An Exploratory Case Study of How High-Performance Team Training Develops Sociomathematical Norms and Differing Levels of Math-Talk

Melanie V. Durfee
Utah State University

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AN EXPLORATORY CASE STUDY OF HOW HIGH-PERFORMANCE TEAM TRAINING DEVELOPS SOCIOMATHEMATICAL NORMS AND DIFFERING LEVELS OF MATH-TALK

by

Melanie V. Durfee

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Education

Approved:

Beth MacDonald, Ph.D. Patricia Moyer-Packenham, Ph.D.
Major Professor Committee Member

Jessica Shumway, Ph.D. Brynja Kohler, Ph.D.
Committee Member Committee Member

James Dorward, Ph.D. Laurens H. Smith, Ph.D.
Committee Member Interim Vice President for Research and Interim Dean of the School of Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

2018
ABSTRACT

An Exploratory Case Study of How High Performance Team Training Develops Sociomathematical Norms and Differing Levels of Math-Talk

by

Melanie V. Durfee, Doctor of Philosophy

Utah State University, 2018

Major Professor: Beth MacDonald, Ph.D.
Department: Teacher Education and Leadership

This exploratory study investigated the influence of High Performance Team (HPT) training on sociomathematical norms and differing levels of the Math-Talk Learning Community framework (Math-Talk) when sixth-grade student teams solved challenging mathematics problems while working in teams. HPT training involved (1) training students on distinct roles in the team problem-solving process, (2) challenging students with complicated mathematical problems, and (3) holding students accountable for contributions to the team. This research project explored the initial stages of the relationship between HPT and student-to-student mathematics conversations though the lens of the Math-Talk Learning Community framework.

The researcher studied four teams (i.e., four cases) with four middle school students in each team/case during a 7-week timeframe. The research study had three phases. The first phase involved gathering baseline data regarding the students’ sociomathematical norms. During the second phase, the students were trained to work in
HPT and then solved challenging mathematics problems in teams. During the last phase, the researcher collected data to explore shifts in sociomathematical norms and student autonomy after the students had the opportunity to be trained and work in HPT. The researcher used descriptive statistics to analyze the quantitative data and open and axial coding to analyze the qualitative data. The analysis included both within- and cross-case analysis.

The descriptive statistics used to analyze the changes in sociomathematical norms and Math-Talk levels indicated that the levels of sociomathematical norms increased when teachers gave students opportunities to participate in mathematics discussion. Specifically, students were most adept in the area of explaining and justifying reasoning and least skilled in the area of indicate when solutions are valid.

The role of the teacher was key to maintaining high levels of Math-Talk. The teachers needed to give appropriate support to maintain these levels in three different areas: (1) select problems that were the appropriate level of complexity and provide scaffolding when needed, (2) ensure students understood the context for the mathematics problems, and (3) teach students how to find their own errors or be ready to give feedback regarding whether students’ answers were correct.

(177 pages)
PUBLIC ABSTRACT

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Melanie V. Durfee

This exploratory study investigated the influence of High Performance Team (HPT) training on sociomathematical norms and differing levels of the Math-Talk Learning Community framework (Math-Talk) when sixth-grade student teams solved challenging mathematics problems while working in teams. HPT training involved (1) training students on distinct roles in the team problem-solving process, (2) challenging students with complicated mathematical problems, and (3) holding students accountable for contributions to the team. This research project explored the initial stages of the relationship between HPT and student-to-student mathematics conversations though the lens of the Math-Talk Learning Community framework.

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I am grateful for the time, effort, expertise, and generosity of my committee members, Drs. Beth MacDonald, Patricia Moyer-Packenham, Jessica Shumway, James Dorward, and Brynja Koehler. Most of all, I would like to thank Dr. MacDonald for the countless hours of conversations regarding this research project, mathematics research, and education in general. I also owe gratitude to my academic writing club members, Jennifer Throndsen and Kami Dupree. Their consistent feedback and support kept me motivated and on track. In addition, this project could not have happened without the support of the amazing teachers of A Pod, Diane, Giselle, Jennifer, and Mindy. Their dedication to teaching their students inspires me to be at my best. I am also thankful for the students of Teams A, B, C-1, and C-2. I appreciate their willingness to participate and enthusiasm for learning math. I also appreciate the tenacity and dedication of Maggie Duersch who worked tirelessly for IRB approval for this project.

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Melanie V. Durfee
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CHAPTER 1
INTRODUCTION

During the past 2 decades, mathematics curriculum reformers have challenged conventional mathematics teaching practices and curriculum development. What emerged were mathematics standards and practices that de-emphasized arithmetic and emphasized conceptual understanding and problem solving (Leinwand, Huinker, & Brahier, 2014). The publication of the National Council of Teachers of Mathematics’ (NCTM) *Principles and Standards for Mathematics* (2000) and the more recent Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices, 2010) gave mathematics educators guidelines for what reformed mathematics teaching and learning should look like in classroom settings. With the new guidelines of practices came new expectations of social norms, types of accepted actions in the classroom (Yackel, 2001). With new expectations of social norms came new expectations of sociomathematical norms, norms focused on mathematics discussions (Cobb & Yackel, 1996), which are specific to the mathematics classroom. Although the practices that mathematics teachers were to adopt were clearly stated, the authors of the CCSSM did not explain how to establish or sustain these norms with students. Interpretation was left up to the mathematics educational community (Rothman, 2011).

**Background to the Problem**

During these reforms, researchers gave practitioners frameworks and guidelines for teacher-to-student discourse. However, teachers were left on their own to know how
to create a classroom environment with sociomathematical norms in which productive student-to-student discourse could emerge and be sustainable. The inconsistency between what mathematics teachers were asked to do and their current skill set created a call to bridge mathematics research to mathematics teaching practices (Arbaugh et al., 2010).

In anticipation of the need for researched-based guidance for practitioners, the NCTM Board of Directors identified linking research and practice as a strategic priority and subsequently sponsored the Research Agenda Project, funded by the National Science Foundation (Arbaugh et al., 2010). The consensus of the committee was that if teachers understood how to provide interventions to students who were having challenges developing mathematics proficiency, teachers could be supported to “plan for and implement the kinds of discourse patterns that help important mathematical ideas surface for discussion” (Arbaugh et al., 2010, p. 27). National discussions emphasized the importance of classroom discourse as a potential intervention to help students who were not proficient in mathematics.

A great deal of research regarding teacher-initiated and sustained discussions is accessible to classroom teachers. Previous research projects regarding teacher-to-student discourse concur that teacher-initiated practices can lead to productive teacher-led classroom discussions and are generalizable to most classrooms (Sherin, 2002; Stein, Engle, Smith, & Hughes, 2008; Walshaw & Anthony, 2008). For example, Herbel-Eisenmann and Cirillo (2009) argue that teachers should use generalizable questioning (i.e., funneling and focusing) and restate students’ words to deepen student understanding of mathematics. However, research regarding effective ways to teach productive student-
to-student discourse is lacking.

Many business communities of learners have demonstrated success in creating an atmosphere for their employees in which discourse and argumentation lead to industrious outcomes (Akindayomi, 2015; Katzenbach & Smith, 1993; Lin, 1997; Lorinkova, Pearsall, & Sims, 2013; Paterson & Sneddon, 2011). An examination of common themes of productive business organizations was used to inform this study on how student teams could be more productive and how student-to-student discourse could be sustained in the mathematics classroom. Katzenbach and Smith describe common themes in productive business organizational teams, called High-Performance Teams (HPT). Although Katzenbach and Smith’s seminal work was published in 1993, many business organizations still use the tenets of its philosophies when training business teams (Bolman & Deal, 2017; Burke, 2017; Levi, 2015).

This study extended the HPT literature to provide more insight as to how student-to-student mathematics discourse could be developed through an integration of mathematics practices with the HPT structures. A common practice of business organizations is to emphasize aspects of teaming that lead to productivity such as: (1) clearly defined roles for each team member, (2) appropriately complicated challenges for each team member, and (3) individual accountability for each team member (Katzenbach & Smith, 1993). Just as members of business organizations have clearly-defined roles according to their specialized skills and training (i.e., accountant, designer, office manager, public relations expert), members of a mathematics classroom can gain expertise in research-based problem-solving strategies (National Research Council, 2001).
and contribute to their classroom teams in distinct ways. The HPT should be challenged with sufficiently complicated work which supports mathematics education research regarding high cognitive demand and the idea that struggling to make sense of mathematics is necessary to learn mathematics with understanding (Hiebert & Grouws, 2007). Providing opportunities for accountability and feedback underscores the research that specific, diagnostic comments to students significantly increases student achievement in mathematics (Black & Wiliam, 2009).

The idea of students’ working together in groups, called cooperative learning groups (Johnson & Johnson, 1994), with assigned roles is not new to the education field. However, the idea of clearly-defined roles that are specific to mathematics has not been examined. For example, the suggested roles of Johnson and Johnson’s cooperative learning theory are reader, checker, encourager, and elaborator. Those four roles are general to all subject areas and not clearly defined with respect to the mathematics classroom. The tenet of HPT is that each student has a clearly defined role that lends itself to more specific roles, particularly those that are researched based.

**Problem Statement**

The ways to facilitate student-to-student mathematical conversations are not clear; although research regarding how mathematics teachers can facilitate teacher-to-student discourse is well-researched. However, the factors that need to be present so that teachers can facilitate the pathways that students need to follow in order to have the autonomy to engage in productive student-to-student discourse is not known. Little research guides
practitioners on the specifics of how to facilitate an environment that is conducive to high levels of student-to-student discourse without a great deal of guess work on the part of the teacher (Wagner & Herbel-Eisenmann, 2014). The ways that teachers should respond to student-to-student discourse cannot be generalized since each situation seems to be context specific (Wagner & Herbel-Eisenmann, 2014). Practitioners need more researched-based strategies from which to draw in order to guide their students toward autonomy that encourages students to take responsibility for their own and their classmates’ learning of mathematics. Those strategies need to be inclusive of the idea that classroom environments are complicated settings in which students are negotiating with their peers to find common and correct mathematical understandings (Cobb & Yackel, 1996). In addition, instructional strategies need to encompass the notion that working together in groups for the attainment of common wisdom is a new idea for students and counter to the competitive culture in which many students may have been raised (Saar & Hargrove, 2013). Components of successful team building (Cobb & Yackel, 1996) in business organizations gave insight for this study into how to enable students to achieve the negotiation skills and autonomy that is necessary for productive mathematical conversations.

Significance of the Study

This study is significant because the results show practitioners how to facilitate student-to-student mathematical conversations. This study informs the field of mathematics education by showing ways to promote classroom sociomathematical norms
that lead to student autonomy. Autonomy is significant because it allows students to take responsibility for their own mathematical learning as well as the learning of their classmates. The results of this research project allow for the focus of teaching and learning to shift away from the teacher as the sole mathematics expert to the students’ sharing the mathematics authority.

**Summary of Research Study Design**

The purpose of this within-case and cross-case exploratory design (Yin, 2009) was to examine the nature of sociomathematical norm development as students worked in teams. In this study, a case was defined as a team of four middle school students as they worked together to solve challenging mathematics problems. The researcher studied four teams (i.e., four cases) with four middle school students in each team/case during a seven-week timeframe. The research study had three phases. The first phase involved gathering baseline data regarding the students’ sociomathematical norms. During the second phase, the students were trained to work in teams and then solved challenging mathematics problems in teams while the researcher collected data. During the last phase, the researcher collected data to explore shifts in sociomathematical norms and student autonomy after the students had the opportunity to be trained and work in teams.

**Research Questions**

Teachers need more specific research-based practices as they work toward facilitating an environment in which students have the autonomy to discuss, argue, and
reason with each other. Since each classroom context has so many varied factors, it is
difficult for researchers to prescribe successful practices for teachers. The purpose of this
study was to investigate how discourse between students developed when engaging in
HPT. To address this purpose, the focus for the study was on the factors that enable
students to discuss mathematics with each other. To investigate these factors, the research
questions for this study were:

   Overarching research question: How does High-Performance Teams training
support the development of sociomathematical norms that lead to in-depth mathematical
conversations among middle school students?

   1. When students are trained in High-Performance Teams (Katzenbach & Smith,
      1993) in classrooms that use research-based mathematics practices, why do
      they perceive and accept particular sociomathematical norms when engaging
      in Math-Talk Learning Communities (Hufferd-Ackles, Fuson, & Sherin,
      2004)?

      a. How and why do the students perceive these norms?

      b. How do the perceptions of these norms change over time?

   2. To what degree are the factors present during High-Performance Team
      mathematics activities (Katzenbach & Smith, 1993) also present when
      students engage in four differing levels of the Math-Talk Learning
      Community Framework (i.e., Questioning, Explaining Mathematical
      Thinking, Source of Mathematical Ideas, and Responsibility for Learning;
      Hufferd-Ackles et al., 2004)?

      a. On which of the four areas do the students rely most often?

      b. How and why do these four areas inform the relationship between High-
         Performance Teams (Katzenbach & Smith, 1993) and Math-Talk Learning
         Communities (Hufferd-Ackles et al., 2004)?

   Assumptions, Scope, and Limitations of Study

   The research for this study was grounded in the constructivist theory of learning,
where it is understood that students learn mathematics through active engagement in mathematics and connections made between new mathematics knowledge and prior knowledge structures (Clements & Battista, 1990). It was assumed that the students would do their best to engage in mathematics and team building lessons as a team in which all members of the team were actively engaged. It was assumed that students would actively solve the mathematical tasks assigned to them by their teachers. Additionally, the researcher assumed that the teachers of the classes in which research was conducted had the skills necessary to conduct productive teacher-to-student and classroom discourse.

This study was not designed for wide generalization to multiple populations (Kelly & Lesh, 2012) such as students of different ages, ethnic groups, and socioeconomic status. The sample of participants was limited to students whose teachers were willing to participate in the study and whose parents gave permission for their students to participate.

This research study did not investigate cooperative groups, which are characterized by small clusters in which students are strategically grouped according to varying abilities. Although some research has been conducted on students’ working in cooperative groups (Ding, Li, Piccolo, & Klum, 2007), no research has been conducted on HPT in the middle school mathematics classroom. Accordingly, this study was designed as a within-case and cross-case exploratory design in order for the issues of these proposed relationships to be explored in their initial stages.

The role of the teacher in originating discourse or shaping students’ thinking was
not addressed in this study. The focus of this study was on student-to-student interactions. Although the classroom teacher is definitely a part of the classroom culture and a critical player during the development of the sociomathematical norms, the norms are established only by what the students perceive and accept (Yackel, 2001). The role of the teacher in shaping sociomathematical norms was beyond the scope of this study.

This study was not interested in the creation and validation of lessons designed to promote discourse in the middle school mathematics classroom. The results of this research study inform teachers on practices that lead to curricula development and enactment. However, the purpose of this research study was to explore and describe the factors associated with the development of sociomathematical norms that may establish and sustain student-to-student discourse in the mathematics classroom. Therefore, the study focused on the factors that enable students to discuss mathematics and not what the teachers would gain specifically from the results of this study.

The researcher also acknowledges that there is a degree of subjectivity in this research project. The researcher was the mathematics instructional coach at this school and had an interest in promoting deeper student engagement at the school in which she conducted the study. To control for bias, she employed member checking and informant feedback (Creswell, 2013).

The researcher was the sole observer in the classroom since the classroom teacher was focused on supporting individual students as they attended to their roles within their teams. To control for subjectivity, after the researcher reviewed her data collections notes, she wrote her reflections in a memo and asked the classroom teacher to give
feedback regarding the episode. Further verification of data analyses included the researcher revisiting video footage in a constant comparison form of analysis.

**Definition of Terms**

The following terms are defined for this study.

*Classroom mathematical discourse (discussion)*: The ways of representing, thinking, talking, agreeing, and disagreeing about mathematical ideas (NCTM, 2000, p. 46); concerns both the process and content of communicating mathematical ideas in a classroom setting (Sherin, 2002).

*Productive mathematical discussion*: One of four types of comments—(1) one that offers a different solution than has already been mentioned, (2) one that is a sophisticated solution, (3) one that offers a more efficient solution than what has already been mentioned, or (4) one that offers an explanation to a previously mentioned solution (Cobb & Yackel, 1996).

*Classroom social norm*: An understanding of social behavior that is shared by all members of a classroom, both students and teacher (Yackel, 2001).

*Sociomathematical norm*: What the collective of the classroom considers to be a different, a sophisticated, or an efficient answer and is specific to the understanding of mathematics by a specific group of students (Cobb & Yackel, 1996); the teachers’ mathematical beliefs and values may develop concurrently with the classroom’s sociomathematical norms (Yackel, Rasmussen, & King, 2000).

*Math Talk Learning Community (Math-Talk)*: A framework with developmental
trajectories that describe the building of a Math-Talk Learning Community; trajectories are: questioning, explaining mathematical thinking, sources of mathematical ideas, and responsibility for learning (Hufferd-Ackles et al., 2004).

*High-Performance Teams (HPT)*: Model of team building that has proven successful in business organizations (Katzenbach & Smith, 1993). Three key areas of focus are: (1) select members for skill and skill potential, not personality; (2) challenge the group regularly with fresh facts and information; (3) exploit the power of positive feedback, recognition, and reward.
CHAPTER 2
LITERATURE REVIEW

Mathematics teachers are tasked with instilling in students the ability to “construct viable arguments and critique the reasoning of others” (National Governors Association Center for Best Practices, 2010, see mathematical practice standard #3). Accordingly, the National Science Foundation funded the Research Agenda Project, a working conference in which the NCTM invited 60 mathematics education researchers to analyze 350 mathematics education practitioner-generated questions. The invited scholars and practitioners identified that “the field should better understand how effective teachers plan for and implement the kinds of classroom discourse patterns that help important mathematical ideas surface for discussion” (Arbaugh et al., 2010, p. 27). A broad spectrum of research has been published regarding mathematics whole-class discussion and the teacher’s role in promoting productive teacher-to-student discourse (Sherin, 2002; Stein et al., 2008; Walshaw & Anthony, 2008).

The teacher obligations regarding productive teacher-to-student discussions are clear. For example, researchers have given guidelines to practitioners regarding how to question students and respond to their answers. However, what is not clear is how to sustain productive student-to-student mathematics discussion. The pathways are not certain that students need to follow to gain sufficient autonomy so that they can participate in productive student-to-student discourse. Components of successful team building in business organizations may give insight as to how to enable students to achieve this autonomy. However, there lacks research to sustain this supposition. We did
not know to what degree students engaged in team building would enact and sustain student-to-student discourse.

This research study proposed to extend the HPT literature to provide more insight as to how student-to-student mathematics discourse could be developed through an integration of mathematics practices with the HPT structures. More specifically, this study investigated the following research questions:

Overarching research question: How does High-Performance Teams training support the development of sociomathematical norms that lead to in-depth mathematical conversations among middle school students?

1. When students are trained in High-Performance Teams (Katzenbach & Smith, 1993) in classrooms that use research-based mathematics practices, why do they perceive and accept particular sociomathematical norms when engaging in Math-Talk Learning Communities (Hufferd-Ackles et al., 2004)?
   a. How and why do the students perceive these norms?
   b. How do the perceptions of these norms change over time?

2. To what degree are the factors present during High-Performance Team mathematics activities (Katzenbach & Smith, 1993) also present when students engage in four differing levels of the Math-Talk Learning Community Framework (i.e., Questioning, Explaining Mathematical Thinking, Source of Mathematical Ideas, and Responsibility for Learning) (Hufferd-Ackles et al., 2004)? On which of the four areas do the students rely most often?
   c. How and why do these four areas inform the relationship between High-Performance Teams (Katzenbach & Smith, 1993) and Math-Talk Learning Communities (Hufferd-Ackles et al., 2004)

Hence, the purpose of this literature review was to present a review of the empirical research regarding social and sociomathematical norms found in the mathematics classroom that related to productive student-to-student discourse. The proposed dissertation study was situated in this literature base and sought to extend the
current literature base by examining the relationship between High-Performance Teams (HPT; Katzenbach & Smith, 1993) and student participation in the highest level of the Math-Talk Learning Community Framework (Math-Talk; Hufferd-Ackles, 2004). It is at this level that students are establishing sociomathematical norms which promote rich engagement with discourse.

**Conceptual Framework**

This research study explored and described the pedagogical factors associated with the development of sociomathematical norms that may establish and sustain student-to-student discourse in the mathematics classroom. This pedagogically focused study examined how HPT and Math Talk affect students’ discourse around mathematics. Figure 2.1 shows the conceptual framework that depicts the relationships between HPT and Math-Talk that the researcher used to investigate the development of student-to-student discourse. The conceptual framework displays the pedagogical factors associated with the shift towards more student autonomy and deeper student-to-student discourse. The purpose of using HPT as a vehicle to establish sociomathematical norms is to explore possible relationships between the factors present in successful business organizations and the highest level of student-to-student discourse as defined by Math-Talk. Through HPT, many business communities of learners have demonstrated success in creating an atmosphere for their employees in which discourse and productive argumentation led to industrious outcomes (Akindayomi, 2015; Katzenbach & Smith, 1993; Lin, 1997; Lorinkova et al., 2013; Paterson & Sneddon, 2011).
Figure 2.1. Conceptual framework explaining possible relationships between high performance teams and Math-Talk.
A common practice of business organizations is to emphasize aspects of teaming that lead to productivity such as: (1) clearly defined roles for each team member, (2) appropriately complicated challenges for each team member, and (3) individual accountability for each team member (Katzenbach & Smith, 1993). These aspects of teaming relate to researched-based mathematical instructional practices. The first, clearly defined roles, relates to research-based mathematical practices (National Research Council, 2001), which are: (1) give representation explaining context of problem, (2) give a graphic representation of mathematics needed to solve the problem, (3) execute the procedures needed to solve the problem efficiently, and (4) check for accuracy during all steps of the problem-solving process. The second, challenging employees with difficult work, corresponds to the instructional practice of giving students access to rich, mathematical tasks (Boaler, 2008; Kosyvas, 2015), and giving regular, specific feedback corresponds to the high effect sizes reported when teachers use formative assessments to give students descriptive feedback (Hattie & Timperley, 2007; see Figure 2.1).

The framework also lists the four levels of Math-Talk. To make the transition from Level 0 to Level 3, students must possess mathematical authority and autonomy. To study the transition from Level 0 to Level 3, the researcher proposes to teach participating students three foci of HPT. As described in Figure 2.1, the three foci are: (1) select members for skills and clearly define their roles, (2) challenge the group regularly, and (3) give positive feedback and ensure individual accountability. The clearly defined roles listed in the conceptual framework reflect research-based problem-solving strategies. Challenging the group regularly with rich mathematical tasks and giving
specific, positive, individual feedback reflect researched-based, best mathematical practices (Boaler, 2008; Hattie & Timperley, 2007; Kosyvas, 2015.) The results of many research studies conclude that students thrive when working on challenging problems when they receive adequate support.

However, no research studies have given guidance for students as to how to learn to give that type of feedback to each other, so it falls on the teacher to give the support. Still, it is physically impossible for the classroom teacher to be able to engage in a conversation that elicits mathematics conversations with each student simultaneously. Students need to gain the ability to provide that support for each other. Each of the researched-based practices listed in the conceptual framework correspond to HPT and may help create a mathematics classroom in which students can develop mathematical judgment and have enough autonomy to engage in productive student-to-student discourse (Lopez & Allal, 2007; Yackel, 2001). This research study proposed to investigate whether the factors associated with HPT related to student enactment of the types of sociomathematical norms that allowed students to engage in the highest level of student-to-student discourse (Hufferd-Ackles et al., 2004), one in which students take full responsibility for their own learning the learning of their classmates. The following review of the literature will explain what has been currently investigated in student-to-student discourse and team building activities.

**Literature Review Summary**

After reviewing the literature, the following four themes relating to the focus of
HPT and the areas of Math-Talk emerged: (1) student-to-student discourse in the mathematics classroom, (2) classroom norms that are associated with productive student-to-student mathematics discussions, (3) student autonomy and the transfer of authority, and (4) aspects of organizational teams in the business industry. A synthesis of results relative to this study will be discussed following the review of the literature.

**Student-to-Student Discourse**

Many teachers are reticent to promote student-to-student discourse because it involves different instructional tactics than more traditional teacher-to-student discourse. For example, since student-to-student discourse employs the negotiation of individual mathematical concepts between peers (DeVries, 1997), the teacher has no assurance that the mathematics that the students are discussing is accurate. This lack of assurance is one of the reasons that many teachers feel a loss of efficacy when allowing students to engage in student-to-student discourse and report difficulty in managing such discourse (Ding et al., 2007). The literature regarding mathematical student-to-student discourse describes environments that may lead to productive, in-depth student-to-student discourse (Berland, 2010). Fuentes (2013) listed 22 specific interventions to try when students are not participating productively (Fuentes, 2013). However, a more specific description of factors present during productive student-to-student discourse could inform the mathematics scholarship community. To explore possible factors that may relate to productive student-to-student discourse, the following discussion begins with definitions of productive student-to-student discourse and then concludes with an examination of two frameworks that have been used to evaluate student-to-student discourse in the
Definitions of mathematical discourse. Sfard (2001) broadly defined mathematics discourse as all utterances when attending to understanding mathematics. Sfard proposed that all mathematical discourse helps solidify mathematical ideas and that student thinking involved in mathematical discourse may be conceptualized as a case of communication with others or communication with oneself. What students think, whether uttered aloud or merely thought, contributes to their understanding of mathematics. Sfard categorized this practice as thinking-as-communicating. Included in Sfard’s definition of discourse were body movements, situational clues, and interlocutors’ histories. Respective examples in mathematics include hand movements, students’ looking around the room to find a model posted on the wall, or stories about previously shared experiences. When students argue with themselves, when they ask questions of themselves, or when they wait for their own responses, they are building mathematical understanding. Sfard proposed that discussion does not just help students think but is indistinguishable to their ability to think.

Cobb and Yackel (1996) gave a more specific definition of productive mathematical discourse: productive student mathematical discourse is one of four types of student comments that may be uttered in a mathematics classroom. A student comment is considered productive if it (1) it offers a different solution than has already been mentioned, (2) it offers a sophisticated solution, (3) it offers a more efficient solution than what has already been mentioned, or (4) the comment offers an explanation to a previously mentioned solution (Cobb & Yackel, 1996). Further, classroom teachers
should work toward teaching individual students what a productive comment is so that students can monitor themselves and contribute with productive mathematical comments (Cobb & Yackel, 1996).

**Frameworks for evaluating student-to-student discourse.** As researchers and practitioners have tried to define what productive mathematics discourse looks like, several frameworks have emerged. The frameworks provide categories for the continuum of student comments as they relate to the level of autonomy of the student. One side of the continuum represents a student merely repeating the mathematics of the teacher. The other side of the continuum represents comments that reflect substantive autonomy (Bishop, Hardison, & Przybyla-Kuchek, 2016). The frameworks help researchers provide generalizations regarding the participation and autonomy of the students. The following two subsections detail two frameworks for categorizing students’ autonomy: The Coding Instrument of Responsiveness (Bishop et al., 2016) and the Math-Talk (Hufferd-Ackles et al., 2004).

*The Coding Instrument of Responsiveness.* The Coding Instrument of Responsiveness (Bishop et al., 2016) provided a framework for examining student and teacher discourse in the classroom. The intent of the framework was to help teachers understand the categories of comments that students were making during mathematics class so that the teachers could make productive in-the-moment decisions regarding mathematics instruction. The authors of the Coding Instrument of Responsiveness (Bishop, et al., 2016) constructed the framework after studying the variations in mathematical responses across seven middle grades classrooms. The researchers posit
that if instructors can better understand student comments by categorizing student responses, instructors can more effectively determine the direction of mathematics lesson. The framework, in its entirety, addresses productive ways for teachers to respond to student discourse.

Of special significance to this study is the method of coding student responses. When using this framework for coding, student discourse contributions are placed into one of four levels: “none,” “minimal,” “considerable,” and “substantive.” The category of none means that students gave no mathematical contribution, generally because of a teacher monologue. The category of minimal means students are performing routine calculations or recalling facts. The category of considerable refers to evidence that students are making sense of mathematical content and are able to share their ideas. The category of substantive includes comments in which students are providing justifications, making generalizations, or participating in mathematical argumentation.

The general purpose of this framework is to enable teachers to make profound in-the-moment decisions regarding how to respond to student mathematical discourse. As a result, the descriptions of the categories of student discourse are brief and do not necessarily allow for complexities that may arise during a student-to-student conversation. A framework that gives more detailed descriptions of categories and several areas of focus within those descriptions could provide additional insight into student-to-student mathematical discussions.

*Math-Talk Learning Community Framework (Math-Talk).* Hufferd-Ackles et al. (2004) created a framework, the Math-Talk Learning Community (Math-Talk), to
examine the process when mathematics teachers shift from teaching in a classroom using
direct instruction to teaching in a classroom in which the students take full responsibility
for their own learning. Math-Talk differs from the Coding Instrument of Responsiveness
in that it allows for four times as many descriptive categories. Math-Talk separates all
student-to-student discourse into four developmental areas, which each has four different
levels of discourse within that area. Each area is reflective of student autonomy and
mathematical thinking. The developmental areas that describe the building of Math-Talk
are: (1) questioning, (2) explaining mathematical thinking, (3) sources of mathematical
ideas, and (4) responsibility for learning (see Table 2.1).

In the area of questioning, the framework allows for descriptions of the transition
that happen as the classroom norms shift from the teacher as the sole questioner to both
the students and teacher as questioners. In the area of explaining mathematical thinking,
the framework allows for descriptions of the transition that happen as students are able to
explain and articulate their own ideas. In the area of source of mathematical ideas, the
framework allows for descriptions of the shift from teacher as the source of all

Table 2.1

<table>
<thead>
<tr>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Course of mathematical ideas</th>
<th>Responsibility for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift from teacher as questioner to students and teacher as questioners.</td>
<td>Students increasingly explain and articulate their math ideas.</td>
<td>Shift from teacher as the source of all math ideas to students’ ideas also influencing direction of lessons.</td>
<td>Students increasingly take responsibility for learning and evaluation of others and self. Math sense becomes the criterion for evaluation</td>
</tr>
</tbody>
</table>
mathematical ideas to students’ ideas also influencing the direction of lessons. Finally, in
the area of responsibility for learning, the framework gives descriptions for students
increasingly taking responsibility for learning and evaluation of others and self.

Each of the four areas (questioning, explaining mathematical thinking, source of
mathematical ideas, and responsibility for learning) has been assigned four different
levels, zero through three, for a total combination of 16 different classifications of
discourse. This level of specificity makes the Levels of Math-Talk a useful tool for
research because it describes students’ comments and actions when they have enough
autonomy to become experts in mathematical reasoning and can take responsibility for
their own and their classmates’ learning.

Table 2.2 shows what each of four areas (questioning, explaining mathematical
thinking, source of mathematical ideas, and responsibility for learning) looks like at the
lowest level, Level 0. Level 0 represents the traditional teacher-directed classroom with
brief answer responses from students. In response to questions, students give short
answers and only respond to their teachers. With respect to explaining mathematical

Table 2.2

<table>
<thead>
<tr>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Source of mathematical ideas</th>
<th>Responsibility for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students give short answers and respond to the teacher only. No student-to-student talk.</td>
<td>No student thinking or strategy-focused explanation of work. Only answers are given.</td>
<td>Students respond to math presented by the teacher. They do not offer their own math ideas.</td>
<td>Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves.</td>
</tr>
</tbody>
</table>
thinking, the student only gives answers, no explanations. For the source of mathematical ideas, the students only respond to the mathematics presented by the teacher and do not follow their own mathematics ideas. In this level, the students do not take any responsibility for their learning, because they only listen passively.

Table 2.3 describes the students’ actions as the teacher begins to pursue student mathematical thinking. At this level, the teacher still plays a central role in guiding the classroom discussions. In the area of questioning, the students answer questions while others passively wait. In explaining mathematical thinking, students only give information about their thinking if they are probed to do so by their teacher. The source of mathematical ideas may come from students, but those ideas are not explored. In the area of responsibility for learning, students showed another student how to do a problem but only by request of the teacher.

The next level in the framework is Level 2 (see Table 2.4) in which the teacher begins to model to students how to help each other. In this level some co-teaching and

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>As a student answers a question, other students listen passively or wait for their turn.</td>
<td>Students give information about their math thinking, usually as it is probed by the teacher (minimal volunteering of thoughts). They provide brief descriptions of their thinking.</td>
<td>Some student ideas are raised in discussions, but are not explored.</td>
<td>Students become more engaged by repeating what other students say or by helping another student at the teacher’s request. This helping mostly involves students showing how they solved a problem.</td>
</tr>
</tbody>
</table>
Table 2.4

**Math-Talk Level 2: Teacher Modeling and Helping Students Build New Roles—Some Co-Teaching and Co-Learning Begins as Student-to-Student Talk Increases. Teacher Physically Begins to Move to Side or Back of the Room**

<table>
<thead>
<tr>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Source of mathematical ideas</th>
<th>Responsibility for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students ask questions of one another’s work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions.</td>
<td>Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing fuller descriptions and begin to define their answers and methods. Other students listen supportively.</td>
<td>Students exhibit confidence about their ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson.</td>
<td>Students begin to listen to understand one another. When the teacher requests, they explain other students’ ideas in their own words. Helping involves clarifying other students’ ideas for themselves and others. Students imitate and model teacher’s probing in pair work and in whole-class discussions.</td>
</tr>
</tbody>
</table>

Co-learning begins, and teachers physically move to the back of the room. In the area of questioning, students ask questions of one another’s work, sometimes at the prompting of the teacher. In the explaining mathematical thinking area, students give information as it probed by the teacher. In the area of source of mathematical ideas, students show confidence about their own thinking. With respect to responsibility for learning, students begin to listen to understand each other.

The highest level of Math-Talk is Level 3 (see Table 2.5). It is at this highest level that students have the autonomy to take responsibility for their own learning and the learning of their classmates. In the area of questioning, the student-to-student talk is initiated by the student, not the teacher. Many questions begin with “why?” and require justification from the person answering. In the area of explaining mathematical thinking, the students describe more complete strategies and are able to justify them. In the area of
Table 2.5

*Math-Talk Level 3: Teacher as Co-Teacher and Co-Learner—Teacher Monitors All That Occurs, Still Fully Engaged. Teacher Is Ready to Assist, But Now in More Peripheral and Mentoring Role (Coach and Assister)*

<table>
<thead>
<tr>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Source of mathematical ideas</th>
<th>Responsibility for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to responses. May questions are “Why?” questions that require justification from the person answering. Students repeat their own or other’s questions until satisfied with answers.</td>
<td>Students describe more complete strategies; they defend and justify their answers will little prompting from the teacher. Students realize that they will be asked questions from other students when they finish, so they are motivated and careful to be thorough. Other students support with active listening.</td>
<td>Students interject their ideas as the teacher or other students are teaching, confident that their ideas are valued. Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.</td>
<td>Students listen to, understand, then initiate clarifying other students’ work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors.</td>
</tr>
</tbody>
</table>

source of mathematical ideas, students interject their ideas as the teacher or other students are teaching, confident that their ideas are valued. In the area of responsibility for learning, students listen to each other’s ideas and ask clarifying questions. Students assist each other in understanding and clarifying errors.

The Levels 0 and 1 emphasize the role of teachers in mathematics discourse. However, Levels 2 and 3 place emphasis on the discourse of the students. Of particular concern is the transition from Level 2 to Level 3, which requires a substantial degree of student autonomy and a shift of mathematical authority from teacher to students (Hufferd-Ackles et al., 2004). As the students and teacher move from Level 2, to Level 3, a significant change in norms and the teacher-student relationships relative to the mathematics occurs. How this change occurs is worthy of further examination (Hufferd-Ackles et al., 2004). An examination of this change begins with an understanding of what
is happening in the Level 2 environment. The Level 2 environment is one in which the teacher is both the co-teacher and the co-learner. The teacher must remain fully engaged and be ready to assist at a moment’s notice. The teacher must be able to do this in a peripheral and monitoring role, not as the central authority. In the Level 3 environment, the students interject their ideas because they are confident that their ideas are valued. Students spontaneously compare, contrast, and build on each other’s ideas, comparing concepts learned from previous mathematics lessons. The role of the teacher in the Level 3 environment is supportive of students as they help one another sort out misconceptions. The teachers follow up only when needed (Hufferd-Ackles et al., 2004).

Huffer-Ackles et al. (2004) were not able to identify the specific factors that were present during a significant shift. The move from students’ being the source of some mathematical ideas (Level 2) to the level in which students take full responsibility for the learning and evaluation of others and self (Level 3) is an important shift, yet one that is difficult to definitively describe (Hufferd-Ackles et al., 2004). The identification of the factors that would promote this shift from the students as the source of some mathematical authority to the having the authority to evaluate their own and their classmates’ mathematics could inform educators how to facilitate productive student-to-student mathematical discourse.

**Norms**

Classroom norm is the general term for the norms that establish what actions are considered normative in any classroom (Yackel, 2001). A specific kind of classroom norm is a social norm, which is the accepted social actions that are considered normative
A more specific social norm, a sociomathematical norm, is one that emerges from both classroom norms and social norms and is specific to a mathematics classroom. Some sociomathematical norms may lead to student autonomy, which allows for a shift of authority in the mathematics classroom (Cobb & Yackel, 1996). This shift explains the transition of authority of the mathematics expert in the classroom from the teacher to students who possess correct mathematical logic (Wagner & Herbel-Eisenmann, 2014). Thus, these norms change the focus away from how the teacher is teaching mathematics to how the students are understanding mathematics. To more clearly describe the relationships found in the literature between classroom norms and sociomathematical norms in learning mathematics, this section is divided into three subsections. The first is a review of social norms and subsequent research regarding social norms in the mathematics classroom. The next subsection is an examination of the research regarding sociomathematical norms. The last subsection reviews literature regarding the transfer of authority from the teacher to student.

Social norms. Social norms have roots in Vygostky’s Sociocultural Learning Theory, meaning that learning is a social endeavor and that teachers have the responsibility to facilitate a setting in which social interaction is conducive to learning. Accordingly, teachers’ roles when developing social norms in the classroom includes choosing instructional tasks that encourage three types of student activity: active involvement, mental activity, and social learning in which communication can take place (Vygotsky, 1978). Teachers should choose curriculum that engages students, is sufficiently challenging, and allows for peer interaction. When teachers engage students
with this type of activity, these teachers are charged with involving students in appropriate learning activities that put students in their appropriate Zone of Proximal Development. The intent of social norms is that every child may develop understanding of taken-as-shared meanings of society, which Vygotsky used to define culturally established concepts. The teachers are expected to emphasize learning mathematics by providing the foundation for social norms with effective communication in the classroom, thus helping students effectively share ideas around their mathematics understandings (Vygotsky, 1978).

Walshaw and Anthony (2008) focused on the effects that social norms could have on classroom mathematics discussion. Results suggest that classroom norms that fostered a sense of cultural identity, citizenship, and exploration, led to more productive and equitable mathematics discourse among students. Further, social norms that encouraged tolerance, fairness, caring, diligence, and generosity produced a classroom environment that allowed students to discuss mathematics freely. Walshaw and Anthony also found that teachers should avoid allowing articulate students to dominate the mathematics discussion, thus reducing the social obligations of quieter students. Results indicated that when teachers allowed more gregarious students to overshadow their counterparts, teachers might inadvertently expect a lower cognitive demand on some students, as they were not expected to engage in the discussion. For instance, Walshaw and Anthony found that the quieter students, who were often those with limited language or slower processing abilities, were provided social norms that did not expect them to engage with the mathematics discussion. Thus, the quieter students were excluded from full
engagement, and their mathematical development did not progress as quickly as it could have.

Boaler (2008) found three student practices to be most effective in establishing these norms: (1) respect for other people’s ideas, leading to positive intellectual relations; (2) commitment to the learning of others; and (3) learned methods of communication and support. Specifically, the learned communications were divided into the following factors:

- Asking good questions
- Rephrasing problems
- Explaining well
- Being logical
- Justifying work
- Considering answers of others (Boaler, 2008, p. 185).

These six factors do not naturally occur in most mathematics classrooms. Students need to learn these methods of communication when discussing mathematics. A good first step is to model them; that may be all a teacher needs to do to instill these practices in some students (Webb, Nemer, & Ing, 2006). Webb et al. found that when teachers communicated with their mathematics students using good questions and requests to justify work, student-to-student conversations followed accordingly. For example, when teachers asked good questions of their students, researchers observed that students, in turn, asked good questions of each other. The teacher’s modeling these attributes is sufficient for some students to engage in productive student-to-student mathematics discussions (Webb et al., 2006), but not all students. Another tactic to instill the above social norms in a classroom involves unpacking each of the six factors above and examining how they relate to one another. Many of the above factors have a reflexive
relationship. For example, asking good questions comes from understanding the problem, which comes about by rephrasing a problem. Asking good questions also leads to students’ explaining their mathematics ideas, which leads to being logical by justifying one’s work. After students justify their work, other students will naturally ask good questions. Thus, the practice of modeling productive communication and being mindful of student conversations can lead to productive social norms in the mathematics classroom. Classroom and social norms are specific to classrooms but not necessarily to mathematics learning. Sociomathematical norms are the norms that are either established or evolved that deal only with mathematics. To understand how these norms relate to mathematics learning, the following section will discuss sociomathematical norms.

**Sociomathematical norms.** A sociomathematical norm is considered a normative understanding of what counts as mathematically significant. Whereas classroom norms refer to teacher and student practices that are normative within a classroom. Specifically, according to Cobb and Yackel (1996), a sociomathematical norm is what the collective of the classroom considers to be a different, sophisticated, or efficient answer and is specific to the understanding of mathematics by a specific group of students. Although forming sociomathematical norms is a collective process, teachers are charged to teach their students autonomy so that students can, individually, make judgments according to what is a different, a sophisticated, or an efficient answer (Cobb & Yackel, 1996). During the evolutionary processes of establishing and maintaining sociomathematical norms, it is not uncommon that the teachers’ mathematical beliefs and values develop concurrently with the classroom’s sociomathematical norms (Yackel et al., 2000).
Cobb and Yackel (1996) account for the idea that students need to be taught how to contribute to the mathematical discussion and not just reiterate what they think their teachers want to hear. Most students do not naturally have the ability to speak with autonomy and are not sure how to contribute productively. Rather they are looking toward teachers who are the authority figures to know how to act or speak appropriately. Sociomathematical norms are significant to student-to-student discourse since many students mirror the types of statements teachers make during their mathematical conversations (Webb et al., 2006). When teachers instruct their students through recitation of procedures and mostly provide answers in discrete steps, the student-to-student conversations follow suit. However, when teachers instruct their students by asking thoughtful questions, student-to-student conversations increase in depth (Webb et al., 2006). Thoughtful questions might include “How can you prove that your answer is correct” or “Could you find the answer in a different way” and establish implicit mathematics expectations in the classroom.

For example, when a third-grade teacher established the sociomathematical norm that her students needed to find one correct method of solving a problem, her students found only one correct method. The students were not interested in discussing the mathematical problem after they had found just one answer. For most of those students, the mathematical discussions were over (Lopez & Allal, 2007). Conversely, a third-grade teacher at the same school saw something quite different with his students. This teacher had established the sociomathematical norm that students were to find the most efficient mathematics strategy to a solution, and as a result the students in his classroom behaved
differently than the students of the teacher who required one correct method. The students in the second classroom engaged in a great deal more discussion regarding solutions because they had to present arguments to their peers to convince each other whose solution path was the most efficient. Lopez and Allal found that the students, who were expected to find the most efficient solution, had much more in-depth mathematical conversations than the students who did not have this sociomathematical norm and that productive sociomathematical norms can be established through teacher expectations of students’ solutions.

Kazemi and Stipek (2001) found three themes in their study regarding relationships between teacher expectations and levels of cognitive demand in which students engage. First, Kazemi and Stipek found that the students in the classrooms who had productive socio-mathematical norms understood that an explanation must consist of a mathematical argument, not merely a description of a procedure. Second, Kazemi and Stipek found that the students who understood that mathematical thinking may involve more than one strategy understood that errors provided opportunities to look at a problem a different way in order to pursue alternative strategies. Finally, Kazemi and Stipek found the students understood that working together effectively involved both individual accountability as well as reaching consensus through discussion. These three themes reinforced the idea that student argumentation involved understanding multiple solution paths and that collaboration with peers could help illuminate which paths may be the most efficient. In addition, students needed to both have individual accountability as well as the ability to listen and collaborate in order to converse with their peers.
As mathematics teachers take on the responsibility to facilitate a setting in which social interaction is conducive to learning mathematics, teachers need to provide the foundation for social norms by modeling effective mathematics communication. Classroom norms that are most likely to result in high-level mathematics discussions foster a sense of cultural identity, citizenship, and exploration among students. Further, students need to be taught how to contribute to the mathematical discussion and not just reiterate what they think the teachers want to hear. Productive student argumentation involves understanding that solving mathematics problems involves attending to multiple solution paths, which can happen with collaboration with peers. In addition, students need to both have individual accountability as well as the ability to listen and collaborate when discussing mathematics with their peers.

**Transfer of authority from teacher to student.** In order for students to engage in productive student-to-student discourse, there needs to be a shift of authority (Yackel, 2001). This means that the teacher can no longer be the only mathematics expert in the room. Students who have sufficient autonomy to make sound mathematical judgments need to hold the mathematics authority. Sociomathematical norms need to be established so that mathematics logic supersedes traditional authority and social position (Wagner & Herbel-Eisenmann, 2014). Teachers need to instill autonomy in students so that the students understand the process of determining what is a correct answer and the judgment to argue and critique the reasoning of others (Cobb & Yackel, 1996).

During student-to-student conversations, the teacher must be consistently attentive to students’ mathematical conversations and the students’ working of
Mathematical problems. However, as attentive as teachers must be, the teachers also need to assume a role that is not absolute authority but rather authority that represents the mathematics community, not necessarily the only person who is capable of deeming an answer or method right or wrong (Yackel & Cobb, 1996). Mathematics instructors need to find a sensible balance between giving students a reasonable amount of autonomy over their mathematics while ensuring that student work is held accountable to mathematics standards (Stein et al., 2008). It is possible for teachers to hold students accountable to the mathematics and relinquish absolute authority. However, in order to do this, teachers must consistently reinforce the classroom social norms and sociomathematical norms of the classroom that have been previously established. The question remains as to what specific factors contribute to the emergence of social and sociomathematical norms that allow students to achieve the autonomy that allows them to become mathematical experts among their peers.

**High Performance Teams Related to Norms**

The subsequent research results regarding student-to-student discourse concur that each mathematics classroom with its accompanying social norms and sociomathematical norms are so varied that finding generalizable factors is difficult. Successful teams in a business environment share some commonalities with effective learning within the mathematics classroom. Successful teams in the business environment acknowledge that the work place is complicated since technological advances, political changes, and economical developments contribute to constant upheaval and opportunity (Saar & Hargrove, 2013). The mathematics classroom is also a complicated environment because
students and teachers must navigate social situations and language barriers as they work toward communicating about and understanding mathematics (Bauersfeld, 1994). Many business organizations rely on team-building research to provide guidance for productivity in industry. The following three themes are prevalent throughout the literature in team-building for organizations: (1) clearly defined roles for members, (2) focus on group objectives, and (3) individual accountability (Akindayomi, 2015; Katzenbach & Smith, 1993; Lin, 1997; Lorinkova et al., 2013; Paterson & Sneddon, 2011). Katzenbach and Smith established HPT as a productive organizational teaming strategy (Solis, Sinfield, & Abraham, 2012; Warrick, 2016). Katzenbach and Smith identified seven areas in which organizations should focus in order to facilitate HPT. Of those seven areas, three can be related specifically to the middle school mathematics classroom. These will be discussed in relation to a mathematics classroom in the following subsections. The first subsection is a discussion of selecting team members for their skill. The next subsection considers challenging the group regularly with novel ideas and information. The last subsection discusses exploiting the power of positive feedback, recognition, and reward.

**Select members for skill.** Selecting members for skill implies that each team member will have a clearly defined role and one in which he or she has been fully trained. Just as members of business organizations have clearly-defined roles according to their specialized skills and training (i.e., accountant, designer, office manager, public relations expert), members of a mathematics classroom can gain expertise in research-based problem solving strategies (National Research Council, 2001) and contribute to
their classroom teams in distinct ways. The proposed roles for HTP in the mathematics classroom are connected to research-based (National Research Council, 2001) practices.

The practices are:

1. Give representation explaining context of problem,
2. Give a graphic representation of mathematics needed to solve the problem,
3. Execute of the procedures needed to solve the problem efficiently, and
4. Check for accuracy during all steps of the problem-solving process.

The corresponding roles for HPT are:

1. Study the problem and draw a picture of the context of the problem,
2. Draw a diagram that illustrates the mathematics of the problem,
3. Solve the problem by performing mathematic procedure(s), and
4. Check for accuracy in the work of the other team members.

Students who can read a problem and know how to draw a diagram to show team members what a question is asking can help the other students in their group understand the context of the problem. Students who know multiple ways of representing the mathematics of a problem can draw diagrams to help their teammates understand the mathematics needed to solve a problem. Students who understand the mathematical procedures necessary to solve a mathematical problem know which calculations should be performed. Students who are focusing on finding mistakes in their teammates’ computations can keep their group from straying from their team objective by giving frequent updates regarding accuracy.

Having distinct roles within a team contributes to social norms that help students effectively share ideas (Vygotsky, 1978). One of the benefits, according to Katzenbach and Smith (1993) and Lin (1997), of emphasizing skills within a team is that it emphasizes the team objective and minimizes the conflicts that can emerge among the
participants. Focusing on what the team could accomplish is more productive than focusing on relationship building (Katzenbach & Smith, 1993; Lin, 1997). Further, productive teams attend to the skills of its members and rely on tacit social contract, the idea that maintaining a focus on the team objectives implicitly tells how group members should treat each other. Productive teams do not focus on personal relationships and only address personality differences on an as-needed basis (Saar & Hargrove, 2013). The practice of selecting students for mathematics teams for skill reinforces the classroom norm that each student is expected to gain the skills in order to contribute to the mathematics learning of their team and class.

**Challenge the group regularly with novel ideas and information.** The second focus of HPT philosophy is that the team is challenged with sufficiently challenging work. Teams that are able to work on complicated problems thrive. Challenging work energizes team members thereby helping the team shape a common purpose (Katzenbach & Smith, 1993). By challenging teams, a leader can cater to diverse member strengths, which positions team members to grow and to remain engaged (Saar & Hargrove, 2013). This theme supports the mathematics research that students need to struggle productively in order to understand the complexities of mathematics (Hiebert & Grouws, 2007). This productive struggle is one of the underlying themes of the CCSSM practices (National Governors Association Center for Best Practices, 2010). Challenging a group of students regularly supports the research that admonishes teachers to give students challenging problems with multiple solution paths (Boaler, 2008; Stein et al., 2008). Giving students challenging problems reinforces the norm that the students are capable of performing
challenging mathematics and that the productive struggle is an important part of learning mathematics. Just as giving students problems that are challenging invigorates their learning, it is also important to scaffold complicated problems so that students have access to understanding the problem (Anghileri, 2006).

**Exploit the power of positive feedback, recognition, and reward.** The third focus of HPT philosophy is that individual members are held accountable and receive feedback. This begins with a leader clearly defining individual expectations and recognizing achievement while also fostering the importance of personal satisfaction when an objective has been completed (Saar & Hargrove, 2013). For example, it is productive for leaders to use awards as recognition and to promote the feeling of the satisfaction of a job well done (Katzenbach & Smith, 1993). In the mathematics classroom, specific, diagnostic comments to students significantly increase the depth of their mathematics understanding (Black & Wiliam, 2009; Hattie & Timperley, 2007). The NCTM asserts that feedback is most useful when it supports the “learning of important mathematics and furnishes useful information to both teachers and student” (National Council of Teachers of Mathematics, 2000, p. 22). Giving feedback, recognition, and reward reinforces the norm that the teacher is actively involved in the students’ learning process and pays careful attention to students’ achievements and misconceptions. As the teacher models this attentive behavior, it will lead to the norm that the other students are expected to be as attentive (just as their teachers are) to the learning of their classmates in their mathematics classroom.
Synthesis of the Review of the Literature

The results of many research studies give direction to practitioners regarding how to discuss mathematics with their students (Sherin, 2002; Stein et al., 2008; Walshaw & Anthony, 2008). The research regarding productive teacher-to-student discourse is prescriptive. However, what is missing is how to teach students the kind of autonomy that allows them to engage in productive mathematics discourse with each other. Researchers have identified classroom norms, social norms, and sociomathematical norms that are present during student-to-student discourse in which students were engaged in deep thinking and understanding of mathematics. Researchers have also identified that students need to possess enough autonomy to individually understand what it means to make sense of mathematics in a community. In addition, mathematics education scholars have created a framework with which to measure the degree of autonomy of student-to-student mathematics discussion. However, there is a need to understand how to facilitate an atmosphere in which all of these elements are present, so that students may engage in productive student-to-student mathematics conversations. The pathways that students need to take in order to engage in high-level mathematics discussions are still not known since the act of student-to-student mathematics discussion has many factors that are difficult to describe let alone control.

It has been established that social and sociomathematical norms are crucial to productive student-to-student mathematics discourse. What is not clear, however, are the specific factors that need to be present in order for mathematics students to take responsibility for their own learning and the learning of their classmates while
maintaining the integrity of the discipline. Mathematics educators could benefit from more clearly defined factors that allow the mathematics authority to shift from teacher to the students and promote student-to-student discourse about mathematics.

The aspects associated with HPT may inform the mathematics community regarding ways that teachers can facilitate student-to-student conversations that promote deep mathematical thinking. The aspect of clearly defined roles in HPT relates directly to four of the eight mathematical practices. Students can be given a distinct role in their team that contributes to their team objective that reinforces one of the eight practices. The HPT focus that group members need to be challenged with sufficiently complicated material supports the idea of the importance of productive struggle in the mastery of mathematics (Hiebert & Grouws, 2007). The individual accountability that is encouraged in HPT mirrors the types of feedback that show high effect sizes in mathematics formative assessment (Black & Wiliam, 2009; Hattie & Temperly, 2007). This research project proposes that if the factors that are consistent with HPT exist while students are engaging in student-to-student mathematics conversations, students can achieve sufficient autonomy to share the mathematics authority with the teacher and take responsibility for their own learning and the learning of their classmates. This autonomy and transfer of mathematics authority will be manifest in student-to-student conversations of mathematics. If the preceding factors can be present during student-to-student mathematical conversations, then students will be able to think deeply about and understand mathematics.
CHAPTER 3
METHODS

This study employed a within-case and cross-case exploratory design to investigate how HPT training related to the development of sociomathematical norms, which lead to student-to-student discourse. Practitioners need more researched-based strategies from which to draw in order to guide their students toward autonomy that encourages students to take responsibility for their own and their classmates’ learning of mathematics.

Constructing viable mathematic arguments and critiquing the reasoning of others is an important component of developing proficiency in mathematics (National Governors Association Center for Best Practices, 2010). Strategically embedding pedagogy that promotes student discourse in the mathematics curriculum has been the object of study for many mathematics education scholars (Sherin, 2002; Stein et al., 2008; Walshaw & Anthony, 2008). Although much research is available to practitioners regarding teacher-to-student discourse, the field of mathematics education is lacking in guidelines for developing social norms and sociomathematical norms for productive student-to-student mathematics discussions. Practitioners could benefit from clear guidelines regarding how to teach their students the types of social and sociomathematical norms that create pathways for students to have autonomy and take responsibility for their own learning as well as the learning of their classmates.
Purpose

Although the factors associated with productive teacher-to-student discourse are clear, the factors that need to be present in order for teachers to facilitate student-to-student discourse are not clear. Teachers need more research-based practices as they work toward facilitating an environment in which the sociomathematical norms exist to allow this to happen. Teachers need to understand ways to instill in their students the autonomy to discuss, argue, and reason with each other. This research study explores and describes the factors associated with the development of sociomathematical norms that may establish and sustain student-to-student discourse in the mathematics classroom. Specifically, this study investigates the relationship between HPT (Katzenbach & Smith, 1993) and student-to-student mathematics discourse. Further, it examines the relationships between sociomathematical norms and student autonomy, which allow students to take responsibility for their own learning and their classmates’ learning.

Research Questions

The research questions that guided this research study were as follows.

**Overarching question:** How does High-Performance Teams training support the development of sociomathematical norms that lead to in-depth mathematical conversations among middle school students?

1. When students are trained in High-Performance Teams (Katzenbach & Smith, 1993) in classrooms that use research-based mathematics practices, why do they perceive and accept particular sociomathematical norms when engaging in Math-Talk Learning Communities (Hufferd-Ackles et al., 2004)?
   a. How and why do the students perceive these norms?
b. How do the perceptions of these norms change over time?

2. To what degree are the factors present during High-Performance Team mathematics activities (Katzenbach & Smith, 1993) also present when students engage in four differing levels of the Math-Talk Learning Community Framework (i.e., Questioning, Explaining Mathematical Thinking, Source of Mathematical Ideas, and Responsibility for Learning) (Hufferd-Ackles et al., 2004)?

   a. On which of the four areas do the students rely most often?

   b. How and why do these four areas inform the relationship between High-Performance Teams (Katzenbach & Smith, 1993) and Math-Talk Learning Communities (Hufferd-Ackles et al., 2004)

**Research Design**

To answer these questions this study used a within-case and cross-case exploratory design (Yin, 2009). An exploratory case study is useful when a great deal of research is not available regarding the issue under investigation (Yin, 2009). What was under investigation in this case study was how High-Performance Teams training might relate to the development of sociomathematical norms, which lead to student-to-student discourse. What was not known was whether the possible relationship between aspects of HPT and shifts in student autonomy that lead to higher levels of participation in the Math-Talk exist (Hufferd-Ackles et al., 2004). This study explored the relationship between HPT training and levels of the Math-Talk. In this study, a case was defined as a team of four middle school students as they worked together to solve challenging mathematics problems. The researcher studied four teams (i.e., four cases) with four middle school students in each team during a seven-week timeframe. The researcher analyzed each case separately and then examined the four cases together to look for
similarities, differences, and patterns.

**Setting, Participants, and Materials**

The school at which the researcher conducted the study had approximately 1,000 students. Throughout the 2016-2017 school year, 355 sixth graders, 314 seventh graders, and 329 eighth graders were enrolled in this particular middle school. Of the 998 enrolled students, 86% of the students identified themselves as White; 36% of the students were identified as coming from low-income households, and 12% of the students were identified as eligible for special education services. This school was selected because it was the school at which the researcher was a mathematics instructional coach and the researcher had convenient access to mathematics teachers and mathematics students. Additionally, the demographics of the participating school were similar to those of other schools statewide, which may strengthen the generalizability of the outcomes of the study at the state level. For example, the average level of proficiency on the sixth-grade mathematics end-of-level exam at the participating school was the same as the score of students statewide, 40%. The average number of students who qualified as low-income at the participating school was 36% while the state reported 35%. Twelve percent of the students at the participating school were classified as eligible to receive special education services and the state average of students who qualified to receive special education services was 11%. The participating school reported that 86% of the students identified themselves as White while the state reported 75% (Utah State Board of Education, 2017). Thus, this school was representative, in demographics and mathematics achievement, of
other schools across the state.

There were 12 sixth-grade teachers at this school, 6 who taught mathematics. The mathematics teachers collectively taught a total of 14 mathematics classes. From these 12 teachers, three teachers participated in the study. Two of the teachers had one student team (Teachers A and B), and one teacher had two student teams. This allowed for four student teams (with four students per team), which, in turn, allowed for 16 participating students. Each of the participating teachers was female. Their ages ranged from 35 to 58 years of age. Their years’ teaching experience ranged from five to 25. Two of the teachers had earned only a bachelor’s degree (Teachers B and C) and one had earned a master’s degree (Teacher A). The three participating teachers placed all of their students into groups according to what groups of students they anticipated would work best together. The teachers agreed that students with somewhat similar characteristics would work best together, because one student would not dominate the group or be left out. To choose what student groups should participate in the study, the teachers provided data regarding the students’ efforts in mathematics class, history of mathematics achievement, and special education classifications. Specifically, the student grades in mathematics, according to the teachers, reflected the effort that students typically put forth in mathematics classes. The summative test scores reflected the students’ academic achievement in mathematics, and the special education classification identified any disabilities that needed to be accommodated while learning mathematics. The researcher’s purposeful selection of participating groups allowed for a variance in effort, achievement, and abilities among the participating groups.
The participating students in this study were 16 sixth-grade students, ranging from 11 to 12 years of age, who were enrolled in mathematics classes at the selected school. The participating students were enrolled in the mathematics classes in which the teachers agreed to teach mathematics using the tenets of High-Performing Teams (Katzenbach & Smith, 1993) and agreed to utilize student-to-student discourse at least two times per week during the duration of the data collection process, approximately seven weeks. The students were recruited based on: (1) their placement in the classroom of a participating teacher, (2) whether their parents provided consent for their students to participate in the study, (3) teacher recommendations regarding which students would work well together, and (4) the students’ varying abilities. Four student groups were chosen, each with four students for a total of 16 participating students. Table 3.1 shows list of participants and their characteristics. Teacher A was the teacher for students 1-4; Teacher B was the teacher for students 5-8; Teacher C was the teacher for two student groups, Students 9-12 and Students 13-16. Students with the highest mathematics achievement historically were students 1-4. Students in the mid-range were students 9-12. The students with the lowest historical achievement were students 13-16 (see Table 3.1).

The participating sixth-grade teachers had access to a variety of materials including their district-assigned textbook, GoMath (Dixon, Larson, Leiva, & Adams, 2012) published by Houghton Mifflin Harcourt, and Texas Instruments TI-108 Solar calculators (one per student). The teacher training occurred during two 30-minute sessions during the teachers’ common preparation time.
Table 3.1

Participants and Their Characteristics

<table>
<thead>
<tr>
<th>Heading</th>
<th>Sex</th>
<th>Grade in math</th>
<th>Summative testing (score out of 4)</th>
<th>Sp. Ed classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher A, 25 years' teaching experience, master’s degree, 59</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alice</td>
<td>F</td>
<td>A</td>
<td>4</td>
<td>None</td>
</tr>
<tr>
<td>Amanda</td>
<td>F</td>
<td>A</td>
<td>4</td>
<td>None</td>
</tr>
<tr>
<td>Annie</td>
<td>F</td>
<td>A</td>
<td>4</td>
<td>None</td>
</tr>
<tr>
<td>Audrey</td>
<td>F</td>
<td>A</td>
<td>4</td>
<td>None</td>
</tr>
<tr>
<td>Teacher B, 18 years' teaching experience, bachelor’s degree, age 43</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bryce</td>
<td>M</td>
<td>A</td>
<td>2</td>
<td>None</td>
</tr>
<tr>
<td>Bill</td>
<td>M</td>
<td>A</td>
<td>3</td>
<td>None</td>
</tr>
<tr>
<td>Bonnie</td>
<td>F</td>
<td>A</td>
<td>3</td>
<td>None</td>
</tr>
<tr>
<td>Barbara</td>
<td>F</td>
<td>A</td>
<td>2</td>
<td>None</td>
</tr>
<tr>
<td>Teacher C, 5 years' teaching experience, bachelor’s degree, age 35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Candace</td>
<td>F</td>
<td>B</td>
<td>1</td>
<td>None</td>
</tr>
<tr>
<td>Christine</td>
<td>F</td>
<td>A</td>
<td>3</td>
<td>None</td>
</tr>
<tr>
<td>Chloe</td>
<td>F</td>
<td>A</td>
<td>3</td>
<td>None</td>
</tr>
<tr>
<td>Carly</td>
<td>F</td>
<td>A</td>
<td>3</td>
<td>None</td>
</tr>
<tr>
<td>David</td>
<td>M</td>
<td>C</td>
<td>1</td>
<td>LD in math</td>
</tr>
<tr>
<td>Dallas</td>
<td>M</td>
<td>B-</td>
<td>1</td>
<td>ELL</td>
</tr>
<tr>
<td>Denton</td>
<td>M</td>
<td>C+</td>
<td>1</td>
<td>ADHD</td>
</tr>
<tr>
<td>Doris</td>
<td>F</td>
<td>B+</td>
<td>2</td>
<td>None</td>
</tr>
</tbody>
</table>

Research Positionality

The researcher was also the mathematics instructional coach for the sixth-grade teachers at the middle school in which the study was conducted. During observations of student teams throughout the study, the researcher assumed a nonparticipant status in the classroom. The researcher acknowledged that there was a degree of subjectivity in this
research project in that she had an interest in promoting deeper student engagement. Therefore, she closely followed all research procedures set forth in this study including member checking and informant feedback (Creswell, 2013).

**Procedures**

The research study consisted of three phases (see Figure 3.1). The purpose of the first phase (1 week) was to acquire baseline information regarding established sociomathematical norms and student and teacher perceptions in the mathematics classrooms. The purpose of the second phase (5 weeks) was to teach students their specific team roles (Katzenbach & Smith, 1993) and observe students solving challenging mathematical problems in teams while noting their levels of autonomy as defined by Math-Talk. The purpose of the third phase (1 week) was to investigate how

*Figure 3.1. Timeline for the three phases of procedures.*
sociomathematical norms and student and teacher perceptions of working in student teams shifted after the students had participated in HPT.

Phase One

The data gathered and analyzed during Phase One addressed research question #1. To gather baseline data and establish the study four activities took place: (1) training of the teachers on HPT, (2) placing students into teams and recruiting participating students, (3) conducting baseline observations of sociomathematical norms, and (4) conducting baseline interviews with teachers and students.

Training of teachers on HPT. During Phase one, the researcher trained the teachers on HPT. Since this study was conducted two months previous to the end-of-level exams that the students were scheduled to take, the teachers requested that the HPT materials reflect all the mathematics that the students had learned during the year, with special emphasis on unit rates and equivalent proportions since many of the students seemed to be struggling with those concepts.

Session one: drawing a picture of the context of the problem. For the first training session, the researcher gave the teachers slips of papers with mathematics word problems. These teachers were instructed to draw pictures of the setting, characters, and dilemmas of the problem. As one teacher drew the picture, the others would guess what the problem was about. The researcher suggested that the teachers teach their students in a similar manner in which the lesson was presented to the teachers.

Session two: drawing a picture of the mathematical concepts. During the second session, the teachers received slips of paper with word problems on them and had to
match those slips of papers to the corresponding drawing of a mathematics concept. For example, a mathematics problem involving a scenario in which there was a need to divide fractions was matched to a bar graphic representing division of fractions.

**Session three: checking for procedural errors.** During the third training session, the teachers were given eight already-worked problems. Each of the problems, however, contained an error. The teachers role-played the misconceptions that their students might have as they examined the problems to find the errors. Further details for the protocol that the researcher used to train teachers is in Appendix C.

**Training the students in HPT.** The teachers trained their students in HPT during three class periods. During the first class period, the teachers trained their students how to draw a picture of the context of a problem. During the second class period, the teachers trained their students to draw pictures to represent different mathematical concepts they had learned. During the third class period, the teachers trained their students how to finding errors in procedural computations.

**Placing students into teams.** During this phase, the teachers put the students into groups according to teacher perceptions regarding who would work well together. The teachers decided to group students who had similar characteristics regarding three criteria. The teachers characterized each student according to (1) their efforts in mathematics as reflected by their grades, (2) their mathematics achievement as reflected by their summative testing scores, and (3) whether the students had a special education classification. After the teachers had assigned students to groups, the researcher was able to choose student teams to participate who had varying levels of abilities and attitudes.
Within the groups, the teachers allowed to students to choose their own roles.

**Conducting baseline observation of socio-mathematical norms.** Also during Phase One, the researcher observed the student teams in their classrooms during a mathematics lesson to acquire a baseline of the sociomathematical norms that had been established. The students may or may not have been engaged in teacher-to-student discourse or student-to-student discourse, depending on the lesson that the teacher presented. The researcher used the research-based Sociomathematical Norm Observation Tool (see Instruments section for greater detail) to record data regarding the sociomathematical norms present when the student teams were in mathematics class.

**Base-line interviews.** Also during Phase One of the study, the researcher conducted semistructured interviews with the participating teachers and each of the student teams to gather data regarding the teachers’ and students’ perceptions of working together to solve challenging mathematics problems as teams. The interview protocols for the semistructured interviews with teachers and students are included in Appendices A and B.

**Phase Two**

During Phase Two (5 weeks), the researcher addressed research question #2. During the first week of Phase Two, the classroom teachers trained all the students on the each of the individual roles they would be assigned when they worked in HPT. During weeks two through five, the students worked in HPT for eight episodes each while the researcher observed each student team solving challenging mathematics problems.

**Training of individual roles.** Each student was trained on all roles. Each of the
lessons was embedded in the specific mathematic standards and objectives that the teachers needed to teach the students as designated by the school district’s scope and sequence of 6th grade mathematics.

Explaining the context of a problem. On day one of the HPT training, the teachers presented lessons to the students on how to make graphic representations of the context of mathematical problems. In order for the students to learn how to draw a graphic representation that showed the context of a mathematics problem, the students participated in an activity similar to a charades-inspired guessing word game. Specifically, the teachers gave student pairs six slips of paper with challenging mathematics problems on them. The teachers instructed one member of the student pair to draw a slip of paper from a bag, read it, and draw a picture while the other student guessed the characters, setting, and dilemma presented in the problem. The students participated in six rounds, with each student receiving three turns to guess the characters, setting, and dilemma. In addition, each student had three turns to read a problem and draw a representation of the context. The selected problems were taken from the student textbook, GoMath (Dison et al., 2013) and the Inside Mathematics (2017) website. The researcher and teachers chose these problems because the students needed a review of ratio tables and expressions and were currently learning one-step equations. The six problems the teachers used for the HPT training are listed in Figure 3.2.

Drawing the mathematics of a problem and performing the mathematical procedure. On day two, the teacher showed the students various ways to represent the mathematics with drawings and what the mathematical procedures for each problem
1. The table shows the number of cups of yogurt needed to make different amounts of a fruit smoothie.

<table>
<thead>
<tr>
<th>Batches, $b$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of Yogurt, $c$</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Jerry used 33 cups of yogurt to make smoothies. How many batches did he make? (Dixon et al., 2012).

2. There are 6 girls in a music class. This represents $\frac{3}{7}$ of the entire class. What is the number of students in a class? (Dixon et al., 2012).

3. There are 9 raisin bagels in a bin. This amount represents $\frac{3}{8}$ of all the bagels in the bin. How many bagels are in the bin? (Dixon et al., 2012).

4. Gail is making costumes for a school play. Each rabbit costume needs one and one half yards of white fur fabric, a yard of blue stripe fabric, and a quarter of a yard of pink felt for the ears. Gail needs to make eight rabbit costumes. How much material does she need?

   White fur fabric: _______________ yards
   Blue stripe fabric: _______________ yards

5. If an architect is designing a building that is 132 feet tall, how many floors can be built? (Dixon et al., 2012).

6. In the town of Pleasant Hill, there is an average of 16 sunny days each month. How many sunny days can a resident of Pleasant Hill expect to have in 9 months? (Dixon et al., 2012).

Figure 3.2. Problems for practicing drawing contexts.

might look like. The teacher gave the students slips of paper with the same problems from the previous lesson as well as slips of paper with a graphic representations of the mathematics used for each of the problems and another slip of paper that had the mathematics procedures used to solve each of the problems. The teacher instructed the students to match the problems with the graphic representations and the mathematics
procedures. When the teachers noticed that students were confused by some of the drawings (specifically, the bagel problem below), the teacher stepped the students through the problem so that it was simpler, and thus easier to understand. The problems, graphics, and procedures needed for day two are listed in Figure 3.3.

**Finding mathematical errors.** On day three of HPT training, the teachers trained the students to check for accuracy. The teachers and researcher discussed common mistakes that students had made with respect to which mathematical concepts (e.g., equivalent proportions, unit rates, order of operations, and one-step equations). The researcher created the Finding Errors sheet (see Figure 3.4) which had eight worked-out mathematical procedures, each with one mistake. Questions 1 and 2 dealt with Finding Equivalent Ratios. On the first question, the mistake was that 4 and 5 were added together instead of multiplied. On question 2, the mistake was that the first ratio was multiplied by $\frac{3}{2}$ instead of $\frac{3}{4}$. The next group of problems dealt with Unit Rates. The error on question 3 was that the unit rate was in the numerator position rather than in the denominator position. The ratio should have been $\frac{512 \text{ grams}}{4 \text{ bananas}}$ which would have given the correct answer of 128 grams per banana. The error in question 4 was in the final computation of $35 \times 3 \div 2.5$. The answer had the decimal in the wrong place. The correct answer was 42 miles, not 4.2 miles. The next heading was Order of Operations. The error in question 5 was that $3^3$ was listed as 9, rather than the correct 27. The error in question 6 was the solution was found by adding first instead of adding and/or subtracting from left to right. Ten should be subtracted from 25 before 3 is added instead of adding 10 and 3 and then subtracting 13 from 25. The last section, Equations, also had two
Problem 1:
The table shows the number of cups of yogurt needed to make different amounts of a fruit smoothie.

<table>
<thead>
<tr>
<th>Batches, b</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of Yogurt, c</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Jerry used 33 cups of yogurt to make smoothies. How many batches did he make? (Dixon et al., 2012)

Math Graphic 1:

Procedure 1:
\[ 3x = 33 \]
\[ x = 11 \]

Problem 2:
There are 6 girls in a music class. This represents \( \frac{3}{7} \) of the entire class. What is the number of students in a class? (Dixon et al., 2012)

Math Graphic 2:

Music Class with 7 students, and \( \frac{3}{7} \) are girls

Music Class with 14 students, and \( \frac{3}{7} \) are girls

Procedure #2:
\[ \frac{3}{7}x = 6 \]
\[ x = \frac{3}{7} \]

Figure 3.3. Problems, graphics, and procedures for day two.
Problem #3

There are 9 raisin bagels in a bin. This amount represents \( \frac{3}{8} \) of all the bagels in the bin. How many bagels are in the bin? (Dixon et al., 2012)

Math Graphic #3

<table>
<thead>
<tr>
<th>Bin with 8 bagels, and ( \frac{2}{3} ) are raisin</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Bagels" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin with 16 bagels, and ( \frac{3}{8} ) are raisin</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Bagels" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin with 24 bagels, and ( \frac{3}{8} ) are raisin</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Bagels" /></td>
</tr>
</tbody>
</table>

Procedure #3

\[
\frac{3}{8} \times 9 = 9
\]

\[
9 + \frac{9}{8} = \frac{81}{8}
\]

\[
9 \times \frac{3}{8} = 24
\]

Problem #4

1. Gail is making costumes for a school play. Each rabbit costume needs one and one half yards of white fur fabric, a yard of blue stripe fabric, and a quarter of a yard of pink felt for the ears. Gail needs to make eight rabbit costumes. How much material does she need?

White fur fabric: \( \_\_\_\_\_\_\_\_\_\_\_\_ \) yards

Blue stripe fabric: \( \_\_\_\_\_\_\_\_\_\_\_\_ \) yards

Pink felt: \( \_\_\_\_\_\_\_\_\_\_\_\_ \) yards (Inside Mathematics, 2017).
Math Graphic #4
White Fur

Blue Stripe

Pink Felt

Procedure #4
White: \( \frac{1}{2} \times 8 = 4 \)
Blue: \( 1 \times 8 = 8 \)
Pink: \( \frac{3}{4} \times 2 \)

Problem #5
If an architect is designing a building that is 132 feet tall, and each floor must be 12 feet tall, how many floors can be built? (Dixon et al., 2012).

Math Graphic #5

Procedure #5
\( 12x = 132 \)
\( 132 \div 12 = 11 \)

(figure continues)
problems. The error in problem 7 was that the divisor was not inverted. The problem should have been solved by multiplying 12 by $\frac{4}{3}$ instead of $\frac{3}{4}$. The error on number 8 was also a result of failing to invert the divisor. The problem should have been solved by multiplying 15 and 5.

Finding errors was the final HPT role on which the teachers trained the students. Now that the students had learned each of the four HPT roles (drawing a picture of the context of the problem, drawing a picture of the mathematics of the problem, performing the proper procedure, and checking for errors), the students were asked to choose which role they would like to use for the first problem-solving episode. During subsequent episodes, the teacher allowed students to choose their role each time but encouraged them to take turns trying out different roles.
Name: Bob

Finding Errors

Directions: For each set of problems, find the student errors. Circle what is wrong.

Equivalent Ratios

Find the number that makes the two ratio pairs equivalent

1. \[ \frac{16}{？} = \frac{4}{5} \]
   \[ \frac{16}{？} \div 4 = \frac{4}{5} \]
   \[ ？ \div 4 = 5 \]
   \[ ？ = 9 \]

2. \[ \frac{25}{4} = \frac{75}{？} \]
   \[ \frac{25 \times 3}{4 \times 2} = \frac{75}{8} \]
   \[ 9 \frac{3}{8} \]

Unit Rates

3. The mass of 4 bananas is 512 grams. Find the unit rate of each banana.
   \[ \frac{4 \text{ bananas}}{512 \text{ grams}} = 0.008 \]

4. Ms. Brown biked 35 miles in 2.5 hours. How many miles could she bike in 3 hours?
   \[ \frac{35 \text{ miles}}{2.5 \text{ hours}} = \frac{？}{3 \text{ hours}} \]
   \[ 35 \frac{3}{2} \div 2.5 = 4.2 \text{ miles} \]

*Figure 3.4. Finding errors sheet for day three.*
Observation of students solving challenging problems in HPTs. Also during Phase Two, the students practiced working in HPT while solving challenging mathematics problems. The researcher observed each group of students for eight episodes each, for a total of 32 observations. To better understand what happened while students were solving problems in teams, the researcher used the Math-Talk Learning Community Framework observation tool (see detail description in Chapter Two). The researcher also

<table>
<thead>
<tr>
<th>Order of Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. $3^3 - 9 + 17 =$</td>
</tr>
<tr>
<td>$9 - 9 + 17 = 17$</td>
</tr>
<tr>
<td>6. $5^2 - 10 + 3 =$</td>
</tr>
<tr>
<td>$25 - 10 + 3 = 25 - 13 = 12$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. $\frac{3}{4}x = 12$</td>
</tr>
<tr>
<td>$12 \div \frac{3}{4} = \frac{12}{1} \cdot \frac{3}{4} = \frac{36}{4} = 12$</td>
</tr>
<tr>
<td>8. $\frac{x}{5} = 15$</td>
</tr>
<tr>
<td>$15 \div \frac{1}{5} = 3$</td>
</tr>
</tbody>
</table>
video recorded all observations and transcribed selections for further analysis.

*Selection of challenging problems.* The classroom teachers and/or the researcher chose the challenging mathematics problems that student teams worked on from the *GoMath* (Dixon et al., 2013) textbook or other Internet resources according to the scope and sequence of the curriculum guide assigned by the mathematics department of the school district. The researchers and teachers chose problems that would fit authentically in the scope and sequence as prescribed by the school district. Whenever possible the problems chosen allowed for multiple solutions paths. However, each of the problems in the curriculum guide allowed for only one correct answer.

Each challenging problem met the following criteria.

1. Had a context that was sufficiently complex so that drawing a diagram of what the question was asking seemed reasonable (National Governors Association Center for Best Practices, 2010),

2. Contained mathematics that was on the 6th grade level (National Governors Association Center for Best Practices, 2010),

3. Was able to be solved with a procedure that the students already knew or one that the students had the background knowledge to construct,

4. Allowed for multiple entry points and varied solution strategies and promoted reasoning and problem solving (Leinwand et al., 2014), and

5. Had a high level of cognitive demand (Leinwand et al., 2014).

In addition, as the researcher and teachers selected problems from the *GoMath* text book (Dixon et al., 2013), they attended to some of the criteria set by Characteristics of High-Quality Task (Lannin, Chval, & Jones, 2013). The criteria that was met by the problems in the school-district adopted text were as follows.

- Aligned with relevant mathematics content standard(s),
- Provides opportunities for students to develop and demonstrate the mathematical practices,
- Allowed entry to the mathematics at a low level (all students can begin the task) but high ceiling) (some students can extend the activity to higher level activities)
- Connected previous knowledge to new learning
- Allowed for multiple solution approaches and strategies
- Includes a relevant and interesting context

Each of the problems selected was selected from the school district-approved *GoMath* (Dixon et al., 2013) text. The problems allowed for multiple solutions paths but only one solution. Each of the problems could be solved using more than one mathematical procedure, or several procedures in differing orders. However, there was only one clear answer to each of the problems. A sample of the list of the challenging problems students solved in HPTs is included in Appendix D. Each episode ranged from 20 to 40 minutes.

**Descriptive feedback from teachers.** As the students work in teams, the teacher gave individual descriptive feedback for each student team member. Specifically, the teacher noted whether and how each student contributed to the team in his/her distinct role (drew a diagram that explained the context of the problem, constructed a graphic representation to show the mathematics of the problem, performed mathematically correct procedures, or checked the problem for accuracy). A tool to help the teacher give descriptive feedback, the Individual Accountability for Students Working in High-Performance Teams, is listed in Appendix F.

**Phase Three**

The activities that happened during Phase Three addressed research question #1.
The researcher observed the participating students in their mathematics classroom while their teacher taught a mathematics lesson to understand and describe the current sociomathematical norms, using the identical protocol that she used to gather baseline data during Phase One. The researcher conducted semistructured interviews with the participating teachers and student teams to investigate the perceptions of teachers’ and students’ regarding working as teams. The researcher interviewed the teachers and students two times, once before the treatment and once after. The researcher used the same interview protocols both times she interviewed the teachers and students.

Data Sources and Instruments

To explore the research questions of this study, the researcher gathered data from five sources: (a) classroom observations using the Sociomathematical Norm Observation Tool (b) semistructured interviews of teachers, (c) semistructured interviews of student teams, (d) classroom observations using Math-Talk and selected transcripts, and (e) student tasks sheets. What follows is a detailed description of data collection instruments and their purposes.

Sociomathematical Norm Observation Tool

The researcher created the Sociomathematical Norm Observation Tool (see Appendix E) by combining research-based sociomathematical norms (Lopez & Allal, 2007; Yackel, 2001) and the criteria for student autonomy as laid out in Math-Talk (Hufferd-Ackles, 2004). The first, the research-based sociomathematical norms, were norms that mathematics education researchers found were present during productive
student-to-student discourse (as discussed in Chapter 2). The second, Math-Talk (Hufferd-Ackles, 2004), is a framework that gives indicators for various levels of student autonomy (also discussed in detail in Chapter 2).

The Sociomathematical Norms Observation Tool allowed the researcher to collect data regarding the degree of student autonomy in four different areas: (1) how students indicate nonunderstanding of a mathematical concept, (2) how students explain and justify their reasoning, (3) how students listen to and attempt to understand and explain others’ explanations, and (4) how students indicate when solutions are valid (Lopez & Allal, 2007; Yackel, 2003). The Sociomathematical Norm Observation Tool allowed the researcher to observe the degree of autonomy in each of the four areas every three minutes and then record a tally under the appropriate designation. After each observation was complete, the researcher could add the tallies in each area and assign a numerical value for each area. Each observation session lasted 24 minutes, which meant that the researcher observed the degree of autonomy using the Math-Talk framework of the participating students eight times during each observation episode. The numerical values ranged from 0 for the least amount of autonomy to 3 for the greatest amount of autonomy. The researcher was able to give a numerical value to each observation session by adding the tallies of the various levels of Math-Talk that were observed during the eight three-minute increments.

The researcher piloted the Sociomathematical Norm Observation tool by observing three teaching episodes on The Teaching Channel, a video-enabled, professional learning platform (Teaching Channel, 2017). The teaching episodes
consisted of middle school teachers discussing a mathematics lesson and then implementing the lesson in a classroom setting. The researcher noted that most of the comments that the students made during the video sessions showed a great deal of student autonomy. The scores were high for the published example lessons, giving information that autonomy was high during those exemplary lessons. However, the pilot study showed that the tool did not lend itself for note taking or to record the time that the researcher made certain observations. As a result, the researcher changed the tool so that there was a place to note the time and other observations. Accordingly, the instrument lists indicators of the degree of autonomy and a place to write the time of the recording and notes regarding the student behavior that relates to the indicators. An example of indicators for the highest degree of autonomy is shown in Table 3.2. To view the fully revised observation tool, see Appendix E.

The researcher used the Sociomathematical Norm Observation Tool during the first phase of the data collection process in order to acquire base-line sociomathematical norms and autonomy of the participating students. Then, after the participating students had worked for five weeks in HPT, the researcher again used the Sociomathematical Norm Observation tool to measure any shifts in sociomathematical norms and student autonomy. The analysis of the two observations (pre and post treatment) address research question #1.

**Semistructured Interviews**

To address research question #1, the researcher conducted semistructured interviews with teachers and participating students during Phase One and Phase Three.
Table 3.2

*Indicators for the Highest Degree of Autonomy for the Sociomathematical Norm Observation Tool*

<table>
<thead>
<tr>
<th>Successful transfer of authority from teacher to student Level 3</th>
<th>Indicate nonunderstanding</th>
<th>Explain and justify reasoning</th>
<th>Listen to and attempt to understand others’ explanations</th>
<th>Indicate when solutions are valid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-to-student talk is student-initiated, not dependent on the teacher. Many are “Why?” questions.</td>
<td>Students describe more complete strategies and defend their answers with little prompting from the teacher.</td>
<td>Students spontaneously compare, contrast and build on ideas and form part of the content of many math lessons.</td>
<td>Students listen to, understand, then initiate clarifying other students’ work and ideas for themselves and for others</td>
<td></td>
</tr>
</tbody>
</table>

Semistructured interviews allowed for open-ended responses as well as consistency in terms of the questions that the interviewer asked participants (Creswell, Hanson, Clark Plano, & Morales, 2007. The researcher video recorded each interview.

**Semistructured interviews with teachers.** The researcher conducted eight semistructured interviews during Phase One to gather baseline perceptions of teachers and student teams regarding sociomathematical norms and student teams. The researcher interviewed the participating teachers again during Phase Three to gather data regarding
the teachers’ perceptions of sociomathematical norms and student teams after the treatment. The researcher created the six-question semistructured interview (Creswell et al., 2007). After introducing herself, the researcher explained the project and asked a warm-up question before moving onto more focused questions about what kinds of student discourse the interviewee had observed in her classrooms and what strategies she had used to help students engage. The researcher asked questions regarding the depth of mathematics understanding the teacher had previously observed when her students engaged in student-to-student mathematics conversations such as how often does it happen and who does most of the talking. A list of interview questions and interview protocol are listed in Appendix A. At the end of the interview, the researcher thanked the participants for their time and ask if they had any further questions. At the end of the five-week time period in which students had opportunity to work in HPT, the researcher interviewed the teachers again with the same protocol to see if there were any shifts in the perceptions of sociomathematical norms.

**Semistructured interviews with student teams.** The researcher interviewed each student team once (for a total of four interviews) during Phase One to gather baseline perceptions of the student teams regarding sociomathematical norms before the students worked in HPT. The researcher created the six-question semistructured interview (Creswell et al., 2007). The researcher began the interview by introducing herself and then describing the project. Then she asked a warm-up question and then more focused questions. The researcher asked questions regarding what types of comments seem to be helpful when solving a mathematics problem and how often they
and other students made those kinds of comments. A list of interview questions and interview protocol for students are listed in Appendix B. At the end of the five-week time period in which students had an opportunity to work in HPT, the researcher interviewed the student teams once again with the same protocol to see if there were any changes in the student perceptions of sociomathematical norms.


To address research question #2, the researcher used the Math-Talk Learning Community Framework (Hufferd-Ackles, 2004) to classify the kinds of comments that students made during mathematics class while working on challenging problems as a team during Phase Two of the study. The researcher used partial interval recording (Merrell, 2000), which is a strategy involving the observation of whether a behavior occurs or does not occur during specified time periods of classroom episodes. The researcher recorded behaviors every three minutes and was guided by the factors listed in Math-Talk, which classifies the types and intensity of autonomy during student-to-student mathematics discourse. The researcher observed eight episodes of each student team and focused on only one student group at a time (even if those student groups happened to be in the same class). After each observation episode, the researcher reviewed the partial interval recording (Merrell, 2000) of the framework rubric (see Chapter Two of this paper), noting the frequency and duration of which kinds of comments were made. The researcher used these notes to decide which portions of the video should be transcribed for further analysis.
Transcripts of Video

Many researchers recommend video recording when examining classroom mathematical discussions (Baxter & Williams, 2010; Hufferd-Ackles et al., 2004). To address research questions #1 and #2, the researcher video recorded all teacher and student interviews and student-to-student mathematics conversations during the observations. The researcher transcribed purposively selected portions of the recordings, choosing sections to be transcribed that provided insight into changes in student discourse. The researcher set up one video recording device to record the groups of student teams as they discussed their mathematical tasks. The recording device was a designated video recorder that was connected to an external microphone. The video recorder had no other purpose other than to record data and then transfer it to a secured drive for storage.

Student Work

To address research question #1, the researcher examined the work that students produced. The researcher collected the student task sheets from the student teams after each student-to-student discourse episode for further reflection and analysis. A sample of the challenging problems the students solved as teams are listed in Appendix E. The student tasks sheets gave insight regarding correct mathematical thinking and depth of understanding. The problems on the task sheets were taken from the GoMath text book or other Internet sites and contained problems with the criteria described in Phase Three of the Procedures section. For a sample of the challenging mathematics problems used in the study, see Appendix D.
Data Analysis

Data analysis for this exploratory case study (Creswell & Clark, 2007) included within-case and cross-case analysis. Within-case analysis allows the researcher to develop an in-depth portrait of a phenomenon or issue that is within a case, attending to elements that may be overlooked when identifying common themes across cases (Patterson, 2011). Cross-case analysis allows the researcher to reinforce validity, support generalizability, and provide underpinnings for theoretical explanations (Burns, 2010). See Table 3.3 for a list of research questions, data sources, and analysis.

The section explaining the data analysis of this study begins with discussion of the validity and reliability measures the researcher employed. Next, the researcher details the analysis used in the within-case analysis. The last subsection explains the analysis the researcher used in the cross-case analysis.

Validity and Reliability

In order to establish validity and reliability in this case study, the researcher employed several methods: (1) data triangulation, (2) member checking, and (3) constant comparison form of analysis. For this study, triangulation of the data (Creswell, 2013) came from the following four sources: video recordings, field notes, semistructured interviews and member checking. Member checking, which involves sharing preliminary results with the participants who then have the opportunity to provide feedback on the accuracy of the researcher’s interpretations, can be done both formally and informally (Creswell, 2013). For further verification of data analysis, the researcher revisited video
Table 3.3

Data Analysis Overview

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>Data Sources</th>
<th>Data Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overarching question: How does High-Performance Teams training support the development of sociomathematical norms that lead to in-depth mathematical conversations among middle school students?</td>
<td>Observations using research-based sociomathematical norms (Yackel, 2003; Lopez &amp; Allal, 2007) Field notes from observations Semistructured interviews with teachers and students Student work</td>
<td>Open coding Axial coding Constant comparative observation (Creswell, 2013) Memoing (Groenewald, 2008)</td>
</tr>
<tr>
<td>1. When students are trained in High-Performance Teams (Katzenbach &amp; Smith, 1993) in classrooms that use research-based mathematics practices, why do they perceive and accept particular sociomathematical norms when engaging in math talk learning communities?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. How and why do the students perceive these norms?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. How do the perceptions of these norms change over time?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. To what degree are the factors present during High-Performance Team mathematics activities (Katzenbach &amp; Smith, 1993) also present when students engage in four differing levels of the Math-Talk Learning Community Framework (i.e., Questioning, Explaining Mathematical Thinking, Source of Mathematical Ideas, and Responsibility for Learning) (Hufferd-Ackles et al., 2004)?</td>
<td>Observation using the Math-Talk Learning Community Framework (Hufferd-Ackles, 2004)</td>
<td>Partial interval recording (Merrell, 2000) Descriptive statistics</td>
</tr>
<tr>
<td>a. On which of the four areas do the students rely most often?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. How and why do these four areas inform the relationship between High-Performance Teams (Katzenbach &amp; Smith, 1993) and Math-Talk Learning Communities (Hufferd-Ackles et al., 2004)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

footage in a constant comparison form of analysis. When the researcher noted a significant shift of student autonomy, she made note of the time on the video recording so she could transcribe a portion of the recording. The researcher referred to the transcripts several times during the analysis.
**Within-Case Analysis**

The within-case analysis used qualitative data, which included analysis, such as, memoing, open coding, and axial coding. This within-case analysis also used quantitative data and analysis using descriptive statistical analysis.

**Qualitative analysis.** The quantitative analysis consisted of memoing, open coding, and axial coding. This enabled the researcher to provide an in-depth description, categorization, and interpretation of the information that the researcher collected from all of the data sources on a case-by-case basis.

**Memoing.** One of the advantages of using a case study design is that the researcher has an opportunity to provide a rich, thick description of the cases (Creswell, 2013). Memoing, a reflection written after data gathering sessions and coding sessions, helps the researcher record events and reflections of those events in an efficient manner (Groenewald, 2008). The researcher used memoing after each instance of data collection in order to reflect on what she observed and how that related to any shifts or emerging patterns. The researcher noted which portions of the episodes showed significant shifts regarding mathematical autonomy of students to inform which portions of the video recordings should be transcribed before the next episodes. The researcher used memoing during the open coding and axial coding phases of analysis to record reflective thoughts regarding codes and potential categories and themes. After writing each memo, the researcher shared the memos with the classroom teacher and students to member-check the validity of the observations (Creswell, 2013).

**Open coding.** The researcher used open coding to discover multiple concepts and
categories, which prevented her analysis from taking on a single perspective (Creswell, 2013). The researcher first imported all data into NVivo, a qualitative software program, and then conducted a preliminary analysis of only a few pages of the data sources to determine the codes for the data. Then, the researcher used these newly created codes to guide the open coding process for the rest of the data sources. The researcher reviewed each data source, line by line, to ensure accuracy in coding, paying particular attention to the Math-Talk Learning Community framework and using assigned numbers or abbreviations for easy identification. From open coding, this researcher was able to develop axial codes.

Axial coding. The axial coding (Stake, 1995) for this study used these the themes generated from the categorization of open codes to explain larger trends in these qualitative data. The researcher reviewed all open codes and looked for categories and themes that related to each other. Text passages with the same codes were compared for similarities or variations, and then the researcher either merged the codes into broader themes, split them up into sub codes, or created new themes. The researcher then reviewed all data sources one more time to make sure that all information was coded and placed into the correct theme and category. Axial codes were also considered in relation to each other and to factors, such as time, students, and task design. Relationships between axial codes and these factors explained how particular aspects of the HPT may explain changes in student-to-student discourse within cases.

Quantitative analysis. The researcher used descriptive statistics (Fraenkel & Wallen, 2003) to analyze measures of central tendencies with the data collected from two
instruments. The first was the base-line data collected using the Sociomathematical Norm Observation Tool. The second was the Math Talk framework, which measured the degree of autonomy and shifts of sociomathematical norms during the eight observation episodes.

Descriptive statistics. For the Sociomathematical Norm Observation tool, the researcher compared the number of instances in which she observed students participating in the varying degrees of autonomy in the base-line observational data with the post-observational data in order to measure the degree of shifts in autonomy or sociomathematical norms. For the analysis of these data collected from the Math Talk framework, the researcher studied the partial interval recordings of Math-Talk framework tool to look for trends in frequencies and patterns in student comments and engagement behaviors. The researcher analyzed the interval recording (Merrell, 2000) tallies to determine the type of mathematics comments the students made and the degree of autonomy the students demonstrated through their engagement. The researcher analyzed the frequencies to identify key parts (e.g., the student comments that preceded a shift of autonomy, the engagement behaviors that preceded a shift in autonomy) of the recordings that should be transcribed for further qualitative analysis.

Cross-Case Analysis

The purpose for using a cross-case analysis for this study was to examine the influence of HPT training on the depth of student-to-student mathematics discourse and look for emerging patterns that may lead to generalizations across cases. After the researcher had completed the analysis of each case, the researcher described the
categories and themes for each case, attending to the areas and categories listed in Math-Talk. After all of the data had been imported into NVivo, coded, and organized into categories and themes, the researcher performed the cross-case analysis.

The researcher created a template (Figure 3.4) in order to analyze shifts in teacher, student, and researcher perceptions regarding sociomathematical norms and student autonomy. This template allowed the researcher to examine each theme in each of the cases to analyze any commonalities across cases. The researcher-created template followed the recommendation of Glaser and Strauss (2009) in completing the following steps for cross-case analysis.

1. Compare incidents across cases in which a category was identified and note the variance of its intensity,
2. Integrate ideas, noting relationships among variations within categories and their variation across cases,
3. Notice patterns of relationships within the categories that became apparent and determine if some relationships have become more evident than others.

The completed template allowed the researcher to analyze similarities and differences and make assertions and generalizations of the cases. The factors that the researcher considered when explaining the differences and similarities between cases were the common and uncommon themes throughout the study. To do so, the researcher compared typical and atypical axial codes.

**Interpretation**

The interpretation of analysis of this study followed a theory in action, which is a set of assumptions about what is required to move an organization from its current state
Figure 3.4. Cross-case analysis template.

to its desired state (City, Elmore, Fiarman, & Teitel, 2009). In this study, the organization was considered a student team, and the current state was the degree of student autonomy and sociomathematical norms as defined by the baseline data. The desired state was a student team working with greater autonomy and taking more responsibility for their learning and the learning of their classmates. The theory of action for this proposal was that after the students had been trained in HPT and had sufficient practice at solving mathematics problems in HPT, their sociomathematical norms would evolve so that the norms more closely resembled those of researched-based classrooms that had high degrees of student-to-student participation. The researcher interpreted that the students’ sociomathematical norms were more aligned with research-based norms if there was an
increase in tally marks between the pre and post observation of sociomathematical norms. The researcher interpreted that the norms had changed if the amount of tally marks changed by 50% or more in a given area.

In addition, the theory of action for this proposal was that students who had been trained in HPT and had sufficient opportunity to practice HPT would have higher degrees of autonomy than they previously had. The researcher interpreted an increase in autonomy if students made comments that were aligned with higher levels of Math-Talk. If the levels that the student teams exhibited increased by 50% or more, the researcher would interpret that students were showing more autonomy and were relying on each other for mathematics authority instead of the teacher.

The analysis of these data was used to interpret the teachers’, students’, and researcher’s perceptions regarding the shifts of sociomathematical norms and student autonomy after the student groups had been trained in HPT and had an opportunity to work in HPT. Specific examples of interpretation of analysis were as follows.

1. If there was evidence that students were having mathematics discussions that exhibited higher levels of Math-Talk, the researcher tried to differentiate whether students were implementing the skills that they learned in HPT training or were naturally becoming more independent.

2. Since it was possible that students would show high degrees of participation in one area of Math-Talk but not the other, the researcher interpreted the analysis of each of the areas of Math-Talk individually. For example, students may have frequently asked in-depth mathematical questions of their teammates but still required regular reassurance from the teacher. Another example is that students would take responsibility for their own learning, yet fail to listen to the mathematical explanations of their teammates.

3. If the researcher noted that there was no change between the pre and post sociomathematical observation data, then the researcher assumed that some shifts were smaller than the instruments allowed for or that some shifts happened in areas that the observation tool was not designed to measure. If
this is the case, the researcher relied on qualitative data to explain the shifts of sociomathematical norms.

4. If the researcher noted a shift that was unobserved by the teacher, the researcher would discuss the observation with the teacher and include elements of that discussion in her notes.

To interpret that analysis of the data, the researcher noted the patterns of relationships within the theme categories. The researcher determined if some relationships had become more evident than others in order to describe the mathematics practices that happened concurrently with shifts of authority from teacher to student and when students were participating in the highest level of student-to-student mathematics discussions.
CHAPTER 4

RESULTS

The purpose of this study was to explore and describe the factors associated with the development of sociomathematical norms that may establish and sustain student-to-student discourse in the mathematics classroom. The overarching research question was: How does HPT training support the development of sociomathematical norms that lead to in-depth mathematical conversations among middle school students? The two subquestions focused on the sociomathematical norms that students accepted while working in HPT and which facets of HPT were present when students engaged in Math-Talk. The subquestions were: (1) How and when do students accept these sociomathematical norms and how do they change over time? and (2) On which Math-Talk areas do students rely on most and why?

The results presented in these sections provide an in-depth description of the changes in sociomathematical norms and the interpretation of the changes as students worked in HPT. The results are organized around the research questions and the forms of data analysis, which are within and cross case. In the within-case analysis, I used quantitative methods with descriptive statistics to analyze the extent of the changes in sociomathematical norms after the students had worked in HPT and also during problem-solving episodes using Math-Talk. An episode was defined as a session lasting from 20 to 40 minutes in which students worked in HPT to solve challenging mathematics problems. Also in the within-case analysis, I used qualitative methods to examine the open and axial codes to explore how and why the students’ Math-Talk levels changed over time. In the
cross-case analysis, I used qualitative methods to examine the open and axial codes, which explored similarities and differences between student teams as they demonstrated different Math-Talk levels that led to changes in sociomathematical norms. The cross-case analysis describes the axial codes for each case and shows how the codes of each team were related or unrelated to the activity of other teams.

**Within-Case Analysis of Student Teams**

Each of the four within-case analyses (Teams, A, B, C-1, and C-2) follows the same organization. Each begins with the quantitative analysis, which includes descriptive statistics comparing pre and post Sociomathematical Norms Observation data. Numerical values (0-3) represent the degree of student autonomy, which I measured eight times in each 24-minute observation. In the next part of the quantitative analysis, I recorded changes in degrees of Math-Talk over time. I used partial interval recording to document student comments and behaviors that related to the indicators every three minutes, assigning numerical values (0-3) to indicate the level of student autonomy (e.g., 0—no student explanations, 1—minimal student explanation, 2—students’ ideas guide the lesson, and 3—talk is initiated by the students not the teacher). The qualitative portion of each within-case analysis describes the themes and patterns that inform the context for the noteworthy shifts in Math-Talk levels over time. The axial codes that emerged during the within-case analysis were: (1) the evolution of relationships among team members, (2) the importance of the selection of appropriate problems, (3) how the students determined correct answers, and (4) teacher support that is required in order to maintain
high levels of Math-Talk. Each within-case analysis concludes with a discussion of how the axial codes may explain the shifts of data exhibited in the descriptive statistics of the quantitative analysis.

**Team A: Alice, Amanda, Annie, and Audrey**

All of the members of Team A were female. Each had received high grades in mathematics and high scores on their summative tests (see Table 3.1).

**Quantitative analysis.** I used descriptive statistics to analyze data from two instruments. The first was the Sociomathematical Observation Tool, and the second was the Math-Talk observation tool.

**Pre and post observation.** I used the Sociomathematical Norm Observation Tool to compare pre and post observation data regarding shifts in sociomathematical norms, which indicated shifts of student autonomy for Team A. Table 4.1 shows the four different levels (0-3) for the four different categories (e.g., indicate when solutions are valid, explain others’ explanations, justify reasoning, and indicate nonunderstanding) for the pre and post observation. The students of Team A exhibited an average autonomy Level 1 for 44% of the observation period and a Level 0 for 56% of the observation period, with the highest area in “indicate nonunderstanding and justify reasoning.” During the post observation, the students exhibited an average autonomy Level 1 for 3% of the observation period and Level 0 for 97% of the observation period with similar ratings in all four areas. These results indicate that the levels of autonomy for Team A were relatively lower from the pre to the post observations. Lower autonomy scores mean
Table 4.1

Pre and Post Sociomathematical Norms for Team A

<table>
<thead>
<tr>
<th>Level</th>
<th>Pre-Treatment</th>
<th>Post-Treatment</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A  B  C  D</td>
<td>A  B  C  D</td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td>0  0  0  0</td>
<td>0  0  0  0</td>
<td>0  0  0  0</td>
</tr>
<tr>
<td>Level 2</td>
<td>0  0  0  0</td>
<td>0  0  0  0</td>
<td>0  0  0  0</td>
</tr>
<tr>
<td>Level 1</td>
<td>5  0  1  8</td>
<td>1  0  0  3</td>
<td>3  0  0  -0.41</td>
</tr>
<tr>
<td>Level 0</td>
<td>3  8  7  0</td>
<td>7  8  8  97</td>
<td>97  8  8  +0.41</td>
</tr>
</tbody>
</table>

N = 4 students.

Note. Categories:
A – Indicate when solutions are valid
B – Explaining others’ explanations
C – Justify reasoning
D – Indicate nonunderstanding

Levels
0 – No student explanations
1 – Minimal student explanation
2 – Student ideas guide lesson
3 – Talk is initiated by students (not teacher)

Tallies made every 3 minutes for 24 minutes for a total of eight tallies.

that the students relied more on the teacher to explain the mathematics than they relied on each other. I will explain possible reasons for this change in the “Interpretation of Data” section of the case of Team A.

Changes in Math-Talk over time. To better understand the nuances in the pre-post observation, I examined the students’ Math-Talk levels over a period of eight episodes. I used the Math-Talk partial interval-recording tool to measure the changes in the four areas of Math-Talk. The line graph in Figure 4.1 shows in which episodes the shifts in Math-Talk levels occurred. There is an increase of one full level between the first and second observation in all four areas and a decrease between the second and third observations in the “indicate nonunderstanding” area. There is an increase between the
third and fourth episodes in all four categories. Other shifts occur during the sixth and seventh episodes and the seventh and eighth. It is interesting to note that during the eighth episode, three areas were lower than the previous episode while one area increased. The area of “indicate when solutions are valid” increased while “listen to and attempt to understand others’ explanations,” “explain and justify reasoning,” and “indicate nonunderstanding” were lower. I discuss interpretations of variances in the levels and areas in the “Interpretation of Data” subsection of Case A. Overall, these changes show that Team A increased in all four areas of Math-Talk from the start of the observation episodes to the finish.

Table 4.2 lists the numerical values of the Math-Talk levels for Team A for each area during each observation period. Math-Talk levels in all four categories during the second episode increased an average of more than one full level (1.25). During the fourth
Table 4.2

**Math-Talk Partial Interval Coding for Team A**

<table>
<thead>
<tr>
<th>Observation episodes</th>
<th>Indicate when solutions are valid</th>
<th>Attempt to understand others' explanations</th>
<th>Explain and justify reasoning</th>
<th>Indicate nonunderstanding</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.88</td>
<td>1.00</td>
<td>1.13</td>
<td>1.25</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>1.86</td>
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<td>2.43</td>
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<td>2.14</td>
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<td>2.75</td>
<td>2.75</td>
<td>2.75</td>
<td>2.75</td>
<td>2.75</td>
</tr>
<tr>
<td>5</td>
<td>2.75</td>
<td>2.5</td>
<td>2.75</td>
<td>2.75</td>
<td>2.69</td>
</tr>
<tr>
<td>6</td>
<td>2.75</td>
<td>3.00</td>
<td>2.75</td>
<td>2.43</td>
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</tr>
<tr>
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<td>3.00</td>
<td>3.00</td>
<td>2.43</td>
<td>2.64</td>
</tr>
<tr>
<td>8</td>
<td>2.20</td>
<td>2.37</td>
<td>2.38</td>
<td>2.29</td>
<td>2.31</td>
</tr>
<tr>
<td>Ave.</td>
<td>2.19</td>
<td>2.37</td>
<td>2.38</td>
<td>2.29</td>
<td>1.07</td>
</tr>
<tr>
<td>Change</td>
<td>+1.32</td>
<td>+1.37</td>
<td>+1.25</td>
<td>+1.04</td>
<td>+1.25</td>
</tr>
</tbody>
</table>

Episode, students’ Math-Talk also increased in all four categories, although not as much as between the first and second episodes (0.62). From the first observation to the last observation, the Math-Talk levels of the students in Team A increased 1.25 levels, with the highest increase coming from the area of “listen to and attempt to understand others’ explanations” (1.37) and the lowest increase from the area of “indicate nonunderstanding” (1.04).

**Qualitative analysis.** I used memoing and open and axial coding of qualitative data, which consisted of interviews, observations, memos, and field notes. The axial codes that emerged during the qualitative analysis for Team A students were: (1) selection of problems and (2) determination of correct answers.

**Selection of problems.** An axial code that emerged was the importance that the teacher selects appropriate problems for the students to solve. Open codes related to this
axial code were “solving problems very quickly,” “discussing topics not related to mathematics,” and “not engaging in solving problems when contexts were unfamiliar.”

These codes explain how students’ Math-Talk levels changed when the teacher selected problems that were the appropriate level of complexity and contained familiar contexts. An example of the Math-Talk levels being lower as a result of students working on problems that were too simple occurred during the first observation episode. At the beginning of the episode, Team A displayed characteristics of the highest levels of Math-Talk, levels two and three. However, the team members solved the problems the teacher gave them in six minutes, leaving 14 minutes remaining. Therefore, the high levels of Math-Talk were not sustained throughout the entire 20-minute episode (see Table 4.2). At the onset of the episode, Annie said, “I think I know what we need to do here. We just need to find out the area of these shapes by moving them around the square.” Alice asked, “why the square?” Audrey answered, “Because that is the only [area] we know.”

The students worked toward solving the problem until they had answered all the questions at which time the students stopped talking about the mathematics. Amanda then asked the group, “Did you guys get your Egypt packet finished?” For the remainder of the episode, the students discussed assignments from other classes. Although the students were engaged in high levels of Math-Talk for the initial six minutes, their over-all Math-Talk rating was lower because they did not talk about mathematics while they were waiting for their teacher to give them further instructions.

During a member-checking session after the first observation, the teacher reported that the problem assigned to the students in Team A was the appropriate level of
complexity for most of the students in her class. However, this particular team may need problems that were more difficult in order to challenge them sufficiently. The teacher decided that she would provide two challenging problems for students in Team A to solve next time. The teacher emphasized that not every student team would be required to solve the extra problem; the additional problem was only available to those students who wanted to extend their learning and was not required of all students. Accordingly, for the second observation, the teacher chose two challenging problems for Team A to solve. The team exhibited higher levels of Math-Talk, because they were engaged for a longer period of time. When the students had finished solving the first problem, Annie called the teacher over and said, “We are finished with the first one. We need the next one.” With additional problems, the students in Team A maintained higher levels of Math-Talk throughout the second episode.

Another theme that emerged related to the appropriate selection of problems was that students exhibited higher levels of Math-Talk when contexts of the math problems were familiar to them. For example, during the seventh episode, the level of student autonomy decreased when students in Team A were presented with a problem with an unfamiliar context. The problem involved a ramp at a skate park (see Figure 4.2). After

![Figure 4.2. Student work for sixth observation for Team A.](image)
reading the problem aloud, Amanda said, “I can’t do this one. I’ve never been to a skate park. Have you?” The other students were confused about the context of the problem as well. The problem described a ramp and asked what the area of the front ramp was. The students debated about what should be considered to be the front of the ramp.

**Amanda:** What part of the ramp is the front?

**Alice:** Just the side part.

**Amanda:** Does it mean looking at it from the top?

**Alice:** [points to the right side of the figure] Or does it mean just this half?

The students continued discussing what was considered the “front of the ramp” and other topics for six more minutes. During that time, the students were not discussing the mathematics but what the purpose of a skate ramp was and what it might look like. The students did not attend to the mathematics of the problem until the teacher came to their group and told them that they needed to find the area of the diagram that was presented with the problem. Thus, this axial code explains why the Math-Talk levels for the students in Team A were higher when the teacher was able to select problems that had familiar contexts.

**Determination of correct answers.** The second axial code evident in Team A, determination of correct answers, explains why possible shifts occurred when students disagreed on their solutions. Open codes related to this axial code were “not knowing how to check for correct answers” and “teacher telling students they had incorrect answers.” These codes explained how students’ Math-Talk levels changed relative to when they found out they had incorrect answers. For example, the levels of Math-Talk
increased during the fourth episode when two of the members in Team A had conflicting answers. The teacher gave Team A students the problem shown in Figure 4.3, which required students to perform a series of calculations that show when dimensions increase the area does not increase proportionately.

Annie quickly answered question four of the problem without attending to questions one through three. Given that Annie skipped these questions, she assumed that the area would increase at the same rate that the side lengths would increase. Audrey, however, completed all questions in the order given to her and had calculated a different answer than Annie. Annie said, “That can’t be right. How is it that 7 times 3 is 21, but the area is [pauses]?” Alice said, “That can work, because you have to add area on both sides and that means you have to multiply both sides.” Alice continued her explanation with frequent interruptions for additional questions and more explanation from the other team members. It was during this episode that the team exhibited its highest levels of Math-Talk of the observation period.

PROBLEM ONE: The dimensions of a 7 cm by 2 cm rectangle are multiplied by 3. How is the area affected?

Checker draws a rectangle that is 7 cm wide and 2 cm tall and then cuts it out.
Reader draws a rectangle that is 21 cm wide and 6 cm tall and then cuts it out.

1. What is the area of the rectangle that is 7 x 2? ______________
2. What is the area of the rectangle that is 21 x 6? ______________
3. By how many times does the area change? ______________ (hint: divide the area of the bigger shape by the area of the smaller shape)
4. How much did each side change? ______________

*Figure 4.3. Problem for the fourth observation of Team A.*
Interpretation of Data

**Pre and post sociomathematical norms.** Understanding the analysis of the pre and post sociomathematical norms data requires some explanation of the context. As previously discussed in the within-case analysis in Team A, the levels of Math-Talk were relatively lower from the pre to the post observations. A likely explanation for this is that in both the pretreatment and the post treatment lessons, the teacher gave little to no chance for student conversations. During both of the lessons, there was little encouragement or opportunity for student discussion.

**Changes in Math-Talk over time.** The students exhibited the highest levels of Math-Talk in the area of “listen to and attempt to understand the reasoning of others.” The students in Team A allowed all of the students in their group to take a turn speaking. The students exhibited the lowest levels in the area of “indicate when solutions are valid.” The only way that students knew if their answers were incorrect was if two or more of the group members calculated different answers or if the teacher told them that their answers were incorrect. The students did not have the skills to tell if their answers were correct on their own. Overall, the students in Team A showed higher levels of Math-Talk when: (1) they had challenging problems to work on, (2) when two or more students had different answers, and (3) when the context of the problems was familiar.

Another variance in the analysis of the data from Team A occurred during the eighth episode. As evidenced in Table 4.1, the students were on an upward trajectory with respect to increasing Math-Talk levels. Their overall average was increasing during each episode. However, during the eighth episode, levels of three of the four Math-Talk
areas decreased. One possible reason for this variance was that the team spent a great deal of time discussing the context of the problem. The main character of the context was a farmer and the setting was his farm. The members of Team A discussed at length what kinds of animals were on the farm and drew sketches of the animals. However, Team A still finished their problem as quickly as the other members of the class. The teacher may have been able to help the students in Team A maintain higher levels of Math-Talk with an additional problem or with a more complex one.

**Team B: Bryce, Bill, Bonnie, and Barbara**

Two of the members of Team B were female and two were male. Each had received high grades in mathematics. The scores on their summative tests were more varied. Two earned scores that deemed them proficient in mathematics, and two were deemed approaching proficient (see Table 3.1).

**Quantitative analysis.** I used descriptive statistics to analyze data from two instruments. The first was the Sociomathematical Observation Tool, and the second was the Math-Talk observation tool.

**Pre- and post-observation.** The Sociomathematical Norms Observation Tool indicated changes in student autonomy when engaged in Math-Talk for Team B (see Table 4.3). During the pre-observation, the students of Team B exhibited an average autonomy Level 1 for 9% of the duration of the lesson and a Level 0 for the remaining 91%. During the post observation, the students showed an average autonomy Level 1 for 3% of the lesson and Level 0 for the remaining 97% of the observation period. The levels of autonomy were relatively lower from the pre to the post observations. I will explain
Table 4.3

Pre and Post Sociomathematical Norms for Team B

<table>
<thead>
<tr>
<th>Level</th>
<th>Pre-Treatment</th>
<th>Post-Treatment</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Level 3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Level 2</td>
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</tr>
<tr>
<td>Level 0</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

N = 4 students.

Note. Categories:
A – Indicate when solutions are valid
B – Explaining others’ explanations
C – Justify reasoning
D - Indicate nonunderstanding

Levels
0 – No student explanations
1 – Minimal student explanation
2 – Student ideas guide lesson
3 – Talk is initiated by students (not teacher)

Tallies made every 3 minutes for 24 minutes for a total of eight tallies.

Possible reasons for the lower levels during the post observation in the “Interpretation of Data” section of the case of Team B.

Changes in Math-Talk over time. The line graph in Figure 4.4 indicates in which episodes the shifts in Math-Talk levels occurred. There is an increase of one full level between the first and second episode in all four areas and a decrease between the second and third episodes in three of the four areas. During the first four episodes, all areas increased and decreased simultaneously. However, during the fifth episode, the areas were somewhat varied. For example, “indicate when solutions are valid” increased during the sixth episode but then decreased during the seventh episode, while “indicate nonunderstanding” decreased during the sixth episode and then increased during the
seventh and eighth episodes. An explanation of the events surrounding these shifts is in the “Qualitative Analysis” subsection of Case B. Overall, the results in Figure 4.4 show that the Math-Talk levels of Team B increased in all areas from the start to the finish of the eight episodes.

Table 4.4 lists the numerical values of the Math-Talk levels for Team B for each area during each episode. During the second episode, Math-Talk levels in all four categories increased more than one full level (1.07). Math-Talk levels decreased during the third episode in the categories of “indicate when solutions are valid” (1.40) and “explain and justify reasoning” (1.20). During the seventh episode there was a decrease in the area “indicate when solutions are valid” (1.88). The area with the highest level was “explain and justify reasoning” (2.43) and the area of the lowest level of Math-Talk was “indicate when solutions are valid” (1.92). The area that showed the greatest change was
Table 4.4

Math-Talk Partial Interval Coding for Team B

<table>
<thead>
<tr>
<th>Observation episodes</th>
<th>Indicate when solutions are valid</th>
<th>Attempt to understand others' explanations</th>
<th>Explain and justify reasoning</th>
<th>Indicate nonunderstanding</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.88</td>
<td>1.00</td>
<td>1.13</td>
<td>1.25</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>2.14</td>
<td>1.57</td>
<td>2.43</td>
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<td>2.04</td>
</tr>
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<td>1.35</td>
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<td>2.71</td>
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</tr>
<tr>
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<td>2.14</td>
<td>2.71</td>
<td>3.00</td>
<td>2.14</td>
<td>2.50</td>
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<td>2.75</td>
<td>3.00</td>
<td>2.43</td>
<td>2.52</td>
</tr>
<tr>
<td>Ave.</td>
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<td>2.26</td>
<td>2.43</td>
<td>1.94</td>
<td>1.07</td>
</tr>
<tr>
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<td>+1.75</td>
<td>+1.87</td>
<td>+1.18</td>
<td>+1.45</td>
</tr>
</tbody>
</table>

“explain and justify reasoning” (1.87), and the area that showed the least change was “indicate when solutions are valid” (1.00).

**Qualitative analysis.** The axial codes that emerged during the qualitative analysis of Team B were: (1) the evolution of team relationships, (2) the determination of correct answers, and (3) teacher support needed in order to maintain high levels of Math-Talk.

**Evolution of team relationships.** An axial code that emerged while analyzing these data for Team B was the evolution of the team relationships. Open codes related to this axial code were “paying attention to Bill not Bryce,” “Bryce being inattentive,” and “Bryce understanding mathematics solutions.” These codes explained how students’ Math-Talk changed throughout the eight episodes. During the first three episodes, Barbara and Bonnie relied heavily on the authority of Bill and discounted the contributions that Bryce made to the group. Bill was calm and serious during the
episodes. When Bill declared an answer, two of the team members, Barbara and Bonnie, were quick to accept the answer as correct. Bryce, on the other hand, seemed inattentive. An example of this group dynamic occurred during the second episode.

**Barbara:** What are we supposed to do?

**Bill:** Wait a sec. [pause] Okay, the answer is one half.

**Barbara and Bonnie:** [writes one-half on her paper.]

**Bryce:** [Begins flipping the manipulatives the students were using to solve the problem across the room].

**Barbara:** Bryce, cool it, bro. Stop flipping things.

**Bryce:** Don’t call me bro.

**Barbara:** Sorry, bro.

Shortly after the conversation, the teacher visited the group and told them their answers were incorrect. At that time, Bryce said, “I knew it was wrong all the time. It’s two.” Barbara asked, “why didn’t you say something earlier.” Bryce said, “It’s two, because two is half of four and the square is four not one.” While Bryce was explaining his answer, he was standing near his chair, dancing.

During the fourth episode, Barbara began to notice that Bryce had many correct answers. Barbara said, “Wait [everyone]. Bryce what do you think the answer is?” Bryce said, “I don’t know, I wasn’t listening. What’s the question? Bonnie read the question back to Bryce, and Bryce gave a valid answer. Bonnie, Barbara, and Bill wrote down Bryce’s answer. Bryce was able to find errors in others’ solutions when he engaged. Initially, Bryce did not have the trust of his teammates that he could solve challenging problems. When the team members realized that Bryce was skilled at solving problems,
their levels in “listen to and attempt to understand others’ explanations” increased. Also, after the teammates realized that Bryce had problem-solving skills that could contribute to the group, the other three team members worked toward redirecting Bryce when he lost focus so that Bryce could contribute more productively.

**Determination of correct answers.** During the third episode, the Math-Talk level in the area of “indicate when solutions are valid” declined. The axial code “determination of correct answers” explained this decline. Open codes related to this axial code were “accepting others’ answers without questioning them,” “two students getting different answers,” “stopping their work when one person had an answer,” and “teacher telling students their answers were incorrect.” These codes explained how students’ Math-Talk levels changed when students realized their answers were incorrect, whatever the method. For example, Team B was asked to find the area of a composite shape (see Figure 4.5).

![Figure 4.5. Student work from third observation of Team B.](image-url)
Bonnie solved the problem incorrectly, and the others assumed she was correct. The other three teammates saw her answer and stopped their own calculations to write down Bonnie’s answer. After the other three students wrote down Bonnie’s answer, they announced they were finished with the problem and talked about subjects not related to mathematics for 7 minutes. When the teacher checked in with the team, she noticed that all four students had the incorrect answer. The teacher said, “that’s wrong—try again,” and left the group. The students studied the problem for 2 minutes before Bill noticed the number of squares had been counted incorrectly. One of the sides was a 9 x 9 square and another was a 9 x 10 rectangle. The students had assumed that both quadrilaterals were rectangles when one was, in reality, a square. After the teacher intervened to tell them that their answers were incorrect, the students began carefully and slowly counting each of the squares to verify their lengths. All four team members engaged in high levels of Math-Talk for 4 minutes.

**Teacher support.** The third axial code, “teacher support” explained the role of the teacher when students were engaged in higher levels of Math-Talk. The open code related to this axial code was “telling the students their answers were incorrect.” This code explained how students’ Math-Talk changed relative to when they found out their answers were incorrect. For example, when students were asked to find the surface area of a square pyramid (see Figure 4.6), Bill calculated that the surface area was 160 cm². The other team members immediately agreed that this was the correct answer and wrote down the incorrect answer on their papers and announced that they were finished. The students then began discussing topics not related to mathematics for 6 minutes until the
teacher came to check on them. When the teacher saw their incorrect answer, she said, “There is a mistake somewhere here” and left the team to check on another team. There was a pause before Bill said, “wait.” Bill then calculated the correct answer by adding the four triangle-shaped faces to the square base, calculating the correct answer of 64 cm$^2$. After Bill finished, he explained his thinking to his teammates. Bryce said, “I knew it.” Barbara said, “Why didn’t you say anything. Bryce shrugged his shoulders. Then Barbara told what her misconception was, “I thought you were right because I thought we had to multiply everything by two.” After this conversation, the students maintained high levels of Math-Talk for the next four minutes. When the teacher told them they made a mistake and the students realized there was an error in their thinking, and they were able to discuss solutions to the problem with each other. However, in this instance, they were not able to find their error without the help of their teacher. When the teacher told them their answer was incorrect, they engaged in high levels of Math-Talk in other areas to solve the problem correctly.
Interpretation of Data

**Pre and post sociomathematical norms.** As previously discussed, the levels of autonomy for Team B were relatively lower from the pre to the post observations. A likely explanation for this is that in the post treatment observation period, the teacher gave a lesson in which little to no chance for student conversations. The students were expected to write down what the teacher wrote on the board and only interrupt if they had questions. During both of the lessons, there was little encouragement or opportunity for student discussion.

**Changes in Math-Talk over time.** The students exhibited high levels of Math-Talk after the team members realized that although Bryce might appear inattentive, he had many mathematical skills. The students seemed to realize this during the fourth episode in which their levels in Math-Talk increased. When the students saw that Bryce had mathematical expertise to contribute, they worked toward helping Bryce focus on the mathematics at hand so that he could help them better solve the mathematics problems.

The students exhibited the lowest levels of Math-Talk in the area of “indicate when solutions are valid.” The ways that students knew their answers were incorrect were if two or more of the group members calculated different answers or if the teacher told them. The students did not have the skills to tell whether their answers were correct on their own. The students exhibited high levels of Math-Talk when the teacher was able to give the students feedback regarding the accuracy of their answers. During a member checking session after the third episode, the teacher discovered the connection between giving feedback quickly and high levels of Math-Talk. After that, she made an effort to
make sure she checked in with the group regularly. However, there were times when she
needed to engage with other teams in her classroom which kept her from consistently
giving the students in Team B the feedback they needed to maintain high levels of Math-
Talk.

Some variances among the four areas of Math-Talk occurred during the sixth and
seventh episodes (see Figure 4.4). Possible explanations for the variances could be that
there was a schedule change for an assembly during the day of the sixth episode, and the
students were unsettled by the change of routine. Another possible explanation could be
that there was a fire drill during the seventh episode, which distracted the students from
focusing on mathematics.

Team C-1: Candace, Christine, Chloe, and Carly

All of the members of Team C-1 were female. Each had received high grades in
mathematics. Three of the four team members earned scores that were deemed proficient
on their summative tests. One of the team members earned a score that deemed her not
proficient (see Table 3.1).

Quantitative analysis. I used descriptive statistics to analyze data from two
instruments. The first was the Sociomathematical Observation Tool, and the second was
the Math-Talk observation tool.

Pre and post observation The Sociomathematical Norms Observation Tool
indicated changes in student autonomy for Team C-1 (see Table 4.5). During the pre-
observation, Team C-1 exhibited an average autonomy Level 1 for 9% of the observation
Table 4.5

Pre and Post Sociomathematical Norms for Team C-1

<table>
<thead>
<tr>
<th>Level</th>
<th>Pre-Treatment</th>
<th>Post-Treatment</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Level 3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Level 2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Level 1</td>
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<tr>
<td>Level 0</td>
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<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

*N* = 4 students.

*Note.* Categories:

A – Indicate when solutions are valid
B – Explaining others’ explanations
C – Justify reasoning
D - Indicate nonunderstanding

Levels

0 – No student explanations
1 – Minimal student explanation
2 – Student ideas guide lesson
3 – Talk is initiated by students (not teacher)

Tallies made every 3 minutes for 24 minutes for a total of eight tallies.

period and a Level 0 for 91% of the observation period. During the post observation, the students exhibited an average autonomy Level 3 for 16% of the observation period, 16% for Level 2, 28% for Level 1, and 48% for Level 0. I will discuss the interpretations of these levels in the “Interpretation of Data” subsection for Case C-1.

**Changes in Math-Talk over time.** The line graph in Figure 4.7 shows in which episodes the shifts in Math-Talk levels occurred for Team C-1. During episode three, the levels in the area of “listen to and attempt to understand others’ explanations” and “explain and justify reasoning” were higher while the other two areas decreased during that episode. The area of “indicate nonunderstanding” decreased steadily after the sixth episode while “listen to and attempt to understand the explanation of others” maintained
the highest level of Math-Talk. I discuss possible reasons for the variance of levels among the four Math-Talk areas in the “Interpretation of Data” subsection of Team C-1. Table 4.6 lists the numerical values of the Math-Talk levels for Team C-1 for each area during each episode. I recorded a shift of Math-Talk levels in all four categories during the second and third episodes, each an average of 1.07. During the seventh episode, the area of “indicate nonunderstanding” dropped to 2.43 and then dropped again during the eighth episode to 2.00. The area with the highest average was “explain and justify reasoning” (2.53), and the lowest was “indicate when solutions are valid” (2.26).

The area that increased the most was “attempt to understand others’ explanations” (2.00), and the area the increased the least was “indicate nonunderstanding” (0.75).

**Qualitative analysis.** The axial codes that emerged during the qualitative analysis were: (1) the evolution of relationships among team members, (2) the appropriate
Table 4.6

Math-Talk Partial Interval Coding for Team C-1

<table>
<thead>
<tr>
<th>Observation episodes</th>
<th>Math-Talk categories</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Indicate when solutions are valid</td>
</tr>
<tr>
<td>1</td>
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<td>2.86</td>
</tr>
<tr>
<td>8</td>
<td>2.40</td>
</tr>
<tr>
<td>Ave.</td>
<td>2.23</td>
</tr>
<tr>
<td>Change</td>
<td>+1.52</td>
</tr>
</tbody>
</table>

selection of problems, and (3) teacher support needed in order to maintain high levels of Math-Talk.

**The evolution of relationships among team members.** The first axial code, “evolution of relationships among team members” explains how the team was able to exhibit higher levels of Math-Talk. Open codes related to this axial code were “one team member did most of the talking during the first few episodes” and “other teams members eventually contributed when they saw that the team member talking was making incorrect mathematical assumptions.” These codes explained how students’ Math-Talk changed relative to the first three episodes. For instance, as the students in Team C-1 talked to each other more often, they were able to listen to each other and explain their thinking in more sophisticated ways. During the pretreatment interview and earlier observations, Christine dominated the discussions. However, Christine did not generally know the most
efficient way to solve a problem or have the background knowledge to understand the context of problems. The following transcript from the second episode gives an example of this.

**Christine:** So, what do you guys think we should do here?

[10-second silence]

**Christine:** [giggles]. Awkward. [pause]. Does anyone remember the formula for finding the area of a parallelogram [looks at her notes]? I think it’s \( B_1 + B_2 \) divided by two times the height.

At this time, Candace was copying down everything that Christine was saying, but Chloe and Carly were writing down the correct formula for the area of a parallelogram and proceeded to solve the problem correctly. When Christine saw that Chloe and Carly were using a different formula, she said, “Oh, that’s right.” I was using the formula for a trapezoid instead of the one for a parallelogram.” Christine erased her work and so did Candace. During this interaction, Christine was the only team member who spoke. The others worked silently.

During the earlier episodes, it was either Chloe or Carly who silently demonstrated the knowledge to solve the problems, either performing calculations on paper or sketching what a context might look like. Since Christine was doing most of the talking, the group was slow to find solutions because those who knew how to approach the problem did not speak. Eventually, Christine noticed the work that Chloe and Carly were silently doing and asked them questions. As Chloe and Carly began speaking more, the group was able to solve problems more efficiently. The classroom teacher of Team C-1 reported that she noticed increased levels of Math-Talk in this team because the team members were becoming more confident in their mathematics and were more willing to
discuss their mathematical ideas. The teacher reported that two of the team members, Chloe and Carly, had rarely spoken at school at all, let alone in a mathematics setting. The teacher said, “although Carly got high scores on her assignments and tests, the other students didn’t know [until she worked in HPT] that she had something to offer.” The teacher attributed Chloe and Carly’s increase in mathematics conversation to confidence that their team members would be interested in their mathematical opinions.

*Selection of problems.* Another axial code that emerged was the “selection of problems.” Open codes related to this axial code were “students did not engage in problems with unfamiliar contexts” and “the teacher needed to explain unfamiliar contexts to students.” These codes explained how students’ Math-Talk levels changed when they were faced with unfamiliar contexts and then again after the teacher explained the contexts. The teacher of Team C-1 learned that the students could maintain high levels of Math-Talk if the students were given contexts that were familiar to them. An example of this happened during the seventh episode. Students were asked to find the surface area of two square pyramids (see Figure 4.8), which was the shape that was on top of a staircase. Finding the surface area of square pyramids was a mathematical procedure with which the students in Team C-1 were familiar. However, the students did not understand how the shape of a square pyramid fit into the context of their problem.

![Figure 4.8. Problem for Team C-1 during the seventh observation.](image)
Christine, said, “Does this mean that the house is shaped like a square pyramid?”
Candace said, “That’s not what the problem says.” After four minutes the teacher
intervened, and told the students, “Read the problem again.”

When the students did not understand the context of the problem after the
problem was read again, the teacher went to another group in the classroom whose
member had drawn a picture of a staircase railing with caps that looked like square
pyramids and showed it to the members in Team C-1. Christine said, “Oh, it’s just a
post.” Chloe said, “That’s what I thought. It’s on the top of the post.” Although the
students knew how to find the surface area of a square pyramid, they would not do so
until they understood how the square pyramid fit into the context of the problem.

**Support of the teacher.** The third axial code that emerged while analyzing the
data for Team C-1 was “support of the teacher.” The role of the teacher in giving students
adequate support to maintain high levels of Math-Talk was critical. The open codes that
related to this axial code were “when student stopped working on problems” and “what
teacher did to enable to students to resume working.” The students needed to understand
the context of a problem before they would engage in solving the problem. For example,
the students were assigned a problem that had a context of a skate park. However, the
students did not solve the problem until the teacher explained to them what a skate ramp
was. The teacher learned that she needed to either provide contexts that she knew were
familiar or take a few minutes at the beginning of the subsequent episodes to ensure that
the students understood the contexts. When teacher was able to do that for them, the
students in Team C-1 were able to maintain high levels of Math-Talk.
Interpretation of Data

Pre and post sociomathematical norms. As previously discussed, the students in team C-1 exhibited higher degrees of autonomy during the post observation of sociomathematical norms. This increase happened because the teacher taught a lesson that encouraged student discussion. The students of Team C-1 had many more opportunities for discussion during the post observation than they did during the pre-observation. The teacher also attributed the increase in Math-Talk levels to increased confidence to talk about mathematics.

Changes in Math-Talk over time. The team members exhibited high levels of Math-Talk when Chloe and Carly began contributing to the mathematics conversations and when the teacher was able to give the students problems with familiar contexts. During the first two episodes, Christine did most of the talking. However, as Carly and Chloe began engaging in conversations more frequently, the team showed higher levels of Math-Talk. The team also exhibited higher levels of Math-Talk when the teacher was able to explain the contexts of the math problems to the students. However, there were times in which the teacher was busy with other groups or when she did not forecast that the students would not understand the contexts, so the students did not always get the support they needed to maintain high Math-Talk levels.

As evidenced by the data in Figure 4.7, the Math-Talk of some levels decreased during the seventh and eight episodes, with “indicate nonunderstanding” decreasing the most. One possible explanation for this was that as Chloe and Carly were participating more, Christine did not talk as much. Christine dominated the conversations in earlier
episodes when she did not understand a mathematics concept but talked less during the later episodes. This could be one possible reason for the variance among areas during the seventh and eight episodes.

**Team C-2: David, Dallas, Denton, and Doris**

Two of the members of Team C-2 were female, and two were male. Each had received Bs and Cs in mathematics. One student earned a score that was approaching proficient on her summative mathematics test. The other three students earned a score that was not proficient (see Table 3.1).

**Quantitative analysis.** I used descriptive statistics to analyze data from two instruments. The first was the Sociomathematical Observation Tool, and the second was the Math-Talk observation tool.

**Pre and post observation.** The Sociomathematical Norms Observation Tool indicated changes in student autonomy when engaged in Math-Talk for Team C-2 (see Table 14.7). During the pre-observation, the students in Team C-2 exhibited an average autonomy Level 1 for 3% of the observation period and a Level 0 for 97% of the observation period. During the post observation, the students exhibited autonomy an average of Level 3 for 3% of the observation period, 9% for Level 2, 31% for Level 1, and 56% for Level 0.

**Changes in Math-Talk over time.** The line graph in Figure 4.9 indicates in which episodes the shifts in Math-Talk levels occurred. The team maintained low levels of Math-Talk for the first five episodes. During the sixth episode and beyond, the team exhibited in increase in levels, with the highest level being in the area of “explain and
Table 4.7

**Pre and Post Sociomathematical Norms for Team C-2**

<table>
<thead>
<tr>
<th>Level</th>
<th>Pre-Treatment</th>
<th>Post-Treatment</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A  B  C   D  %</td>
<td>A   B  C  D  %</td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td>0  0  0  0  0</td>
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<td>+1</td>
</tr>
<tr>
<td>Level 2</td>
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</tr>
<tr>
<td>Level 1</td>
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<td>+9</td>
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<tr>
<td>Level 0</td>
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<td>3  4  6  5  56</td>
<td>-15</td>
</tr>
</tbody>
</table>

N = 4 students.

*Note.* Categories:
A – Indicate when solutions are valid
B – Explaining others’ explanations
C – Justify reasoning
D - Indicate nonunderstanding

Levels
0 – No student explanations
1 – Minimal student explanation
2 – Student ideas guide lesson
3 – Talk is initiated by students (not teacher)

Tallies made every 3 minutes for 24 minutes for a total of eight tallies.

**Figure 4.9.** Changes in Math-Talk levels over time for Team C-2.
justify reasoning” and lowest in “indicate when solutions are valid.”

Table 4.8 lists the numerical values of the Math-Talk levels for Team C-2 for each area during each episode. The team exhibited levels of 0 in two areas during the first episode, “explain and justify reasoning” and “indicate nonunderstanding.” During episode seven, three areas increased while one decreased. The areas that increased were “listen to and attempt to understand others’ explanations” (1.17), “explain and justify reasoning” (2.17), and “indicate nonunderstanding” (1.14). The area that decreased during episode seven was “indicate when solutions are valid (0.33). I discuss the interpretations of these variances in areas in the “Interpretation of Data” subsection of Team C-2. The area with the highest average was “explain and justify reasoning” (0.83) and the lowest was “indicate when solutions are valid” (0.46).

Table 4.8

Math-Talk Partial Interval Coding for Team C-2

<table>
<thead>
<tr>
<th>Observation episodes</th>
<th>Indicate solutions are valid</th>
<th>Attempt to understand others' explanations</th>
<th>Explain and justify reasoning</th>
<th>Indicate nonunderstanding</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>3</td>
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<td>0.57</td>
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</tr>
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</tr>
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<td>5</td>
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<td>0.17</td>
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<td>1.00</td>
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<td>+2.14</td>
<td>+1.17</td>
<td>+1.14</td>
</tr>
</tbody>
</table>
Qualitative analysis. The axial codes that emerged during the qualitative analysis of Team C-2 were: (1) student engagement, (2) selection of problems and (3) teacher support.

Student engagement. The first axial code, “student engagement” explains the circumstances surrounding when the students did and did not engage in solving the problems. Open codes related to this axial code were “students took too long (18 minutes) to initially engage in solving a problem,” “the teacher needed to address the students’ stated accommodations in their individualized education plans,” and “students initially engaged only when their teacher was able to sit with them as part of their group.” These codes explained how students’ Math-Talk levels were low for the first five episodes but then began a steady incline after. For example, during the first episode, the teacher realized that there was an issue with student engagement. During the first episode, the students did not engage for 18 minutes in spite of the teacher’s attempts to involve them by asking probing questions. The problem presented in the first episode was an exploration into area of two-dimensional shapes. The students were given several shapes cut out of cardstock and asked to find the area of the shapes in relation to each other. One of the areas of the shapes was given, and the students were charged with finding out the area of the other shapes by comparing sizes of shapes to each other. The teacher checked in with the students in Team C-2 every few minutes and asked strategic questions. The students gave short answers to the teacher, but stopped talking about the problem as soon as the teacher left. A partial transcript of the episode follows:

Teacher: [pointing to the square] Dallas, what can you tell me about this shape?

Dallas: It’s a shape.
Teacher: [pointing to the square] What about you Doris, what can you tell me about this shape.

Doris: It’s a square.

Teacher: [To the whole team] How big is the square?

Dallas, Doris, David, and Denton: [look at their papers]

Teacher: Who is in charge of drawing a picture of what is happening in the problem, the context.

Dallas, Doris, David, and Denton: [look at their papers]

Teacher: [pointing at the part of the problem that gives the area of the square] Doris, can you read what the area of the square is [indicating a square shape that was still in the plastic bag the teacher previously handed to all members of the classroom]? It is given in the problem.

Doris: [pause] It is four inches square.

Teacher: Correct, Doris. If the area of this square is four inches squared, how big are these two smaller triangles? [pause] I’ll come back and check on you in a few minutes to see what you have found out.

The teacher checked on the team two more times, having conversations similar to the one in the above transcript. After 18 minutes, the students, in the team, opened their bags of shapes and began to compare sizes. During the member-checking session with the teacher, the teacher explained that she needed to check in more frequently with the students in Team C-2 to see if additional help from her would improve their engagement.

During the second episode, the teacher was able to attend to the students in Team C-2 more often. The group became engaged after ten minutes which is eight minutes sooner than they did in the first problem. During the member-checking session after the second episode, the teacher decided that she needed to spend even more time with this team. During each subsequent episode, the other students in the class (other than the students in Team C-2) demonstrated more and more autonomy, which allowed the
teacher to attend to C-2 team more frequently. By the time that the students in Team C-2 worked in their sixth episode their teacher was able to sit with group most of the time. In fact, the sixth episode, the members of C-2 began working on their problem before the teacher had finished giving the whole-class instruction regarding the problem they were to solve.

**Selection of problems.** The second axial code that emerged was the need for the appropriate “selection of problems.” Open codes related to this axial code were “students only initially engaged when problems were relatively easy” and “students eventually engaged in challenging problems.” These codes explained how students’ Math-Talk changed relative to the first episode. During the first episode, students did not engage for 18 minutes, and during the last episode they engaged with no prompting from the teacher. During the last episode, the students trusted that the teacher was giving them problems they could understand. In order for students to maintain high levels of Math-Talk, the teacher needed to ensure that the problems were appropriate level of complexity and also that they problems contained the appropriate scaffolding.

**Teacher support.** The third axial code that emerged was the need for increased teacher support. Open codes related to this axial code were “student participation increased when the teacher sat with students,” “the accommodations set forth in the student’s individual education plans needed to be considered,” “the teacher needed to provide adequate scaffolding for students,” and “the teacher could only attend to the Team C-2’s needs when the rest of the class worked autonomously.” These codes explained how students’ Math-Talk changed relative to the first episode. The students in
Team C-2 engaged most when the teacher was able to accommodate their special needs, provide appropriate scaffolding, and sit with the students.

An example that illustrates the need for attention to student accommodations involves Denton. During the first episode, at the 23-minute mark, Denton seemed agitated. I noticed he was nodding his head vigorously and intermittently standing up and down. I knew this student was classified as ADHD and considered that the environment of the classroom with all the students talking in their groups may be over stimulating him, causing him distress. I removed Team C-2 from the noisy classroom into a small teacher workroom. Later, the teacher and I decided that this team would need to meet in a separate room for the next few episodes in order to minimize the stimulation of Denton. During the second episode, with the team in a quieter area, Denton did not appear agitated.

In episode seven, the students’ Math-Talk levels increased when the teacher explicitly showed Doris how to check for accuracy. The students were given a grid in which they were to draw five quadrilaterals. For each quadrilateral, three points were given, and the students were assigned to find the fourth point. The end result was a graphic representation of a smiling face (see Figure 4.10). Initially, the students in Team C-2 drew what they thought the mouth should look like without checking that the shapes they had drawn were those that were specified in the assignment. When the teacher saw that the mouth was not two parallelograms, she asked them several questions to help them realize that they had made an error and needed to erase part of their work.
Figure 4.10 Student work for seventh observation for Team C-2.

Teacher: This can’t be right here [pause]. Can you see that this shape is not a parallelogram?

David: [sighs and begins to crumple his paper]

Teacher: We can either erase this part, or I can get you a brand new paper so you don’t have eraser marks.

David: [begins erasing]
Doris: Did I do this right?

Teacher: Pretty close. Can you count these squares on this side?

Doris: [counts] There are 7.

Teacher: So, now make sure there are 7 on the other side.

Doris: [Counts other side]. Oh, I see. It looks like a parallelogram now.

Teacher: Right. Can you show David what to count to make sure his shape is also a parallelogram?

Doris: Ya [counts with David].

After the teacher left to attend to another group in the classroom, Doris helped some of the other members of her team check to make sure the lines were parallel which increased the Math-Talk levels of the group for several minutes after the teacher stopped interacting with them.

Interpretation of Data

Pre and post sociomathematical norms. As previously discussed in the within-case analysis of Team C-2, the students exhibited higher levels of autonomy during the post observation of sociomathematical norms. The likely explanation for the increase is that students knew that they were expected to contribute during class and were given many opportunities to do so. The teacher of team C-2 said, “I believe they talked more because they had more confidence. Since doing HPT, I can’t get Denton to stop talking about math.”

Changes in Math-Talk over time. The students in Team C-2 exhibited low levels of Math-Talk in the earlier episodes because they did not engage in solving the problem for several minutes. The teacher reviewed the accommodations that were listed
in the students’ individual education plans and gave the team an alternate place to meet that was away from the noise of the rest of her class. However, it was not until the teacher sat with the students, as one of the team members, that the students began to fully engage and increase their Math-Talk levels. The group’s levels increased when the teacher was able to show Doris how to check for accuracy and then assigned Doris to check the other team members’ work. At times, the teacher was able to give her full attention to the students in Team C-2. However, she still needed to attend to the other teams in her classroom and was pulled away from Team C-2 at times. The team members were slow to initially engage and needed more support than the other teams in the classroom. In order for the team to increase their levels of Math-Talk, the teacher needed to intervene frequently.

Cross-Case Analysis of Student Teams

The purpose for using a cross-case analysis for this study was to examine the influence of HPT training on the depth of student-to-student mathematics discourse and look for emerging patterns that may lead to generalizations across cases. This section begins with a quantitative analysis of the descriptive statistics and then a qualitative analysis of the open codes that emerged during the analysis.

Quantitative Analysis

Reliance on Math-Talk areas. The averages of the extent to which each of the teams accepted the four areas of Math-Talk are listed in Figure 4.11. All four groups relied most on the same Math-Talk area, “explain and justify understanding.” One
possible reason for this was in HPT, the students took turns talking, so everyone was encouraged to explain their ideas. The teachers in this study did not put any limitations on what the students said during their mathematics discussions. For example, one of the students in Team C-2, Denton, gave an explanation to the other members of his team regarding why he had the misconception that $3 \times 20$ was equal to 80 instead of the correct answer 60. Although this was a lengthy explanation, the other teammates were obligated to listen to him.

The area that the students relied upon least often was also the same for the four different cases, “indicate when solutions were valid.” One possible reason for this was that they seemed untrained in how to check their work and how to see whether the answers they provided made sense in a given context. Another possible reason was that
students seemed reticent to find errors in other students’ work, as though that may be impolite. The students did not seem to have the expectation that they were responsible to find mathematical errors although it was listed as one of the HPT roles. When one student had an answer, the others stopped working on the problem, assuming that the student was correct. The members in Team B indicated that they were more motivated to complete the task quickly than to ensure they had the correct answer.

Qualitative Analysis

I used a template (see Figure 4.12) to examine common and atypical changes throughout the study. By comparing typical and atypical axial codes I found: (1) the evolutions of team relationships was important; (2) in order to maintain higher levels of Math-Talk, the teacher needed to be deliberate about choosing appropriate problems; (3) students needed help in determining whether answers were correct; and, (4) the role of the teacher during the problem solving episodes was key to maintaining high levels of Math-Talk. The subsections that follow describe the results that emerged.

Evolution of team relationships. The relationships among two of the four teams (Team B and C-1) notably changed during the eight episodes. The relationships of members in Team B changed when the students had enough opportunity to listen to the feedback from one particular student, Bryce. During the first three episodes of Team B, two of the team members looked to Bill for leadership, for he seemed confident and calm. However, Bill made mathematics mistakes. Bryce, on the other hand, seemed unfocused and inattentive but could offer more mathematics insight than Bill. When the team members realized that Bryce had much to contribute, they began to encourage Bryce to
Figure 4.12. Completed cross-case template.
pay better attention to the team discussions so that he could help them solve problems.

The relationships among the members in Team C-1 changed over time because two of the team members, Carly and Chloe, began to talk more. In the beginning, only Christine talked. However, Christine did not have as much mathematical insight as Carly and Chloe, but since she was the only teammate who was talking, other viewpoints were not presented. As Carly and Chloe began to share their mathematical insight, Christine was able to discuss mathematical issues with other students.

Relationships of students in Teams A and C-2 did not change throughout the eight episodes. In the case of Team A, the students already had an academic respect for each other, so they had already earned each other’s trust. For instance, during their initial interview, they told that they were excited to be in a group with students who “tried hard and didn’t goof around.” They mentioned that their usual groupings involved students who “wouldn’t pay attention and were hard to help.” In the case of Team C-2, the students did not have enough time in the eight episodes to be able to earn each other’s trust. The students in Team C-2 only started engaging in solving problems during the sixth episode. The relationships of the members in Team C-2 may have evolved had there been more than eight episodes. It was not until the seventh episode that the students began to discuss problems with each other without direct intervention from the teacher. Had the students more time together, they may have learned to rely on each other for mathematics discussions more.

Selection of problems. The appropriate selection of problems emerged as an important theme during the cross-case analysis. The selection of problems was found to
explain when students were most likely to engage in the highest level of Math-Talk. The two themes regarding selection of problems were (a) the importance of selecting problems that were of the appropriate level of complexity and (b) problems with unfamiliar contexts led to low levels of Math-Talk.

**Problems that were of the appropriate level of complexity.** Two of the four teams (Teams A and C-2) had higher levels of Math-Talk when the teacher chose problems that were of the appropriate level of difficulty. In the case of Team A, the teacher needed to select appropriately complex problems in order for the team members to maintain high levels of Math-Talk. When the members in Team A were given problems that were too simple or too few, the team showed lower levels of Math-Talk. Conversely, the teacher of Team C-2 needed to select problems that were appropriately simple or had built-in scaffolding in order for the students in Team C-2 to engage. However, the Math-Talk levels of the members in Teams B and C-1 did not seem to be dependent on the level of complexity of their problems. One possible explanation for this was that the problems that the teacher initially choose were the appropriate level of difficulty for these students and there was no reason to make any changes to the problems.

**Problems with unfamiliar contexts.** Two of the four teams (Teams A and C-1) exhibited higher levels of Math-Talk when the teacher presented problems with contexts that were familiar to the students or when the teacher explained the contexts to the students. The students did not engage in solving the problem until the teacher intervened to explain the context.

**Determination of correct answers.** The students of all teams exhibited the
lowest Math-Talk level in the area of “indicate when solutions are valid.” The students were not particularly skilled at finding errors unless two or more students calculated different answers or a teacher told them their answers were incorrect. Immediately after students realized that they had incorrect answers, their levels of Math-Talk increased. However, the students were not regularly able to check their own work.

**Teacher support.** Another important finding was that each team required different kinds of support from their teachers in order to increase or maintain their Math-Talk levels. The members in Team A had high Math-Talk levels when the teacher checked on the group often enough to ensure they had sufficiently challenging problems. However, since the members in Team A worked quietly, the teacher’s attention was frequently drawn to the students in other groups who were talking loudly. The members in Team B exhibited high levels of Math-Talk when their teacher checked their answers frequently to tell them whether their answers were correct. The teacher did not need to explain why an answer might be incorrect. The students were able to find their error after they knew their answer was wrong. The students in Team C-1 had high levels of Math-Talk when the teacher helped them with unfamiliar contexts. The Math-Talk levels of Team C-2 increased when the teacher attended to the students’ stated accommodations and gave the students more overall support. Whenever possible, the teacher of Team C-2 sat with the team members and worked through the problems with them. Figure 4.13 lists the themes found through my analysis of the four team observations.
<table>
<thead>
<tr>
<th>Theme</th>
<th>Team</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Evolution of Relationships</strong></td>
<td>A</td>
<td>Little evolution of relationships; students had academic trust before working in HPT</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Bonnie and Barbara began trusting judgment of Bryce (even though he seemed inattentive)</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>During the 4th episode, Chloe and Carly started talking more</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>Little evolution; team members only started working together during the 6th episode</td>
</tr>
<tr>
<td><strong>Selection of Problems</strong></td>
<td>A</td>
<td>Needed challenging and extra problems to stay engaged; also needed to have unfamiliar contexts explain</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>The problems selected were appropriate</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>Needed unfamiliar contexts explained to them</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>Needed relatively easy problems and scaffolding</td>
</tr>
<tr>
<td><strong>Determination of Correct Answers</strong></td>
<td>A</td>
<td>Showed high levels when 2 or more students had different answers or when the teacher told them their answer was incorrect</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Showed high levels when 2 or more students had different answers</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>Showed high levels when 2 or more students had different answers</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>Relied on the teacher to tell when an answer was correct and required additional explanation</td>
</tr>
<tr>
<td><strong>Teacher Support</strong></td>
<td>A</td>
<td>Needed challenging problems and help with unfamiliar contexts</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Needed to know when answer was correct</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>Needed help with unfamiliar contexts</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>Needed near constant support</td>
</tr>
</tbody>
</table>

*Figure 4.13. Themes found through analysis of the four team observations.*

**Summary**

The results of the pre and post sociomathematical observation tool showed that the levels of autonomy in students in Teams A and B did not increase from the pre to post observation. The students did not have much opportunity to talk during the post observation. Conversely, the autonomy levels of the students in Teams C-1 and C-2 were
higher during the post observation since the teacher of those students allowed for a great deal of discussion during the lesson. The results of the eight episodes using the Math-Talk tool show that the each of the student teams had the highest levels in the area “explain and justify reasoning.” The area that was the lowest for all teams was “indicate when solutions are valid.”

The within-case analysis explored events surrounding shifts of levels of Math-Talk. Shifts for Team A occurred when the teacher selected the appropriate level of complexity of problems, when those problems had familiar contexts, and when students had different answers. Shifts for Team B occurred when students had differing answers and when the team members learned to rely more on one particular student, Bryce. Shifts in levels of team C-1 occurred when the students had problems with familiar contexts and when two of the students, Carly and Chloe, began talking more. Shift in levels of Team C-2 occurred when the teacher attended to the students’ accommodations set forth in their individual education plans and when the teacher was able to sit with the students while they worked as a group.

During the cross-case analysis, four themes emerged: (1) evolution of team relationships, (2) selection of problems, (3) determination of correct answers, and (4) the role of the teacher. The students in Team B and Team C-1 experienced changes among their relationships with each other. In both cases, the students learned to rely on all members of their teams, not just those who seemed more serious or those who talked the most. In the cases of Team A and Team C-2, the selection of problems with the appropriate complexity level and problems with familiar contexts was a factor in
maintaining high levels of Math-Talk. Determining incorrect answers was also a recurring theme. When students knew they had incorrect answers, their levels of Math-Talk increased. However, since the students were not adept at finding their own errors, they needed to rely on their teachers to tell them if their answers were correct or wait to see if other students had different answers. The role of the teacher to maintain high levels of Math-Talk differed among the four cases. In the cases of Teams of A and C-2, the teacher needed to be diligent in selecting appropriate problems. In the case of Team B, the teacher needed to tell the students whether their answers were correct. In the case of Team C-2, the teacher needed to attend to the accommodations listed in the students’ individual education plan and give near constant support.
CHAPTER 5
DISCUSSION

The factors associated with productive teacher-to-student discourse are clear. However, the factors that need to be present in order for teachers to facilitate student-to-student discourse are not. Teachers need more research-based practices as they work toward facilitating an environment in which sociomathematical norms exist that encourage productive discourse. Teachers need to understand ways to instill in their students the autonomy to discuss, argue, and reason with each other.

This research study explored and described the factors associated with the development of sociomathematical norms that establish and sustain student-to-student discourse in the mathematics classroom. Specifically, this study explored how and why students accepted sociomathematical norms that allowed for student-to-student discourse. It also investigated the relationship between High-Performance Teams and Math-Talk (productive student-to-student discourse). This chapter will focus on the following four sections (1) acceptance of sociomathematical norms, (2) influence of HPT on Math-Talk, (3) limitations, and (4) implications and future research.

Acceptance of Sociomathematical Norms

Research question #1 asks which sociomathematical norms the students most readily accepted and how their acceptance changed over time. The sociomathematical norms were: (1) indicate nonunderstanding, (2) explain and justify reasoning, (3) listen to and attempt to understand others’ explanation, and (4) indicate when solutions are valid.
Of the four sociomathematical norms, the one that was most often accepted by the students who engaged in HPT was “explain and justify reasoning.” A possible reason for this norm being so prevalent was that none of the three participating teachers placed restrictions on the types of mathematics conversations in which the students engaged, as long as the discussion pertained to the problem. Among all teams, no explanation was deemed irrelevant by any other teammate or by the teacher. The students did not have any criteria given to them by their teachers as to what was considered a productive comment. Cobb and Yackel’s (1996) criteria for a productive comment to a mathematical conversation is that each comment should offer one of the following: (1) a different solution than has already been mentioned; (2) a solution that is sophisticated; (3) a more efficient solution than what has already been mentioned; or (4) an explanation to a previously mentioned solution. The teachers had not placed restrictions such as these on students when making mathematical comments. The sociomathematical norm that students accepted least was “indicate when solutions are valid.” A possible explanation for this was that the teams were reticent to trust their judgment even though they had the skills to be able to determine whether a solution was correct. In addition, students seemed as though it was impolite to tell their team members that their answer was incorrect.

Changes Over Time

Research question #1 also asked how and why the acceptance of these
sociomathematical norms changed over time. The acceptance of the norms changed according to three themes which were: (1) the evolution of team relationships, (2) the selection of appropriate problems, and (3) strategic teacher support. The relationships of Teams B and C changed over time. In the case of Team B, the members of the team initially did not trust the mathematics authority of Bryce. However, as Bryce began to explain his mathematical understanding more often, the group was able to accept the desired sociomathematical norms. In the case of Team C-1, two members did not initially speak much. After the fourth episode, those students started explaining their mathematical thinking more, and the group was able to accept the desired sociomathematical norms.

The selection of appropriate problems was noteworthy because the students in Teams A, C-1, and C-2 conversed with high levels of Math-Talk when the teacher presented problems that were the appropriate level of complexity and when the students understood the contexts of the problems. It took a few episodes and member checking sessions for the teachers to understand what problems were appropriate for what teams. In the case of Team A, the teacher knew after the first episode that the problems needed to be more complex. This underscores the literature, which states that when students are challenged with rich, mathematical tasks, they have a higher engagement in mathematics (Boaler, 2008; Kosyvas, 2015). The teacher of Team C-1 learned after the fourth episode to make sure that students understood the problem context. The teacher of Team C-2 learned after the sixth episode the degree to which she needed to scaffold the problems in order for students to maintain the desired sociomathematical norms. This connects to the
literature that suggests that students can only be engaged when their learning activities fit within in their Zone of Proximal Development (Vygotsky, 1978).

As the teachers learned the types and degrees of support their students needed, the acceptance of sociomathematical norms changed over time. In the case of Team B, the teacher learned after the third episode that she needed to tell the students whether their answers were correct in order to maintain the desired sociomathematical norms. Prior research shows that giving students specific, diagnostic feedback during formative assessments leads to higher effect sizes in mathematics learning (Black & Wiliam, 2009; Hattie & Temperly, 2007). In the case of Team C-2, the teacher needed to give students a different type of support. The teacher of Team C-2 eventually learned how best to provide accommodations according to the students’ individual education plans and that she needed to sit with the group in order for them to accept the desired sociomathematical norms. As the teachers observed what factors of HPT led to the students’ acceptance of desired sociomathematical norms, the teachers were able to adapt their teaching practices to provide those environments in their classrooms.

**Influence of High Performance Team on Math-Talk**

Research question #2 asked to what degree were the factors that were present during HPT also present when students engaged in Math-Talk. The factors of HPT were: (1) clearly defined roles for the students, (2) problems that were the appropriate level of complexity, and (3) individual accountability of team members. Of the three factors, it was most crucial that the teacher follow the second aspect (selecting problems that were
the appropriate level of complexity) exactly as specified in the HPT training. With respect to the other two areas (clearly defined roles and individual accountability), the teachers allowed the student teams to create their own team norms that followed the philosophy of HPT but not necessarily the specific guidelines set forth in the training. For example, in the first area, clearly defined roles, the teams kept the philosophy of full participation and engagement but did not always keep their roles distinct.

**Clearly Defined Roles**

The intention of the first area of HPT, clearly defined roles, was two-fold: first that the teams would have the expertise in each group to solve the challenging problems and, second, that students would engage in and contribute to solving the problems with their teams. The clearly defined roles corresponded with accepted mathematical proficiencies (National Research Council, 2001). Although each team learned with the same HPT curriculum, each found different means to ensure that team members were fully participating. As the norms for each team emerged, the students’ mathematics conversations became more in depth. For example, in the first episode, Team A decided as a group that they would each do every step so that everyone would be able to participate. The members in Team A found that solving the problems together was more engaging than keeping the roles distinct. The students in Team B kept their roles distinct until the fourth episode when they realized that contributing to the group at will was more efficient than taking turns contributing. In the case of Team C-1, as Carly and Chloe earned the respect of their peers, the distinct roles became blurred. The student conversations became more fluid, and the students spoke whenever they had a question or
contribution instead of when it was considered their turn. However, Team C-2 strictly adhered to the distinct roles throughout the entirety of the eight episodes. One possible explanation for this is that it took several episodes and a great deal of support from the teacher for the students in Team C-2 to understand what each role entailed. After the group members understood their roles, they seemed to take comfort in knowing exactly how they could contribute to their group. Since the intent of distinct roles within HPT was to ensure that all students participated and that the team had all the skills they needed to solve mathematics problems, when the students veered away from the specifics of HPT they could still maintain high levels of Math-Talk.

Table 5.1 lists each of the clearly defined roles and their corresponding intentions. The intention of the role of drawing a picture of the contexts of the mathematics problem is that at least one person on the team understands the context of the problem. The intention of the role that draws a picture of the mathematics is that at least one person on the team has sufficient conceptual insight to understand how to solve the problem. The intention of the role of performing procedures means that at least one person on the team can carry out the procedures require to solve the problem. And last, at least one person on the team needs to have the skills to be able to check the solution for errors.

**Choosing Problems at the Appropriate Level**

The teachers in this study adhered rigorously to the second area of HPT, finding the appropriate level of complexity of problems. The HPT aspect of access to challenging problems supports the idea of the importance of productive struggle in the mastery of
Table 5.1

*High Performance Team Roles and Their Underlying Philosophies*

<table>
<thead>
<tr>
<th>Specific high performance team role</th>
<th>Underlying philosophy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw picture of context</td>
<td>Understand the context of the problem and what it is asking</td>
</tr>
<tr>
<td>Draw a picture of the mathematics needed to solve the problem</td>
<td>Demonstrate conceptual understanding</td>
</tr>
<tr>
<td>Perform the mathematical procedures</td>
<td>Understand the procedures needed to solve the problem</td>
</tr>
<tr>
<td>Check for errors</td>
<td>Look for errors in others' thinking and work</td>
</tr>
</tbody>
</table>

Three of the four teams (A, B, and C-1) thrived with challenging problems. However, the students in Team C-2 did not engage in solving challenging problems until the teacher provided additional steps for the students. The interventions of the teacher of Team C-2 underscores the literature which states that students learn only when the learning activities are within their reach or in their Zones of Proximal Development (Vygotsky, 1997). Scaffolding challenging problems gives students, particularly those with learning disabilities, access to understanding challenging mathematics problems (Anghileri, 2006).

**Individual Accountability**

The third idea of HPT is that students are held accountable to learn the mathematics. In order for HTP to mirror the types of feedback that show high effect sizes in mathematics formative assessment (Black & Wiliam, 2009; Hattie & Temperly, 2007), each of the teams was trained to fill out the Individual Accountability for Students Working in High-Performance Teams form (Appendix F) while solving problems with
their teams. Although all groups began the first episode using the accountability form they received in their HPT training, none of the teachers continued to use the form past the first two episodes. During member checking sessions, teachers indicated that they felt that the forms were no longer useful since most of the students were engaging in HPT as long as they had sufficient support.

The teachers followed only one of the areas of HPT strictly, namely giving students problems that were the appropriate level of complexity. In the other two areas, the teachers allowed students to create their own team norms that satisfied the intent of these areas. The teachers used the three areas of HPT as a framework to know where to begin to teach students how to discuss mathematics with each other. The classroom environments in this study had many of the same characteristics as the environments that lead to productive, in-depth student-to-student discourse as cited in the literature (Berland, 2010).

**Likelihood of Replicability**

In the investigation of the factors that need to be present in order HPT to be replicated, three perceptions emerged. The first perception is that the HPT training of students needs to be refined. The second is that teachers need to have appropriate expectations. The third is the role of the instructional coach in helping the teachers lay the groundwork to be able to manage those expectations.

**High Performance Team Training of Students**

The training of HTP to students would have been more effective if the duration
were longer than two class periods. Further, students may to need to learn one aspect of HPT at a time and then have opportunity to practice and refine their skills in the context of their mathematics class before moving on to the next aspect. Each of the teams’ Math-Talk Levels Over Time line graphs show that there was a large jump in Math-Talk levels between the first and second episodes. Had the students been trained more comprehensively before the first episode, the levels between the first and second periods would have more likely been similar.

**Teacher Expectations**

For some of the teachers, HPT was their first experience teaching mathematics using student teams. Some teachers were not proficient in classroom management skills related to small groups such as assigning students to groups efficiently, establishing procedures for orderly moving of desks to various parts of the classroom, and managing noise levels. As a result, some teachers did not give the students clear behavior expectations and did not anticipate the initial chaos that ensued. In addition, some of the teachers struggled dividing their attention among the eight teams during the HPT episodes. Another expectation that did not align was that some of the teachers seemed to consider that group work that was autonomous and did not require teacher support. During the early episodes, some teachers worked at their desks and did not consider their support role as students worked in teams. As teachers learned in later episodes what their role in supporting students would entail, the students’ stayed engaged consistently.
Role of Instructional Coaches

This research study shows that HPT has many variables that need to be present for successful enactment. Instructional coaches can help teachers conceptualize what variables need to be present, such as tactics that allow for teachers to monitor eight teams at a time and more intense training of specific HPT roles. Coaches can also help guide teachers toward choosing appropriate problems and determining which student teams need what kinds of supports.

This study investigates the idea that high levels of Math-Talk could be present when HPT is enacted. However, it is possible that HPT is not necessary for students to engage in Math-Talk. Rather, it may be that the development of sociomathematical norms has a greater role in influencing Math-Talk. Further research could investigate the other variables that need to be present to develop sociomathematical norms that contribute to high levels of Math-Talk.

Limitations

The limitations of this study involve other variables that were not included in the study that may have been mediating the levels of Math-Talk. For example, the conceptual framework for this study (see Figure 2.1) indicates that certain sociomathematical norms need to be present in order for students to have high levels of Math-Talk. However, these sociomathematical norms were not always present. For example, during the post sociomathematical norm observation for Team A and B, the teacher gave a lesson that allowed for little to no student discussion. Another example was when there were class
and school interruptions. Whenever a member of the office staff called into a classroom to ask the teacher a question or request that a student come the office, many students in the class began talking about topics not related to the mathematics problem, and the teacher needed to visit teams in order to get them engaged again and could not attend the students’ mathematical learning during that time.

As with all studies, there were limitations that affected the generalizability of these results. This study used qualitative methods, the results of which are not as generalizable as other methods. All data was collected during a seven-week period. Had this study been a longitudinal study, the results may have been able predicted larger trends. Another limitation was that the population of the students at the school at which this study was conducted was a homogenous population and may not be generalizable to a more diverse population.

**Implications and Future Research**

The results of this study bring forth the questions “What can educators do to help students feel more independent when they discuss mathematics?” and “How can educators provide an environment in which all students, regardless of level of mathematics understanding and language abilities, would be able to participate in meaningful student-to-student mathematics conversations?” Some of the participating students in this study readily accepted the sociomathematical norms that lead to productive student-to-student discussion, and some students needed more time and support from their teacher. Some preliminary connections were found between teacher
interventions while students were working in HPT and higher levels of Math-Talk. This study may give guidance to practitioners who want to create an environment conducive to deep student-to-student mathematics conversations; however, a more refined method than was presented in this study might lead to greater depth more quickly. Since student teams require varying types and intensities of support, teachers need guidance and practice regarding how to give support more efficiently and with more intensity. Further research could also address how various student groups react to HPT. Those student groups could include those who are classified with special needs, group members that are the same or different genders, or students who speak non-English languages in their homes. Another variable that could be explored is the willingness of the teachers to allow for more interactive direct instruction as opposed to some of the teachers in this study who requested that their students not interrupt unless it was important.

**Conclusion**

Attending to the HPT philosophies of participation and engagement leads to sociomathematical norms that allow for participation in meaningful student-to-student mathematics conversations. It is possible to create a classroom environment that is conducive to these sociomathematical norms. It does, however, require training of the students and teachers, such as giving students clear expectations and helping teachers discover what supports each student team needs. Students needed to be trained in some type of structure that allows (1) all students to participate, (2) ensures that the teams have the appropriate skills for the problems they are assigned to solve, and (3) gives
accountability to the student. Specifically, when teachers were able to engage in two teacher moves, their students were able to increase their Math-Talk levels. First, the teacher needed to choose problems that matched the skills of the team. Second, the teachers needed to observe the dynamics of the teams to ensure that the students were given an opportunity to participate and received appropriate support when they needed it. HPT allowed for student and teacher training that accomplished those objectives. Further, HPT gives a structure for teachers to follow in which sociomathematical norms can evolve so that students can participate in deep mathematical conversation with each other (Kazemi & Stipek, 2001; Lopez & Allal, 2007; Yackel, 2001).
REFERENCES


Sfard, A. (2001). There is more to discourse than meets the ears: Learning from mathematical communication things that we have not known before. *Educational Studies in Mathematics, 46*(1/3), 13-57.


APPENDICES
Appendix A

Semistructured Pre-Post Interview Protocol for Teachers
Researcher: Thank you for participating in this research study regarding student-to-student conversations in math class. This session will be video recorded so that I can refer back to this interview when I am doing analysis on this research project. I would like to get your impressions of the levels of your students’ math discussions both before and after we teach them some of the aspects of High-Performance Team Building. Before we begin, do you have any questions?

1. How often do your students participate in math discussions?
   a. During those discussions, is the discourse conducted by you, the teacher, or do the students initiate the conversations with each other?
   b. What strategies in keeping the students on task and engaged in the math discussions have you found success with?

2. When your students engage in conversations about math, are their discussions more shallow or deep? Shallow means check for the correct answer or procedures, and deep means explaining or arguing about math concepts.

3. When students tell you that they would rather work alone than in groups, do you encourage them to work with their classmates or do you quickly agree to let them work how they want?

Researcher: Do you have any closing comments that you would like to share about high-performance teams and how they relate to your students?
Appendix B

Semistructured Pre-Post Interview Protocol for Students
Researcher: Thank you for participating in this research study regarding talking about math during math class. This session will be video recorded so that I can refer back to this interview when I am doing analysis on this research project. I would like to get your impressions on working in teams with your classmates and talking about the math you are working on. Before we begin, do you have any questions?

1. How often did you participate in math discussions last year, when you were in 5th grade?
   a. When you worked in teams, did everyone talk an equal amount or did some students talk more than others?
   b. What would members of the group do when there was a disagreement?
   c. Were the disagreements about math or about other topics not related to math?

2. When you are working in teams, how do you know if what you say will help the group solve the problem?

3. Can you give an example of something that you could say that would help the group solve the problem and understand math better?

4. Can you give an example of a comment that someone could say that would make it so that others don’t learn as much?

Researcher: Do you have any comments that you would like to tell me about working in teams during math class and talking about solving math problems?
Appendix C

Protocol for Training Teachers in High Performance Teams
Three Components of High Performance Teams

Each group member has distinct role (Classroom teachers train students on this):
- Explain the context of a problem (see handout one)
- Make a graphic representation of the mathematics needed to solve a problem (see handout two)
- Know procedures needed to solve a problem efficiently (see handout two)
- Check for accuracy (see handout three)

Members are held accountable and receive specific feedback regularly
(Researcher trains teachers how to do this). Teachers may find this form useful:

<table>
<thead>
<tr>
<th>Role</th>
<th>Student name</th>
<th>What student did</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw a picture of what the problem is asking</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Give graphic representation of math involved</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do the procedures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check for mistakes</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Access to challenging problems (Researcher and/or teacher may choose the problems):
- Have a context that is sufficiently complex so that drawing a diagram of what the question is asking seems reasonable,
- Contain mathematics that is on the 6th grade level,
- Be able to be solved with a procedure that the students already know or one that the students have the background knowledge to construct,
- Allow for multiple entry points and varied solution strategies and promotes reasoning and problem solving, and
- Have a high level of cognitive demand

Timeline

Phase One: Interview teachers, select student groups, interview student groups, observe the level of autonomy of student groups during class, train on HPT.
**Phase Two:** Observe 4 student groups solving problems 8 times each (32 in total).

**Phase Three:** Interview teachers and student groups again. Observe the level of autonomy of students groups during math class again.

<table>
<thead>
<tr>
<th>Student Role in High-Performance Teams</th>
<th>Learning Objective of Instructional Episodes</th>
<th>6th Grade CCSS Standard</th>
<th>Curriculum Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw a pictorial representation that represents the context of a problem</td>
<td>Students learn to draw pictures representing contexts of problems</td>
<td>6.RP.3 Using ratio reasoning to convert measurement units</td>
<td></td>
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<tr>
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<td></td>
<td></td>
<td><img src="Learnzillion.png" alt="Image" /></td>
</tr>
<tr>
<td>Draw a pictorial representation to reflect mathematics discussed</td>
<td>Students learn how models can reflect mathematics thinking</td>
<td>6.NS.1 Division of fractions by fractions</td>
<td><img src="otra.jpg" alt="Image" /></td>
</tr>
<tr>
<td>Perform mathematical procedures</td>
<td>Students learn procedures to solve mathematical problems</td>
<td>6.RP.3 Using ratio reasoning to convert measurement units</td>
<td><img src="otra.jpg" alt="Image" /></td>
</tr>
<tr>
<td>Proof all group mathematical work for errors</td>
<td>Students will practice finding errors in fraction simplification and multiplication problems which have deliberate errors</td>
<td>6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities</td>
<td><img src="otra.jpg" alt="Image" /></td>
</tr>
</tbody>
</table>

In this sample, the correct method and answer are provided. In many of the problems, the students will be given work with errors.
Appendix D

Challenging Problems Students Solved While Working in High Performance Teams
Note: These problems are a sample of what a mathematical tasks look like. The tasks that teachers chose were dependent on the timing as they related to the school district's scope and sequence.

Maria baked a loaf of bread. She used $\frac{3}{4}$ cups of flour. She has $\frac{3}{2}$ cups of flour left. How much flour did Maria have to start out with? Show how to find the answer at least two different ways.

(Dixon et al., 2012).
In the Playground
This problem gives you the chance to:
• work with areas

The playground committee decides to make a sandbox area for toddlers. For safety reasons, the sandbox must be surrounded by a strip of rubber matting that is 2 feet wide.

Here is a scale drawing of the sandbox.

1. Find the area of the sandbox and the area of the rubber matting.
   Sandbox area: __________ square feet          Rubber matting area: __________ square feet

More children are using the playground, so the committee decides to double the area of the sandbox.

2. Design a new rectangular sandbox that has double the area of the original sandbox.
   On the grid below, make a scale drawing of the new sandbox and the surrounding rubber matting.

3. How many square feet of rubber matting will they need? ______________ square feet

4. What is the length and width of the new sandbox? length __________ feet

(Mathematics Assessment Project, 2018)
Trapezoids

Directions: Choose 3-5 trapezoids below that you want to work with. Using tracing paper and different colored paper, transform the trapezoids into shapes you know how to find the area of, rectangles, triangles and parallelograms. Then, answer the question at the bottom of the page.

Question: What rule(s) can you use to find the area of a trapezoid?

(Adapted from Hindle, 2018)
Appendix E

Sociomathematical Norm Observation Tool
<table>
<thead>
<tr>
<th>No autonomy Level 0</th>
<th>Indicate nonunderstanding</th>
<th>Explain and justify reasoning</th>
<th>Listen to and attempt to understand others' explanations</th>
<th>Indicate when solutions are valid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students give short answers and respond to the teacher only. No student-to-student talk.</td>
<td>No student thinking or strategy-focused explanation of work. Only answers are given.</td>
<td>Students respond to math presented by the teacher. They do not offer their own math ideas.</td>
<td>Students attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves.</td>
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</tbody>
</table>

| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

<table>
<thead>
<tr>
<th>Little autonomy Level 1</th>
<th>Indicate nonunderstanding</th>
<th>Explain and justify reasoning</th>
<th>Listen to and attempt to understand others' explanations</th>
<th>Indicate when solutions are valid</th>
</tr>
</thead>
<tbody>
<tr>
<td>As a student answers a question, other students listen passively or wait for their turn.</td>
<td>Students give information about their math thinking, usually as it is probed by the teacher.</td>
<td>Some student ideas are raised in discussions, but are not explored.</td>
<td>Students help each other mostly by showing how they solved a problem.</td>
<td></td>
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</tbody>
</table>

| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

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| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |

<p>| Time: Notes: | Time: Notes: | Time: Notes: | Time: Notes: |</p>
<table>
<thead>
<tr>
<th>Students ask questions of one another's work on the board, often at the prompting of the teacher.</th>
<th>Students take a position and articulate in response to probes while students listen supportively.</th>
<th>Student ideas sometimes guide the direction of the math lesson.</th>
<th>When the teacher requests, students explain other students' ideas in their own words, clarifying other students' ideas for themselves and others.</th>
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<tbody>
<tr>
<td>Time:</td>
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</table>

<table>
<thead>
<tr>
<th>Student-to-student talk is student-initiated, not dependent on the teacher. Many are &quot;Why?&quot; questions.</th>
<th>Students describe more complete strategies and defend their answers with little prompting from the teacher.</th>
<th>Students spontaneously compare, contrast and build on ideas and form part of the content of many math lessons.</th>
<th>Students listen to, understand, then initiate clarifying other students' work and ideas for themselves and for others.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time:</td>
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Appendix F

Individual Accountability for Students Working in High Performance Teams
Note: The teacher may fill this observation sheet out independently of the students or may ask the students to fill it out as they complete the tasks, depending on the amount of autonomy the classroom teacher decides to give the students.

<table>
<thead>
<tr>
<th>Role</th>
<th>Student name</th>
<th>What student did</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
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<td>Draw a picture of what the problem is asking</td>
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<tr>
<td>Check for mistakes</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CURRICULUM VITA

MELANIE VALENTINE DURFEE

Business Address:       Home Address:
Cedar Middle School       25 O Street #2
250 W. 500 S        Salt Lake City, UT 84103
Salt Lake City, UT  84111       (435) 592-5704
(801) 538-7719
Email: melanie.durfee@schools.utah.gov

EDUCATION

Ph.D.  2018
Curriculum and Instruction, Emphasis in Math Education and Leadership, Utah State
University Logan, UT

Admin License

M.A.Ed.  2003
MED Learning and Technology, Western Governors University

Certificate  1991
Secondary English, Math, University of Texas at Dallas
Dallas, TX

B.A.  May 1986
English, Brigham Young University
Provo, UT

LEADERSHIP AND TEACHING

Employment:
Digital Teaching and Learning Achievement Specialist (August, 2018 to present)
Utah State Board of Education
Salt Lake City, UT
Responsibilities include working directly with districts and charter schools to implement their personalized
digital teaching and learning plans, overseeing the development of professional learning communities
within the Digital Teaching and Learning community to help foster the spread of best practices.

Employment:
Mathematics Instructional Coach, Grade 6 (2016 to May, 2018)
Iron County School District, Cedar Middle School
Cedar City, UT
Responsibilities include leading mathematics professional learning communities, consulting with teachers
regarding remediation practices, conducting mathematics support groups for 6th grade students.
Professional Development Course Facilitator, Grade 7 (2016 to present)
Utah State Board of Education
Salt Lake City, UT
Responsibilities include facilitating USBE-created courses to districts throughout rural Utah

Secondary School Math Teacher, Grade 7 (2013 to 2016)
Iron County School District, Cedar Middle School
Cedar City, UT
Responsibilities include planning and teaching 7th grade math and co-teaching 7th grade math with a certified special education teacher and working collaboratively with academic and grade-level teams to coordinate remediation and enrichment for students.

Iron County School District, Cedar High School
Cedar City, UT
Responsibilities included all aspects of running a high school theatre program (producing several major productions yearly, coaching students as they prepared plays for publication, training all technical personnel, instructing students on sewing and prop making), teaching English Common Core (11th and 12th-grade), and teaching Computer Technology (including piloting the Microsoft Office Certification programs).

Technology Trainer (1999 to 2005)
Southwest Educational Development Center
Cedar City, UT
Responsibilities included giving technology integration presentations in Beaver, Kane, Garfield, Iron, and Washington counties using Word, Excel, PowerPoint, Dreamweaver, Photoshop, Pioneer Library and UEN on-line tools; co-teaching with classroom teachers in south west Utah to model technology integration in a classroom setting; and presenting technology integration with K12 core subjects at state and national conferences.

Secondary School Teacher, Grade 10, 11, 12 (1991 to 1997)
Jordan School District, Brighton High School
Salt Lake City, UT
Responsibilities included teaching 10th and 11th-grade English, journalism, and coaching the school newspaper staff, The Barb.

Endorsements:
- Mathematics Level 4 (highest K-12 certification), Utah State University, May 2013
- Secondary Theatre (6-12), Utah State Office of Education, May 2008
- Computer Technology, IC3, Exam, May 2006
- Web Development, Exam, May 2003

PRESENTATIONS

Durfee, M., Durfee, R. (2016). How to Get the History Teacher to Teach Math. MidSchool Math, Santa Fe., NM.


RESEARCH

Publications:
Are they using the data? Teacher perceptions of, practices with, and preparation to use assessment data. *International Journal of Education*, 8(3), 50


“Making math magic in the classroom: Lessons from improv” (Spring 2016). *Journal of the Utah Council of Teachers of Mathematics*.

“Promoting a social math class” (under review)

Research Interests:
- Understanding the relationships between conceptual understanding and procedural fluency in middle grade mathematics students
- Creating curriculum and establishing pedagogy that utilizes the existing social structures in classrooms to motivate middle grade students to achieve higher in math.
- Creating curriculum and establishing pedagogy that promotes social justice in mathematics

Research Assistant:
*Active Learning Lab*, Utah State University (July, 2015 to present)
Compiling data for STEM Action Center Technology Grants

*The Utah Middle School Math Project* (September, 2013 to present)
Contributed to 6th grade math textbook, chapter 2
Piloted 7th grade math textbook, edited Chapter 4 of 7th grade text, compiled technology links for all 7th and 8th grade chapters, compiled research regarding usage of textbook throughout the nation, drafted chapters 1 and 2 of 6th grade textbook.

SERVICE

*American Institutes for Research* (June, July, 2014)
Assisted in reviewing 7th grade SAGE math rubric and wrote test items for 7th grade SAGE math test

Southern Utah University Preservice Teachers (January – April, 2015)
Created and coordinated the *Iron Schools Mindset Math* Program in which preservice teachers gave a series of mini-lessons regarding mathematics mindset to secondary mathematics students in Iron County

Referee:
NCTM’s *Journal of Research in Mathematics Education* - journal referee – January 2015 to present

Webmaster:
Utah Theatre Association

Professional and Academic Association Memberships:
- National Council of Teachers of Mathematics
- Utah Council of Teachers of Mathematics