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Water Entry Cavity Dynamics

Nathan B. Speirs
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WATER ENTRY CAVITY DYNAMICS

by

Nathan B. Speirs

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Mechanical Engineering

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2018
ABSTRACT

Water entry cavity dynamics

by

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Utah State University, 2018

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When a solid or liquid object impacts the surface of a pool of water a crater or air cavity often forms behind the impacting object as it descends into the pool. This phenomena has been studied for over a century, possibly due in part to the natural beauty of such events and related naval and industrial applications. Despite the extensive study, many facets of water entry remain poorly understood or even undiscovered. This dissertation investigates four of these areas using high-speed photography as the primary means of experimental investigation. First, it examines the impact of streams of multiple droplets, uniformly sized and spaced, on the surface of a pool of water. The cavity dynamics of such successive impact events are related to the cavity dynamics of jet and solid object impact and predictions for several cavity characteristics (depth, diameter, velocity and shape) are developed. Second, the impact of steel spheres on a pool consisting of a water-surfactant mixture is examined. Surfactant is shown to alter the critical velocity for cavity formation. When bubbles form on the pool surface we see that cavities form at all impact velocities tested, and elucidate the mechanism of such cavity formation. Third, the influence of surface coatings on the cavity type formed is investigated and previously unknown conditions leading to cavity formation are described. Fourth, the discovery of a method for drastically reducing the initial impact force on an impacting sphere is discussed. This method precedes the sphere impact with a
jet of water, which accelerates the liquid onto which the sphere impacts, thus reducing the force from added mass.

(116 pages)
PUBLIC ABSTRACT

Water entry cavity dynamics

Nathan B. Speirs

When a sphere or a stream of water hits the surface of a pool of water and enters a crater or air cavity often forms. This topic has been studied, both formally and informally, for a long time. This dissertation investigates four areas of water impact that are still poorly understood using high-speed photography. First, it examines a stream of droplets impacting on a pool of water, similar to a faucet drizzling into a full bucket. For these types of impacts we predict the depth, diameter, velocity, and shape of the cavities that the droplet stream forms. Second, it examines what occurs when a sphere impacts a pool of soapy water, such as a bubble bath or kitchen sink. The minimum velocity for a cavity to form decreases when soap is present. If the water has bubbles on the surface, the sphere will always form a cavity. Third, it examines how different coatings on a sphere (car wax, etc.) affect whether the sphere forms a cavity, and it shows how the coatings affect the shape of that cavity. Fourth, when objects impact a water surface they experience a large force, which many people have noticed when participating in cliff jumping, high diving, and belly flop competitions. We show that the force of impact can be reduced by 75% simply by allowing a mass of water to impact in front of the object.
To my wife, Andrea Speirs, who put up with me delaying a real job for three more years and many evenings where I could not stop thinking about my research.
ACKNOWLEDGMENTS

I would like to thank Tadd Truscott and Jesse Belden for mentoring me throughout my graduate studies. Their time and coaching helped to build my strengths and strengthen my weaknesses. I would also like to thank all of the members of the splash lab who helped me with experiments and listened as I talked out potential explanations of my findings. I am grateful for and acknowledge funding from the Office of Naval Research, Navy Undersea Research Program (grant N0001414WX00811), monitored by Ms. Maria Medeiros and from the Utah State University Research and Graduate Studies Development Grant Program. This enabled me to focus on my research and finish so quickly.

Nathan B. Speirs
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CHAPTER 1
INTRODUCTION

The impact of solid bodies and liquids on a liquid pool has been studied extensively [1, 6, 8, 14, 15, 18–21]. The natural beauty of such events leads to intrinsic interest by many researchers, but interest is also generated by naval and industrial applications, such as torpedo water entry, boat slamming, surface coating, metallurgy and foam creation. In this research I expand upon previous research discussing impact onto a pool surface (freesurface) by investigating the impact of multi-droplet streams, jets, and solid spheres. I describe their similarities and differences and explain their cavity dynamics.

This dissertation is organized as a compilation of four papers with each of chapters 2 through 5 comprising a self-contained study. As such, each contains the background, experiments, theory, and results relevant to the specific subject matter. In chapter 2, I look at the impact of multi-droplet streams on a pool of water. In chapter 3, I investigate the effect of adding surfactant to the pool. In chapter 4, I examine the influence of a sphere’s surface coatings on the cavity type formed. In chapter 5, I demonstrate a method for reducing the large initial impact force experienced during the early stages of sphere entry. Chapter 6 concludes this dissertation, and discusses the main findings and contributions.

1.1 General overview of water entry

When a sphere impacts a water surface a splash crown forms as the sphere begins to displace the water in the pool (Fig. 1.1, \( t = 12.4 - 22.4 \) ms). The sphere pulls air beneath the free surface as it descends, forming a cavity (\( t = 12.4 - 42.4 \) ms). The cavity walls collapse inward due to surface tension and hydrostatic forces and meet in what is known as a pinch-off event (\( t = 52.4 - 62.4 \) ms). The high pressure at pinch-off forms two jets, one that shoots upward into the atmosphere and one that shoots into the lower portion of the cavity impacting the sphere (\( t = 72.4 \) ms).
Fig. 1.1: An 18 mm steel sphere impacts the water surface at $t = 0$ ms and forms an air cavity with a) showing the above-water view and b) showing the below-water view. A splash crown forms on the surface at impact ($t = 12.4 \pm 22.4$ ms). The cavity collapses inwards and closes or pinches-off at $t = 62.4$ ms. After the pinch-off an upward and downward jet forms ($t = 72.4$ ms).

Although this basic description of events is quite simple, small changes in the impact conditions create a variety of different cavity behaviors and flow dynamics. Decreasing the density of the sphere results in smaller cavities and faster pinch-offs [2]. Using different materials or surface coatings changes the wettability of the sphere, which controls whether a cavity forms or not [7]. The size and geometry of the impactor also matters, with discs forming larger diameter cavities that take longer to pinch-off [9] and long slender bodies (torpedo-like) forming narrow cavities that collapse on themselves for certain nose shapes [4]. At extremely high velocities cavitation occurs, which greatly increases the size of the cavity as it fills with water vapor [17]. If the object is deformable, the cavity takes on a wavy shape and the trajectory is significantly altered [11]. Liquid impactors form cavities with similarities, but the cavities have many differences as the liquid deforms continuously throughout the impact event [8, 12, 21].

Prior research on liquid and solid body water entry has shown that inertial, gravitational, viscous and surface tension effects influence the cavity dynamics [18]. The relative importance of these effects are generally characterized by four nondimensional numbers. The Bond number is a ratio of gravitational to surface tension forces and is often used to
describe the effects of the size of the impacting object [1]. It is defined as \( Bo = \rho g L^2 / \sigma \), where \( \rho \) is the liquid density, \( g \) is the acceleration of gravity, \( \sigma \) is the surface tension, and \( L \) is a length scale (usually the diameter or radius of the impacting object). The Weber and Froude numbers are ratios of inertial to surface tension and gravitational effects respectively. They are defined as \( We = \rho U^2 L / \sigma \) and \( Fr = U^2 / g L \), where \( U \) is the impact velocity (the Froude number can also be defined as \( Fr = U / (gL)^{1/2} \)). At low \( We \) solid objects do not enter the pool [1], liquid droplets bounce or float on the surface [13] and jets enter the pool without forming a cavity [12]. Many studies examine water impact in high Weber and Froude numbers regimes, in which inertia dominates. In this regime the Weber and Froude numbers scale various characteristic of the cavity size and pinch-off event [1, 6, 14]. Note that \( Bo, We \) and \( Fr \) are interrelated with \( Bo = We / Fr \). The Reynolds number is defined as \( Re = \rho U L / \mu \), where \( \mu \) is the dynamic viscosity, and is a ratio of inertial to viscous forces. Water entry studies generally occur at high \( Re \) where viscous effects are negligible and hence most studies ignore its effects [18], although some have found it to be useful in describing specific phenomena [16].

Although the topic of water entry has been studied by several groups, many areas of research remain unexplored. First, with the exception of a few multi-droplet papers, most studies investigate the impact of a single object (solid or liquid) onto an initially quiescent pool. But we can reasonably suspect that altering the pool surface or flow prior to impact will alter both the cavity dynamics and the impact forces. Second, previous studies also examine impact onto a pool of a pure liquid without additives commonly present in natural and industrial processes that alter the fluid properties. Third, the effect of wettability has been shown to affect cavity formation, but it effect on the types of cavities that form has not previously been elucidated. Fourth, the impact of solid and liquid objects have been studied separately. However, the pre-wetting of solid objects or the presence of a solid inside an impacting liquid mass is likely to alter its wetting characteristics, flow dynamics and associated forces. This dissertation seeks to fill these holes in the literature with the following objectives.
1.2 Objectives

The purpose is of this dissertation is to improve our understanding of various aspects of the impact of solids and liquids on pools of water. The main focus will be on the types of cavities formed for various impact condition, but I also investigate other facets of the cavity dynamics and impact forces. My research is presented in this dissertation in a multi-paper format, with each of my four papers forming a separate chapter. The primary research objectives are:

1. Understand the cavity dynamics of multiple, successive impacts of droplets (i.e. multi-droplet streams).

2. Investigate the effect of the addition of surfactant to the pool liquid.

3. Examine the influence of the sphere's surface coating or contact angle on the cavity type formed.

4. Study the change in the impact force when a sphere impacts inside a falling mass of water.

I preform four studies to accomplish these objectives.

The first study investigates the impact of a stream of multiple droplets, uniformly sized and spaced, on the surface of a pool of water. The cavity dynamics of such successive impact events are related to the cavity dynamics of jet and solid object impact. Using impact conditions (droplet diameter, velocity and frequency) I predict the cavity types formed, cavity depths, the cavity diameter, and cavity velocity. I also predict these cavity behavior for jet impacts, some of which have not been studied previously. The results of this study are published in the *Journal of Fluid Mechanics* and comprise chapter 2.

The second study examines the impact of steel spheres on a pool consisting of a water-surfactant mixture. It focuses on the effect of surfactant on the critical velocity for cavity formation. I then examine the effect of adding a bubble layer to the pool surface, as bubbles commonly form on the surface of water-surfactant mixtures and find a change in the critical velocity for cavity formation. The results of this study comprise chapter 3.
My third study examines the influence of surface coatings or contact angle on the cavity type formed. Cavity formation is dependent on static contact angle and impact velocity, but the types of cavities formed have only previously been studied for high contact angles. Hence, in this study I employ different surface coatings on steel spheres to achieve various contact angles. I vary the diameter and impact velocity of the spheres near transitional Bond and Weber numbers to discover the different regimes or cavity types formed. The results of this study comprise chapter 4.

The last study stems from a desire to reduce the large initial impact force of an object when striking a pool surface. To achieve this I accelerate the pool surface downward, prior to sphere impact. This is accomplished by placing a sphere inside a jet of water. The jet impacts the pool, forming a cavity, into which the sphere falls, impacting the moving liquid at the cavity bottom. I compare these impact forces to the forces experienced by the sphere when impacting a quiescent pool surface and find significant reductions. The results of this study comprise chapter 5.

Chapter 6 summarizes the results of these separate studies and details how the research objectives have been met.
REFERENCES


CHAPTER 2
THE WATER ENTRY OF MULTI-DROPLET STREAMS AND JETS

2.1 Prologue

In order to gain the most general overview of cavity dynamics I first present my study on the impact of streams of droplets and jets on a pool of water and relate it to the impact of solid spheres. The data for this study was collected and processed at Utah State University. Analysis and modeling occurred at Utah State University and at the Naval Undersea Warfare Center in Newport, Rhode Island. The study was published in the Journal of Fluid Mechanics, volume 844, pages 1084-1111 on 10 June 2018 under the title “The water entry of multi-droplet streams and jets.” The authors listed are: Nathan B. Speirs, Zhao Pan, Jesse Belden and Tadd T. Truscott. The article in its entirety is presented below.

2.2 Abstract

Water entry has been studied for over a century, but few studies have focused on multiple droplets impacting on a liquid bath sequentially. We connect multi-droplet streams, jets, and solid objects with physical based scaling arguments that emphasize the intrinsically similar cavities. In particular, the cavities created by the initial impact of both droplet streams and jets on an initially quiescent liquid pool exhibit the same types of cavity seal as hydrophobic spheres at low Bond number, some of which were previously unseen for jets and droplet streams. Low frequency droplet streams exhibit an additional three new cavity seal types unseen for jets or solid spheres that can be predicted with a new nondimensional frequency. The cavity depth and cavity velocity for both droplet and jet impact are rationalized by an energy scaling analysis and the Bernoulli equation.
2.3 Introduction

The water entry of solid bodies and liquid entering liquid, in the form of droplets or continuous jets, has been studied extensively [1, 7, 8, 17, 19, 29–31]. In some cases, the sub-surface air cavities formed by liquid jets and solid bodies (e.g., spheres) are remarkably similar (Fig. 2.1c,d). While a single liquid droplet forms a much shallower cavity (Fig. 2.1a), a stream of multiple millimetric sized droplets impacting in rapid succession can form deep, narrow cavities resembling those formed by solid spheres and continuous jets (Fig. 2.1b). In this paper, we investigate the water entry of such multiple droplet streams, characterizing the cavity physics and drawing comparisons with jet and sphere water entry.

Prior research on liquid and solid body water entry has shown the cavity dynamics may be influenced by inertial, gravitational, viscous and/or surface tension effects depending on the scales involved in the problem [29]. The relative importance of these effects are
characterized by the Bond number \((Bo = \rho g L^2/\sigma)\), the Weber number \((We = \rho U^2 L/\sigma)\), the Froude number \((Fr = U^2/gL)\), and the Reynolds number \((Re = \rho U L/\mu)\), where \(\rho\) is the fluid density, \(g\) is the acceleration of gravity, \(\sigma\) is the surface tension, \(L\) is a length scale (specified for our study in §3), \(U\) is the impact velocity, and \(\mu\) is the fluid dynamic viscosity.

Previous studies on single droplet impact inform our study on multi-droplet cavity formation. Research on single droplets impacting a deep pool has found several areas of focus, with the objective to understand the underwater sound of rain [10, 21], spray creation [32], and near surface air entrainment [28]. Engel [8, 9] studied the cavity formation of large droplets (droplet diameter \(d_d = 4.5\, \text{mm}\)) impacting at high velocities (above 9 m/s) and predicted the cavity depth by equating the kinetic energy of the droplet to the potential energy of the cavity formed. Rodriguez and Mesler [26] found that the fall height and shape of the droplet at impact affect the cavity shape. Leng [14], Morton et al. [20], Cole [5], and Ray et al. [25] characterized a range of cavity behaviors including jet formation, vortex ring formation, coalescence, bubble entertainment and splash dome-over, using \(We\) and \(Fr\) to classify the physics.

The cavity physics associated with liquid jets impacting liquid pools are quite different from single droplet impact. When a continuous jet of water impacts normal to an initially quiescent free-surface, the impact forms a cavity that is driven deeper into the pool by the jet. The evolution of this cavity can include many features of rigid body impact as indicated in Fig. 2.1c&d. The current study focuses on the initial impact of a jet onto a quiescent free surface up until the cavity collapses. Several common topics persist in studies on cavity forming jet impacts. Cavity dimensions including the final depth of the cavity, the depth of pinch-off and the cavity radius are most prominently discussed by [22] and [33]. They find that the final cavity depth and the pinch-off depth are functions of the Froude number and that the cavity radius is approximately twice the jet radius. Most studies agree that the downward cavity velocity is equal to one half the jet velocity [3, 22–24, 33] but Kersten et al. [12] and Soh et al. [27] argue to the contrary. A few studies also discuss the motion and deformation of the jet fluid after impact showing that it coats the surface.
of the cavity [12, 22, 27, 33]. It is worth noting that following the transients associated with jet impact onto a quiescent pool, steady-state air entrainment may occur at later times if the jet is allowed to continuously flow into the pool [13, 16].

Bick et al. [2] performed one of the first studies on multi-droplet impact in which they examined the bubble entrainment caused by the impact of the first two droplets created from a jet undergoing break up. They found that bubble entrainment is affected by droplet diameter, impact velocity, and timing between impacts. Hurd et al. [11] were among the first to show that if the temporal frequency of the multi-droplet stream is high enough sequential droplets hitting the free surface in the same place create nested cavities, with each successive droplet forming a cavity at the base of the preceding cavity. They named this a matryoshka cavity after the Russian nesting dolls. We will also refer to the nested cavities created by multi-droplet impacts as matryoshka cavities. Bouwhuis et al. [4] studied the same event for micrometer sized droplets impacting with frequencies in the range of 10-30 kHz using boundary integral simulations with some experimental data for validation. In the regime studied, they showed that the expansion of the cavity is driven by inertia, but the collapse of the cavity is governed by surface tension (i.e., low Bo). They also showed that the downward cavity velocity approaches jet-like behavior as the spacing between the droplets decreases. Finally, they suggested that the maximum cavity radius and cavity depth at pinch-off scale with a modified We. We show that their scaling only applies to one type of cavity seal (shallow seal) and does not generalize to our extensive experimental data set (§ 2.9)

The existing body of work on rigid object impact has shown a variety of cavity shapes depending on the parameter space [29]. The depth of the cavity at pinch-off and the pinch-off time have been found to be functions of We and Bo [1, 7]. Aristoff and Bush [1] found four pinch-off types which all fall into distinct locations on a Bo-We plot. At the lowest We they describe quasi-static seal, in which pinch-off occurs on or very near the sphere surface. As We increases shallow seal is seen, in which a much larger cavity forms that collapses near the pool surface under the influence of surface tension. Increasing We results in deep seal,
where the cavity pinches-off approximately halfway between the surface and the sphere due to hydrostatic pressure. At the highest We a *surface seal* occurs, wherein the splash created upon impact collapses due to air pressure and surface tension [18] sealing off the cavity from further air entrainment and separating the cavity from the pool surface. Bouwhuis et al. [4] revealed a shallow seal cavity type formed by successive micro-droplets similar to that formed by rigid sphere impact [1]. On the other hand, prior work on impacting liquid jets has observed only deep seal closure modes [12, 22-24, 27, 33]. However, one might ask if impacting jets and multi-droplet streams can produce all of the cavity seal types observed for rigid sphere entry. Further, we suggest that other types of cavity seal exist that are unique to multi-droplet streams.

The aim of this experimental study is to investigate the dynamics of cavities created from multi-droplet streams and jets of water impacting on a deep pool. We explore multi-droplet and jet impacts over a much larger parameter space than has previously been investigated. The parameter ranges of multi-droplets and jets (Table 2.1) span the dimensionless numbers: $Bo \sim O(10^{-2}-10^0)$, $We \sim O(10-10^3)$, $Fr \sim O(10^2-10^4)$, and $Re \sim O(10^3-10^4)$. These ranges suggest that for our experiments surface tension, gravitational, and inertial forces are important, but viscous effects are negligible. The experiments reveal that $We$ and $Bo$ scaling separate the cavity regimes and show that multi-droplet stream cavities are consistent with those formed by continuous jets. These cavity types encompass those seen previously for solid sphere entry, with three additional types for multi-droplet streams.

### 2.4 Experimental setup and description

Figure 2.2a shows the experimental set up used for this study. A nozzle is placed 100-200 mm above a tank of water that is $730 \times 280 \times 350$ mm$^3$. Water flows to the nozzle through tubing that connects to a pressure reservoir that controls the velocity of the water exiting the nozzle. The nozzle forms a continuous stream, which is captured in a reservoir to prevent it from hitting the surface of the bath. We then remove the reservoir and allow the stream to fall continuously throughout the experiment. This method cannot control the length of the stream nor the final number of droplets. The diameter of the water jet
The droplet diameter $d_j$ (Fig. 2.2a) is varied by changing the nozzle diameter. The velocity of the jet exiting the nozzle ($U_s$) is measured by imaging 50 μm tracer particles that are mixed with the water; the average velocity across the jet profile is reported herein. The cavity velocity ($U_c$) is measured by tracking the bottom of the cavity over time and fitting a line to the data; $U_c$ is taken as the slope of this line. In order to create multi-droplet streams with droplet diameter $d_d$, the nozzle is attached to a large shaker with adjustable frequency $f$ and amplitude to induce jet break-up as shown in Fig. 2.2a. The impact of the jet and droplet streams is viewed at 6000 fps using two high speed cameras. One images below the surface to capture characteristics of the air cavity. The other views above the surface, from which diameters, velocities and frequencies of droplets are measured. These measurements reveal that the velocity of jets or droplets just prior to impact with the free-surface are equal for a given nozzle diameter and water flow rate; this impact velocity is referred to as $U_s$.

The achievable output parameters for multi-droplet streams (i.e., frequency, droplet diameter and velocity) are interrelated. Fig. 2.2b and Table 2.1 show the range of combinations of $f$, $d_d$ and $U_s$ that can be produced for each nozzle diameter. The relationship of these variables can be derived from conservation of mass yielding,

\[
d_d = \left( \frac{3d_j^2U_s}{2f} \right)^{1/3}.
\]

This relationship is shown by the dashed line in Fig. 2.2b. The data follow this prediction very well except for the largest jet diameter ($d_j = 1.49$ mm), for which satellite droplets are formed but often land away from the stream impact location, thus reducing the average volume of the main droplets in the stream. Note, due to limitations of the setup, not all combinations of $d_d$, $f$ and $U_s$ can be achieved.

Figure 2.3a shows an ideal matryoshka cavity created by a multi-droplet stream with relevant parameters given in Table 2.2. The droplet spacing can be defined as $U_s/f$, where $f$ is the frequency of the droplet stream in the lab reference frame and is equivalent to the shaker frequency. The cavity geometry is defined by the depth from the undisturbed
Fig. 2.2: a) Schematic of the experimental setup with the inset indicating the mechanism for producing multi-droplet streams via jet breakup. The velocity of droplets \( U_s \) has been observed experimentally to be largely constant over the fall height, and is equal to the jet velocity. b) The diameter of droplets \( d_d \) in the multi-droplet streams is controlled by the nozzle diameter \( d_j \), stream velocity \( U_s \), and shaker frequency \( f \) (Eq. 2.1). For a given nozzle diameter, the droplet diameter increases with decreasing \( f \) and increasing \( U_s \).

<table>
<thead>
<tr>
<th>Jet Diameter ( d_j ) (mm)</th>
<th>Droplets ( d_d ) (mm)</th>
<th>( U_s ) (m/s)</th>
<th>( f ) (Hz)</th>
<th>Jets ( U_s ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.34 - 0.66</td>
<td>4.04 - 7.9</td>
<td>1000 - 7000</td>
<td>–</td>
</tr>
<tr>
<td>0.21</td>
<td>0.45 - 0.80</td>
<td>3.61 - 10.5</td>
<td>1000 - 5000</td>
<td>–</td>
</tr>
<tr>
<td>0.41</td>
<td>0.85 - 1.62</td>
<td>2.78 - 7.51</td>
<td>300 - 4000</td>
<td>4.24 - 7.83</td>
</tr>
<tr>
<td>0.48</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3.12 - 6.51</td>
</tr>
<tr>
<td>0.71</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>3.94 - 9.69</td>
</tr>
<tr>
<td>0.81</td>
<td>1.40 - 2.25</td>
<td>1.85 - 7.26</td>
<td>200 - 2500</td>
<td>2.91 - 5.32</td>
</tr>
<tr>
<td>1.14</td>
<td>1.89 - 3.08</td>
<td>1.5 - 8.02</td>
<td>80 - 2500</td>
<td>3.69 - 8.01</td>
</tr>
<tr>
<td>1.49</td>
<td>2.60 - 4.11</td>
<td>1.92 - 5.64</td>
<td>100 - 500</td>
<td>2.57 - 10.25</td>
</tr>
</tbody>
</table>

Table 2.1: Experimental parameter ranges for each jet diameter \( d_j \) (Fig. 2.2a).
free-surface $h(t)$ and the cavity diameter $d_c(z, t)$. Additionally, there are three important cavity depths associated with relevant events. The first two can be seen in Fig. 2.4a at time 32.0 ms when pinch-off occurs. The depth at which cavity seal or pinch-off occurs, is denoted $h_p$. The depth of the bottom of the cavity at this time will be denoted as $h_b$. After all of the droplets from the stream have impacted and expanded the bottom of the cavity a new cavity depth is reached $h_c$ (Fig. 2.4 at $t = 42.0$ ms).

For droplet streams impacting with low frequency, $f$, we will show that there is a dependent nondimensional number that can help define the cavity regimes discussed more in § 2.5.1. We coin this useful nondimensional parameter the Matryoshka number and define it as

$$Mt = f_ap T_{max}$$

(2.2)

where $f_ap$ is the apparent frequency between droplet impacts and $T_{max}$ is the time it takes for a cavity from a single droplet to reach maximum size. The apparent frequency experienced by the droplets impacting the bottom of the cavity is $f_ap = (1 - U_c/U_s) f < f$, where $U_c$ and $U_s$ are depicted in Fig. 2.3a. Measured values of $U_c$ and $U_s$ from image-based tracking methods are used to calculate $f_ap$ directly from experiments. When a single droplet impacts a pool of water it creates a cavity which expands radially outward from the point of impact. $T_{max}$ occurs when the maximum cavity size is reached (Fig. 2.3b). [14] studied single droplet cavity formation and provided equations for estimating $T_{max}$ ($T_{max} = 0.524 \frac{d_d}{U_s} Fr^{0.625}$), which we use for all cases where $Mt > 10$. When $Mt < 10$ our own single droplet impact data was taken and the measured value of $T_{max}$ used to calculate $Mt$. More accurate values of $T_{max}$ are required at low $Mt$ since the measured vs. predicted values of $T_{max}$ can vary by up to a factor of two or three. Given the equations for $T_{max}$ and $f_ap$ above it is important to note that $Mt$ is a dependent nondimensional parameter as $f_ap$ and $T_{max}$ are functions of $Fr$, $\rho'$, $\dot{f}$ (defined below), $d_d$, $U_s$, and $f$. Nevertheless $Mt$ is useful for classifying certain cavity types, especially cavities that occur when $Mt < 4$ as will be shown (§ 2.5.1).

To give more physical intuition to the significance of $Mt$, consider the phenomena depicted in Fig. 2.3. The individual impacts of successive droplets divides the cavity into
Fig. 2.3: a) An idealized cavity created by a multi-droplet stream consists of several sub-cavities. b) A single droplet impact causes the cavity to expand at velocity $U_c$. The time of maximum cavity size ($T_{max}$) corresponds to $U_c = 0$; for $t > T_{max}$ the cavity is collapsing. c) The matryoshka number ($Mt$) is defined by the timing of the impact of the second droplet with respect to the maximum cavity size of the first. For example, for $Mt = 1$ the second droplet impacts the cavity at $T_{max}$. d) The dimensionless frequency $\tilde{f} = f d_d / U_s$ defines the ratio of droplet diameter to spacing between droplets.

multiple sub-cavities. As the first droplet impacts the pool it creates a portion of the matryoshka cavity labeled sub-cavity 1. The second droplet creates sub-cavity 2, the third creates sub-cavity 3 and so on (Fig. 2.3a). If droplets in a multi-droplet stream impact the bottom of the matryoshka cavity while the lowest sub-cavity is expanding then $Mt > 1$ (Fig. 2.3c). If droplets impact exactly at the time of momentum reversal of the lowest sub-cavity then $Mt = 1$, and if droplets impact after the momentum of the sub-cavity has already reversed then $Mt < 1$. The matryoshka number can be thought of as the number of droplets that impact the bottom of the matryoshka cavity before the cavity from a single droplet impact has reached its maximum size.

A second important dimensionless frequency, denoted $\tilde{f}$ by Bouwhuis et al. [4], is a
### Table 2.2: List of variables and their description.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>diameter</td>
<td>${iap}$</td>
<td>apparent</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
<td>${ib}$</td>
<td>bottom of cavity at pinch-off</td>
</tr>
<tr>
<td>$h$</td>
<td>depth</td>
<td>${ic}$</td>
<td>cavity</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>${id}$</td>
<td>droplet</td>
</tr>
<tr>
<td>$U$</td>
<td>velocity</td>
<td>${ij}$</td>
<td>jet</td>
</tr>
<tr>
<td>$L, l$</td>
<td>stream length</td>
<td>${ip}$</td>
<td>pinch-off</td>
</tr>
<tr>
<td></td>
<td></td>
<td>${is}$</td>
<td>stream (droplet or jet)</td>
</tr>
</tbody>
</table>

**Fig. 2.4:** a) Four images from the impact of a multi-droplet water stream onto a pool of water. The pinch-off frame is shown at $t = 32$ ms with $h_p$ indicating the depth of the pinch-off point and $h_b$ indicating the depth of the cavity at this time. At $t = 42$ ms all of the droplets from the multi-droplet stream have impacted the cavity base creating a depth of $h_c$. At $t = 63$ ms all of the droplets from the downward jet created at pinch-off have impacted the cavity. b) The maximum cavity diameter over time is measured at discrete depths, yielding $d_{c_{\text{max}}}(z)$. The average of these max diameters over all depths defines $d_c$ (red dotted line).
ratio of droplet diameter to the center to center spacing between droplets, \( \bar{f} = f d_d / U_s \), taking the same form as the Strouhal number. As explained by Bouwhuis et al. [4] and shown in Fig. 2.3d, if \( \bar{f} = 1 \) droplets touch end to end, and the stream is reminiscent of a jet. If \( \bar{f} = 1/3 \), a third of the droplets remain in the stream. As \( \bar{f} \) approaches zero the droplets in the stream become separated by such a large distance that by the time the second droplet impacts the cavity has already become a flat surface again.

### 2.4.1 Uncertainty

Uncertainty in all image based measurements are calculated and the uncertainty bands in the figures represent the 95% confidence interval of the measurement [6]. The uncertainty in calculated variables was often found to increase linearly with the variable. Where applicable, two bands are placed on the extremes of the axes of a figure to show that the error is linearly increasing (e.g., Fig. 2.5), when only one band is present the mean uncertainty is shown.

### 2.5 Cavity formation

The basic sequence of events involved in a multi-droplet stream impact can be seen in Fig. 2.4a and supplemental video 1. As the first droplet hits the free surface a hemispherical cavity is formed. The next droplet impacts the base of the first cavity causing it to expand downward. This process continues until the cavity pinches together on the droplet stream sealing it off from more incoming droplets. Even after this pinch-off event occurs droplets in the lower portion of the cavity continue impacting the bottom, pushing it deeper into the pool while maintaining roughly the same volume since pinch-off. Once all of the droplet stream has impacted, droplets formed from the downward jet created at pinch-off will continue to impact the cavity bottom, pushing it even deeper into the pool. These droplets created from the downward jet are usually much less consistent in diameter and trajectory and thus cause the cavity to evolve in a much more chaotic manner. Our experimental observations conclude that the downward jet does not affect the droplet stream as it forms after the stream has passed the pinch-off point and has a velocity approximately less than
or equal to the stream velocity (e.g., supplemental videos 1 and 7). This sequence of events is similar for the impact of a jet (supplemental video 2).

Figure 2.5 shows that the ratio of the cavity diameter to jet or droplet diameter increases linearly with impact velocity. Oguz et al. [22] and Zhu et al. [33] claim that $d_c/d_j$ is constant around 2.41 for jets, irrespective of impact velocity. This finding is inconsistent with the data presented here, but could be explained by the small velocity ranges studied in those works, 0.88–2.64 m/s and 0.96–1.87 m/s respectively.

The cavity diameter produced by jets is always larger than the cavity diameter formed by multi-droplet streams at the same velocity (Fig. 2.5). We find a common factor of approximately 1.75 by taking the ratio of the least squares regressions of the jet data to the multi-droplet data. Because the cavity diameter affects other cavity dynamics, such as the relative magnitude of the gravitational, inertial, and surface tension forces, we will include this scaling factor by defining a modified droplet diameter of $d^*_d = d_d/1.75$. The inset of Fig. 2.5 shows that this re-scaling collapses the multi-droplet data onto the curve for jets, where $d_s = d^*_d$ for droplets and $d_s = d_j$ for jets. Based on this empirical finding, we redefine the Bond, Weber, and Froude numbers of the multi-droplet streams as

$$Bo^* = \rho gd_s^2/\sigma,$$

$$We^* = \rho U_d^2 d_s/\sigma,$$

$$Fr^* = U_d^2 /gd_s,$$

respectively. For the remainder of the paper, we may use the * symbol for jets and droplets for ease of discussion, but when used for jets the $d_j$ is not modified. While we are currently unable to offer an analytic explanation for the factor of 1.75, the empirical evidence shows self-consistency within our wide ranged experimental data set.

Given previous work on the water impact of spheres [1], one might expect different classes of cavities to form for multi-droplet impact, depending on the parameter regime. Indeed, Bouwhuis et al. [4] observed cavities formed by micro-droplet streams that are best described as shallow seal. Here we observe six distinct cavity types, three occurring for multi-droplet streams at low matryoshka number and three more that occur for both droplet streams and jets (Fig. 2.6a). The first three occur only when droplets impact close to the maximum cavity size of the preceding sub-cavity (i.e., when $Mt \sim O(1)$). In these
Fig. 2.5: Normalizing cavity diameter \( (d_c) \) by jet diameter \( (d_j) \) collapses the jet data onto a line that is a function of impact velocity \( (U_s) \). A line with different slope and intercept is found for multi-droplet cases. We define a modified droplet diameter \( d_d^* = d_d/1.75 \), which collapses the normalized cavity diameter data onto a single line for multi-droplet and jet streams (inset).

Cases, sub-cavities maintain some independence from the matryoshka cavity as a whole and their independent behavior determines when and where cavity seal will occur. Thus, we call them sub-cavity seals. The other three occur for both droplet streams and jets and are the same type of cavity seals or pinch-offs observed by Aristoff and Bush [1] for hydrophobic spheres at low Bond number. We will discuss each of these in turn.

2.5.1 Sub-cavity collapse

As shown in Fig. 2.3b, as a single droplet impacts the water surface it creates a cavity that expands downward and radially outward from the point of impact [8]. After reaching its maximum size the bottom of the cavity collapses radially inwards and upwards while the top widens under the flattening influence of surface tension. When \( Mt < 1 \), the momentum of the first sub-cavity has already reversed and is in the upward direction by the time the second droplet impacts its base (Fig. 2.7a, \( t = 14.7 \) ms). Upon impact the second droplet spreads and transfers downward momentum to the cavity but only over the small area that
Fig. 2.6: a) Multi-droplet streams produce six cavity seal types, three of which - shallow seal, deep seal and surface seal - are common to jet streams and rigid sphere impact. Scale bars at the bottom of each image are 3 mm. b) The six cavity seal types for multi-droplet streams can be separated by a $Mt-We^*$ regime diagram. The horizontal dashed line represents the lowest $We$ found by Franz [10] for which dome over occurs for a single droplet (line also shown in c; note that $We$ from Franz [10] has been modified using $d_0^*$. Sub-cavity 1 collapse only occurs for $Mt < 1$, while sub-cavity 2 collapse only occurs between $1 < Mt < 4$. Sub-cavity dome over occurs for $We^* > 350$ and $Mt < 4$; the horizontal dotted line is to guide the eye. c) Shallow seal, deep seal and surface seal are better sorted on a $Bo^*-We^*$ plot. The solid line in c) is found by equating the dimensionless pinch-off times for deep and shallow seal (Eq. 2.3). Using the modified diameter $d_0^*$ causes the cavity seal regime boundaries to align for multi-droplet and jet streams. Data from previous studies are plotted and match the expected cavity seal regimes; [a] [2], [b] [4], [c] [24], [d] [33], [e] [27], [f] [12], and [g] [23]. Symbols and data are the same for all figures throughout the paper.
Fig. 2.7: Two types of sub-cavity collapse: sub-cavity 1 collapse a), and sub-cavity 2 collapse b). a) For $Mt < 1$, the second droplet impacts the cavity at $t = 14.7$ ms after sub-cavity 1 has begun to collapse ($f = 80$ Hz, $U_s = 1.77$ m $s^{-1}$, $d_d = 3.08$ mm, $We^* = 77$, $Bo^* = 0.4$, $Mt = 0.88$, and $\bar{f} = 0.14$). The dashed line indicates the location of the bottom of sub-cavity 1 at its maximum depth. b) For $1 < Mt < 4$, the second droplet impacts the first sub-cavity at $t = 4.0$ ms, the third droplet impacts at $t = 10.0$ ms and so on. The overall cavity is still expanding when each successive droplet impacts the cavity, thus, enabling the cavity to grow larger in the vertical direction ($f = 300$ Hz, $U_s = 1.80$ m $s^{-1}$, $d_d = 2.18$ mm, $We^* = 56$, $Bo^* = 0.21$, $Mt = 1.55$, and $\bar{f} = 0.36$). The cavity collapse occurs just below the bottom of sub-cavity 1 but above the bottom of sub-cavity 2. Dashed lines indicate the bottoms of sub-cavities 1 and 2 just prior to impact by the subsequent droplet. Numbers at the bottom of each frame indicate the time after initial impact in milliseconds. Corresponding positions in the regime diagrams can be seen in Fig 2.6b. Videos for a) and b) are shown in supplemental videos 3 & 4 respectively.

it impacts. The rest of the first sub-cavity is not impacted by the second droplet and continues its upward collapse while the second sub-cavity expands downward (Fig. 2.7a, $t = 16.7 - 20.7$ ms). These opposing velocities cause sub-cavities 1 and 2 to separate from each other (Fig. 2.7a, $t = 21.7$ ms). In this case cavity seal always occurs between the undisturbed free surface and the bottom of sub-cavity 1 and hence we will call this type of cavity seal sub-cavity 1 collapse.

If we maintain $We^* \lesssim 350$ and increase the matryoshka number, $1 < Mt < 4$, each successive droplet impacts while the preceding sub-cavity is still expanding downward (Fig. 2.7b, $t = 4$ ms). This gives the droplet more time to spread over the base of the sub-cavity, transferring the momentum to a larger area and counteracting the impending momentum reversal in the vertical direction, in contrast to $Mt < 1$. Although this transfers downward momentum it does not counteract the impending momentum reversal.
in the radial direction. The pinch-off location is determined by which sub-cavity is first to collapse (1 or 2). The difference between sub-cavities 1 and 2 is in the boundary conditions. The top of sub-cavity 1 resides on the free-surface which allows it to widen for a longer time. The widening of the top of sub-cavity 1 combined with the expansion of the bottom due to the impact of the second droplet delays the collapse of sub-cavity 1. On the other hand, the top of sub-cavity 2 is driven radially inward by surface tension and hydrostatic pressure, resulting in a collapse time that is sufficiently small to pinch-off before sub-cavity 1 (Fig. 2.7b, \( t = 18 \) ms). In this regime (\( 1 < Mt < 4, We^* \lesssim 350 \)) pinch-off always occurs between the bottoms of sub-cavity 1 and 2. Hence, we call this pinch-off type sub-cavity 2 collapse.

Increasing \( We^* \) above 350 while maintaining \( Mt < 4 \), we see in Fig. 2.8 that a splash crown forms at the bottom of each sub-cavity upon impact by the subsequent droplet. When conditions are right, the splash crown of each sub-cavity will collapse inward on itself and dome over, sealing off air flow between sub-cavities. We call this event sub-cavity dome over. In Fig. 2.8a we examine a cavity with \( Mt = 1.84 \) and \( We^* = 527 \). At \( t = 31 \) ms the splash crown at the base of the second sub-cavity is about to dome over. At 33.5 ms a droplet has impacted this dome over event morphing into a downward moving splash. Meanwhile the bottom of sub-cavity 2 continues its upward motion and the cavity collapses inward near the dome over event eventually leading to a full pinch-off that detains further droplets from entering into the lower portion of the cavity (\( t = 36 - 41 \) ms).

We see different behavior looking at a case with a slightly higher \( Mt \), but still below four. In Fig. 2.8b at 33.2 ms the splash crown at the base of the third sub-cavity domes over. As the next droplet continues downward, it breaks through this domed-over splash crown and morphs into a downward moving splash (\( t = 34.8 \) ms), which then impacts the dome over event at the base of the next sub-cavity, sub-cavity 4 (\( t = 34.8 - 36.5 \) ms). Because \( Mt \) is higher than the previous case, the sub-cavities have insufficient time to collapse. Hence, incoming droplets continue to break through each sub-cavity dome over event and the cavity eventually pinches off with a deep seal, as discussed below in § 2.5.2, and seen
Fig. 2.8: Two cases of sub-cavity dome over. When $Mt < 4$ and $We^* \gtrsim 350$ splash crowns form at the base of each sub-cavity and sometimes dome over. a) When $Mt$ is at the lower end of this range the cavity fully pinches off at the dome over positions detaining further droplets from entering into the lower portion of the cavity ($f = 100 \text{ Hz}, U_s = 4.41 \text{ m/s}, d_d = 3.42 \text{ mm}, We^* = 527, Bo^* = 0.52, Mt = 1.84$, and $\bar{f} = 0.08$). b) As $Mt \rightarrow 4$ droplets break through the sub-cavity dome overs reopening the cavity and preventing a full pinch-off ($f = 200 \text{ Hz}, U_s = 5.45 \text{ m/s}, d_d = 3.76 \text{ mm}, We^* = 886, Bo^* = 0.63, Mt = 3.27$, and $\bar{f} = 0.14$). Numbers at the bottom of each frame indicate the time after impact in milliseconds. The first image in each sequence shows a zoomed out view of the cavity and each following image shows a zoomed in view of the cavity of the area indicated by the dashed box. Corresponding positions in the regime diagrams can be seen in Fig 2.6b. Videos for a) and b) are shown in supplemental videos 5 & 6 respectively.
in supplemental video 6. The division between these three sub-cavity regimes can be seen on the \( Mt \) verse \( We^* \) plot shown in Fig. 2.6b. Note that for \( Mt < 4 \) and \( We^* > 914 \), it is unclear if the sub-cavity dome over regime persists; however, based on the work of Franz [10] we suspect that sub-cavity 1 would begin to dome over in this region effectively producing a surface seal.

### 2.5.2 Sphere-like cavity collapse

When \( Mt > 4 \), the cavity velocity becomes more stable (§ 2.6) and the matryoshka cavity assumes a more continuous form as droplets impact in rapid succession on newly formed sub-cavities. In this regime the three different types of cavity seals, seen for both multi-droplet streams and jets, are best shown on a \( Bo^*-We^* \) plot in Fig. 2.6c. These three cavity seal types are also seen in the water entry of hydrophobic spheres [1] and thus we call them sphere-like cavity seals.

The first of the sphere-like cavities occurs at the lowest \( We^* \) and \( Bo^* \) and is called shallow seal. Fig. 2.9 shows an image sequence of this event for a multi-droplet stream and a jet. As the droplet stream or jet impacts, a long, narrow, cylindrical cavity forms. A capillary wave forms near the surface, which travels down the cavity walls eventually causing it to pinch together at a depth on the order of the capillary length, as shown at 5.5 ms in Fig. 2.9a for a droplet stream and at 13.3 ms in Fig. 2.9b for a jet. This is the same sequence of events described by Aristoff and Bush [1] for shallow seal of cavities created by small spheres. This event is dominated by capillary forces as described by the small \( Bo^* \) for which it occurs. Although not specifically called shallow seal in their paper, this is the experimental regime that Bouwhuis et al. [4] studied. Following the shallow seal event, the cavity may experience a deep seal pinch-off as shown in Fig. 2.9b at 16.3 ms.

An interesting phenomenon that can occur for jets in the shallow seal regime is the break up of the jet after pinch-off due to disturbances that grow from the Rayleigh-Plateau instability [15]. When the cavity walls pinch-off and impact the sides of a small diameter jet, the disturbance can be sufficiently large to cause the jet to break up into an uneven droplet stream more quickly than normal. This break up is seen in Fig. 2.9b at 14.8 ms.
Fig. 2.9: Shallow seal shown for two stream types. A capillary wave forms near the surface and travels downward until pinch-off occurs at a depth on the order of the capillary length. a) A cavity created by a multi-droplet stream pinches off with a shallow seal for small \( \text{Bo}^* \) (\( f = 6000 \) Hz, \( U_s = 6.39 \) m s\(^{-1} \), \( d_d = 0.39 \) mm, \( \text{We}^* = 128 \), \( \text{Bo}^* = 0.0069 \), \( Mt = 39.7 \), and \( \delta = 0.37 \)). b) A cavity created by a jet experiences first shallow and then deep seal (\( U_s = 5.78 \) m s\(^{-1} \), \( d_d = 0.41 \) mm, \( \text{We}^* = 190 \), and \( \text{Bo}^* = 0.023 \)). After shallow seal pinch-off, the jet is perturbed and begins to break up into a droplet stream from the Rayleigh-Plateau instability (\( t \geq 14.8 \) ms). Numbers at the bottom of each frame indicate the time after impact in milliseconds. Corresponding positions in the regime diagrams can be seen in Fig 2.6b & c. Videos for a) and b) are shown in supplemental videos 7 & 8 respectively.
and explains why the smooth cavity created by the jet becomes irregular after pinch-off.

As $We^*$ and $Bo^*$ increase, inertial and pressure forces become more important and cavities enter the deep seal regime. Unlike the shallow seal event, the cavity walls pinch together approximately halfway between the surface and the bottom of the cavity at the time of pinch-off (Fig. 2.10). The predominate force driving this event is hydrostatic pressure as described by the higher $Bo^*$. While the jet literature cited in the introduction does not specify the type of cavity seal, the images and range of $Bo$ and $We$ reported in these references indicate deep seal cavities [12, 23, 24, 27, 33]. As shown in Fig. 2.6c, these prior studies fall in the deep seal regime on the $Bo^*-We^*$ diagram.

The division between the shallow and deep seal regimes shown by the solid line in Fig. 2.6c are found by equating the dimensionless pinch-off times for shallow and deep seal [1]. Pinch-off time is nondimensionalized as $t_p U_s / d_s$ in Fig. 2.11. Shallow seal is driven by surface tension, thus the dimensionless pinch-off time can be predicted using $We^*$ (Fig. 2.11a), whereas deep seal is driven by hydrostatic pressure and thus pinch-off time scales with $Fr^*$ (Fig. 2.11b). Equating the shallow and deep nondimensional pinch-off times gives the equation for the solid black line in Fig. 2.6c separating the shallow and deep seal regimes as

$$We^* = \frac{m_{deep}^2}{m_{shal}^2} (Bo^*)^{-1}, \quad (2.3)$$

with $m_{shal}$ equal to the slope of the pinch-off time fit for shallow seal and $m_{deep}$ equal to the slope of the fit for deep seal, as shown in Fig. 2.11. It is worth noting that modifying the droplet diameter as discussed in § 2.5 causes the dimensionless pinch-off times to fall on a common line for both jets and droplet streams. This again indicates that the cavity diameter governs the pinch-off characteristics.

An interesting phenomenon can occur for multi-droplet impacts that border on three cavity seal regimes. For example, on Fig. 2.6c, the blue circle inside the black square corresponds to a shallow seal event with $Bo^* = 0.073$ and $We^* = 234$, yet the location of the symbol falls in the deep seal regime. Furthermore, for this case $Mt = 4.84$, which places the case close to the sub-cavity 2 collapse regime. Because the pinch-off for shallow
Fig. 2.10: Deep seal shown for two stream types. Pinch-off occurs approximately halfway between the free-surface and cavity bottom. a) A cavity formed by a multi-droplet stream with higher Bo* and We* pinches off in a deep seal. \( f = 1500 \text{ Hz}, U_s = 6.59 \text{ m/s}, d_d = 2.17 \text{ mm}, We^* = 748, Bo^* = 0.21, Mt = 16.9, \) and \( f_l = 0.49 \). b) A cavity formed by a jet at a moderate Bo* pinches off in a deep seal \( (U_s = 7.09 \text{ m/s}, d_j = 0.71 \text{ mm}, We^* = 527, \) and \( Bo^* = 0.068) \). Because Bo* is relatively small for this jet impact, we still observe the downward moving capillary wave near the surface seen for shallow seal. Numbers at the bottom of each frame indicate the time after impact in milliseconds. Corresponding positions in the regime diagrams can be seen in Fig 2.6b & c. Videos for a) and b) are shown in supplemental videos 1 & 2 respectively.
Fig. 2.11: Dimensionless pinch-off time for a) shallow seal and b) deep seal. The slopes of the respective fits are used in Eq. 2.3 to compute the boundary between shallow and deep seal shown on Fig. 2.6c. Symbols are outlined in the legend of Fig. 2.6a.

Seal and sub-cavity 2 collapse occurs at approximately the same depth below the surface, we suggest that the two mechanisms associated with these two types of cavity seal have an additive effect resulting in a shallow pinch-off. In this case shallow seal was closely followed by deep seal, reflecting the intersection of three regimes of cavity seal for this case.

As $We^*$ increases above that seen for deep seal, the low air pressure created by the velocity of the stream and surface tension forces cause the splash crown to collapse inward on itself, or dome over, sealing the cavity off from further air entrainment [18]. This regime is called surface seal. The cavity then pulls away from the free surface entering deeper into the pool. Deep seal often follows surface seal splitting the cavity in two. This sequence of events can be seen in Fig. 2.12 with dome over occurring at 12.0 ms in a) for the multi-droplet stream and at 8.3 ms in b) for the jet. Even after the dome over has occurred the stream of water continues to penetrate into the cavity, but has now been distorted by passing through the splash crown. The cut off between the deep seal and surface seal regimes occurs at $We^* = 914$, which is the same $We^*$ (following modification of the droplet diameter) at which Franz [10] saw dome over beginning to occur for single droplet impacts.
Fig. 2.12: Surface seal for two types of stream impacts. a) The splash crown of a cavity created by a multi-droplet stream domes over at $t = 12.0$ ms resulting in surface seal ($f = 2500$ Hz, $U_s = 7.51$ m s$^{-1}$, $d_d = 2.06$ mm, We$^* = 923$, Bo$^* = 0.19$, Mt = 28, and $f = 0.69$). b) The same phenomenon is observed for a cavity created by a jet with dome over occurring at $t = 8.3$ ms ($U_s = 9.69$ m s$^{-1}$ and $d_d = 0.71$ mm, We$^* = 921$, and Bo$^* = 0.068$). Numbers at the bottom of each frame indicate the time after impact in milliseconds. Corresponding positions in the regime diagrams can be seen in Fig 2.6b & c. Videos for a) and b) are shown in supplemental videos 9 & 10 respectively.

This implies that the dome over event is predominately caused by the impact of the first droplet and thus can be thought of as either a sub-cavity 1 dome over or a surface seal event from a sphere-like water entry.

### 2.6 Cavity velocity

We will now examine the downward growth rate of the cavity. In Fig. 2.13a the unsteady growth of four different cavities in time is presented. The effect of each individual droplet impact can be seen in the cavity depth evolution, with the downward velocity varying over the period of expansion of each sub-cavity, $T_{max}$. Thus, in describing the downward velocity of the cavity we can discuss two parts: the average cavity velocity (captured by linear fits)
and the oscillating cavity velocity.

Based on the work of Birkhoff and Zarantonello [3] (pg. 16) we can develop a model that predicts the cavity velocity, $U_c$. To begin we neglect gravity during impact and consider the impact of a jet in a reference frame moving at the velocity of the cavity bottom. Noting that the pressure of the two media (jet and pool) at the stagnation point are balanced and applying the Bernoulli equation along the axial stream lines in both media, the unknown pressure at the stagnation point can be eliminated. This approach yields

$$\frac{1}{2} \rho (U_s - U_c)^2 = \frac{1}{2} \rho U_c^2, \quad (2.4)$$

and rearranging gives

$$\frac{U_c}{U_s} = \frac{1}{2}. \quad (2.5)$$

Bouwman et al. [4] used computation and experimental methods to show that for multi-droplet streams $U_c/U_s$ approaches 1/2 as $\tilde{f}$ approaches unity. We can now modify the energy balance in Eq. 2.4 accounting for the intermittent droplet impact by multiplying the left hand side by $\tilde{f}$

$$\frac{1}{2} \rho (U_s - U_c)^2 \tilde{f} = \frac{1}{2} \rho U_c^2. \quad (2.6)$$

Now solving this for $U_c/U_s$ we get

$$\frac{U_c}{U_s} = \frac{\sqrt{\tilde{f}}}{1 + \sqrt{\tilde{f}}}, \quad (2.7)$$

where $\tilde{f} \in [0, 1]$ (Fig. 2.3d). As shown in Fig. 2.13b, this modified theory accurately captures the experimentally measured cavity velocities. As $\tilde{f}$ approaches one, the multi-droplet stream becomes more jet-like approaching the theoretical value of 1/2. Experimental values for all of the 65 jets studied have a mean value of 0.492 as marked. Most studies agree that the jet cavity velocity equals half of the jet velocity [22–24, 33].

To describe the oscillations in the cavity we consider the black line in Fig. 2.13a,
Fig. 2.13: Downward cavity velocity of multi-droplet streams and jets. a) Depth of the cavity in time for four values of $Mt$. The black line represents a multi-droplet stream with $\tilde{f} = 0.14$ and for a case with sub-cavity 1 collapse. The blue curve represents a multi-droplet stream with sub-cavity 2 collapse and $\tilde{f} = 0.26$. A linear fit is applied to estimate the average cavity velocity (red dashed line). The magenta line represents a multi-droplet stream with $\tilde{f} = 0.27$, which displays shallow seal behavior. Note that as $Mt$ increases, the oscillations in the cavity velocity become less pronounced. A jet (green line) also forms a shallow seal cavity with velocity nearly identical to the $Mt = 16.5$ multi-droplet case. b) The average downward cavity velocity ($U_c$) normalized by the stream velocity ($U_s$) as a function of $\tilde{f}$. The black dashed line represents Eq. 2.7 and the red dotted line represents the theoretical velocity ratio for a jet. The mean velocity ratio for all 65 jet cases is marked by the grey dot and a standard box and whisker plot ($\tilde{f} = 1$). c) The amplitude of oscillation ($U_a$) normalized by the average cavity velocity ($U_c$) as a function of $Mt$. The vertical dotted lines indicate transitional $Mt$ numbers of 1 and 4. The large circles in b) and c) outline data points represented by the corresponding colors of the lines in a).
which represents a cavity formed by the impact of a multi-droplet stream with $Mt = 0.88$. Taking the numeric derivative of this curve we obtain the downward cavity velocity over time, which is found to be periodic over the time between impact of successive droplets (appearing similar to a sine wave). The oscillating amplitude of the cavity velocity is then averaged by finding the mean amplitudes of all periods ($U_a = \frac{1}{N} \sum_{n=1}^{N} (u_n^{max} - u_n^{min})/2$, where $u_n^{max}$ and $u_n^{min}$ are the maximum and minimum velocities within the $n^{th}$ period). Because $Mt < 1$ we can see the first sub-cavity reach its maximum size at $T_{max}$. The growth of a multi-droplet-stream cavity can be modeled by constructing a superposition of curves from portions of $h(t)$ vs. $t$, where each curve represents the growth of a sub-cavity. When $Mt = 1$, the second droplet impacts just as the curve reaches its apex and the parabolic shaped growth is repeated (Fig. 2.13a black line). When $Mt = 2$, the second droplet impacts at $T_{max}/2$ and only this first part of the curve is repeated (not labeled). Thus, the portion of the parabola that is repeated is from $t = 0$ to $t = T_{max}/Mt$. When $Mt > 4$ the portion of the curve that is repeated is nearly a straight line and hence oscillations in the cavity velocity are small (Fig. 2.13a magenta line). Not only can this be seen in Fig. 2.13a but it can also be seen by comparing the murals of the sphere like cavities in Figs. 2.9a, 2.10a, and 2.12a, against the uneven growth rate of cavities created by multi-droplet streams with low $Mt$ as seen in Figs. 2.7 and 2.8. Quantitatively this means that the average amplitude of oscillation of the cavity velocity, $U_a$, normalized by the average cavity velocity, $U_c$, depends on $Mt$ (Fig. 2.13c). At $Mt < 4$ the cavity velocity becomes very unsteady, and around a $Mt = 1$ the amplitude of oscillation is on the order of the average cavity velocity.

2.7 Cavity dimensions

In this section we will discuss the various cavity dimensions including pinch-off depth $h_p$, the depth of the cavity bottom at pinch-off $h_b$, the cavity depth after the entire stream has impacted $h_c$, and the average cavity diameter $d_c$ as shown in Fig. 2.4.

2.7.1 Pinch-off depth

The pinch-off depth ($h_p$) is affected by the type of cavity closure, and thus depends on
$Mt$, $We^*$ and $Bo^*$. Fig. 2.14a plots the dimensionless pinch-off depth as a function of $Mt$, illustrating this dependence on the closure regime. At $Mt < 1$, when sub-cavity 1 collapse occurs the pinch-off depth is approximately equal to $d_d$ (non-modified droplet diameter). At $1 < Mt < 4$ sub-cavity 2 collapse occurs and the nondimensional pinch-off depth doubles to approximately $2d_d$. In this same matryoshka number range, but at $We^* > 315$ sub-cavity dome over occurs. The depth of sub-cavity dome over is unpredictable and multiple sub-cavities may dome over within one matryoshka cavity. At $Mt > 4$ we see the three pinch-offs common to solid-sphere water entry (i.e., surface, shallow and deep seal). Surface seal occurs at the surface $h_p/h_c = 0$. Shallow seal is on the order of the capillary length, approximately three to six droplet diameters ($3 < h_p/d_d < 6$) below the surface and increasing slightly as $Mt$ increases. Deep seal occurs approximately halfway between the surface and the bottom of the cavity ($h_p/h_b \approx 1/2$).

In the case of deep seal, the ratio of pinch-off depth to cavity depth after the entire stream has impacted the cavity reveals a relatively constant ratio $h_p/h_c \approx 1/3$ (Fig. 2.14b), which is the same as reported by Oguz et al. [22] for jets. [3] showed theoretically that when the density of the jet and pool are equal, the depth of the cavity equals the length of the jet used to form it (i.e., $h_c = L_j$). We can use this to explain the ratio $h_p/h_c = 1/3$. The jet is cut off when pinch-off occurs and the lower portion of the cavity contains a portion of the jet that is $1/2 h_b$ long. This jet will add its remaining length to the depth of the cavity $(h_c = h_b + 1/2 h_b)$, thus, $h_p = 1/2 h_b = 1/3 h_c$. A similar relationship between $h_p$, $h_b$ and $h_c$ is derived for multi-droplet streams in § 2.7.3.

### 2.7.2 Predicting cavity depth and diameter using an energy balance

Cavity dimensions can be predicted by an energy analysis similar to that performed by Engel [8]. The kinetic energy of a multi-droplet stream of water can be expressed as

$$KE_d = \frac{\pi}{12} \rho d_d^3 U_s^2 N, \quad (2.8)$$
Fig. 2.14: Pinch-off depth of multi-droplet streams. a) Nondimensional pinch-off depth for all multi-droplet streams as a function of $Mt$. Pinch-off depth is normalized by the non-modified droplet diameter $d_d$. b) Pinch-off depth as a function of cavity depth for the deep seal regime only for multi-droplet streams and jets. The upper bound for the theoretical range is $h_p = \frac{1}{2} h_c$, as predicted in §2.7.3. The lower bound predicted for droplets is set by the line $h_p = \frac{3}{4} h_c$, which is also the theoretically predicted pinch-off depth for jets. The least squares fit to the droplet data has a slope of 0.319 and the fit to the jet data 0.277. Symbols and data are the same as in Fig. 2.6.
where \( N \) is the number of droplets that impact the cavity, and \( U_s \) is the velocity of the stream. The jet kinetic energy can be expressed by

\[
KE_j = \frac{\pi}{8} \rho d_j^2 L_j U_s^2. \tag{2.9}
\]

In this study, the stream enters the cavity until the pinch-off event blocks further flow. Hence, \( N \) can be calculated as

\[
N = f t_p - f h_p/U_s, \tag{2.10}
\]

where \( f t_p \) represents the number of droplets that pass the undisturbed free surface before \( t_p \) and \( f h_p/U_s \) represents the number of droplets that do not make it into the lower portion of the cavity. The jet length, \( L_j \), can be similarly calculated as

\[
L_j = U_s t_p - h_p. \tag{2.11}
\]

The potential energy of the cavity, which is equal to the work done on the pool by the stream, can be broken up into two portions, the energy due to the hydrostatic pressure surrounding the cavity, \( PE_{pr} \), and the energy due to the generated surface area, \( PE_\sigma \). We model the maximum cavity size described in § 4.4 as a long narrow cylinder of constant diameter, \( d_c \), and depth \( h_c \). The potential energy from pressure (\( PE_{pr} = \int_V \rho gz dV \)) simplifies to

\[
PE_{pr} = \frac{\pi}{8} \rho gh_c^2 d_c^2. \tag{2.12}
\]

The potential energy of the cavity due to generated surface area is

\[
PE_\sigma = \pi d_c h_c \sigma. \tag{2.13}
\]

Equating the kinetic (Eq. 2.8) to the potential energies (Eqs. 2.12 and 2.13) for a multi-droplet stream and rearranging yields

\[
(d_c^2 \rho g)h_c^2 + (8d_c \sigma)h_c - \frac{2}{3} \rho d_j^3 U_s^2 N = 0. \tag{2.14}
\]
Fig. 2.15: The nondimensional cavity size for both jets and multi-droplet streams is collapsed by a function of $Bo$ and $Fr$ computed from an energy balance. The theoretical predictions of Eqs. 2.15 and 2.16 are shown by the dotted line, as both equations collapse to a single curve. Note that the droplet diameter used in this analysis is $d_d$, the non-modified droplet diameter. Symbols and data are the same as in Fig. 2.6.

Assuming a positive solution to the quadratic formula, nondimensionalizing and simplifying we find

$$\frac{h_c d_c}{d_d^2} = -\frac{4}{Bo} + \sqrt{\left(\frac{4}{Bo}\right)^2 + \frac{2N}{3}Fr} \equiv \Omega_d,$$

(2.15)

for droplets. Performing the same analysis for a jet yields

$$\frac{h_c d_c}{d_j^2} = -\frac{4}{Bo} + \sqrt{\left(\frac{4}{Bo}\right)^2 + \frac{L_j}{d_j}Fr} \equiv \Omega_j.$$

(2.16)

The left hand side of Eqs. 2.15 & 2.16 can be thought of as a characteristic cavity size, which is a function of $Bo$ and $Fr$. Experimental data in Fig. 2.15 shows good agreement with this relationship. Eqs. 2.15 & 2.16 only tell us about the product of $h_c$ and $d_c$. If we want to predict the cavity depth ($h_c$) we must know the cavity diameter ($d_c$). However, the ratio $d_c/d_{j,d}$ was empirically shown to be a linear function of impact velocity (Fig. 2.5).

2.7.3 Predicting cavity depth using the Bernoulli equation
We will now discuss another way that the depth of the cavity, $h_c$, can be predicted. Recall that Birkhoff and Zarantonello [3] showed mathematically that when the density of the jet and the density of the pool are equal, then $h_c = L_j$. The ratio of the cavity depth to the jet length for a given time can be expressed as

$$\frac{h(t)}{l_j(t)} = \frac{U_c t}{(U_s - U_c)t}, \quad (2.17)$$

where $l_j(t)$ is the length of jet that has impacted the cavity by time $t$. By inserting $U_c = \frac{1}{2}U_s$ from Eq. 2.5 and simplifying we get:

$$\frac{h(t)}{l_j(t)} = 1. \quad (2.18)$$

We can apply the same analysis to a multi-droplet stream by solving for $U_c$ in Eq. 2.7 and placing it into Eq. 2.17 yielding

$$\frac{h(t)}{l_d(t)} = \sqrt{\bar{f}}, \quad (2.19)$$

where $l_d(t)$ is approximately equal to the distance from the first droplet that impacts to the last droplet that has impacted by time $t$. These equations work well to predict the depth of the cavity, $h_c$ for jets in Fig. 2.16a and droplets in Fig. 2.16b. Eq. 2.18 slightly over predicts the depth of the cavity for jets. This can be explained by recalling that in Eq. 2.4 the effect of gravity was neglected. Thus, the hydrostatic pressure, which pushes against the impinging jet is not taken into account; therefore, departure from Eq. 2.18 increases with increasing depth. One could modify these equations to include gravity but the overall conclusion remains the same.

We can now revisit the relationship between the cavity depths $h_p$ and $h_c$. The total cavity depth for a multi-droplet stream can be written as $h_c = h_b + \frac{1}{2}h_b\sqrt{\bar{f}}$ leading to $h_p = \frac{1}{2}h_b = h_c/(\sqrt{\bar{f}} + 2)$ where $h_p \in [\frac{1}{3}h_c, \frac{1}{2}h_c]$ which is closely supported by the trends of Fig. 2.14b. It is likely that the difference between the theoretical estimate and the experimental result is that in the experiment it is possible for a portion of the jet or a
Fig. 2.16: a) The length of the jet that impacts the cavity bottom equals the depth of the cavity that it creates. b) An equivalent expression for multi-droplet streams works to predict the depth of the cavity, $h_c$. The dashed lines in a and b represent Eqs. 2.18 and 2.19 respectively. The gray circle in b) represents the average $h_c/L_j$ for all of the jet cases with the uncertainty bands at two standard deviations. $L_s$ represents the length of either the multi-droplet stream or jet stream. Symbols and data are the same as in Fig. 2.6.

droplet to get ‘caught’ in the pinch-off event, and not descend downward with the rest of the fluid, thus reducing the effective full stream cavity depth.

Finally, the depth of the cavity can be estimated using Eq. 2.11 by substituting $h_c$ for $L_j$ and assuming a semi-infinite jet that is cut short by the pinch-off event

$$h_c = U_s t_p - h_p.$$  \hspace{1cm} (2.20)

We can then replace $h_p$ depending on the cavity type: for deep seal we use $h_p = \frac{1}{2}h_c$, for shallow seal $h_p$ is approximately equal to the capillary length ($\lambda$) and for surface seal $h_p = 0$, thus each of these cases becomes

$$h_{c,jets} = \begin{cases} 
\frac{3}{4}U_s t_p & \text{deep} \\
U_s t_p - \lambda & \text{shallow} \\
U_s t_p & \text{surface.} 
\end{cases}$$  \hspace{1cm} (2.21)
Applying this same method to multi-droplet streams, but using $L_d = \frac{h_c}{\sqrt{f}}$, the equations for deep, shallow, and surface seal become

$$h_{c, \text{droplets}} = \begin{cases} \frac{U_st_p \sqrt{\bar{f}}}{1 + \frac{\bar{f}}{f}} & \text{deep} \\ (U_st_p - \lambda) \sqrt{\bar{f}} & \text{shallow} \\ U_st_p \sqrt{\bar{f}} & \text{surface}. \end{cases} \tag{2.22}$$

These analyses show that the dimensions of the cavity for jets can be used to unravel the multi-droplet cavity dimensions and that the two different stream types are indeed related physically. For instance, if $\bar{f} = 1$, which is the case for a jet, Eqs. 2.22 simplify to the jet equations Eqs. 2.21.

We emphasize the differences between the models presented in §2.7.2 & 2.7.3. The energy balance based model in §2.7.2 predicts the cavity size (a non-dimensional measurement including both cavity depth ($h_c$) and cavity diameter ($d_c$)) by considering all of the kinetic energy of the incoming stream and comparing it to the potential energy of the entire cavity that it forms. In order to predict cavity size $h_c d_c / d_c^2$ and obtain some information on the cavity diameter $d_c$ surface tension and gravity must be considered as both resisting the radial expansion of the cavity and contributing to the cavity’s potential energy. If one desires to obtain the cavity depth using this energy balance based model the average cavity diameter $d_c$ must be known. On the other hand the Bernoulli based approach in §2.7.3 predicts $h_c$ directly by using an energy balance solely at the cavity bottom. Surface tension and gravitational parameters are unnecessary in this method because the energy balance occurs on a short section of a single streamline (the axis of the flow), where the effect of these parameters is negligible. To compare the fidelity of these two methods in predicting cavity depth we plot measured versus predicted values of $h_c$ in Fig. 2.17. The energy based method predicts $h_c = \Omega_s d_c^2 / d_c$ (Eqs. 2.15 & 2.16 rearranged), where $d_c$ is evaluated from experimental measurements (Fig. 2.4b). The Bernoulli method predicts $h_c = l_s \sqrt{f}$ (Eqs. 2.18 & 2.19 rearranged). The Bernoulli method results in a slightly more accurate prediction of $h_c$ without considering surface tension and gravity. In fact, introducing gravity and
Fig. 2.17: A comparison of the two methods for predicting cavity depth $h_c$ is shown. The y-axis is the measured value of the cavity depth $h_c$, while the x-axis is the predicted value of $h_c$ based on two different approaches. The first term on the x-axis comes from the energy based method ($h_c = \Omega \sigma \sqrt{d_c/\gamma}$, Eqs. 2.15 & 2.16 rearranged), while the second term is from the Bernoulli based method ($h_c = \sqrt{\bar{f}}$, Eqs. 2.18 & 2.19 rearranged) as marked in the legend. An exact prediction is represented by the dashed line.

surface tension does not improve the model; implying that the surface tension and gravity may not be dominant contributors to the downward growth of the cavity. However, they are important when considering the radial growth and collapse of the cavity as outlined in section § 2.5.

2.8 Conclusion

Many past water impact studies have focused separately on the initial impact of single droplets, jets or solids into a deep pool of liquid. Some groups have loosely connected multi-droplet stream impact to jet and solid sphere impact [4, 11], yet have not necessarily studied an expansive range of parameters nor threaded the underlying physical scaling laws together quantitatively. Despite the fact that the initial impact of multi-droplet streams create different cavity aesthetics (rough ridges) when compared to the cavities made by jets, or solid hydrophobic spheres (Fig. 2.1), our experimental work shows that all initial impact
types create intrinsically similar cavities that can be connected by physical arguments.

Empirically, we show that the diameter of a cavity made by a multi-droplet stream matches that made by a jet if the diameter of the droplets are 1.75 times the jet diameter. This modification in diameter is valid over a large range of impact parameters. The cavity types can be separated by introducing a new dimensionless number we coin the matryoshka number, $Mt$. When $Mt < 4$ sub-cavities behave independently from the cavity as a whole, forming cavities reminiscent of single droplet impact. In this region we find three new cavity shapes: subcavity 1 collapse, subcavity 2 collapse and subcavity dome over. When $Mt > 4$ the multi-droplet streams act like jets creating well known cavities similar to ones created by hydrophobic spheres and distinguished by their pinch-off locations: deep seal, shallow seal and surface seal. Further, the cavities of multi-droplet streams become more smooth, jet-like, and approach the jet cavity velocity and depth as the dimensionless droplet stream frequency $\tilde{f} \to 1$. Each of the regimes can also be accurately predicted by the $Bo^*$ and $We^*$ numbers as one might expect. Finally, the cavity dimensions (i.e., pinch-off depth, cavity depth and cavity diameter) can be predicted from a simple energy balance or by expanding the work of Birkhoff and Zarantonello [3] on jets.

The scope of future work might include the effect of increasing jet and droplet sizes (if possible), predicting when each sub-cavity will dome over and finding a theoretical explanation for the scaling of the cavity diameter between multi-droplet streams and jets (e.g., 1.75 found in Fig. 2.5). There is also the question of continuous impact of the jet or multi-droplet stream, and how the underwater bubble field will interact with the incoming stream and whether similar cavities can be formed after the initial impact and collapse of the first cavity. One application of these findings might be the impact of multiple objects and object types in succession upon the water surface to influence the cavity shape and resultant forces.

2.9 Appendix: Alternative scaling

Bouwhuis et al. [4] studied multi-droplet stream impact on a pool using boundary integral simulations with some experimental data for validation. They studied the shallow seal
Fig. 2.18: Prediction of $d_c$ and $t_p$ with a modified Weber number, $We_m = \rho U_c^2 d_d/\sigma$, as discussed by Bouwhuis et al. [4]. a) The nondimensional cavity diameter is said to scale as $d_c/d_d = 0.1625 We_m$ shown by the dashed line. b) The pinch-off time nondimensionalized by a capillary time scale is said to scale with $We_m^{3/2}$ with the dashed line showing $t_p \sqrt[3]{8\sigma/\rho d_d^3} = 0.1 We_m^{3/2}$. Symbols are outlined in the legend of Fig. 2.6a.

regime for micrometer-sized droplets with $Bo \sim O(10^{-4})$ and developed scaling arguments using a modified Weber number defined by the cavity velocity, $We_m = \rho U_c^2 d_d/\sigma$, to predict $d_c$ and $t_p$. To verify the validity of these trends over a more expansive parameter range we plot our experimental data for their proposed scaling arguments. Fig. 2.18 shows that the scaling of $d_c$ and $t_p$ works for the shallow seal data in each case, but does not work well for the other pinch-off regimes. The dimensionless cavity diameter was better predicted by plotting it against impact velocity as shown in Fig. 2.5 and the nondimensional pinch-off time is better predicted with $We^*$ as shown in Fig. 2.11.
REFERENCES


CHAPTER 3
ENTRY OF A SPHERE INTO A WATER-SURFACTANT MIXTURE AND THE EFFECT OF A BUBBLE LAYER

3.1 Prologue

The purpose of this study is to investigate the effect of the addition of surfactant to the pool liquid. The data for this study was collected, processed and analyzed Utah State University. The study is currently in review in Physics of Fluids under the title “Entry of a sphere into a water-surfactant mixture and the effect of a bubble layer” and is currently in review. The authors listed are: Nathan B. Speirs, Mohammad M. Mansoor, Randy C. Hurd, Saberul I. Sharker, Wesley G. Robinson, B. J. Williams and Tadd T. Truscott. The article in its entirety is presented below.

3.2 Abstract

A rigid sphere entering a liquid bath does not always produce an entrained air cavity. Previous experimental work shows that cavity formation, or the lack thereof, is governed by fluid properties, wetting properties of the sphere and impact velocity. In this study, wetting steel spheres were dropped into a water-surfactant mixture with and without passing through a bubble layer first. Surprisingly, in the case of a water-surfactant mixture without a bubble layer, the critical velocity for cavity formation becomes radius dependent. When a soap bubble layer is present subsurface cavities form at all impact velocities. Our analysis shows that the bubble layer wets the sphere prior to impact with a patchy coating of droplets and bubbles. The droplets alter the splash and create an aperture for air entrainment which leads to cavity formation at wetted locations on the sphere surface. The water-surfactant entry behavior of these partially wetted spheres results in a progression of cavity formation regimes with increasing Weber number, similar to the cavity regimes of hydrophobic spheres.
entering water. Nonuniform droplet coatings create cavity asymmetries altering transitions between these regimes.

3.3 Introduction

When a rigid sphere impacts a liquid surface with sufficient velocity it creates a splash crown and an air-filled subsurface cavity [1]. Duez et al. [2] showed that the presence or absence of a subsurface cavity for a smooth sphere depends on liquid surface tension ($\sigma$), viscosity ($\mu$), static wetting angle ($\theta$) and sphere impact velocity ($U$). For hydrophilic or wetting spheres ($\theta < 90^\circ$) cavities form when the impact occurs above a constant critical velocity, $U_{cr}$. For hydrophobic or non-wetting spheres ($\theta > 90^\circ$), the value of $U_{cr}$ decreases with increasing $\theta$ [2]. Zhao et al. [10] expanded this work by showing that cavity formation also depends on sphere roughness $R_z$ with the critical velocity for cavity formation decreasing with increasing roughness.

Aristoff & Bush investigated cavities formed by small non-wetting spheres, identifying four distinct cavity types named quasi-static, shallow, deep and surface seal cavities [3]. Spheres that are half wetting and half non-wetting produce asymmetric cavities and curved trajectories beneath the free-surface as shown by Truscott & Techet [4] and Bodily et al. [5] for slender torpedo-like bodies. Although the wettability of an object is important to the water entry behavior, one might wonder what will happen if an object is partially wetted before entry. For instance, vegetables being dropped into industrial washing basins, and the washing of dishes in the kitchen sink. However, cavity formation characteristics in surfactant mixtures, pools covered with a bubble layer or partially pre-wetted projectiles have not been addressed previously.

Herein, we first examine the effect of a surfactant on the critical velocity necessary for a sphere to form a cavity during free-surface entry. Second, bubbles commonly form in surfactant mixtures, and observations show that spheres that pass through a bubble layer before free-surface entry create cavities at lower impact velocities than anticipated. Hence, we examine the principle mechanism by which bubbles cause these unanticipated cavities (i.e., partial wetting). We then report on the cavity formation types.
3.4 Experimental setup

A schematic of the experimental setup is represented in Fig. 4.2. Experiments were performed with smooth stainless steel spheres \( (\rho_s = 7750 \text{ kg/m}^3) \) with radii \( R_s = 1.59 \) to 12.70 mm \( (\pm 0.0025 \text{ mm}) \). Spheres were dropped into a glass tank \( (40 \times 40 \times 122 \text{ cm}^3) \) from an electromagnet. Drop height was varied to alter impact velocity \( U \) at the water free surface from approximately 0.44 to 7.67 m/s, representing a parameter space where sphere deceleration is negligible during cavity formation and collapse [6]. The impact event was filmed using three Photron SA3 high speed cameras with diffuse backlighting. Two cameras were used to film both above and below the free surface from the side and a third camera captured the entry event from above for a top-down view. Close-up color images were taken with a Phantom v2511. Critical dimensionless numbers and and their ranges for this study include: Froude number \( (Fr = U^2/gR_s; \ 1.6-3,800) \), Weber number \( (We = \rho_l U^2 R_s/\sigma; \ 11-23,000) \) and Bond number \( (Bo = \rho_l g R_s^2/\sigma; \ 0.91-58) \).

![Schematic diagram](image)

Fig. 3.1: (a) Stainless steel spheres of radius \( R_s \) and density \( \rho_s \) were dropped from an electromagnet into a glass tank, impacting the liquid surface with velocity \( U \). The tank was filled with a water-surfactant mixture having density \( \rho_l \), viscosity \( \mu \) and surface tension \( \sigma \). For some experiments the water-surfactant mixture was covered with a bubble layer of height \( h_B \) composed of average bubble diameters \( d_B \).

The glass tank was filled with a water-surfactant mixture made with Ajax dish soap.
The water-surfactant mixture was characterized by the following physical properties: density $\rho_l = 999 \text{ kg/m}^3$, viscosity $\mu = 1.09 \pm 0.01 \times 10^{-3} \text{ Pa} \cdot \text{s}$, surface tension $\sigma = 27.3 \pm 0.2 \text{ mN/m}$, and stainless steel advancing static contact angle $\theta = 30^\circ \pm 4^\circ$. Experimental work with the surfactant mixture was not performed for more than three days. Liquid properties ($\mu, \sigma & \theta$) were measured with each new water-surfactant mixture, with 95% confidence of the mean values listed above. Surface bubble layers were created for heights $h_B$ ranging 5-100 mm which comprised of bubble diameters $d_B$ in the range of 1-20 mm.

3.5 Results

Figure 3.2 displays images of smooth, wetting spheres impacting the water-surfactant mixture at various impact conditions. In (a) a sphere with radius $R_s = 4.76 \text{ mm}$ impacts the pool with a velocity of $U = 5.42 \text{ m/s}$ without forming a cavity. This impact velocity is much higher than the critical velocity for cavity formation for the water-surfactant mixture as predicted by Duez et al. [2] ($U_{cr} = 0.1\sigma/\mu \approx 2.5 \text{ m/s}$). In (b) a larger sphere ($R_s = 11.11 \text{ mm}$) impacts at the same velocity forming a cavity. This shows that the critical velocity for cavity formation is dependent on $R_s$ in a water-surfactant mixture. In (c) a sphere with radius $R_s = 4.76 \text{ mm}$ impacts the pool at $U = 2.43 \text{ m/s}$ without forming a cavity, but in (d) a sphere with the same radius and velocity forms a cavity after first passing through a bubble layer resting on the pool surface.

We first examine the effect of surfactant on the critical velocity for cavity formation $U_{cr}$ for a clean free-surface (no bubbles). As shown in Fig. 3.3(a), cavity formation in the water-surfactant mixture does not occur at a constant $U_{cr}$. Rather, as shown in the inset, $U_{cr}$ varies with $R_s$, where $U_{cr}$ is approximately equal to the critical velocity in pure water for small $R_s$ and decreases towards the predicted critical velocity [2] in the surfactant mixture as $R_s$ increases. Hence, $U_{cr}$ for a water surface mixture lies between the Duez prediction for water and a water-surfactant mixture. It seems that the surfactant changes the dynamic wetting properties, greatly altering the predictability of cavity formation [2].

As the entry event transitions from non-cavity forming (Fig. 3.3(b)) to cavity forming
Fig. 3.2: Images show the impact of smooth, wetting stainless steel spheres onto the water-surfactant mixture. (a) A sphere of radius $R_s = 4.76$ mm impacts the mixture at $U = 5.42$ m/s without forming a cavity. (b) A larger sphere ($R_s = 11.11$ mm) impacts the pool at the same velocity forming a messy cavity. (c) A sphere of radius $R_s = 4.76$ mm impacts the pool at $U = 2.43$ m/s without forming a cavity. (d) A sphere with the same impact conditions as in (c) passes through a bubble layer prior to the free surface impact and creates an asymmetric subsurface cavity. See supplemental videos 1-4 that correspond to (a)-(d) respectively.

cases (Fig. 3.3(e)), an intermediate stage is seen in which two types of partial cavities form, similar to the observations of Marston et al. [11] in the water entry of Leidenfrost spheres. The first occurs when cavity formation initiates only at small localized sections of the sphere leading to a rapid pinch-off and a small asymmetric air pocket as seen in Fig. 3.3(c). The second generally occurs for larger sphere radii when the splash moves up the sphere sides in a nonuniform manner leading to an asymmetric closure at the sphere apex and a passage for a small amount of air to be entrained under the surface (Fig. 3.3(d)).

When a bubble layer rests on the pool surface, cavities form at all impact velocities tested (Fig. 3.3(a)), regardless of varying $h_B$ and $d_B$. To investigate why the presence of a bubble layer leads to cavity formation we dropped a sphere through a bubble-filled tube and examined it while exiting into the air as shown in Fig. 3.4. As the sphere passes through the bubble layer, ruptured soap films adhere to the sphere forming small droplets and bubbles.
Fig. 3.3: The critical velocity for cavity formation by spheres impacting onto a watersurfactant mixture for both a clean pool surface (no bubbles) and with a bubble layer resting on the surface is shown in (a). At low impact velocities on a clean surface, spheres do not form cavities as shown in (b) although small bubbles may be pulled under the surface. A transitional region is seen in which small asymmetric air pockets (c) and small cavities (d) form, which we call partial cavities. At the highest velocities full cavities form (e). The inset in (a) shows that the critical velocity for cavity formation on a clean pool surface decreases as the sphere radius increases. Cavities form at all impact velocities when spheres first pass through a bubble layer. All the spheres are made of the same steel with static contact angle $\theta = 30^\circ \pm 4^\circ$, but the data is spread between $26^\circ < \theta < 34^\circ$ for readability. Theoretical estimates for the critical velocity for cavity formation ($U_{cr}$) for water (...) and the water-surfactant mixture (---) are based on Duez et al. [2]. Uncertainty bands show 95% confidence intervals for the mean uncertainty in impact velocities in (a) and the inset.
Fig. 3.4: This sequence of images shows a sphere ($R_s = 4.76 \text{ mm}$) exiting a bubble-filled tube ($t = -8 \text{ to } 0 \text{ ms}$) and impacting the pool surface ($t = 4 \text{ to } 12 \text{ ms}$). Several bubble films are ruptured when the sphere passes through the bubbles, wetting the sphere surface with small droplets and bubbles. When this wetted sphere impacts the pool a cavity forms. See supplemental videos 5 & 6.

The bubble layer thus partially wets the sphere prior to the free surface impact resulting in cavity formation.

To examine the mechanism by which small droplets on the sphere surface initiate cavity formation we place a single droplet of miscible red dye (food coloring, $\mu = 2.50 \times 10^{-3} \ \text{Pa}\cdot\text{s}$, $\sigma = 55.5 \ \text{mN/m}$, and $\theta = 80^\circ \pm 2^\circ$) near the equator of a clean sphere before dropping it into tap water. Two separate impact events were recorded from top and side views (Fig. 3.5(a) & (b)) and aligned from the time of impact ($t = 0$). When the sphere is approximately half submerged ($t = 1 \text{ ms}$), the dye droplet impacts the pool causing it to deform into a thin sheet, extending upward into a splash and initiating cavity formation (Fig. 3.5(c)). As the droplet deforms it pushes water away from the sphere near the equator in a manner reminiscent of non-wetting coatings [1]. While water advances up the un-wetted portion of the sphere, the detachment created by the dye droplet results in a splash and a means for air entrainment, leading to cavity formation in the droplet vicinity. This localized cavity formation results in an asymmetric cavity that resembles those created by the water entry of half-wetting spheres [4] and produces lateral motion. As the sphere descends further
Fig. 3.5: Image sequences of two independent events recorded from (a) top and (b) side views where a stainless steel sphere ($R_s = 4.76$ mm) enters tap water with a droplet of red dye placed at its equator prior to release. The spheres impact water with identical velocities $U$, less than the critical velocity for air entrainment $U_{cr}$. The droplet deforms upon impact ($t = 1$ ms), spreading into the splash (a) and left edge of the cavity (b) to initiate cavity formation. The cavity and splash form only in the vicinity of the droplet with water climbing up the sphere surface in all other locations. The contact line moves towards the sphere apex ($t = 2 - 3$ ms), the cavity expands and moves down the other side towards the equator at $t = 4 - 5$ ms. Supplemental videos 7 & 8 correspond with (a) and (b) respectively. (c) The schematic shows droplet deformation pushing water away from the sphere near the equator in a manner reminiscent of non-wetting coatings [1].
into the liquid, the droplet of dye continues to coat the cavity wall and deflect water away from the sphere ($t = 2 - 3$ ms). The contact line, initially existing on only one sphere side, expands upward to un-wet the sphere as it moves towards the apex and down the other side; the cavity expands and effectively shifts contact from the upper-left side of the sphere (Fig. 3.5(b)) to the trailing side ($t = 2 - 5$ ms). A similar sequence of events is observed when a sphere falls through a bubble-filled tube followed by an air gap before impacting a clean pool surface (as seen in Fig. 3.4).

The cavity formed by placing a droplet of dye on a clean sphere initially resembles that formed by a single droplet impact [7]. When a single droplet impacts a pool it spreads out on the surface, pushing the fluid both downward and outward with the droplet liquid spreading over the surface of the newly formed cavity. The initial impact of a liquid jet on a pool behaves in the same way [8,9]. The combination of the two impact types (solid-liquid and liquid-liquid) causes wetting spheres to form cavities similar to those formed by non-wetting or rough spheres.

A similarity between non-wetting spheres and wetting spheres that pass through a bubble layer prior to impact is noted in the cavity types observed in Fig. 4.3. For the lowest $We$ values, pinch-off occurs on or very near the sphere surface which is described as quasi-static seal (Fig. 4.3(a)). As the Weber number reaches $We \approx 800, 400$ and 2,300, for $Bo = 0.91, 8.2$ and 58 respectively a larger cavity forms a shallow seal (Fig. 4.3(b)). When $We \approx 6,000$ for $Bo = 58$, pinch-off occurs approximately midway between the sphere and free surface, resulting in a deep seal (Fig. 4.3(c)). At the highest values, $We \gtrsim 1,300, 2,400$ and 9,000 for $Bo = 0.91, 8.2$ and 58 respectively, the splash crown domes over leading to surface seal (Fig. 4.3(d)). These cavity regimes were identified by Aristoff & Bush [3], who obtained the same progression of regimes with increasing $We$ for small non-wetting spheres with a similar comparison for $h_p$ and $We$ as seen in Fig. 4.3(e).

Pre-wetting of wetting spheres does not lead to a perfect overlap of pinch-off regimes found by Aristoff & Bush [3]. For instance, the shallow seal events observed occurring at $Bo = 58$ (Fig. 4.3(e)), do not correspond with previously published results [3]. The
Fig. 3.6: Cavity regimes observed with increasing Weber numbers transitioning from (a) quasi-static seal to (b) shallow seal, to (c) deep seal, and finally (d) surface seal shown for $Bo = 58$ ($R_s = 12.70$ mm). All cavity types were formed by stainless steel spheres ($\theta = 30^\circ$) entering a pool of water-surfactant mixture through a bubble layer resting on the free surface. Supplemental videos 9-12 correspond with (a) through (d) respectively. (e) The nondimensional pinch-off depth, $h_p/R_s$, is plotted as a function of $We$ with symbol size increasing for increasing $Bo$ ($Bo = 0.91, 8.2 \& 58$). Hollow symbols represent impact cases with bubbles on the pool surface and solid symbols represent cases where spheres passed through a bubble tube elevated above the pool surface.
discrepancy is brought about in part by nonuniform wetting as the sphere passes through the bubble layer, resulting in asymmetric cavities. This non-uniformity can in turn lead to an asymmetric cavity collapse as seen in Fig. 4.3(b) (evidenced by the wide pinch-off point). The asymmetries are most prominent near the pool surface, before the cavity has migrated to the sphere wake. This effect can lead to much narrower cavity diameters near the surface affecting the quasi-static, shallow, and surface seal regimes more significantly. The phenomenon is more pronounced for the two smaller sphere radii where asymmetries near the pool surface cause shallow seal to occur rather than deep seal. Although the cavity asymmetries are caused by the asymmetric adhesion of droplets on the sphere surface, no trends were observed in the cavity types with changes in bubble height ($h_B$) or bubble diameter ($d_B$).

3.6 Conclusion

In conclusion, our experimental results show that the addition of a surfactant to a pool of water alters the critical velocity for cavity formation, causing it to vary with sphere radius. The presence of a bubble layer resting on the surface of a water-surfactant mixture leads to the formation of subsurface cavities at all impact velocities tested. By observing a sphere falling through a bubble layer suspended above the free surface, we note bursting bubbles lead to the formation of small droplets and bubbles on the sphere surface. Rather than enhancing the wettability of a sphere, these droplets disrupt the advancing fluid and alter the splash, which leads to air entrainment and cavity formation under conditions where this would not normally be expected. The pre-wetted spheres mimic the water-entry behavior of non-wetting spheres; forming the same four cavity regimes (i.e., quasi-static, shallow, deep and surface seal). But the non-uniform droplet coatings cause cavity asymmetries that disrupt transitions between these regimes. It is also possible that the surfactant may disrupt the transitions, something we did not isolate in this study. The presence of a bubble layer on a liquid free surface thus causes spheres to exhibit similar water-entry characteristics as those shown by non-wetting spheres.
REFERENCES


CHAPTER 4
THE WATER ENTRY OF SPHERES AT VARIOUS CONTACT ANGLES

4.1 Prologue

The purpose of this study is to examine the influence of the sphere’s surface coating or contact angle on the cavity type formed. The data for this study was collected, processed and analyzed Utah State University. The study is in preparation to be submitted to the Journal of Fluid Mechanics under the title “The water entry of spheres at various contact angles”. The authors listed are: Nathan B. Speirs, Mohammad Mansoor, Jesse Belden and Tadd T. Truscott. The article in its entirety is presented below.

4.2 Abstract

The water entry of spheres causes cavity formation above a critical impact velocity, which is a function of the contact angle [4]. Aristoff and Bush [1] showed that four different cavity shapes or regimes (quasi-static, shallow, deep and surface) appear at one specific contact angle depending on the Bond and Weber numbers. We experimentally investigate how these cavity regimes change for different contact angles and find cavity formation to occur below the critical velocity described by Deve et al. [4]. In addition, we find an alternate scaling for the Bond and Weber numbers to predict the cavity formation regimes for various impacting bodies (e.g., spheres, multi-droplet streams and jets) on the same regime diagram. Our findings show that solid-liquid impact is quite similar to liquid-liquid impact.

4.3 Introduction

At the turn of the millennium a resurgence of interest occurred in water entry studies. During this time, two foundational papers on cavity formation and cavity dynamics were published, which describe the regimes into which all other water entry studies fall. The first
Fig. 4.1: A 2 mm diameter sphere impacting the water surface creates various cavity types depending on the impact conditions. In a) the sphere is coated in turtle wax ($\theta = 101.0^\circ$) and impacts at 4.43 m/s without forming a cavity. In b) through e) the sphere is coated in Glaco Mirror Coat Zero ($\theta = 141.1^\circ$) and impacts with velocities 0.24 m/s, 1.40 m/s, 2.80 m/s, and 4.43 m/s forming quasi-static, shallow, deep, and surface seal cavities respectively.

was written by Duez et al. [4] who investigated when impacting spheres form air cavities, as shown in figure 4.1b-e, and when they do not, as shown in figure 4.1a. The second paper, written by Aristoff and Bush [1], came along a couple of years later discussing the four different shapes or types of cavities that form once the appropriate conditions are met for cavity formation (shown in figure 4.1b-e). We will now examine each of these papers in turn, discussing their findings, an unaddressed discrepancy between the two, and how this paper expands our understanding of the conditions in which these five water entry regimes occur.

Duez et al. [4] found that cavity formation of smooth spheres occurs above a critical velocity $U_{cr}$ that is a function of the advancing static contact angle $\theta$. Hydrophilic spheres ($\theta < 90^\circ$) form cavities above $U_{cr} \approx 7.2$ m/s in water. The critical velocity decreases for hydrophobic spheres ($\theta > 90^\circ$) going to zero as $\theta$ goes to 180°. They explain this finding by discussing the contact-line stability of the thin, upward-moving film or splash that forms around the circumference of the sphere upon impact. When the splash adheres to the sphere
(below the critical velocity) no cavity forms. When it separates from the sphere, air gets pulled behind the sphere and a cavity forms.

Aristoff and Bush [1] studied the water entry of spheres with one contact angle, $\theta = 120 \pm 5^\circ$, and various impact velocities and sphere diameters. Their expansive data set found cavities forming at all impact velocities with four distinct shapes defined by their collapse or pinch-off location, each of which occurs at a specific location on a Bond-Weber plot. At the lowest Weber number, $We$, they describe quasi-static seal, in which pinch-off occurs on or very near the sphere surface (figure 4.1b). At higher $We$ both shallow and deep seal are seen. Shallow seal occurs at lower Bond number, $Bo$, where surface tension dominates and the pinch-off depth is on the order of the capillary length (figure 4.1c). Deep seal occurs at higher $Bo$, where gravitational forces dominate and pinch-off occurs about half way between the pool surface and the sphere (figure 4.1d). Surface seal occurs at the highest $We$, wherein the splash created upon impact collapses inward due to air pressure and surface tension [7] sealing at the pool surface (figure 4.1e).

According to Duez et al. [4], at the contact angle used by Aristoff and Bush [1] ($\theta = 120^\circ$), cavities should not form below $U_{cr} = 2.14$ m/s. Yet Aristoff & Bush report quasi-static, shallow and deep seal cavities at velocities below this value. We can explain this discrepancy by the high surface roughness of the coating used by Aristoff & Bush, Cytonix WX2100, which after our own testing have found to have a large ten-point-mean roughness of $R_s = 50.2 \pm 21.4 \mu m$. Zhao et al. [13] found that $U_{cr}$ is also a function of the sphere roughness $R_s$, with increasing roughness leading to lower values of $U_{cr}$. Cavities should always form at the above value of $R_s$ and hence the cavity formation described in work of [1] is due to both the high contact angle and high roughness.

Since these foundational works, several other important studies have come forth. Important topics include: the water entry of spinning spheres [9, 12], the effect of sphere density [2], the occurrence of multiple pinch-off events [6], the buckling instability in the crown [7], the effects of deformability [5], the unsteady forces during entry [11], and many more described in the annual review by Truscott et al. [10].
Although these works have contributed much to our understanding of water entry, we will return our focus to the foundational works discussed above and examine how the cavity formation regimes respond to experimentally varying the wetting angle $\theta$, the sphere diameter $d$ and the impact velocity $U_o$ for smooth spheres. We will explain the physics using the Bond, Weber, and Froude numbers, which we define as $Bo = \rho gd^2/\sigma$, $We = \rho d^2U^2/\sigma$ and $Fr = U^2/gd$ respectively, where $\rho$ is the liquid density, $g$ is the acceleration of gravity and $\sigma$ is the surface tension. We use the diameter $d$ instead of the sphere radius as the appropriate length scale in defining the above dimensionless numbers as it results in transitional behaviors around a value of one. In § 4.6 we will also examine an alternate method of defining the Bond, Weber, and Froude numbers that allows us to predict the cavity types for various impacting bodies on the same regime diagram.

4.4 Experimental setup and description

Figure 4.2 shows the experimental setup used in this study. Various diameter stainless steel spheres ($d_s = 1$ to 18 mm) of specific gravity 7.83 are dropped from an electromagnet onto a tank of water. Two high-speed cameras record the impact on the free-surface, imaging at 2,500 frames per second both above and below the surface from the side. The height of the electromagnet controls the impact velocity of the spheres $U_o$, which is varied from 0.24 to 10.39 m/s. To vary the advancing static contact angle $\theta$ three different coatings (or the lack thereof) are used: clean steel, Turtle Wax Super Hard Shell car wax, and Glaco Mirror Coat Zero. Values of $\theta$ and the ten point mean roughness $R_z$ are shown in table 4.1. Roughness measurements are obtained using a profilometer. The spheres are prepared by first washing with soap and water and then rinsing with ethyl alcohol. Coatings are then applied and allowed to dry before testing. After each test the spheres are dried and then recoated to ensure consistent surface properties.

From the high-speed videos we determine whether or not a cavity forms. If a cavity does form, the cavity type is determined using the definitions described by Aristoff and Bush [1]. Measurements are also taken from the videos to find cavity depths, diameters, and the time to pinch-off. The pinch-off depth $h_p$ is defined as the distance from the
Fig. 4.2: A schematic of the experimental setup is shown. A stainless steel sphere is attached to the electromagnet and dropped when the magnet turns off. The height of the electromagnet above the pool surface is adjusted to vary the impact velocity. Two high-speed cameras image the impact event from above and below the free surface of the pool with diffuse backlighting.

<table>
<thead>
<tr>
<th>Coating</th>
<th>θ</th>
<th>( R_z ) (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Steel</td>
<td>86.1 ± 2.2°</td>
<td>0.6 ± 0.3</td>
</tr>
<tr>
<td>Turtle Wax Super Hard Shell</td>
<td>101.0 ± 4.7°</td>
<td>0.9 ± 0.2</td>
</tr>
<tr>
<td>Cytonix WX2100</td>
<td>116.6 ± 7.0°</td>
<td>50.2 ± 21.4</td>
</tr>
<tr>
<td>Glaco Mirror Coat Zero</td>
<td>141.1 ± 3.8°</td>
<td>1.0 ± 0.5</td>
</tr>
</tbody>
</table>

Table 4.1: List of coatings with their advancing static contact angle θ and ten point mean roughness \( R_z \), with the mean and 95% confidence window reported.
undisturbed free surface to the location where the cavity walls or splash pinch together or collapse (positive downward). The depth of the bottom of the sphere at the time of pinch-off, \( h_b \), is also measured from the undisturbed free surface. The pinch-off time \( t_p \) is defined as the time from impact to the pinch-off event. In cases where cavities do not form, we define the pinch-off time \( t_p \) as the time when the splash closes on the top of the sphere and the pinch-off depth \( h_p \) as the location of the top of the sphere at this time. The cavity diameter is measured at discrete depths and times and the maximum cavity diameter over time is found at discrete depths, yielding \( d_{c_{\text{max}}}(z) \). The average of these maximum diameters over all depths defines \( d_c \).

4.5 Cavity formation and types

Multiple sphere diameters were tested over a large range of impact velocities for each contact angle. From this data we produce \( Bo-We \) plots similar to Aristoff and Bush [1] to examine how the cavity regimes change with the contact angle (figure 4.3). The regime diagram for \( \theta = 86.1^\circ \) is shown in figure 4.3a. Duez et al. [4] predict the critical velocity for cavity formation for a hydrophilic sphere to be \( U_{cr} = 0.1\sigma/\mu = 7.2 \text{ m/s} \), which is represented by the dashed line in figure 4.3a. Surface seal cavities occur above this dashed line (as shown in figure 4.1e) and no cavity formation is observed just below it (figure 4.1a). Once \( We \) has decreased below about 240, cavities start to form again, which is unexpected in light of the work of Duez et al. [4]. Below \( We \approx 240 \) spheres always form quasi-static seal cavities.

Looking at the regime diagram for \( \theta = 101.0^\circ \) (figure 4.3b) we see a similar trend to \( \theta = 86.1^\circ \) with the exception of a few differences. The critical velocity for cavity formation as predicted by Duez et al. has decreased due to the increased contact angle and is defined by \( U_{cr} = \frac{\gamma}{2\pi\mu}(\pi - \theta)^3 = 4.89 \text{ m/s} \). The critical \( We \) below which cavities form again does not change with the increase in \( \theta \). Below \( We \approx 240 \) mostly quasi-static seal cavities form with the exception of a few shallow seal cavities at low \( Bo \), for which the volume of air entrained with the sphere is small, but approximately equal to the volume of the sphere (this is consistent with the cut-off defined by Aristoff and Bush [1]).
Fig. 4.3: The cavity regimes for various contact angles are shown for a) \( \theta = 86.1^\circ \), b) \( \theta = 101.1^\circ \), c) \( \theta = 120^\circ \) and \( R_s = 50 \mu m \) (remade from [1]) and d) \( \theta = 141.1^\circ \). All of the spheres are smooth according to Zhao et al. [13] except c) (table 4.1). The dotted and dashed lines in a) and b) represent splash formation and Duez cavity formation respectively. The regime separation lines in c) and d) are not the same as in a) and b). Instead they come from the predictions of Aristoff and Bush [1]. Pictures of the different regimes can be seen in figure 4.1.
To understand why cavities form below a $We \approx 240$ we look at what happens as the velocity or $We$ increases for one sphere diameter or $Bo$. Figure 4.4a shows the impact of a 10 mm diameter sphere at various $We$ when the sphere is approximately half submerged. At the lowest $We$ a short, thick rim forms around the edge of the sphere. This rim does not have enough upward velocity to climb up the surface of the sphere and meet itself at the pole to prevent cavity formation [4]. Hence, as the sphere descends, the free-surface is pulled down with the sphere and a quasi-static seal cavity forms in its wake (figure 4.4b & c, $We = 7.9$). As $We$ increases the rim thins and grows taller due to its increasing upward velocity, which allows it to begin to climb the surface of the sphere as seen in figure 4.4a at $We = 109$. The upward velocity and adherence of the rim to the sphere directly competes with the rate of sphere submergence. This causes the water to move up and around the top of the sphere faster as it descends below the original free-surface plane, resulting in less air entrainment and the formation of increasingly smaller cavities. Hence, the pinch-off time $t_p$, depth of the sphere at pinch-off $h_b$, and the pinch-off depth $h_p$, all decrease with increasing $We$ which can be seen qualitatively in figure 4.4 and quantitatively in figure 4.5. Once $We \approx 240$ (dotted line) the rim has formed into a splash which has enough upward velocity to reach the top of the sphere by the time the sphere has descended to the level of the undisturbed free-surface ($h_p = 0$ and $h_b = 1$, figure 4.5b & a inset), as seen at $We = 269$ in figure 4.4b & c. Hence, the formation of the splash (which adheres to the sphere) suppresses cavity formation and we will call the dotted lines in figures 4.3a & b and 4.4 the splash formation line.

As $We$ increases above the splash formation line the splash climbs up the sphere surface faster causing $h_p$ to gradually rise above the free-surface (figure 4.5b), and decreasing both $t_p$ (figure 4.5a) and $h_b$ (figure 4.5a inset). Around $U_{cr}$, a small asymmetric cavity forms with the splash quickly doming over the top (figure 4.4, $We = 2696$). Once $U_{cr}$ is reached, the splash has enough velocity to separate from the sphere and a large surface seal cavity forms (figure 4.4, $We = 8081$). Hence, we see that cavity formation or suppression is governed by the formation and separation of the splash. Also, that cavity formation occurs.
Fig. 4.4: The development of the splash and progression of cavity regimes are shown for increasing $We$, with constant sphere diameter $d = 10$ mm, and contact angle $\theta = 101.1^\circ$. Each column shows different times and/or views of the same impact event with $We$ labeled at the top. In a) we show the development of the rim or splash when the sphere is approximately half submerged. In b) and c) we show the frame just prior to pinch-off or closure of the splash above the sphere for both above and below the pool surface respectively. The dotted and dashed lines represent splash formation and Duez cavity formation respectively and correspond with the dotted and dashed lines in figure 4.3b.
Fig. 4.5: a) The nondimensional pinch-off time $t_p$, depth of the bottom of the sphere at pinch-off $h_b$ (inset) and b) the pinch-off depth $h_p$ scale with $We_a$ for the quasi-static and no cavity regimes for $\theta = 86.1^\circ$ and $101.0^\circ$. Figure 4.4 shows these same trends qualitatively except that here the surface seal data is not shown as transition occurs at a constant velocity and the data does not collapse with $We_a$. The dotted line represents the splash formation line. The Weber number and dimensionless pinch-off time have been modified slightly using the average velocity $U_a$ from initial impact to full submergence. This accounts for the acceleration of the sphere over the submergence time, which is significant for large spheres at the lowest drop heights (e.g., an 18 mm sphere falling 3 mm before impact) ($We_a = \rho U_a^2 d/\sigma$). The legend in b) applies to all three plots.
in two regimes; at low enough $We$ that a splash does not form, and at high enough impact velocity that the splash separates from the sphere. The boundaries of the no cavity regime are defined by splash formation and what we will call Duez cavity formation (dashed lines in figure 4.3a & b and 4.4).

Surface seal is caused by the collapse of the splash crown. In the ideal cases typically depicted in the literature, the collapse of the crown, also know as dome over, causes a complete seal between the air in the cavity and the atmosphere. Figure 4.6 shows an event in which a complete seal does not occur during dome over. At $t = 6.4$ ms the splash crown has domed over and the air cavity behind the sphere begins to pull away from the free surface of the pool. At $t = 8.0 - 9.6$ ms air continues to enter the cavity as evidenced by the small conical structure that forms and connects the top of the cavity to the surface of the pool. The conical structure then collapses radially slightly below the surface providing a complete seal ($t = 11.2$ ms). Although in this study we cannot directly see the mechanism by which air continues to enter the cavity after dome over, we believe that it is caused by small holes that form in the collapsing splash crown which allow a small amount of air to continue to enter the cavity through the conical structure after dome over. The formation of holes in the crown was observed by Marston et al. [7] who describe the formation of thin films or bags in the crown that pop leaving holes. These partial surface seal events occur most commonly just above the critical velocity for cavity formation and at lower contact angles.

At the highest contact angle tested, when $\theta = 141.1^\circ$ Duez et al. [4] predict that cavities should form above a critical velocity of $U_{cr} = \frac{7}{270} \frac{\sigma}{\mu} (\pi - \theta)^3 = 0.58$ m/s. This velocity gives $We < 240$ for all sphere diameters tested. Hence, the splash separation line lies below the splash formation line at all $Bo$ tested and we would expect cavities to form at all impact velocities. This is indeed the case as shown in figure 4.3d. Comparing figure 4.3d to the data obtained by Aristoff and Bush [1] for rough spheres with $\theta = 120^\circ$ (shown in figure 4.3c) we see that the regime locations for the two coatings are very similar. Discrepancies in the cut off between quasi-static seal and shallow or deep seal are likely due to the lower
Fig. 4.6: A sphere with $d = 10$ mm and $\theta = 101^\circ$ impacts the pool surface at $U = 6.26$ m/s forming a surface seal cavity. The above water view is shown in a) and below the water is shown in b). The splash crown does not always provide a complete seal of the cavity during dome over. Sometimes it mostly seals, but let some air continue to enter the cavity as shown by the conical air pocket near the surface and above the main portion of the cavity at $t = 9.6$ ms. The conical portion of the cavity collapses radially ($t = 11.2 - 12.8$ ms). This partial surface seal is most common just above the critical velocity for air entrainment and at lower contact angles. The size of the conical section decreases and then disappears as the velocity and/or contact angle increase.
atmospheric pressure in Logan, Utah (elevation of 1382 m) where our experiments were performed and Cambridge, Massachusetts (elevation of 40 m) where the experiments of Aristoff and Bush [1] were performed, with the lower pressure leading to less air entrained and thus quasi-static seal cavities.

Spheres rebound off of the pool surface at the lowest $Bo$ and $We$ tested for all three contact angles. Figure 4.3 shows that transition from water entry to rebound occurs when $We = 1$ and $Bo < 1$. In this parameter space surface tension dominates over both inertial and gravitational forces and hence, neither the sphere's inertia nor its weight cause it to enter the water surface. Figure 4.3 also shows that rebound is slightly dependent on the sphere's contact angle, with higher contact angles leading to rebound at higher $Bo$ and $We$.

4.6 A new scaling

When defining dimensionless numbers it is always difficult to pick the appropriate length and velocity scales to describe the physics of the problem. Historically in water entry research, the sphere diameter or radius has been chosen as the length scale and the initial impact velocity for the velocity scale [10]. As the cavity collapse is likely to be a function of the cavity characteristics, it could be insightful to redefine the appropriate dimensionless numbers using cavity length and velocity scales. We define the cavity Weber number as $W_{ec} = \rho U_{c}^2 d_c / \sigma$, the cavity Bond number as $Bo_{c} = \rho g d_{c}^2 / \sigma$ and the cavity Froude number as $Fr_{c} = U_{c}^2 / g d_{c}$, where $d_c$ is the cavity diameter defined in §4.4 and $U_c$ is the downward cavity velocity, which we set equal to the initial sphere impact velocity $U_o$ as they are approximately the same. Plotting a regime diagram with the cavity scaling for $\theta = 141.1^\circ$, we see in figure 4.7 that $Bo_{c}$ and $W_{ec}$ separates the cavity types (hollow symbols).

Speirs et al. [8] investigated the water entry of multi-droplet streams and jets and found that shallow, deep and surface seal cavities occur for liquid-liquid impact as well. In that paper we predicted the cavity seal types for both multi-droplet streams and jets on the same $Bo$-$We$ regime diagram using a scaling based on the cavity diameter and no alteration of the impact velocity. We can collapse this data onto the $Bo_{c}$-$W_{ec}$ regime diagram for
Fig. 4.7: The cavity regimes can be predicted for various impacting bodies using the cavity diameter $d_c$ and cavity velocity $U_c$ to define $Bo_c$ and $We_c$. The hollow symbols are for spheres and the solid symbols are for multi-droplet streams and jets. The dash-dotted line separating the shallow and deep seal regimes is found by equating the dimensionless pinch-off times for shallow and deep seal (figure 4.8). The dashed line dividing deep and surface seal is drawn by modifying the cutoffs found by Aristoff and Bush [1] and Birkhoff and Isaacs [3].

spheres using $d_c$ and $U_c$ to define $Bo_c$ and $We_c$ as in figure 4.7. The cavity velocity $U_c$ is set equal to one half the impacting stream velocity for simplicity, which is shown to be a good approximation for jets by Speirs et al. [8]. Using this scaling, figure 4.7 shows that the regimes for all three water entry types can be predicted in the same $Bo_c$-$We_c$ parameter space. This scaling suggests that we can predict the pinch-off type of a cavity if we know its diameter and downward velocity, regardless of the type of impacting body used.

The cavity nondimensional pinch-off times, $t_p U_c / d_c$, of the sphere, multi-droplet stream and jet data can be predicted on the same plot for shallow and deep seal using $We_c$ and $Fr_c$ respectively. Figure 4.8a shows that $t_p U_c / d_c$ scales with $We_c^{1/2}$ for shallow seal and figure 4.8b shows that $t_p U_c / d_c$ scales with $Fr_c^{1/3}$ for deep seal. Equating these nondimensional pinch-off times and rearranging gives the cut-off between the shallow and deep seal regimes as shown by the dash-dotted line in figure 4.7 ($We_c = 1,525 Bo_c^{-2}$).

To predict the cut-off for surface seal we look at previous works. Aristoff and Bush [1]
used an empirical fit of $We = 640$ to define the cut-off for surface seal at low $Bo$ while at high $Bo$ Birkhoff and Isaacs [3] predicted the cut-off to occur at $Fr = 1/12800(\rho/\rho_a)^2$, where $\rho_a$ is the air density. We can use these results to find the surface seal cut-off in terms of $We_c$ and $Fr_c$. At low $Bo_c$, the cut-off is $We_c = 640 (d_c/d)_{mean}$, where $(d_c/d)_{mean} \approx 2.5$ is the mean cavity to sphere diameter ratio for the deep and surface seal data just above and below the transition at low $Bo_c$. This leads to a transition at $We_c = 1600$. At high $Bo_c$, the cut-off is $Fr_c = 1/12800(\rho/\rho_a)^2 (d/d_c)_{mean}$, where $(d/d_c)_{mean} \approx 1.6$ is the mean for the deep and surface seal data just above and below the transition at high $Bo_c$. This leads to a transition at $Fr_c = 44$. These transitional lines are shown in figure 4.7 with the dashed line and appropriately divide the deep and surface seal regimes. It is interesting to note that the shallow, deep and surface seal transition lines intersect at $Bo_c = 1$, indicating that when surface tension dominates over gravitational forces, shallow seal will always occur instead of deep.

Seeing the importance of the cavity diameter in calculating $Bo_c$ and $We_c$, we now scale $d_c$ for each pinch-off type. Plotting the nondimensional cavity diameter $d_c/d$ against $We$ we
Fig. 4.9: The cavity diameter for the impact of spheres with $\theta = 141^\circ$ scales with $We$ for shallow and surface seal a) and $Fr$ for deep seal b). The solid line in a) is a linear fit for the shallow seal data and the dashed line in a) is a fit for the surface seal data. The curved dotted line in b) is a fit for the deep seal data, but at small $Fr$ the cavity diameter approaches the sphere diameter, $d_{cav}/d = 1$ and diverges from the fit.

see in figure 4.9a that the cavity diameter for the shallow seal data is a function of $We$ and can be predicted with a linear fit of $d_c/d = 0.0074We + 1$, where the y-intercept is forced to equal one sphere diameter. We can also predict the cavity diameter for the surface seal data using $We$ with the fit $d_c/d = 4952We^{-1.24} + 1.64$. As commonly seen, the deep seal data scales better with $Fr$ as shown in figure 4.9b and can be predicted $d_c/d = 0.73Fr^{0.18}$ for high $Fr$, but below $Fr \approx 7$ the ratio $d_c/d$ asymptotes to one.

4.7 Conclusion

Cavity formation is dependent on the formation and behavior of the splash crown. Three crown behaviors exist. 1) At low $We$ a slow-moving, thick rim forms around the sphere, which allows air to entrain in the wake of the sphere forming small cavities. 2) At higher $We$ the crown thins and gains velocity which allows it to adhere to the sphere, climb the surface, and meet at the apex to preventing cavity formation. 3) Once the critical velocity for cavity formation is reached the splash crown separates from the sphere forming
the classical cavities discussed above and in previous works. The cutoffs between these behaviors are defined by splash formation and Dues cavity formation. For hydrophilic and slightly hydrophobic spheres, the inception of splash formation and adherence to the sphere decreases the cavity size, compared to higher contact angles, leading to quasi-static seal and small shallow seal cavities. When the contact angle is high enough ($\theta \gtrsim 140^\circ$) or the sphere is rough [13], cavities form at all impact velocities because the splash crown either does not form or it separates from the sphere. These cavity formation regimes are predicted by Aristoff and Bush [1]. When cavity formation is not inhibited by the splash, the pinch-off type can be predicted by the cavity diameter and downward velocity regardless of the type of impacting body (e.g., sphere, jet, or multi-droplet stream). This forms a more complete picture, linking the impact of solids and liquids on liquid pools.
REFERENCES


CHAPTER 5
REDUcing water entry impact forces

5.1 Prologue

This last study stems from a desire to reduce the large initial impact force of an object when striking a pool surface. The data for this study was collected, processed and analyzed at the Naval Undersea Warfare Center in Newport, Rhode Island and at Utah State University. The study was submitted for publication in the Journal of Fluid Mechanics on 22 June 2018 under the title “Reducing water entry impact forces.” The authors listed are: Nathan B. Speirs, Jesse Belden, Zhao Pan, Sean Holekamp, George Badlissi, Matt Jones and Tadd T. Truscott. The article in its entirety is presented below.

5.2 Abstract

The forces on an object impacting the water are extreme in the early moments of water entry and can cause structural damage to biological and man-made bodies alike. These early-time forces arise primarily from added mass, peaking when the submergence is much less than one body length. We experimentally investigate a means of reducing impact forces on rigid spheres by making a jet of water strike the quiescent water surface prior to the object impacting. The water jet accelerates the pool liquid and forms a cavity into which a sphere falls. Through on-board accelerometer measurements and high speed imaging, we quantify the force reduction compared to the case of a sphere entering a quiescent pool. Finally, we find the emergence of a critical jet volume required to maximize force reduction; the critical volume is rationalized using scaling arguments informed by near-surface particle image velocimetry (PIV) data.

5.3 Introduction

Free surface impact has been investigated for over a century [23] with most studies
examining solid or liquid impact on a quiescent pool. In this study, we examine the phenomenon of a solid body descending through a transient air cavity that is formed by a liquid jet, as shown in Fig. 5.1. The impact of the liquid jet greatly alters the flow field into which the sphere enters, which in turn dramatically changes the forces on the sphere during entry. In this paper, we examine these forces and find that the very initial impact force can be greatly reduced when the sphere impacts the bottom of a jet cavity.

Prior research on solid impact on a free surface informs our study on the impact forces. One of the first to study the forces during free surface impact was Thompson [19], who experimentally investigated the maximum pressure on sea plane floats during landing. Von Karman [22] followed up this study by developing a formula to apply Thompson’s experimental results to different shaped floats and impact velocities and was one of the first to model the impact force using added or virtual mass. Shiffman and Spencer [17] studied the impact of spheres on water up to a submergence depth of one radius and mathematically predicted the drag coefficient as a function of submergence depth using added mass arguments. They found that the maximum drag coefficient occurs when the sphere is submerged between ten and twenty percent of its radius. Others have shown similar trends for other geometries, with added mass being the dominant source of large peak forces for
small body submergence [8, 11]. Further theoretical developments on modeling the very initial impact force have been reviewed by Korobkin and Pukhnachov [10] with Miloh [13] and Faltinsen and Zhao [6] making significant contributions since. Moghisi and Squire [14] experimentally validated the work of Shifman and Spencer [17] for low viscosity liquids and impact velocities between 1 and 3 m/s. They also found that the impact force varies with the square root of the depth for depths less than ten percent of the radius. Further work on the initial impact force was performed by Bodily et al. [3] who studied the water entry of slender axisymmetric bodies. They showed that the impact force is a function of nose geometry.

The forces experienced after the very initial stages of impact depend on whether the sphere pulls air under the surface with it or enters without air entrainment [20]. In cases where cavities form as the falling sphere comes in contact with the water, the water is repelled away from the sphere near the sphere’s equator and an air cavity forms in its wake. If the sphere enters the water without entraining air the water will travel up the sides of the sphere meeting at the sphere’s apex, thus preventing entrainment. While cavity formation can be suppressed at low velocity if the static contact angle is less than 90° [5] and the sphere is smooth [24], cavities always form with sufficiently large sphere impact velocity \( U_0 > 7.3 \text{ m/s in water} \) which decreases as the contact angle or roughness increase. Once a cavity forms, the balance between the inertial, gravitational, and surface tension forces, described by the Bond, Weber, and Froude numbers (defined below), dictate its dynamics and cause it to take on one of four shapes or regimes [1]. The two applicable cavity regimes for this study are the deep seal regime, in which hydrostatic pressure forces the cavity to close approximately halfway between the sphere and water surface, and the surface seal regime in which the splash collapses inwards sealing off further air flow into the cavity [1]. In the current study, cavities always form when impacting a quiescent pool due to the high surface roughness of the sphere and both deep and surface seals are seen.

Other studies have focused on the forces experienced after the initial impact. May and Woodhull [12] found the average drag coefficient of steel spheres during the entrance cavity
phase, while Shepard et al. [16] studied the effect of sphere density on the drag coefficient for the same time phase. Truscott et al. [20] showed that the forces during these later stages of impact are very unsteady and depend on whether the sphere forms a cavity. In non-cavity forming cases vortices shed in the wake of the sphere cause large impulses in the sphere acceleration. When cavities form the trailing air bubble suppresses vortex shedding and the sphere experiences other forces caused by the pinch-off event [3]. Other studies on water impact include: Glasheen and McMahon [7] who studied the impact forces of circular disks to understand how basilisk lizards and shore birds run along the water surface, Baldwin [2] who studied cones, and Tveitnes et al. [21] who studied wedges.

In this study we focus on a method for reducing the initial, impulsive impact force experienced by a sphere during water impact. As described above, the body of literature shows that this early time force, occurring before the sphere is fully submerged, is predominately caused by the sphere having to accelerate the surrounding water; i.e., added mass. Here we suggest that the impact force can be reduced by accelerating a volume of water just below the free-surface prior to sphere impact. To test this experimentally, we allow a jet of water to strike the surface prior to the sphere impacting. On-board measurements of acceleration using custom inertial measurement units (IMUs) confirm that the large initial impact force is reduced, and that the subsequent forces and cavity dynamics throughout entry are also altered. We investigate this effect over a range of impact velocities and water jet lengths, and place our findings in the context of prior work on rigid sphere impact on a quiescent pool.

5.4 Experimental setup and description

Figure 5.2a shows the setup used for this study. A polycarbonate pipe of inner diameter 51 mm is held above a tank of water and the bottom opening of the pipe is sealed by smashing an inflated party balloon against it using a metal ring. The pipe is filled with water to the desired height and then a 50 mm diameter sphere is placed at the top of the water in the pipe. When the balloon is popped it quickly moves out of the way and the water and sphere fall towards the pool surface as seen in Fig. 5.2b. As the jet impacts
Fig. 5.2: A schematic of the experimental setup is shown in a) with a zoomed in view of the IMU nestled in the interior. b) When the balloon pops the jet and sphere fall towards the pool and the jet forms a cavity into which the sphere falls (time between images is 20 ms). In the upper part of the images the ring and popped balloon can be seen.

When the sphere hits the pool it spreads on the surface and forms an air cavity into which the sphere falls. The sphere impacts the bottom of the cavity and the impact event is viewed at 1000 fps with a high-speed camera imaging below the pool surface with diffuse back lighting. In some cases a second synchronized camera views the jet and sphere above the pool surface. The same sphere is also dropped without a jet and impacts a quiescent pool. Measurements are taken from these videos to find the sphere impact velocity, cavity velocity, and the length of jet in front of the sphere.

The sphere consists of an outer shell, weights, and an inertial measurement unit (IMU), as shown in Fig. 5.2a. The outer shell of the sphere is 3D printed in two parts using Vero plastic. This provides a hydrophilic surface with wetting angle $\theta = 80 \pm 8^\circ$ and surface roughness $R_s = 7.2 \pm 1.2 \, \mu m$ at 95% confidence. The steel weights are placed in the lower half of the sphere with the IMU firmly attached to them. This helps the sphere to fall inline with its vertical axis and to minimize the sphere rotation during free fall and impact; the magnitude of the total acceleration vector is computed from measurements in the three axes and is reported herein. The two pieces of the sphere are pressed together and the seam and top hole sealed with Colorimetrics gray putty tape to prevent water from entering. The seam between the two pieces of the sphere shell is located about two thirds of the sphere radius from the bottom of the sphere so as to minimize its influence on the dynamics of the
water impact event. The specific gravity of the sphere as a whole is 2.253 ±0.007.

The IMU was built in house and has two three-axis accelerometers, that separately record each impact event at 1000 Hz. The low range accelerometer is set to a maximum range of ±16 g and the high range accelerometer is set to a maximum range of ±100 g. When possible the data from the low range accelerometer is used as it results in less noise, but the measurements of the two separate accelerometers are comparable. Because the sphere experiences small rotations during free fall and impact the three components of acceleration are summed and the magnitude of the acceleration vector is reported herein.

Using the setup described the impact velocity at the cavity bottom or quiescent pool surface, \( U_o \), was changed from 1.83 to 9.34 m/s by varying the drop height. The length of the jet impacting in front of the sphere, \( L_f \), varied from 0 to 55 cm. This resulted in nondimensional parameters with the following ranges: \( Re = \rho U_o R_s/\mu \) between 40,000 and 200,000, \( We = \rho U_o^2 R_s/\sigma \) between 1,100 and 31,000, and \( Fr = U_o/\sqrt{gR_s} \) between 3.6 and 18.9, where \( \rho \) is the liquid density, \( R_s = 25 \text{ mm} \) is the radius of the sphere, \( \mu \) is the dynamic viscosity of the liquid, \( \sigma \) is the liquid-air surface tension, and \( g \) is the acceleration of gravity. Some limitations of the setup are that the sphere and jet radii must be equal and that the drop height of the cases with jets could not be increased above 4 m as the jet front becomes more distorted with increased falling distance.

5.4.1 Uncertainty

Uncertainty in all measurements is calculated and the uncertainty bands in the figures represent the 95% confidence interval of the measurement [4]. The uncertainty in calculated variables was often found to scale monotonically with the variable. Where applicable, two or more bands are placed on the extremes of the figure axes or with the data set to show how the uncertainty scales (e.g., Fig. 5.4b), when only one band is present the mean uncertainty is shown.

5.5 Results and discussion

Figure 5.3a shows an image sequence of a sphere impacting a quiescent pool of water
at $U = 4.39$ m/s with the corresponding acceleration of the sphere shown in Fig. 5.3c. In
the very early stages of impact the sphere accelerates a portion of the surrounding water
(added mass) [17], which causes a large, but short lived peak in the acceleration of the
sphere (Fig. 5.3c, $t = 0$ to 0.01 s). A cavity then forms expanding downwards into the
pool. At 25 ms after impact the splash crown domes over with no immediately noticeable
influence on the sphere acceleration. At approximately 100 ms a deep seal occurs causing
ripples and volume oscillations in the lower portion of the cavity which give rise to the
oscillations seen in the sphere acceleration with approximate amplitude of 0.45 g [3, 9] At
approximately 175 ms a bubble sheds from the lower portion of the cavity increasing the
amplitude of the oscillations in the sphere acceleration to about 0.59 g.

If a sphere is placed inside a falling jet of water, the jet impacts the pool prior to
sphere impact and forms an air cavity into which the sphere falls. This is shown in Fig. 5.3b
for approximately the same impact velocity as for the quiescent impact case shown in a.
Immediately after impact the acceleration of the sphere increases but not to as large a
value as seen in the quiescent impact case (Fig. 5.3c, $t = 0$ to 0.05 s). The sphere enters
the pool without forming a cavity because the sphere is already immersed inside the jet.
The cavity previously formed by the water jet collapses in a deep seal at 35 ms after sphere
impact. The large bubble formed by the pinch-off event oscillates leading to oscillations in
the sphere acceleration, which for the case shown begins with an amplitude of about 1 g
and decrease exponentially as the sphere descends away from the bubble. These oscillations
are superimposed on the increase of acceleration from $t = 0.05$ to 0.15, which is caused by
a vortex shed from the sphere as discussed by Truscott et al. [20].

As the jet significantly reduces the maximum impact force experienced by the sphere
during the very initial stage of impact (Fig. 5.3c $t = 0$ – 0.01 s and $t = 0$ – 0.05 s) we
now focus on this time period and, in particular, the maximum acceleration during this
eyear stage. The maximum measured acceleration $a_{max}$ is normalized by $g$ and plotted as
a function of $Fr$ in Fig. 5.4a. For the sphere impacting on a quiescent pool, the maximum
acceleration increases quadratically with $Fr$. We can predict this behavior from a force
Fig. 5.3: a) A 50 mm diameter sphere impacts a quiescent pool surface with velocity $U = 4.39 \text{ m/s}$ forming a subsurface air cavity, that experiences surface seal, deep seal and cavity shedding at around, 25, 100, and 175 ms respectively. b) A 50 mm diameter water jet impacts a pool surface forming a subsurface air cavity. At $t = 0$, the 50 mm diameter sphere impacts the bottom of the jet cavity at velocity $U = 4.35 \text{ m/s}$ without forming a cavity. c) The total acceleration $a_{total}$ vs. time $t$ is plotted for the sphere impacting a quiescent surface in a) and for the sphere impacting behind a jet in b). See supplemental movies 1 & 2.
balance including total drag, 
\[ a_{max} = \frac{1}{2} C_{d \text{max}} A_s \rho U_s^2 / \rho_s V_s \] where \( C_{d \text{max}} \) is the peak drag coefficient, and \( A_s \) and \( V_s \) are the sphere cross-sectional area and volume, respectively. Nondimensionalizing and rearranging the above equation we obtain

\[ \frac{a_{max}}{g} = \frac{3}{8} \frac{\rho}{\rho_s} C_{d \text{max}} Fr^2. \]  
\[ (5.1) \]

Based on the work of Shiffman and Spencer [17] we take \( C_{d \text{max}} \approx 1 \), which is reasonable for our spheres with density ratio \( \rho_s / \rho = 2.26 \) (see Fig. 5.4b). The dotted line in Fig. 5.4a plots (5.1) evaluated for the experimental conditions herein and is found to be a good approximation of the peak acceleration for spheres impacting on a quiescent surface. A large amount of uncertainty is found in the maximum impact acceleration for spheres impacting on quiescent pools at high velocities. This occurs because the duration of the impact peak becomes very short and the sampling frequency of the accelerometer is limited to 1000 Hz. Hence, the maximum acceleration during this peak occurs between data points leading to the large and asymmetric uncertainty bands for the quiescent impact cases with high velocity as illustrated in Fig. 5.4. Uncertainty is estimated using the \( C_d \) verse depth curve from Shiffman and Spencer [17] and considering the largest possible values our 1000 Hz sampling could have missed (see Appendix A §5.7).
The spheres impacting inside a jet have lower peak accelerations for all $Fr$ compared to the quiescent impact cases (Fig. 5.4a). To rationalize these lower max accelerations, we note that the bottom of the jet cavity moves downwards with velocity $U_c$, which is about half the jet velocity $U_j$ (i.e., $U_c = \frac{1}{2} U_j$, see [15, 18] for more details) during sphere impact, thus changing the relative impact velocity of the sphere ($U_{rel} = U_o - U_c$). For sufficiently large drop heights the distance that the jet falls before impact is approximately equal to the distance that the sphere falls before impact and thus, $U_o \approx U_j$ and $U_{rel} \approx \frac{1}{2} U_o$. Invoking the derivation of (5.1) using $U_{rel}$, the maximum acceleration of a sphere impacting behind a jet is

$$\frac{a_{max}}{g} \approx \frac{1}{4} \left( \frac{3}{8} \frac{\rho}{\rho_s} C_{d_{max}} Fr^2 \right).$$

(5.2)

Therefore, if the sphere impacts a pool of water behind a jet the impact force can theoretically be reduced by up to 75% from the quiescent impact case. We can also examine the effect of the jet by looking at the change in the drag coefficient. Equation (5.2) can be rewritten as $a_{max}/g \approx \frac{3}{8} \frac{\rho}{\rho_s} C_{eq} C_{d_{max}} Fr^2$, where $C_{eq} = \frac{1}{4} C_{d_{max}}$ is the equivalent drag coefficient. This implies that the maximum drag coefficient when impacting behind a jet is about a quarter that of a sphere impacting on a quiescent pool. Fig. 5.4b shows that the peak drag coefficient is significantly reduced when the sphere impacts in the wake of a jet.

Figure 5.3c shows that not only is the maximum impact acceleration lower for the jet cases, but the peak is much wider. This extended duration is quantified here by taking the peak width at half height $t_{hh}$, which is then made nondimensional using the sphere radius $R_s$ and the absolute impact velocity $U_o$ for the quiescent cases or the relative impact velocity $U_{rel}$ for the jet cases. When $t_{hh} U_o/2R_s$ or $t_{hh} U_{rel}/2R_s = 1$ this represents the time it takes for the sphere to be fully submerged in the water. For both cases this nondimensional time is found to be relatively constant for all impact velocities with $t_{hh} U_o/2R_s = 0.29 \pm 0.09$ for the quiescent cases and $t_{hh} U_{rel}/2R_s = 1.23 \pm 0.11$ for the jet cases. Hence, the peak in the sphere acceleration subsides before the sphere is fully submerged for the quiescent cases, but extends beyond the point of full submergence for the jet cases. We consider the effect of the extended peak duration on the total impulse of the sphere during this time. The magnitude
of the total impulse is computed as \( I = \int_0^{t_f} \rho_s V_s a_{total} dt \) by numerically integrating the acceleration curves in Fig. 5.3c; \( t_f \) corresponds to the half height time following the maximum acceleration. Nondimensionalizing the impulse by the initial momentum \( \rho_s V_s U_o \) for both cases reveals relatively constant values of \( I/\rho_s V_s U_o = 0.067 \pm 0.003 \) for the quiescent impact cases and \( I/\rho_s V_s U_o = 0.175 \pm 0.006 \) for the jet cases. Thus, although the spheres that impact inside a jet experience much smaller maximum acceleration magnitudes, the width of the initial impact peak is much larger leading to nearly three times the total impulse magnitude and, in turn, a larger reduction in velocity over the initial impact force duration.

To gain a better understanding of why the presence of a jet changes both the height and width of the initial acceleration peak, we use planar particle image velocimetry (PIV) to investigate the velocity field of the water under the sphere impact, a jet impact, and a sphere preceded by a jet (Fig. 5.5). The PIV images were taken with inter-frame spacing of 0.5 ms and processed with four passes at 64×64 pixel and two passes at 32×32 pixel interrogation regions using DaVis software. In Fig. 5.5a we see the first moments of impact, in which the sphere accelerates the fluid directly below itself \( (t = 0.6 \text{ ms}) \). The velocity of the fluid directly in front of the sphere decreases from the sphere velocity to zero as the distance from the sphere increases. As the sphere descends further into the pool the mass of accelerated fluid in front of the sphere increases and the radius of the fluid mass stays approximately equal to the radius of the submerged portion of the sphere \( (t = 1.2 - 2.4 \text{ ms}) \). When a jet of the same radius and velocity impacts, a larger local moving pool forms (Fig. 5.5b, note that the scaling of b differs from a). The velocity of the fluid directly in front of the jet cavity decreases from the jet velocity to zero as the distance from the cavity increases. This is the velocity field into which the sphere enters when impacting behind a jet, as shown in Fig. 5.5c. When the sphere first impacts the cavity bottom, the fluid that it passes through first has a velocity just smaller than its own and therefore the maximum impact acceleration is less than in the quiescent case. As the sphere continues to descend, the velocity of the fluid that it passes through gradually decreases. Hence the relative velocity between the sphere and the surrounding fluid gradually increases, extending the
duration of impact.

Given that the primary source of the large initial impact force is added mass, and that the jet reduces this force by accelerating a mass of fluid in the pool, one would expect the amount of water contained in the jet to affect the peak accelerations. To attempt to approximate the required mass of fluid we start by asking the question: how much water in a jet is required to accelerate a large enough local moving pool with a velocity equal to \( U_j/2 \) (the cavity velocity)? Approximating the jet as a cylinder and maintaining the radius of the jet equal to \( R_s \), we define the jet volume as \( \pi R_s^2 L_{cr} \), where \( L_{cr} \) is the critical or minimum jet length for maximum force reduction. We approximate the local moving pool as a hemisphere of radius \( \kappa R_s \) and equate the momentum of the impacting jet with the momentum of the local moving pool as follows:

\[
\rho U_j \pi R_s^2 L_{cr} \approx \frac{\rho U_j}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \pi (\kappa R_s)^3.
\]

Solving for \( L_{cr} \) we find that \( L_{cr} \approx \frac{4}{3} \kappa^3 R_s \). To approximate \( \kappa \), we look at the velocity field in the pool created by the impact of a jet (Fig. 5.5b at \( t = 16 \) ms) and find the distance from the bottom of the cavity, along the axis of the jet, over which the average velocity equals \( U_j/2 \). Setting that distance equal to \( \kappa R_s \) we find that \( \kappa = 1.4 \) which yields \( L_{cr} \approx 0.91 R_s \) or approximately one sphere radius \( R_s \).

To validate \( L_{cr} \) experimentally we vary the length of the jet impacting in front of the sphere \( L_j \) and examine its effect on the maximum sphere acceleration. To do this we plot the nondimensional jet length \( L_j/R_s \) against the max acceleration experienced by the sphere impacting inside a jet \( a_j \) normalized by the max acceleration experienced by a sphere impacting a quiescent pool \( a_q \) at the same absolute velocity \( U_o \). Fig. 5.6 shows that as \( L_j \) increases from zero to about one sphere radius, the maximum impact acceleration decreases for all impact velocities tested (\( U_o = 2.55, 4.23, \) and \( 5.75 \) m/s), but when \( L_j \gtrsim R_s \) no further reduction is achieved. If \( L_j = R_s \) the mass of liquid falling in front of the sphere is approximately equal to \( V_s \rho/2 \), which is same as the added mass of a fully-submerged sphere. Thus, the most efficient jet that will reduce the force by 75% has mass on the order
Fig. 5.5: The flow fields created upon impact of a sphere, a jet and a jet followed by a sphere are shown in a) through c) respectively. The flow fields were measured using particle image velocimetry (PIV) and the thin red lines show the location of the masking, which covers the spheres and air cavities. The radii of the spheres and jets are 25 mm in each case. a) A sphere impacts an initially quiescent pool at 4.23 m/s accelerating the liquid in front of it. b) A jet with the same velocity impacts a quiescent pool, deforms and creates a large, local downward flow. c) A sphere at 4.45 m/s impacts the bottom of a cavity formed by a jet with the same impact conditions as in b). Only the left half of each image is shown. The length scale bar and velocity vector scale arrow shown in b) apply to c) as well. The coloring of the images shows the vertical velocity of the fluid \( u_z \) with positive defined in the upward direction as shown in the color bar on the right. The bar in b) at \( t = 16 \) ms shows the radius of the local moving pool, \( \kappa R_s \), used to predict \( L_{cr} \). See supplemental movies 3 through 5.
Fig. 5.6: Increasing the jet length in front of an impacting sphere \( L_j \) decreases the maximum acceleration experienced by the sphere during the very initial stages of impact for \( L_j < R_s \). Once \( L_j > R_s \), no further force reduction is achieved by increasing \( L_j \). The maximum acceleration of a sphere impacting inside a jet \( a_j \) is nondimensionalized by the maximum acceleration experienced by a sphere impacting a quiescent pool \( a_q \) at the same absolute velocity \( U_o \). The uncertainty of the nondimensionalized acceleration differs for each impact velocity, with the uncertainty bands shown next to the corresponding legend entry.

of the sphere’s added mass, which makes sense when one considers that the added mass is the fluid accelerated by the sphere.

5.6 Conclusion

The water impact forces experienced by a falling body can be violent, due primarily to the fact that the body has to accelerate a mass of water from rest. If a liquid jet is made to impact prior to the body, then the forces can be significantly reduced by up to 75%. The jet accelerates the previously quiescent water thereby reducing the added mass effect on the impacting body. A jet length comparable to the sphere radius is sufficient to achieve this effect. This information could lead to a reduction in the impact force on objects that are dropped or launched into water such as torpedoes, sonobuoys, and space craft water landings.

5.7 Appendix A: Uncertainty of the maximum acceleration

A large amount of uncertainty is found in the maximum impact acceleration for spheres
impacting on quiescent pools at high velocities. This occurs because the duration of the peak in the acceleration is very short and the sampling frequency of the accelerometer is limited to 1000 Hz. Hence, the maximum acceleration during the initial peak nearly always occurs between data points so that the maximum acceleration read by the accelerometer is lower than the true maximum acceleration. To estimate the uncertainty of the maximum acceleration we use the prediction of the drag coefficient as a function of the submergence depth found by [17] and validated by [14]. Dimensionalizing the drag coefficient using the impact velocity, sphere radius and sphere mass and converting the submergence depth to time from impact using the impact velocity we obtain a prediction of the acceleration over time. Using this curve, we find the two points with the same acceleration that straddle the peak and are separated by one millisecond. Taking the difference between the acceleration of these points and the maximum acceleration from the curve gives an estimate of the uncertainty caused by insufficient sampling frequency. This uncertainty was combined with the uncertainties of the accelerometer leading to the asymmetric uncertainty bands seen in Figs. 5.4a & 5.6.
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CHAPTER 6
CONCLUSIONS

This dissertation investigates several aspects of the impact of solids and liquids on pools of water. It expands our understanding of the types of cavities formed for various impact condition, and other facets of the cavity dynamics and impact forces. The primary research objectives were:

1. Understand the cavity dynamics of multiple, successive impacts of droplets (i.e. multi-droplet streams).

2. Investigate the effect of the addition of surfactant to the pool liquid.

3. Examine the influence of the sphere’s surface coating or contact angle on cavity type formed.

4. Study the change in the impact force when a sphere impacts inside a falling mass of water.

I will now show how my research presented in the the previous chapters has fulfilled these research objectives.

In Chapter 2 I discussed the free-surface impact of multi-droplet streams and jets. I showed that six different cavity types can form for multi-droplet streams and three different cavity types can form for jets. Three of these cavity types, shallow, deep and surface seal, are common for multi-droplet streams, jets and solid spheres and can be predicted with the Bond and Weber numbers. The other three cavity types are only seen for multi-droplet streams and occur when the impact frequency is low relative to the expansion time of a single sub-cavity (the cavity formed by the impact of a single droplet). For these cavity types, pinch-off occurs when a sub-cavity collapses. A new dimensionless number I developed and coined the matryoshka number, which along with the Weber number can predict which
type of sub-cavity collapse will occur. I have also found scaling laws for several cavity characteristics including: the cavity velocity, diameter, pinch-off depth, final cavity depth, and pinch-off time, which can all be predicted with input parameters. The results of this study meet research objective 1.

In Chapter 3 I investigated the effect of the addition of surfactant on water entry phenomena. Surfactant is found to decrease the critical velocity for cavity formation and cause the critical velocity to decrease with the sphere radius. When bubbles rest on the surface of the pool cavities form at all impact velocities. This occurs because the sphere pops the bubbles, which wet the sphere surface with droplets. When the wetted sphere impacts the pool the droplets deform and push the pool fluid away from the sphere causing a cavity to form. The results of this study meet research objective 2.

In Chapter 4 I examined the influence of the sphere's surface coating or contact angle on the cavity type formed. Cavity formation is found to depend on the formation and behavior of the splash. When the impact of a sphere forms a splash, which adheres and travels up the sphere sides, the splash meets at the pole and suppresses cavity formation. Hence cavities form under two separate conditions: first when spheres impact at such low Weber numbers that no splash is formed, and second when spheres impact at high enough velocity that the splash separates from the sphere and allows air to entrain in its wake. The cutoff of these two regions is defined by the splash formation line and the Duez splash separation line. For hydrophilic and slightly hydrophobic spheres, the inception of splash formation and adherence to the sphere decreases the cavity size, compared to higher contact angles, leading to quasi-static seal and small shallow seal cavities. When the contact angle is high enough (θ ≥ 140°) or the sphere is rough, cavities form at all impact velocities because the splash either does not form or it separates from the sphere. When cavity formation is not inhibited by the splash, the pinch-off type can be predicted by the cavity diameter and downward velocity regardless of the type of impacting body (e.g., sphere, jet, or multi-droplet stream). The results of this study meet research objective 3.

In Chapter 5 I study the change in the impact force when a sphere impacts inside
a falling mass of water. The water impact forces experienced by a sphere are large, due primarily to the fact that the body has to accelerate a mass of water from rest. If a liquid jet is made to impact prior to the body, then these dramatic forces can be reduced by up to 75%. The jet accelerates the resting water thereby reducing the added mass effect on the impacting body. A length of jet comparable to the sphere radius is sufficient to achieve this effect. The results of this study meet research objective 4.
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3. Doctoral Student Researcher of the Year for the College of Engineering, USU (2017)