SECONDARY MATHEMATICS TEACHERS' RESPONSES

TO PIVOTAL TEACHING MOMENTS

by

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ABSTRACT

Secondary Mathematics Teachers' Responses to Pivotal Teaching Moments

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This study used a multiple case study design to explore the occurrence of pivotal teaching moments, teachers' responses to these moments, and teachers' own perceptions of the impact of these moments on their own knowledge development. The participants in this study were six practicing secondary mathematics teachers. The researcher collected data from teacher created lesson plan outlines, observations of the same lesson delivered to two different classes, semi-structured participant interviews, and teacher reflection journals. The procedures included evaluating the lesson plan outlines to ascertain what teachers anticipated as part of their lesson delivery and observing an initial delivery of a mathematics lesson to one class and a subsequent delivery of the same lesson to a second class. During these observations, the researcher recorded observed pivotal teaching moments and corresponding teacher responses to these moments. The researcher also noted observed instructional changes between the two observed lessons. The procedures also included interviews, conducted during the same school visit, to allow the researcher to identify in-the-moment teacher thinking and teachers' motivations for

their responses to pivotal teaching moments. Finally, the researcher used ongoing teacher reflections to deepen and refine insights related to teachers' classroom actions and learning.

The results confirmed and built upon existing classifications of pivotal teaching moments and teachers' responses to them, while also identifying seven themes related to teacher motivations for their responses. Teacher perceptions of changes in their pedagogical content knowledge were manifest in changes made to observed lessons, teacher statements about future anticipated changes to lessons, and general comments about their own in-the-moment learning. Teachers' perceptions of changes in their content knowledge aligned to shifts in their understanding of mathematical definitions and mathematical structures. Overall, results suggest a relationship between pivotal teaching moments and teacher knowledge development. However, more research is needed to explore how pivotal teaching moments are created, how individual teacher-student interactions shape teacher knowledge development, and to examine the role of teachers' reflections about their practice in their knowledge development.

(143 pages)

PUBLIC ABSTRACT

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Kami M. Dupree

This study used a multiple case study design to explore the occurrence of pivotal teaching moments, teachers' responses to these moments, and teachers' own perceptions of the impact of these moments on their own knowledge development. The participants were six practicing secondary mathematics teachers. The researcher collected data from teacher created lesson plan outlines, observations of the same lesson delivered to two different classes, participant interviews, and teacher reflection journals. The researcher reviewed the lesson plan outlines prior to observations to understand teachers' anticipations. During observations, the researcher recorded observed pivotal teaching moments, corresponding teacher responses to these moments, and instructional changes between the two observed lessons. Interviews allowed the researcher to identify in-the-moment teacher thinking and teachers' motivations for their responses. Teacher reflection journals provided insights related to teachers' classroom actions and learning.

The results confirmed and built upon existing classifications of pivotal teaching moments and teachers' responses, while also identifying seven themes related to teacher motivations for their responses. Teachers' perceptions of changes in their own knowledge base occurred for their content knowledge as well as their pedagogical content knowledge. Future research should explore how pivotal teaching moments are created, how teacher-student interactions shape teacher knowledge development, and examine the role of teachers' reflections about their practice in their knowledge development.

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Kami M. Dupree

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CHAPTER I

INTRODUCTION

Success in a career field requires knowledge that is shared with other disciplines and yet unique to the specific career. For example, while both the surgeon and the biologist share knowledge about human anatomy, the knowledge required to perform medical surgery is vastly different from the knowledge a biologist uses to explore human reactions to stimuli. Considering that much of the knowledge possessed by both the surgeon and the biologist originates in a classroom, it becomes evident that the knowledge required for *teaching* human anatomy differs even more markedly from that of the surgeon or the biologist. The use of a medical analogy here does not preclude the truth of similar analogies in other disciplines. In truth, although all professional knowledge is itself specialized, the knowledge necessary for teaching is more specialized because teachers must simultaneously possess content knowledge and knowledge pertaining to how to help others learn and attain similar content knowledge.

To provide high quality preparation and ongoing professional development for teachers, it is important to understand how teachers' own knowledge develops. Further, because teachers' practice their trade in the context of active classrooms and in relation to unanticipated student questions and misunderstandings, it is important to understand the impact of in-the-moment circumstances and events on teachers' knowledge development. The primary focus of this study was to explore how teacher knowledge develops in relation to the complexities associated with the ever-changing dynamics of a classroom.

Background of the Problem

In the late twentieth century, Shulman (1986) introduced the construct of *pedagogical content knowledge*, prompting educational researchers to examine both "what" teachers know and "how" they know what they know. While society has always expected teachers to possess a certain level of knowledge, there has historically been a difference of opinion as to what that knowledge looks like, what it should include, and which aspects are most important to the work of teaching (Hill, Sleep, Lewis, & Ball, 2007; Shulman, 1986).

Near the end of the twentieth century, Shulman (1986) lamented that evaluation of teachers had shifted from focusing on what they knew about what they were teaching (content knowledge) to what they knew about teaching in general (pedagogical knowledge). Shulman argued the existence of a "blind spot with respect to content" among the educational research community of his time and urged a focus on the *content* of teachers' instructional lessons and efforts toward understanding where and how teachers learn what kinds of questions to ask, what kinds of representations to use, and how to deal with student misunderstandings.

In the ensuing years, many have attempted to answer Shulman's call to action by offering their own conceptualizations of teacher knowledge (Ball, Thames, & Phelps, 2008; Fennema & Franke, 1992; Rowland, 2005; Rowland & Turner, 2007). Each conceptualization uses unique terminology to describe facets of teacher knowledge, but researchers generally agree that teacher knowledge is comprised of two unique components. First, teachers must have the ability to transform what they know about content into a form that allows students to learn the content, and second, anticipating and

addressing student thinking related to the content and possible misunderstandings are key aspects of the knowledge teachers must demonstrate in their work. To use Shulman's (1986) terms, to achieve the highest levels of teacher effectiveness, teachers' *subject matter knowledge* and *pedagogical content knowledge* must coexist in complementary ways.

In thinking about knowledge and learning, theorists espousing a constructivist view have argued the importance of experience as a source of learning (Dewey, 1998; Kolb, 1984), noting that internal reflection and conceptualization of concrete experiences leads to continuous cycles of learning and knowledge growth. Other scholars have highlighted the important role of reflection in the development of professional knowledge, suggesting that a professional's internal reflections about her responses to concrete experiences and the subsequent outcomes of those responses lead the professional to add to, or eliminate response strategies from her professional repertoire. (Schön, 1984).

In fact, many acknowledge the complex environments in which teaching occurs and consequently embrace a situated view of teacher knowledge development. Such a view suggests that at least portions of teacher knowledge exist and develop within the classroom (Ball et al., 2008; Fennema & Franke, 1992; Rowland, 2005; Rowland & Turner, 2007; Schön, 1984). For researchers adopting this perspective, aspects of teacher knowledge evolve during the practice of teaching, making teaching simultaneously an act of knowledge demonstration and knowledge creation.

Previous efforts to understand teachers' knowledge have largely focused on teaching generally (as opposed to subject-specific study) (Shulman, 1986), or, in mathematics education research, have overwhelmingly examined elementary and/or preservice teachers (Ball et al., 2008; Rowland, 2005; Rowland & Turner, 2007). There are at least three problems with such approaches. First, although it is possible to glean some understanding of teacher knowledge through a general treatment across subjects, the knowledge required to teach different subject matter is necessarily unique. Thus, in an effort to understand the development of teachers' knowledge for teaching mathematics, one may draw on general principles of teachers' knowledge but must also stay grounded in knowledge of and about mathematics. Second, while some measures of effective teaching are consistent across grade levels, the mathematical knowledge held by teachers in elementary grades, where teachers act as content generalists, differs from that of mathematics teachers in secondary grades, because secondary teachers generally have more preparation in mathematics. Finally, studying the knowledge of preservice teachers is unlikely to account for the situated nature of teaching mathematics because preservice teachers typically have little to no prolonged experience in actual classrooms. Stated more succinctly, teachers who have previously taught particular mathematics content will exhibit different moves in the classroom than teachers who have not (Berliner, 1988).

Statement of the Problem

Little research exists exploring how specialized mathematics knowledge develops among practicing mathematics teachers during in-the-moment interactions in secondary mathematics classrooms. There exists a sizeable research base exploring and elaborating

upon the construct of mathematics teachers' pedagogical content knowledge (Depaepe, Verschaffel, & Kelchtermans, 2013) and the development of expertise in the face of professional practice (Dreyfus & Dreyfus, 1986; Schön, 1984). Additionally, research in the sciences has begun to explore how the development of pedagogical content knowledge is shaped by in-the-moment teaching experiences (Alonzo, Kobarg, & Seidel, 2012; Cayton, Hollebrands, Okumus, & Boehm, 2017; Chan & Yung, 2015; Stockero and Van Zoest, 2013). Recently, researchers have also identified and classified *pivotal teaching moments*, or moments when "interruptions in the flow of a lesson provide an opportunity to modify instruction to improve students' mathematical understanding" (Stockero and Van Zoest, 2013, p. 127). Still, despite wide acknowledgement of content knowledge and pedagogical content knowledge as specialized components of teachers' knowledge, little research exists exploring how these types of knowledge develop during in-the-moment interactions in secondary mathematics classrooms.

Significance of the Study

This study is significant because it addresses two major gaps in the current literature. First, this study responds to Sowder's (2007) call for a more comprehensive understanding of how teachers' pedagogical content knowledge develops. Second, this study addresses Arbaugh's (2010) suggestion for research providing insight into the professional learning and knowledge growth of secondary mathematics teachers.

By focusing on teachers' knowledge development in the moment, this study is significant to mathematics education research in at least three ways. First, understanding how in-the-moment classroom interactions relate to teacher knowledge development can

help researchers understand which elements of mathematics teachers' knowledge are teachable in traditional teacher preparation programs and which require more direct and authentic classroom teaching experiences.

Second, providing teachers with such experiences requires understanding the "trajectory of teachers' evolution of their competencies" (Arbaugh, 2010, p. 19). This study explored the evolution of content knowledge and pedagogical content knowledge in response to pivotal teaching moments, or classroom events that force teachers to think on their feet. Understanding how pivotal teaching moments relate to teacher knowledge growth may allow those responsible for teacher preparation programs to more effectively plan and provide opportunities for preservice teachers to experience and respond to pivotal teaching moments prior to entering the classroom (Taylan & da Ponte, 2016).

Finally, existing literature has helped inform the research community regarding how to structure elementary preservice teacher preparation to improve mathematical content knowledge for teaching. This study adds to the knowledge base regarding how to improve similar preservice programs and the ongoing preparation of secondary mathematics teachers.

Study Purpose and Research Questions

The purpose of this study was to explore how secondary mathematics teachers responded to pivotal teaching moments (PTMs) and how the teachers perceived that their own content and pedagogical content knowledge was related to PTMs and their responses to them. To accomplish this purpose, this study answered the following research questions:

1. How do six practicing secondary mathematics teachers respond to pivotal teaching moments?
2. How, if at all, do six practicing secondary mathematics teachers perceive that their own knowledge development relates to pivotal teaching moments and their responses to them?
3. What are the similarities and differences among six secondary mathematics teachers' with varying years of teaching experience in their (a) responses to pivotal teaching moments and (b) perceptions of how their responses and knowledge development are related?

Summary of Research Design

The researcher conducted this study using an exploratory multiple case study design (Creswell, 2013; Yin, 2014). Because the literature has only recently addressed how teacher knowledge develops in the moment, this study sought only to explore, rather than to explain, the phenomena of teachers' responses to pivotal teaching moments. A case study approach allowed for a more detailed exploration of the teacher-student interactions occurring in the classroom. The use of multiple cases allowed for examination of teacher knowledge development from multiple perspectives and identification of common themes within and across cases.

Assumptions

A few assumptions were key to the design of this study. First, for the purposes of this study, the researcher assumed that those who met the qualifications to teach

secondary mathematics in the state in which they were practicing possessed an acceptable level of mathematics content knowledge for inclusion. Second, the researcher assumed that the interview responses and reflections of teachers accurately captured the essence of a teacher's actual thoughts and perceptions about classroom experiences.

Delimitations

In an effort to narrow the focus of this study, the researcher put certain constraints in place. First, while understanding the impact of pivotal teaching moments on aspects of teaching, such as teachers' choice of instructional strategies, representations, or questioning strategies, may be of interest, the scope of this study was limited to exploring the impact of pivotal teaching moments on teachers' knowledge development. Second, many studies focus more formally on the nature and development of teachers' mathematics content knowledge. Such studies are important in fully understanding teacher knowledge, but in this study, the researcher limited examination of teacher knowledge development to teachers' own perceptions of their knowledge development. Third, instructional strategies employed by teachers varied widely, with some teachers relying on multiple strategies during a single lesson. To help ensure quality over quantity in identifying pivotal teaching moments and subsequent teacher responses, the researcher limited her focus to identifying pivotal teaching moments during portions of a lesson in which the teacher was engaging with the entire class, or intentionally created smaller groups of students, as opposed to when the teacher was interacting with individual students.

Summary

Existing research on teacher knowledge has led many to theorize about the nature of teacher knowledge and to explore its unique characteristics. Shulman's (1986) construct of pedagogical content knowledge has become a widely accepted form of knowledge unique to teaching. Both constructivist and experiential learning theorists argue that individual experiences are critical to learning and knowledge acquisition (Dewey, 1998; Kolb, 1984). Inasmuch as a teacher is simultaneously teaching and learning, exploring the knowledge development of teachers requires consideration of the role pivotal teaching moments may play in their knowledge development.

This study examined how six secondary mathematics teachers responded to pivotal teaching moments and how such moments related to teachers' perceptions of their own content knowledge and pedagogical content knowledge development. The focus of the data collection was to identify teacher responses to pivotal teaching moments and to solicit teachers' perspectives about both their responses in such moments, and their perceptions of how such moments modified, refined, or changed their own content knowledge and pedagogical content knowledge. The results of this study may inform educational leaders regarding how to best support preservice and practicing teachers in their knowledge development and growth.

Definition of Terms

Pivotal Teaching Moment: A moment during a classroom lesson in which the flow of the lesson is interrupted, providing the teacher an opportunity to "modify instruction

in order to extend or change the nature of students' mathematical understanding" (Stockero & Van Zoest, 2013, p. 127).

Mathematics Content Knowledge: Knowledge of the concepts and underlying structures of the mathematics typically comprised within the secondary mathematics curriculum. Secondary mathematics teachers generally acquire this knowledge throughout their experiences as students (Rowland & Turner, 2007). Stated more broadly, the mathematics that is known by a teacher, including her own understandings of mathematics generally and her knowledge of the "why's" associated with the mathematics (Shulman, 1986).

Mathematical Pedagogical Content Knowledge: Knowledge evidenced by teachers' ability to (a) represent mathematics in multiple ways, (b) respond to pivotal teaching moments quickly and in mathematically accurate ways, and (c) refine and alter future teaching to better facilitate student learning of mathematics (Shulman, 1986).

Teacher Knowledge Development: Collective term simultaneously referring to the development of teachers' mathematics content knowledge and mathematical pedagogical content knowledge (Rowland & Turner, 2007; Shulman, 1986).

CHAPTER II

LITERATURE REVIEW

The purpose of this study was to explore how secondary mathematics teachers responded to pivotal teaching moments (PTMs) and how the teachers perceived that their own content and pedagogical content knowledge was related to PTMs and their responses to them. The literature reviewed and summarized in this chapter provides insight into present understandings about teacher knowledge generally, the development of teacher knowledge during in-the-moment interactions between students and teachers, and the identification of PTMs and teacher responses to them.

This chapter consists of five major sections. The first section offers a brief history of the original introduction of the construct of pedagogical content knowledge (PCK) and addresses two of the most common criticisms of Shulman's (1986) original articulation of PCK. The second section presents relevant components of three other conceptualizations of teacher knowledge most often referenced in the mathematics education literature. The main purpose of this summary is to highlight that, although researchers have used different terms in describing the components of teacher knowledge, all have included components that closely mirror that of PCK. The third section includes a detailed discussion of Stockero and Van Zoest's (2013) articulation of pivotal teaching moments, as well as a summary of research on the development of PCK in the moment. The fourth section summarizes theoretical perspectives on the relationship between experience, reflection, and learning that help inform how teacher knowledge may develop

in the act of teaching. The final section explains the conceptual framework guiding this study.

History and Criticisms of Pedagogical Content Knowledge

Near the end of the twentieth century, as people became increasingly concerned with teacher quality and the knowledge teachers need to be effective, Shulman (1986) lamented that the pendulum had swung from a focus on teacher knowledge of content to teacher knowledge of pedagogy. According to Shulman, the focus had shifted from focusing on what teachers knew about what they were teaching (content knowledge) to what they knew about teaching in general (pedagogical knowledge). Suggesting that the educational leaders of the time had a “blind spot with respect to content,” Shulman (1986) argued for a return toward focusing on the aspects of knowledge unique to teaching.

Shulman suggested that teacher knowledge is comprised of specialized knowledge in three domains. The first domain, *subject matter knowledge*, includes the “amount and organization of knowledge … in the mind of the teacher” (p. 9). This includes a teacher’s knowledge of “how a subject is” and develops as one learns *that* something is true, as well as *why* it is true. This type of knowledge is perhaps what we hope to instill in future teachers of mathematics by requiring completion of certain college-level mathematics courses seen as being critical to the development of mathematical knowledge. In this dissertation study, the term *mathematics content knowledge* refers to this type of knowledge.

Recognizing that merely knowing a subject is insufficient for teaching others to know the subject, Shulman (1986) introduced a second knowledge domain, *pedagogical content knowledge* (PCK), which he defined as a special kind of knowledge that combines knowledge of content with knowledge of how to represent the content and make it comprehensible to others. Shulman's articulation of PCK bridges the dichotomy of content knowledge and pedagogical knowledge by raising questions regarding how teachers know how to represent content, how to question students about it, and how to deal with student misunderstandings.

It is important to highlight that Shulman did not view PCK as being separate from content knowledge, nor separate from pedagogical knowledge. Instead, PCK is a "*particular form of content knowledge* that embodies the aspects of content most germane to its teachability" (p. 9, emphasis added). Thus, PCK is part of a teacher's content knowledge base, but also a part of her pedagogical knowledge base. PCK is thus manifest in the act of blending what one knows about the subject with helping others to know the subject, and any attempt to understand PCK development cannot be isolated from observing what teachers actually *do* during their instructional practice.

The third domain of teacher knowledge identified by Shulman (1986) was *curricular knowledge*, or knowledge of the tools and resources available for teaching the subject. This includes additional knowledge that allows a teacher to relate content in a given course or lesson to those being discussed in other courses (*lateral curriculum knowledge*) and to related content across years of study (*vertical curriculum knowledge*). While Shulman viewed this as a separate domain of teacher knowledge, this study adopts

the perspective that all such types of curricular knowledge are part of what allows teachers to exhibit particular responses in the face of pivotal teaching moments and consequently makes no distinction between curricular knowledge and pedagogical content knowledge.

Addressing Criticisms of the PCK Construct

Shulman's (1986) conceptualization of teacher knowledge has been widely referenced in the teacher knowledge literature. Ball et al. (2008) highlight that more than 1,200 refereed journal articles reference Shulman's work and Depaepe, Verschaffel, and Kelchtermans (2013) found 811 results in searching just three databases using *pedagogical content knowledge* and *mathematics* as keywords. Although the original articulation of PCK was not limited to the discipline of mathematics, the PCK construct has been widely accepted, and debated, in the mathematics education literature. Despite such embracement of the construct, however, PCK is not without its critics.

Arguing that the PCK construct fails to address the fully situated dynamic of teaching, Fennema and Franke (1992) were openly critical of PCK. They proposed that Shulman's articulation of PCK is too rigid to allow for consideration of the ever-evolving dynamic nature of teacher knowledge. However, it is interesting to note a significant degree of overlap between Fennema and Franke's (1992) knowledge domain termed *knowledge of students' cognitions* and Shulman's (1986) articulation of PCK. The similarities are discussed later in this chapter, so for now, suffice it to say that Shulman considered teacher knowledge from a theoretical, as opposed to practical, standpoint. Thus, even though Shulman did not discuss specific classroom dynamics in introducing

PCK, it is certainly possible to discuss and examine PCK within classrooms.

Another criticism of PCK came from Meredith (1995) who claimed that although the transformation of subject matter knowledge into forms to facilitate teaching is at the heart of PCK, PCK implies only a teacher-directed, didactic model of pedagogy. Her argument is that PCK fails to encompass a variety of teaching approaches such as group work, investigative activities, or problem solving. This criticism is easy to dismiss when one recognizes that the original articulation of PCK *includes* why teachers choose the representations (and presumably the activities) they do and thus does not "preclude teaching approaches" as Meredith claims. In fact, this researcher argues that PCK *is* what allows a teacher to determine whether a didactic, group, or investigative instructional technique is most suited to accomplishing the goals for learning. Thus, studying PCK includes looking at the instructional methodologies teachers use and how those methodologies may change as teachers develop PCK. In this dissertation study, the researcher observed teacher delivery of the same lesson to two different classes to explore the degree to which identified pivotal teaching moments in the first lesson led to shifts in teachers' instructional approaches during the second lesson. The researcher considered such lesson modifications from one class to another as indications of PCK development.

It is important to acknowledge the situated nature of classroom teaching. Teacher knowledge development necessarily occurs in the context of classrooms with fluid, rather than static, environmental factors. Researchers acknowledging this dynamic in their work (Ball et al., 2008; Rowland, 2005; Rowland & Turner, 2007) have typically

identified knowledge domains that are both similar to and different from those articulated by Shulman (1986). The next section highlights aspects of three additional conceptualizations of teacher knowledge that informed the goals of this study and that support the PCK construct.

Additional Conceptualizations of Teacher Knowledge

Since the introduction of the pedagogical content knowledge construct, many researchers have offered additional perspectives regarding the domains of teacher knowledge. Some of these views built on and supported Shulman's work (Ball, Thames, & Phelps, 2008), but others offered differing views on the domains of teacher knowledge (Fennema & Franke, 1992; Rowland, 2005). Within the realm of mathematics education research, three main views of teacher knowledge are highly referenced in the literature. This section summarizes key elements of each of these views, with an emphasis on demonstrating that despite the use of different language to describe aspects of teacher knowledge, researchers agree that both mathematics content knowledge and mathematical pedagogical content knowledge are necessary for effective mathematics teaching.

Fennema and Franke's Perspective

Embracing a situated perspective of mathematics teacher knowledge, Fennema and Franke (1992) propose that one cannot examine teacher knowledge adequately outside of its complex relation to classroom experiences, both planned and unplanned. Although Fennema and Franke's view of teacher knowledge is comprised of four

interacting domains (*knowledge of the content of mathematics, knowledge of students' cognitions, knowledge of pedagogy,* and *teacher beliefs*), only the first two of these domains fit within the scope of this study.

Knowledge of the content of mathematics. This domain includes the "concepts, procedures, and problem solving processes within the domain…*as well as in related content domains*" (Fennema & Franke, 1992, p. 162, emphasis added). Thus, although mathematics content knowledge is included here, Fennema and Franke (1992) add knowledge about how mathematics content connects with other domains to their conceptualization. This dissertation study considers knowledge that enables teachers to make connections to disciplines other than mathematics as part of teachers' mathematical pedagogical content knowledge. Further, as will be discussed later in this chapter, teachers who are able to connect mathematics topics to other disciplines are exhibiting one of the five key responses to pivotal teaching moments identified by Stockero and Van Zoest (2013).

Knowledge of students' cognitions. Included in this domain are both general and content specific knowledge related to how students learn (Fennema & Franke, 1992), including knowledge of where students will likely struggle as well as where they are likely to experience success. As mentioned earlier, Fennema and Franke opposed elements of Shulman's PCK construct, yet Shulman (1986) argued,

> "pedagogical content knowledge also includes an *understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them* to the learning of those most frequently taught topics and lessons" (Shulman, 1986, p. 9, emphasis added).

Thus, despite their disagreement with Shulman and consequent use of different terminology, this researcher is unable to see a distinction between Fennema and Franke's *knowledge of student cognitions* domain and Shulman's PCK. Consequently, this dissertation study considered evidence of teacher knowledge of how to manage content trouble spots for students as an indicator of mathematical pedagogical content knowledge.

Summary. Despite the use of different terminology, Fennema and Franke (1992) agreed that both mathematics content knowledge and mathematical pedagogical content knowledge are critical aspects of teacher knowledge. Further, at least part of teacher knowledge evolves during the practice of teaching.

> "Within a given context, teachers' knowledge of content interacts with knowledge of pedagogy and students' cognitions and combines with beliefs to create a unique set of knowledge that *drives classroom behavior*" (Fennema & Franke, 1992, p. 162, emphasis added).

In this way, teaching involves the creation of knowledge as well as the demonstration of knowledge, and teachers' knowledge develops within and because of classroom interactions with the subject matter and the students. This dissertation study attends directly to teachers' perceptions of their mathematical content knowledge and mathematical pedagogical content knowledge development by exploring how PCK develops in dynamic classroom environments.

Rowland and Turner's Perspective

Developed from the observation of lessons delivered by preservice teachers during their school-based placements, the Knowledge Quartet (Rowland, 2005) is a synthesis of 18 codes representing observed behaviors linked to content knowledge.

Rowland (2005) and Rowland and Turner (2007) proposed four main types of teacher knowledge: *foundation, transformation, connection,* and *contingency*.

Foundation knowledge. According to Rowland and Turner (2007), the knowledge base teachers possess by virtue of their own personal education and preparation becomes a form of foundational knowledge upon which they rely while teaching. The present study assumed some degree of consistency in foundational knowledge across teachers as part of their formal preparation in mathematics. At the same time, this dissertation study proposed that the degree to which teachers rely on this foundational knowledge during instruction may be manifest by differences in teacher responses to pivotal teaching moments.

Transformation knowledge. Meant to comprise a teacher's choice of examples, representational tools, and instructional demonstration, transformation knowledge looks at teacher behaviors directed toward students. It consists of the deliberate judgments and decisions made that, although possibly informed by foundation knowledge, offer insight into how a teacher transforms her knowledge into an accessible form. The present study treated this type of knowledge as part of teachers' mathematical pedagogical content knowledge.

Connection knowledge. The attempt to integrate either portions of a single lesson, or information across multiple lessons, constitutes connection knowledge. This category includes an anticipation of the complexity of the material, decisions about sequencing, and recognition of conceptual appropriateness in addition to making connections. Connection knowledge closely aligns to Shulman's curricular knowledge

and was considered part of teachers' mathematical pedagogical content knowledge in the present study.

Contingency knowledge. Recognizing the ever-changing nature of teaching associated with unexpected events, Rowland and Turner (2007) include deviation from the agenda, responding to student's ideas, and use of opportunities as part of contingency knowledge. This category is concerned with how teachers respond to classroom events that were unanticipated in the planning phases of a lesson. To some extent, contingency knowledge represents teachers' ability to "think on their feet" and adjust instructional directions accordingly. Thus, the present study supposed that those with deeper contingency knowledge would likely exhibit more effective responses to pivotal teaching moments.

Summary. As articulated by Rowland (2005) and Rowland and Turner (2007), teachers' foundation knowledge consists of the mathematical content knowledge acquired in their experiences *as* students, while transformation, connection, and contingency knowledge are knowledge types exhibited in working *with* students. The present study did not directly rely on the Knowledge Quartet to explain observations, but instead considered these knowledge types more broadly as mathematics content knowledge and mathematical pedagogical content knowledge.

Ball, Thames and Phelps' Perspective

Undoubtedly, the most comprehensive effort to explore the knowledge necessary for teaching in the field of mathematics has come from Ball et al. (2008) who proposed the Mathematical Knowledge for Teaching (MKT) framework shown in Figure 1.

Separating knowledge for teaching into two main domains (*subject matter knowledge* and *pedagogical content knowledge*), the MKT framework as a whole closely mirrors Shulman's (1986) original views regarding teacher knowledge. However, the MKT framework further informs the research community regarding teacher knowledge, by refining the aspects of subject matter knowledge and pedagogical content knowledge that teachers possess.

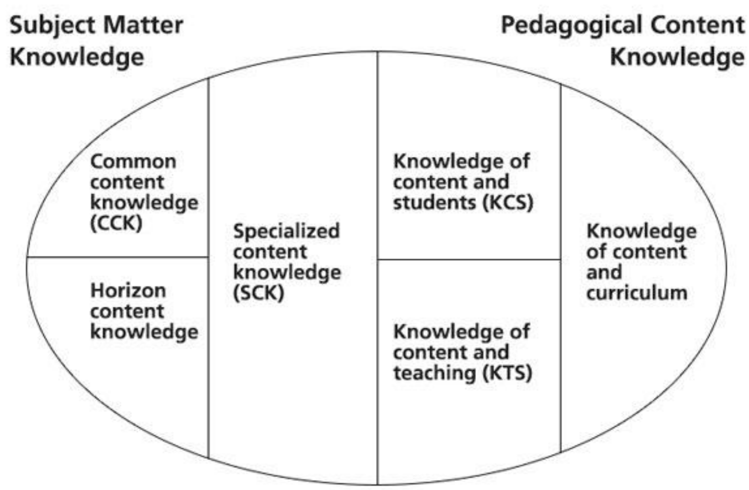


Figure 1. *Domains of Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008)*

In the interest of brevity, a detailed discussion of the subdomains of the MKT framework is not included here. However, it is important to point out that the subdomains of the MKT framework broadly encompass the knowledge types mentioned in the preceding conceptualizations of teacher knowledge. For example, despite the different subdomains of subject matter knowledge shown in Figure 1, *common content*

knowledge (CCK) and *specialized content knowledge* (SCK) both consist of what teachers know about mathematics, with the major difference being whether such content knowledge is shared by others outside of teaching (CCK) or unique to teaching itself (SCK). This dissertation study does not differentiate between, for example, the knowledge teachers need to be able to respond to a student's misunderstanding and the knowledge needed to represent the content in a comprehensible way. Instead, the present study considers both as evidence of teachers' mathematical pedagogical content knowledge.

The MKT model proposed by Ball et al. (2008) has been widely accepted in mathematics education research with documented research on each of the six domains. It would be difficult to ignore the influence of such a widely used model in explaining the knowledge necessary for teaching mathematics. However, it should be noted that this model was developed from work with elementary teachers and may be lacking in its ability to explain teacher knowledge outside of this context, a limitation acknowledged by Ball et al. and other researchers (Fauskanger, 2015; Speer, King, & Howell, 2015). The present study addresses gaps in the existing research base by focusing on teacher knowledge development in the secondary grades.

Summary

Knowledge of subject matter is a key aspect of each conceptualization of teacher knowledge presented above. Although the terminology used to describe subject matter knowledge differs, all researchers agree that teachers must be knowledgeable about the discipline of mathematics. This dissertation study refers to this type of knowledge as

mathematics content knowledge and the researcher assumes that in attaining a license to teach secondary mathematics, teachers possess a reasonable level of mathematics content knowledge, including a knowledge of the concepts and underlying structures of the mathematics typically taught in the secondary mathematics curriculum in the United States. In addition, the researcher assumes that attainment of this type of knowledge is requisite for licensure to teach mathematics and is thus a precursor to the development of mathematical pedagogical content knowledge (PCK). That is, one must first have a reasonable knowledge of mathematics before one can teach it effectively.

One challenge to exploring the development of pedagogical content knowledge (PCK) is that there exists no universally accepted definition of the construct (Hashweh, 2005; Van Driel, Verloop, and de Vos, 1998). However, in their evaluation of five varying conceptualizations of PCK (including that of Shulman), Van Driel, Verloop, and de Vos (1998) suggested a general agreement that PCK consists of two key elements: knowledge of how to represent the subject matter, and knowledge related to student difficulties, conceptions, and misconceptions related to the subject matter. This view is supported in subsequent research (Alonzo, Kobarg, and Seidel, 2012; Chan and Yung, 2015; Park and Oliver, 2008) and seems to be the basis of many present efforts to understand the development of PCK. This dissertation study espouses a similar view and, as shown in Figure 2, suggests that despite the use of different terminology in describing the facets of teacher knowledge, researchers have broadly described domains of teacher knowledge linked to mathematics content knowledge (those domains depicted

in bold) and mathematical pedagogical content knowledge (those domains depicted in italics).

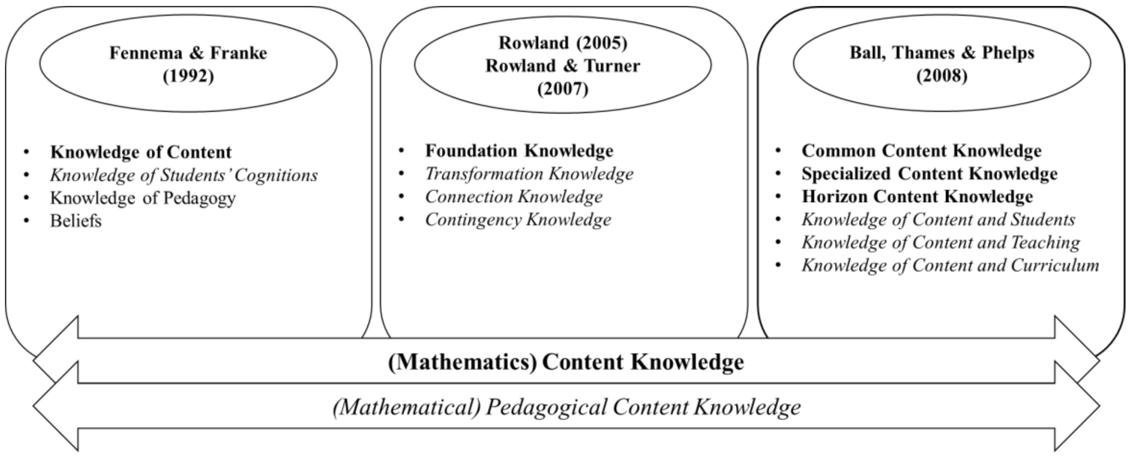


Figure 2. How articulated domains of teacher knowledge relate to content knowledge and pedagogical content knowledge.

Pivotal Teaching Moments and In-The-Moment PCK Development

Although the research base examining teacher knowledge is vast, the exploration of the development and exhibition of pedagogical content knowledge in the moment is in its infancy and much of the work in this area has taken place outside the field of mathematics. This section begins with a detailed summary of Stockero and Van Zoest's (2013) classification of pivotal teaching moments and teacher responses to them. This is followed by a more general discussion of what is known about in-the-moment teacher knowledge development.

Pivotal Teaching Moments and Teacher Responses to Them

As used in this dissertation study, and consistent with Stockero and Van Zoest (2013), a *pivotal teaching moment* (PTM) is defined as "an instance in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding" (p. 127). To respond effectively to a PTM, teachers must first recognize that a PTM has occurred and then respond to it in an instructionally appropriate way. However, as Stockero and Van Zoest point out, a PTM is constituted by the *opportunity* to respond rather than the response itself. This means that although teachers may respond to many PTMs in the course of a lesson, others may go unnoticed. Still, a PTM can be considered as having taken place, regardless of whether it is recognized by the teacher.

In their analysis of more than 45 hours of video from six beginning teachers' classroom lessons, Stockero and Van Zoest (2013) first identified pivotal teaching moments and then classified five types of circumstances that generate them:

- *Extending:* students offer questions or comments grounded in, but extending beyond, the mathematics being discussed.
- *Incorrect mathematics:* an incorrect solution or incorrect mathematical thinking occurs in public and the error offers the opportunity to improve student understanding. Errors, such as computational errors that offer little more than a need for correction, do not constitute a PTM.
- *Sense-making*: students' efforts to make sense of mathematical content in a lesson provide the opportunity to clarify critical aspects of the mathematics in the lesson.

- *Mathematical contradiction*: a seeming contradiction, such as competing interpretations of a mathematical situation, creates an opportunity for a teacher to bring student attention to the nature of mathematics and the aspects needed to resolve the contradiction.
- *Mathematical confusion*: students can express and articulate mathematically things about which they are confused.

Exploring the impact of PTMs in the development of teacher knowledge requires not just identification of the moment(s), but identification and understanding of the teacher response(s) to the moment. In their study, Stockero and Van Zoest (2013) further identified five actions teachers exhibit in response to PTMs:

- *Ignores or dismisses*: teacher fails to acknowledge a PTM, or rejects it outright.
- *Acknowledges, but continues as planned*: teacher acknowledges a PTM, but only in a superficial way.
- *Emphasizes meaning*: teacher attempts to highlight mathematical meaning using definitions, or the mathematics underlying the procedures.
- *Pursues student thinking*: teacher attempts to find out more about what students who initiate PTMs are thinking.
- *Extends/makes connections*: teacher goes beyond the topic of the present lesson to build connections to past learning or lay a framework for future learning.

A summary of the PTM and teacher response classifications identified by Stockero and Van Zoest (2013) is shown in Table 1.

Table 1

Summary of Pivotal Teaching Moment and Teacher Response Classifications

Pivotal Teaching Moment	Teacher Response
Extending	Ignores or dismisses
Incorrect Mathematics	Acknowledges, but continues as planned
Sense-Making	Emphasizes meaning
Mathematical Contradiction	Pursues student thinking
Mathematical Confusion	Extends/makes connections

Note. PTM and teacher responses to PTMs as identified by Stockero and Van Zoest (2013). PTM = pivotal teaching moment

Teachers must possess a strong knowledge base to effectively recognize and respond to PTMs, and, although more PTMs are likely to occur in classrooms where students are more actively engaged in the lesson, PTMs are found in classrooms where a more didactic approach is taken (Stockero & Van Zoest, 2013). Stockero and Van Zoest advocate for additional research that includes interview data with experienced teachers as a means of more completely understanding teacher responses to PTMs. This dissertation study addressed this call to action, while also exploring how teacher knowledge development related to PTMs.

In-the-Moment Development of Teacher Knowledge

Studies exploring PCK have the potential to inform not only teacher preparation efforts, but student achievement efforts as well. Alonzo, Kobarg, and Seidel (2012)

found that although students had entered a course with approximately equal levels of pre-course content knowledge and interest, students of the teacher exhibiting higher levels of PCK had significantly higher post-course achievement levels as well as higher interest levels.

Although Alonzo et al. (2012) acknowledged the limited generalizability of their study and several potential confounding factors, their results suggest that teachers with higher levels of PCK exhibit greater flexibility in their use of content knowledge. According to Alonzo et al., teachers who are familiar with different ways of expressing content, who have a flexible understanding of the content, who have built up a repertoire of examples and representations, and who have learned about common student difficulties may be better equipped to encourage student gains in content knowledge.

Although several researchers have posited means for studying and assessing PCK development, a complete understanding of how PCK develops remains elusive. There is some evidence that initial PCK develops through teacher reference to textbook presentations of material whereas a more refined development of PCK requires authentic classroom experiences (Van Driel et al., 1998). In fact, Van Driel et al.'s (1998) findings suggested that authentic simulation of classroom experience in which teachers respond to student responses and reflect on their own responses is a key aspect in teachers' PCK development. Prior research has recognized, as teachers themselves suggest, that questions posed by students, student behavior during lessons, and examination of student work provide the most profound opportunities for their PCK growth (Van Driel, de Jong, & Verloop, 2002).

The above results were supported by Park and Oliver (2008) who suggested that PCK develops in the face of four main student-centered activities. First, challenging questions posed by students that force teachers to consider things they had not previously thought about. Second, as teachers informally assess student participation in instructional episodes, they are led to adjust current instructional strategies and create new plans for future instructional design of lessons. Third, student responses to assessment questions often motivate teachers to validate or refute unfamiliar approaches, further deepening teachers' PCK. Finally, student misconceptions and teacher efforts to resolve misunderstandings that arise when subject matter knowledge conflicts with direct real-life experience lead to moments wherein both teachers and students refine their understanding of the content. This refining influences future teacher actions in preparing to teach the same content.

Although each of the above studies suggest that authentic classroom experience is essential to PCK development and support that teacher PCK can develop in the process of teaching, these studies have focused exclusively on secondary science teachers. This dissertation study adds to the existing literature by exploring the role of pivotal teaching moments in the development of mathematical pedagogical content knowledge.

Experience, Reflection and Learning, and Teacher Knowledge

Theorists in multiple fields of study have addressed and debated the foundational aspects of how learning occurs through experience (Bednarz & Proulx, 2009; Dewey, 1998; Kolb, 1984) and others have argued for reflection as a component in learning

(Schön, 1984). This section offers a summary of relevant aspects of experiential learning and Schön's (1984) constructs of reflection-in-action and reflection-on-action.

Dewey (1998) argued that teacher-directed learning was the opposite of true learning and that for learning to occur, an intimate relationship between experience and education is necessary. Although Dewey's ideas held students as learners, research suggests that teachers are themselves learners and that teacher knowledge develops during the act of teaching (Bednarz & Proulx, 2009). Consequently, there is value in placing teachers in the role of learner and reconsidering the applicability of Dewey's perspectives.

Suggesting that a person's past experience translates into strategies for effectively dealing with the future and that this happens largely independent of an individual's desire or intent, Dewey argued:

> Every experience affects for better or worse the attitudes which *help decide the quality of further experiences, by setting up certain preference and aversion, making it easier or harder to act for this or that end*. Moreover, every experience influences in some degree the objective conditions under which further experiences are had (pp. 29-30, emphasis added).

Arguably, as we pursue this perspective, teachers' engagement with students in moment-to-moment classroom interactions not only shapes teachers' responses *in* the moment but also teachers' future responses and actions *beyond* the moment. Consequently, it is possible that a pivotal teaching moment and the ensuing teacher response have an impact not only in that moment but also on the teacher response exhibited later under similar circumstances. Thus, the initial pivotal teaching moment and subsequent teacher response have the power to shape the knowledge development of the teacher.

Although Dewey's views of experience tie directly to education, there is support for Dewey's ideas in other areas as well. Kurt Lewin's views are a result of his study of group dynamics. Kolb (1984) cites Lewin's work as leading to a discovery that "there is a dialectic tension and conflict between immediate concrete experience and analytic detachment" (Kolb, 1984, p. 9). In Lewin's work, this tension manifested itself in dynamic conversations between trainees that occurred when their immediate experiences conflicted with the conceptual models of the training staff. Extending such circumstances to education, similar tensions occur frequently in classrooms where teachers' mathematical content knowledge is in conflict with students' immediate learning experiences. For example, when a teacher's attempt to teach a mathematical concept fails to transfer effectively into the intended learning, students are likely to seek to make sense of the mathematics, perhaps by asking questions, or expressing their mathematical confusion directly. Such situations often result in definable pivotal teaching moments. In this dissertation study, examining teachers' perceptions regarding the relationship between the moment itself, the enacted response, and the knowledge development of the teacher were of primary interest.

The Lewinian view of learning, shown in Figure 3, suggests that concrete experiences are followed by observations and reflection (whether formal or informal) and that as learners attempt to synthesize meaning from their observations and reflections, they informally develop abstract generalizations about how to act. These informal generalizations are then "tested" in the face of new concrete experiences and the cycle begins anew. Schön (1984) suggested that all practitioners experience a similar learning

cycle as they reflect on what is happening and experiment with possible actions while in the process of practicing their trade. For Schön "our knowing is *in* our action" (p. 49, emphasis in the original) and effective practitioners constantly engage in reflection about what they are doing while they are doing it and doing so is essential to dealing with uncertain and unexpected situations. Thus, *reflection-in-action* necessarily occurs within and prompts active experimentation on the part of the practitioner. As practitioners carry out a choice of action(s), they move from reflection-in-action to *reflection-on-action* (Schön, 1984) wherein the focus shifts from "what I'm doing" to "what is happening because of what I am doing."

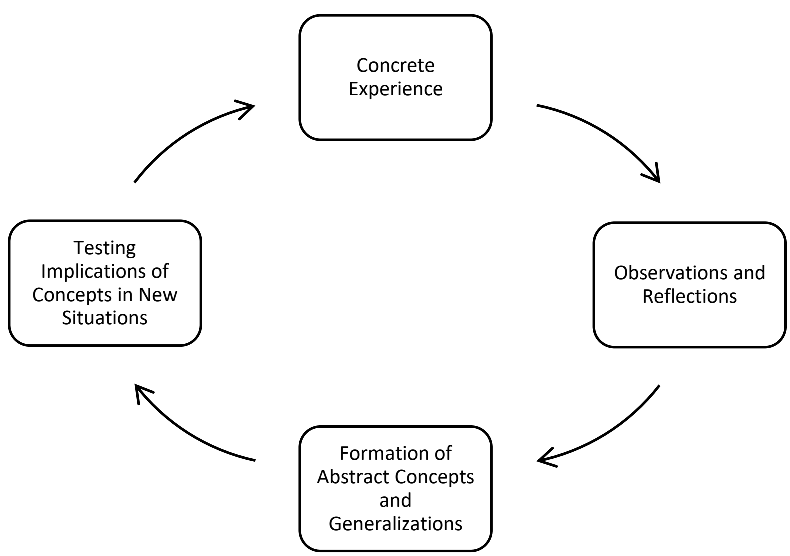


Figure 3: The Lewinian Experiential Learning Model

When a pivotal teaching moment occurs, teachers must quickly resolve the tension between this concrete experience and past experiences (i.e., their knowledge

base), quickly determining how to respond. After enacting a determined response, teachers must evaluate the results of that response in an instant, which leads to an informal assessment regarding whether the chosen response was “successful” (i.e., should be repeated), or not.

It seems somewhat logical to assume that teachers who have had more opportunities to navigate the complexities associated with teaching specific content, may have developed deeper mathematical pedagogical content knowledge than teachers who have had fewer teaching opportunities. Under this premise, how long teachers have been teaching may relate to how easily teachers respond to pivotal teaching moments (Stockero and Van Zoest, 2013) and how much personal knowledge growth they attribute to pivotal teaching moments. This study explored similarities and differences in responses and teacher perceptions of knowledge development among teachers with varying years of classroom teaching experience.

Empirical evidence exists to support that teacher reflection both during and after teaching plays a critical role in the development of pedagogical content knowledge and that analysis of in-the-moment teaching provides an opportunity to get inside the head of the teacher (Taylan & da Ponte, 2016). Particularly compelling in light of the present study is the following statement taken from Park and Oliver (2008):

> PCK as knowledge-in-action became salient in situations where a teacher encountered an *unexpectedly challenging moment in a given teaching circumstance*. In order to transform the challenging moment into a teachable moment, the teacher had to integrate all components of PCK accessible at that moment and apply them to students through an appropriate instructional response (p. 268, emphasis added)

This points to the importance of more closely examining how pivotal teaching moments relate to the development of mathematical pedagogical content knowledge as well as the need for better understanding of what teachers are thinking in these moments.

Understanding what teachers are thinking during pivotal teaching moments and how that thinking relates to teacher knowledge development requires soliciting teacher perspective regarding identified moments. Other researchers have examined aspects of professional learning experiences that teachers have perceived as having contributed to their knowledge development (Herro, D. & Quigley, 2017; Wilkie & Clarke, 2015). Viewing in the moment classroom experiences as a professional learning opportunity, this dissertation study also explored teachers' perspectives regarding the impact of pivotal teaching moments on their knowledge development.

Conceptual Framework

This section brings together teacher knowledge, pivotal teaching moments, and the Lewinian Learning Cycle to present the conceptual framework that guided this dissertation study. The conceptual framework hinges on three major premises. Below, a discussion of these premises precedes the presentation and formal explanation of the conceptual framework.

Guiding Premises

The first premise is that actual classroom teaching experience is key to the development of teacher knowledge. This premise is the reason practicing teachers (as opposed to pre-service teachers) were the focus of this study. Inasmuch as teacher

knowledge develops in the act of teaching, one simply cannot examine teacher knowledge development without appreciating the role that classroom teaching experience plays in that development. It is possible that teachers who have had more exposure to pivotal teaching moments related to specific mathematical content will exhibit different responses, and experience knowledge development in ways very different from those who have not.

The second premise is that pivotal teaching moments are a major driver of in-the-moment knowledge development. Although Stockero and Van Zoest's (2013) introduction of *pivotal teaching moments* is relatively recent, other researchers have acknowledged that pedagogical content knowledge develops in the face of unexpected classroom moments and events (Park & Oliver, 2008; Van Driel et al. 2002). Although these researchers use different terminology, all agree that moments that require teachers to "think on their feet" are those with the most to offer in the way of teacher knowledge development.

The third premise inherent in the conceptual framework of this study is that teacher responses to pivotal teaching moments are evidence of their progression through the Lewinian Learning Cycle and only in seeking to "get into the mind of the teacher" can we more fully understand teachers' full progression through the cycle. To arrive at a more thorough understanding of the development of teacher knowledge in relation to pivotal teaching moments, this study sought teachers' perspectives and perceptions, something largely absent from the present research base on this topic.

Explanation of Conceptual Framework

The Lewinian Learning Cycle forms the basis for the researcher's overall conceptualization of how teacher knowledge develops. As conceptualized in this study and as shown in Figure 4, teacher knowledge development begins with an identified pivotal teaching moment. Upon identifying the pivotal teaching moment, a teacher internally processes all available information in light of this pivotal teaching moment and, drawing on her mathematics content knowledge and mathematical pedagogical content knowledge, formulates a response. These two stages of the cycle occur "behind the scenes," within the mind of the teacher. Only the teacher knows much of what transpires in these stages of the cycle. However, in this study, the researcher used interviews and journal reflections to help explore teacher thinking in these stages. During the final stage of the cycle, the teacher enacts her chosen response and informally evaluates the outcome of the chosen response. The ultimate result of this process is the generation of new knowledge. This new knowledge is added to the teacher's existing knowledge base, and (assuming an acceptable outcome has been achieved) the cycle terminates until a new pivotal teaching moment begins a new cycle. In the event that the initial enacted response fails to elicit an acceptable outcome, the teacher returns to the processing stage, formulating a revised response. In this way, a single pivotal teaching moment may result in several iterations of the learning cycle depicted in Figure 4, with each iteration contributing to the teacher's knowledge base.

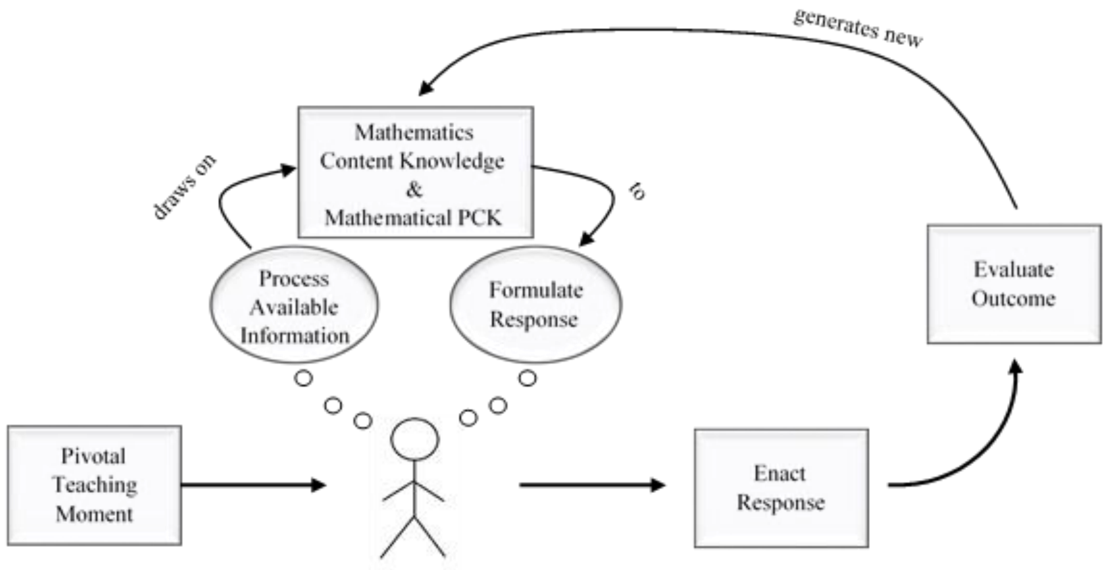


Figure 4: Conceptual framework. A pivotal teaching moment begins teachers' progression through the Lewinian learning cycle that involves their drawing on their own knowledge base to formulate, enact, and evaluate responses. This cycle is continuous and ongoing during the act of teaching.

Chapter Summary

There exists general agreement among researchers that content knowledge and pedagogical content knowledge are two key components of teacher knowledge. Research suggests that unexpected events akin to pivotal teaching moments offer unique opportunities to examine and understand the development of teacher knowledge. Experiential Learning Theory supports that teacher knowledge develops in the practice of teaching as well as in reflecting upon that practice. The researcher built this dissertation study on current research exploring the development of these two components of teacher knowledge, and sought to understand how the situated nature of the classroom influenced its development. The study used an experiential learning cycle to explore how teachers'

mathematics content knowledge and mathematical pedagogical content knowledge development were related to pivotal teaching moments.

CHAPTER 3

METHODS

The purpose of this study was to explore how secondary mathematics teachers responded to pivotal teaching moments (PTMs) and how the teachers perceived that their own content and pedagogical content knowledge was related to PTMs and their responses to them.

Research Design

This study used an exploratory multiple case study research design. In a multiple case study design, the researcher explores a single phenomenon from the perspective of multiple cases (Creswell, 2013). In selecting this design, the researcher acknowledges three key aspects of prior research examining in-the-moment development of pedagogical content knowledge. First, such studies have overwhelmingly used qualitative research designs (Alonzo et al., 2012; Chan & Yung, 2015; Park & Oliver, 2008; Taylan & da Ponte, 2016; Van Driel et al., 2002). Second, researchers have directly argued for the use of multiple case study designs in examining the complexities associated with teacher knowledge (Park and Oliver, 2008). Third, the situated nature of the classroom environment warrants exploratory case studies to facilitate detailed and dynamic descriptions of the factors of the study and perspectives of the participants (Chan and Yung, 2015) and the teacher-student interactions that occur within the classroom (Alonzo et al., 2012; Chan & Yung, 2015; Van Driel et al., 2002).

How teacher knowledge develops in the moment has only recently become a focus of education research and much of the existing literature explores in-the-moment teacher knowledge development in disciplines outside of mathematics (Alonzo et al., 2012; Chang & Yung, 2015; Taylan & da Ponte, 2016). Contrastingly, research characterizing and exploring pivotal teaching moments exists solely within mathematics education. To date, however, the focus of such studies is limited to novice teachers and the impact of PTMs and teacher responses on student learning (Stockero and Van Zoest, 2013), or instruction that utilizes technology (Cayton, Hollebrands, Okumus, & Boehm, 2017). This study filled a void in the research by providing an exploration of how practicing secondary mathematics teachers' knowledge developed in relation to pivotal teaching moments. In the present study, a case was defined as a single participating secondary mathematics teacher and the classroom observations, interviews, and journal reflections associated with the teacher.

Research Questions

To accomplish the purpose of this study, the researcher collected data to answer the following research questions:

1. How do six practicing secondary mathematics teachers respond to pivotal teaching moments?
2. How, if at all, do six practicing secondary mathematics teachers perceive that their own knowledge development relates to pivotal teaching moments and their responses to them?

3. What are the similarities and differences among secondary mathematics teachers' with varying years of teaching experience in their responses to pivotal teaching moments and perceptions of how their responses and knowledge development are related?

As used above, the word *practicing* differentiates teachers with actual classroom teaching experience from preservice teachers and the term *knowledge development* refers to development of teachers' content knowledge and pedagogical content knowledge.

Participants

The participants in this study were six secondary mathematics teachers in Utah, one male and five females. Three teachers taught in a public junior high school (Grades 7-9), one taught in a K-8 charter school, and two taught in public high schools (Grades 9-12). Because this study sought to explore themes related to the knowledge development of teachers, both generally and across years of teaching experience, the researcher purposefully selected six participants with various years of teaching experience in mathematics and who represented a variety of grade levels. To ensure that participants had previously demonstrated some degree of acumen in teaching mathematics, the researcher only included teachers who had been teaching mathematics for a minimum of three years. This resulted in six participants whose overall experience teaching mathematics averaged 12 years, but whose experience teaching at their current grade level averaged only 6 years. Table 2 offers a summary of participant characteristics.

Table 2

Summary of Participant Characteristics

Participant Pseudonym	Age	Education	Teaching Experience	Grade Level	Grade Level Experience
Arthur	42	MS Math Education	14	8	1
Claire	43	MS Mathematics	19	11	12
Kathy	37	MS Mathematics	13	9	2
Linda	50	BS Mathematics	8	9	3
Melissa	50	BS Mathematics	13	7/8	13
Rachel	28	BS Mathematics	5	9	5

Note. Shows participant age, education, years of experience teaching mathematics, current grade level assignment, and years of experience teaching at that grade level. MS = Master of Science, BS = Bachelor of Science

Use of a multiple case study design warranted consideration of what Yin (2014) refers to as *replication*. “Each case must be carefully selected so that it either (a) predicts similar results (a *literal replication*) or (b) predicts contrasting results but for anticipatable reasons (a *theoretical replication*)” (Yin, 2014, p. 57, italics in original). Inclusion of teachers who were teaching the same grade level demonstrates the researcher’s consideration of literal replication, while inclusion of teachers with varying years of teaching experience demonstrates consideration of theoretical replication.

Data Sources

While qualitative studies often involve observation data, observation data alone fails to capture rationale for teacher decisions, and thus cannot completely capture teachers’ PCK development (Alonzo et al., 2012). Other researchers have included reflections focused on teacher intentions for and anticipations about a lesson (Taylan & da Ponte, 2016), and interview and reflection evidence to address questions related to what a teacher does, what a teacher knows, and why a teacher does what she does (Park

& Oliver, 2008). To allow for consideration of multiple perspectives in this study, the researcher used four data sources: lesson plan outlines, classroom observations, teacher perspective interviews, and participant reflection journals. An overview of the research questions, data source(s), and procedures used appear in Table 3 and detailed descriptions follow.

Table 3

Overview of Research Questions, Data Sources, and Data Collection Procedures

Research Question	Data Source(s)	Data Collection Procedures
How do six practicing secondary mathematics teachers respond to pivotal teaching moments?	Lesson Plan Outline	*Quantity:* 4 per case *When:* Distributed at least 48 hours prior to classroom visit; collected within 24 hours prior to classroom visit *Purpose*: Help differentiate anticipated teacher responses from unanticipated responses
	Identification Observation	*Quantity*: 4 per case *When*: Once per classroom visit *Purpose*: Identify PTMs and teacher response(s) to them
How, if at all, do six practicing secondary mathematics teachers perceive that their own knowledge development is related to pivotal teaching moments and their responses to them?	Teacher Perspective Interview	*Quantity*: 4 per case *When*: Once per classroom visit (between observations when possible) *Purpose*: Obtain teacher perspective on (a) the reason(s) for her response(s) to identified PTMs, (b) how PTMs inform changes to content knowledge, and (c) how PTMs inform changes to PCK
	Participant Reflection Journals	*Quantity*: 4 per case *When*: Distributed within 24 hours after each classroom visit; collected prior to next classroom visit *Purpose*: Obtain additional teacher perspective(s) on knowledge development
What are the similarities and differences among secondary mathematics teachers' with varying years of teaching	Lesson Plan Outline	*Quantity*: 4 per case *When:* Distributed at least 48 hours prior to classroom visit; collected within 24 hours prior to classroom visit *Purpose*: Within and cross-case comparisons

experience in their responses to pivotal teaching moments and perceptions of how their responses and knowledge development are related?	Identification Observation	*Quantity*: 4 per case *When*: Once per classroom visit *Purpose*: Within and cross-case comparisons
	Teacher Perspective Interview	*Quantity*: 4 per case *When*: Once per classroom visit (between observations when possible) *Purpose*: Within and cross-case comparisons
	Evaluation Observation	*Quantity*: 4 per case *When*: Once per classroom visit (following interview when possible) *Purpose*: Within and cross-case comparisons
	Participant Reflection Journals	*Quantity*: 4 per case *When*: Distributed within 24 hours after each classroom visit; collected prior to next classroom visit *Purpose*: Within and cross-case comparisons

Note. Shows the per case quantities of each data source collected, when each data source was used, distributed, and/or collected, and the purpose of each data source.

Lesson Plan Outlines

In this study, the term “lesson” refers to a single class period (typically 50 to 90 minutes in duration) and the term “lesson plan outline” refers to the plan that a teacher created to teach one mathematics class lesson. The researcher used the lesson plan outline to capture the essence of what a teacher planned and had anticipated as part of her lesson delivery. Capturing teachers’ anticipations about a lesson was important because researchers have argued that it is more difficult to determine when an experienced teacher is generating an in-the-moment response because their experience allows them to respond more quickly than novice teachers (Stockero & Van Zoest, 2013).

Teachers completed the lesson plan outline (see Appendix A) prior to each of four scheduled school visits, resulting in 24 completed lesson plan outlines for the six

teachers. As part of the lesson plan outline, participating teachers provided the following information:

- the topic(s) they planned to address in each lesson,
- the instructional strategies they planned to use,
- reason(s) for including each instructional strategy,
- expectation(s) regarding what students would find easy and difficult about the lesson,
- anticipated student question(s) about the content of the lesson, and
- anticipated response(s) to the anticipated student questions.

While the depth and specificity of teacher's lesson plan outlines varied, the researcher used these data to inform her perspective during classroom observations.

Classroom Observations

Classroom observations served as a second data source for this study. During data collection, the researcher visited each participating teacher's classroom four times, observing two separate instructional periods at each visit. Thus, the study included eight classroom observations for each of the six participants, for a total of 48 classroom observations. The researcher conducted two different types of observations during each visit: an identification observation and an evaluation observation.

Identification observation. Each participant's schedule consisted of at least two instructional periods of the same course on a given day. During the first instructional period, the researcher conducted an identification observation to identify and record pivotal teaching moments (PTMs) that forced the teacher to generate an in-the-moment

response and the teacher's response(s) to the PTMs. The researcher conducted 24 identification observations, four for each of the six teachers.

Evaluation observation. During the same school visit, the researcher observed the teacher a second time during later delivery of the same lesson. During this evaluation observation, the researcher again identified and recorded the occurrence of and teacher response(s) to PTMs. Additionally, the researcher also watched for evidence of teacher knowledge development evidenced by things such as changes in the teacher's instructional approach, questioning strategies, or responses to pivotal teaching moments that mirrored those that occurred during the identification observation. The researcher conducted 24 evaluation observations, four for each of the six teachers.

Teacher Perspective Interviews

As part of each school visit, the researcher conducted a semi-structured teacher perspective interview with each teacher. These interviews occurred after school, or at a time when the teacher had a preparation period on the day of the school visit. This resulted in interviews for three participants taking place between the identification observation and the evaluation observation, and interviews for three participants taking place after both observations were complete. The researcher conducted one interview during each school visit, resulting in 24 teacher perspective interviews, four for each of the six teachers.

Teacher perspective interviews served three main purposes: First, the interview allowed the researcher to determine teachers' motivations regarding instructional moves and in-the-moment responses to each identified pivotal teaching moment, an element

lacking in previous studies on this topic (Stockero & Van Zoest, 2013). Second, by questioning teachers regarding changes to their own understanding of a lesson's mathematical content, the researcher gained insight into teachers' perceptions of relationships between PTMs and teachers' mathematics content knowledge development. Finally, the interview allowed the researcher to question teachers on how (if at all) teachers perceived that the identified PTMs would shape their future teaching of the same content. By asking a teacher if, and how she planned to modify future instructional approaches to a lesson, the researcher identified emergent themes regarding how teachers' pedagogical content knowledge develops in relation to PTMs.

Participant Reflection Journals

The final data source was a participant reflection journal, which the researcher used to explore knowledge development that occurred in light of teachers' continued reflection about the lesson and its content. Some teachers continued to reflect on the lessons and PTMs beyond the interview period and the day of the school visit. Such *reflection-on-action* (Schön, 1984) provided further insight into how pivotal teaching moments related to teachers' knowledge development.

Each participating teacher had the opportunity to complete four reflection journals (one for each of the four school visits). Not all teachers completed reflection journals, and not all teachers engaged in the same level of reflection. The researcher collected 18 reflection journals during the study. Both the quantity and quality of teacher reflections provided the researcher with relevant data for addressing the second and third research questions.

Procedures

Three main research activities occurred during the study: participant selection, participant orientation, and data collection. A visual overview of the chronology and a brief description of these activities appears in Figure 5. This section provides detailed descriptions of the procedures associated with each activity.

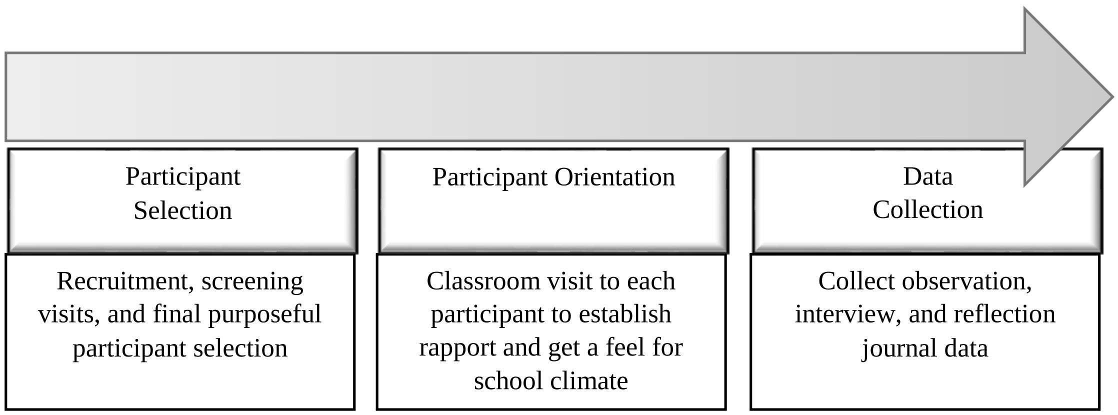


Figure 5. Visual overview, chronology, and description of research activities.

Participant Selection Procedures

Selection of participants occurred in two phases. This section provides procedural details regarding each phase and Table 4 summarizes the activities for each phase.

Table 4

Summary of Participant Selection Activities

Phase	Activities
1	Initial email soliciting interest, establishing and sorting pool of eligible participants and obtaining informed consent from eligible participants.

2	Informal screening visits to identify final participants and final participant selection.

Note. Summarizes the activities associated with participant selection.

Phase 1. To recruit participants, the researcher sent email invitations to secondary mathematics (i.e., grades 7-12) teachers throughout Utah. Twenty-one teachers responded indicating interest in participating in the study. The researcher sent a second email to these respondents requesting completion of the questionnaire shown in Appendix B. Responses to the questionnaire helped the researcher determine which interested teachers had a bachelor's degree or higher in mathematics, statistics, mathematics education, or statistics education and at least three full years of prior experience teaching mathematics. These initial screening criteria ensured that the teachers selected for participation had a reasonable level of mathematics content knowledge and teaching experience. Responses to the questionnaire also provided the researcher with demographic data related to the age, gender, ethnicity, present teaching assignment, and activities associated with a typical instructional period for each potential participant. Teachers also provided informed consent at this stage of the study. Nine interested teachers failed to meet the initial screening criteria and two failed to respond to the researcher's emails regarding further participation in the study. Thus, the researcher removed 11 interested teachers from consideration during Phase 1 of participant selection.

Phase 2. During the second phase of participant selection, the researcher scheduled an informal, screening classroom observation and brief 10-15 minute teacher visit with the 10 eligible teachers remaining from Phase 1. The primary purpose of the

screening observation was to ensure that the final purposeful sample included teachers whose classrooms and circumstances were free from barriers that would preclude quality data collection. Such barriers encountered by the researcher included a significant lack of opportunities for PTMs, planned teacher absence from school for a prolonged time during the study period, and travel concerns related to the teacher's schedule and/or the location of the school. The researcher excluded four teachers for participation based on these barriers.

A secondary purpose of the screening observations was to identify six participants representing a variety of grade levels, schools, and overall teaching experience. Additionally, the researcher selected participants whose schedules allowed for the observation of the same lesson twice during the instructional day and an interview session either after school, or at a time between the instructional observations.

Participant Orientation Procedures

Once the researcher identified six teachers for participation, she conducted an informal participant orientation visit with each participating teacher. The researcher did not collect formal data during these visits but instead used these visits to establish rapport with the participating teachers and identify established school procedures for visitors. Each of these six visits took approximately 30 minutes, during which time the researcher oriented participants to the overall purpose and expectations associated with participation and answered any general questions participants had relative to their participation. During this visit, the researcher asked each teacher to distribute a letter of information to the students in each classroom to inform parents about the study. The researcher allowed

approximately one week for the distribution of these letters and subsequent parent decisions to allow their child to opt out of the study. There were no instances of parents opting out of the study. Following this waiting period, the researcher scheduled the first formal classroom visit with each teacher.

Data Collection Procedures

During data collection, the researcher made 24 school visits (4 per teacher), conducted 48 observations (8 per teacher), and conducted 24 interviews (4 per teacher). Each school visit included an identification observation, a teacher perspective interview and an evaluation observation. Based on the variability in teachers' schedules, interviews for three of the cases took place between the two observations, and interviews for three of the cases took place after the two observations. Because teachers were identified for participation at various times, and to accommodate for differences in schedules of teachers on block schedules versus those on a 7-period schedule, the researcher scheduled school visits with each teacher so that there was a minimum of one week between school visits. This allowed the researcher to collect data from teachers on a time schedule most convenient for them and, in most cases, allowed the researcher to visit two or three teachers each week. Details regarding the activities before, during, and after each school visit appear in the subsections below and Table 5 provides a summary.

Table 5

Summary of Activities Before, During, and After Each School Visit

Before Each School Visit

School visit scheduled at least one week in advance.
LPO emailed to teacher and returned to researcher.
Researcher reviews LPO to acquaint herself with the intentions of the planned lesson.
During Each School Visit
Observed teacher's initial presentation of a planned lesson (IO).
Interviewed teacher regarding PTMs and influence on CK and PCK (TPI).
Observed a second presentation of the same lesson (EO).
After Each School Visit
Email containing PRJ sent to teacher.
PRJ from each lesson collected prior to next scheduled school visit.

Note. Summarizes activities before, during, and after each school visit. LPO = lesson plan outline, IO = identification observation, CK = content knowledge, PCK = pedagogical content knowledge, TPI = teacher perspective interview, EO = evaluation observation, PRJ = participant reflection journal.

Before the school visit. The researcher scheduled each formal school visit a minimum of one week in advance. At least two working days prior to each school visit, the researcher emailed a lesson plan outline (see Appendix A) to the participating teacher with instructions to return the outline by email prior to the day of the scheduled visit. Upon receiving the completed outline from the teacher, the researcher reviewed it, familiarizing herself with the mathematical content of the lesson and the teacher's anticipations regarding students. The researcher brought the completed lesson plan outlines with her to the classroom observations to use for reference as needed.

During the school visit. During each school visit, the researcher observed two class periods of instruction and conducted a participant interview. The researcher used a printed version of the observation protocol (see Appendix C) to capture the essence of both observations and each observation was audio-video recorded. All interviews were audio-recorded.

During the first observation period (i.e., identification observation), the researcher identified each PTM and the teacher's response(s) to the PTM. The researcher also recorded the researcher's perspective about the PTM and notes related to these moments during the lesson. Researcher notes included general questions the researcher wanted to ask the teacher during the interview, as well as questions intended to clarify aspects of the lesson and the teacher's response. In most cases, the observed lesson was the first time that the teacher taught the lesson to a group of students. There were two exceptions: one teacher taught only three students during her first lesson delivery and the other taught only half of the planned lesson to a group of students she saw every day as opposed to every other day.

The researcher conducted an evaluation observation after each identification observation. Both observations focused on the same lesson plan outline created by the teacher. During the evaluation observation, the researcher noted any deviations made to the lesson from the first observation. She was particularly interested in responses to PTMs that were similar, or even identical to, those documented during the identification observation. Additionally, the researcher recorded any additional PTMs and subsequent teacher responses that arose during the second lesson. The evaluation observation allowed the researcher to determine whether teachers made any instructional changes based on reflection about the PTMs during the initial lesson delivery.

During the same school visit, the researcher conducted a semi-structured teacher perspective interview to seek each teacher's perspective regarding each identified pivotal teaching moment and to question teachers regarding their own knowledge development.

While each interview varied based on the events the researcher observed, and her notations and ponderings related to the observation, interviews broadly included several iterations of three main parts. First, the researcher briefly summarized her observation(s) around a particular pivotal teaching moment and asked the teacher for her perspective on that moment in the lesson (e.g., “What was happening for you in that moment of the lesson? What were *you* thinking?”). Second, when relevant, the researcher questioned the teacher for insights into how the event challenged or changed her own understanding of the mathematical content of the lesson (e.g., “How did that moment impact your understanding of the mathematical content?). Finally, the researcher asked the teacher about whether and in what ways she might foresee altering future instruction on the topic in light of the pivotal teaching moment. When the interview took place after the teacher had delivered both lessons, the researcher replaced the third question above with questions about instructional changes the researcher noticed between the two lessons. Asking the teacher to reflect on changes she may, or did, make to future instruction, provided insight into how teachers’ instructional strategies, and anticipation of student trouble spots was related to the PTM, two critical aspects of pedagogical content knowledge identified by Shulman (1986).

After the school visit. Within 24 hours of each school visit, the researcher sent a follow-up email, thanking the participating teacher for her time. This email also included the participant reflection journal page (Appendix D) as an attachment, with instructions inviting the teacher to record any additional perspectives she had upon further reflection about the lesson, the content, or future plans for teaching the lesson again. The

researcher collected completed reflection journals from each participant prior to the next scheduled school visit.

Data Analysis

Analysis of qualitative data can be more ambiguous than quantitative data analysis and is comparable to assembling a jigsaw puzzle (LeCompte, 2000). First, one creates piles of similar pieces of data, which one then assembles into clusters, before assembling the clusters to form a completed picture. When formally analyzing data, Eisenhardt (1989) suggests that case study analysis should begin with analysis of data within a single case. This, she argues, allows the researcher to become intimately familiar with the details of a single case before conducting subsequent comparative analyses. Drawing on these ideas, the researcher conducted all analysis for this study in ways that allowed understanding elements of the story within a single case before conducting cross-case comparisons. This section begins with an overview of how the data analysis progressed and then provides more specific details related to three phases of data analysis.

Overview of the Data Analysis

The researcher conducted the data analysis in three main phases. At the conclusion of each phase, the researcher was prepared to answer one of the three research questions. Table 6 provides an overview of the purpose of each phase along with the analysis performed in each phase and the sections that follow provide more detail regarding the analyses conducted.

Table 6

Overview of the Purpose of and Analysis in Each Phase of Data Analysis

Phase 1	Purpose: Classify PTMs and teacher responses to PTMs • Analyzed observation protocols within each case • Analyzed observation protocols across cases
Phase 2	Purpose: Develop a participant profile for each participant • Analyzed remaining data sources within a case and related to a single school visit to generate four school visit sub-profiles for each participant. • Create participant profiles synthesizing data from the school visit sub-profiles.
Phase 3	Purpose: Synthesize data within and across cases • Within-case analysis of participant profiles • Cross-case analysis of participant profiles

Note. Provides a summary of the purpose and major analysis activities during three phases of data analysis

Phase 1 of Data Analysis

During Phase 1 of data analysis, the researcher reviewed the observation protocols from both the identification and evaluation observations for each participating teacher and used provisional coding (Saldaña, 2016) to classify the observed PTMs and teacher responses based on the categorizations identified by Stockero and Van Zoest (2013). Upon completion of the provisional coding cycle, the researcher returned to the observation protocols for each teacher and identified any PTMs or teacher responses to PTMs that remained unclassified. Using a sequence of initial coding and theming of the data, the researcher identified new classifications for one additional PTM type and three additional teacher response types. Once coding was complete, the researcher identified the frequency of occurrence for each PTM type and teacher response.

Phase 2 of Data Analysis

During the next phase of data analysis, the researcher analyzed the lesson plan outlines, video recordings of observed classes, field notes, audio recordings of interviews, and participant reflection journals for each participant. In total, the researcher reviewed 24 lesson plan outlines, 46 video recordings of observed classes totaling approximately 37 hours, 24 audio recordings of interviews totaling approximately 6 hours, and 18 participant reflection journals. Owing to technical difficulties, two video recordings of observed classes were unavailable for later analysis. The researcher relied on field notes for analysis of these lessons. The researcher reviewed each data source related to a single school visit for each participant until she had summarized four school visit sub-profiles for each participant. A description of the analysis used to generate each of these sub-profiles appears below and a summary of the mathematical content of each observed lesson can be found in Appendix E.

Analysis of lesson plan outlines. The researcher began by analyzing each participant's four lesson plan outlines. During this analysis, the researcher examined teacher responses to each question in the lesson plan outline, underlining key words and phrases the teacher used to describe her anticipations. This allowed the researcher to descriptively code and generate themes (Saldaña, 2016) summarizing each teacher's anticipations. The researcher then broadly synthesized the data across four categories: anticipated instructional strategies, general anticipations about the lesson, anticipated student questions, and anticipated responses to student questions.

Analysis of observation field notes and video recordings. Next, the researcher reviewed her field notes and video recordings of each identification observation as well as each evaluation observation for a participant. During this step, the researcher used structural coding (Saldaña, 2016) to identify themes related to instructional changes teachers did, or did not make during the evaluation observation as compared to the identification observation. This structural coding allowed the researcher to summarize the general observations of each classroom visit in preparation to find thematic evidences of teachers' knowledge development.

Analysis of interviews. When analyzing recorded interview data, the researcher again used structural coding (Saldaña, 2016) to identify statements made during the interview that related to teachers' motivations for their responses to PTMs, changes they anticipated making in future lessons, and general statements made indicating changes to their own content, or pedagogical content knowledge. This analysis, allowed the researcher to identify themes helpful for answering the second and third research questions.

Analysis of participant reflection journals. The researcher used data from participant reflection journals to identify themes related to changes teachers anticipated making to future lessons. The researcher used emerging themes to provide insight into the development of teachers' pedagogical content knowledge.

The analysis performed during this phase culminated in the creation of six participant profiles each summarizing a teacher's anticipations about a lesson, aspects of her lesson delivery, changes made between lessons, her motivations for responses to

PTMs, and additional anticipations about how she might modify future lesson development and delivery. The researcher used these participant profiles to conduct within and cross-case comparisons during the final phase of data analysis.

Phase 3 of Data Analysis

During the final phase of data analysis, the researcher used themes identified in the participant profiles generated during Phase 2 to conduct within and cross-case synthesis (Yin, 2018). The goal of cross-case synthesis is to compare and synthesize within-case patterns across cases (Yin, 2018). The researcher used the participant profiles as a holistic description of each case and explored emergent patterns in teachers' motivations for their responses to PTMs, instructional changes observed by the researcher between identification and evaluation observations, changes teachers anticipated making to future lessons, and comments made related to teachers' knowledge development. The analysis conducted during this phase provided the foundation for answering the second and third research questions.

Approaching data analysis as presented in this section represents an adaptation of Boeije's (2002) proposed steps to a purposeful constant comparative analysis and is consistent with cross-case synthesis analysis as proposed by Yin (2018).

Validity and Reliability

High-quality research designs address issues of validity and reliability. Terrell (2015) suggests that issues of validity and reliability in qualitative studies require

consideration of issues of credibility, transferability, and confirmability. This section addresses each of these factors as they relate to this study.

Credibility

Credibility, the qualitative equivalent to internal validity, establishes the believability of the research from the participants' perspective (Terrell, 2015). The researcher established credibility in this study in multiple ways. First, the researcher spent several hours in each participant's classroom both before and during formal data collection. This prolonged engagement allowed the researcher to establish amicable working relationships with each participant. Additionally, this study relied on multiple sources of evidence, allowing for the triangulation of results, another important aspect of establishing validity in qualitative studies (Terrell, 2015; Yin, 2014). Finally, the researcher member checked (Creswell, 2013) her summaries with each participant by providing each of them with draft copies of her summaries of the data analysis and welcoming their perspective as to the accuracy of the researcher's summaries.

Transferability

Rich descriptions of the details of the study helps establish external validity in qualitative research (Creswell, 2013). The researcher considered and addressed issues of transferability by comprehensively describing the participants, analysis, and results and clearly articulating all procedures, allowing readers to ascertain to what degree the results presented here are generalizable.

Confirmability

Discussing how a researcher achieved neutrality in a study helps establish the confirmability of a qualitative study (Terrell, 2013). Triangulation of data sources helped accomplish this neutrality by checking the researcher perspectives against those of the participants and external documents. Thus, the researcher's use of field notes, interview recordings and participant reflections helped achieve triangulation. Additionally, the researcher made every effort to maintain accurate records of study procedures and processes, and to collect, analyze and report data in a neutral way, maintaining constant diligence to adhere to approved study procedures.

Researcher Positionality

In conducting this study, the researcher acknowledges the contribution of her own experiences to the data analysis. Although the researcher's role in this study was that of an observer, rather than a participant-observer, her experiences teaching both secondary and college mathematics have shaped her view of, and interest in, teacher knowledge development. The researcher has 19 years of combined experience teaching mathematics and statistics in high school (12 years) and university (7 years). Additionally, the researcher has two years' experience working as a K-12 mathematics curriculum and instruction supervisor for a large school district. Her interest in studying teacher knowledge comes primarily from her own experience wherein it took her eight years to realize that she taught *students*, not mathematics. Stated differently, she was well into her career before she recognized that her ability to blend her mathematics content knowledge and mathematical pedagogical content knowledge were key in determining her effectiveness as a teacher. Teacher learning opportunities inherent within teaching

activities continue to fascinate her and often encouraged her discussion of lesson content with participants in this study outside the scope of relevant data collection. The researcher acknowledges that her experiences have shaped her views and beliefs about mathematics teaching and learning and that these views may have played an unconscious role in the analysis process.

Chapter Summary

The researcher selected participants for this study from secondary mathematics teachers who had a minimum of three years' experience teaching mathematics. Selection of these participants allowed for representation of a variety of grade levels and teaching experience. The researcher used data from lesson plan outlines, observations, interviews, and reflection journals to generate a participant profile for each teacher. Creation of the participant profile relied on use of multiple coding methods and multiple coding cycles. The researcher used these profiles to synthesize data within and across cases. To help ensure the validity and reliability of the study, the researcher maintained prolonged engagement with the participants, triangulated data sources, has richly described the details of the study, member checked researcher summaries, and maintained relevant and accurate records related to study procedures and processes.

CHAPTER 4

RESULTS

This study explored (a) how secondary mathematics teachers with classroom teaching experience responded to pivotal teaching moments (PTMs), and (b) how the teachers perceived that their own content and pedagogical content knowledge related to PTMs and their responses to them. To accomplish this purpose, the researcher collected data to answer three research questions:

1. How do six practicing secondary mathematics teachers respond to pivotal teaching moments?
2. How, if at all, do six practicing secondary mathematics teachers perceive that their own knowledge development relates to pivotal teaching moments and their responses to them?
3. What are the similarities and differences among six secondary mathematics teachers' with varying years of teaching experience in their (a) responses to pivotal teaching moments and (b) perceptions of how their responses and knowledge development are related?

In this chapter, the researcher presents the relevant results related to each research question.

Research Question 1: Teacher Responses to Pivotal Teaching Moments

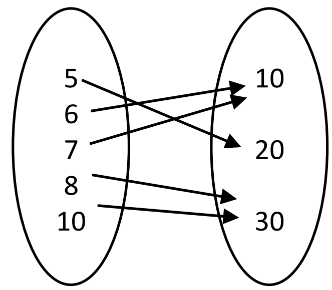
The purpose of this section is to present results to answer the first research question: How do six practicing secondary mathematics teachers respond to pivotal

teaching moments? Prior research on pivotal teaching moments experienced by preservice teachers identified five types of pivotal teaching moments: extending (E), incorrect mathematics (IM), sense making (SM), mathematical contradiction (MCT), and mathematical confusion (MCF) (Stockero & Van Zoest, 2013). Table 7 provides a description of each of these classifications as well as an example instance of this type of PTM as identified in Stockero and Van Zoest's work as well as an example from the present study. Because the researcher in this study remained open to the possibility of the manifestation of additional types of PTMs, she identified one additional PTM classification: *unconventional, but accurate, mathematical process* (U). This type of PTM is manifest when a student presents a solution process that is mathematically accurate, but differs from how most students, and perhaps even teachers, would approach a problem.

Table 7

Description and Examples of Observed Pivotal Teaching Moments

PTM Code	Description	Example (Stockero & Van Zoest, 2013)	Example Present Study
E	Student comment or question is grounded in, but goes beyond, the planned mathematical conversation. (Stockero & Van Zoest, 2013)	Teacher was focused specifically on explaining how the "m" and "b" in the equation $y = mx + b$ can be found from the graph of a linear function. A student asked if it were possible to have more than one *y*-intercept.	While watching a video clip of someone pumping air into a football, students are tasked with drawing the graph of the air pressure of the ball as a function of time. A student raises the issue of how the air pressure capacity of the football impacts the graph.

IM	Incorrect mathematical thinking, or an incorrect solution requiring more than a simple correction occurs in public. (Stockero & Van Zoest, 2013)	When asked to construct a graph to model a situation where a soccer ball was kicked into the air, one student's "distance-time graph" was a picture of someone kicking a ball in the air and the path of the ball as it returned to the ground.	Students are asked if the mapping shown represents a function or not. Several students indicate it does not 
SM	Students are trying to make sense of the mathematics in a lesson and their attempts to verbalize this creates opportunities to highlight critical aspects of a lesson. (Stockero & Van Zoest, 2013)	A student who was trying to conceptually understand what was being presented as a purely procedural explanation raised a question about why the procedure works.	Student asks whether $f(2) = 6$ is the same as $f(x) = 2$.
MCT	May be two different answers to a problem that should clearly have only one answer, or two competing interpretations of a mathematical situation. The contradiction provides an opportunity for a teacher to bring students' attention to the nature of mathematics that makes such contradictions unacceptable. (Stockero & Van Zoest, 2013)	When calculating the sine of an angle using an overhead calculator, students' calculators give different answers than the teacher's owing to some calculators being in radian mode and others being in degree mode.	Given $f(x) = \frac{x+7}{x^2+16x+63}$, some students examine the graph of the given function, while others evaluate the graph of the simplified form $f(x) = \frac{1}{x+9}$, resulting in a discussion about whether these functions should have the same or different graphs.
MCF	Students are confused and can articulate mathematically what they are confused about. (Stockero & Van Zoest, 2013)	When attempting to simplify expressions containing exponents, one student is able to point to the second step of another student's simplification as the point of her confusion.	In finding the average rate of change between (2, -1) and (6, -13), differences in which point is used as (x_1, y_1) result in the calculation 2 – 6 = -4 for the denominator. A student questions why the result is negative if 6 – 2 = 4.
U	Student presents a solution process that is mathematically accurate, but differs from how other students, or teachers,	NA	Posed with $\frac{dm^{-2}}{3} + \frac{5d}{m^2}$, a student proposes that this can be viewed as $\frac{\frac{1}{3}d}{m^2} + \frac{5d}{m^2} = \frac{5\frac{1}{3}d}{m^2}$

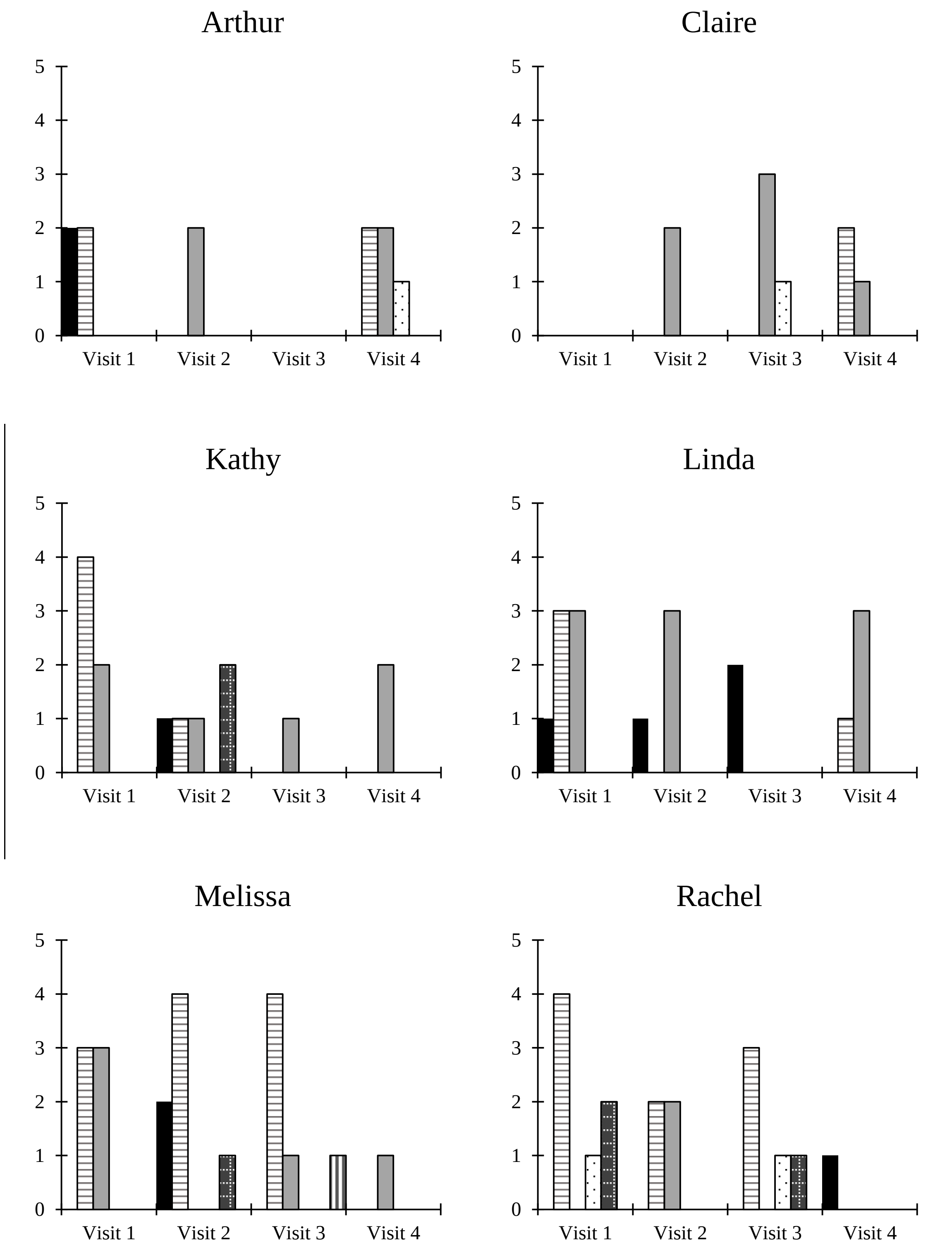
	would typically approach a problem.

Note. Observed PTM types and examples. PTM = pivotal teaching moment. E = extending, IM = incorrect mathematics, SM = sense making, MCT = mathematical contradiction, MCF = mathematical confusion, U = unconventional, but accurate, mathematical process, NA = not applicable.

As the examples in the rightmost column of Table 7 show, the researcher was able to confirm, through the observation of practicing teachers, the same types of pivotal teaching moments introduced by Stockero and Van Zoest (2013) in working with beginning teachers, while also identifying a sixth type of PTM.

Observed PTMs

The researcher conducted 24 total school visits (4 for each participant) and 48 total observations (24 identification observations and 24 evaluation observations). During these observations, the researcher observed and documented 88 total pivotal teaching moments. In the results below, the researcher has assigned the following pseudonyms to the six teacher participants: Arthur, Claire, Kathy, Linda, Melissa, and Rachel. Figure 6 provides a visual summary of the types of PTMs observed for each teacher disaggregated by school visit.


Arthur
Claire
Kathy
Linda
Melissa
Rachel
0
1
2
3
4
5
Visit 1
Visit 2
Visit 3
Visit 4

■ E ⊟ IM ▩ SM ⊡ MCT ▦ MCF ▥ U

Figure 6. Summary of observed pivotal teaching moments by participant, type, and school visit. E = extending, IM = incorrect mathematics, SM = sense making, MCT = mathematical contradiction, MCF = mathematical confusion, U = unconventional, but accurate mathematical process.

Figure 6 shows that the researcher observed a variety of PTMs and that the frequency and types of PTMs observed varied between participants and across school visits. Only two school visits resulted in no observed PTMs (Arthur's third visit and Claire's first visit). Still, the researcher observed between three and five different types of PTMs in each participant's classroom. The teachers with the highest frequency of PTMs were Melissa (20), Rachel (17) and Linda (17), while the fewest number of PTMs occurred in the classrooms of Arthur (14), Kathy (11), and Claire (9). Sense making (SM) and incorrect mathematics (IM) PTMs accounted for 76% of the total PTMs. Interestingly, in classrooms where teachers' instructional strategies tended to rely primarily on direct instruction (i.e., Melissa and Rachel) the most frequent PTM type was incorrect mathematics (IM), while teachers employing exploratory, or a blend of instructional strategies (i.e., Claire, Kathy, and Linda) experienced primarily sense making (SM) PTMs.

Because each school visit included an observation of a lesson taught early in the day (identification observation) and a second observation of the same lesson taught later in the same day (evaluation observation), several of the same PTMs occurred during both lessons. Table 8 provides a summary of the sequential occurrence of observed PTMs while also indicating whether the PTM occurred during the identification observation or

evaluation observation associated with each visit. Additionally, Table 8 identifies the occurrence of the same PTM during both observations using an asterisk.

Table 8

Sequential Summary of Pivotal Teaching Moment Occurrence by Observation Type

Participant	Visit	Identification Observation	Evaluation Observation
Arthur	1	E; IM; E; IM	
	2	SM	SM
	3		
	4	IM*	MCT; SM; IM*; SM
Claire	1		
	2	SM	SM
	3	SM*; MCT; SM	SM*
	4	SM	IM; IM
Kathy	1	IM; IM*; SM	SM; IM; IM*
	2	E; SM; IM; MCF; MCF	
	3	SM	
	4	SM; SM	
Linda	1	E; SM*; IM*	SM; SM*; IM; IM*
	2	SM; SM	SM; E
	3	E	E
	4	IM; SM; SM*	SM*
Melissa	1	SM; SM; IM*; IM	SM; IM*
	2	E; IM*; IM; IM;	E; MCF; IM*
	3	SM; U; IM; IM*; IM	IM*
	4	SM	
Rachel	1	IM; IM*; MCT*; MCF	IM*; MCF; MCT*
	2	SM*; IM	SM*; IM
	3	MCF; IM; IM*	IM; IM*
	4		E

Note. Shows observed PTMs disaggregated by participant, classroom visit, and classroom observation. * denotes that the same PTM occurred during both the identification observation and evaluation observation. PTM = pivotal teaching moment, E = extending, IM = incorrect mathematics, SM = sense making, MCT = mathematical contradiction, MCF = mathematical confusion, U = unconventional, but accurate, mathematical process.

The results shown in Table 8 indicate that 54 PTMs occurred during the initial identification observations, while 34 occurred during the evaluation observations. Of the 34 PTMs observed during the evaluation observations, 38% of these represented PTMs identical to those observed during the identification observation. In all but one instance, these identical PTMs were sense-making (SM) or incorrect mathematics (IM) moments.

Observed Teacher Responses

In addition to identifying five types of PTMs, Stockero and Van Zoest (2013) identified five types of teacher responses to PTMs: ignore or dismiss (ID), acknowledge, but continue as planned (AC), emphasize meaning (EM), pursue student thinking (PT), and extends or makes connections (EC). A description of these classifications, an example from Stockero and Van Zoest's work, and an example from the observations in this study appears in Table 9. Just as the researcher remained open to the possibility that practicing teachers may encounter different types of PTMs, the researcher also considered the possibility that practicing teachers would respond in ways that differed from those identified by Stockero and Van Zoest. Based on the observations, the researcher identified three additional teacher responses to PTMs: *provides additional examples* (AE), *asks guiding questions* (GQ), and *refer to others* (RO). The last three rows of Table 9 provide a description and example from the observations for these additional classifications.

Table 9

Description and Examples of Observed Teacher Responses to Pivotal Teaching Moments

Response Code	Description	Example (Stockero & Van Zoest, 2013)	Example Present Study
ID	Teacher does not notice, ignores, or dismisses the PTM. (Stockero & Van Zoest, 2013)	As students point out a difference in the problem being worked in class and the corresponding problem in students' textbooks, the teacher tells them to forget about one of the problems and focus on the one they are working as a class.	Student gives an incorrect answer to a question, but teacher continues with the lesson, ignoring the error.
AC	Teacher gives attention to PTM, but only in a superficial way. (Stockero & Van Zoest, 2013)	Teacher responds to PTM by affirming that a student has posed a good question, or comment before moving on with the lesson as if the PTM had not occurred.	Given $f(x) = \frac{x+7}{x^2+16x+63}$, some students examine the graph of the given function, while others evaluate the graph of the simplified form $f(x) = \frac{1}{x+9}$. Teacher acknowledges the graphs should be different, but doesn't say anything about what the differences are.
EM	Teacher focuses directly on definitions, rules, or algorithmic procedures underlying the PTM. (Stockero & Van Zoest, 2013)	Students are creating a graphical model for a rabbit jumping over a 3-foot-high fence and leaving and returning to the ground 4 feet from either side of the fence ask, "All the values only gonna be on our line, though, right?" The teacher comments and considers points not on the curve, emphasizing the meaning of domain.	When asked to calculate the odds, student gives probability and teacher responds with "[your answer] sounds like a probability to me, remember the definition of odds."
PT	Teacher finds out more about what students initiating the PTM are thinking. (Stockero & Van Zoest, 2013)	Following a student response of "I know how to do it, but still don't understand why. I just don't see the logic in it." the teacher asks a series of questions to try to pinpoint where the breakdown in the student's understanding occurred.	Teacher asks "tell me why you think that" or "tell me more about how you're thinking about this."
EC	Teacher goes beyond the lesson topic being studied to connect to prior learning or lay a foundation for future learning. (Stockero & Van Zoest, 2013)	As students investigate vertical translations of quadratic functions (e.g., $y = x^2 + 3$), a student asks "What if it was like $x^2 + 3x$?" The teacher validates the student's example as also being quadratic, but comments that for right now they are just	As students are examining an *xy*-table to determine if it represents a function, the teacher encourages graphing of the coordinates and examination of previously learned vertical line test.

		going to look at adding a constant.	
AE	Teacher provides additional, unplanned examples to reinforce student understanding.	NA	After discussing issues with order of operations involving computing *f*(-2) for $f(x) = 12x^2 + 1$, teacher creates additional examples in the moment for student practice.
GQ	Teacher poses additional questions back to student(s) initiating the PTM and uses effective questioning to lead student(s) to correct or build upon their own thinking.	NA	After student states that $(3x)^3 = 9x^3$ the teacher asks “are you squaring or cubing?”
RO	Teacher poses questions back to the class as a whole, or another student(s).	NA	In evaluating *f*(2) for *f*(*x*) = 8 student answers “doesn’t the two mean you have two *x*’s, so it would be 16?” The teacher, speaking to the entire class, replied “what do you guys think about [Johnathan’s] question?”

Note. Observed teacher responses to PTMs and examples. PTM = pivotal teaching moment. ID = ignore or dismiss, AC = acknowledge, but continue as planned, EM = emphasize meaning, PT = pursue student thinking, EC = extends or makes connections, AE = provides additional examples, GQ = asks guiding questions, RO = refer to others, NA = not applicable.

Justification for three additional classifications of teacher responses occurred as the researcher was unable to apply Stockero and Van Zoest’s (2013) original descriptions to the observed teacher responses during the provisional coding phases of analysis. Support for the classification provides additional examples (AE) was evident as Rachel exhibited a series of responses in the face of a mathematical contradiction (MCT) PTM involving evaluating a quadratic function for a negative domain value. After presenting the function $g(x) = 12x^2 + 1$ and asking students to find *g*(-2), a mathematical contradiction PTM arose when some students failed to properly square the negative value

(i.e., evaluating 12(-4) + 1 instead of 12(4) + 1). While Rachel's initial response was to emphasize the meaning (EM) inherent in squaring a value, a secondary response came as she posed several follow up examples involving evaluating the function for both positive and negative domain values. Presumably, this response was an effort to ensure that her emphasis on the meaning of the squared expression was understood by students by asking them to immediately attempt new examples. Consequently, the researcher determined that providing additional examples (AE) became a secondary response to the initial response of emphasizing meaning.

Support for the classification of asks guided questions (GQ) was evident in Melissa's response to an incorrect mathematics PTM. In attempting to create an expression for the volume of a shipping container shaped like a cube with side length of $3x$, a student gave an incorrect response of $9x^3$, obtained by only squaring the 3, but cubing the *x*. Melissa's response was to state, "Oh! So close. But you did 3^2, x^3. So, [for volume], you want...[pause]...are you squaring or cubing?" At first, this response seemed to be an emphasizes meaning (EM) response. However, because Melissa did not directly answer the question for the student by referring to rules, procedures, or definitions, but instead prodded the student in the direction of discovering, and correcting, his own error, the researcher determined this response differed from EM and warranted its own classification.

Support for the refer to others (RO) classification was most strongly evident in Kathy's responses to multiple types of PTMs. It was common for Kathy, in response to student utterances, to withhold her own evaluative judgment of the utterance, instead

presenting a question, or statement back to another student, or to the class for their comment. For example, in evaluating $f(2)$ for $f(x) = 8$, a student answers “doesn’t the two mean you have two x’s, so it would be 16?” Kathy, speaking to the entire class, replied, “What do you guys think about [Johnathan’s] question?” Thus, rather than pursue the thinking of the student initiating the PTM, what Stockero and Van Zoest (2013) classified as a pursues student thinking (PT) response, Kathy pursued the collective thinking of the group by referring the comment back to the group for evaluation.

As the above examples and those in the rightmost column of Table 9 show, while the researcher observed the same types of teacher responses to pivotal teaching moments introduced by Stockero and Van Zoest (2013), she also observed that the teachers in this study responded in ways not captured in the original classifications.

The researcher observed and recorded 104 total teacher responses to PTMs during this study. The total number of responses is higher than the total number of PTMs because, like Stockero and Van Zoest (2013), this researcher found that teachers often exhibited multiple responses in connection with a single PTM. Figure 7 visually summarizes participant responses by type and visit.

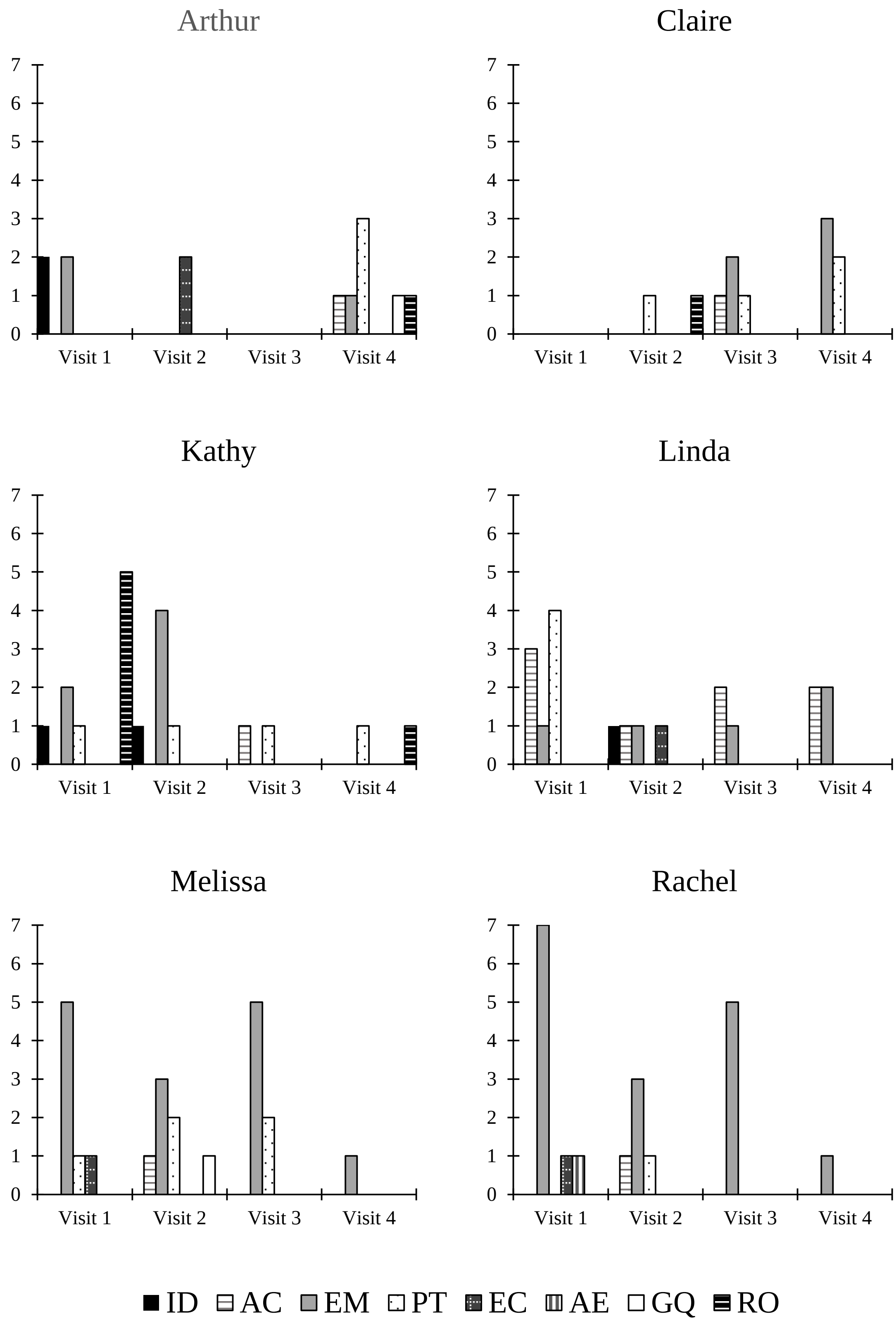

Arthur
Claire
Kathy
Linda
Melissa
Rachel
0 1 2 3 4 5 6 7
Visit 1
Visit 2
Visit 3
Visit 4
ID
AC
EM
PT
EC
AE
GQ
RO

Figure 7. Summary of observed teacher responses to pivotal teaching moments by participant, type, and school visit. ID = ignore or dismiss, AC = acknowledge, but continue as planned, EM = emphasize meaning, PT = pursue student thinking, EC = extends or makes connections, AE = provides additional examples, GQ = asks guiding questions, RO = refer to others.

As summarized in Figure 7, while the researcher observed a variety of teacher responses within and across classroom visits, an overwhelming number of teacher responses involved emphasizing meaning (EM). For three of the six teachers (Claire, Melissa, and Rachel) this response was so frequent it accounted for nearly half, to more than half, of their responses. Only Linda exhibited a different response (AC) more frequently than EM. Kathy's responses provided the strongest support for additional PTM response classifications, with nearly one-third of her responses classified as refer to others (RO).

There were 16 instances where teachers exhibited a sequence of responses to a single PTM. Table 10 offers a sequential summary of teacher responses to PTMs and summarizes the instances where the researcher observed a sequence of responses (→), the observation (identification or evaluation) in which the PTM occurred, and when the same PTM elicited a different teacher response (*).

Table 10

Sequential Summary of Observed Teacher Responses by Observation Type

Participant	Visit	Identification Observation	Evaluation Observation
Arthur	1	ID; EM; ID; EM	
	2	EC	EC
	3		
	4	PT→EM*	PT; AC; PT→GQ*; RO

Claire	1		
	2	RO	PT
	3	EM; AC; PT	EM
	4	EM	PT→EM; PT→EM
Kathy	1	PT→RO; RO*; RO	RO→EM; RO→EM; ID*
	2	EM; EM; PT→ID; EM; EM	
	3	PT→AC	
	4	PT; RO	
Linda	1	AC; AC*; EM*	PT; PT→AC*; PT; PT*
	2	ID; AC	EM; EC
	3	AC; AC→EM	
	4	EM; AC; AC	EM
Melissa	1	EM; EC; EM; PT→EM	EM; EM
	2	AC; EM*; EM; GQ	PT; EM; PT*
	3	PT→EM; PT; EM; EM; EM	EM
	4	EM	
Rachel	1	EM; EM*; EM→AE*; EM	EC→EM*; EM; EM*
	2	AC*; PT→EM;	EM*; EM
	3	EM; EM; EM	EM; EM
	4		EM

Note. Shows observed teacher responses to PTMs by participant, classroom visit, and response type. → denotes that a teacher exhibited a series of responses to the same PTM. * denotes an identical PTM that elicited a different response. PTM = pivotal teaching moment. ID = ignore or dismiss, AC = acknowledge, but continue as planned, EM = emphasize meaning, PT = pursue student thinking, EC = extends or makes connections, AE = follows up with additional examples, GQ = asks guiding questions, RO = refer to others.

Table 10 shows that each teacher exhibited a variety of responses to PTMs. Further, at certain times, all teachers exhibited a sequence of responses to a single PTM. In 10 of the 16 instances where the researcher observed a sequence of responses, the teacher response culminated in emphasizing meaning (EM), or relying on definitions, procedures, and/or rules to address the PTM.

Responses by PTM Type

In examining the types of responses teachers exhibited following a particular type of PTM, Table 11 shows participants' responses disaggregated by PTM type rather than school visit. Boldface responses indicate teachers' responses observed during the evaluation observation.

Table 11

Summary of Observed Teacher Responses by Type of Pivotal Teaching Moment

Participant	PTM Type	Observed Teacher Response
Arthur	E	ID; ID
	IM	EM; EM; PT→EM; **PT→GQ**
	SM	EC; **EC**; **AC**; **RO**
	MCT	**PT**
	MCF (none observed)	
	U (none observed)	
Claire	E (none observed)	
	IM	**PT→EM; PT→EM**
	SM	RO; **PT**; EM; PT; **EM**; EM
	MCT	AC
	MCF (none observed)	
	U (none observed)	
Kathy	E	EM
	IM	PT→RO; RO; **RO→EM**; **ID**; PT→ID
	SM	RO; RO→EM; EM; PT→AC; PT; RO
	MCT (none observed)	
	MCF	EM; EM
	U (none observed)	
Linda	E	AC; **EC**; AC; AC→EM
	IM	EM; **PT; PT**; EM
	SM	AC; **PT**; **PT→AC**; ID; AC; **EM**; AC; AC; EM
	MCT (none observed)	
	MCF (none observed)	
	U (none observed)	
Melissa	E	AC; **PT**
	IM	EM; PT→EM; EM; EM; EM; GQ; **PT**; EM; EM; EM; EM
	SM	EM; EC; **EM**; PT→EM; EM
	MCT (none observed)	

	MCF	EM
	U	PT
Rachel	E	**EM**
	IM	EM; EM; **EC→EM**; PT→EM; **EM**; EM; EM; **EM; EM**
	SM	AC; **EM**
	MCT	EM→AE; **EM**
	MCF	EM; **EM**; EM
	U (none observed)	

Note. Shows observed teacher responses to PTMs disaggregated by PTM type. Boldface indicates PTMs that occurred during an evaluation observation. → denotes that a teacher exhibited a series of responses to the same PTM. PTM = pivotal teaching moment. ID = ignore or dismiss, AC = acknowledge, but continue as planned, EM = emphasize meaning, PT = pursue student thinking, EC = extends or makes connections, AE = follows up with additional examples, GQ = asks guiding questions, RO = refer to others. E = extending, IM = incorrect mathematics, SM = sense making, MCT = mathematical contradiction, MCF = mathematical confusion, U = unconventional, but accurate, mathematical process.

The results in Table 11 indicate that in all six instances of mathematical confusion (MCF) PTMs, teachers exhibited an EM response. Additionally, teachers faced with incorrect mathematics (IM) PTMs, overwhelmingly relied on EM (57.8%) and PT (24.4%) responses. One notable exception to this was Kathy, whose primary response to incorrect mathematics (IM) PTMs were to pursue student thinking (PT) and refer to others (RO). Sense-making (SM) PTMs seemed to generate the widest variety of responses from the teachers in this study and extending (E) PTMs generated the highest percentage of ignores or dismisses (ID) responses.

Motivation for Teacher Responses

Prior studies examining pivotal teaching moments and teachers' responses to them have acknowledged the absence of information on teachers' motivations for their

responses (Stockero & Van Zoest, 2013). The researcher conducted the interviews in this study to provide insight into *why* teachers choose to respond to PTMs in certain ways. The researcher was able to identify seven themes present in teachers' comments regarding why they exhibited certain responses to PTMs. This section summarizes each motivating theme and provides information on which types of responses teachers explained as being due to these motivations.

Failure to notice. The first theme, evident in comments from Kathy, Arthur, and Linda, indicated that teacher responses to ignore, dismiss, or only superficially acknowledge a PTM were often a result of their failure to notice the PTM, or hear a student utterance.

Inaccurate anticipations about student trouble spots. Arthur and Rachel both indicated that during portions of their lessons they anticipated students would be struggling with different things than they actually were. For Arthur, this resulted in a response to pursue student thinking about PTMs, while Rachel's response was to emphasize meaning.

Misunderstanding what a student was asking. Rachel, Melissa, and Linda admitted that at certain points in a lesson they were unsure exactly what a student was asking. For Rachel and Melissa, this resulted in referring back to rules, definitions, or procedures (emphasizes meaning response), while Linda's response was to acknowledge, but continue with the lesson as planned.

Maintaining, or building the flow of a lesson. Rachel indicated that her decision to acknowledge, but continue in response to a sense making PTM was motivated

by her feeling as though her instruction was proceeding in a particular direction, or with a flow that she did not want to interrupt by fully acknowledging the PTM. Instead, she returned later in the lesson to the PTM, to emphasize meaning. In a similar way, Arthur and Rachel both acknowledged that their responses (to acknowledge, but continue as planned, and offer additional examples respectively) were deliberate choices made in light of knowing what was coming later in the lesson. Their knowledge of how particular lessons progressed led them to make decisions to postpone direct discussion of elements related to the PTM, or respond to prepare students for what was coming.

Consideration of student or class individualities. A common motivation given for multiple and various PTM responses dealt with teacher consideration of individual students' and/or classroom dynamics. Five of the six teachers in this study acknowledged that often their response to a particular PTM was influenced by their knowledge about the student who generated the PTM. Teachers admitted that they may respond differently to the same PTM if it were initiated by another student, or during another class. Kathy, Melissa, and Arthur directly acknowledged that classes with high numbers of advanced students, or special education students, often motivated them to respond differently.

Pushing students to do the thinking. Kathy, Claire, and Linda all indicated that their responses to refer to others, pursue student thinking, and superficially acknowledge a PTM came as a result of their wanting students to continue to think and generate ideas. Speaking about her inclination to pursue student thinking, Claire said, "I use this approach because then I know they're actually thinking about it. If they're doing the work, they're learning. If I am, they may not be learning." Kathy echoed this sentiment

in response to why she so often responded by referring to others: "I want kids to identify what is correct and incorrect. This helps me see if kids are listening to each other and to me, and to determine when and if they are confused." Linda also cited wanting students to do the thinking as a reason for her responses to acknowledge, but continue as planned, indicating that the less decisive she was in placing judgment on student comments, the more they continued to think and the freer they were with their responses.

Responding based on experience. All of the teachers in this study indicated that they often predicated their responses on their prior experience teaching the lesson. In some cases, teachers referred to having taught the course "last year" or "for so many years" that they knew what to expect and were prepared to respond in ways they otherwise might not. Other times, this experience was as recent as the previous lesson, with teachers acknowledging that their response came in light of "what happened last period."

The themes outlined above show that teachers' motivations for their responses to PTMs vary. Teachers often factor in knowledge they have of the flow of a lesson, the students themselves, and other factors not known to an observer when determining how to respond to a PTM. This suggests that teacher responses to PTMs are more complex than a response classification may make them seem.

Research Question 2: Teacher Perceptions of their own Knowledge Development

This section presents results to answer the second research question: How, if at all, do six practicing secondary mathematics teachers perceive that their own knowledge

development relates to pivotal teaching moments and their responses to them? As defined in this study, the term, *teacher knowledge development* refers collectively to the development of teachers' content knowledge (CK) and pedagogical content knowledge (PCK). However, for the purposes of this section, results related to teachers' PCK development appear first, followed by results related to teachers' CK.

Teacher Perceptions of PTM Relations to PCK Development

The researcher analyzed interview and reflection journal data to gain insight into how teachers perceived that their pedagogical content knowledge developed in the moment and how it related to the occurrence of PTMs. While the teachers in this study did not directly mention the term pedagogical content knowledge when talking about their own in-the-moment learning, their comments led the researcher to identify three main evidences of teacher PCK development: changes made between the two observed lessons, teacher anticipated changes to future lessons, and direct comments referencing teachers' "in the moment" learning. Within each of these three classifications, the researcher identified several themes relating to the kinds of changes, or learnings, teachers experienced.

Changes made between the two observed lessons. Because the researcher observed two deliveries of the same lesson, she was able to identify changes teachers made between the lessons. While some alteration occurs naturally between classes due to differing dynamics (e.g., individual groups of students, class size, time of day), for this analysis, the researcher limited her focus to substantive changes in the delivery, sequencing, or content of the lesson. The researcher identified these changes in all six

teachers' classrooms and on multiple visits, indicating that these changes were not isolated events, but rather possible indications of ongoing teacher PCK development. The researcher identified three themes capturing the types of changes observed: clearer instructions and/or questioning, altered sequencing or scaffolding, and intentional adjustments to instruction or discussion.

Clearer instructions and/or questioning. In many cases, the researcher noticed that teachers provided clearer instructions for portions of a lesson during the evaluation observation than they had during the identification observation. For example, before turning time over to students to explore features of radical functions, Claire was intentional about clearing up aspects of the activity with which students in an earlier class had struggled. Consequently, she provided detailed guidelines about what she wanted students to look for and discuss during the activity that she had not provided to the first class.

In other cases, teachers asked questions in one class that encouraged thinking differently, or more deeply about mathematical content. For instance, in trying to determine which function would map the inputs 2, 5, and -3 to outputs 4, 25, and 9 respectively, students in one class indicated that the function was "multiplying a number by itself." While correct, students struggled to express this function algebraically as $y = x^2$. In a later class, however, some students identified the algebraic representation, while others suggested that the input was being "multiplied by itself." During this second lesson, Arthur asked students whether these two ideas were the same thing, prompting a brief discussion in which students in the second class were able to make the connection

between the two interpretations. Arthur did not make this connection directly during the first lesson.

Altered sequencing or scaffolding. It was common for teachers to alter the sequencing of tasks within their lesson, or to provide a different level of scaffolding for one class compared to another. Sometimes this manifested itself in teachers deleting from or adding to their notes, presentation slides, or other instructional resources between classes. Other times it consisted of teachers doing more examples with one class, or directly altering expectations for one group compared to another. Occasionally this was effective in helping students better understand the task or lesson content. Rachel effectively used pre-questioning and scaffolds before students entered into an exploratory problem that helped prevent errors that had arisen in the earlier class. However, under similar circumstances, Kathy felt that her increased scaffolding was a disservice to the students, saying, "I lowered the cognitive demand for [second period] because I was worried about time. I provided too much scaffolding. I feel like I did [them] a disservice by not asking all the questions." This instance suggests that, although there are some instances when a teacher learns from one class and it results in a more refined delivery during the second lesson, a teacher's alteration to the lesson does not always result in instructionally "better" moves from one class to another.

Intentional adjustments to instruction or discussion. During the researcher's classroom observations, it was evident that sometimes teachers made intentional adjustments to how they taught, or how they guided discussion within a lesson. Linda admitted that during her early delivery of a lesson, she learned "what was and wasn't

important" in the lesson and so in her subsequent classes she was able to more carefully guide the lesson toward a desired goal. In this instance, the result was a much deeper discussion on discrete and continuous functions in the last period of the day than had occurred during earlier lessons. Arthur also indicated this kind of learning:

> When it's the first time you're teaching a lesson in a class, for you, it just feels so terrible. But by the time I got to fourth period, the questions students came up with were ones I really liked. … [their questions] were more interesting than my intended focus, so I was learning right along with them.

Thus, at times, PTMs and in-the-moment classroom circumstances encouraged teachers to change direction, resulting in different, often deeper, conversation and learning for both students and teachers.

Teacher anticipated changes to future lessons. Teacher interview and reflection data suggested that teachers often completed a lesson anticipating they would make future changes in one of two ways: to their instructional approach within a lesson, or to the lesson itself.

Anticipated instructional changes. Teachers often anticipated that they would make changes to how they approached the observed lesson in the future. Anticipated changes included altering the sequencing within a lesson, or the sequence of the lesson itself in relation to other lessons taught during the school year. Teachers suggested that such changes would help make parts of the lesson flow better, or would allow them to connect different parts of the curriculum. Making more connections between parts of a lesson, or to other aspects of the curriculum was also a commonly anticipated change. In one case, Linda questioned whether her discussion on average rate of change might fit well with a discussion on lines of best fit, allowing students to explore connections

between the two. Other teachers (Kathy, Arthur, and Rachel) suggested they would make better use of student work, increasing or decreasing emphasis on certain strategies, or representations used by students to solve problems. Claire often anticipated following up on ideas that surfaced during a lesson that she hadn't initially intended to discuss, indicating flexibility in letting the flow of a lesson guide future lesson development.

Anticipated lesson changes. Five of the six teachers in this study indicated that they anticipated making changes to the lesson itself. In some instances, teachers anticipated changing the lesson format in some way to involve more student discussion, or to involve students more directly in discovering relationships, or connecting new learning to prior learning. Other teachers anticipated including a wider variety, or different examples to guide student learning.

Direct comments related to "in the moment" teacher learning. In addition to commenting on anticipated instructional or lesson changes, several teachers commented on how they learn along with their students and in light of PTMs and other classroom factors. Teachers also commented on how being asked about their anticipations for a lesson (in the lesson plan outline), or their reflections after a lesson (in interviews and the participant reflection journals) aided their own learning.

Teachers' learning in the moment. During separate school visits, Arthur commented on how students' approaches to the content shaped his understanding of their thinking and how he might better approach the topic in the future.

> As time went on, I think I learned where students were going to struggle, so I could jump right there in later classes. … part of what makes a good teacher is recognizing what makes a problem easier and more accessible to students, so

> when I saw [students using particular approach to a problem] I thought 'I need to add that to my repertoire.'

Claire echoed this sentiment: "I was startled by how they approached some problems, so in the moment I had to try to figure out exactly what I wanted them to say and where I wanted them to get" and Linda confirmed, "I love seeing how the kids think because that changes my teaching. When I can see what their perspective is, it give *me* a different perspective and I teach things differently" (emphasis in the original).

These statements indicate that teachers might not always know ahead of time where a particular lesson will lead, or in what direction student insights might take the lesson. Consequently, they must draw on their own PCK to pull the conversation back, guide it in the intended direction, or allow it to morph into something unintended, but worthwhile. Perhaps the most profound insight offered in this regard came from Kathy:

> When I do it the first time, in first period, I think I know what's going to happen and I think I know what kinds of questions I'm going to ask. But after I've gone through the whole first period, what's going through my mind is 'this is a great question to ask,' 'oh, I need to make sure I'm focusing on this aspect of it because in the future we're going to be talking about this,' or 'I need to make sure that I clarify this because I didn't do it well in first period, so I need to do it better in second or third period.' So, all of those things, all those observations of my teaching,…the way that I format my lesson changes from how I taught it first period because this is how I envision my lesson going, but then reality hits and this is how it actually goes. So first period, for me, is always more rough just because it's taking my vision and putting it into practice, but then second period is *taking my experiences from my vision and how I did it and then refining it* for second period. And then, third period is taking what I did in first and second period, *learning from those experiences and refining it even more*. So, for me, teaching is about continual refining … (emphasis added).

This supports that teaching is simultaneously an act of knowledge demonstration and knowledge creation, but the knowledge creation referenced by Kathy goes beyond the

specific mathematical content to teachers' learning about how to best present the content, that is, a teachers mathematical PCK.

Teachers learning from anticipations and reflections. Teachers in this study also acknowledged that they learned in the face of having to think about the lesson prior to and beyond its delivery. For example, Melissa stated: "Things change when I think about it ahead of time and write it down," and Linda commented, "Talking to you [the researcher] kinda helps me solidify my ideas and think, 'Okay, here's how I'm going to go about it tomorrow.'"

While the depth of reflection exhibited by teachers in this study through their reflection journals varied, the researcher often found that the conversations between the teachers and the researcher, both during and following the interview, provided insights that teachers had not previously considered. These instances contributed to teachers' content knowledge development and pedagogical content knowledge development.

Results presented in this section confirm that understanding how teacher knowledge develops is not a simple task (Ball, Thames, and Phelps, 2008) and that teachers' reflection-in-action (Schön, 1984) leads them to make subtle, as well as more complex changes to a lesson, as they effectively experiment with how to best present the content. Further, reflection-on-action (Schön, 1984) provides teachers with the opportunity to internalize what they have learned from one class to another and to make appropriate changes to future instruction.

Teacher Perceptions of PTM Relations to CK Development

Teacher perceptions on the development of their content knowledge were less prolific than perceptions on their pedagogical content knowledge development. However, four teachers indicated that their content knowledge evolved in some way during at least one lesson. Throughout the participant interviews, the researcher noticed that teachers often indicated that PTMs raised issues that were not "new" to them, but that made them think differently about the content. Such comments seemed to suggest that teachers did not associate their thinking about content differently, or more deeply, as affecting what they themselves knew about mathematics. However, Shulman (1986) suggested that *subject matter knowledge* includes the "amount and organization of knowledge … in the mind of the teacher" (p. 9). Consequently, the researcher considered instances where PTMs led teachers to think differently about mathematical content as evidence of a relationship between PTMs and the development of teachers' mathematics content knowledge. The researcher identified two themes (definitions and structure) summarizing six instances of PTMs promoting teacher content knowledge development. A summary of each theme appears below.

Definitions. Melissa and Linda both experienced PTMs that prompted reflection around definitions and the refining of their understanding of mathematical definitions. In Melissa's lesson on probabilities of independent and dependent events, a student, confused about what constitutes independent events asked about whether tossing a coin and getting heads and then tails would be dependent. This caused some confusion for Melissa who paused to try to figure out how to explain the definition differently. Following the lesson, Melissa admitted that she had forgotten that students confuse

independence of individual coin tosses with the overall outcome of the two tosses and that they consequently seek to identify the *outcome* of the tosses as dependent or independent, instead of identifying the individual tosses themselves as being independent. At the conclusion of the interview, the researcher sensed some residual uncertainty on Melissa's part as she commented that she would have to "come back" to this idea in the future to clarify this nuance with students.

Linda experienced similar uncertainty surrounding identifying a function as being discrete or continuous. The researcher noticed throughout the lesson that Linda seemed reluctant to provide evaluative conclusions related to whether examples presented in class represented discrete or continuous functions. When asked why this was the case, Linda admitted:

> I'll be honest. I think that before today's lesson, I think that in my mind I felt like I knew when a graph should be discrete and when it should be continuous, and to me, before today's lesson, it was based on the independent variable only. But then today during a lesson, it was based on the dependent variable, so then I started thinking maybe I don't know what forces something to be discrete or continuous, and so, I kinda thought, maybe I better not definitively tell the kids because I wasn't sure.

Following the interview, continued conversation with the researcher on this topic seemed to refine Linda's understanding of discrete and continuous functions further and she commented in her reflection journal that she spent time that night researching the topic and further refining her understanding. According to Linda, this energized her to think even more deeply about other facets of functions.

Structures. Three of the teachers experienced content knowledge growth related to their understanding of mathematical structures. However, the depth of this knowledge

development varied widely between participants. Following PTMs related to function notation and the similarities and differences inherent in notations such as $f(x)$, $f(2)$, $f(x) = 2$, and $f(x) - g(x)$, Rachel commented:

> This is probably the first year that I realized the different use of parentheses for $f(5) = 3(5) + 1$ versus $f(x) - g(x) = (3x + 1) - (5x - 7)$. When plugging in a number, I think students got used to parentheses; however, putting the parentheses around the function being substituted was not as clear because it doesn't match the parentheses on the left side of the equation.

This statement indicates an evolution in Rachel's awareness of how, in the context of function notation, parentheses are used to indicate the independent variable, or its value within the function, as well as to separate different functions when performing operations on functions. While perhaps a trivial recognition, this contributed to Rachel's own CK and her understanding of how to help students struggling with the diverse use of parentheses (i.e., her PCK).

Melissa also learned to see mathematical structures differently in the face of a student question regarding identifying whether two variables are directly proportional. The approach of the lesson was to suggest that any relationship in the form $y = kx$ represents a direct relationship between y and x and that k is referred to as the constant of proportionality. This traditional interpretation of direct variation was challenged by a student who questioned whether it was necessary to have this form, because a relationship in the form $x = ky$ would also exhibit direct variation. In the interview, Melissa commented that she had not realized that direct variation exists anytime one variable is a multiple of the other and that the constant of proportion is just different. In

light of this, Melissa observed, "My understanding of the math evolves as I'm forced to look at how kids look at it."

Finally, Kathy's mathematical content knowledge evolved on two levels related to writing and solving linear inequalities. The first instance occurred while students were exploring other ways of writing inequalities expressed as conjunctions and disjunctions. Presented with the disjunction $x < 3$ or $x > 6$, some students wanted to write $3 > x > 6$. During the interview, Kathy expressed that she did not like the classroom conversation that occurred with this example and she removed it from later lessons. However, in subsequent conversation with the researcher during and after the interview, Kathy questioned whether consolidating a disjunction in this way is mathematically accurate, or inaccurate. She remained incredibly uncomfortable with this idea and seemed to settle on avoiding the conversation in future discussions with students.

A related instance of content knowledge development for Kathy came following a student question about why the inequality symbol(s) switched directions when dividing by a negative number during the solution process (e.g., why $-x < 5$ becomes $x > -5$). Her response to the student and subsequent discussion with the researcher in the interview indicated a tentative and complex understanding of why this happens. Kathy even stated that, while there was a reason why this happened, and she felt like she understood the reason, it was a complicated thing to try to explain to ninth graders. A post-interview discussion with the researcher on this topic, and further reflection, culminated in Kathy coming to a different, but more straightforward, understanding of the "why" behind this PTM.

The above summaries support that PTMs can act as a catalyst for refining and deepening teachers' knowledge of mathematics and that reflection upon PTMs further aids teacher content knowledge development.

Research Question 3: Similarities and Differences among Participants

This section presents results relevant to answering the final research question posed in this study: What are the similarities and differences among six secondary mathematics teachers' with varying years of teaching experience in their (a) responses to pivotal teaching moments and (b) perceptions of how their responses and knowledge development are related? To facilitate the cross-case comparisons necessary to answer this question, this section begins with a participant profile summary for each of the six teachers who participated in this study. These profiles form the basis for the comparisons presented in this section. A summary of the profiles appears in Table 12.

Table 12

Summary of Participant Profiles

Participant	Teaching Experience	School Type	Observed Instructional Approach(es)
Arthur	14	Public 7 – 9	Direct instruction, whole group discussion, and group/partner work
Claire	19	Public 9 – 12	Guided discovery with small groups
Kathy	13	Public 7 – 9	Task-based launch, explore, discuss cycles
Linda	8	Public 7 – 9	Direct instruction, whole group discussion
Melissa	13	Charter K – 8	Direct instruction – Saxon Math ®
Rachel	5	Public 9 – 12	Guided notes, group activity and/or partner work

Note. Shows years of experience teaching mathematics, type and grade level of school, and observed instructional approaches.

Participant Profile Summaries

The researcher collected data related to each participant's pre-lesson anticipations (lesson plan outline), during lesson actions (identification and evaluation observations), and post-lesson reflections (reflection journals). Consequently, the participant profiles presented here include demographic data as well as a summary of each of the above referenced components. Profiles appear in alphabetical order.

Arthur. Arthur is a 42-year-old white male teaching at a public junior high school serving students in Grades 7-9. He has a master's degree in mathematics education, 14 years of experience teaching mathematics, and has been at his present school for five years. This is his first year teaching 8^{th} Grade mathematics. In the observations comprising this study, Arthur's instructional approach was a blend of lecture with efforts at Socratic discussion. He employed a mixture of direct instruction, class discussion, and group/partner work. Arthur's anticipations about his lessons indicated that he believed students would find visual aspects of the lessons easy while struggling with the more algebraic aspects of the lessons. He anticipated students would ask primarily procedural questions and planned to address student questions by exploring and sharing student thinking. Arthur's post-lesson reflections gravitated toward changes he would, or might make to future lessons and the potential impact changes would have on the student experience.

Claire. Claire is a 43-year-old white female teaching at a public high school serving students in Grades 9-12. She has a master's degree in mathematics, 19 years of experience teaching mathematics, and has been at her present school for 12 years. She

has been teaching 11th Grade mathematics for at least the last 12 years. In the observations comprising this study, Claire's instructional approach was guided discovery in which small groups of students worked together and she acted as facilitator for their exploration of mathematical ideas. Claire anticipated primarily procedural questions as trouble spots for students and planned to address each by referring to examples, or using questioning to refine and guide student thinking. Claire's post-lesson reflections were limited, with most of her reflection taking place in real-time during the interviews as she discussed what went well or poorly and what was surprising to her.

Kathy. Kathy is a 37-year-old white female teaching at a public junior high school serving students in Grades 7-9. She has a master's degree in mathematics, 13 years of experience teaching mathematics, and has been at her present school for 2 years. This is her second year teaching 9th Grade mathematics. Kathy relied on an instructional approach that involved the launching, exploration, and discussion of selected tasks and models instructional ideas introduced in *5 Practices for Orchestrating Productive Mathematical Discussion* (Stein, Smith, and National Council of Teachers of Mathematics, 2011). Kathy's anticipations about her lessons indicated that procedural aspects of the lessons would typically be what students found easy, while conceptual ideas would be more challenging. She planned to address student questions using methods matching the type of questions students asked. Kathy rarely offered post-lesson written reflections, but was highly reflective in discussions with the researcher during interviews.

Linda. Linda is a 50-year-old white female teaching at a public junior high school serving students in Grades 7-9. She has a bachelor's degree in mathematics, 8 years of experience teaching mathematics, all of which are at her present school. This is her third year teaching 9th Grade mathematics. In the observations comprising this study, Linda relied almost exclusively on direct instruction with periods of whole class discussion. Linda's anticipations about her lessons indicated that she anticipated students would ask primarily procedural questions and she planned to address student questions by referring to examples, rules, or processes. Linda's post-lesson reflections were deeper with more conceptual lessons and often extended beyond the day of my visit. She was open to talking things through with others to refine her understanding of mathematics.

Melissa. Melissa is a 50-year-old white female teaching at a charter school for students in Grades K-8. She has a bachelor's degree in mathematics and 13 years of experience teaching mathematics, all of which are at her present school and with students in Grades 7 and 8. In the observations comprising this study, Melissa's instructional strategies were exclusively lecture-based. She attributed this to a school-wide emphasis on adhering to a Saxon Math® curriculum. Melissa's anticipations about her lessons centered exclusively on procedural aspects of the lesson and she anticipated responding to student questions by providing instruction that pre-empted questions and then relied on examples to clarify ideas. Melissa's post-lesson reflections were limited and she seemed primarily to reflect on ways to improve a lesson.

Rachel. Rachel is a 28-year-old white female teaching at a public high school serving students in Grades 9-12. She has a bachelor's degree in mathematics, 5 years of

experience teaching mathematics, and has been at her present school for 4 years. This is her fifth year teaching 9th Grade mathematics. In the observations comprising this study, Rachel relied on guided notes followed by a group activity, or partner work. Rachel's anticipations about her lessons focused on conceptual aspects of the lesson, with her anticipating students would find low-level conceptual ideas easy and higher-level conceptual ideas challenging. She anticipated addressing student questions by emphasizing meaning and making connections to prior learnings. In her post-lesson reflections, Rachel often brainstormed thoughts about what she might do in the future and how she might help students think about the underlying concepts.

Similarities and Differences in Response to PTMs

To examine the similarities and differences in responses to PTMs across participants, Table 13 shows each participant's responses as a percentage of their total responses. Consistent with results presented in answering the first research question, the largest percentage of responses for all but one participant (Linda) were emphasizes meaning responses (EM). The overwhelming reliance of the teachers in this study shows that despite a teacher's experience in the classroom, or the instructional strategies employed, a primary response is to turn to definitions, rules, and procedures to address PTMs. However, the fact that a pursues student thinking (PT) response was the next most frequent response suggests that teachers in this study often solicited more insight about what students were thinking before responding to a PTM.

Table 13

Percentage of Pivotal Teaching Moment Responses Exhibited by Participant

	Response Type							
Participant	ID	AC	EM	PT	EC	AE	GQ	RO
Arthur	15.4	7.7	23.1	23.1	15.4	–	7.7	7.7
Claire	–	9.1	45.5	36.4	–	–	–	9.1
Kathy	10.5	5.3	31.6	21.1	–	–	–	31.6
Linda	5.3	42.1	26.3	21.1	5.3	–	–	–
Melissa	–	4.5	63.6	22.7	4.5	–	4.5	–
Rachel	–	5	80	5	5	5	–	–

Note. Shows the percentage of PTM responses by type for each participant. – indicates no observed PTMs of a given type. PTM = pivotal teaching moment. ID = ignore or dismiss, AC = acknowledge, but continue as planned, EM = emphasize meaning, PT = pursue student thinking, EC = extends or makes connections, AE = follows up with additional examples, GQ = asks guiding questions, RO = refer to others.

Two differences in Table 13 are of particular interest. First, although Linda's primary response was to acknowledge and continue as planned (AC), three of her eight responses of this type occurred during the first school visit during which she admitted that her own understanding of the mathematical content was tentative. Accounting for these instances as evidence of internal uncertainty about the mathematical content resulted in Linda's most common responses being to emphasize meaning (EM) and to acknowledge, but continue as planned (AC). Overall, teachers in this study overwhelmingly relied on an EM response to PTMs. Interestingly, however, during this same lesson, Linda commented that her initial responses to reserve judgment and acknowledge, but not directly address PTMs actually led to deeper student conversations. According to Linda, her effort to leave "loose-ends" by not immediately responding to PTMs became more deliberate as the day progressed because she noticed that, by reserving judgment, more students became engaged in the conversation and this led to

deeper mathematical conversations. This suggests that, what may appear to be an undesirable response to a PTM (AC) may, if handled carefully, lead to other PTMs and desirable student outcomes. In other words, it may be possible that teachers' chosen response to a PTM actually generates new PTMs. Additional evidence for this came from Kathy's classroom where her purposeful selection of which students to call to the front of the classroom to share their thinking often allowed her to ask deeper questions and shape instruction from basic to more complex understandings.

The second difference of interest is that Kathy, who tended to begin her classes with a context before progressing toward more abstract thinking, responded by referring to others (RO) with equal frequency as emphasizing meaning (EM) and with higher frequency than any other teacher in the study. This raises the possibility that Kathy's exploratory task-based instructional approach allowed her to respond to PTMs differently than those teachers employing more direct instructional approaches.

Similarities and Differences in Perceptions of Knowledge Development

All of the teachers in this study acknowledged that their own knowledge developed in light of PTMs and other classroom factors. Several teachers commented that because they had taught a particular lesson before they were better prepared to know what students would find problematic and effectively address issues. At the same time, teachers also lamented that, because they *had not* taught a lesson before, they were still trying to figure out how to best present it in a way that was accessible to students. This study found no evidence to suggest that such comments occur in relation to how many years of experience teachers had been teaching mathematics. However, Arthur, Linda,

and Kathy, at times, indicated that because they had not *taught this particular class* for very long, they were still working out kinks in their approach to the lesson. This is significant because, of the six teachers in this study, only these three teachers have been teaching at their current grade level for three years or less. While overall teaching experience may not be related to teachers' knowledge development, years of experience at a particular grade level (i.e., with particular content) may influence how teachers' knowledge evolves.

Chapter Summary

The results presented in this chapter support the PTM and teacher response classifications identified by Stockero and Van Zoest (2013) while also identifying one additional PTM type and three additional types of teacher responses. Teachers with more direct instructional approaches tended to experience a higher number of incorrect mathematics (IM) PTMs while teachers with more blended or exploratory approaches experienced primarily sense making (SM) PTMs. The most common type of teacher response observed in this study was emphasizes meaning (EM) and this response occurred independent of teaching experience, instructional strategy used, or other teacher demographics.

Results from this study suggest seven motivating themes for teacher responses to PTMs: failure to notice, inaccurate anticipations about student trouble spots, misunderstanding what a student was asking, maintaining or building the flow of a lesson, consideration of student or class individualities, pushing students to do the thinking, and responding based on experience. Despite identification of these themes,

motivations varied among teachers and were often more complex than a simple categorical classification.

The teachers in this study also experienced the development of their content knowledge (CK) and their pedagogical content knowledge (PCK). PTMs and subsequent reflection on them, at times, acted as a catalyst for teacher CK development. For teachers in this study, the researcher identified CK development related to mathematical definitions and mathematical structures. The researcher also identified three evidences of teachers' PCK development in the face of PTMs: changes made between observed lessons, teacher anticipated changes to future lessons, and comments related to learning in the moment. Consequently, results of this study suggest that it is possible for both CK and PCK to develop in relation to PTMs and teachers' responses to them.

Finally, in examining similarities and differences among the teachers in this study, results suggest that it may be possible for teachers themselves to generate PTMs by simply reserving their responses to an initial PTM. Further, while this study did not identify any indicators that teachers who taught mathematics for a different number of years experienced PTMs or knowledge development differently, there was evidence to support that teachers' instructional strategies (direct instruction versus exploratory or task-based), and their level of experience teaching specific mathematical content, may relate to their responses to PTMs and their development of PCK.

CHAPTER 5

DISCUSSION

The purpose of this study was to explore how secondary mathematics teachers responded to pivotal teaching moments (PTMs) and how the teachers perceived that their own content and pedagogical content knowledge was related to PTMs and their responses to them. This chapter offers a summary of the study as well as a discussion of the results, limitations, conclusions, and recommendations for further research.

Study Overview

The research questions addressed in this study were:

1. How do six practicing secondary mathematics teachers respond to pivotal teaching moments?
2. How, if at all, do six practicing secondary mathematics teachers perceive that their own knowledge development relates to pivotal teaching moments and their responses to them?
3. What are the similarities and differences among six secondary mathematics teachers' with varying years of teaching experience in their (a) responses to pivotal teaching moments and (b) perceptions of how their responses and knowledge development are related?

The researcher collected observation, interview, and reflection journal data from six, practicing secondary mathematics teachers and analyzed the data using several coding methods. Initial coding cycles culminated in the generation of themes related to data

sources and participants. The researcher used these themes to generate participant profiles that she used to answer the research questions. The results inform mathematics education professionals regarding the existence of pivotal teaching moments (PTMs), teacher responses to PTMs and their motivations for those responses, and how teachers' own knowledge development may be related to PTMs and teachers' responses to them.

Discussion of Results

The results of this study suggest that practicing secondary mathematics teachers respond to PTMs in identifiable ways and that their responses vary and are motivated by many factors. This study confirmed the existence of six types of PTMs, and eight classifications for teacher responses to PTMs and identified seven themes related to teachers' motivations for their responses to PTMs. These results confirm prior classifications of PTMs and PTM responses while also extending the research and offering insights into teachers' motivations for responding in identified ways. This study also provides support for the development of teacher knowledge in response to PTMs. Although the study examined data from secondary mathematics teachers with varying levels of experience teaching mathematics, only teachers' experience teaching particular content seemed to differentiate the teachers in terms of their development of mathematics content knowledge and mathematical pedagogical content knowledge. The remainder of this section provides further discussion of these results.

The Existence of PTMs

Results of this exploratory study confirm that practicing teachers experience the same types of PTMs and in large part, exhibit the same types of responses to PTMs as do beginning teachers. The types of PTMs observed in this study overwhelmingly support the validity of the framework proposed by Stockero and Van Zoest (2013). All but one of the 88 observed PTMs fell within their classifications. At the same time, the identification of a sixth PTM classification in this study (*unconventional, but accurate, mathematical process*) suggests that future research may continue to identify additional, and refine existing, PTM classifications.

Because teachers in this study employed both direct instructional strategies as well as more student-centered, exploratory strategies, the results confirm Stockero and Van Zoest's (2013) assertion that PTMs can occur in all types of classrooms. Further, the variety of PTMs experienced by the teachers in this study suggests that manifested PTM types may be independent of particular instructional strategies employed, or how long a teacher has been teaching mathematics.

Teacher Responses to PTMs

This study identified 93 teacher responses to PTMs that fit within the framework offered by Stockero and Van Zoest's (2013). However, the researcher also identified 11 teacher responses that warranted three additional teacher response classifications (*provides additional examples, asks guiding questions,* and *refer to others*). This suggests that classifying teacher responses may involve more nuance than classifying PTM types, a conclusion supported by the fact that in some cases teachers exhibited a sequence of varied responses to a single PTM. Alternatively, because the teachers in this

study had more experience than those in previous studies, it may be that teachers with more classroom experience exhibit different responses to PTMs than preservice or beginning teachers.

In their work with beginning teachers, Stockero and Van Zoest's (2013) found a nearly uniform distribution across teacher response types, while teachers in this study were significantly more likely to exhibit an emphasizes meaning (EM) response and significantly less likely to exhibit an ignores and dismisses (ID) or extends or makes connections (EC) response. Based on the analyses conducted in this study, it was not possible to determine whether these variations were attributable to differences in the total number of observed responses, to the participants themselves (e.g., to teaching style, or years of teaching experience), or to some other factor (e.g., observer bias). These results highlight the need for more research on *why* teachers exhibit certain responses to PTMs.

Teacher Motivations for PTM Responses

Results from this study led to the identification of seven themes explaining teachers' motivations for their responses to PTMs. Sometimes teachers' responses were attributable to a failure to notice the PTM. While failure to notice a PTM does not remove the significance of its occurrence, it does help explain why teachers sometimes fail to respond in ways that an observer might deem effective. Additionally, the results suggest that ineffective teacher responses are also attributable to teachers' inaccurate anticipations about aspects of a lesson students would find difficult, or misunderstandings about what students are asking. This highlights the importance of helping teachers to respond in ways that allow them to further clarify what, or how, students are thinking

about the mathematics (i.e., a pursues student thinking (PT) response). Perhaps when teachers more clearly understand student mathematical thinking, they are better able to respond to PTMs in effective ways. Effective implementation of a response plays a key role in how the PTM affects student learning (Stockero & Van Zoest, 2013). Thus, more than just soliciting student thinking, teachers must learn to do so effectively to have the greatest impact on their students' learning.

Many times in this study, teachers who chose to acknowledge a PTM only superficially did so because they were trying to build the flow of a lesson, or maintain the existing flow of a lesson. In some of these instances, teachers returned to the idea(s) put forth in an initial PTM at a later point in the lesson. Thus, a teacher's anticipations and knowledge of how a lesson will progress from beginning to end may provide motivation for their responses to PTMs. Similarly, teachers in this study often made decisions to respond to PTMs in ways that accounted for student or class individualities. Thus, a teacher's knowledge of her students and of dynamics related to particular groups of students may lead a teacher to respond differently to the same PTM under differing circumstances. This suggests that effective teachers are able to weigh student and classroom factors in the face of unanticipated classroom moments and respond according to those unique factors. In other words, effective teaching requires adaptive, rather than static responses to in-the-moment events.

Teachers in this study also frequently responded to PTMs in ways that they believed would push students to do the mathematical thinking. Here again, it must be acknowledged that what may appear to be an ineffective response to an onlooker, may in

actuality be an attempt by the teacher to refrain from “spoon feeding” content to her students. This response may lead to visible impact (positive or negative) on student learning in the immediate moment, but if students are pushed to think more deeply, the full impact of a teacher’s response to a PTM may not manifest itself until later lessons. Thus, while Stockero and Van Zoest (2013) attempted to analyze how effectively a teacher implemented a response to a PTM, results from this study suggest that caution is in order when making such evaluations because the impact of a teacher’s decision to push student thinking may not immediately manifest itself.

Finally, there were several instances in this study where teachers’ responses to PTMs came because of their prior experience with the content or with a particular lesson. While this experience had sometimes come recently (i.e., the previous class period), teachers in this study also cited their learning from prior years as a motivating factor in their responses to PTMs. This result points to connections between teacher knowledge development and their responses to PTMs.

Teacher Knowledge Development in Relation to PTMs and Teacher Responses

This study explored teacher perspectives on their own knowledge development in the face of PTMs and in connection with their responses to PTMs. Results suggest that teachers may experience changes to their mathematics content knowledge (CK) and mathematical pedagogical content knowledge (PCK) during teaching. More specifically, there exists a relationship between teachers’ knowledge development and PTMs.

Teachers in this study frequently made intentional changes to their lessons between one class and another, providing clearer instructions, asking deeper questions of

students, or altering sequencing of content. Additionally, teachers in this study anticipated making instructional changes, or directly commented on their own knowledge development along with their students' knowledge development during a lesson. Much of current research questions how teacher PCK develops (Chan & Yung, 2015; Park & Oliver, 20008; Shulman, 1986; van Driel, de Jong, & Verloop, 2002). This study lends credibility to growing evidence that PCK, at least in part, develops during teaching.

Separate instances of teacher content knowledge development in this study suggest that PTMs can act as a catalyst for refining and deepening teachers' CK related to definitions and to mathematical structures. This result is significant because it confirms that teaching is simultaneously an act of knowledge demonstration and knowledge acquisition. Additionally, while an increasing number of scholars have found support for the development of PCK in the moment (Chan & Yung, 2015; Hashweh, 2005; van Driel, de Jong, & Verloop, 2002), this study lends credibility to the idea that some elements of CK also develop in the moment.

Teachers in this study acknowledged that their knowledge continued to evolve as they reflected (either in written form or during interviews) about their lesson planning, lesson execution, and responses to PTMs. Despite the differing levels of reflection exhibited by teachers in this study, the results support Park and Oliver (2008) who found that PCK develops due to reflection-in-action as well as reflection-on-action. Further, results of this study confirm that, even if teachers are unaware of their knowledge development in the moment, reflection-on-action has the potential to draw teachers' awareness to their knowledge development later (Taylan & da Ponte, 2016).

Many studies exploring the development of PCK in teachers have been conducted with pre-service, or early-career teachers (Taylan & da Ponte, 2016; van Driel, de Jong, & Verloop, 2002). Researchers conducting these studies, and studies related to PTMs and teachers' responses to them, have argued for the examination of how mid- and late-career teachers' knowledge develops. While this study included participants representing a range of career experience, the only observed difference in teacher knowledge development was in relation to how long the teachers had taught particular content, rather than the teachers' overall teaching experience. This result highlights a need for continued examination of the knowledge development of teachers at various levels of their career.

Limitations, Recommendations, and Conclusions

This section describes the limitations associated with this study, recommendations for future research, and offers conclusions based on the results.

Limitations

One limitation of this study was that the results relied on the researcher's subjective observations and interpretations of those observations. While the researcher made efforts to adhere to established observation and interview protocols, the results presented here still reflect the researcher's interpretation of the meaning behind what she observed and discussed in participant interviews. The researcher did not solicit teachers' motivation behind their response to every PTM, and may inadvertently overlooked some reasons behind teachers' decisions. More objective methods may exist for ascertaining

potential relationships between teacher knowledge development, PTMs and teacher responses to PTMs.

A second limitation relates to a lack of diversity in examining teachers from different regions of the country, or different ethnic backgrounds. All of the teachers in this study were Caucasian and all taught in schools with limited number of minority students. A study with teachers and classrooms representing a variety of backgrounds and socioeconomic standing would provide a well-rounded understanding of how teacher knowledge develops in diverse settings.

Finally, this study examined only teachers' interactions with a whole class, or small groups of students. The researcher observed several one-on-one interactions between students and teachers, but did not identify PTMs and teacher responses associated with these interactions. It is possible that teachers exhibit different responses to individual students than they do during whole-group instruction. Consequently, future research should include examination of these interactions as well.

Recommendations

Based on the results of this study, this section presents three recommendations for developing greater understanding of how teacher PCK develops in the moment.

Generally, for teachers in this study, the second delivery of a lesson was smoother and more refined. Teachers generally agreed that, as the day progressed, they became more comfortable with the lesson as well as how to enact it more effectively. Most would agree that teachers bear responsibility for student knowledge development and that a major part of teaching mathematics involves teacher efforts to improve student

understanding of mathematical content. However, if student generated PTMs help teachers refine their own knowledge of the content, then the level of student engagement in a lesson may help drive teacher learning. In other words, teachers may be a driving factor behind student learning, but students may also be a driving factor behind teacher learning, making classroom teaching a symbiotic relationship wherein teachers and students share power to influence learning. Future research exploring potential relationships between instructional strategies and the development of both teacher and student knowledge could provide insights into the nature of this symbiotic relationship as would exploration of potential relationships between specific types of PTMs, or teacher responses to teachers' content knowledge and pedagogical content knowledge development.

While this study was able to confirm the existence of PTMs and support that they influence teachers' knowledge development, a remaining uncertainty is whether a teacher's actions can actually lead to the purposeful creation of PTMs. During classroom observations, the researcher noticed that sometimes PTMs followed a teacher's well-crafted sequencing of classroom activities or tasks. When asked about these moments during the interview, teachers rarely stated they had purposely intended to solicit a particular question (or PTM). A suggestion for future research is to explore the degree to which teachers' choice or sequencing of instructional tasks generates PTMs. That is, do teachers' decisions about how to scaffold a lesson actually lead to the creation of PTMs? If so, how do teachers' responses to these somewhat anticipated PTMs differ from their responses to unanticipated PTMs?

Finally, to understand the role of teacher reflection in connection to teacher knowledge development, future research should examine whether teacher knowledge develops differently in the presence or absence of various types of teacher reflection. Results of this study support that teacher reflection-on-action through interviews played a role in teacher knowledge development. However, it remains unclear whether, and to what degree, teachers' natural inclination to reflect, and their effectiveness in doing so, results in deeper or more superficial teacher knowledge development.

Conclusions

This study confirmed that teachers often face unanticipated events requiring in the moment responses. Such pivotal teaching moments (PTMs), and teachers' responses to them have the potential to influence teacher understanding of mathematical content as well as their development of pedagogical content knowledge. Teachers' motivations for their responses to PTMs vary widely and it can be difficult for an observer to ascertain in the moment whether a particular response is ultimately effective or ineffective.

A key takeaway from this study is that the types of PTMs experienced by teachers, and the ways in which teachers respond to PTMs do not necessarily differ between teachers with varying years of experience teaching mathematics. However, teachers' familiarity with particular content and experience teaching specific content may have an influence on the degree to which their PCK develops in relation to the PTMs. Further, the degree to which a teacher reflects on her action seems to have a potential role in the nature of her knowledge development.

For those responsible for the preparation and professional development of secondary mathematics teachers, the implications of this study are that providing teachers with information on how to recognize and respond to PTMs may help further the development of teachers' knowledge. Further, providing preservice and practicing teachers with opportunities to observe and reflect with each other may serve to deepen teachers' CK and PCK.

Understanding how teacher knowledge develops remains key to preparing effective teachers. This study provides a foundation suggesting that it is possible to identify moments that influence teacher knowledge development and that teacher knowledge development occurs during the act of teaching. Still, further research on the role of instructional strategies, choices regarding instructional sequencing and the larger role reflection plays in teacher knowledge development will do much to refine our understanding of PTMs, teachers' responses to them, and the relation of each to teacher knowledge development.

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APPENDICES

Appendix A: Lesson Plan Outline

Date: __________ *Class/Period:* _________ *Teacher:* __________

Lesson Topic(s)	
List each instructional strategy you anticipate using to teach this lesson? (e.g. lecture, discussion, activity or task, group work, etc.)	Briefly describe ***why*** you plan to use each instructional strategy.

What do you anticipate students will find easy in this lesson? (Please be as specific as possible)

What do you anticipate students will find difficult in this lesson? (Please be as specific as possible)

What kinds of question(s) do you anticipate students will have during this lesson? (Please be as specific as possible)

How do you plan to address the question(s) you anticipate above? (Please be as specific as possible)

Appendix B: Participant Questionnaire

1. Are you currently licensed to teach secondary mathematics in Utah?

   Yes No

2. Which of the following best represents your preparation to teach secondary mathematics?
   - University-sponsored teacher preparation courses
   - Bachelors degree in mathematics and/or statistics
   - Masters degree in mathematics and/or statistics
   - Doctoral degree in mathematics and/or statistics
   - Bachelors degree in mathematics education and/or statistics education
   - Masters degree in mathematics education and/or statistics education
   - Doctoral degree in mathematics education and/or statistics education
   - Other (please specify): ____________________________________

3. What grade level mathematics courses do you **currently** spend **the majority** of your time teaching?
   - 7th Grade
   - 8th Grade
   - 9th Grade
   - 10th Grade
   - 11th Grade
   - 12th Grade
   - College level courses (AP, IB, CE)
   - Other (please specify): ______________________________

4. How many **consecutive years** have you been teaching at the grade level mentioned above?

5. What is the name of the school where you presently spend the majority of your time teaching?

6. How many consecutive years have you been teaching at this school?

7. With which gender to you most closely identify?

8. With which ethnic group(s) do you most closely identify?

9. What is your age?

10. Please complete the table below indicating your anticipated teaching assignment for the months of August through December of 2018.

Period	**Class Title**	**Approximate Times**

11. Please briefly describe what a typical day in your class looks like.

Appendix C: Observation Protocol

Date: ____________ *Class/Period:* ___________*Teacher:*_____________ *Int. Type:* I E

Identified PTM	Teacher Response	PTM Related Researcher Notes:
Extending Incorrect Mathematics Sense-Making Contradiction Confusion Other: _________________	Extends and/or Makes Connections Pursues Student Thinking Emphasizes Meaning Acknowledges, But Continues as Planned Ignores or Dismisses Other: ___________________________	
General Comments:		

Appendix D: Participant Reflection Journal

*Teacher:*___________________________________

In the time between now and our next scheduled observation, it is possible that you will have additional, or altered perspectives regarding the teaching moments we discussed in yesterday's interview. For example, you may have clearer perspective on your response(s) to the moments we discussed, or you may have an "ah-ha" moment regarding either the mathematical content the moments were centered around, or regarding how you might approach the instruction of the content differently in the future.

Please record any and all such thoughts here. I will collect these perspectives before my next visit.

Appendix E: Mathematical Content of Observed Lessons

Teacher	Visit 1	Visit 2	Visit 3	Visit 4
Arthur	Slopes of Lines in Context	Point-Slope Form of a Line	Review of Lines	Introduction to Functions
Claire	Analyzing Graphs of Radical Functions	Simplifying, Multiplying, and Dividing Rational Expressions	Review of Function Analysis	Approximating Surface Area and Volume of Real World 3D Shapes
Kathy	Functions and Features of Functions	Solving and Graphing Compound Inequalities	Multiplication of Matrices	Representing Systems of Linear Inequalities Visually
Linda	Write & Graph Two-Variable Equations from Real-Life Scenarios	Average Rates of Change	Rules of Exponents	Simplifying Radicals
Melissa	Probabilities of Independent and Dependent Events	Simplifying and Evaluating Expressions Using the Power Property of Exponents	Simplifying Rational Expressions with Like Denominators	Identifying, Writing, and Graphing Direct Variation
Rachel	Functions & Function Notation	Writing Sequences from Context	Average Rates of Change	Solving Systems of Linear Equations (Elimination Method)

Appendix F: IRB Approval Letter

Institutional Review Board

USU Assurance: FWA#00003308

UtahState University
OFFICE OF RESEARCH AND GRADUATE STUDIES

Full Accreditation

Expedite #6 & #7

Letter of Approval

FROM:

Melanie Domenech Rodriguez, IRB Chair

Nicole Vouvalis, IRB Administrator

To: Patricia Moyer-Packenham, Kami Dupree

Date: June 25, 2018

Protocol #: 8422

Title: A Multiple Case Study Of Secondary Mathematics Teachers' Responses To Pivotal Teaching Moments

Risk: Minimal risk

Your proposal has been reviewed by the Institutional Review Board and is approved under expedite procedure #6 & #7 (based on the Department of Health and Human Services (DHHS) regulations for the protection of human research subjects, 45 CFR Part 46, as amended to include provisions of the Federal Policy for the Protection of Human Subjects, November 9, 1998):

#6: Collection of data from voice, video, digital, or image recordings made for research purposes.

#7: Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

This approval applies only to the proposal currently on file for the period of one year. If your study extends beyond this approval period, you must contact this office to request an annual review of this research. Any change affecting human subjects must be approved by the Board prior to implementation. Injuries or any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Institutional Review Board.

Prior to involving human subjects, properly executed informed consent must be obtained from each subject or from an authorized representative, and documentation of informed consent must be kept on file for at least three years after the project ends. Each subject must be furnished with a copy of the informed consent document for their personal records.

4460 Old Main Hill Logan, UT 84322-4460 PH: (435) 797-1821 Fax: (435) 797-3769 WEB: irb.usu.edu EMAIL: irb@usu.edu

CURRICULUM VITAE

Kami M. Dupree

Business Contact:
Utah State University
Brigham City Regional Campus
989 S. Main St.
Brigham City, UT 84302
1-435-919-1284

Personal Contact:
98 S 100 W
Mantua, UT 84324
1-801-554-2368

Education

Ph.D. April 2019
Education, Utah State University
Specialization: Curriculum & Instruction
Emphasis: Mathematics Education and Leadership

MStat May 2009
Master of Statistics, University of Utah
Specialization: Business
Secondary Business Teaching Certification

B.A. May 2000
Bachelor of Science, Weber State University
Mathematics Education
Secondary Mathematics Teaching Certification

Employment History

University Teaching

Utah State University

Lecturer, Mathematics & Statistics (2012 – present)
Department of Mathematics & Statistics
College of Science
Present and past responsibilities include teaching mathematics, statistics, and mathematics content courses for teachers in live, online, and via an interactive video conferencing broadcast system, supervising and mentoring of Math for America student fellows, and serving as regional campus course coordinator for Math 0990, Stat 1045, and Stat 4010.

University of Utah

Adjunct, Operations & Information Systems (2010 – 2012)
Department of Operations & Information Systems
School of Business Management

Taught business statistics courses (OIS 2340 and OIS 3440) to undergraduate and graduate students. Used and taught Excel, Minitab, and other statistical software programs as an analysis tool.

Public School Teaching & Leadership

Davis County School District

K-12 Mathematics Curriculum Supervisor (2010 – 2012)
Farmington, UT
Responsible for the professional development and support of K-12 teachers throughout the second largest school district in Utah. Supervised the implementation of mathematics curriculum, including the Common Core State Standards. Worked collaboratively to administer the state USTAR grant and to manage budgets.

High School Mathematics Teacher (2000 – 2011, 2012)
Bountiful High School, Bountiful, UT
Taught various high school mathematics courses including calculus, dual enrollment college algebra (Math 1050), dual enrollment trigonometry (Math 1060), precalculus, intermediate algebra, geometry, beginning algebra, and remedial algebra. Also served as coordinator for implementation of professional learning communities in the school.

Related Work Experience

Utah State Board of Education, Salt Lake City, UT

Secondary Mathematics Professional Development Designer & Facilitator (2016)
Collaboratively developed curriculum to train 7th grade teachers in mathematical content in the Utah Core State Standards.

Core Academy Designer & Facilitator (2011 – 2012)
Collaboratively developed curriculum to train 8th grade teachers in mathematical content in the Common Core State Standards. Facilitated three sessions of a four-day training for 8th grade teachers.

University Teaching Experience

Utah State University, Logan, UT (2012 – present)
Department of Mathematics & Statistics

Mathematics Courses Taught

<u>MATH 0990 – Elements of Algebra (Face-to-Face & Broadcast)</u>
Remedial Undergraduate Course. Instruction in elementary concepts of algebra and basic skill associated with a first course in algebra, including real numbers, algebraic expressions, graphing, and solving equations and inequalities; operations on polynomials; factoring polynomials; rational expressions and equations, and systems of equations.

<u>MATH 1010 – Intermediate Algebra (Face-to-Face & Broadcast)</u>

Undergraduate Course. Instruction in linear equations and inequalities, polynomials and exponents, rational expressions, roots and radicals, quadratic equations, lines and systems of linear equations.

MATH 1050 – College Algebra (Face-to-Face & Broadcast)
Undergraduate Course. Instruction in graphs, transformations, combinations, and inverses of quadratic, polynomial, rational, exponential, and logarithmic functions. Course included applications of functions, systems of equations, matrices, and partial fractions.

MATH 1060 – Trigonometry (Broadcast)
Undergraduate Course. Instruction in trigonometric functions, equations, identities, and applications.

MATH 2010 – Algebraic Thinking and Number Sense for Elementary School Teachers (Broadcast)
Undergraduate Course. A course designed to lead preservice elementary school teachers to develop a deep conceptual understanding of the foundations of algebra and numeration necessary for them to lead elementary school students to learn mathematics as specified in elementary school mathematics curricula in a manner consistent with National Council of Teachers of Mathematics Principles and Standards for School Mathematics.

MATH 2020 – Euclidean Geometry and Statistics for Elementary Education School Teachers (Broadcast)
Undergraduate Course. A course designed to lead preservice elementary school teachers to develop a deep conceptual understanding of Euclidean geometry and statistics necessary for them to lead elementary school students to learn mathematics as specified in elementary school mathematics curricula in a manner consistent with National Council of Teachers of Mathematics Principles and Standards for School Mathematics.

MATH 4500/6500 – Methods of Secondary School Mathematics Teaching (Broadcast)
Undergraduate and Graduate Course. A teaching methods course required of prospective secondary school mathematics teachers and inservice teachers seeking status as highly qualified. Instruction in research-based methods for effective mathematics instruction.

MATH 3300/4300/6300 – School Laboratory for Mathematics Teachers Level I (Broadcast)
Undergraduate and Graduate Course. Supervised field experiences for prospective secondary school mathematics teachers.

Statistics Courses Taught

STAT 1040 – Introduction to Statistics (Broadcast)
Undergraduate Course. Instruction in descriptive and inferential statistical methods with an emphasis on conceptual understanding and statistical thinking.

STAT 1045 – Introduction to Statistics with Elements of Algebra (Broadcast & Online)
Undergraduate Course. Instruction in descriptive and inferential statistical methods with an emphasis on conceptual understanding and statistical thinking, as well as algebra concepts related to statistical computation and analyses.

STAT 2000 – Statistical Methods (Online)
Undergraduate Course. A quantitative intensive course that introduces statistical concepts, graphical techniques, probability distributions, estimation, one and two sample testing, chi-square tests, and simple linear regression, one-way ANOVA.

STAT 2300 – Business Statistics (Hybrid/Flipped & Online)
Undergraduate Course. A quantitative literacy course that introduces descriptive and inferential statistics, probability, sampling, estimation, tests of hypotheses, linear regression and correlation, chi-square tests, analysis of variance, and multiple regression with an emphasis on making sound business-related decisions from data. Used R programming for some analyses.

STAT 3000 – Statistics for Scientists (Broadcast & Online)
Undergraduate Course. A quantitative intensive course that introduces statistical concepts, graphical techniques, discrete and continuous distributions, parameter estimation, hypothesis testing, and chi-square tests for those pursuing science intensive majors.

STAT 4010 – Probability and Statistics for Teachers (Broadcast)
Undergraduate Course. An introductory statistics course taught to prospective and current secondary mathematics teachers. Introduces descriptive and inferential statistics, probability, sampling, estimation, tests of hypotheses, linear regression and correlation, chi-square tests, and analysis of variance with an emphasis on preparing teachers to teach statistical concepts in the secondary mathematics curriculum.

University of Utah, Salt Lake City, UT (2010 – 2012)
Department of Operations and Information Services

Courses Taught

OIS 2340 – Business Statistics
Instruction in fundamental statistical concepts of collection, analysis, and interpretation of business and economic data; measures of central tendency and dispersion; probability theory and probability distributions; sampling distributions and statistical inference, including estimation and hypothesis testing. Used Microsoft Excel for computation and descriptive purposes.

OIS 3440 – Applications of Business Statistics
Instruction in essential tools and concepts of Six Sigma, including design of experiments, goodness of fit, contingency tables, correlation analysis, nonparametric statistics, and an introduction to statistical process control. Also included instruction in the hands-on development and interpretation of regression models. Used Microsoft Excel to create graphical and numerical outputs.

Course Development

Utah State University, Logan, UT (2013 – 2015)

Undergraduate Course Development

STAT 4010 (2019) – Probability & Statistics for Teachers
Developed an introductory probability and statistics course aimed at providing current and future secondary mathematics teachers with a fundamental understanding of the probability and statistics content in the Utah Core Standards. Course design included an emphasis on introducing students to probability and statistical concepts, identifying connections between statistical ideas and their mathematical/algebraic counterparts, and developing student pedagogical content knowledge by exposing common misconceptions, teaching and assessment strategies, and integration of technology.

Online STAT 1045 (2017) – Introduction to Statistics with Algebra
Developed Stat 1045 (described below) as a fully online course.

STAT 1045 (2014) – Introduction to Statistics with Algebra

I led a collaborative effort between myself, as a distance campus representative, and a main campus instructor to create STAT 1045, a statistics course designed to address the needs of students with weaker algebra backgrounds who were not intending to major in mathematics or science. This course was developed in an effort to reduce the number of courses these students need to satisfy quantitative literacy by combining essential aspects of algebra that are necessary to understand basic statistical concepts and analyses. The course design aligned instruction for STAT 1045 with STAT 1040 to ensure that students completing STAT 1045 met the same university requirements as those on the typical qualitative literacy pathway of STAT 1040. I taught the pilot course, and continue to refine it to meet the needs of the students.

Research

Dissertation

Secondary Mathematics Teachers' Responses to Pivotal Teaching Moments

Other Research Interests

Improving the Pedagogical Content Knowledge of Pre-service and Practicing Teachers
Improving Teacher Preparation to Teach Mathematics and Statistics

Presentations

State and Regional Presentations

Dupree, K., (2017, August). Learning is a Process, Not a Destination. *Empowering Teaching Excellence*, Utah State University, Logan, UT

Dupree, K., (2016, November). Myths of Mnemonics. *Utah Council of Teachers of Mathematics*, Layton, UT

Dupree, K., (2016, June). The Myth of Mnemonics. *Utah Multi-Tiered Systems of Supports Conference,* Layton, UT

Dupree, K., (2014, November). Execute Aunt Sally and Teach Order of Operations. *Utah Council of Teachers of Mathematics*, Layton, UT

Publications

Nadelson, L.S., Throndsen, J., Campbell, J. E., Arp, M., Durfee, M., Dupree, K., Poll, T., & Schoepf, S. (2016). Are they using the data? Teacher perceptions of, practices with, and preparation to use assessment data. *International Journal of Education, 8*(3), 50.

Dupree, K. M. (2016). Questioning the order of operations. *Teaching Mathematics in the Middle School*, *22*(3), 152-159.

Christiansen, M. A., Lambert, A. M., Nadleson, L. S., Dupree, K. M., Kingsford, T. A. (2016). In-Class versus at-home quizzes: Which is better? A flipped learning study in a two-site synchronously-broadcast organic chemistry course. *Journal of Chemical Education,* 94, 157-163. doi: 10.1021/acs.jchemed.6b00370

Grant Work

USTAR Grant – Davis School District

Worked in collaboration with science curriculum supervisor to implement grant programs and finances, analyze data and report on outcomes.

Service

Utah State University

Mathematics and Statistics Education Committee Member (2013-present)
Work with other mathematics educators to better the universities teacher preparation program and courses. Collaborate on bringing university courses in line with State Teacher Licensure requirements.

Stat 1045 Regional Campus Course Coordinator (2014-2016)
Collaborate with main campus supervisor on the direction of the course, creation and distribution of testing materials to all instructors on regional campuses. Provide support for regional campus instructors.

Math for America Regional Support Administrator (2013-2015)
Mentor Math for America fellows. Mentoring overlapped with supervising student teaching efforts for these students and providing continued support as they moved through their first years of teaching.

Math 0990 Regional Campus Course Coordinator (2012-2014)
Collaborate with main campus supervisor on the direction of the course, creation and distribution of testing materials to all instructors on regional campuses. Provide support for regional campus instructors.

National Council for Teachers of Mathematics (NCTM) (2015 – Present)

Manuscript Reviewer for *Mathematics Teaching in the Middle School* (2015 – present)
Review and submit feedback on manuscripts submitted for publication on an as-needed basis.

Manuscript Reviewer for *Teaching Children Mathematics* (2015 – present)
Review and submit feedback on manuscripts submitted for publication on an as-needed basis.

Resource and Materials Reviewer for *Teaching Children Mathematics* (2015 – present)
Review and submit feedback on resources (books, technology, etc.) on an as-needed basis.

Professional Affiliations & Memberships

National Council of Teachers of Mathematics
Utah Council of Teachers of Mathematics

Other Work-Related Skills

Microsoft Excel, Word, and PowerPoint – Proficient
GeoGebra, SPSS, R, Minitab – Experienced