INCORPORATING TECHNOLOGY IN MATHEMATICS EDUCATION:
A SUITE OF E-ACTIVITIES FOR THE MODERN
MATHEMATICS CLASSROOM

by

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ABSTRACT

Incorporating Technology in Mathematics Education:
A Suite of E-Activities for the Modern Mathematics Classroom

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National studies indicate major deficiencies in students’ understanding of mathematics. Research suggests that students tend to view mathematics as a set of computational rules rather than a process of discovery and a tool for problem-solving. Most students fail to grasp the concepts behind the computations.

Technology provides a partial solution to this problem. Over the past decade, computers have emerged as a powerful tool in education. Computers place the control of action in the learning process with the student. They allow students to experiment with, explore, and discover mathematics at their own pace. With computers, students can consider more examples than are possible with a pencil and paper. The graphic capability of computers aids students in concept visualization; the computational
capacity allows them to focus on concepts while the computer executes the tedious computations.

The purpose of this thesis is to facilitate the effective use of computers in mathematics education. The primary component of this thesis is a CD-ROM containing a suite of computer manipulatives intended for use in the mathematics classroom. An explanation of the manipulatives accompanies the CD-ROM, as does a description of the creation process.
ACKNOWLEDGMENTS

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I would especially like to thank Dr. Heal and Dr. Cannon for inviting me to work on the Mathematical and Teaching Technology Innovations (MATTI) project, of which this thesis is a part. Both Dr. Heal and Dr. Cannon have served as mentors to me throughout my entire postsecondary education. Dr. Heal deserves special recognition as my major professor and my advisor of eight years. He has influenced and shaped my education more than any other teacher.

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Finally, I would like to thank my parents for instilling in me a love of education and a hunger for knowledge. They have served as examples to me and have supported me through every phase of my education. My deepest gratitude goes to them.

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BACKGROUND AND INTRODUCTION

This thesis is part of a large-scale, ongoing project that is being supported by a grant from the National Science Foundation (NSF). The project is a three-year endeavor aimed at developing a library of uniquely interactive, web-based virtual manipulatives for mathematics instruction. Most of these computer activities are in the form of Java applets, small programs designed to run on the web. To date, several manipulatives have been created for the project. Some of the applets are still in the process of development, but most of the applets are relatively polished. A library of the manipulatives that have been created thus far can be found at the project’s web site: http://matti.usu.edu.

The applets in the project’s "Virtual Manipulatives" library are organized into five main categories: Number and Operations, Data Analysis and Probability, Algebra, Geometry, and Measurement. Each of these categories corresponds to one of the standards set by the National Council of Teachers of Mathematics (NCTM) in their Standards 2000 project (NCTM). Within each category, the manipulatives in the library are further broken down according to grade level.

This project, which is centered at Utah State University, is dubbed the Mathematical and Teaching Technology Innovations (MATTI) project. The principal investigators on the project are Lawrence O. Cannon, James T. Dorward, E. Robert Heal, and Leo Edwards. Ultimately the MATTI project team will make these computer materials available at several sources on the Internet, creating a national library from

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which teachers may freely draw to enrich their mathematics classrooms.
MOTIVATION

The driving philosophy behind this thesis (and the larger project of which this thesis is a part) is that students learn mathematical concepts far more effectively when they are allowed to discover the concepts themselves. The control of action in the learning process must be with the student. The role of the teacher should be to guide and direct student exploration and to help students solidify and recognize what they have seen and learned.

Although some educators cling to tradition, research indicates that traditional educational methods fall short. The third National Assessment of Educational Progress (NAEP) (Carpenter et al.) reveals major deficiencies in students' learning of mathematics. The NAEP results indicate that students show "a lack of understanding of basic concepts and processes in many content areas" (28). According to Carpenter et al., it appeared that "most students in the study did not learn basic problem-solving skills, and attempted instead to mechanically apply some calculation to whatever numbers were given in a problem" (28).

The NAEP study attributes the deficiencies in students' learning to lack of student interaction. Carpenter et al. reported that students "perceive their role in the mathematics classroom to be primarily passive.... They feel they have little opportunity to interact with their classmates about the mathematics being studied, to work on exploratory activities, or to work with manipulatives" (36).

Technology provides a partial solution to this problem. Computer technology, in
particular, facilitates student interaction and exploration. Papert (quoted in Clements) is a proponent of computer technology in the classroom. He asserts:

I believe with Dewey, Montessori, and Piaget that children learn by doing and by thinking about what they do. And so the fundamental ingredients of educational innovation must be better things to do and better ways to think about oneself doing these things. I claim that computation is by far the richest known source of these ingredients. We can give children unprecedented power to invent and carry out exciting projects by providing them with access to computers.... (12)

In *Principles and Standards for School Mathematics* (NCTM), the authors stress the importance of using technology in the classroom. They list technology as one of the six main principles describing features of high-quality mathematics instruction. They state: "Technology is essential in teaching mathematics; it influences the mathematics that is taught and enhances students’ learning" (24).

Technology is especially valuable in assisting students with visualization, computation, exploration, and discovery. The authors of the standards maintain that students can learn "more mathematics more deeply" with technology (25).

Traditional mathematics classrooms in which teachers give lectures and students memorize algorithms are archaic. Learning and internalizing mathematics requires student engagement. Technology provides educators with an important key to achieving student engagement and interactivity in their classrooms.
DEFINING VIRTUAL MANIPULATIVES

Manipulatives are physical objects that help students visualize relationships and understand mathematical applications. Physical manipulatives, such as base-ten blocks and tangrams, have been utilized in classrooms for decades to actively involve students in the learning process. Virtual manipulatives are computer simulations of physical manipulatives that address the same goals (NCTM 26).

Until now, there has been very little done to create good computer-based manipulatives or learning tools at elementary and middle school levels with any degree of interactivity. The virtual manipulatives that are being created for this project are highly interactive, web-based, instructional units. Some of the virtual manipulatives are software versions of common physical manipulatives. Most of them, however, reach far beyond the limitations of a physical manipulative. Unlike physical manipulatives, virtual manipulatives are not awkward to use, hard to store, or expensive. Virtual manipulatives are much more versatile than their physical counterparts. A single electronic manipulative can be used for several different classroom activities and at various grade levels.

Each virtual manipulative created by the MATTI project team is a Java applet. A Java applet is a small program designed to run on the web.

Java has clear advantages over other programming languages for this type of project. The primary advantage Java offers is web-based accessibility. Java applets are designed to be placed on web pages. The applets are downloaded over the Internet and
executed right inside the web page by a browser that supports Java. Currently, most browsers do support Java.

Another advantage Java offers is platform independence, the capability of moving easily from one computer system to another. Patterson Hume and Stephenson explain that lack of standardization across hardware platforms, operating systems, and software applications is a major stumbling block for software developers (2). Incompatibility forces programmers to develop multiple versions of a single piece of software in order to meet differing machine requirements. Java compilers overcome this problem by generating bytecode rather than native machine code. Bytecodes are sets of instructions that look a lot like machine code, but are not specific to any one processor (Lemay and Perkins 7). Compilers for most programming languages translate programs into machine codes that are specific to the processor the computer is running. The Java development environment has two parts: a compiler and an interpreter. Java compilers translate programs into bytecode that the web page downloads. Java interpreters in the browser read the bytecodes and execute the programs. Java’s inherent portability is what makes Java so well-suited to the Internet.

Java’s other strengths include its object-oriented environment and its simplicity. Java’s object-oriented nature allows software developers to create flexible, modular programs and reusable code. For a project of this magnitude, having reusable code is desirable. Java’s simplicity is ideal for educational programming. Patterson Hume and Stephenson explain that Java syntax is based on C++ syntax, but that many of the difficult and error prone parts of C and C++, such as pointers and operator overloading,
have been eliminated. Java has additional features which are not a part of C or C++. One such feature that assists educational programmers is its inclusion of graphical user interface classes.

The MATTI project team currently programs using the *IBM VisualAge for Java* 2.0 programming environment. This programming environment assists programmers in organizing and editing their Java code. The chief advantage of the *IBM VisualAge* environment is that it compiles programs incrementally. When a programmer makes changes to a Java program, *IBM VisualAge* compiles only the changed code, not the entire code base. Although incremental compilation requires more memory than standard compilation, it affords the programmer immediate editing capabilities. *IBM VisualAge* provides programmers with a convenient way to enter, edit, and keep track of Java programs.
DESCRIPTION OF MANIPULATIVES

The virtual manipulatives that make up this thesis will ultimately be included in the MATTI library of computer tools. The manipulatives that are included in this thesis are the following: *Turtle Geometry*, a programming tool designed to help students develop reasoning skills and geometric understanding; *Towers of Hanoi*, a virtual version of a classic mathematical puzzle; *Peg Puzzle*, also a virtual version of a classic puzzle; *Derivative*, a graphing tool to help students understand the interpretation of the derivative as the slope of the tangent line; *Pie Chart*, a data organization tool; *Spinner*, a tool for conducting simple probability experiments; *Mastermind*, a virtual version of a classic logic game; and *Color Patterns*, a simple applet designed to introduce young students to pattern recognition. The sections that follow describe each of the above manipulatives and explain how they can be used to satisfy the standards set by the NCTM.

*Turtle Geometry*

*Turtle Geometry* (see Figure 1) is a computer tool similar to *Logo*, a simple programming language that children use to generate geometric shapes and figures. *Logo* was created by Seymour Papert (see Solomon), a scholar with a background in computer science, mathematics, philosophy, and psychology. Papert's philosophy in creating *Logo* centers on the theory that learning best occurs in an active environment in which students embark on self-chosen and self-directed projects. Since Papert believes
that "doing mathematics" consists, in part, of constructing objects, both mental and physical, and "debugging" them, he believes that students can learn to "do math" by developing programming skills (Solomon 12).

In Papert’s *Logo* programming language, students issue a series of typed commands for a "turtle" on the screen to execute. For example, the command FOR 3 moves the turtle forward 3 spaces on the screen. *Turtle Geometry* has many of the advantages of Papert’s *Logo*, but removes much of the unnecessary abstraction that confuses younger children.

In *Turtle Geometry*, students also navigate a turtle about a screen by issuing a
series of commands. Unlike Logo, however, Turtle Geometry offers the advantage of a child-friendly, graphical interface. To enter commands into a Turtle Geometry "program," students simply press buttons which place command icons in a window below the main screen. To watch the turtle execute their program, students press the play button at the top of the applet. Students choose from commands that step the turtle forward (up to 5 steps at a time), turn the turtle right or left (in 15-degree increments), change the color of the trail that the turtle leaves, or direct the turtle to leave no trail at all. Instead of walking the turtle through a blank screen, students may choose to navigate the turtle through a maze (see Figure 1) or an obstacle course. They also have the option of bringing up pictures on the screen and writing programs that direct the turtle to trace over the pictures. The versatility added by these options makes Turtle Geometry engaging for children.

The pedagogical benefits of Logo have long been documented. An evaluation by the National Science Foundation (NSF) (see Solomon) indicates that Papert’s approach to helping students develop mathematical reasoning and problem-solving skills through the use of Logo is effective (Solomon 13). Studies aimed at understanding how students’ geometric understanding changes through the use of Logo have also been positive. Kieran, Hillel, and Gurtner cite one study in which researchers employed the van Hiele theories about the levels of geometric understanding to develop "Logo-motion" geometry tasks for seventh graders. The results of this research suggest that children do learn geometry through such activities and that they gain sophistication in talking about geometric objects and processes.
Like Logo, Turtle Geometry helps students develop reasoning skills and geometric understanding. The goals of Turtle Geometry correspond with the goals set forth in the NCTM 2000 Standards in the categories of Problem Solving and Geometry (NCTM).

_Towers of Hanoi_

_Towers of Hanoi_ (see Figure 2) is an interactive graphical version of a popular physical manipulative. The _Towers of Hanoi_ puzzle (also known as the "Tower of Brahma" or the "three-peg puzzle") is a well-known logic puzzle that reportedly appeared first as a wooden toy in France in the late 1800s. The puzzle consists of three poles, with a tower of disks on the first pole. The objective of the puzzle is to transfer the entire tower of disks to either the second or the third pole, moving only one disk at a time. Each disk that is moved must be taken from the top of a stack and no disk can be placed on top of a disk smaller than itself.

Several versions of the toy have been created in the last century. The reason for the toy's popularity is the challenge it presents to the user. The two disk tower is trivial to solve. It can be done in only three moves. The three disk tower is also easy, requiring only seven moves. As the number of disks increases, however, the minimum number of moves required to solve the puzzle grows rapidly. Already, with eight disks, the solution requires a minimum of 255 moves. The growth is exponential.

At Utah State University, students use this applet in the math for preservice elementary teachers courses. The class reads a legend upon which the puzzle may have
been based. The legend states that the world will vanish with a thunderclap when a group of monks finishes transferring a 64 disk tower in a certain temple. The instructor poses the question to the class: "How long would it take to transfer the 64 disk tower if the monks transfer the disks at the rate of one disk per second?" The students work with the applet to determine the minimum number of moves for towers with one, two, three, four, and five disks. They then use inductive reasoning to generalize to \( n \) disks. Most students are surprised to discover that the 64 disk tower in the legend would take over 580 billion years to transfer.

According to the NCTM Standards for Number and Operations, students should be able to "judge the effects of operations such as...computing powers...on the
magnitudes of quantities" (NCTM 393). As students discover the exponential growth inherent in the *Towers of Hanoi* problem, they discover the effects of powers on magnitudes of quantities. More explicitly, *Towers of Hanoi,* promotes the development of problem-solving and reasoning skills. The NCTM Problem-Solving Standard states that instructional programs should enable students to "monitor and reflect on the process of mathematical problem solving" (NCTM 402). Students experience mathematical problem solving as they solve this puzzle. They get a taste for mathematical discovery as they uncover the recursive algorithm for solving the puzzle and as they determine a formula for the minimum number of moves given the number of disks. Many students who work with *Towers of Hanoi* in classes at USU comment that they spent hours "playing with" the applet and that their husbands or roommates were also drawn in by the manipulative. *Towers of Hanoi* gives students a chance to learn meaningful math in a fun way.

*Peg Puzzle*

*Peg Puzzle* is a manipulative similar to *Towers of Hanoi.* Like *Towers of Hanoi,* *Peg Puzzle* is based on a physical manipulative. The puzzle consists of a row of red pegs and a row of blue pegs placed in a slotted board (see Figure 3). The blue pegs are separated from the red pegs by a single space. The objective of the puzzle is to trade the blue pegs and the red pegs places, moving only one peg at a time. The pegs may move forward (i.e., toward the opposite side) into an empty hole or they may jump forward over a single peg of the opposite color if there is an empty space in which to land. No
other move is allowed. No peg may move backwards.

Peg Puzzle encourages the development of reasoning and problem-solving skills in the same way that Towers of Hanoi does. By experimenting with the applet, students can discover a recursive algorithm for solving the puzzle. Since the puzzle can be modified to have two, four, six, or eight pegs (by clicking the buttons at the bottom of the applet), students can explore the relationship between the number of pegs and the minimum number of moves it takes to solve the puzzle.

As electronic tools, Peg Puzzle and Towers of Hanoi have advantages over their physical counterparts. A mouse click, drag, and release is much quicker and less awkward than a physical transfer of a wooden disk or peg. In addition, physical manipulatives do not lend themselves to being quickly returned to their original state,
but computer manipulatives reset in an instant. This feature encourages user persistence.

*Derivative*

The *Derivative* applet (see Figure 4) is geared toward students in high school or college. It is a graphing tool designed for use in an introductory calculus class. This applet helps students internalize the concept of rate of change by looking at it graphically. The NCTM Algebra Standard (NCTM) asserts that students in the upper grade levels should be able to "interpret representations of functions of two variables" and to "approximate and interpret rates of change from graphical and numerical data" (395). The *Derivative* applet helps students develop these skills. It introduces them to one of the fundamental concepts of calculus: the concept of the derivative.

Students in a first calculus class generally have a difficult time understanding the definition of the derivative. One interpretation of the derivative of a function at a point is the slope of the line tangent to the graph of the function at that point. When students are simply told how to calculate this slope, the definition of the derivative is meaningless. When they discover how to calculate the slope of the tangent line on their own, the definition comes alive for them. Using the *Derivative* applet, students can discover how to calculate the slope of a line tangent to a function at a point. The applet allows the user to graph a function in a chosen viewing window along with its tangent line at any point. A slider at the bottom of the graph moves the point back and forth along the function; the tangent line changes accordingly. The user can approximate the
slope of a given tangent line by looking at secant lines that get closer and closer to the tangent line. The Derivative applet makes it easy to look at secant lines. To bring up a secant line, the user simply checks the box labeled "show secant line," and the secant line appears in red. The sliders at the bottom of the screen drag the points defining the secant line. By dragging the sliders at the bottom of the screen, it is easy for students to discover that the slope of the tangent line is the limiting value of the slopes of the secant lines. This dynamic, graphical representation is much more convincing than a graph sketched on a chalkboard.

Another check box at the bottom-left of the applet gives students the option of displaying the graph of the derivative of the function. This is a useful option because
students often have a hard time thinking of the derivative of a function as another function. By experimenting with this option, students can see how the derivative of a function relates to the function itself.

Much of the power of the Derivative applet lies in its ability to generate many examples quickly. The authors of the NCTM Standards 2000 point out that "the graphic power of technological tools affords access to visual models that are powerful but that many students are unable or unwilling to generate independently" (NCTM 25). The Derivative applet graphs a wide variety of functions including trigonometric, exponential, and logarithmic functions. In typical calculus textbooks, students are restricted to three or four examples of a given concept because there is not room to print more. The Derivative applet provides students with an unlimited supply of self-chosen examples. It gives them the opportunity to experiment with functions themselves. The Derivative applet really takes advantage of the graphic power of technology.

**Pie Chart**

According to the NCTM Standards for Data Analysis and Probability, mathematics instruction should enable students to "select, create, and use appropriate graphical representations of data" (NCTM 401). **Pie Chart** (see Figure 5) is a data representation tool that aids students in doing this. In **Pie Chart**, the user enters data categories along with the number in each category. When the user presses the "draw chart" button, the computer draws a pie chart representing the student’s data. The applet also calculates and displays the percentages in each category of the pie chart.
Pie Chart provides students with a quick visualization of their data. Students can experiment with different data sets to determine what type of data sets are most appropriately represented with a pie chart. Because the computer generates pie charts quickly with little effort required on the part of the user, students can produce many more examples than they could with a paper and pencil. Students can practice their estimation skills by sketching a pie chart on their own before having the computer draw the chart. Comparing their estimate with the chart that the computer generates helps students understand connections between percentages and parts of a whole.
Spinner

Spinner (see Figure 6) is a simple tool for conducting probability experiments. When a user presses the "spin" button on this applet, the spinner pseudo-randomly selects one of the colored sections. The power of this applet lies in its adaptability to hundreds of different probability experiments. The user can customize the spinner to his or her specific probability experiment. For example, a user who wishes to simulate drawing with replacement from a bag with two red balls and one black ball can create a spinner with two red sections and one black section. The "change spinner" button at the bottom of the applet brings up a box that prompts the user to specify the section colors and names for the spinner (see Figure 7). The spinner can have up to twenty sections colored from fifteen different colors.

The Spinner applet supports the NCTM Data Analysis and Probability Standard,
which states: "Instructional programs...should enable all students to use proportionality and basic understanding of probability to make and test conjectures about the results of experiments and simulations" (NCTM 401).

Mastermind

The NCTM Standards for Reasoning and Proof assert that students should be able to "recognize reasoning and proof as fundamental aspects of mathematics" (NCTM 402). Deductive reasoning skills are requisite to participating in real mathematics. *Mastermind* is a classic game of deduction. The *Mastermind* applet (see Figure 8)

![Mastermind applet](image-url)
included in this thesis is an electronic version of the classic game.

In the Mastermind applet, the task of the user is to break the code chosen by the computer. The computer randomly picks a sequence of four colors from the six colors shown at the right of the applet. The user's challenge is to guess what color sequence the computer picked. One row at a time, the user fills in the rows with guesses by pressing the peg buttons at the right of the applet. After filling a row, the user submits his guess. Upon the submission of each guess, the computer responds by indicating how many colors in the guess are exactly where they belong (indicated by the number of small black pegs) and how many colors in the guess are right, but in the wrong position (indicated by the number of small white pegs.) The user is allowed up to eight guesses. The computer reveals the code if the user has not yet discovered it after eight guesses.

Mastermind is a pedagogical tool disguised as a game. The user–knowingly or unknowingly–engages himself in a valuable logic exercise.

Color Patterns

Several of the NCTM Standards mention pattern recognition as a necessary skill for students. The Algebra Standard asserts that students should be able to "recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns" (NCTM 394). The authors of the standards explain:

Early experiences with classifying and ordering objects are natural and interesting for young children. Teachers might help children notice that red-blue-blue-red-blue-blue can be extended with another red-blue-blue sequence. Initially students may describe the regularity of patterns verbally rather than with mathematical symbols.... Algebra is more than
moving symbols around. (38)

*Color Patterns* (see Figure 9) is an extremely simple applet that assists children in the development of pattern recognition. When the applet starts up, a sequence of colors presents itself dynamically on the screen. The child then continues the pattern by filling in the missing bubbles with the color choices at the right of the screen. Because the applet pseudo-randomly generates the color sequences, it has, in effect, an endless supply of patterns to choose from.

Although the applet is simple, it is invaluable for students in the early grades. It teaches them pattern recognition skills that can be built upon in later grades.

Figure 9: *Color Patterns* applet.
CONCLUSION

Student engagement is essential in the modern mathematics classroom. Virtual manipulatives are an important innovation that promote student involvement.

Because researchers and educators have done little in the past to create interactive, computer-based manipulatives at elementary and middle school levels, experimentation was a necessary part of the creation process for this project. The process was a team effort involving several stages of design, trial, and redesign. My experience on this project led me to the following conclusions:

- Computer applications geared toward elementary and secondary students should be as simple to use as possible. Unnecessary components of computer tools—bells and whistles, so to speak—can act as motivators as long as they do not complicate the application.

- The style of the interface for a given computer manipulative should depend on the grade level for which the manipulative is intended. For example, students in older grades can benefit from a busier interface with more functionality (as in the Derivative applet), while students in younger grades function better with a less cluttered interface (as in Color Patterns).

- For a group of computer tools, consistency is important. In this group of applets, consistency is maintained in the general layout, color, and basic functionality of the applets.

- Computer tools such as these have serious potential. The preservice elementary
teachers at Utah State University (USU) who currently use these applets in their courses are enthusiastic about the manipulatives. Rebecca Gonzalez, a Ph.D. student at USU, is conducting research for her dissertation on the effectiveness of the MATTI computer tools. Preliminary results are positive. 

As interactive software of this nature is developed in the future, several questions could be addressed. Some questions that may be explored in the future are:

- What types of computer tools are most effective? How does individual applet use affect student understanding?
- What is the most appropriate use of computer tools such as these? Do students benefit both from teacher demonstration of applets and from individual exploration of an applet?
- Do students’ attitudes about mathematics change through the use of computer tools such as these?
- How does student understanding of concepts change through the use of computer tools such as these?
- How can these kind of computer tools be used to assist students with various learning disabilities?
- What implications might various student impairments have on the design of the computer tools? (For example, how could a manipulative such as Color Patterns be modified to accommodate a student who is blind or color blind?)

The virtual manipulatives that make up this thesis provide teachers with exciting, new approaches to interactive mathematics instruction. With more research and
development, computer tools such as these have the potential to revolutionize mathematics education.
REFERENCES


