line for empirical powers for the original F ratios in Figure 10 indicates some departures from the theoretical values. Again these departures were not viewed as serious problems and likely due to graphical interpolation errors.
Table 3

Empirical Powers for Original F' Using 
\( \alpha = 0.1, 0.05 \) and \( 0.01 \) Tests

<table>
<thead>
<tr>
<th>((1- \beta))</th>
<th>(\alpha = 0.1)</th>
<th>(\alpha = 0.05)</th>
<th>(\alpha = 0.01)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.94</td>
<td>0.88</td>
<td>0.70</td>
</tr>
<tr>
<td>0.8</td>
<td>0.89</td>
<td>0.76</td>
<td>0.51</td>
</tr>
<tr>
<td>0.7</td>
<td>0.79</td>
<td>0.68</td>
<td>0.39</td>
</tr>
<tr>
<td>0.6</td>
<td>0.80</td>
<td>0.63</td>
<td>0.39</td>
</tr>
<tr>
<td>0.5</td>
<td>0.59</td>
<td>0.46</td>
<td>0.17</td>
</tr>
<tr>
<td>0.4</td>
<td>0.55</td>
<td>0.44</td>
<td>0.16</td>
</tr>
<tr>
<td>0.3</td>
<td>0.53</td>
<td>0.37</td>
<td>0.18</td>
</tr>
<tr>
<td>0.2</td>
<td>0.40</td>
<td>0.26</td>
<td>0.09</td>
</tr>
<tr>
<td>0.1</td>
<td>0.24</td>
<td>0.16</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Table 4

Empirical Powers Defined as the Proportion of 100 Sample F Ratios which Exceed $F_{0.05, 2.27} = 3.35$ for $\alpha = 0.05$ Tests for Five Different F Ratios

<table>
<thead>
<tr>
<th>(1- $\beta$)</th>
<th>Original $F'$</th>
<th>$F$ for ranked data</th>
<th>$F$ for jittered ranked data</th>
<th>$F$ for jittered data</th>
<th>$F$ for Bootstrap data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.90</td>
<td>0.90</td>
<td>0.87</td>
<td>0.77</td>
<td>0.90</td>
</tr>
<tr>
<td>0.8</td>
<td>0.78</td>
<td>0.73</td>
<td>0.72</td>
<td>0.66</td>
<td>0.75</td>
</tr>
<tr>
<td>0.7</td>
<td>0.69</td>
<td>0.69</td>
<td>0.65</td>
<td>0.56</td>
<td>0.69</td>
</tr>
<tr>
<td>0.6</td>
<td>0.64</td>
<td>0.61</td>
<td>0.60</td>
<td>0.53</td>
<td>0.61</td>
</tr>
<tr>
<td>0.5</td>
<td>0.53</td>
<td>0.49</td>
<td>0.48</td>
<td>0.45</td>
<td>0.52</td>
</tr>
<tr>
<td>0.4</td>
<td>0.38</td>
<td>0.40</td>
<td>0.38</td>
<td>0.25</td>
<td>0.34</td>
</tr>
<tr>
<td>0.3</td>
<td>0.32</td>
<td>0.33</td>
<td>0.31</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>0.2</td>
<td>0.24</td>
<td>0.23</td>
<td>0.22</td>
<td>0.13</td>
<td>0.25</td>
</tr>
<tr>
<td>0.1</td>
<td>0.12</td>
<td>0.12</td>
<td>0.11</td>
<td>0.08</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Empirical Powers for F Ratios Based on the Original Data Using $\alpha = 0.1, 0.05$ and 0.01.
Figure 9

Empirical Powers for the Original Data F Ratios, the Jittered Data F's and the Bootstrap F's.
Figure 10

Deviations of the Empirical Powers from the Specified Powers for Five Different F Ratios.
Generate Ranked Data

The nonparametric rank method suggested by Conover and Iman (1981), replaces each of the 30 observations with its rank in the total sample. An F ratio is then computed for the ranks. Table 5 and Figures 11 and 12 display the average rank F ratios, $F_{\text{rank}}$, the expected values $E(F')$, the sample $S^2_{\text{rank}}$ and the $V(F')$ for the 100 samples taken for each value of $\phi^2$. From the plots, the $F_{\text{rank}}$'s appear close to $E(F')$. The $S^2_{\text{rank}}$'s are generally close to the $V(F')$, but for large theoretical variances are almost always greater than the variance of the $F'$ was expected to be. This means that the rank method may be too liberal for the generated data.

$F'$ Score for Ranked Data

To assess the "closeness" of the $F_{\text{rank}}$ statistics is to the corresponding noncentral F distribution, $F'$ score plots as discussed earlier were formed. From Figures 13, 14 and 15, it appears that the F's for the ranked data are very close to the F' distribution, since the lines are close to straight. Again there appears to be a slight "hook" near the upper percentiles, as found in the $F'$ score plots for the original data. This again was not deemed a serious flaw.

Although the distribution of the F ratios computed for the ranks do not appear to be an exact noncentral F ratio, the $F'$ score plots and the comparison of the first two central moments with the theoretical moments suggest that they may be "correctable". A two
Table 5
Average F' Values and Sample Variances of F' Values for Ranked Data, E(F') and V(F') for 100 Samples from the Simulated CRD Experiments for Indicated Powers

<table>
<thead>
<tr>
<th>(1 - β)</th>
<th>Average F' values for ranks</th>
<th>Expected F' values</th>
<th>Sample variances for ranks</th>
<th>Population variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>9.045</td>
<td>8.821</td>
<td>32.593</td>
<td>26.210</td>
</tr>
<tr>
<td>0.8</td>
<td>7.088</td>
<td>6.941</td>
<td>24.582</td>
<td>19.218</td>
</tr>
<tr>
<td>0.7</td>
<td>6.484</td>
<td>5.784</td>
<td>19.705</td>
<td>15.223</td>
</tr>
<tr>
<td>0.6</td>
<td>4.865</td>
<td>4.713</td>
<td>11.779</td>
<td>11.729</td>
</tr>
<tr>
<td>0.5</td>
<td>4.363</td>
<td>4.229</td>
<td>9.078</td>
<td>10.217</td>
</tr>
<tr>
<td>0.4</td>
<td>3.196</td>
<td>3.500</td>
<td>6.350</td>
<td>8.014</td>
</tr>
<tr>
<td>0.3</td>
<td>3.319</td>
<td>3.211</td>
<td>8.976</td>
<td>7.168</td>
</tr>
<tr>
<td>0.2</td>
<td>2.504</td>
<td>2.256</td>
<td>4.592</td>
<td>4.472</td>
</tr>
<tr>
<td>0.1</td>
<td>1.797</td>
<td>1.773</td>
<td>3.269</td>
<td>3.169</td>
</tr>
</tbody>
</table>
Figure 11

Average and Expected F' Values for 100 Ranked Samples from the Simulated CRD Experiments for Indicated Powers.
Figure 12

Sample and Expected Variances of F' Values for 100 Ranked Samples from the Simulated CRD Experiments for Indicated Powers.
Figure 13

F' Score Plots for F Ratios from Ranked Samples for Power (1 - \( \beta \)) = 0.9. The Fairly Straight Line Indicates that the F' Values Calculated for the Ranked Samples are Close to the Target Noncentral F Distribution with \( \lambda = 7.168 \), \( \nu_1 = 2 \) and \( \nu_2 = 27 \).
Figure 14

F' Score Plots for F Ratios from Ranked Samples for Power \((1 - \beta) = 0.5\). The Fairly Straight Line Indicates that the F' Values Calculated for the Ranked Samples are Close to the Target Noncentral F Distribution with \(\lambda = 2.916\), \(v_1 = 2\) and \(v_2 = 27\).
Figure 15

F' Score Plots for F Ratios from Ranked Samples for Power (1- \( \beta \)) = 0.1. The Fairly Straight Line Indicates that the F' Values Calculated for the Ranked Samples are Close to the Target Noncentral F Distribution with \( \lambda = 0.642 \), \( v_1 = 2 \) and \( v_2 = 27 \).
moment approximation may be used to improve the characteristics of this test.

Empirical Powers for Ranked Data

Empirical powers discussed earlier were computed for the ranked F procedures and are plotted in Figure 9 and listed in Table 4. From the plot, it appears that for $\alpha = 0.05$, the empirical powers for the ranked data are slightly smaller than the empirical powers for the F ratios calculated for the original data for specified powers of 0.2, 0.5, 0.6 and 0.8. For specified powers of 0.3 and 0.4, the empirical powers for the F procedures based on the ranks are slightly larger than the empirical powers for the original F. For powers of 0.1, 0.7 and 0.9, the empirical powers for the F procedures based on the ranks are equal to the empirical powers for the original F. These values are not too far from the diagonal line, which indicates that the empirical powers are close to these specified powers.

Based on these plots, the rank procedure appears to provide a test with power close to that provided by the F test for the original data.
CHAPTER IV
JITTERED SAMPLES

Generate Jittered Data

The original samples were jittered according to the procedures described by Chambers et. al. (1983) by adding random "noise" to the data to simulate measurement errors. After some preliminary work, the noise for runs in this study was generated from a normal distribution with zero mean and variance 9. Table 6 and Figures 16 and 17 display the F ratios for the jittered data, $F_{\text{jitter}}$, against $E(F')$, and $S^2_{\text{jitter}}$ against $V(F')$ for the 100 samples taken for each power. From these plots, the $F_{\text{jitter}}$'s are close to $E(F')$ and $S^2_{\text{jitter}}$'s are also close to $V(F')$, although they are consistently lower than the expected values. This indicates that this jittering process tends to decrease the both the value and variability of the F ratios.

$F'$ Score for Jittered Data

To assess the "closeness" of the F ratios computed for the jittered data, $F'$ score plots as discussed in earlier were formed for 3 different runs. From the plots in Figures 18, 19 and 20, it appears that the F's for the jittered data may be close to the F' distribution, since the lines are again "close" to straight. The "hook" for these $F'$ score plots does not appear any more severe than the $F'$ score plots for the original data. Again, no specific reason for the departure is known, and again, the departures do not appear to be important. Since the means and variances for the jittered data F ratios are low, a 2 moment approximation may also be needed here.
<table>
<thead>
<tr>
<th>(1 - β)</th>
<th>( \bar{F}_{\text{jitter}} )</th>
<th>E(( F' ))</th>
<th>( S^2_{\text{jitterF}} )</th>
<th>V(( F' ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>6.770</td>
<td>8.821</td>
<td>17.792</td>
<td>26.210</td>
</tr>
<tr>
<td>0.8</td>
<td>5.844</td>
<td>6.941</td>
<td>14.228</td>
<td>19.218</td>
</tr>
<tr>
<td>0.7</td>
<td>4.737</td>
<td>5.784</td>
<td>11.029</td>
<td>15.223</td>
</tr>
<tr>
<td>0.6</td>
<td>3.485</td>
<td>4.713</td>
<td>7.140</td>
<td>11.729</td>
</tr>
<tr>
<td>0.5</td>
<td>3.703</td>
<td>4.229</td>
<td>6.708</td>
<td>10.217</td>
</tr>
<tr>
<td>0.4</td>
<td>2.499</td>
<td>3.500</td>
<td>3.269</td>
<td>8.014</td>
</tr>
<tr>
<td>0.3</td>
<td>2.616</td>
<td>3.211</td>
<td>6.091</td>
<td>7.168</td>
</tr>
<tr>
<td>0.2</td>
<td>2.165</td>
<td>2.256</td>
<td>3.901</td>
<td>4.472</td>
</tr>
<tr>
<td>0.1</td>
<td>1.613</td>
<td>1.773</td>
<td>2.729</td>
<td>3.169</td>
</tr>
</tbody>
</table>
Average and Expected $F'$ Values for 100 Jittered Samples from the Simulated CRD Experiments for Indicated Powers.
Figure 17

Sample and Expected Variances of F' for 100 Jittered Samples from the Simulated CRD Experiments for Indicated Powers.
F' Score Plots for F Ratios from Jittered Samples for Power (1- β) = 0.9. The Fairly Straight Line Indicates that the F' Values Calculated for the Jittered Samples are Close to the Target Noncentral F Distribution with λ = 7.168, v₁ = 2 and v₂ = 27.
Figure 19

F' Score Plots for F Ratios from Jittered Samples for Power \((1-\beta)\) = 0.5. The Fairly Straight Line Indicates that the F' Values Calculated for the Jittered Samples are Close to the Target Noncentral F Distribution with \(\lambda = 2.916\), \(v_1 = 2\) and \(v_2 = 27\).
Figure 20

F' Score Plots for F Ratios from Jittered Samples for Power (1- \( \beta \)) = 0.1. The Fairly Straight Line Indicates that the F' Values Calculated for the Jittered Samples are Close to the Target Noncentral F Distribution with \( \lambda = 0.642 \), \( v_1 = 2 \) and \( v_2 = 27 \).
Empirical Powers for Jittered Data

The empirical powers for the jittered data F ratios were computed, plotted in Figure 9, and listed in Table 4. From this plot, it appears that the empirical power for jittered data F ratios is the smallest of the five procedures considered in this study. The empirical powers for jittered ranks data F ratio are all larger than the empirical powers for jittered data, but all less than the empirical powers for the F procedures based on the original data and ranks. For powers 0.3, and 0.4, the empirical powers for jittered rank data are slightly larger than the empirical powers for bootstrap data, and equal to the empirical power for bootstrap data when power is 0.1. For powers of 0.2, 0.5, 0.6, 0.7, 0.8 and 0.9, the empirical powers for jittered rank data are less than the empirical powers for bootstrap data. From Figure 10, it appears that the empirical powers for the jittered data are the lowest of the five procedures considered in this study.
CHAPTER V

BOOTSTRAP FOR ORIGINAL DATA

Generate Bootstrap Data

For each data set, the bootstrap procedure used in this report took 100 samples with replacement of size 30 from each of the original samples. The usual F ratio was then computed for each bootstrap sample.

Table 7 and Figures 21 and 22 display the $F_{boot}$ against $E(F')$ and the $S_{bootF}^2$ against $V(F')$ for the 100 samples taken for each power. From these plots, the $F_{boot}$'s are not close to $E(F')$ nor are the $S_{bootF}^2$ close to $V(F')$.

$F'$ Score for Bootstrap Data

As a second indicator of the "closeness" of the $F_{boot}$ statistics to the noncentral F distribution, $F'$ score plots as discussed earlier were formed. From the plots in Figures 23, 24 and 25, it appears that the F's for the bootstrap data do not follow the $F'$ distribution, since the lines are not straight.

QQ plots for several distributions were tried using Minitab, and it appears that the Bootstrap data follows the Normal distribution, since the QQ plots comparing the Bootstrap F's with a normal distribution produced a line that was close to straight, as seen in Figures 26, 27 and 28. This would also explain the large differences between the $F_{boot}$ and $E(F')$ and between $S_{bootF}^2$ and $V(F')$, since the distribution of the bootstrapped F ratios seems to be governed by the central limit theorem.
Table 7

Average Bootstrap $F'$ Values and Expected $F'$
Sample Variances of Bootstrap $F'$ Values and
Population Variance for 100 Samples from the
Simulated CRD Experiments for Indicated Powers

<table>
<thead>
<tr>
<th>$(1-\beta)$</th>
<th>$\bar{F}_{\text{Boot}}$</th>
<th>$E(F')$</th>
<th>$S^2_{\text{boot}F}$</th>
<th>$V(F')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.1017</td>
<td>24.524</td>
<td>0.0142</td>
<td>108.61</td>
</tr>
<tr>
<td>0.9</td>
<td>1.0806</td>
<td>8.821</td>
<td>0.0144</td>
<td>26.210</td>
</tr>
<tr>
<td>0.8</td>
<td>1.0742</td>
<td>6.941</td>
<td>0.0109</td>
<td>19.218</td>
</tr>
<tr>
<td>0.7</td>
<td>1.0860</td>
<td>5.784</td>
<td>0.0139</td>
<td>15.223</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0962</td>
<td>4.713</td>
<td>0.0148</td>
<td>11.729</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0800</td>
<td>4.229</td>
<td>0.0130</td>
<td>10.217</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0666</td>
<td>3.500</td>
<td>0.0128</td>
<td>8.014</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0843</td>
<td>3.211</td>
<td>0.0150</td>
<td>7.168</td>
</tr>
<tr>
<td>0.2</td>
<td>1.0900</td>
<td>2.256</td>
<td>0.0146</td>
<td>4.472</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0768</td>
<td>1.773</td>
<td>0.0112</td>
<td>3.169</td>
</tr>
</tbody>
</table>
Figure 21

Average and Expected $F'$ Values for 100 Bootstrap Samples from the Simulated CRD Experiments for the Indicated Powers.
Figure 22

Sample and Expected Variances of $F'$ Values for 100 Bootstrap Samples from Simulated CRD Experiments for the Indicated Powers.
Figure 23

F' Score Plots for F Ratios from Bootstrap Samples for Power (1 - β) = 0.9. The Curve Shape Indicates that the F' Values Calculated for the Bootstrap Samples are not Close to the Target Noncentral F Distribution with λ = 7.168, v₁ = 2 and v₂ = 27.
AVGBSF

- * * * * * * * * * *
1.28+ ** ** ** ** **
- 22
- 22
- *3423*
1.12+ *43
- 553
- *5
- 55
- 65*
0.96+ 4
- *
- *
- 2
- *
0.80+ *

F’ Score Plots for F Ratios from Bootstrap Samples for Power 
(1-\(\beta\)) = 0.5. The Curve Shape Indicates that the F’ Values 
Calculated for the Bootstrap Samples are not Close to the Target 
Noncentral F Distribution with \(\lambda = 2.916\), \(v_1 = 2\) and \(v_2 = 27\).
F' Score Plots for F Ratios from Bootstrap Samples for Power \((1 - \beta) = 0.1\). The Curve Shape Indicates that the F' Values Calculated for the Bootstrap Samples are not Close to the Target Noncentral F Distribution with \(\lambda = 0.642\), \(v_1 = 2\) and \(v_2 = 27\).
Figure 26

Nscore Plots for F Ratios from Bootstrap Samples for Power(1 - β) = 0.9.
Fig. 27

Nscore Plots for F Ratios from Bootstrap Samples for
Power(1- \(\beta\)) = 0.5.
Figure 28

Nscore Plots for F Ratios from Bootstrap Samples for Power(1 - \( \beta \)) = 0.1.
Empirical P-values for Bootstrap Data

To compare the bootstrap method with the other procedures, empirical p-values were computed for each of the 4 bootstrap procedures: the bootstrap for the original data, the bootstrap for the ranks, the bootstrap for the jittered data and the bootstrap for the ranks of the jittered data.

These empirical p-values were computed as the proportion of bootstrap samples with F ratios that exceeded the original F value and are listed in Table 8 and plotted in Figure 29. From these we see very close agreement. There does not appear to be any consistent pattern of differences for the four techniques. As far as empirical p-values are concerned, there is little difference between the 4 procedures. Jittering the data seems to have little effect, suggesting that the p-values are robust with respect to measurement errors.

As a consequence of these small differences, the only empirical powers for bootstrap samples that were computed used only the original F ratios. There appears to be no consistent pattern to the empirical powers for the bootstrap samples compared to the other procedures. It is generally close to those of the original F ratios, and almost always better than that of the jittered data.
Table 8
Average P-values for Four Different Bootstrap Procedures

<table>
<thead>
<tr>
<th>$\phi^2$</th>
<th>(1- $\beta$)</th>
<th>p-values for bootstrap samples</th>
<th>p-values for ranked bootstrap samples</th>
<th>p-values for jittered bootstrap samples</th>
<th>p-values for ranked jittered bootstrap samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.796</td>
<td>0.9</td>
<td>0.0156</td>
<td>0.0216</td>
<td>0.0161</td>
<td>0.0216</td>
</tr>
<tr>
<td>3.61</td>
<td>0.8</td>
<td>0.0503</td>
<td>0.0612</td>
<td>0.0512</td>
<td>0.0638</td>
</tr>
<tr>
<td>2.89</td>
<td>0.7</td>
<td>0.0489</td>
<td>0.0563</td>
<td>0.0482</td>
<td>0.0549</td>
</tr>
<tr>
<td>2.25</td>
<td>0.6</td>
<td>0.0863</td>
<td>0.0981</td>
<td>0.0872</td>
<td>0.0955</td>
</tr>
<tr>
<td>1.932</td>
<td>0.5</td>
<td>0.1637</td>
<td>0.1780</td>
<td>0.1620</td>
<td>0.1717</td>
</tr>
<tr>
<td>1.488</td>
<td>0.4</td>
<td>0.1819</td>
<td>0.2023</td>
<td>0.1870</td>
<td>0.2070</td>
</tr>
<tr>
<td>1.232</td>
<td>0.3</td>
<td>0.1931</td>
<td>0.1958</td>
<td>0.1926</td>
<td>0.1887</td>
</tr>
<tr>
<td>0.723</td>
<td>0.2</td>
<td>0.2422</td>
<td>0.2601</td>
<td>0.2398</td>
<td>0.2527</td>
</tr>
<tr>
<td>0.423</td>
<td>0.1</td>
<td>0.3784</td>
<td>0.3802</td>
<td>0.3728</td>
<td>0.3766</td>
</tr>
</tbody>
</table>
Figure 29

Average P-values for Four Bootstrap Procedures.
CHAPTER VI
CONCLUSIONS

Recommendations

From the empirical powers for the above four procedures (Table 4) and the differences from the theoretical powers 0.1 thru 0.9 (Figure 10) for these four procedures, it appears that there is little difference between the empirical power of the Bootstrap procedure compared to the other theoretical powers. The empirical power for the jittered procedure is furthest from the theoretical power. The rank procedure seems to do well. Since the bootstrap procedure provides its own p-value without making any distribution assumptions, it is the nonparametric procedure of choice. The rank procedures may also be satisfactory, but if the test statistic is to be compared to an F distribution, then the F for the ranks appears to be too large, making the test too liberal. This means there may be an excess of type I errors using the ranked data F ratios. A two moment approximation could possibly be used to eliminate this problem.

The jittering procedure generally degraded the performance of all procedures to which it was applied. The bootstrap appears to have been least effected however. This suggests that the bootstrap technique is fairly robust to measurement errors as simulated by jittering.

Future Research

For this paper, the standard deviation is 3, for each of the 3 groups and there were 10 observations in each group. An $\alpha = 0.05$ was
used to find the power and the parameter $\phi^2$ from the charts given in Dixon and Massey(1969). 100 bootstrap samples were taken for each of the 100 simulations. These parameters for this study suggest several avenues for future research: we could use different standard deviations, different $\alpha$'s, more groups, more observations, more bootstrap samples, more simulations, and different underlying distribution besides the normal. At this point, the bootstrap technique appears to be a good nonparametric procedure which is robust to measurement errors. The rank procedure also appears to be a good, but slightly too liberal procedure.
REFERENCES


APPENDIX

Appendix 1: Program Listing

PROGRAM TRY(INPUT, OUTPUT, OUTFILE1, OUTFILE2);
CONST
PI = 3.1415927;
G = 3;
N = 10;
NBS = 100;
(* 3 GROUPS *)
(* 10 OBSERVATIONS IN EACH GROUP *)
(* 100 BOOTSTRAP SAMPLES *)
TYPE
VECT = ARRAY[1..5] OF REAL;
TABLE = ARRAY[1..100] OF REAL;
VAR
BEP : VARYING [20] OF CHAR;
BB : VARYING [20] OF CHAR;
CC : VARYING [20] OF CHAR;
DD : VARYING [20] OF CHAR;
D : VARYING [20] OF CHAR;
REM, R, C, O, Q, L, BSJ, RBSI, JJ, III, JJ, RJI, OI: INTEGER;
X, Y, Z, NO_SIMU : INTEGER;
OUT_FLAG, SIMNO, BSI, P, I, K, M, J, DFA, DFE : INTEGER;
RANKED_ORIG, TTY, JITTERED_DATA, RANK_JITTERED: TABLE;
DATA, DATA1, ORIG, BSORIG, RBSORIG, RJBSSORIG : TABLE;
FIND 95 : VECT;
BS_PVAL, RBS_PVAL, JBS_PVAL, RJBS_PVAL : REAL;
TEST, EMPROB : REAL;
CRITIF : ARRAY[1..3] OF REAL;
EP : ARRAY[1..3, 1..5] OF REAL;
SEED : INTEGER;
JORIG_F, JRANK_F, ORIG_F, RANK_F, AVGBS_F : REAL;
AVGBS_F, AVGJBS_MSE, AVRJBS_MSE, AVGRJBS_MSE : REAL;
AVGBS_MSE, AVRBS_MSE, AVGRBS_MSE : REAL;
MU, SIGMA : VECT;
JITTER_WIDTH, N1, N2, SSA, SSE, MSA, MSE, F : REAL;
NRAN_FLAG : BOOLEAN;
PAR_FILE, OUTFILE1, OUTFILE2, OUTFILE3 : TEXT;
PAR_TITLE, TITLE : PACKED ARRAY[1..30] OF CHAR;
FIRSTTIME : [STATIC] BOOLEAN := TRUE;

(* RANDOM NUMBER FUNCTION *)
FUNCTION RANDOM: REAL;
FUNCTION MTH$RANDOM(VAR SEED : INTEGER): REAL; EXTERN;
BEGIN
IF FIRSTTIME THEN
BEGIN
SEED := CLOCK * 2 + 1;
FIRSTTIME := FALSE;
END;
RANDOM := MTH$RANDOM(SEED);
END;
(* INPUT 3 DIFFERENT MEANS *)
PROCEDURE INPUT_PARAMETERS;
VAR I : INTEGER;
BEGIN
  FOR I := 1 TO G DO
    READ(PAR_FILE,MU[I]);
    READLN(PAR_FILE);
    FOR I := 1 TO G DO
      SIGMA[I] := 3.0;
  END;

(* TRANSFORMATION FOR UNIFORM DISTRIBUTION TO GET NORMAL RANDOM NUMBER *)
FUNCTION NRAN:REAL;
VAR
  U1,U2 : REAL;
BEGIN
  IF NRAN_FLAG THEN
    BEGIN
      TEST := RANDOM;
      WHILE TEST = 0.0 DO
        TEST := RANDOM;
      U1 := SQRT(-2.0 * LN(RANDOM));
      U2 := RANDOM;
      N1 := U1 * COS(2.0 * PI * U2);
      N2 := U1 * SIN(2.0 * PI * U2);
      NRAN_FLAG := FALSE;
      NRAN := N1;
    END_FLAG THEN
    BEGIN
      TEST := RANDOM;
      WHILE TEST = 0.0 DO
        TEST := RANDOM;
      U1 := SQRT(-2.0 * LN(RANDOM));
      U2 := RANDOM;
      N1 := U1 * COS(2.0 * PI * U2);
      N2 := U1 * SIN(2.0 * PI * U2);
      NRAN_FLAG := FALSE;
      NRAN := N1;
    END
  ELSE
    BEGIN
      NRAN_FLAG := TRUE;
      NRAN := N2;
    END
  END;

(* INDEX NUMBER FOR BOOTSTRAP *)
FUNCTION SAMPLE(VAR UP : INTEGER) : INTEGER;
BEGIN
  SAMPLE := TRUNC(RANDOM * UP) + 1;
END;

(* GENERATE RANDOM NUMBER *)
PROCEDURE GENERATE_DATA(VAR ORIG:TABLE);
VAR I,J,K : INTEGER;
BEGIN
  FOR I := 1 TO G DO
    BEGIN
      FOR J := 1 TO N DO
        BEGIN
          K := (I - 1) * N + J;
        END
    END;
(* SORT FOR BOOTSTRAP EMPIRICAL POWER *)
PROCEDURE SORTED(VAR A:VECT;MM:REAL);
VAR X,Y,Z:INTEGER;
P,PP:REAL;
BEGIN
  FOR X:=1 TO 5 DO
    BEGIN
      FOR Y:= X TO 5 DO
          BEGIN
            P:=A[X];
            A[X]:=A[Y];
            A[Y]:=P;
          END;
      END;
END;

(* RANK PROCEDURE *)
PROCEDURE RANK(VAR DATA,RANKS : TABLE);
VAR TEMP : REAL;
TY,SY: TABLE;
I,M,K : INTEGER;
BEGIN
  FOR K:= 1 TO G*N DO
    TY[K] := DATA[K];
  FOR I := 1 TO G*N DO
    BEGIN
      FOR M := 1 TO G*N-1 DO
        BEGIN
          IF TY[M] > TY[M+1] THEN
            BEGIN
              TEMP := TY[M];
              TY[M] := TY[M+1];
              TY[M+1] := TEMP;
            END;
        END;
    END;
  FOR I := 1 TO G*N DO
    BEGIN
      TTY[I] := TY[I];
      TTY[I] := I;
    END;
  FOR I := 1 TO G*N DO
    BEGIN
      SY[I] := DATA[I];
      FOR J := 1 TO G*N DO
        BEGIN
          IF SY[I] = TY[J] THEN
            BEGIN
              SY[I] := TTY[J];
            END;
        END;
    END;
END;
FOR I:=1 TO G*N DO
    RANKS[I]:=SY[I];
END;

(* ANALYSIS OF VARIANCE *)
PROCEDURE ANOV(VAR DATA: TABLE);
VAR TOTAL,TEMP,CF,SUM1,SUM2,SUM3 : REAL;
    A1,A2,A3,T1,T2,T3,T,A : REAL;
    I : INTEGER;
BEGIN
    SUM1 := 0.0;
    SUM2 := 0.0;
    SUM3 := 0.0;
    A1 := 0.0;
    A2 := 0.0;
    A3 := 0.0;
    T1 := 0.0;
    T2 := 0.0;
    T3 := 0.0;
    FOR I := 1 TO N DO
        BEGIN
            SUM1 := SUM1 + DATA[I];
            T1 := T1 + DATA[I] * DATA[I];
        END;
    A1 := (SUM1 * SUM1) / N;
    FOR I := N + 1 TO 2 * N DO
        BEGIN
            SUM2 := SUM2 + DATA[I];
            T2 := T2 + DATA[I] * DATA[I];
        END;
    A2 := (SUM2 * SUM2) / N;
    FOR I := 2*N+1 TO 3*N DO
        BEGIN
            SUM3 := SUM3 + DATA[I];
            T3 := T3 + DATA[I] * DATA[I];
        END;
    A3 := (SUM3 * SUM3) / N;
    TOTAL := SUM1 + SUM2 + SUM3;
    CF := (TOTAL * TOTAL) / (G * N);
    A := A1 + A2 + A3;
    T := T1 + T2 + T3;
    SSA := A - CF;
    SSE := T - A;
    DFA := G - 1;
    DFE := G * (N - 1);
    MSA := SSA / DFA;
    MSE := SSE / DFE;
    F := MSA / MSE;
END;

(* JITTER DATA *)
PROCEDURE JITTER(VAR DATA: TABLE; JWDTH: REAL);
VAR I : INTEGER;
BEGIN
  FOR I:=1 TO G*N DO
    DATA[I]:=DATA[I]+(RANDOM - 0.5)*JWDTH;
END;

(* BOOTSTRAP DATA *)
PROCEDURE BOOTSTRAP(VAR DATA, DATAl: TABLE);
VAR K, I, J, M, P : INTEGER;
BEGIN
  K := 0;
  FOR I := 1 TO G DO
    BEGIN
      FOR J := 1 TO N DO
        BEGIN
          K := K + 1;
          M := G * N;
          P := SAMPLE(M);
          DATAl[K] := DATA[P];
        END;
    END;
END;

(* ******************************************* MAIN PROGRAM ******************************************* *)
BEGIN
  OPEN(PAR_FILE,'PAR.DAT', OLD);
  RESET(PAR_FILE);
  OUT_FLAG := 0;
  IF (OUT_FLAG = 1) THEN
    BEGIN
      WRITE('ENTER FILE NAME : '); READLN(TITLE);
      OPEN(OUTFILE1,TITLE , NEW);
      REWRITE(OUTFILE1);
    END;
  NO_SIMU := 100;
  EMProb := 1.0/NO_SIMU;
  CRITIF[1] := 2.51; (* ALPHA IS EQUAL TO 0.1 *)
  CRITIF[2] := 3.35; (* ALPHA IS EQUAL TO 0.05 *)
  CRITIF[3] := 5.49; (* ALPHA IS EQUAL TO 0.01 *)
  BB := 'APS697.';
  BEP := 'EMPOWER.';
  FOR R := 48 TO 57 DO
    BEGIN
      D := CHR(R);
      CC := BB + D;
      DD := BEP + D;
      Writeln('CC=',CC);
      Writeln('DD=',DD);
      OPEN(OUTFILE2,CC,NEW);
      REWRITE(OUTFILE2);
      OPEN(OUTFILE3,DD,NEW);
      REWRITE(OUTFILE3);
    END;
INPUT_PARAMETERS;
NRAN_FLAG := TRUE;

(* INITIALIZE EMPIRICAL POWERS *)
FOR X := 1 TO 3 DO
  FOR Y := 1 TO 5 DO
    EP[X,Y] := 0.0;

(* DO 100 SIMULATION SAMPLES *)
FOR SIMNO := 1 TO NO_SIMU DO
  BEGIN
    GENERATE_DATA(ORIG);
    ANOV(ORIG);
    ORIG_F := F;
    (* CALCULATE EMPIRICAL POWERS FOR ORIGINAL DATA *)
    FOR Z := 1 TO 3 DO
      IF(ORIG_F > CRITIF[Z]) THEN
    RANK(ORIG,RANKED_ORIG);
    ANOV(RANKED_ORIG);
    RANK_F := F;
    (* CALCULATE EMPIRICAL POWERS FOR RANKED DATA *)
    FOR Z := 1 TO 3 DO
      IF(RANK_F > CRITIF[Z]) THEN
    JITTER(RANKED_ORIG, 2*((N*N-1)/12)**0.5));
    ANOV(RANKED_ORIG);
    JRANK_F := F;
    (* CALCULATE EMPIRICAL POWERS FOR JITTERED RANKED DATA *)
    FOR Z := 1 TO 3 DO
      IF(JRANK_F > CRITIF[Z]) THEN
    BS_PVAL := 0.0;
    RBS_PVAL := 0.0;
    JBS_PVAL := 0.0;
    RJBS_PVAL := 0.0;
    AVGBS_F := 0.0;
    AVGBS_MSE := 0.0;
    AVGRBS_F := 0.0;
    AVGRBS_MSE := 0.0;
    AVGJBS_F := 0.0;
    AVGJBS_MSE := 0.0;
    AVGRJBS_F := 0.0;
    AVGRJBS_MSE := 0.0;
    JITTER_WIDTH := 0.0;
    FOR L := 1 TO G DO
      JITTER_WIDTH := JITTER_WIDTH + 2*(SIGMA[L]/G);
    (* DO 100 BOOTSTRAP SAMPLES *)
    FOR BSI := 1 TO NBS DO
      BEGIN
        BOOTSTRAP(ORIG,BSORIG);
        ANOV(BSORIG);
        IF OUT_FLAG = 1 THEN
          WRITE(OUTFILE1,F:6:2,MSE:6:2);
        IF ( F > ORIG_F) THEN
          BS_PVAL := BS_PVAL + 1;
AVGBS_F := AVGBS_F + F/NBS;
IF BSI <= 5 THEN
    FIND_95[BSI] := F
ELSE
    SORTED(FIND_95,F);
    AVGBS_MSE := AVGBS_MSE + MSE/NBS;
    RANK(BSorig,RBSorig);
    ANOV(RBSorig);
    IF OUT_FLAG = 1 THEN
        WRITE(OUTFILE1,F:6:2,MSE:6:2);
    IF F > RANK_F THEN
        RBS_PVAL := RBS_PVAL + 1;
        AVG_RBS_F := AVG_RBS_F + F/NBS;
        AVG_RBS_MSE := AVG_RBS_MSE + MSE/NBS;
        RANK(BSorig,JBSorig);
        ANOV(JBSorig);
    IF OUT_FLAG = 1 THEN
        WRITE(OUTFILE1,F:6:2,MSE:6:2);
    IF F > ORIG_F THEN
        JBS_PVAL := JBS_PVAL + 1;
        AVG_JBS_F := AVG_JBS_F + F/NBS;
        AVG_JBS_MSE := AVG_JBS_MSE + MSE/NBS;
        RANK(ORIG,RJBSorig);
        ANOV(RJBSorig);
    IF OUT_FLAG = 1 THEN
        WRITE(OUTFILE1,F:6:2,MSE:6:2);
    IF F > RANK_F THEN
        RJBS_PVAL := RJBS_PVAL + 1;
        AVG_RJBS_F := AVG_RJBS_F + F/NBS;
        AVG_RJBS_MSE := AVG_RJBS_MSE + MSE/NBS;
    END;
    SORTED(FIND_95,-9999.99);
    JITTER(ORIG,JITTER_WIDTH);
    ANOV(ORIG);
    ORIG_F := F;
FOR Z := 1 TO 3 DO
    IF(ORIG_F > CRITIF[Z]) THEN
    BS_PVAL := BS_PVAL/NBS;
    RBS_PVAL := RBS_PVAL / NBS;
    JBS_PVAL := JBS_PVAL / NBS;
    RJBS_PVAL := RJBS_PVAL / NBS;
    FOR III := 1 TO N*G DO
        JITTERED_DATA[III] := ORIG[III];
        JITTER(JITTERED_DATA,JITTER_WIDTH);
        ANOV(JITTERED_DATA);
        RANK(JITTERED_DATA,RANK_JITTERED);
        ANOV(RANK_JITTERED);
        WRITE(OUTFILE2,ORIG_F:6:2,RANK_F:6:2,JRANK_F:6:2,
            ORIG_F:6:2,AVGBS_F:5:2,AVGRBS_F:5:2,
            AVG_JBS_F:5:2,AVGRJBS_F:5:2);
WRITE(OUTFILE2,AVGBS_MSE:6:2,AVGRBS_MSE:6:2,
AVG_JBS_MSE:6:2,AVGRJBS_MSE:6:2,
RBS_PVAL:5:2,JBS_PVAL:5:2,RJBS_PVAL:5:2);
WRITELN(OUTFILE2);
(* CALCULATE EMPIRICAL POWERS FOR BOOTSTRAP DATA *)
IF ORIG_F > FIND_95[1] THEN
END;
I := 2;
   FOR J := 1 TO 5 DO
      WRITE(OUTFILE3,EP[I,J]:6:3);
      WRITELN(OUTFILE3);
   END;
CLOSE(OUTFILE2);
CLOSE(OUTFILE3);
END;
### Appendix 2: One Run Sample Output Listing

<table>
<thead>
<tr>
<th>Column Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Original F ratio</td>
</tr>
<tr>
<td>2</td>
<td>F ratio for ranked sample</td>
</tr>
<tr>
<td>3</td>
<td>F ratio for jittered ranks sample</td>
</tr>
<tr>
<td>4</td>
<td>F ratio for jittered original sample</td>
</tr>
<tr>
<td>5</td>
<td>F ratio for average bootstrap sample</td>
</tr>
<tr>
<td>6</td>
<td>P-value for bootstrap sample</td>
</tr>
<tr>
<td>7</td>
<td>P-value for ranked bootstrap sample</td>
</tr>
<tr>
<td>8</td>
<td>P-value for jittered bootstrap sample</td>
</tr>
<tr>
<td>9</td>
<td>P-value for ranked jittering bootstrap sample</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.76</td>
<td>2.50</td>
<td>2.53</td>
<td>1.58</td>
<td>0.94</td>
<td>0.07</td>
<td>0.07</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>3.76</td>
<td>3.05</td>
<td>2.91</td>
<td>2.66</td>
<td>1.16</td>
<td>0.06</td>
<td>0.11</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0.87</td>
<td>0.80</td>
<td>1.31</td>
<td>0.77</td>
<td>0.89</td>
<td>0.36</td>
<td>0.47</td>
<td>0.36</td>
<td>0.40</td>
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<tr>
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<td>0.98</td>
<td>0.76</td>
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<td>0.58</td>
<td>1.05</td>
<td>0.42</td>
<td>0.42</td>
<td>0.46</td>
<td>0.55</td>
</tr>
<tr>
<td>5</td>
<td>5.52</td>
<td>4.50</td>
<td>4.14</td>
<td>3.60</td>
<td>1.00</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>1.53</td>
<td>1.16</td>
<td>0.68</td>
<td>2.97</td>
<td>1.05</td>
<td>0.21</td>
<td>0.29</td>
<td>0.22</td>
<td>0.31</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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</tr>
<tr>
<td>8</td>
<td>8.01</td>
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<td>11.64</td>
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<td>1.09</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>3.87</td>
<td>3.86</td>
<td>5.55</td>
<td>3.28</td>
<td>1.07</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>5.60</td>
<td>5.05</td>
<td>5.13</td>
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<td>1.05</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>11</td>
<td>4.73</td>
<td>3.63</td>
<td>3.01</td>
<td>3.81</td>
<td>0.94</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>12</td>
<td>7.15</td>
<td>8.45</td>
<td>8.00</td>
<td>6.35</td>
<td>1.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>13</td>
<td>4.69</td>
<td>5.62</td>
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